Orebody Modelling and Strategic Mine Planning

Uncertainty and Risk Management Models

Second Edition

Edited by

Roussos Dimitrakopoulos

The Australasian Institute of Mining and Metallurgy
Spectrum Series No 14

Published by

THE AUSTRALASIAN INSTITUTE OF MINING AND METALLURGY
Level 3, 15 - 31 Pelham Street, Carlton Victoria 3053 Australia

All papers published in this volume were peer reviewed prior to publication.

ISBN 978 1 920806 77 4

Kalgoorlie Consolidated Gold Mines Pty Ltd (KCGM) are acknowledged for supplying the centre image for the front cover.

Desktop published by:
Angie Spry, Kristy Pocock and Jenni Stiffe for The Australasian Institute of Mining and Metallurgy

Compiled on CD ROM by:
Visual Image Processing Pty Ltd
PO Box 3180
Doncaster East VIC 3109

Foreword

'*Orebody modelling and strategic mine planning*' are arguably the backbone of our industry and represent an intricate, complex and critically important part of mining ventures. They have a profound effect on the value of a mine, as well as determining the technical plan to be followed from mine development to mine closure.

It is most gratifying to introduce the second revised edition of this Spectrum Series volume on the above topic, following the depletion of all copies of the first edition within about a year of its publication. The present volume includes new developments since the first edition, ranging from integrated mine evaluation and mine management under uncertainty presented by DeBeers, to BHP Billiton's recent efforts in jointly optimising ore extraction and in-pit dumping, to the stochastic simulation of orebody geology or wireframes with multi-point spatial statistics. Undoubtedly, this edition is a reaffirmation of the continuing commitment to our field of work by a large number of individuals, mining companies and professional organisations.

Uncertainty and risk management models are the underlying themes of this volume. Several of the papers presented refer to the staggering statistics of mining risk and the recognised importance to strategic mine planning of geological ore reserve risk. The reality is that few projects perform as expected due to problems in orebody modelling and ore reserve estimates. Surveys suggest that nearly three quarters of mining projects fail to meet expectations, leading to capital investment losses in the order of hundreds of millions of dollars. The aim of this volume is to foster not only an understanding of the adverse effects of risk, but also address the potential limitations encountered in traditional methods and demonstrate how to technically manage risk to perform better. Adding value to the industry means demonstrating how the quantification of uncertainty and risk management can be used to capture maximum upside potential while minimising downside risk in assessing the value of mining assets.

The papers in this volume are grouped according to key themes. *Why strategic risk management?* is the opening theme of this volume. Papers examine the following: issues for optimisation methods, large geological risk modelling case studies that look at effects throughout the chain of mining, financial models that include integration of risk from various sources, new approaches to optimisation with quantified risk, and new models and their associated issues for reserve modelling and stochastic simulation. *New practical conditional simulation methods* for modelling large orebodies are highlighted in the next section, which puts an emphasis on new multi-point as well as very efficient methods that are practical for routine use in the industry environment. A group of papers on *advances in conventional mining optimisation* complements the previous section. *Integrated large-scale applications* includes both major case studies using new technologies and new approaches, and their application in various commodities and mining methods. *Geological uncertainty and mineral resources/ore reserves* is a section focusing on the issues underpinning the sustainability of the mining industry. *Geotechnical risk and mine design* raises the need to further integrate geotechnical risk into modelling and optimisation. *Case studies and blending optimisation* includes optimisation of multiple operating policies in complex resource exploitation. *New concepts, technologies and directions* is the final section of this volume and deals with new, broadly applicable risk-based frameworks for optimising under uncertainty. As well as documenting the concepts, it explores the distinct financial advantages of risk-based optimisation through case studies.

A key impetus for the preparation of this volume was the outstanding success of an international symposium on the same topic, held in Perth, Western Australia, in November 2004, and the commitment of over 260 participants from around the globe, as well the mining industry sponsors and organisers to further the transfer and dissemination of emerging leading edge technologies and new promising results from research, development and applications in recent years. Thus, this volume includes selected upgraded papers from that symposium, several new contributions which complement this topic, and new papers that have been added along with previously published papers, recently revised for this second edition. At a time when demands for improved performance in sustainability, responsibility and economic growth are accelerating, technical uncertainties (geological and mining) and uncertain mineral market forecasts have traditionally been seen as limitations on the sector's ability to 'do better'. This need not be the case, as is demonstrated in this volume.

In particular, several papers represent a technical articulation of a paradigm shift based on the use of information that can be gained by applying sophisticated mathematical models to data where there is inherent uncertainty. This type of modelling enables quantification and analysis of multifaceted risk, and facilitates the identification of major changes that can result in improved resource assessment, mine planning and mining operations. It is hoped that our intellectual capital investment in 'mineral resource management and mining under uncertainty', along with the outcomes presented in this second edition Spectrum Series volume, will not only contribute to and encourage a shift in the way we approach and solve problems in the mining sector, but also contribute to the dissemination of new technologies for modelling uncertainty, mine design, production scheduling and options valuation.

Education underpins the transfer and acceptance of new technologies and concepts to both the current generation of mining professionals, as well as the next. I am particularly indebted to The AusIMM, whose collaboration in preparing and producing this volume for the second time, as well as several other professional development activities, has been most effective and greatly appreciated over the years. With the ongoing globalisation of the mining sector, the contribution and collaboration of the CIM (Canada), the SME (USA) and the SAIMM (South Africa) has also been critical to the success of our efforts. This volume is, I believe, the continuation of the effort of the above Institutes to enhance professional excellence in the critical field addressed by this Spectrum Series publication in both of its editions to date.

If this volume makes a contribution to our profession, it is due to the combined efforts of many professionals over several years. In particular, I would like to thank our colleagues and international experts: Jeff Whittle, Gavin Yeates, Peter Ravenscroft, Peter Forrestal, Allen Cockle, Wynand Kleingeld, Jean-Michel Rendu, Peter Monkhouse, Martin Whitham, Georges Verly, Olivier Tavchandjian, Duncan Campbell, Peter Dowd, Jean-Paul Chilès, Andre Journel and Paul Greenfield whose support over many years has been invaluable. In addition, I would like to update this list and express my gratitude to include the following colleagues who have supported and contributed to our broader efforts as well as this second edition: Rick Allan, Edson Ribeiro, Brian Baird, Ian Douglas, David Whittle, Kapila Karunaratna, Jaimie Donovan, Peter Stone and Malcolm Thurston.

I would further like to acknowledge the diligent work of the reviewers who are listed on page iii, and thank the authors for the high quality of their contributions. Last, but not least, I wish to acknowledge the multifaceted support of the sponsors of both the first edition: AngloGold Ashanti, BHP Billiton, De Beers, Hamersley Iron, Newmont, Rio Tinto, Whittle Programming and Xstrata Copper, as well as the sponsors of this second edition: Companhia Vale do Rio Doce (CVRD) and Barrick Gold. This volume is in your hands thanks to them.

Roussos Dimitrakopoulos
Professor
COSMO – Stochastic Mine Planning Laboratory
Department of Mining, Metals and Materials Engineering
McGill University
Montreal QC Canada

Contents

Why Strategic Risk Management?

New Practical Conditional Simulation Methods and Applications

Advances in Conventional Mining Optimisation and Applications

Integrated Large-Scale Applications

Geological Uncertainty and Mineral Resources/Ore Reserves

Geotechnical Risk and Mine Design

Case Studies and Blending Optimisation

New Concepts, Technologies and Directions

Appendix

Beyond Naïve Optimisation

P H L Monkhouse[1] and G A Yeates[2]

ABSTRACT

Most practitioners would regard the maximising of the net present value (NPV) of a mine by changing mining schedules, push-backs, cut-off grades, ultimate pit shells and stockpile rules and procedures as encompassing current best practice in mine planning. This optimisation is typically carried out for a single set of assumptions about:

- orebody tonnes and grade,
- processing methods and costs,
- maximum sales volumes in the case of bulk commodities,
- commodity prices, and
- discount rates.

About the only thing we can be sure of is that the assumptions on all these factors will be wrong, yet we continue to naïvely optimise our mine plan. This paper argues that this approach is inherently flawed. Recognising that our assumptions will be wrong, and that our actions can alter over time as new information is made available, means that the mine plan that is 'optimal' under a single set of assumptions may well be suboptimal in the real and uncertain world.

INTRODUCTION

Best practice is a fuzzy term; when applied to mine planning it can mean many things. Current best practice in mine planning, as viewed by most practitioners, encompasses the maximising of the net present value (NPV) of a mine by changing mining schedules, push-backs, cut-off grades, ultimate pit shells and stockpile rules and procedures. This analysis is typically performed for a single set of assumptions, which we can almost guarantee will be wrong. Assumptions typically cover: orebody tonnes and grade; processing methods and costs; maximum sales volumes in the case of bulk commodities; commodity prices; and discount rates.

Planning for a single set of assumptions that turn out to be incorrect will result in a suboptimal, or naïve, mine plan. There are two possible responses to this. The first is to try harder to correctly estimate (forecast) the future. The second response is to recognise that the future is in many respects unknowable, and to subsequently develop mine plans that have the flexibility to respond to changes to assumptions in the future. This flexible – or robust – mine plan will continue to give high mine values over a wide range of input assumptions (both optimistic and pessimistic), rather than a plan that only gives optimal results over a very small range of assumptions.

The key to addressing these issues is understanding uncertainty and risk, and developing methods to incorporate them into the mine planning process. This allows us to value flexibility and the benefit derived from robust mine plans. Whilst acknowledging that this is difficult, we propose that solutions can be found by combining the research from two broad but quite different areas, those of mine planning and real options. Even if robust or flexible plans are developed, the organisational challenge is to act effectively. For example, how many copper mines changed their mine plans when the copper price doubled over a relatively short period of time? How many of these operations are still working to the cut-off, the schedule and ultimate pit that were in place when the copper price was half what it is today? A mine with flexibility, with exposed ore and with surplus stripping capacity would be able to respond by raising the cut-off, raising the head grade and thereby producing more copper during periods of higher prices and hence capturing value during the price spike. How much value is being destroyed by not changing our current operating plans in light of new information?

In this paper, current industry practice in regard to mine planning is briefly reviewed and the generic assumptions that strongly influence the final mine plan are then discussed. Two key sources of uncertainty – orebody uncertainty and price uncertainty – are then reviewed in some detail. A discussion follows regarding current practices within BHP Billiton before concluding with some suggestions for future developments in this area.

CURRENT INDUSTRY PRACTICE

The current practice in industry is to take a single estimate (model) of the orebody, using a single set of mining assumptions, along with a single set of deterministic external economic assumptions, to come up with an 'optimal' ultimate pit design, extraction sequence, and schedule. The term 'optimal' usually means the maximising of a single variable, usually NPV or its proxy, for a given set of assumptions. The optimised model typically defers stripping, brings forward revenue (high grade) and often extends mine life by dynamically changing cut-off grade over time. Sometimes additional effort is applied to look for the potential of additional value in the stockpiling of low-grade material.

The first step in a mine optimisation typically involves coming up with final pit limits. The tool commonly used is the Whittle pit optimisation, the nested pit version of the Lerchs-Grossmann algorithm (Lerchs and Grossmann, 1965; Whittle, 1988; Muir, 2007, this volume). The mine planner's dilemma in using these techniques is that they focus on the final limits. Given that the decision about the final limit is usually far into the future and heavily reliant on external economic assumptions, such as the price at the time the final pushback will be mined, the decision is fraught with difficulty. While this decision is likely to be refined during mine life, key investment decisions are often made on the basis of this information. The next steps in mine optimisation are encapsulated in the seminal book in this area, *The Economic Definition of Ore* (Lane, 1988) with the general approach being considered as established practice in the industry.

Unfortunately, the big picture is often lost and the mine planning process blindly followed in the beliefs that the assumptions are right and that the resultant plan is optimal in reality. The key concept regarding all of these factors is that they are only optimal for a given set of assumptions (inputs) – today's optimised mine plans have no flexibility to respond to changed circumstances. This is usually due to the stripping being deferred, all exposed ore being minimised, all stockpiles cut to near zero by the accounting drive to minimise working capital, and material movement matched to the fleet capacity thereby eliminating sprint capacity. Further, if we consider current practice in use at most of our mining operations, the mine plan is often not revised, even when we have significant changes to external assumptions.

SOURCES OF UNCERTAINTY OR KEY ASSUMPTIONS

The key sources of uncertainty that affect the final mine plan are as follows:

1. Vice President – Business Strategy for Carbon Steel Materials, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: peter.hl.monkhouse@bhpbilliton.com

2. FAusIMM(CP), Global Manager – Mineral Resource Development, Business Excellence, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: gavin.yeates@bhpbilliton.com

Orebody uncertainty: The three-dimensional distribution of grade over the orebody is estimated by relatively limited drill hole data coupled with a geological interpretation, which may or may not be correct. This uncertainty, however, is often ignored in the mine planning process. This issue is discussed in more detail in a subsequent section.

Processing uncertainty: Just as methods for modelling grade now exist, so do advances in the modelling of what is now called 'geometallurgical' performance. It is now possible to deterministically model variables such as ore hardness, flotation or leach recovery, concentrate grade, and ultimately dollars per hour through the mill (eg Wooller, 1999). Ultimately, these variables can also be simulated to describe the range of possible outcomes that may be encountered in the future operation. This is essentially modelling the current performance through a given process plant (Flores, 2005).

Uncertainty in changing technologies: Another significant uncertainty far more difficult to model is a major technology change; these step changes could well have major impacts on future mine plans. Examples include atmospheric leaching of nickel ores, leaching of chalcopyrite ores, and the use of high phosphorous iron ore in steel plants. The key uncertainties for these particular changes are threefold: Will the breakthrough occur? If so, when will it occur? If it occurs what will be the size of the step change in cost, recovery and therefore reserve definition?

Volume uncertainty: London Metals Exchange (LME) commodities effectively exhibit no volume uncertainty, as product can always be sold and delivered to LME warehouses. However, non-LME commodities, such as coal and iron ore, can only be sold to traders or customers, thereby introducing volume or sales uncertainty. The ability to sell the material is also influenced by its quality.

Price uncertainty: The price forecast we enter into our computer models is problematic, especially when the only certainty is that the price forecast we use will be wrong. This will be discussed in more detail later.

Discount rate uncertainty: The issue of interest rate uncertainty is more subtle, but no less important, in that it affects what discount rate we use. It affects the trade-off decision between future benefits versus current benefits. Again, the only thing we know about our forecast of interest rates, and hence discount rates, is that they will change over time. Political risk, often allowed for in the discount rate, further complicates this issue. Should we allow for a country risk premium on our annual discount rate that declines with time, as we learn to operate in a country? Or does country risk keep growing exponentially, as is implied in a constant per period discount rate?

OREBODY UNCERTAINTY

The traditional approach has been to provide mine planners with a single 'best' interpretation of the orebody. This single geological interpretation is then treated as fact. This approach gives no indication of the uncertainty in the interpretation, nor does it communicate the risk that the interpretation could be wrong or the likely range of possible outcomes. Geologists are dealing with imperfect knowledge, they know that the data on which the interpretation is based is incomplete, imprecise and inaccurate. They also know that there are multiple possible interpretations, each of which is valid. Some may have greater probability than others, but each is valid if it can explain the available data. It is now possible to quantify and model some aspects of the geological uncertainty. The use of simulation techniques is well-developed for modelling the grade uncertainty, but also well known is the critical nature of geological interpretation that controls the grade. There are limited examples of quantifying the range of geological interpretations and hence the grade (eg Jackson *et al*, 2003; Khosrowshahi, Shaw and Yeates, 2007, this volume; Osterholt and Dimitrakopoulos, 2007, this volume).

Dimitrakopoulos, Farrelly and Godoy (2002) illustrate a case where, for a range of equally probable geological outcomes, the mine plan developed on a single estimate of the orebody is excessively optimistic. This is partly driven by any misestimation of grades – resulting in a loss of value either by ore being classified as waste and an opportunity loss suffered – or waste being classified as ore and additional processing costs incurred. This resulting 'bias' is what makes many deterministic plans optimistic. It should be noted, however, that the opposite may also occur unpredictably, to stress the limits of the current modelling and optimisation technologies. This finding has been confirmed by internal research at BHP Billiton Technology (Menabde *et al*, 2007, this volume). Further, and more importantly, this work shows consistently that a mine plan can be developed considering the uncertainty in the geological input assumptions, and this mine plan will have a higher NPV on average (ie over a wide range of inputs), a finding independently observed in Godoy and Dimitrakopoulos (2004); and Ramazan and Dimitrakopoulos (2007, this volume).

PRICE UNCERTAINTY

To illustrate the problems with current best practice, the following hypothetical mine development is used.

A simplified example

Consider a mining company that requires an optimal mine plan for a copper orebody shown (simplistically) in Figure 1.

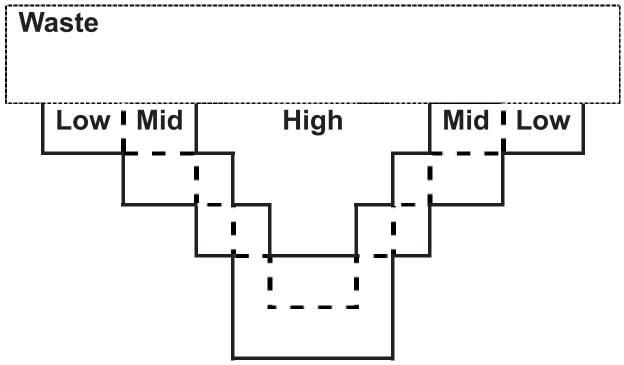


FIG 1 - A simplistic hypothetical copper orebody.

For the high-grade block, assume:

1. A grade of 1.25 per cent copper and containing 20 million pounds of copper. At a copper price of US$1/lb this block will produce US$20 M revenue.
2. The total cost of mining and processing for this high-grade block is US$12 M, split US$6 M for the waste removal and US$6 M for the mining and treatment of the ore. Mining and processing should occur in year 1.

For the mid-grade block, assume:

1. A grade of one per cent copper and containing 12 million pounds copper. At a copper price of US$1/lb it will produce US$12 M revenue.
2. The total incremental cost is US$12 M, split between additional waste removal (US$2 M) and mining and processing mid-grade. If mining were to be undertaken, the mining and processing should occur in year 2.

For the low-grade block, assume:

1. The low-grade block is not drilled because the Promoter wants the orebody open at depth, but George the Geologist is convinced it has a grade of 0.65 per cent Cu, containing 12 M pounds copper, for revenue of US$12 M.

2. The incremental cost of removing the low-grade block is estimated at US$14 M, split US$2 M for additional waste removal and US$12 M for mining and processing the ore. If undertaken, the mining and processing of this low-grade block should occur in year 3.

Furthermore, assume that all the waste must be extracted in year 0, and that once this decision is made it is very expensive to go back, in either cost and/or time, and re-strip the additional waste.

The problem facing the company

The problem for the mining company is that a decision needs to be made today on what to mine. If the company forecasts the copper price to be US$1/lb:

- Should the company only mine the high-grade block?
- Should it mine the mid-grade block?
- Should it trust George the Geologist and plan to mine the low-grade block?

If our assumption was that the forecast copper price was US$1/lb then we would apply the approach outlined by Lane (1988). Primarily because of the effects of discounting – with cost of waste removal being incurred in year 0 and revenue in years 1, 2 and 3 – we would only extract the high-grade block. An alternate approach may be to use a break-even cut-off (and ignore the effects of discounting), where at US$1/lb copper and for the costs outlined previously, a break-even cut-off grade for the high-grade block is 0.75 per cent copper, the mid-grade block is one per cent copper, and the low-grade block is some 0.76 per cent copper. Accordingly, using this approach the company would have mined the high- and mid-grade blocks.

Under what circumstances would the company plan on mining all the blocks? How would the company develop a robust (or flexible) mine plan that allows them to respond to changing circumstances? To highlight the impact of price uncertainty, discount rate uncertainty and geological uncertainty, how would the decision change if:

- Analysis of the futures market indicated there was a 50 per cent chance the copper price would exceed US$1.50 in three years' time?
- The deposit was located in a country with a corrupt dictator that may expropriate the operation at any time?
- An independent review of George the Geologist's work indicated there is a 95 per cent chance he is right.

Intuitively, all these assumptions should change the optimal mine plan, yet current best practice would struggle to include these assumptions. It is suggested that the 'best' mine plan should be one that maximises value over a 'reasonable' range of input assumptions.

Framing the questions in the language of real options

To determine what we mean by 'best' and a 'reasonable' range of assumptions, the previous example will be re-stated.

For the high-grade block, assume:

1. A grade of 1.25 per cent copper containing 20 million pounds of copper. At a copper price of US$1/lb this will produce US$20 M revenue.
2. Total cost of mining and processing the high-grade block is US$12 M, split US$6 M for waste removal and US$6 M for mining and treating the ore. The waste removal will occur in year 0 with mining and processing to occur in year 1.

For the mid-grade block, assume:

1. A grade of one per cent copper containing 12 million pounds copper. At a copper price of US$1/lb it will produce US$12 M revenue.
2. For the cost of additional stripping in year 0 of some US$2 M, we have the option to mine and process the mid-grade block in year 2 at a cost of some US$10 M.

For the low-grade block, assume:

1. The low-grade block is not drilled because the Promoter wants the orebody open at depth. George the Geologist is convinced the grade is at least 0.65 per cent copper, and contains 12 million pounds of copper, which would produce revenue of US$12 M if he is correct.
2. for the cost of additional stripping in year 0 of another US$2 M, we have the compound option to mine and process the low-grade block in year 3 at a cost of some US$12 M. It is a compound option because it is conditional on us mining the mid-grade block in year 2. In this example, the low-grade block is only mined if the mid-grade block is already mined. Compound options are highly non-linear and the effects are complex. In general, the second option (on the low-grade block) has the effect of increasing the value of the first option (on the mid-grade block). However, compound options are not that difficult to value.

Considering this scenario, does the company now mine the high grade block? Does the company now buy the (real) option for US$2 M to mine and process the mid-grade block in two years hence? Does the company buy the (real) option over the low-grade block costing a further US$2 M? Unless the options (or flexibility) can be valued, or the benefits of a robust mine plan can be valued, it is unlikely that mine planning will be successful in moving forward. The keys are properly modelling uncertainty and risk, and understanding the value of preserving options and flexibility.

In our example the two key questions are: What options should be purchased? When, if at all, should options be exercised? To answer the first question the company must know the cost of purchasing the option – in the above example this is US$2 M to undertake the additional stripping. The harder question is: What is the value of acquiring this option, or flexibility? If the option is worth more than it costs, then the company will want to purchase it, and develop a flexible, or robust mine plan. Yet there are limits to the amount of flexibility that should be acquired. To answer the second question about when to exercise the options, the company needs to know the value of keeping the option alive, and the value of exercising the option. Again, we will exercise the option, or mine the mid- and possibly the low-grade blocks if the value of exercising the option is greater than the value of keeping the option alive. The harder issue is valuing the option, not the value of exercising it (developing the mine).

Valuing the real options for price uncertainty

Price uncertainty can be modelled in a real options framework by building a price tree. To simplify the mathematics in this example, it is assumed that the prices will be constant for one year, and then may vary. It is further assumed that the price distribution is log-normal† and that the volatility of the copper price is 20 per cent per annum. It is also assumed that this price tree is a risk-neutral price tree, as obtained from futures data. It is **not** the price tree of expected copper price movements. This distinction is very important to ensure price risk is handled properly. With these assumptions, the up price factor is 1.2214 and the down price factor is the reciprocal, or 0.8187. Assuming a five per cent per annum risk-free rate (continuously compounded) and these up and down factors it follows that the risk-neutral probability of an up price movement is 0.5775 and the risk-neutral probability of a down price movement is 0.4225.

† This assumption is discussed in detail in corporate finance textbooks (eg Brealey and Myers, 2003, Chapter 21; Hull, 2000, Chapter 9).

Copper price tree

Given the above assumptions, and assuming the current copper price is US$1/lb, the copper price tree is shown in Table 1.

TABLE 1

Copper price tree with and copper price at US$1/lb.

Now	Year 1	Year 2	Year 3
			1.82
		1.49	
	1.22		1.22
1.00		1.00	
	0.82		0.82
		0.67	
			0.55

Value of high-grade block

Given there are 20 M pounds of copper and the mining and processing costs are US$6 M, the cash flows from mining the high-grade block (assuming the waste removal has already occurred in year 0) is shown in Table 2.

TABLE 2

Cash flows from mining the high-grade block (assuming the waste removal has already occurred in year 0).

Now	Year 1	Year 2	Year 3
	18.43*		
	10.37		

* Calculated as (1.2214*20)-6.0

Assuming the risk-free interest rate is five per cent per annum (continuously compounded) and that the waste removal has already occurred, the value tree is shown in Table 3.

After spending US$6 M on waste removal we should have a value of US$14.29 M. Thus, before we even start the project it has a value of some US$8.29 M and indicates that the high-grade block should be mined.

Value of mid-grade block

Using the same price tree as above and given there are 12 M pounds of copper and the mining and processing costs are US$10 M, the cash flows from mining the mid-grade block (assuming the waste removal has already occurred in year 0) are shown in Table 4.

Assuming the risk-free interest rate is five per cent per annum (continuously compounded) and that the waste removal has already occurred, the value tree is shown in Table 5.

Spending US$2 M on additional waste removal should give us a value of US$3.27 M. Thus the project, before we start, has a value of some US$1.27 M and means the company should at least undertake the prestrip for the mid grade block. However, we will only mine the mid-grade zone if the copper price is US$1/lb or above. We will not mine the mid-grade zone if the copper price is the low price in year 2 of US$0.67/lb. Ultimately, it is the ability to defer this mining decision that is creating the value, and thus facilitating the mining of the mid-grade zone in some circumstances.

TABLE 3

Value tree, assuming the risk-free interest rate is five per cent per annum (continuously compounded) and that the waste removal has already occurred.

Now	Year 1	Year 2	Year 3
	18.43		
14.29*			
	10.37		

* Calculated as (18.43*0.5775+10.37*0.4225)/exp(0.05). The exponential term is because the interest rate is expressed on a continuously compounded basis.

TABLE 4

Cash flows from mining the mid-grade block (assuming the waste removal has already occurred in year 0).

Now	Year 1	Year 2	Year 3
		7.90	
		2.00	
		-1.96	

TABLE 5

Value tree, assuming the risk-free interest rate is five per cent per annum (continuously compounded), and that the waste removal has already occurred.

Now	Year 1	Year 2	Year 3
		7.90	
	5.14		
3.27		2.00	
	1.10		
		0.00	

A possible counterintuitive result is also evident from this example. Consider the case where the copper price remains at US$1/lb through the mine life. In this case the company will end up mining the mid-grade block because:

- the option analysis commits the company to undertake the prestrip, as the copper price might rise; however
- when the company gets to make the mining decision it decides to mine even if the copper price is only US$1/lb because the **prestripping is now a sunk cost** and is excluded from the analysis.

More of the deposit is mined if the copper price turns out to be a constant US$1/lb under the robust mine planning framework compared to a current ‘best practice’ framework. This is despite the fact that if we had perfect foresight we would not have committed to this prestripping and the mining of the mid-grade block. This is of obvious benefit to the host country.

Value of low-grade block

Now let us repeat this procedure for the low-grade block. The price tree is the same as in the previous example. Given that there are 12 M pounds of copper and the mining and processing costs are US$12 M, the cash flows from mining the low-grade block (assuming the waste removal has already occurred in year 0) are shown in Table 6.

TABLE 6

Cash flows from mining the low-grade block (assuming the waste removal has already occurred in year 0).

Now	Year 1	Year 2	Year 3
			9.87
			2.66
			-2.18
			-5.41

Assuming the risk-free interest rate is five per cent per annum (continuously compounded), and that the waste removal has already occurred, the value tree is shown in Table 7.

TABLE 7

Value tree, assuming the risk-free interest rate is five per cent per annum (continuously compounded) and that the waste removal has already occurred.

Now	Year 1	Year 2	Year 3
			9.87
		6.49	
	4.15		2.66
2.60		1.46	
	0.80		0.00
		0.00	
			0.00

Spending US$2 M on additional waste removal should give us a value of US$2.60 M. The project, before we start, therefore has a value of some US$0.60 M. This means the company should do the prestrip for the low-grade block as well, but will only mine the low-grade zone if the copper price is above US$1.22/lb. The low-grade zone will not be mined if the copper price is only US$0.82/lb or less. At the risk of labouring the point, it is the ability to defer this mining decision that is creating the value, and thus facilitating the mining of the low-grade zone in some circumstances.

Note that more of the deposit is mined under the robust planning framework than under the current 'best practice' framework. The expected amount of material mined at the start of the mining operation is greater under the robust mine planning framework than any other framework, with significant benefits to the company, shareholders and the host country.

INTRODUCING ADDITIONAL SOURCES OF UNCERTAINTY IN THE ANALYSIS

The simplified example shown previously introduced an additional source of uncertainty. Should the company trust George the Geologist's intuition and plan to mine the low-grade block? What about the risk that George is wrong? Should this risk be allowed for in the analysis? Before discussing this in more detail we need to introduce another concept from corporate finance, namely diversifiable risk and non-diversifiable risk. The key issue is that some (non-diversifiable) risks are priced (investors will pay to avoid them, eg commodity price risk, interest rate risk), and other (diversifiable) risks are unpriced (investors are indifferent about bearing them, eg geological uncertainty). This concept forms the bedrock of the Capital Asset Pricing Model or CAPM (Brealey and Myers, 2003, Chapters 7 and 8).

If George's estimate of the grade is truly a central estimate then because geological risk is, at least to a first order approximation, unpriced, we should not introduce any additional value reduction because of the 'risk', even if the distribution of possible outcomes is incredibly wide. The key issue is whether George's estimate is a central estimate because, unlike copper price, the risk of the possible outcomes does not enter the valuation.

MORE GENERAL COMMENTS ON UNCERTAINTY AND RISK

The latter example has introduced two key corporate finance concepts: namely real options[‡], and diversifiable and non-diversifiable risk, but this paper cannot do justice to these concepts[§]. Together these two concepts allow for the classification of risks into priced and non-priced risks, and where they are priced an analytical tool to evaluate them is provided. It allows the valuation of mine plans (and risk) from the perspective of shareholders and allows the company to then compare the cost of acquiring flexibility, versus the value of having flexible mine plans.

Failure to adequately address risk (such as using expected spot prices instead of risk-neutral prices) means that we get the garbage-in-garbage-out problem, a very large problem. Properly valuing the risk introduced by real options is complex. We can quickly end up in the world of stochastic differential equations, or large-scale numerical methods. Yet failing to properly value risk means we are wasting our time. The authors believe that we are better off relying on our intuition than doing some pseudo-maths that does not properly allow for risk.

POSSIBLE CRITICISMS OF THE PROPOSED APPROACH

In these examples, a flexible or robust mine plan means removing all the waste in year 0, which goes beyond standard practice in the industry. One possible criticism of this approach is that the decision to prestrip is made up-front and is artificial. In practice you could go back and prestrip for the mid- and the low-grade blocks if the price spiked. While this is to some extent correct, it can be argued that:

1. going back and undertaking additional prestripping will contribute to cost and time penalties, although these can be modelled if considered appropriate;

‡ The Nobel Prize in Economics in 1997 was awarded to Scholes and Merton for adequately handling risk in (financial) option valuations. The earlier tool of the Capital Asset Pricing Model – while important and underpinning all NPV analysis – does not allow risk to be accurately valued when we have option-type pay-offs. The seminal option paper by Black and Scholes (1973) effectively provided a numerically quantifiable way of handling non-diversifiable (or priced) risk in option-type pay-offs. This concept has since been extended to real options.

§ The application of real options is discussed in Copeland and Tufano (2004). The application of real options to a mining example is discussed in McCarthy and Monkhouse (2003).

2. in a real-world approach you need to model mean reversion in the commodity prices, which means that any time delays suffered could well cause a significant value loss; and
3. in any mining operation, the time taken to do any additional stripping is measured in years.

In any event, the mere fact that we are thinking how we will respond to changed economic circumstances is the whole point of this paper. The aim, in real options talk, is to acquire flexibility for less than its inherent value – if that can be done by alternative and lower cost means then so much the better. It could be argued that all this is too hard and that sensitivity analysis will get us most of the way there, but at a fraction of the complexity. To the extent that sensitivity analysis builds intuition, then that is a great outcome. But of itself, sensitivity analysis will have limited benefit in generating a robust or flexible mine plan as it will be unable to justify the cost of investing in flexibility. This can only be achieved by implementing real options analysis as described previously.

STATE OF PLAY IN BHP BILLITON

Within BHP Billiton it is well-recognised that there are limitations to optimising a mine plan for a given set of assumptions that will inevitably turn out to be incorrect. Further, it is accepted that this approach will lead to suboptimal outcomes, for both our shareholders and the host country. Overcoming this deficiency is crucial; it requires the development of new mine planning techniques, and – just as importantly – it requires the development of management systems to facilitate changes to the mining operations in response to changing economic conditions. At BHP Billiton we are developing robust and flexible mine plans, and we have adjusted budgets and incentives to reflect changed economic circumstances. We believe we already have a competitive edge in this area, but we are the first to admit that there is a lot more work to be done.

CONCLUDING REMARKS

This paper has discussed current best practice in mine planning and has identified a key shortcoming. The fact that the key assumptions underpinning our mine plans will inevitably prove to be incorrect means that our mine plans are no longer optimal over a reasonable range of real world outcomes. Possible sources of uncertainty were highlighted and discussed. The paper then focused on two key sources of uncertainty: price uncertainty and geological uncertainty. By using a simplified example it was shown that mine plans will change if price uncertainty is explicitly recognised. The issue of geological uncertainty was also introduced in the simplified example and it was indicated that plans will likely change to extract more ore. Perhaps counterintuitively, it was argued that the risk of geological uncertainty did not affect the mine plan and was of a fundamentally different character to that of commodity price risk. Possible criticisms of the proposed approach were also discussed.

What needs to be remembered is that every day mine planners are making decisions about:

- What is waste and what is ore?
- How much exposed ore should we carry?
- When should we run down our levels of exposed ore?
- What sequence of push-backs should we use?
- What stockpiles should we carry?
- How much ‘excess’ mining capacity we should carry?

We cannot stop the mining operations to perform the analysis. We have uncertainty regarding geology, processing, new technologies, market, prices and discount rates; the opportunity cost of suboptimal mine plans is large. At BHP Billiton we are mindful of the limitations of conventional optimisation techniques, and are developing methods and tools to assist us in valuing flexibility and ultimately developing robust mine plans.

REFERENCES

Black, F and Scholes, M, 1973. The pricing of options and corporate liabilities, *Journal of Political Economics,* 81:637-659.

Brealey, R A and Myers, S C, 2003. *Principles of Corporate Finance,* seventh edition (McGraw Hill Irwin: New York).

Copeland, T and Tufano, P, 2004. A real-world way to manage real options, *Harvard Business Review*, March, 90:9.

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Flores, L, 2005. Hardness model and reconciliation of throughput models to plant results at Minera, Escondida Ltda, Chile, in *Proceedings 37th Canadian Mineral Processors Conference,* Ottawa, 18 - 20 January.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions*, 316:43-50.

Hull, J C, 2000. *Options, Futures and Other Derivatives,* fourth edition (Prentice Hall).

Jackson, S, Frederickson, D, Stewart, M, Vann, J, Burke, A, Dugdale, J and Bertoli, O, 2003. Geological and grade risk and the Golden Gift and Magdala gold deposits Stawell, Victoria, Australia, in *Proceedings Fifth International Mining Geology Conference*, pp 207-213 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Khosrowshahi, S, Shaw, W J and Yeates, G A, 2007. Quantification of risk using simulation of the chain of mining — A case study at Escondida Copper, Chile, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 33-41 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Lane, K, 1988. *The Economic Definition of Ore* (Mining Journal Books: London).

Lerchs, H and Grossmann, L, 1965. Optimum design of open-pit mines, *Trans CIM*, LXVII, pp 17-24.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

McCarthy, J and Monkhouse, P H L, 2003. To open or not to open – Or what to do with a closed copper mine, *Journal of Applied Corporate Finance*, Winter, pp 63-73.

Muir, D C W, 2007. Pseudoflow, new life for Lerchs-Grossmann pit optimisation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 113-120 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Osterholt, V and Dimitrakopoulos, R, 2007. Simulation of orebody geology with multiple-point geostatistics — Application at Yandi Channel iron ore deposit, WA and implications for resource uncertainty, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 51-59 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Whittle, J, 1988. Beyond optimisation in open pit design, in *Proceedings Canadian Conference on Computer Applications in the Mineral Industries*, Rotterdam, pp 331-337.

Wooller, R, 1999. Cut-off grades beyond the mine – optimising mill throughput, in *Strategic Mine Planning Conference,* pp 217-229 (Whittle Programming Pty Ltd: Melbourne).

Integrated Mine Evaluation — Implications for Mine Management

G D Nicholas[1], S J Coward[1], M Armstrong[2] and A Galli[2]

ABSTRACT

Mine management is often expected to make rapid evaluation decisions at different stages of projects based on limited and uncertain data. The challenge is exacerbated by having to distil technical complexity into a financial model that is usually designed to produce only one or two key indicators, such as NPV and IRR. Mining is a complex environment with many sources of uncertainty ranging from sampling to economics. In order to optimise investment decision-making, an appropriately structured evaluation framework must be utilised. An evaluation framework should be designed to encapsulate and integrate the complexity across the evaluation cycle, that is, sampling, resource estimation, mine planning and treatment, and financial and economic modelling. This complexity is diverse and ranges from sampling support, scale effects to understanding the impact of variability, uncertainty and flexibility on operational efficiency and economic viability. These complexities, combined with time and capital constraints, usually do not allow all facets of evaluation to be integrated into the model. The model must strike a balance between simplified estimation techniques and sufficient incorporation of aspects of the project that will make a material difference to the investment decision.

This paper demonstrates the impact of the scale of measurement on the valuation of a mineral project. NPV comparisons are made between global estimation averages using a top-down approach and local estimates using a bottom-up approach. Three sampling campaigns were conducted on a virtual orebody to compare the relative NPV accuracies. Stochastic forward models were run on foreign exchange rates and are compared with the results from a fixed foreign exchange rate model.

INTRODUCTION

This paper explores the impact of the measurement scale on the estimate of a mineral project's NPV. Scale of measurement refers to dimensions in both space and time that are related to the key variables of the project, such as ore volume (thickness), grade, density, costs, revenue, foreign exchange rates, etc. Why is this important? Given a complex geological deposit and volatile price environment, it is suggested that the valuation of a mineral project may be materially affected by the use of large scale, annual average estimates for major variables. An integrated mine evaluation approach should be adopted using short-term, operational scale numerics that are accumulated into annual estimates to derive more realistic NPVs.

Many of the well-established resource and reserve classification codes refer to a mineral resource as having some 'reasonable and realistic prospects for eventual economic extraction' (JORC, 1999; SAMREC, 2000; NI43-101, 2001). These codes offer guidelines for assessing the criteria required to define mineral reserves but do not stipulate any quantitative confidence limits associated with tonnages, grade and revenue estimates. The selection of measurement scales is ultimately based on the judgement of a competent person. In order to quantify the impact of the selected scale on valuation, it is recommended that the process incorporate a quantitative assessment of the impact of these effects. This assessment should include both the modelling of unsystematic (specific) risks for resources and reserves, and systematic (market) risks, such as foreign exchange variability and costs of commodities such as oil, steel, concrete, etc. This would facilitate the setting of confidence limits around project valuation.

1. De Beers, Mineral Resource Management R&D Group, Mendip Court, Bath Road, Wells BA5 3DG, United Kingdom.

2. Cerna, Ecole des Mines de Paris, 60 Boulevard Saint-Michel, 75272 Paris Cedex 06, France.

It is unrealistic to create predictions of resource and reserve estimates on a small block scale when sample data are limited and spread out over a large area. Thus, in many cases production estimates of tonnages and grades are computed on an annual basis rather than a shorter-term scale (eg daily, weekly, etc). The sum of the local reserve depletions in a year is not equal to the total expected production derived from the average global reserve depletions. This is true for mineral projects that have a high degree of short-scale geological and mineralisation variability but only limited sampling data. The effect is amplified when resource variability has a substantial impact on mining rate and treatment efficiencies. The problem is further exacerbated for marginal projects which usually cannot afford the cost and potential time delays of spending additional evaluation capital on attaining close-spaced sampling data.

As the scale of data acquisition changes (ie more or less data are acquired), the mean and dispersion of the data will change. The impact of scale on a single variable is largely dependent on the distribution of the underlying phenomenon, eg for grade or density. If many sample data were acquired, the shape of the distribution (specifically, the means and variances) for each variable would be well defined. In most cases of evaluation, however, only limited sampling data are acquired and as a result, changes in the means and variances of individual resource variables could have a material impact on the project value. As variances are additive, the cumulative impact could result in over- or under-estimation of the NPV.

Two different evaluation approaches are selected in this paper to demonstrate the impact of measurement scale, viz. top-down versus bottom-up techniques. The former refers to annual forecasts that are calculated from depleting resource estimates through a global mine plan. Average expected values per annum are used as inputs into the mine plan to produce a NPV estimate. An alternative approach utilises a bottom-up evaluation technique whereby additional sampling data allow finer resolution resource models to be created. These finer scale models provide a way to carry out a quantitative assessment of the impact that resource variability has on daily mine output. Annual cash flow forecasts are derived from accumulations of daily depletions based on localised resource estimates.

While it may appear that these two methods would produce similar NPV results, there are cases where they do not. A case-study of an underground mine in Canada is presented where diamonds are contained in an irregular dyke that intruded into a fractured granitic host rock. Two sources of uncertainty were modelled. Firstly, geology was evaluated as a form of unsystematic (specific) risks due to the uncertain thickness of a mineralised dyke and its undulating top surface. Secondly, economic uncertainty, in the form of foreign exchange rate volatility between the US dollar and the Canadian dollar, was integrated into the evaluation model as a systematic (market) risk.

A virtual orebody (v-bod) was created using a non-conditional geostatistical simulation based on actual sampling data to provide a method of comparing the top-down and bottom-up approaches with 'reality' in the form of a v-bod. Comparisons were made between the two techniques and the v-bod. Three sampling campaigns were conducted on the v-bod and resource and reserves estimates were recalculated each time using the additional information to assess the impacts on differences between the top-down and bottom-up approaches.

EVALUATION PRACTICES

Project evaluation comprises three main components: project uncertainty, project structure, and value numerics (Samis and Davis, 2005). The authors of this paper focused solely on project uncertainty. Firstly, because in their experience technical complexities and correlations between variables cannot be captured easily in the typical evaluation of mineral projects; and secondly, because the impacts of technical uncertainty and variability are not clearly communicated to management.

Geostatistical techniques are routinely used to estimate grade, geology and density resource models for most mineral commodities, Matheron (1973) and Krige (1951). Since geostatistical simulations were developed (Matheron, 1973; Journel, 1974), they have been used to model the inherent variability and compare the impact of different mining methods or support sizes on resources and reserves. Early work (Dowd, 1976; Dumay, 1981; Chica-Olmo, 1983; and Fouquet De, 1985) focused on understanding the influence of technical aspects related to complex mining constraints and on quality control during production. As computer power increased, more simulations could be run and different types of simulation methods were developed that allowed more complex types of geology to be modelled.

Since the 1990s, the impact of uncertainty on project economics became increasingly important as more marginal projects were discovered. Ravenscroft (1992), Berckmans and Armstrong (1997), Dowd (2000), Dimitrakopoulos *et al* (2002), Godoy and Dimitrakopoulos (2004), and in this volume Ramazan and Dimitrakopoulos (2007), Menabde *et al* (2007), and Dowd and Dare-Bryan (2007) have used a combination of objective functions and geostatistical techniques to evaluate the impact of resource risks on the mine plan and determine their probabilistic impacts on NPV. These techniques incorporate resource uncertainty in the scheduling optimisation algorithm compared to traditional mine planning methods which could result in suboptimal reserves.

Over the past 15 to 20 years the techniques used in financial valuation of mineral projects have also evolved. Standard discounted cash flow (DCF) is used as the baseline for decision-making, but most mining companies now understand its limitations Davis (1995) and Smith (2000). Firstly, the technical and financial parameters used as input in NPV calculations are subject to uncertainty; secondly, mine management can and do react to changing circumstances (eg rising or falling commodity prices) by adapting the mine plan. Monte Carlo simulations coupled with geostatistical orebody simulations overcome the first limitation; real options were developed to overcome the second one.

According to Brealey and Meyers (2003) the first person to have recognised the value of flexibility was Kester (1984) in an article in the Harvard Business Review. The following year, Mason and Merton (1985) reviewed a range of applications to corporate finance and in their seminal paper, Brennan and Schwartz (1985) applied option pricing techniques first developed in finance to the evaluation of irreversible natural resource investments using Chilean copper mines to illustrate the procedure. To simplify the mathematics, they assumed that the reserves were perfectly homogeneous and that the grades were perfectly known. From a mining point of view, these assumptions may be unrealistic. Galli and Armstrong (1997), Carvalho *et al* (2000), and Goria (2004) address this by combining geostatistics with option pricing. Other mining aspects are presented by others, including Blais *et al* (2007, this volume) and Monkhouse and Yeates (2007, this volume).

In their paper, Brennan and Schwartz (1985) used a geometric Brownian motion based on Black and Scholes (1973) method with a convenience yield proportional to price in order to model the copper price. This was necessary to try and reproduce the natural variability of commodity prices over time. In contrast to many other commodities, diamond prices are not as volatile. Factors like the oil price and the exchange rate are more volatile and have a material impact on project value; the oil price affects costs and the exchange rate influences the company's revenue. The authors have chosen to focus on the exchange rate for this study.

Many models have been developed for interest rate and foreign exchange rates, ranging from simple extensions of Black and Scholes (1973) through Vasicek (1997) and on to the latest models with stochastic volatility. The book edited by Hughston (1996) provides a good overview of the subject. The authors chose to use the Garman and Kohlhagen (1983) which is a simple extension of Black and Scholes. In this model the drift term is replaced by the difference between the domestic and foreign interest rates. If St denotes the spot exchange rate at time t and rd and rf are the domestic and foreign interest rates, then

$$dS_t = (r_d - r_f)S_t dt + \sigma_S S_t dW_t$$

where:

σ_S is the volatility of the exchange rate

dW_t is a Brownian element

Two advantages of this model are that the exchange rates generated are lognormally distributed and hence positive, and that the parameters are easy to estimate.

The evaluation of a mineral project is a complex and technically challenging process, further complicated by numerous estimates of variables, covariance relationships and associated uncertainties. This paper captures a few crucial aspects of the evaluation pipeline, summarised in the following four sections, sampling and resource modelling, estimation of reserves (mine-planning and treatment), financial evaluation and economic modelling, and analysis and interpretation of results.

SAMPLING AND RESOURCE MODELLING

Sampling data in any evaluation model are fundamental in producing estimates that reflect reality. Although including more samples reduces uncertainty associated with both the mean and variance of resource estimates, it does not alter the natural variability within the deposit. The limitations of designing a sampling campaign for multiple variables have been discussed before Kleingeld and Nicholas (2007). Three variables were considered in this evaluation model, the

- geometrical variability of the top surface of the dyke (v1),
- thickness related to the volume of the dyke, and
- grade (in carats per hundred tonnes).

Core drilling was used to delineate geological variability on three different grid densities; 75 by 75 m, 50 m by 50 m and 25 m by 25 m, creating scenarios 1, 2 and 3, respectively. A 50 m by 50 m drilling grid was used to sample for grade, using large diameter drilling (LDD). Grade was not deemed to have any significant variability between scenarios and therefore, a single sampling campaign sufficed. The same grade estimates were applied to each scenario. A virtual orebody (v-bod) was created using a non-conditional geostatistical simulation based on data gathered from a combination of drilling information, bulk-samples and face mapping from an exposed part of the dyke. It is assumed to be the 'reality' on which the various sampling campaigns were conducted to generate sample data. Table 1 describes the design of the simulated sampling campaigns on a virtual orebody; sampling occurred at point support and simulation grid nodes were 4 m by 4 m in dimension.

The limitation of this approach is that only a single v-bod was created due to the time constraints and all conclusions are directly a function of both the data used to seed the v-bod and the design of the subsequent sampling campaigns. Sample data were used as input to generate kriged estimates and spatial simulations of grade, dyke thickness and geometric surface undulations of

TABLE 1

Sampling campaign design – summarising the three sampling campaigns and the v-bod.

	V-bod	Scenario 1	Scenario 2	Scenario 3
Description	Reality	Wide-spaced	Moderate	Detailed
Grid dimensions	4 m × 4 m	75 m × 75 m	50 m × 50 m	25 m × 25 m
No of samples/ nodes	399 360	1136	2556	10 224
Sample per cent of v-bod	-	0.28%	0.64%	2.56%

the dyke. A single mine plan was created based on the kriged estimates and overlain onto each estimate and simulation to determine the reserves. All output was fed into the financial model. Base maps of the v-bod and each sampling campaign are shown in Figure 1. Table 2 shows the statistical differences between the v-bod and each scenario for grade, dyke thickness and the geometrical variability of the dyke surface (v1).

RESERVES

The degree of resource complexity will have little impact on an operation's financial outcome for models that are generally unconstrained in terms of mining and treatment thresholds (assuming that the resource estimates have been accurately estimated). This applies to scenarios where abundant flexibility is included in the mining plan so that no bottlenecks occur in the extraction or treatment processes. The rate and scale of mining would deviate very little from plan as a result of resource variabilities. In contrast, mining operations (such as the underground example in this paper) that operate under strict geotechnical and geohydrological constraints in environmentally sensitive areas do not have the luxury of unlimited mining and treatment flexibilities. These mines cannot easily respond to changes in tonnages or grades as a function of resource variability. In the case of marginal operations with limited capital expenditure, the impact of this limited responsiveness is further exacerbated by the presumption of 'smoothed' ore horizons due to kriging with limited sampling data. The impact of this 'smoothing' will be demonstrated in this paper.

There are multiple factors to consider at this stage, ranging from resource uncertainties, mining and treatment constraints, financial cost per tonne data and economic volatilities with respect to commodity pricing and consumable costs. Identifying those factors that have the biggest impact on project value is essential but can be a very complex and time consuming process. This is largely driven by the number of variables that have to be considered and the complex interaction between variables, which are associated with different uncertainties and variabilities. While legal, social, political and environmental factors may influence managerial decision-making, the authors have elected to concentrate on the mining and treatment components of this model as discussed below.

Mine – planning and design

In this example, a conventional room and pillar underground method is considered with an option of slashing and drifting, depending on whether the dyke thickness was less than a specified mining threshold. An average extraction rate of 75 per cent was used. Each mining block of size 250 m by 250 m was depleted based on a combination of rim tunnels, stope tunnels and stope slashing. An average daily call of 3150 treatment tons was imposed on the project by management. The mine plan and treatment plant were designed to meet this production requirement on average per year.

The tabular nature of this deposit and mining, geohydrological and geotechnical restrictions severely limit the sequencing and optimisation of extraction. Simplistic assumptions were made regarding the selection sequence of blocks based on the highest value blocks being extracted first to maximise the time value of money. While the authors recognise the work done by Dimitrakopoulos and Ramazan (2004), Godoy and Dimitrakopoulos (2004), Grieco and Dimitrakopoulos (2007a), Grieco and Dimitrakopoulos (2007b), Ramazan and Dimitrakopoulos (2007), Menabde *et al* (2007) all in this volume; and others, involving the optimisation of the extraction sequence of blocks given resource and reserve uncertainties, there was insufficient time to include this in the study. The mine plan provided an opportunity to understand the interaction of the spatial nature of the reserves with the temporal realisation of its value. Mine blocks were depleted at a smallest mining unit (SMU) scale of 4 m by 4 m with a minimum mining height requirement of 2.0 m for equipment access into stope tunnels. Maximum mining heights of stopes were constrained to 2.2 m while rim tunnels were 3.5 m; rim tunnels were 4 m × 4 m × 3.5 m (height), stope tunnels were 4 m × 4 m × minimum 2.0 m (height), stope blocks were 4 m × 4 m × minimum 1.0 m (height). Pillar dimensions varied depending on the support required but no span greater than 8 m was created.

Recovery modelling

The estimation of the mean recovery factor and its variance is critical in determining the quantity of recovered material at a predetermined throughput treatment rate. The recovery factor depends largely on three key considerations. The characteristics of the ore type, its liberation and separation properties, and the design and interaction of the treatment process in relation to this ore type. The challenge of achieving efficient recoveries is to understand these complex three-way interactions. Due to the time constraints, simplistic assumptions were made regarding a linear relationship between the proportion of kimberlite ore and the waste.

The impact of the recovery factor on the recovered carats can be very marked especially if there are constraints on the system. For example, if the cut off grade is close to the statistical mean, subtle variations in the mean cut-off grade could significantly impact the project NPV. If the cut-off grade is raised, the average

TABLE 2

Resource simulation output showing the statistical differences between the v-bod and each scenario for grade, dyke thickness and the geometrical variability of the dyke surface (v1).

	V-bod	Scenario 1 (75 m)		Scenario 2 (50 m)		Scenario 3 (25 m)	
		Kriged	Sim 1	Kriged	Sim 1	Kriged	Sim 1
Mean thickness	1.70	1.70	1.66	1.70	1.71	1.70	1.69
Variance thickness	0.23	0.11	0.18	0.11	0.15	0.13	0.20
Mean v1	1.88	1.90	1.90	1.90	1.91	1.90	1.91
Variance v1	0.18	0.09	0.17	0.09	0.10	0.09	0.17
Mean grade	191	195	187	195	195	195	192
Variance grade	1985	1062	2860	1062	1523	1062	2004

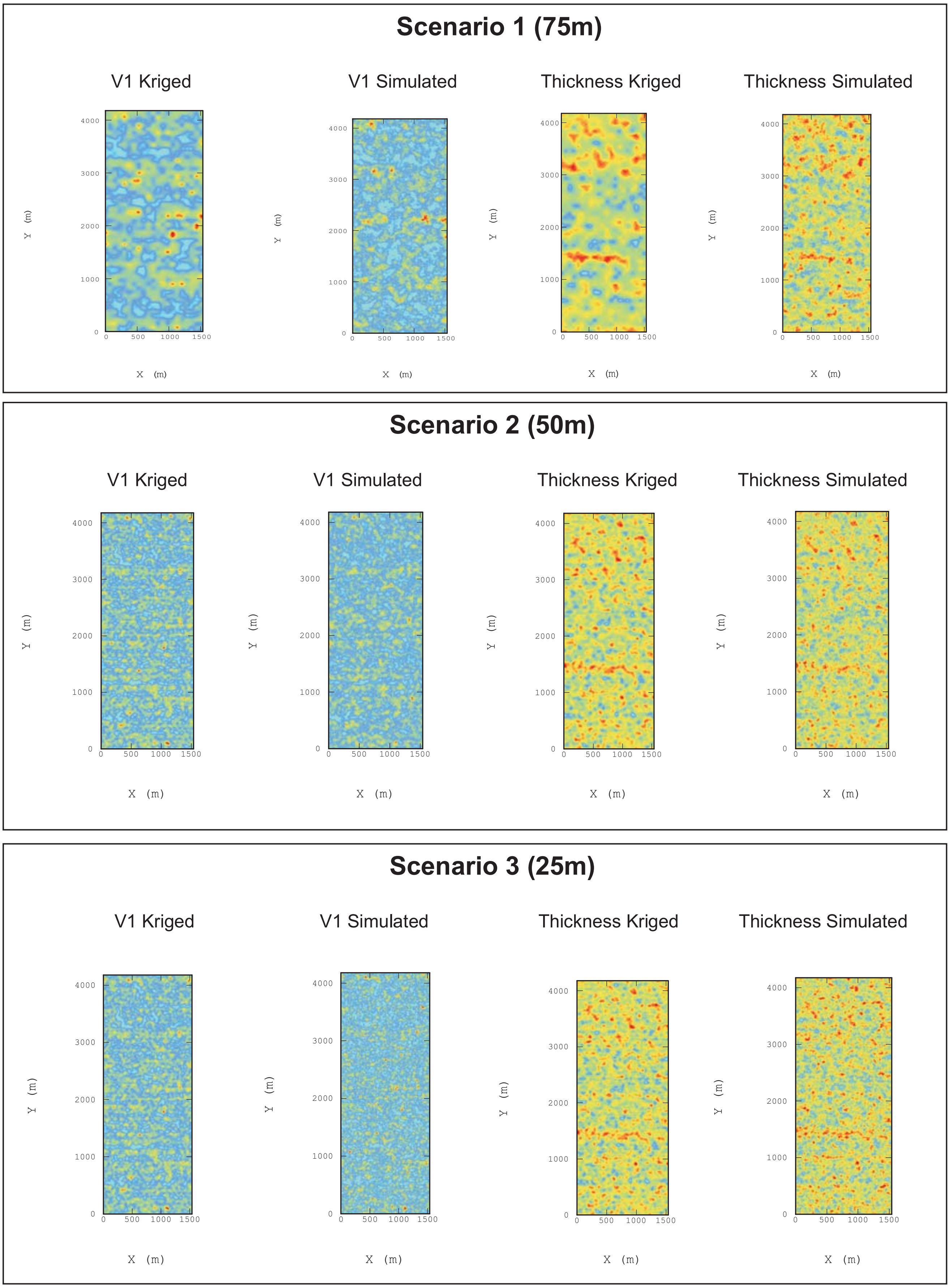


FIG 1 - The thickness and v1 base maps for the kriged and simulated outputs of each scenario with that of the v-bod. Grade was held constant between scenarios (warmer colours represent higher values while darker colours are low values).

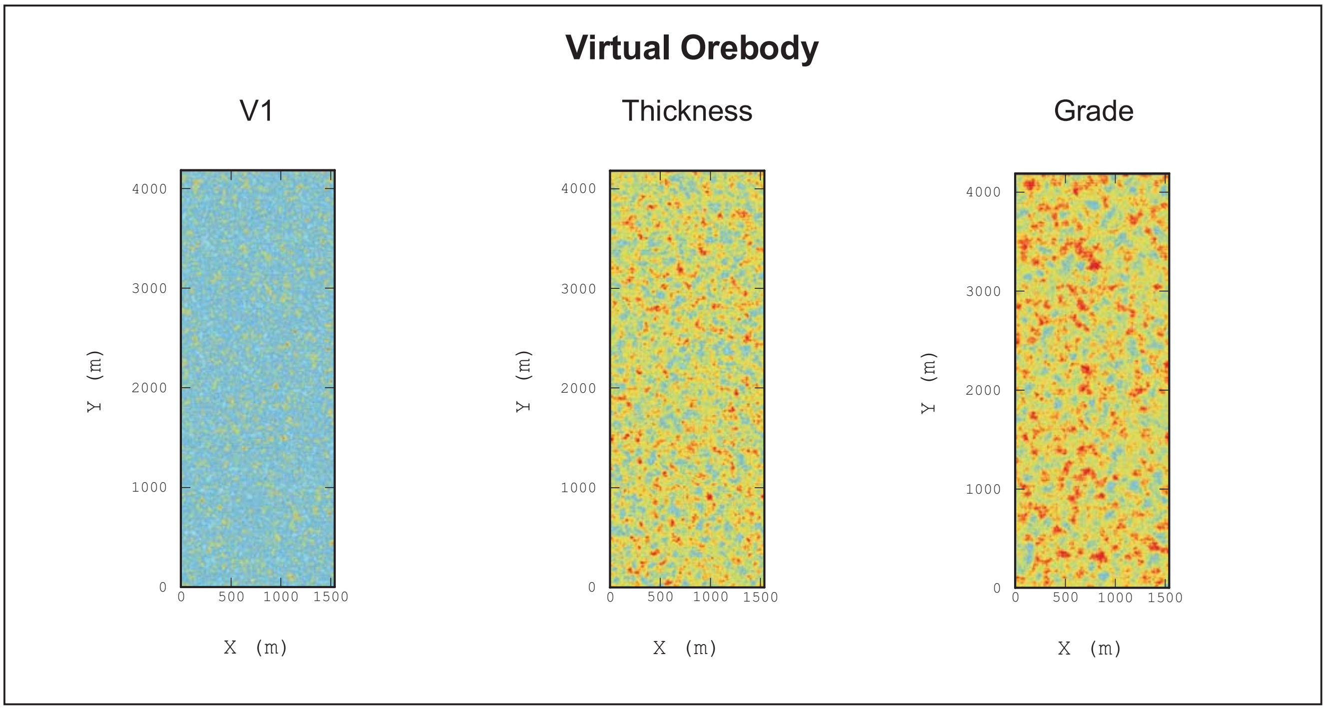


FIG 1 - The thickness and v1 base maps for the kriged and simulated outputs of each scenario with that of the v-bod. Grade was held constant between scenarios (warmer colours represent higher values while darker colours are low values).

grade above cut-off increases which may require mining that is too selective using the current mine design and equipment. Plant design, by its nature, requires a best fit for the 'average expected feed' and hence cannot incorporate the daily feed variation that may occur over the project's LOM. Conventional approaches to plant optimisation Parker (1997) usually entail, adapting the plant to accept the variability, installing a stockpile blending system, as well as adapting the mining method to increase the number of faces or draw points and use smaller equipment to improve selectivity.

The example in this study is fairly fixed in terms of its mine design and equipment selection. In addition, environmental policies limit the creation of large stockpiles. A total stockpile capacity of 3000 tons was created, which included capacity from an underground storage bin. While some degree of flexibility was available to adapt the plant settings to the ore variability, this was more suited to weekly and monthly fluctuations but would not cater for daily variations in the system. While dynamic simulations are considered as a possible means to estimate the short-scale variability in the recovery efficiency, time constraints did not allow for this. A simpler, pragmatic approach was sought to ascertain the impact.

In this model, depletions of the simulated 4 m by 4 m SMUs provided the ore-waste proportion information. A simplistic, linear relationship was imposed on treatment recoveries in relation to the proportion of kimberlite and waste; recovery efficiency improved as the proportion of kimberlite increased. A plant surge capacity constraint was included to assess the impact of varying dyke thickness (on a 4 m by 4 m SMU scale) on the feed rate variability using an 'event-based' simulation. The principle strategic levers that were considered in this mining and treatment sections were as follows: Annual mining rate in order to produce 3150 tons per day; bin storage capacity of 3000 tons; SMU selection (4 m × 4 m × height); the maximum mining ramp angle (17 degrees); a threshold imposed on the waste/kimberlite proportion (70/30) and if any blasted block had more than 70 per cent waste, it was not sent to the treatment plant.

Mine plan and treatment output

The daily production variations for scenario 3 are shown in Figure 2 together with resource variability in relation to mining and treatment constraints. The recovered carats after deducting all losses due to the waste threshold vary considerably on a daily basis. Output from the mining and treatment phase on an annual basis is tabulated in Table 3 for the v-bod and each of the three scenarios. More specifically, Table 3 shows the annual production output for the v-bod and the three scenarios. Recovered carats and grade are shown after all deductions.

FINANCIAL MODELLING

Given the uncertainties associated with each component of the model, the conventional practice of quoting a single NPV output is deemed idealistic and often, misleading. Conversely, running hundreds (or thousands) of stochastic realisations to quantify uncertainty in each component may be excessively time consuming and expensive and could result in superfluous data that have little material impact on the NPV. A balance must be struck. The financial model must be sufficiently flexible to accommodate multiple input scenarios for both global and local estimates yet quick when generating NPV outputs. Financial models are often designed as disparate systems, usually in a spreadsheet form, to compute the financial value of a project based on hard-coded production output from mine plans. So they have difficulty in capturing dynamic, technical linkages between resource, mining, treatment and economic models. The evaluation framework of conventional models allows limited risk and sensitivity analyses to be conducted as they do not assess the impact of correlated variables across the evaluation pipeline.

In conventional sensitivity analyses, all parameters, except the one in question, are held constant in the evaluation model. While this helps to identify which variable has the highest influence on the NPV, it cannot capture the range and probability of realistic scenarios when parameters vary simultaneously. Monte Carlo simulations (MCS) are a useful tool but should be used in

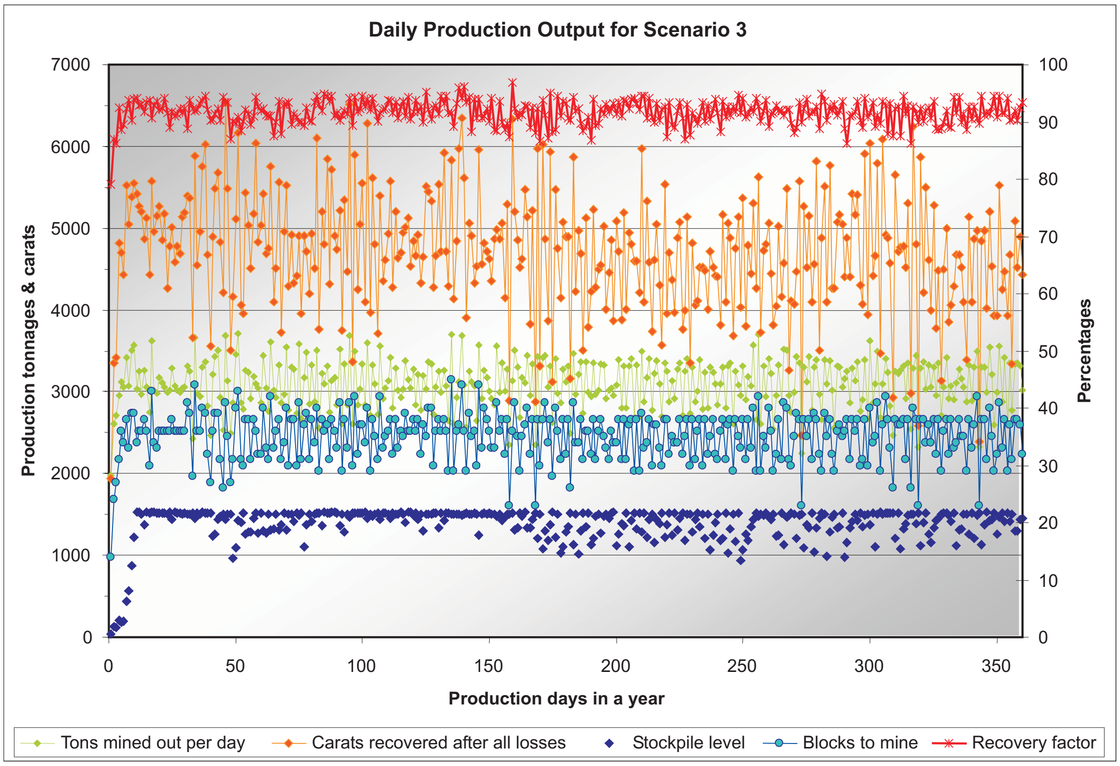


FIG 2 - An example of the daily production output in one year for scenario 3.

TABLE 3

LOM production output showing the annual production output for the v-bod and the three scenarios (recovered carats and grade are shown after all deductions).

	V-Bod	Kriged results			Simulated results		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
Total tons (million)	10.8	10.8	10.8	10.8	10.8	10.8	10.8
Recovery factor	92.5	93.6	93.3	93.1	92.1	92.5	93.0
Recovered carats (million)	16.2	16.6	16.6	16.5	15.7	16.2	16.4
Recovered grade	149.6	153.8	153.7	153.2	145.7	149.8	151.9

conjunction with other geostatistical and economic modelling tools to model the spatial and time-based variabilities. Examples of economic stochastic variables are foreign exchange rate prices, commodity prices, oil and diesel prices, etc.

A few of the key financial concepts are discussed below forming the building blocks of an integrated mine evaluation approach. Financial values have been adjusted to maintain confidentiality of the Canadian diamond mine.

Bottom-up versus top-down evaluation

Temporal scale is one of the most important aspects to be considered in the design of a financial model. The time interval in which cash flows are estimated must correspond with the time interval in which mining and treatment production data are measured and accumulated. These reserves in turn, depend upon the mine plan's ability to react to resource variability at the appropriate operational short scale. In addition to the unsystematic (project specific) risks, the financial model should also take due cognisance of systematic (market, economic related) risks by incorporating these stochastic variables at the appropriate time scale (support size). This section of the paper demonstrates that cash flow constituents derived from annual estimates in a top-down approach will not correctly reflect the asymmetries due to operational variability on a local, daily basis.

A more accurate way of deriving annual cash flow estimates needed to make decisions on projects would be to accumulate the appropriate values from a bottom-up approach, ie daily, monthly, quarterly then derive annual estimates for NPV forecasts.

The bottom-up approach entailed estimation (via geostatistical kriging techniques) of the main resource variables into a fine resolution grid (SMUs of 4 m by 4 m) based on sampling data from each campaign. Each SMU was analogous to a mining blast that was assessed to ascertain if it met the necessary mining and plant criteria, before either contributing to the daily plant call of 3150 tons per day or being trammed to the waste bin if it comprised more than 70 per cent waste. These daily accumulations were added together to form monthly, quarterly and annual production totals forming inputs into the cash flow models to derive NPVs for each scenario.

For the top-down approach, it was assumed that the mine plan only incorporated sufficient detail to deplete large-scale mine blocks of dimensions 250 m by 250 m. This implied that local mine plans (within each large-scale mine block) were not available to allow sequential depletion of the SMUs to accumulate tonnages and carats in a given year. Although the resource was modelled on a finer resolution (SMUs of 4 m by 4 m), these values were averaged into larger 250 m by 250 m mine blocks. The mine plan was designed to deplete on average 3.3 large-scale mine blocks per annum.

The average resource values for each year were run through this mine plan, assuming a fixed daily plant call of 3150 tons per day could be attained. Total recovered carats were calculated as a function of depleting the average estimated tonnages (per large-scale mine block) at a fixed throughput rate of 3150 tons per day, then multiplying the depleted carats with an average recovery factor per large-scale mine block. The carats per large-scale mine block were accumulated into annual cash flow models to produce global NPV estimates for each of the three kriged scenarios. Table 4 shows the differences between the global NPV, using a top-down approach versus that of the NPV annual based on a bottom-up approach (all values were calculated using a flat forex rate).

Technical discount rate

Many approaches have been developed to include technical risks in projects. Davis (1995) and Samis *et al* (2006) have argued against using a single discount factor to the aggregate net cash flows; they favour discounting each component as a function of its specific risk level. The authors, however, elected to use a single discount rate for the following reasons:

- It is still used in practice today as a baseline metric for financial comparisons.
- It allowed uncomplicated calculations of the NPV and the principles of this study are applicable to any other approach used.
- This study assessed the evaluation of a single project rather than a portfolio of projects. Project (technical) risks could be diversifiable if a large portfolio of projects were considered. The overall variance of the portfolio would reduce as a function of the number of projects in the portfolio, Markowitz (1952).

In this study, a ten per cent discount rate has been used. The standard NPV formula is well known where *CF* refers to the cash flow in each period *i* and *r* is the discount rate (see Equation 1). This equation can be rewritten as a weighted sum to illustrate the impact of the discount rate on the variance of the DCF (see Equation 2).

$$NPV = \frac{\sum CF_i}{(1+r)^i} - I_0 \tag{1}$$

$$DCF = \sum CF_i * \left(\frac{1}{(1+r)^i}\right) \text{ or DCF} = \sum CF_i * w_i \tag{2}$$

When risk analyses are conducted to ascertain the impact of the uncertain cash flows on a project's NPV, the mean net cash flow in each period, *i*, will be reduced by the weighting factor, *w*. This penalises cash flows in later years. The variance of the cash flows also reduces but by the square of the weighting factor, *w*, so this has an even greater effect on the variance than on the mean. In this example there are two opposing effects; on the one hand the variance reduces over time but as the sampling information is sparser in later years, less knowledge exists about the continuity of the dyke or the grade variability in those years.

DCF Analysis and time windows

NPV is a metric to assess whether the project makes a profit after all debts, invested capital and interests have been repaid. Once the NPV estimate has been determined, the second step is to plot the annual DCFs as this shows when the major proportion of cash flows fall and whether there are any irregularities over the LOM. The annual, locally-derived NPVs using the kriged estimates for scenario 1 and 3 are CAD 32.9 million and CAD 28.3 million, respectively. Figure 3 compares the annual cash flows and DCF values for these two scenarios.

Figure 3 shows that the period between (2008 and 2012) accounts for more than 60 per cent of the project's positive annual cash flow and 70 per cent of the DCF value. As cash flows generated after 2012 are discounted at values of 50 per cent and higher, management would have to make significant operational changes in order to increase net cash flows beyond 2012. Money would be better spent on attempting to improve the net cash flows earlier on to maximise the NPV. Risk mitigating controls could be implemented such as mining or treatment modifications or by reducing the technical risk proportion in the discount rate through further sampling.

Economic (forex) uncertainty

Two scenarios considering forex uncertainty were integrated into the evaluation model. In both cases, the forex rate was applied only to the revenue component as sales from diamonds were in notional US$ whereas all costs were assumed to be sourced locally. The first scenario assumed a flat rate of 1.21 CAD$ to a US$. This corresponds to a forward forex price. Transaction costs were ignored. The NPV results of the three scenarios relative to the v-bod using the flat rate were shown in Table 4. The second scenario assumed that the project management team would expose the project to the forex rate volatility. Forex stochasticity was modelled using a Garman and Kohlhagen (1983) to incorporate mean reversion and volatility parameters. A total of 100 simulations were run over a ten-year period emulating the forex uncertainty (Figure 4). Each of the 100 simulations was incorporated into the financial model to produce NPV estimates for each scenario and for the v-bod. NPV histograms and cumulative probability plots for the v-bod are shown in Figure 5 and Figure 6. NPV comparisons incorporating the forex rate simulations are tabulated in Table 5 for each scenario and for the v-bod. Table 5 shows the maximum, minimum and 50th percentile NPVs of the three scenarios relative to the v-bod after including forex rate modelling (per

TABLE 4

Financial output (in CAD$) showing the differences between the global NPV, using a top-down approach versus that of the NPV annual based on a bottom-up approach (all values were calculated using a flat forex rate).

	V-Bod	Kriged		
		Scenario 1	Scenario 2	Scenario 3
Global annual NPV	-	91.6	80.1	73.9
Local annual NPV	2.1	32.9	31.4	28.3
Differences	-	58.8	48.7	45.6
	V-Bod	**Simulated**		
		Scenario 1	**Scenario 2**	**Scenario 3**
Global annual NPV	-	12.0	39.7	58.1
Local annual NPV	2.1	(26.1)	3.6	18.1
Differences	-	38.0	36.1	40.0

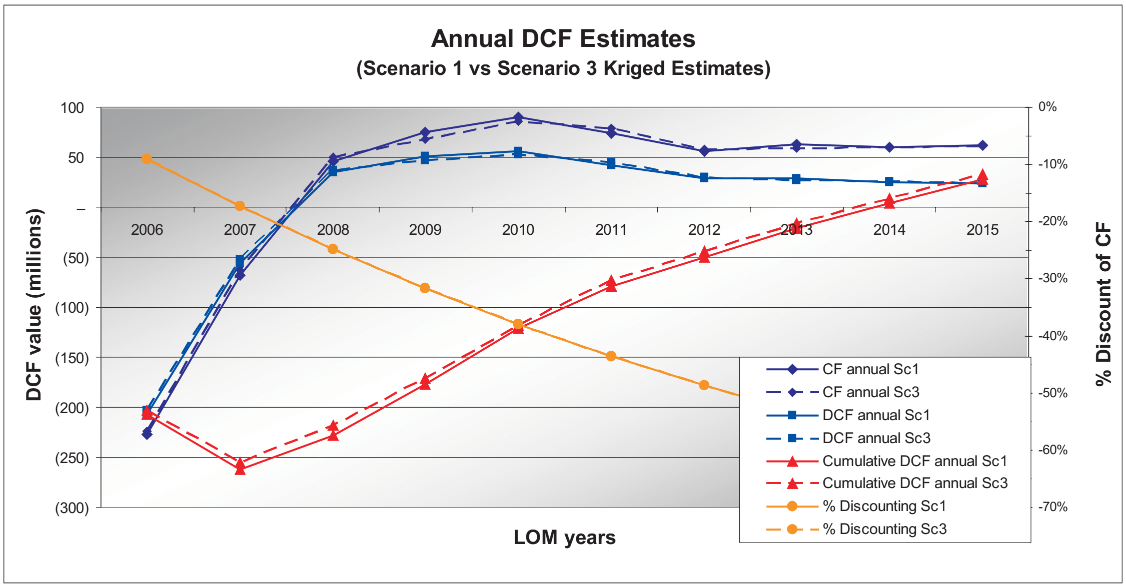


FIG 3 - Comparison of the net cash flows (CF), discounted cash flows (DCF), cumulative discounted cash flows and the percentage discounting applied to the net cash flows for scenario 1 and scenario 3.

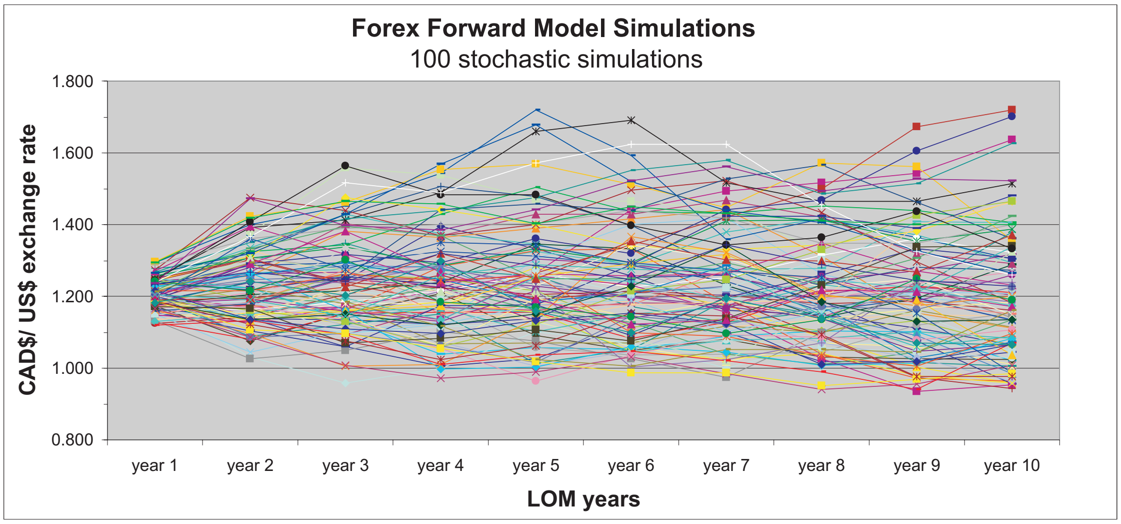


FIG 4 - Forex rate stochastic output per year from 100 simulations.

cent differences are relative to the v-bod P50 value). All values shown were calculated using the local estimation technique (bottom-up approach).

ANALYSIS AND INTERPRETATION

The case study demonstrated the impact of resource and economic stochasticities on a project's NPV as a function of both sampling and temporal uncertainties. A v-bod was constructed from actual sampling data, derived from a Canadian mine, to provide a method of comparing scenarios against a simulated version of reality. Three sampling campaigns at grids of 75 m, 50 m and 25 m were conducted on the v-bod to produce scenarios 1, 2 and 3. It was shown that global annual NPV estimates derived in a top-down fashion, markedly under-estimated the v-bod NPV. Comparisons between scenarios showed material differences in the NPV estimates.

Global NPVs derived from kriged estimates for the three scenarios (75 m, 50 m and 25 m) were CAD 91.6 million, CAD 80.1 million and CAD 73.9 million, respectively.

As drilling grid densities increased from 75 m to 50 m and 25 m intervals, the uncertainty of v1 and dyke thickness reduced and the estimates improved relative to the actual v-bod NPV (CAD 2.1 million). Nonetheless, all global estimates over-estimated the v-bod NPV estimate by a magnitude of 43 to 35 times (75 m to 25 m scenarios). Local NPVs derived from kriged estimates for the three scenarios (75 m, 50 m and 25 m) were CAD 32.9 million, CAD 31.4 million and CAD 28.3 million, respectively. Similarly, the NPV estimates improved as more samples were taken. Local estimates over-estimated the v-bod NPV estimate by a magnitude of 15 to 13 times (75 m to 25 m scenarios). Note that the number of samples are significantly large (1136 samples for the 75 m scenario, 2556 samples for the 50 m scenario and 10 224 samples for the 25 m scenario). The more complex a deposit is (in terms of geological structures and mineralisation dispersion), the more sample holes will be required to reduce uncertainty and produce more accurate estimates of the statistical means and variances of relevant variables. Greater NPV differences between sampling scenarios would be expected if fewer samples were taken.

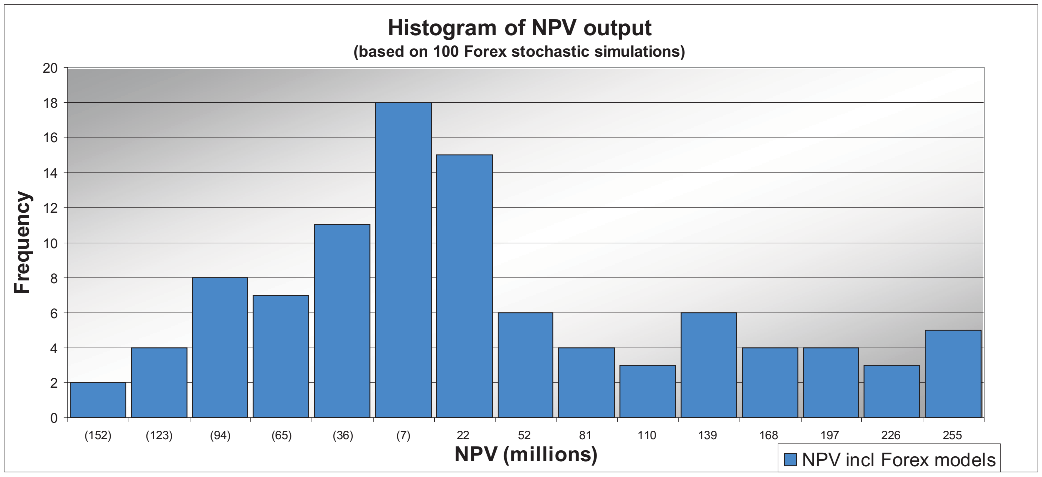


FIG 5 - An NPV histogram for v-bod after including 100 forex simulations.

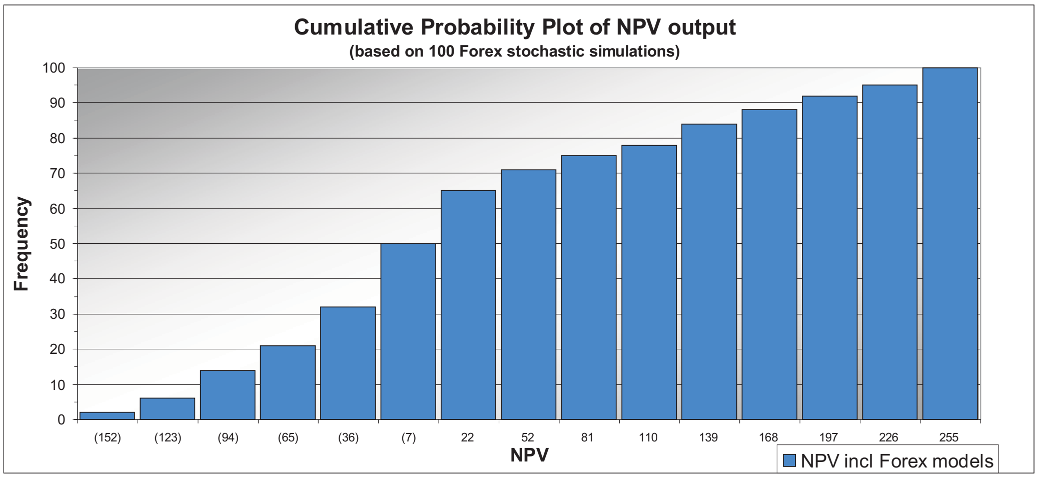


FIG 6 - The cumulative probability plot of the NPV for v-bod after including 100 forex simulations.

TABLE 5

Economic forex output (in CAD$) showing the maximum, minimum and 50th percentile NPVs of the three scenarios relative to the v-bod after including forex rate modelling (per cent differences are relative to the v-bod P50 value).

	V-bod	Scenario 1	Scenario 2	Scenario 3
Maximum NPV (annual)	255.2	292.5	291.5	287.6
Minimum NPV (annual)	(177.4)	(150.3)	(152.8)	(155.2)
NPV P50 (annual)	(6.7)	24.5	22.7	19.6
P50 difference	-	468%	440%	394%

While kriging exercises produced the best unbiased estimates for key variables, they tend to provide 'smoothed' resource estimates based on limited data. It is this 'smoothing effect' that results in over-estimation of the grades, thickness and v1 variables. NPV estimates would be over-estimated relative to the actual deposit. Contrary to kriging, spatial simulations provide a better indication of the range of variabilities to be expected. Insufficient time was available to generate a range of simulated realisations for comparison purposes. Thus, only a single simulated realisation was selected as an example of the expected differences in mean values.

Local NPVs based on conditional simulated estimates for the three scenarios (75 m, 50 m and 25 m) were negative CAD 26.1 million, CAD 3.6 million and CAD 18.1 million, respectively. These simulated outcomes are significantly lower than the kriged estimates and closer to the actual v-bod NPV. This may give the impression that conditional simulations provided more accurate estimates than kriging, but these simulations represent only one extraction from a range of simulations. This could represent the tenth or 90th percentiles (P10 or P90) of the simulated distribution outputs. Further work is necessary to generate the e-type estimate from a complete range of conditional simulations and compare it with the kriged result. The use of a flat forex rate was compared with a stochastic forward model that considered forex rate volatility. A fixed forex rate of 1.21 was used (February 2006 CAD:US$ rates) to derive a v-bod NPV of CAD 2.1 million. Table 5 shows the probable range in NPVs for the v-bod and three kriged scenarios when each of the 100 forex models were run through the financial model. The medians (ie 50th percentile or P50) for scenarios 1, 2 and 3 were CAD 24.5 million, CAD 22.7 million and CAD 19.6 million, respectively.

Using the variable forex rates, the P50 of the v-bod NPV reduced from CAD 2.1 million to negative CAD 6.7 million (four times less). This would imply that the project is susceptible to forex rate volatility. However, as shown in Figures 5 and 6, there is considerable upside opportunity when the 50th to 90th percentiles are considered. Projects that are particularly revenue or cost sensitive may benefit by conducting forward modelling of the forex rate as it allows management to gain an improved understanding of the range of probable NPVs. The costs of hedging against downside risks of forex rate fluctuations should be weighed against the negative impact that it may have on project value.

The estimation of resources strives to create a view of the quantity of *in situ* material that can reasonably be mined. It is this 'reasonable expectation' of 'mineability' that implies it is impossible to estimate resources totally independently of all external factors. These factors include the economic and technological limits that have to be imposed, and the scale and rate of mining.

As noted from this study, the optimal operational strategy of a mine is related to a number of key factors that need to be defined at an appropriate temporal scale:

- resource complexity, in terms of the continuity of mineralisation within geological structures; and thickness of the ore zone;
- design of sampling campaign(s) to detect the means and variances of selected variables; specifically considering sample support size and quantity of samples;
- resource modelling; kriged estimates to determine the means of grade, thickness, etc and geostatistical simulations to assess the probabilistic impact of variabilities on the evaluation model;
- design of the mine plan in response to resource complexities;
- mining and treatment logistical, environmental and financial constraints;
- financial cash flow model with respect to revenues, costs and other aspects, such as taxes and royalties that emphasise asymmetries in cash flows;
- economic stochasticity of foreign exchange rates and commodity prices for steel, diesel, concrete costs, etc; and
- encapsulation of different sources of technical risks in the evaluation model.

While this study focused exclusively on a diamond mine example, it is believed that the key aspects mentioned above are true for most mineral projects that have complicated resource models but only limited sampling data, and restrictive mining and treatment constraints.

CONCLUSIONS

Mine evaluation requires an integrated, holistic approach as the valuation of intangible resource and reserve assets are based on uncertain data that are linked to several components of the valuation pipeline. The complexity of valuing mineral projects lies in evaluating a number of spatial and time-dependent variables, within an appropriate time scale. These variables may or may not be correlated with each other. There is usually a high degree of uncertainty about the true means and variances of these variables which complicates the design of an optimal integrated evaluation system. As noted from this study, selection of the appropriate time measurement scale in which to evaluate a number of diverse variables in a mineral resource project is critical in attaining realistic NPV estimates. Further analysis demonstrated the knock on effects that both uncertainty and variability have on the evaluation pipeline. For this reason, the evaluation model components cannot be optimised individually; the synchronisation of resource, mining and treatment, and financial components is required in order to achieve an optimal balance of the system. There are three main effects that have been investigated in this model.

The first is the impacts that a complex and uncertain resource has on the design of an optimal mine plan. The effectiveness of the design is usually determined by a combination of the inherent, stochastic variability of the deposit and the uncertainty of predictions of this complexity that arises from limited sampling data. If restrictive mining and treatment constraints are imposed onto a complex resource model, the adaptability of the mine will decrease. Where possible, the flexibility in the mine plan should be matched to both the estimated degree of resource complexity and the uncertainty that the mine design team has about that complexity.

Secondly, synchronisation between the treatment plant and the mining extraction process has a huge impact on the asymmetries in the cash flow model. If mining constraints, such as mining rate of advance, development tonnes, dilution, etc are not aligned with the treatment constraints in terms of storage bin capacity or plant throughput, the mine could produce more tonnes at a time when the plant cannot treat it, or conversely, the mine will produce less tonnes at a time when the plant's capacity exceeds that of the mine. Imbalances in these constraints result in time wastage and inevitably, lost profit opportunities.

Lastly, the process of integrating this model revealed which resource, mining and treatment parameters have the biggest impact on project value. This process would assist the competent person in identifying those areas which are uncertain and could lead to a material difference in valuation. Once an integrated development system has been developed, it will be possible to explore the upside potential of optimally synchronising economic forecasts with mining and treatment parameters. Forward models of cost and revenue data should be at an appropriate time scale that is aligned with the estimates of cash flow forecasts and reserve calculations. Changes in revenue as a function of commodity price or exchange rates, or costs related to oil and diesel, steel and concrete prices could have a material impact on a project's value. The derivation of reserves will also be influenced by these economic stochasticities.

While a balance was sought between a pragmatic yet sufficiently detailed evaluation model, inclusion of realistic mining and treatment constraints necessitated the construction of a more complex evaluation model to reflect the value of additional sampling data. Mining and treatment constraints in response to resource variability defined the key relationships within the evaluation model that resulted in different NPVs between scenarios.

Further work is pending in the following areas:

- modelling spatial correlations between thickness, geometrical surfaces of the dyke and grade using the latest sampling data;
- mine plan sequencing and optimisation in response to resource uncertainty;
- dynamic recovery modelling with particular emphasis to liberation, separation and their interaction with the ore properties;
- economic stochastic modelling related to oil, steel and concrete prices;
- response of the evaluation model (feed-back and feed-forward loops) to different sources of uncertainty;
- real options valuation to ascertain the impact of flexibility in the model; and
- development of an integrated software platform to rapidly evaluate projects.

> *As a last remark, beware that uncertainty … arises from our imperfect knowledge of that phenomenon, it is data-dependent and most importantly model-dependent, that model specifying our prior concept*

(decisions) about the phenomenon. No model, hence no uncertainty measure, can ever be objective – Goovaerts (1997).

ACKNOWLEDGEMENTS

Support and input from De Beers Canada, De Beers GME and the De Beers MRM R&D Group, especially Dr Malcolm Thurston, Marty Rendall, Andy Wood and Fanie Nel are gratefully acknowledged. The authors would also like to specifically thank Dr Wynand Kleingeld for sharing his valuable experience on the concepts of a v-bod with them and Professor Peter Dowd for his contributions on developing a pragmatic evaluation model.

REFERENCES

Berckmans, A and Armstrong, M, 1997. Geostatistics applied to the Monte Carlo analysis of mining projects, in *Geostatistics Wollongong '96* (eds: E Y Baafi and N A Schofield) Vol 2, pp 743-754 (Kluwer Academic Publishers: Dordrecht).

Black, F and Scholes, M, 1973. The pricing of options and corporate liabilities, *Journal of Political Economy,* 81:637-654.

Blais, V, Poulin, R and Samis, M, 2007. Using real options to incorporate price risk into the valuation of a multi-mineral mine, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 21-27 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Brealey, R A and Meyers, S C, 2003. *Principles of Corporate Finance*, international edition (McGraw-Hill Irwin: New York).

Brennan, M J and Schwartz, E S, 1985. Evaluating natural resource investments, *Journal of Business,* 58:135-157.

Carvalho, R M, Remacre, A Z and Suslick, S B, 2000. Geostatistical simulation and option pricing: A methodology to integrate geological models in mining evaluation projects, in *Proceedings Sixth International Geostatistical Congress – Geostats 2000* (eds: W J Kleingeld and D G Krige) Vol 1, pp 1-10, Capetown, South Africa.

Chica-Olmo, M, 1983. Approche géostatistique de la caractérisation des ressources en charbon. Thèse de Doct-Ing Centre de Geostatistique, Ecole des Mines de Paris.

Davis, G A, 1995. An investigation of the under pricing inherent in DCF valuation techniques, paper presented at SME Annual Meeting, Denver, USA.

de Fouquet, C, 1985. L'Estimation des réserves récupérées sur modèle géostatistique de gisements non-homogènes. Thèse de Doct-Ing Centre de Géostatistique Ecole de Mines, Paris.

Dimitrakopoulos, R, Farrelly, C and Godoy, M C, 2002. Moving forward from traditional optimization: Grade uncertainty and risk effects in open-pit mine design, *Transactions of the IMM*, Section A, Mining Technology, 111:A82-A89.

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions*, 316:1-9.

Dowd, P A, 1976. Application of dynamic and stochastic programming to optimise cut off grades and production rates, *Transactions of the IMM*, Section A, Mining Technology, 85:A22-A31.

Dowd, P A, 2000. MINVEST financial evaluation package for mining projects.

Dowd, P A and Dare-Bryan, P C, 2007. Planning, designing and optimising production using geostatistical simulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 363-377 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dumay, R, 1981. Simulations d'exploitations miniers sur modèles géostatistiques de gisements. Thèse de Doct-Ing Centre de Géostatistique Ecole de Mines, Fontainebleau.

Galli, A and Armstrong, M, 1997. Option pricing: Estimation versus simulation for the Brennan and Schwartz natural resource model, in *Geostatistics Wollongong '96* (eds: E Y Baafi and N A Schofield) Vol 2, pp 719-730 (Kluwer Academic Publishers: Dordrecht).

Garman, M B and Kohlhagen, S W, 1983. Foreign currency option values, *International Money Finance*, 2:231-237.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions,* 316:43-50.

Goovaerts, P, 1997. *Geostatistics for Natural Resource Evaluation* (New Oxford University Press: New York).

Goria, S, 2004. E'valuation D'un Projet Minier: Approche Bayesienne et Options Reélles. Ecole Des Mines de Paris.

Grieco, N and Dimitrakopoulos, R, 2007a. Managing grade risk in stope design optimization: Probabilistic mathematical programming model and application in sublevel stoping, *Transactions of the IMM*, Section A, Mining Technology, 116(2):49-57.

Grieco, N and Dimitrakopoulos, R, 2007b. Grade uncertainty in stope design — improving the optimisation process, in *Proceedings Orebody Modelling and Strategic Mine Planning*, second edition, pp 167-174 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hughston, L, 1996. *Vasicek and Beyond: Approaches to Building and Applying Interest Rate Models* (Risk Publications: London).

JORC, 2004. Australasian Code for Reporting of Mineral Resources and Ore Reserves (The JORC Code), The Joint Ore Reserves Committee of The Australasian Institute of Mining and Metallurgy, Australian Institute of Geoscientists and Minerals Council of Australia [online]. Available from: <http://www.ausimm.com.au/main/about/docs/jorc0105.pdf> [Accessed: 7 May 2007].

Journel, A G, 1974. Geostatistics for conditional simulation of orebodies, *Economic Geology,* 69:673-687.

Kester, W C, 1984. Today's options for tomorrow's growth, *Harvard Business Review,* 62:153-160.

Kleingeld, W J and Nicholas, G D, 2007. Diamond resources and reserves — Technical uncertainties affecting their estimation, classification and valuation, in *Proceedings Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 227-233 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Krige, D G, 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand, *Journal of the Chemical, Metallurgical and Mining Society of South Africa*, 52:119-139.

Markowitz, H M, 1952. Portfolio selection, *Journal of Finance,* 7:77-91.

Mason, S P and Merton, R C, 1985. The role of contingent claims analysis in corporate finance, in *Recent Advances in Corporate Finance* (eds: E Altman and M Subrahmanyam), pp 7-54 (Richard D Irwin Inc: Homewood).

Matheron, 1973. The intrinsic random functions and their application, *Advances in Applied Probability,* 5:439-468.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Monkhouse, P H L and Yeates, G A, 2007. Beyond naïve optimisation in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 3-8 (The Australasian Institute of Mining and Metallurgy: Melbourne).

NI43-101, 2001. National Instrument 43-101, Standards of Disclosure for Mineral Projects. Canadian Institute of Mining (CIM).

Parker, H, 1997. Applications of geostatistical methods in exploration programme design (National Council for United States – China Trade Technical Exchange: Peking).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ravenscroft, P, 1992. Risk analysis for mine scheduling by conditional simulation, *Transactions of the IMM*, Section A, Mining Technology, 101:A104-A108.

Samis, M and Davis, G A, 2005. Using real options to value and manage natural resource projects. Course notes. Vancouver, Canada.

Samis, M, Davis, G A, Laughton, D and Poulin, R, 2006. Valuing uncertain asset cash flows when there are no options: A real options approach, *Resources Policy,* 30:285-298.

SAMREC, 2000. South African Code for Reporting of Mineral Resources and Mineral Reserves, 2000. South African Mineral Resource Committee (South African Intstitute of Mining and Metallurgy: Johannesburg).

Smith, L D, 2000. Discounted cash flow analysis, methodology and discount rates, in *CIM/PDAC Mining Millennium 2000*, pp 1-18.

Vasicek, O A, 1997. An equilibrium characterisation of the term structure, *Journal of Financial Economics,* 5:177-188.

Using Real Options to Incorporate Price Risk into the Valuation of a Multi-Mineral Mine

V Blais[1], R Poulin[2] and M R Samis[3]

ABSTRACT

The mining industry is increasingly focused on using a consistent approach to determine the effect of risk on project value and operating policy. Valuation techniques from the financial industry are being adapted so that a mine valuation model can successfully integrate market information about risk with a detailed description of project structure. The real option method is one such import that is finding increasing use in the mining industry. However, real option models are often built with only one source of uncertainty – namely, the primary output mineral. This can produce misleading valuation results when secondary minerals are also recovered.

This paper extends the application of real options in the mining industry by developing a Monte Carlo valuation model of an undeveloped mine that can produce two minerals. We compare the results to a Monte Carlo discounted cash flow model and demonstrate the importance of explicitly recognising the unique risk and uncertainty characteristics of each mineral within the value calculation. We consider the industry practice of converting secondary minerals into metal equivalents and highlight project situations where this may be acceptable and others in which it is not. In particular, copper-gold prospects are shown to be unsuitable since differences in uncertainty characteristics may cause metal equivalents to produce results that overstate or understate project value and incorrectly identify price levels at which operating policy changes. Our results show that it is important that mine valuation professionals and qualified persons be aware of the important consequences associated with ignoring the uncertainty characteristics of secondary minerals.

INTRODUCTION

The mining industry is has been working towards a consistent approach to determine the effect of project risk on value and operating policy. With a variety of valuation techniques, project analysts are attempting to build mine valuation models that successfully integrate market information about risk with a detailed description of project structure. The real option approach is one such method that is finding increasing use, since it can consider project development and operation alternatives in conjunction with the unique risk and uncertainty characteristics of each output mineral within the value calculation.

However, real option models often include only one source of uncertainty because the most commonly used numerical techniques have difficulty including multiple sources of uncertainty. Some numerical techniques used by real option practitioners have limitations that force many valuation professionals to consider the primary output mineral as the only underlying state variable. Any secondary minerals are either converted into primary mineral equivalents or are treated as a pre-set risk-discounted revenue stream. This can produce misleading valuation results since the unique risk and uncertainty characteristics of secondary minerals can have important value consequences. This paper extends the application of real options in the mining industry by using Monte Carlo simulation to value a project with multiple sources of mineral price uncertainty and abandonment flexibility.

The potential problems associated with using metal equivalents are demonstrated with an example valuation of a copper-gold prospect where gold is an important secondary output and there is no flexibility. A detailed cash flow model of the full project life cycle is presented. A net present value (NPV) is calculated using both the discounted cash flow (DCF) method and the real option (RO) method in which gold production is converted into copper equivalents. The gold copper equivalents are calculated using the ratio between expected gold and copper prices at each production time.

The DCF method of calculating project NPV is the most widely accepted valuation method in the mining industry. However, the conventional DCF practice of using a constant corporate discount rate to value a range of projects is problematic. The primary criticism of this practice is that it implicitly assumes that net cash flow uncertainty varies across time and projects in a constant manner. This is a careless treatment of the fundamental valuation principle that uncertainty is an important value influence, since uncertainty often varies across time and projects in a non-constant manner. Samis, Laughton and Poulin (2003) discuss this limitation of the conventional DCF method and demonstrate how the RO valuation method recognises variations in net cash flow uncertainty. The application of DCF and RO methods in evaluating and selecting from different open pit mine designs in the presence of both cash flow and metal uncertainty is described in Dimitrakopoulos and Abdel Sabour (2007).

The ability to abandon the project at any time is incorporated into the project model and a RO NPV is calculated in which gold output is expressed in terms of copper equivalents. This model is then re-configured for two state variables such that it is not necessary to translate gold output into copper equivalents and a price path is generated for each metal. Project NPVs are calculated when these price paths are independent and over a range of negative and positive copper-gold price correlations. However, for an actual project valuation, a valuation analyst would likely use a low correlation between copper and gold price movements since copper is mined for its industrial uses while gold is mined primarily as an investment asset with some secondary industrial uses. Project values are calculated over a range of price correlations to demonstrate the value effect of price correlation. The potential value impact of using the metal equivalent simplification for copper-gold prospects can be observed from comparing the various value results.

THE COPPER-GOLD PROJECT

Mining is a highly capital-intensive business in which project cost structure has a large number of fixed components. Significant upfront capital expenditures in the form of exploration, pre-production and development costs as well as large investments in fixed assets are necessary when establishing a mine. The project considered in this paper has development and operating cost components in both American and Canadian dollars. Initial capital of CAD$320 million and US$70 million is invested between 2004 and 2007. Production begins in 2008 at a

1. Towers Perrin Risk Capital (Canada) Inc, 1600, 111-5 Avenue SW, Calgary AB T2P 3Y6, Canada.
Email: vincent.blais@towersperrin.com

2. Université Laval, Faculté des Sciences et génie, 1774, Pavillon Adrien-Pouliot, Cite Universitaire QC G1K 7P4, Canada.
Email: Richard.Poulin@vrex.ulaval.ca

3. Director of Financial Services (Mining and Metals), AMEC Mining and Metals, AMEC Americas Limited, Suite 700, 2020 Winston Park Drive, Oakville ON L6H 6X7, Canada.
Email: michael.samis@amec.com

constant rate of two million tonne of ore per year over 12 years. Copper and gold grades decrease in a linear manner that reflects a mine design where operations move from high quality to low quality reserves. More complex patterns of ore grades can be used without increasing the model's complexity. Details of the copper-gold prospect are presented in Table 1.

The annual real risk-free rate of 2.0 per cent is used for RO NPV calculations and a risk-adjusted discount rate (RADR) of 10.0 per cent is used for DCF NPV calculations. A constant exchange rate of CAD$1.37:US$1.00 is used to convert Canadian dollar costs into American dollars. Exchange rate uncertainty can be added easily as an additional state variable but this is not done here in order to retain a focus on the practice of using metal equivalents.

STOCHASTIC PROCESS OF MINERAL PRICES

Microeconomic theory highlights that the price of an industrial commodity, such as copper or nickel, should be linked to its marginal production cost in the long term. Expectations of future prices revert back towards the marginal production cost even when price shocks force the current spot price away from this equilibrium. Conversely, investment assets such as financial stocks or gold do not exhibit price reversion in that there does not appear to be a long-term equilibrium price towards which future price expectations trend.

Gold and copper prices in this paper are modelled by the single factor stochastic process (details of this process can be found in Laughton and Jacoby, 1993; Salahor, 1998; Samis, 2000):

$$dS = \left[\alpha^* + \frac{1}{2}\sigma^2 - \gamma \ln\left(\frac{S}{S^*}\right)\right] Sdt + \sigma Sdz$$

where:

S is the current mineral spot price

S^* is the current long-term price median

α^* is the short-term growth rate of the price medians

σ is the short-term price volatility

γ is the reversion factor

dz is the standard Wiener increment

The strength of economic forces pulling spot prices back towards a long-term equilibrium price is measured by a reversion factor. This factor is determined with the formula:

$$\gamma = \frac{\log(2)}{HL}$$

where HL is the mineral price half-life in years. Price half-life measures the length of time required for a price shock of X per cent to dissipate by one half. For example, a price shock pushing a mineral spot price 20 per cent above the long-term equilibrium price would cause the three-year expected prices to be ten per cent above the equilibrium price if the mineral's half-life is three years. Metals that do not demonstrate price reversion, such as gold, can also be modelled with this process by setting the half-life of gold to 1 000 000 years. The stochastic process becomes a geometric Brownian process when this is done.

TABLE 1

Project model.

Year	2004	2005	2006	2007	2008	2009	2010	2011
Project time	0	1	2	3	4	5	6	7
Ore reserve (thousands t)				24 000	22 000	20 000	18 000	16 000
Production (thousands tpy)					2000	2000	2000	2000
Copper grade (%)					3.00	2.95	2.90	2.85
Gold grade (oz/t)					0.080	0.078	0.076	0.074
Canadian operating costs (CAD$M/t)					60.00	60.00	60.00	60.00
Canadian CAPEX costs (CAD$M)	50.00	70.00	110.00	90.00				
US operating costs (US$M/t)					10.00	10.00	10.00	10.00
US CAPEX costs (US$M)	10.00	20.00	30.00	10.00				
Total operating costs (US$/t)					53.80	53.80	53.80	53.80
Total CAPEX costs (US$M)	46.50	71.09	110.29	75.69				
Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
Project time	8	9	10	11	12	13	14	15
Ore reserve (thousands t)	14 000	12 000	10 000	8000	6000	4000	2000	0
Production (thousands tpy)	2000	2000	2000	2000	2000	2000	2000	2000
Copper grade (%)	2.80	2.75	2.70	2.65	2.60	2.55	2.50	2.45
Gold grade (oz/t)	0.072	0.070	0.068	0.066	0.064	0.062	0.060	0.058
Canadian operating costs (CAD$M/t)	60.00	60.00	60.00	60.00	60.00	60.00	60.00	60.00
Canadian CAPEX costs (CAD$M)								
US operating costs (US$M/t)	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
US CAPEX costs (US$M)								
Total operating costs (US$/t)	53.80	53.80	53.80	53.80	53.80	53.80	53.80	53.80
Total CAPEX costs (US$M)								

Exchange rate CAD$/US$: 1.37

DCF RADR (%): 10.0

Annual real risk free rate (%): 2.0

An important difference between conventional DCF and RO valuation methods is found in risk discounting. Conventional DCF applies a risk representing overall project or corporate risk to the project's net cash flow stream, while RO applies a risk adjustment to the source of uncertainty and then filters risk-adjusted uncertainty through to the net cash flow stream. This difference requires the previous stochastic process to be modified to include a risk-adjustment before it can be used for RO Monte Carlo value calculations. The modified process includes an additional variable *RiskRate* to produce the risk-adjusted stochastic process:

$$dS = \left[\alpha^* + \frac{1}{2}\sigma^2 - \gamma \ln\left(\frac{S}{S^*}\right) + RiskRate\right] Sdt + \sigma Sdz$$

The new parameter *RiskRate* is determined by the formulas:

$$RiskRate = \frac{\log(RDF_{Mineral})}{dt},$$

$$RDF_{Mineral} = \left(\exp - PRisk_{Mineral} \times \sigma \times \left(1 - \exp(-\gamma dt)\right) / \gamma\right),$$

$$PRisk_{Mineral} = PRisk_{Mkt} \times \rho_{Mkt,\,Mineral}$$

where:

$\rho_{Mkt,\,Mienral}$ is the correlation between mineral and the market

$PRisk_{Mkt}$ is the price of market risk

$PRisk_{Mineral}$ is the price of mineral risk

$RDF_{Mineral}$ is the risk discounting factor for the mineral

The stochastic price parameters used in this paper are detailed in Table 2. The stochastic process parameters used here have been retrieved from older published sources or selected to reflect personal understanding of metal price movements. They should not be used in an actual project valuation. It is recommended that a professional econometrician be retained to parameterise metal price processes for any valuation exercise. Note that the long-half life of gold converts the reverting process into a classical geometric Brownian stochastic process that is often used in uncertainty models of investment assets. Refer to Oksendal (1995) for an exhaustive discussion of stochastic differential equations.

ANALYSIS PROCEDURE

The copper-gold project is valued using the following three valuation models:

1. a conventional DCF NPV model with copper and gold expressed as copper equivalencies,
2. a RO NPV model with a risk-adjusted copper price stochastic process and gold output expressed as copper equivalent, and
3. RO NPV model incorporating copper and gold price stochastic processes that are risk adjusted and include a range of correlations.

An early closure option is also included in both RO calculations since managers have the ability to permanently exit a project when economic conditions turn unfavourable. This option is expressed in terms of metal prices and closure costs.

TABLE 2

Stochastic process parameters for copper and gold †.

Stochastic process parameters	Copper (Cu)	Gold (Au)
S	1.08 US$/lb	400.00 US$/oz
S^*	0.90 US$/lb	400.00 US$/oz
α^*	0%	-1.1%
σ	23.3%	15.0%
HL	1.875 years	1 000 000 years
$\rho_{Mkt,Mineral}$	0.8	0.1
$PRisk_{Mkt}$	0.4	0.4

† The stochastic process parameters used here have been retrieved from older published sources or selected to reflect personal understanding of metal price movements.

REAL OPTION VALUATION METHOD

Many tools are available for valuing both financial and real options but most of these are only appropriate in specific situations. Blais and Poulin (2004) provide a critical review of the various option valuation methods. They highlight that option pricing methods can be divided into four main classes: analytical solutions for European-type claims as proposed by Black and Scholes (1973), lattice methods first introduced by Cox, Ross and Rubinstein (1979), finite difference methods for solving partial differential equations (PDEs) such as used by Brennan and Schwartz (1985), and stochastic Monte Carlo simulations first presented by Boyle (1977).

The Monte Carlo method has traditionally been used to value options with one exercise decision point. Longstaff and Schwartz (2001) recently extended the Monte Carlo method so that options with multiple exercise decision points can also be valued. Their approach combines Monte Carlo simulation with least squares linear regressions to determine continuation value and the optimal stopping time, both necessary to apply an optimal exercise decision policy.

Stochastic Monte Carlo simulation should be the preferred numerical technique in the majority of practical valuations because it has the ability to calculate option values in multidimensional economical environment without constraint. An extended version of the simulation algorithm proposed by Longstaff and Schwartz (2001) is used in this paper because singularity problems are often encountered when using the original least squares linear regression. They also recognised this stability problem and suggested use of a quadratic algorithm (QR-algorithm) to perform the least squares regression. However, the QR-algorithm may still experience singularity problems even though the results tend to be more stable. The accuracy of the Longstaff and Schwartz algorithm may also be affected when it is extended to estimate solutions of overdetermined systems of linear equations via iterative-refinement algorithms and pseudoinverses, since the inverted matrix may almost be singular. A discussion about generalised inverses of linear transformations is available in Meyer and Campbell (1991).

The pseudoinverse extension used in this paper solves the singularity difficulties of the Longstaff and Schwartz approach and allows options with multiple exercise decision points to be valued.

RESULTS

The value estimates from the three NPV calculation approaches are presented and discussed in the next subsections. The estimates from the stochastic Monte Carlo simulations generate project mean NPV, standard deviation, and confidence intervals. Three thousand simulations of 5000 experiments each were completed for each valuation configuration. All values are stated in American dollars.

Conventional DCF valuation of NPV with copper uncertainty and gold output expressed as copper equivalent

A conventional DCF NPV calculation using a ten per cent RADR and treating gold output as copper equivalent estimated the project value to be $70.250 million with a standard deviation of $2.152 million. Table 3 presents the results of the value simulation.

TABLE 3

DCF valuation results (US$M).

Stochastic simulation results	Number of bath: 3000 Number of replication: 5000
DCF	70.250
σ_{DCF}	2.152
$\alpha_{10\%}$	(70.205, 70.306)

The mining industry predominantly uses conventional DCF techniques to estimate project value. This may lead some valuation analysts to prefer the DCF NPV as the correct estimate of value. However, there are strong reasons to reject this value as being misleading. First, net cash flow uncertainty varies with price scenarios since high price scenarios tend to produce net cash flows that are less uncertain than low price scenarios and varies with project structures since reserves with small profit margins (high unit operating costs) tend to generate net cash flows that are more uncertain than those with large profit margins. Second, the copper price is modelled as reverting towards a long-term equilibrium price that results in copper price uncertainty saturating in the long term. Uncertainty saturation leads to per period copper price uncertainty growing at decreasing rates each year until at some point in the future overall copper price uncertainty can be considered constant for valuation purposes. Finally, the early closure option allows management to limit downside price risk and fundamentally change the structure of net cash flow uncertainty.

The conventional DCF method assumes that project uncertainty grows at a constant rate when a constant RADR is used. Each of the previous reasons for rejecting the DCF value highlights that net cash flow uncertainty can vary tremendously over the life of the project due to changes in cost structure, price levels, uncertainty characteristics and operating strategy. A fundamental principle of valuation theory is that investors are concerned with net cash flow uncertainty and require compensation for being exposed to risk (ie risk adverse). This suggests that the assumption of net cash flow uncertainty growing at a constant rate is problematic since it violates the principle of investor risk aversion where the net cash flow uncertainty is not increasing at a constant rate.

Note that this limitation of the conventional DCF method has important implications for qualified person valuation reports. It may become necessary in the future for qualified persons to state why they accept this limitation of the DCF method value when they could use RO, which is able to easily recognise changes in net cash flow uncertainty within the value calculation.

Real option with copper price uncertainty and gold output expressed as copper equivalent

The second valuation model uses RO to estimates project value with a risk-adjusted copper price stochastic process and gold output converted into copper equivalents. When there is no early closure option, the project value is estimated to be $36.025 million with a standard deviation of $3.302 million. This indicates that the RO approach considers the project net cash flows to be much riskier than the ten per cent RADR used by the conventional DCF model.

The ability to limit downside risk with an early closure option increases project value by an estimated $51.315 million, producing a total estimated project value of $87.340 million. The extended Longstaff and Schwartz Monte Carlo (LSM) algorithm is used to perform the optimal control of sample paths during the calculation of project value. Table 4 presents the project values when there is no early closure option and when such an option available.

TABLE 4

Real option valuation results (US$M).

Stochastic simulation results	Number of bath: 3000 Number of replication: 5000
Real option without flexibility	36.025
$\sigma_{RONOFLEX}$	3.302
Value of the option to abandon	51.315
σ_{FLEX}	1.244
$\alpha_{10\%}$	(33.029, 36.102)

The additional value added by flexibility may seem surprising to some. However, a study conducted by Kester (1982) shows the value of flexibility associated to real options is sometimes worth more than half the value of large firms. Even though his study bears only on large market capitalisations, the same characteristics also exist for small firms whose potential growth represents an even more important part of their value.

This analysis is an improvement over the conventional DCF model because it recognises variation in net cash flow uncertainty and the ability to close the project early in response to low metal prices. However, it misses the importance of describing the dynamics of gold price uncertainty with a separate price process. Gold prices have a low correlation with general market uncertainty and as such do not require a risk-adjustment as large as that applied to the copper price. Some gold mining companies use this argument to justify use of a small RADR to calculate conventional DCF project values.

Real option with copper and gold prices generated by independent stochastic price processes

The third valuation model of the copper-gold project goes one step further and uses risk-adjusted stochastic price processes to describe both metal price movements independently so that their unique risk characteristics are captured. In this model, the gold revenues are calculated and risk-adjusted with a stochastic process that reflects the uncertainty and risk characteristics of gold and not copper. The gold risk-adjustment is much smaller than the copper risk-adjustment because gold is modelled to have a much lower correlation (a correlation coefficient of 0.1) with general economic uncertainty than does the copper price (a correlation coefficient of 0.8). The estimated project NPV increases to $92.106 million (a 255 per cent increase) when there is no early closure option and gold price is modelled as a separate price process. This increase in value over estimated NPV from the previous RO model ($36.025 million) is solely due to the recognising the differences between gold and copper risk characteristics.

The estimated project NPV when there is an early closure option increases to $132.272 million. The value of the closure option decreases to $40.166 million. This is the logical consequence of it being less likely that the early closure option will be exercised given that the underlying project NPV is higher. Table 5 presents the project NPVs when metal prices are each modelled by a separate stochastic process.

The results presented to this point demonstrate that the use of metal equivalents with either the conventional DCF method or the RO method over simplifies the unique risk characteristics of each metal. The widespread mining industry practice of expressing secondary metals in terms of primary metal equivalents can lead to large valuation errors.

The results also highlight that ignoring operational flexibility such as early closure can generate misleading project NPV estimates. Managers often have the ability to manage project risk with operational strategies and this ability can have significantly effect on project NPV. An extensive review of managerial real options is provided in Trigeorgis (1996).

TABLE 5

Real option results for copper and gold (US$M).

Stochastic simulation results	Number of bath : 3000 Number of replication : 5000
Real option without flexibility	92.106
$\sigma_{RONOFLEX}$	3.722
Value of the option to abandon	40.166
σ_{FLEX}	1.081
$\alpha_{10\%}$	(92.019, 92.193)

CORRELATED MINERAL PRICES

Copper and gold price were assumed to be independent in the previous section when in reality they may exhibit some correlation. Valuation simulations assuming price independence may produce misleading estimates if there is some degree of correlation between prices. Metal price correlations can be introduced into the valuation model by conducting an Eigenvalue decomposition on the price correlation matrix. An Eigenvalue decomposition is used even though it is more difficult to implement than a Cholesky decomposition because it can handle matrices that are not positive definite and that contain hundreds of variables. The Cholesky decomposition can barely handle more than ten. A short note outlining how to generate correlated random variables is provided in this section due to the importance of this concept.

The first step is to analyse time series data of all state variables or sources of uncertainty to determine the correlations between them. Regression packages can do this work easily once a data file to extracted numbers from has been constructed. Correlation coefficients are required in a matrix form.

The procedure to generate correlated random variables begins with the decomposition of the correlation matrix into matrices Λ and E such that:

$$C = E^T \Lambda E$$

where:

C is the correlation matrix,

$$C = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \rho_{23} & & \rho_{2N} \\ \rho_{31} & \rho_{32} & 1 & & \rho_{3N} \\ \vdots & & & \ddots & \vdots \\ \rho_{N2} & \rho_{N2} & & \cdots & 1 \end{bmatrix}$$

The coefficients ρ_{12} and ρ_{21} have the same value and represent the correlation coefficient between the first and second state variable. Λ is a diagonal matrix whose entries are the eigenvalues:

$$\Lambda = \begin{matrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_N \end{matrix}$$

The matrix E when multiplied by its transpose E^T produces the identity matrix.

The correlation matrix can be re-written with these results as follows:

$$C = B^T B$$

where the matrix B is be obtained in the following manner:

$$B = \Lambda^{1/2} E$$

The matrix B is multiplied by a vector of standard normal random variables to generate correlated random variables, t, such that:

$$F = \bar{Z} B$$

In the general case of N underlying assets, the transformation of independent standard normal random variables in correlated random variables can be accomplished with the linear algebra operation:

$$\left[\delta f_1 \ \delta f_2 \ldots \delta f_N\right] = \left[Z_1 \ Z_2 \ldots Z_N\right] \begin{bmatrix} e_{11}\sqrt{\lambda_1} & \cdots\cdots & e_{1N}\sqrt{\lambda_1} \\ e_{21}\sqrt{\lambda_2} & \cdots\cdots & e_{2N}\sqrt{\lambda_2} \\ \vdots & \ddots & \vdots \\ e_{N1}\sqrt{\lambda_N} & \cdots\cdots & e_{NN}\sqrt{\lambda_N} \end{bmatrix}$$

The previous step is repeated as many times as there are time steps in the simulation, so as to generate a sample path characterising each state variable. For instance, the project time horizon of 15 years has been divided into 150 time steps or ten time steps per year. The previous exercise is performed as many times as the required number of replications. 5000 replications are used to value the project valued in this paper. The generation of random variables having the desired correlation properties is possible using appropriate coding software such as MatLab.

Impact of correlated mineral prices on project value

Time series of copper and gold prices (see Figures 1 and 2) has been analysed from 1998 to 2004 to estimate the correlation coefficient for these metals. A correlation coefficient of 0.5107 was calculated, suggesting that stochastic Monte Carlo simulations should be performed with correlated diffusion processes. The econometric calculations conducted here are likely not appropriate and require more extensive validation by a professional econometrician before being used in an actual mining project valuation.

A sensitivity analysis has been conducted to verify the impact of the correlation coefficient on the value of early closure and on RO project NPVs. Estimated RO NPV when there is no early closure option is graphed in Figure 3 for correlation coefficients ranging from zero to one. The estimated RO NPV increases from \$92 million when the correlation coefficient is zero to \$123 million when the coefficient is one.

The value of the early closure option is plotted against correlation coefficient in Figure 4 for correlation coefficients ranging from zero to one. This figure shows that option value decreases from \$40 million when the coefficient is zero to approximately \$28 million when the coefficient is one.

The net effect on estimated project NPV is an increase in value from \$132 million to approximately \$151 million as the correlation coefficient rises from zero to one.

CONCLUSION

An extension to the RO Monte Carlo simulation algorithm introduced by Longstaff and Schwartz (2001) was presented to assist mine valuation professionals improve their assessment of project value. The use of the Longstaff and Schwartz algorithm permitted a reassessment of the industry practice of using metal equivalents to combine primary and secondary revenue streams in a project valuation. This paper demonstrated that converting secondary metal output into primary metal equivalents may lead to large valuation errors, as this practice ignores the unique uncertainty and risk characteristics of the secondary mineral. The results also showed that the correlation between primary and secondary metal prices can also have important value effects.

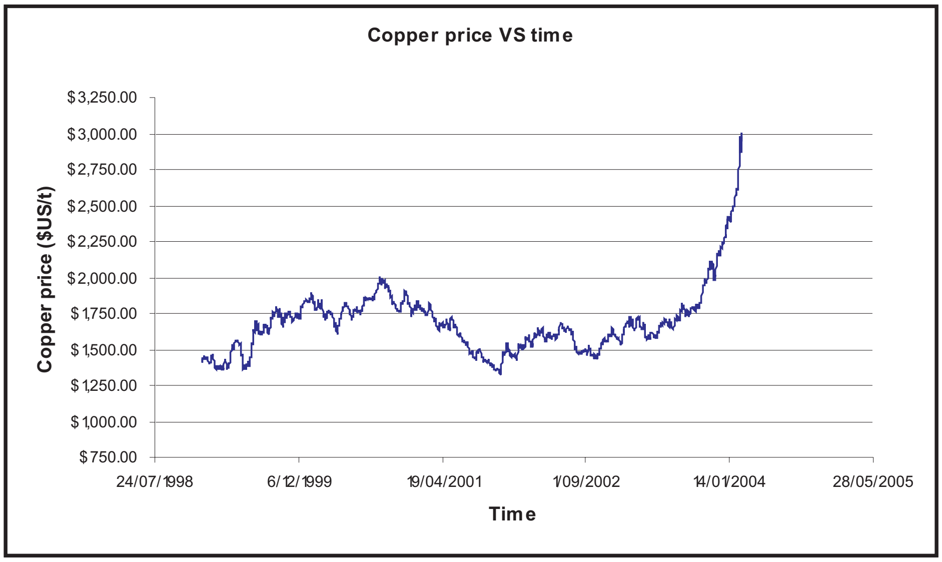


FIG 1 - Copper price time series.

FIG 2 - Gold price time series.

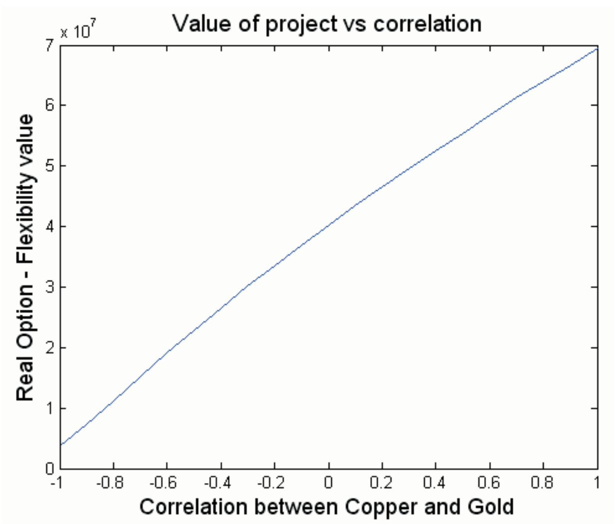


FIG 3 - Flexibility value versus gold and copper correlation.

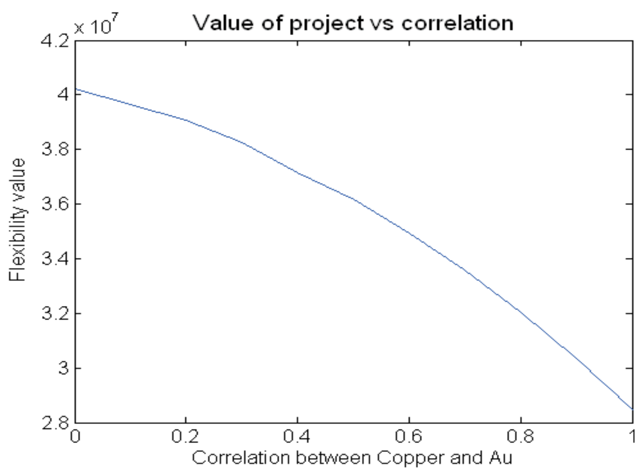


FIG 4 - Flexibility value versus gold and copper correlation.

The model presented here can be extended to include other forms of management flexibility and uncertainty. One such uncertainty is surely foreign exchange rate risk, since mineral deposits cannot be shifted when exchange rates move adversely. Movements in foreign exchange rates can only be managed in the medium to long term with operating strategies such as temporary closure of marginal reserves. The adaptation of the Longstaff and Schwartz RO Monte Carlo algorithm presented in this paper can incorporate the myriad of uncertainties and operating flexibilities that are part of any mining project.

REFERENCES

Black, F and Scholes, M, 1973. The pricing of options and corporate liabilities, *Journal of Political Economy*, 81:637-659.

Blais, V and Poulin, R, 2004. Stochastic pricing of real options, paper presented to 13th Annual Conference of the Mineral Economics and Management Society, Toronto, 21-23 April.

Boyle, P P, 1977. Options: a Monte Carlo approach, *Journal of Financial Economics*, 4:323-338.

Brennan, M J and Schwartz, E, 1985. Evaluating natural resource investments, *Journal of Business*, 58(2):135-157.

Cox, J C, Ross, A S and Rubenstein, M, 1979. Option pricing: a simplified approach, *Journal of Financial Economics*, 7:229-263.

Dimitrakopoulos, R and Abdel Sabour, S A, 2007. Evaluating mine plans under uncertainty: Can the real options make a difference? *Resources Policy*, 40:32-44.

Dixit, A and Pindyck, R, 1994. *Investment Under Uncertainty*, 468 p (Princeton University Press: New Jersey).

Kester, W C, 1982. Evaluating growth options: a new approach to strategic capital budgeting, Working paper, 45 p, Harvard Business School.

Laughton, D and Jacoby, H, 1993. Reversion, timing options and long-term decision-making, *Financial Management*, 22(3):225-240.

Longstaff, F A and Schwartz, E S, 2001. Valuing American options by simulation: a simple least-squares approach, *The Review of Financial Studies*, 14(1):113-147.

Meyer, C D and Campbell, S L, 1991. *Generalized Inverses of Linear Transformations*, 288 p (Dover Publications: Mineola).

Oksendal, B, 1995. *Stochastic Differential Equations*, 326 p (Springer: London).

Salahor, G, 1998. Implications of output price risk and operating leverage for the evaluation of petroleum development projects, *Energy Journal,* 19(1):13-46.

Samis, M, 2000. Multi-zone mine valuation using modern asset pricing (real options) techniques, unpublished PhD dissertation, Department of Mining and Mineral Process Engineering, University of British Columbia.

Samis, M, Laughton, D and Poulin, R, 2003. Risk discounting: the fundamental difference between the real option and discounted cash flow project valuation methods, KMC working paper, 23 p [online]. Available from <http://papers.ssrn.com> [Accessed: 7 May 2007].

Trigeorgis, L, 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, 427 p (MIT Press: Cambridge).

Roadblocks to the Evaluation of Ore Reserves — The Simulation Overpass and Putting More Geology into Numerical Models of Deposits

A G Journel[1]

INTRODUCTION

Many factors including data scarcity, volume support effects, information effect, accessibility and pervasive uncertainty, make the early prediction of recoverable reserves a challenge that cannot be addressed by mere estimation or interpolation algorithms. There is the illusion that as long as one uses the 'best' estimation algorithm based on quality data and sound geological interpretation, one would provide the best possible evaluation. Unfortunately, a set of locally accurate ('as best as they can be') estimated values does not generally make for a good, or even an unbiased base on which to assess future recoverable reserves. The dichotomy between local accuracy and global representation is at the source of many arguments and severe prediction errors. A discussion on the various factors affecting the reliability of reserves prediction may help in focusing efforts on what matters, marking common pitfalls, then stress what must be done, such as building into the deposit numerical models geological interpretation beyond mere variogram models. It is suggested that the essential components of a mining operation could be simulated from such numerical models, like the performance of the wings of a future plane is simulated in a wind tunnel.

LOCAL VERSUS GLOBAL ACCURACY

The illusion that a sound estimation algorithm suffices for ore reserves evaluation comes from the lack of understanding of the trade-offs involved when defining the goodness criterion of any estimator. No estimation algorithm, unless trivially based on exhaustive accurate data, can be good for all purposes. Most estimation algorithms, and kriging is no exception, aim at local accuracy, that is providing an estimate $z^*(\mathbf{u}_i)$ as close as possible to the true and unknown value $z(\mathbf{u}_i)$, irrespective of its relation with any other estimated value $z^*(\mathbf{u}_j)$, $j \neq i$. The attribute z could be any variable, say the mineral content of a given volume centred at a location of coordinates vector $\mathbf{u}_i$. Local accuracy would suffice if the estimation was so good as to allow the approximation: $z^*(\mathbf{u}_i) \approx z(\mathbf{u}_i)$ and $z^*(\mathbf{u}_j) \approx z(\mathbf{u}_j)$, in which case the pair of estimated values $\{z^*(\mathbf{u}_i), z^*(\mathbf{u}_j)\}$ would reflect the continuity in space of the true values $\{z(\mathbf{u}_i), z(\mathbf{u}_j)\}$. Or, more generally, the estimated map would reflect accurately the true patterns of spatial continuity. Unfortunately, the data available at the time of mine planning and reserves prediction are never sufficient to assume that the map of estimated values accurately reflects the spatial variance of the true values. This is the well known smoothing effect of estimation, a smoothing effect made worse by being non-stationary. This effect is minimal next to the data locations, maximal away from the data and may create patterns that are artefacts of the drill hole locations. An example of a potentially misleading effect on mine planning of otherwise locally accurate orebody models is shown in Dimitrakopoulos, Farrelly and Godoy (2002).

What makes a mine feasible is not only the tonnage of potential payable ore but also how that potential is distributed in space, allowing economical recovery. Hence, a correct assessment of the actual spatial distribution of grades and relevant morphological properties of the deposit is critical, more critical than local accuracy. Local accuracy is critical only at the time of selection, when the mine is already operating. In addition, that selection is typically performed from different data not available at the time of reserves prediction.

Thus, for recoverable reserves estimation, one should trade, or at least balance, the local accuracy criterion for a criterion ensuring accurate depiction of the patterns of heterogeneities prevailing over the actual study area, whether that area is the entire deposit, a bench or a mining panel, within which selective mining will take place. In geostatistics, the traditional measure of spatial variability is the variogram model $\gamma(\mathbf{u}_i\text{-}\mathbf{u}_j)$. Thus, we should require that the estimated values reproduce that model; the qualifier 'simulated' is then used instead of 'estimated'. In advanced geostatistics, we aim at reproducing patterns of heterogeneities involving multiple locations at a time, as opposed to reproducing a mere variogram, the latter being but a two-point $(\mathbf{u}_i,\mathbf{u}_j)$ statistic. The name multiple-point (mp) geostatistics is therefore given to that advance, see Appendix and Strebelle (2002).

Stochastic simulation trades poorer local accuracy for a better global or 'structural' accuracy as defined by a prior model of spatial variability, whether that model is limited to a histogram plus a variogram as in traditional geostatistics, or that model is given as a training image as in mp geostatistics. In the presence of limited data, it is suggested to forfeit any attempt at locating precisely each ore block or Selective Mining Unit (SMU). Instead, one should aim at providing a spatial representation of the grades distribution that mimics the spatial patterns of the true grades, those patterns that may affect the mine plan and recovery. Since stochastic simulation trades off local accuracy, any one of the simulated patterns is likely, though probably not at its true location. Hence simulation should provide many alternative representations or realisations of that spatial distribution, all consistent with the few local data available. No result taken from any single simulated realisation should be used as a local estimate. By definition, results should be collected from multiple simulated realisations, that is, a distribution of results should be provided. *A single simulated realisation should not be used,* in lieu of say a kriging map, for any local decision; yet a set of simulated realisations could replace that kriging map for such a local decision, which then leads to a probabilistic decision (Srivastava, 1987).

Although it is unreasonable, from sparse data, to try locating and hence estimating any single recoverable SMU, estimation of large panels or homogeneous zones can be attempted because one could capitalise on the averaging of errors over large volumes. However, within-panel or within-zone recovery should be approached through simulation of the spatial patterns of grades distribution within each panel or zone. No localisation of the within-panel recovery is yet possible, nor is such detail needed for mine planning.

1. Geological and Environmental Earth Sciences Department, Stanford University, Stanford CA 94305, USA.
Email: journel@pangea.Stanford.edu

DATA SCARCITY

In a simulation approach data are needed for two purposes:

1. delineation of homogeneous mineralisation zones, each defined such that its grade distribution could be characterised by a stationary model, typically limited to a histogram and variogram, or better by a training image that includes the two previous statistics; and
2. rough localisation of ore patches within the previous zones.

The data required for the first purpose does not need to all come from drilling; they can be structural and interpretative in nature. The delineation of homogeneous zones is typically guided from geological interpretation, possibly borrowing structural information from outcrops or similar formations mined elsewhere. In modern geostatistics, multiple-point statistics can include information beyond the variogram by borrowing from geological drawings (training images), the patterns of grade variability deemed to prevail in the actual deposit. In the presence of uncertainty about the style of variability, alternative training images can be considered, each leading to a possibly different recovery of the same global tonnage. This is tantamount to varying the variogram model.

Each simulated realisation is then anchored to whatever local data are available. However, here a shortage of data is less consequential because no local accuracy is required, nor should any single simulated result be used as a local estimate.

THE VOLUME SUPPORT EFFECT

Future mining selection will operate on selective mining units whose geometry and volume support may vary considerably. The volumes are typically beyond the resolution of the data available at the time of mine planning and reserves estimation. Within each large homogeneous zone, a histogram of SMU grades is needed to evaluate the proportion of such SMUs that could be recovered as ore. However, that histogram cannot be built from estimated SMU grades because of the smoothing effect of estimation. The solution is not to attempt an awkward analytical correction of the histogram of estimated values, but to simulate the grade distribution at the quasi-point support volume of the data composite used. These simulated point values can then be averaged into simulated grades for SMUs of possibly different sizes, then the selection process can be simulated on the spatial distribution and histogram of the simulated SMU grades. Sensitivity of ore recovery to SMU size and more generally to the mine selection process can then be easily performed. The utilisation of a common quasi-point support realisation ensures consistency of all results, no matter which SMU size is chosen.

THE INFORMATION EFFECT

Possibly the most important contribution of the simulation approach is the assessment of the impact of misclassification on recovery. No present estimation-based geostatistical approach, whether by indicator kriging or uniform conditioning, offers that flexibility. Selective mining calls for small SMUs of varying support volumes, far below the resolution of the data available at the time of mine planning. Indeed, SMUs will be sorted on their ultimate estimated values based on future data not yet available, but it is the corresponding true grades that are sent to the mill and contribute to actual recovery. Misclassification is an unavoidable and often critical aspect of any selective mining; its rigorous evaluation cannot be ducked.

One can simulate the future selection data, for example blasthole data, together with the SMU grades $z_v^{(s)}$ from the point-support simulated grade realisation $z^{(s)}$. The simulated blasthole data are then combined into 'simulated future' SMU estimated values $z_v^{(s)*}$. The superscript (s) stands for simulated, a star * is added for estimated, and subscript v represents the SMU support volume. Availability of the simulated pairs $\{z_v^{(s)}(\mathbf{u}), z_v^{(s)*}(\mathbf{u})\}$, true SMU grade and selection estimate, at any location $\mathbf{u}$, allows an assessment of the impact of misclassification. Again, sensitivity analysis to various aspects of that information effect can be easily performed, say the type and density of the future data available for ore/waste selection, the geometry of the mine dig lines, etc. Consistency of the various results is ensured by the common quasi-point support of any one of the simulated base realisations.

A lot of heat in the debate about the cause and remediation for 'conditional bias' would be reduced if the information effect was better understood. Any set of estimates, kriging being no exception, is conditionally biased if used to predict a recovery that is performed on another set of estimates. What is needed is the joint distribution of the actual selection estimates versus the true values, these are yet unknown but can be simulated and were previously denoted as $\{z_v^{(s)}(\mathbf{u}), z_v^{(s)*}(\mathbf{u})\}$. Improving the kriging procedure, say by culling some data or increasing the search neighbourhood, or designing yet another estimator, say through indicator or disjunctive kriging, would not solve the problem.

ACCESSIBILITY

There is rarely, if ever, free selection: the economic worth of a block *in situ* depends not only on its metal content but also on the cost of accessing it and then mining it, the total cost involved being shared with other neighbouring blocks. The decision to mine a block as ore or waste depends on the mine plan, which itself depends on the estimated grades at the time of selection. Estimation of recoverable reserves and mine planning are closely related endeavours that call for a difficult optimisation problem.

Unfortunately, with some notable exceptions (Godoy and Dimitrakopoulos, 2004), that optimisation is rarely fully addressed. Instead, and too often, mine plan and design are based on rough, large-scale estimates of grade distributions, with little or no account for the impact of the smoothing effect and future misclassification. Fortunately, such large-scale estimates are not significantly affected by the smoothing effect if based on sound prior geological zoning. As for the impact of future misclassification, it usually is dealt with through dilution factors.

I suggest that simulated realisations of both the distributions of mineral zones and their mineral grades could provide the data bases necessary for testing and fine-tuning alternative mining scenarios, accounting for the all-important support and information effect. There will come a time when mine planning will reach the level of rigour and scientific repeatability of the design of a new aircraft. At that time, simulated numerical models of the distribution of grades and rock properties will be needed, and once again global or structural accuracy of the model will prevail over its local accuracy; that is, stochastic simulation will prevail over estimation.

UNCERTAINTY ASSESSMENT

Evaluation of recovered reserves from early development data, not the data used for actual selection, is an extremely challenging task fraught with uncertainty at each step. Not only should it be ensured that all known biases are avoided, but a final assessment of uncertainty about the reserves figures should be provided. It is clear that such uncertainty assessment is beyond any estimation or combined kriging variance, because:

1. Kriging variances are independent of the data values; they are no different whether the SMU is selected as ore or sent to the waste.
2. A variance does not suffice to characterise a distribution unless an arbitrary, and here inappropriate, Gaussian-related distribution is assumed. Simulation approaches can, however, provide this uncertainty assessment.

CONCLUSION

There is no practical alternative to a simulation approach if critical biases are to be avoided and if the uncertainty about global reserves figures is to be assessed. The paradigm is simple, but its application is difficult. One generates alternative data sets, called simulated realisations, on which the process of imperfect selection is simulated. Provided that the simulated realisations mimic reasonably those traits of the actual grade distribution that most affect the recovery of reserves, and provided that the simulation of the future selection process and its related misclassification is possible, a probabilistic distribution (histogram) for the simulated recovery numbers can be obtained, thus providing a model of uncertainty and confidence intervals. Note that for both simulation processes (geology and mining) various scenarios can and should be considered. Given an early and sparse data set, there can be alternative geological scenarios/interpretations and many alternative options for the mining plan.

All previous provisos render the simulation approach extremely demanding, but correspondingly rewarding, an endeavour that befits the critical importance of reserves assessment.

ACKNOWLEDGEMENTS

This paper borrows from a recent book co-authored by this author, Journel and Kyriakidis (2004). Thanks to Roussos Dimitrakopoulos who has been persistent in calling me back to my mining geostatistics roots.

REFERENCES

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions,* 316:43-50.

Journel, A G and Kyriakidis, P C, 2004. *Evaluation of Mineral Reserves: A Simulation Approach*, p 216 (Oxford Press: New York).

Remy, N, 2004. S-GeMS – Geostatistical earth modeling software: User's Manual, Stanford University, 87 p [online]. Available from: <http://sgems.sourceforge.net> [Accessed: 7 May 2007].

Srivastava, R M, 1987. Minimum variance or maximum profitability, *CIM Bulletin*, 80(901):63-68.

Strebelle, S, 2002. Conditional simulation of complex geological structures using multiple-point statistics, *Mathematical Geology*, 34(1):1-22.

APPENDIX: PUTTING MORE GEOLOGY INTO NUMERICAL MODELS OF DEPOSITS

Most reserves evaluation and mine planning start with a numerical model of the spatial distribution of the deposit mineral zones. Yet no model is better than the algorithm from which it is built, the algorithm that relates the data to the unknowns. Should the estimation or simulation process include explicitly additional structural information indicated but not included in the data? We suggest that it is that additional information, beyond the actual drill hole data, which determines the quality of a mine model, and hence of its reserves forecasts. Local data, particularly when sparse in an early development stage, are less consequential than the structural/geological information used to tie them to the unsampled locations.

Research in mineral deposit modelling should focus on developing algorithms capable of including more geology in the numerical models. Ignoring prior geological interpretation on grounds that it is uncertain or too subjective is not only counterproductive, it is also conceptually wrong. Better an inaccurate geology than an automatic interpolation algorithm, whether geostatistical or not, that replaces all geology by its own canned universal structure, one that is most often maximum entropy forbidding geological organisation. Accordingly, the major source of uncertainty is the geological interpretation.

Recent developments on multiple-point geostatistics have adopted that route (Strebelle, 2002; Remy, 2004), replacing the two-point variogram by pattern statistics lifted directly from

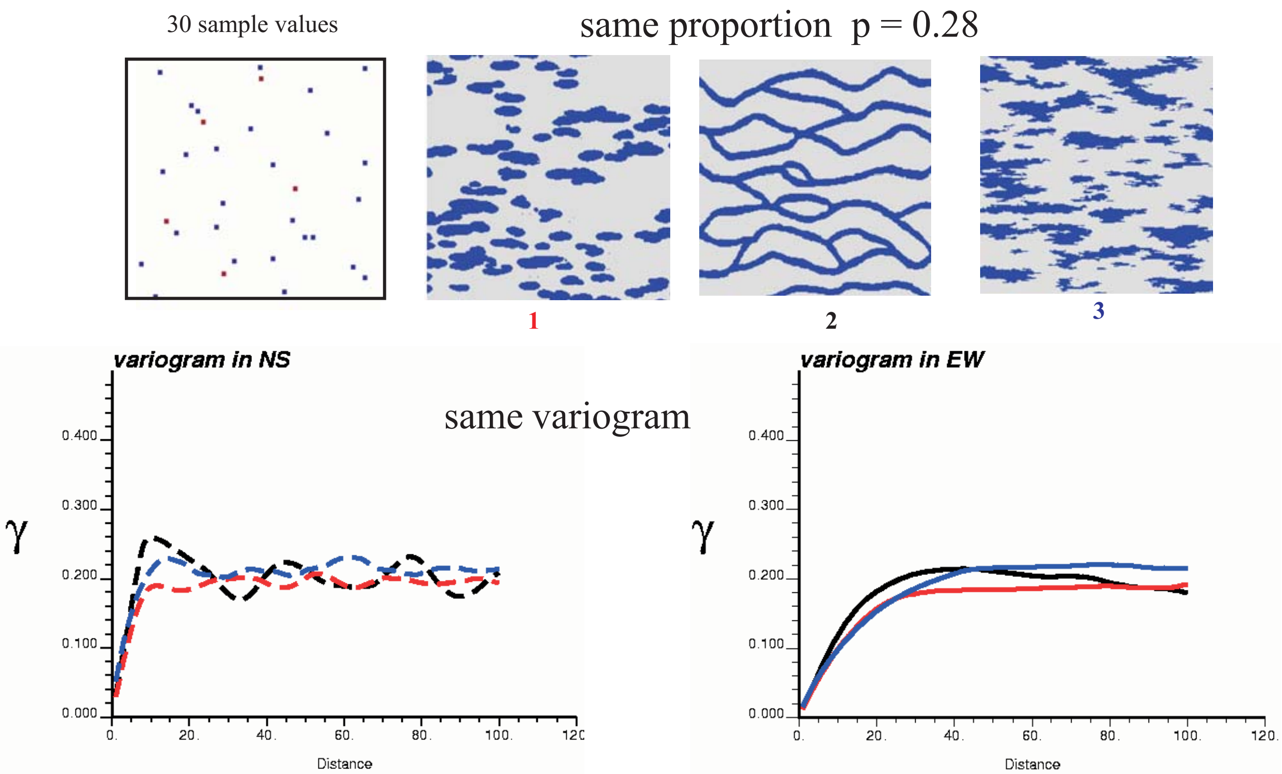


FIG 1 - Widely different patterns, same statistics up to order two.

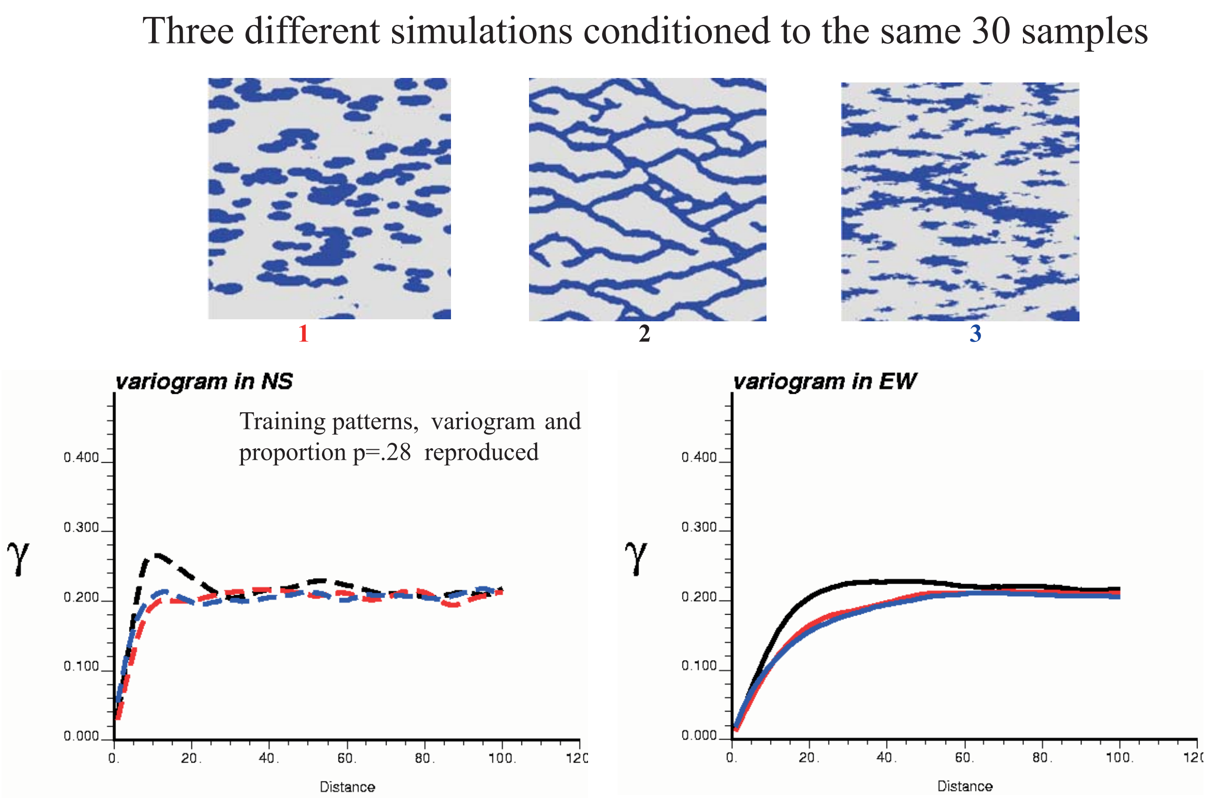


FIG 2 - Simulated realisations, using for training images the patterns of Figure 1.

prior training images proposed by geologists to represent their prior concept about facies or rock type geometry and spatial distribution.

These conceptual geometrical patterns are morphed and anchored to the actual local data. Only when the architecture of the deposit has been built on sound geological considerations can grade interpolation or simulation be performed using the traditional variogram-based algorithms.

An eye opener example

Figure 1 gives images of three different binary facies distributions, say the dark grey facies represent the high-grade mineralisation. The three images are conditioned to the same 30 sample data shown at the left of the figure. Although the three facies distributions are clearly different leading possibly to different mining dilution hence recovery, their exhaustive (indicator) variograms in both EW and NS directions are about the same. Had those variograms been calculated from the 30 sample data they would be all identical! The point made is that a variogram, and more generally two-point statistics, does not suffice to characterise complex spatial patterns.

These three images are now used as training images for conditional simulation with an algorithm based on multiple-point (mp) statistics; conditioning is to the same 30 samples. The results are shown at the top of Figure 2: mp simulation has succeeded to distinguish the three types of spatial patterns; as for variogram reproduction (bottom of Figure 2) it is as good as, or better than, what would be provided by any traditional variogram-based simulation algorithm. In mp geostatistics, the variogram structural function is replaced by multiple-point spatial patterns lifted from a training image and anchored to the hard conditioning data. The challenge for the geologist is to provide such training images corresponding to their geological interpretation of the data available; alternative geological scenarios could and should be considered. This challenge is no different from that of inferring a variogram model.

Quantification of Risk Using Simulation of the Chain of Mining — Case Study at Escondida Copper, Chile

S Khosrowshahi[1], W J Shaw[2] and G A Yeates[3]

ABSTRACT

Quantification of risk is important to the management team of any rapidly expanding mining operation. Examples of areas of concern are the likelihood of not achieving project targets, the impact of a planned drilling program on uncertainty and the change in the risk profile due to a change in the mining sequence. Recent advances in conditional simulation and the practical use of such models have provided the opportunity to more fully characterise mineral deposits and to develop empirical estimates of the recoverable resources and ore reserves. This allows meaningful quantification of risk (and upside potential) associated with various components of a mining project.

This paper presents an approach referred to herein as 'simulation of the chain of mining' to model the grade control and mining process. Future grade control sampling, mining selectivity and other issues that impact on the final recoverable tonnes and grades are incorporated. The application of this approach to Escondida, a large-scale open pit copper mining operation in Chile, provided a definitive way to assess the expected risk of a number of alternative development strategies on operational performance of the project. This approach is gaining acceptance as one of the most important steps in developing short-term mining models. The concepts developed here also have implications for assessing the ore that will be recovered from ore reserves during mining.

INTRODUCTION

The Escondida open pit copper mine is located 140 km south east of Antofagasta, Chile. The mine started production in the late 1990s and by 2004 the annual production reached 82.4 Mton of sulfide ore; generating 1 005 200 ton of copper concentrate, 152 300 ton of cathode copper, 179 800 oz of gold and 4.5 Moz of silver. The orebody is a porphyry copper formed by two major stages of sulfide and one stage of oxide mineralisation. The supergene enrichment blanket of the deposit is defined by chalcocite and minor covellite with remnant chalcopyrite and pyrite that reaches a thickness of several hundred metres in places. The largest contributor of mineralised tonnage in the deposit is an Oligocene porphyritic intrusive hosted by andesites, combined with less significant hydrothermal and igneous breccias occurring throughout the deposit.

This study was conducted to assess the risk associated with the use of the Escondida resource model as a basis for developing mine schedules, forecasts and budgets of mineable ore. In addition, it was used to define the impact of risk for the first five years of the Phase IV Expansion and identify the alternative mine schedules that present less risk. The study was based on the construction of a large conditional simulation model, covering a significant part of the Escondida copper mine and the analysis of this model through a 'transfer function' or mining process termed the Chain of Mining (CoM).

More specifically, a geostatistical conditional simulation (CS) model was developed for a large part of the Escondida sulfide resource that contained five years of scheduled mining from the start of year one to the end of year five. The CS model consisted of 50 realisations that independently defined the lithology (andesite or non-andesite), the mineralisation zones (High Enrichment, Low Enrichment and Primary) and the grade (per cent copper as total copper and soluble copper) dependent on the previous two geological variables. A Chain of Mining approach was then used to model the errors impacting upon the translation of the *in situ* resource to a recoverable ore reserve. A number of CoM models were developed and analysed to determine the parameters that would match actual mining performance at Escondida. The impact of various contributing errors was modelled using parameters for blasting movement, sampling and assaying precision, sampling and assaying bias, and mining selectivity. The CoM models were examined in relation to all available reconciliation results. From available production data it was evident that the Escondida resource model available at that time significantly over-predicted the tonnage that was realised during mining. A base case Chain of Mining model was selected that appeared to best capture the real performance indicated by the production data. This case was used to predict the performance of the current mining practice within the volume defined by the planned next five years of mining. The analysis was done on a quarterly basis and a pushback basis for two alternative (north and east) mining options.

The approach presented herein is based on sequential conditional simulation (eg Journel and Huijbregts, 1978; Goovaerts, 1997; Benndorf and Dimitrakopoulos, 2007, this volume; Nowak and Verly, 2007, this volume) and the concept of 'future' grade control data for recoverable reserve estimation detailed in Journel and Kyriakidis (2004). Related aspects are discussed in the next sections, which start with the description of available data and conditional simulation modelling at Escondida, followed by the CoM approach (Shaw and Khosrowshahi, 2002), a calibration of the resulting models and a comparison with production. Conclusions and comments follow.

DATA AND DATA ANALYSIS

Data sets used for analysis were based on 15 m bench composites for exploration data and grade control data. Subsequent analysis was based on the High Enrichment (HE), Low Enrichment (LE) and Primary (PR) zones. The lithology was considered as two domains, Non-Andesite porphyry/breccia and Andesite. Thus, there were six modelling domains for preliminary analysis, including univariate statistics of exploration and grade control data for total copper (CuT) and soluble copper (CuS).

To assess continuity trends for the characterisation of anisotropies in the data prior to variography, maps of grade and grade indicators were constructed. The interpolated maps were not constrained by the lithology or mineralogical zones and, therefore, reflect an isotropic interpolation of the data in 3D. The maps were used for the preliminary identification of grade continuity trends in order to further the definition of domains and for variographic analysis. The plan view maps indicated different grade continuity trends on either side of the north-south line at 16 300 E. On the eastern side, grade continuity has a NE orientation. This differs from the western side, which shows a NW continuity. An indicator defining the samples coded as andesite or non-andesite was also mapped in the same way.

1. MAusIMM(CP), Principal Consultant, Golder Associates, Level 2, 1 Havelock Street, West Perth WA 6005, Australia. Email: sia@golder.com.au

2. FAusIMM(CP), Principal Ore Evaluation Services, Golder Associates Pty Ltd, Level 2, 1 Havelock Street, West Perth WA 6005, Australia. Email: wshaw@golder.com.au

3. FAusIMM(CP), Global Manager – Mineral Resource Development, Business Excellence, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: gavin.yeates@bhpbilliton.com

Variography of the exploration and grade control data sets for total copper and for the ratio of soluble copper to total copper (ratio) was carried out for each of the HE, LE and PR mineralogical zones with subdivision by lithology (andesite and non-andesite, ie porphyry) that was separated into east and west at 16 300 E. Preliminary variograms of exploration data did not provide a good definition of short-scale structures. This is mainly due to the exploration data density, which does not allow accurate and detailed variogram definition over small distances. The exploration variograms generally characterised large-scale structures, but these are not as critical to risk assessment as the characterisation of short-scale continuity. It was found that variograms of grade control data generally showed less continuous behaviour, and a far clearer definition of short-scale variability. Accordingly, it was decided to model variograms of grade control data for all domains containing sufficient data to characterise this short-scale variability for simulation purposes. Exploration variograms were also modelled to determine the sensitivity of the study to this approach. For the Primary zone, grade control data was scarce and the variograms were based on exploration data, although this generally produced poorly defined variograms for the west domains.

The enrichment surfaces were based on the HE, LE and PR codes in the exploration and grade control data (Figure 1). For this analysis, it was considered necessary to use a combined grade control and exploration hole surface data set for each of HE, LE and PR for variographic purposes to ensure that maximum coverage was provided of the spatial data.

GENERATION OF THE CONDITIONAL SIMULATION MODELS

First, the enrichment surfaces were simulated using sequential Gaussian simulation, followed by the simulation of the two lithologies, andesite and non-andesite, using sequential indicator simulation. These models were merged resulting in simulated models, each with its own lithology and enrichment surface. Next, these models were populated with simulated CuT and CuS grades.

Simulation of the enrichment surfaces

An example of the final simulated enrichment surfaces are provided in Figure 2. The influence of the conditioning data is evident when comparing the simulated images of the HE, LE and PR surfaces. The lower number of conditioning data points for the PR surface leads to greater variability in the simulated surface. Variography was carried out for the mineralogical contacts described by the geological interpretation (enrichment surfaces). Variography of the surfaces was performed in 2D (Figure 3) with the variable analysed being the RL coordinate.

Simulation of lithological data

The dominant rock type for the Escondida deposit is porphyry. Grades in the andesite west of the 16 300 coordinate line are generally recognised to be lower than those in the porphyry lithologies and metallurgical recoveries are lower. The data was examined and it was decided, for the purpose of this study, to define two lithologies, namely andesite and non-andesite (or porphyry), which is used for porphyry/breccia and all other non-andesite lithologies. The lithology variography was based on indicators for andesite (and porphyry) for all data below the top of the HE zone. The indicators were defined from the drill log codes in the grade control and exploration data sets. As for the grade variography, the lithology variography was carried out for separate populations east and west of 16 300 E.

The lithological data was simulated as a categorical variable (Figure 4). The presence of andesite was defined in the drill hole data using an indicator value of 1 with the absence of andesite (ie the presence of porphyry) assigned an indicator value of 0. The conditioning data set used for simulation of this categorical variable was the 15 m composited exploration data combined with the grade control 15 m blasthole data. The coded lithology data and the indicator variogram parameters were used to generate a sequential indicator simulation 3D model of the lithology as defined by the distribution of the andesite indicator.

Generation of the geological conditional simulation model

The 50 two-dimensional simulated realisations of each of the three enrichment surfaces and the 50 three-dimensional simulated realisations of andesites in two separate domains (east and west) were then merged into a single geology conditional simulation model comprising all simulated outcomes. Thus, there were 50 simulations each with a different lithology and Minzone outcome (Figure 5).

Simulation of grades for CuT and CuS

Twelve separate domains were considered for simulation of the percentage of copper as CuT and CuS grades. The conditioning data for each domain was the 15 m exploration composite data set. For each domain, appropriate data belonging to that domain was extracted. The sequential Gaussian simulation approach was used to simulate grades (Figure 6) and simulated realisations for each domain were validated by checking the reproducibility of the weighted histogram of the exploration data, and the normal score variogram model from the grade control data.

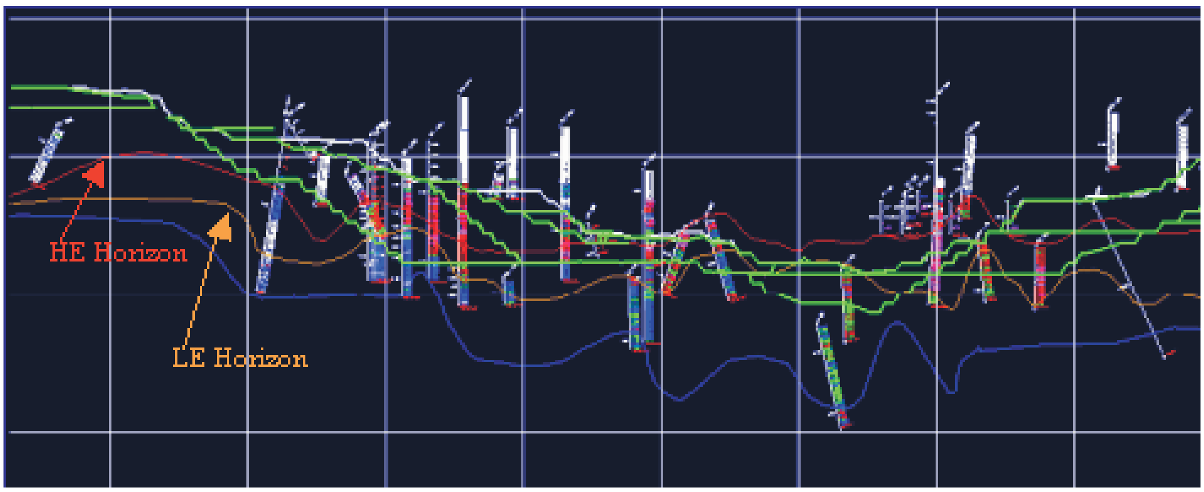


FIG 1 - Typical cross-section at Escondida copper.

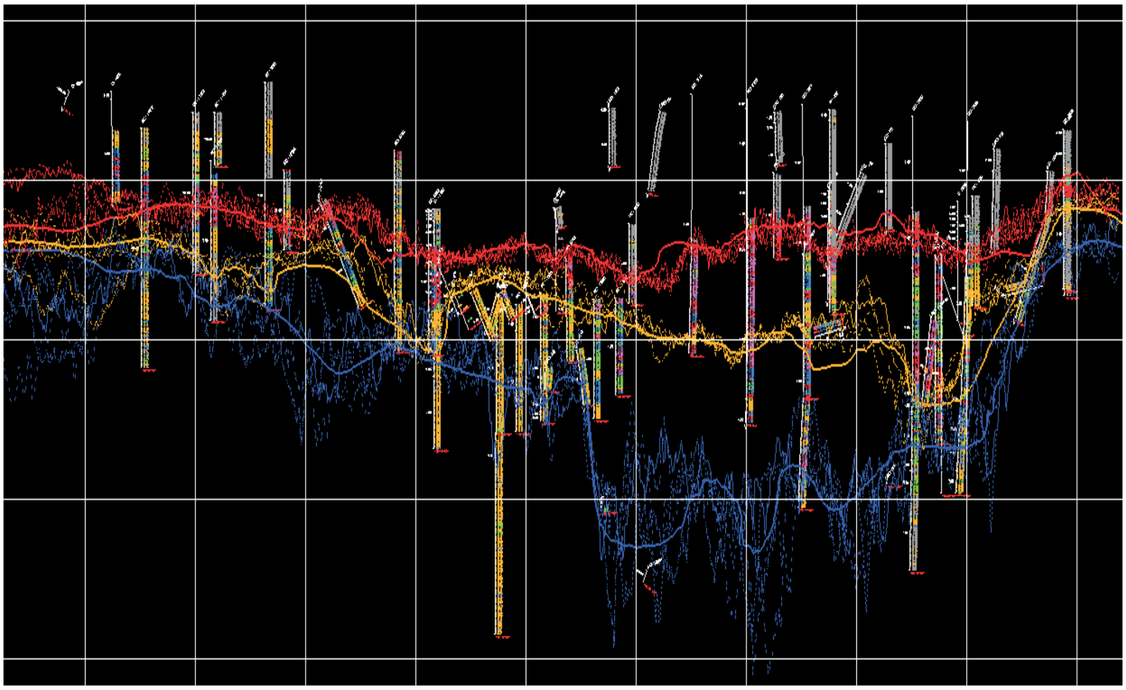


FIG 2 - Example of simulated image of the enrichment profiles.

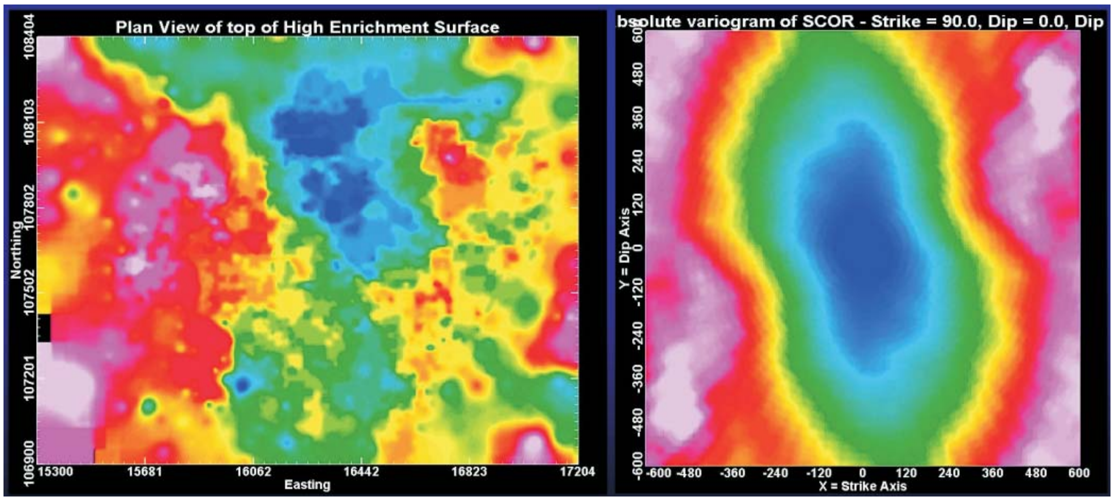


FIG 3 - Example of High Enrichment elevation contour and associated variogram map.

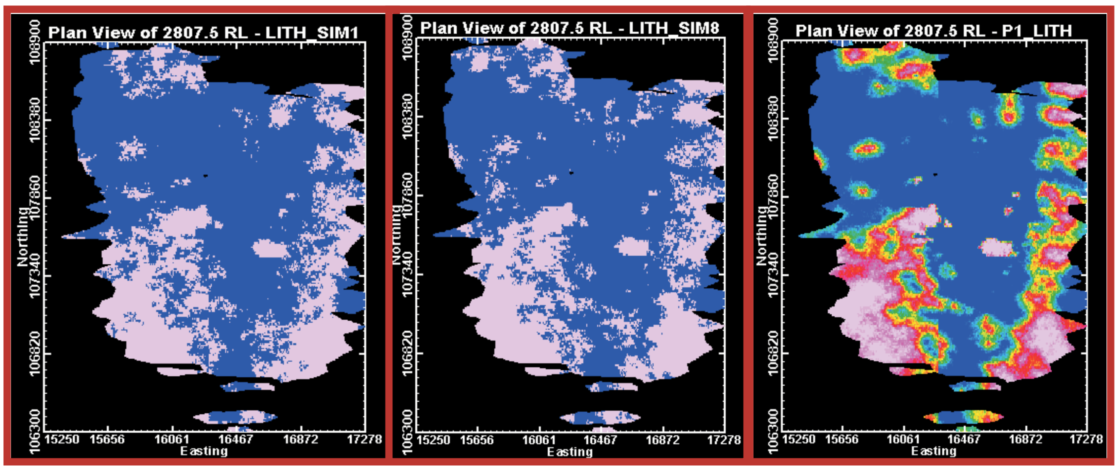


FIG 4 - Various simulated lithological data with associated probability map.

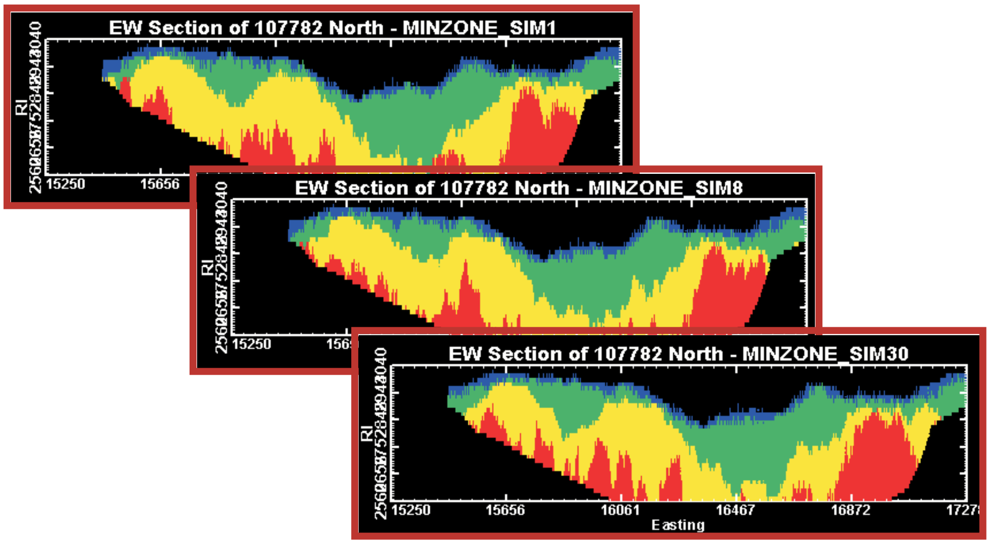


FIG 5 - Example of combined simulated geological data.

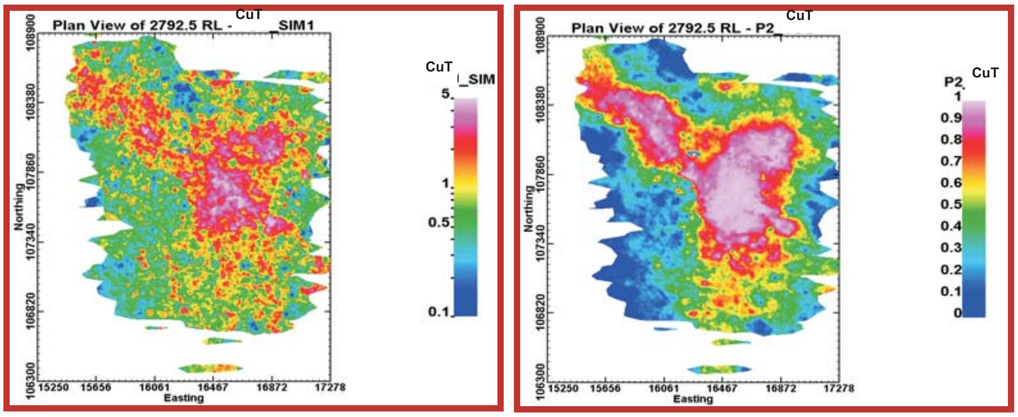


FIG 6 - Typical simulated image for CuT and associated probability map.

It is impossible to produce a perfect representation of any deposit as a resource model since the geological knowledge, the sampled data, and the assumptions made during estimation are all imperfect. If a model was perfect it could be used as the basis for mining without any requirement for further mapping or sampling. Collectively, these imperfections are termed the *information effect* and can never be overcome completely. During mining, decisions are made based on similar imperfect data. Geological mapping, sampling and assaying are used to provide a basis on which the ore boundaries are defined and mined. Estimates of grades within the ore blocks must be made from the best available data. The impact of such estimates causes dilution (material below the cut-off grade being sent to the mill) and ore loss (ore incorrectly being sent to low grade stockpiles or waste dumps). Imperfect knowledge of the deposit again plays a part, but to this is now added imperfect mining practices. Even if the cut-off grade boundary could be defined perfectly it could not be mined perfectly every time at a practical mining scale. To differentiate between the impact on resource modelling and the impact on mining, these imperfections are collectively termed the *grade control effect* and, again, can never be overcome completely.

THE CHAIN OF MINING APPROACH

For any measurable value, the term error can be used to indicate the difference between an estimate and the true value. During the process of defining an ore block for mining, a number of measured values are used, such as the location of the ore in 3D space, the representativity of the sample, the quality of the sample, the grade of the sample, and the cut-off boundary of the ore block boundary to be mined. For each of these attributes a 'true' value and an 'estimated' value can be defined.

Mining decisions are in all cases based on the estimated value. However, the results of mining are in all cases determined by the true value. For example, the placement of an ore block boundary and the predicted grade of that ore block might be defined solely by the sampled grades in and around that block. Errors in the sampling process (which leads to imperfect delineation of boundaries) and during mining (which leads to imperfect mining of the planned boundaries) both result in dilution and ore loss such that the grade of the ore delivered to the mill is invariably lower than that predicted by the estimated values. This is because the application of a cut-off grade alters the impact of the distribution of errors. Waste incorrectly sent to ore is by definition always of lower grade than ore incorrectly sent to waste.

There are various approaches that can be taken to solving this nexus between 'predicted' and 'actual' mining performance. For the present study, a series of parameters that model the differences between the predicted and actual mining performance were measured. To define these parameters, the various stages where errors can occur in measured values were considered. The mining process as a whole was considered to be a chain of events with the consequences of each event impacting on the next measurement in sequence. The term Chain of Mining is used to underscore the dependence of the eventual mining result on each link in the process (Shaw and Khosrowshahi, 2002; Shaw *et al*, 2002; Khosrowshahi and Shaw, 1997). Figure 7 provides a schematic of the process to characterise the generation of recoverable resource estimates.

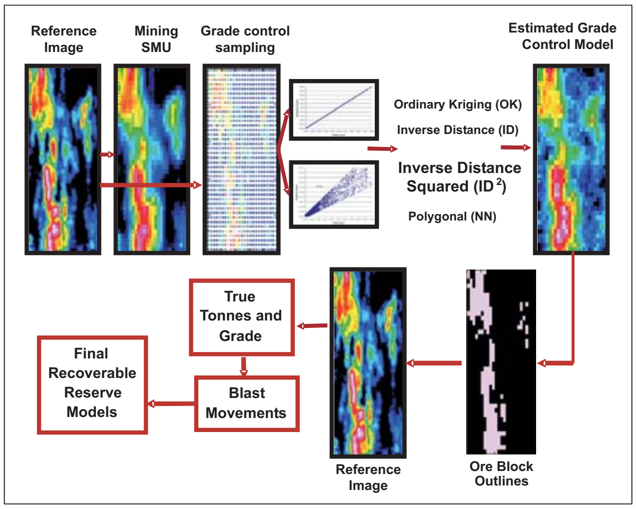


FIG 7 - Using the Chain of Mining process on a simulation model to characterise recoverable reserve estimates.

Sources of error during mining

It was apparent that there were four possible sources of error that contributed to the grade control effect and which could be modelled, namely, sampling and assaying errors of precision, sampling and assaying errors of bias, movement due to blasting as lateral displacement or heave, and mining selectivity. It was recognised that it would be impractical to attempt to define parameters in detail for every possible source of error at Escondida. In addition, due to the large and very complex nature of such a mining operation, there is always the possibility that one or more practices will change in time. Instead, an empirical approach was taken. Error models were developed where observation on site indicated that this would be appropriate and these various error models were tested to determine their impact.

Error due to sampling and assaying precision

The grade control sampling at Escondida is done using vertical blastholes. The ore is blasted and dug on 15 m high benches. The blastholes are drilled with large rotary air blast equipment, drilled to a depth of 15 m (one mining bench) plus subdrill of approximately 2.5 m. Sampling errors that will lead to a difference between the actual grade of the material in the cone of blasthole cuttings and the true grade of the ore in the ore block are not quantifiable (since they are frequently not repeatable). Nevertheless, these errors exist and include both the sample delimitation error due to subdrill material remaining in the cuttings cone, and sample extraction error due to contamination and loss during the open hole rotary drilling, and due to dust loss.

The subsampling of the spoil cone is done manually after drilling using a tube sampler and eight increments are collected. The sample is then further crushed and subsampled in the MEL site laboratory. The errors that impact on the predicted grade include:

1. the grouping and segregation error that is due to splitting of the spoil cone (in this case due to the tube splitter); and
2. error due to the relationship between particle size and grade, known as the Fundamental Sampling Error (Gy, 1979) that results from the process of splitting, crushing and pulverising to reduce the 2 t sample spoil cone to a 200 g pulped sample submitted for assay.

The first type of error is not quantifiable, and every subsampling system incurs the second type of error. The total impact of all these errors was modelled in two scenarios:

Low sampling error

A relative sampling precision of ±20 per cent was assumed as the base case. This incorporates the measured precision of ±10 per cent demonstrated by repeat sampling and assaying of blasthole cones (Figure 8). An allowance for additional error was made due to the drill sampling method. This scenario assumes high quality grade control sampling is available.

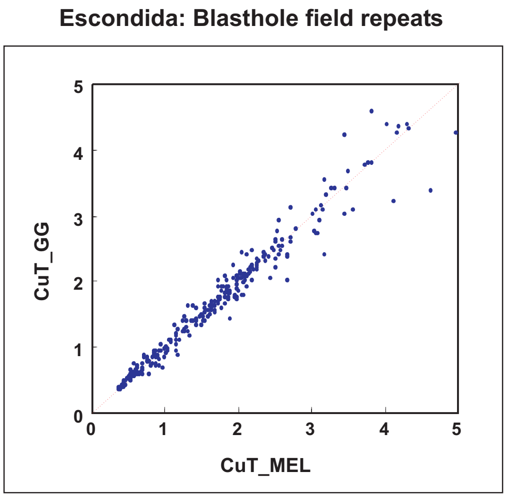


FIG 8 - Field duplicates of Escondida blasthole cone sampling from which a relative precision of ±10 per cent was obtained.

High sampling error

A relative sampling precision of ±60 per cent was assumed as the high error case to indicate the typical level of sampling repeatability that occurs in twinned blasthole drill sampling. No data for this estimate was available. The nearest such data was paired blasthole and resource hole estimates where a precision of ±40 per cent was obtained. The high error case was adopted to allow for the impact of the blasthole subdrill and accounts for the local variability typically seen in blasthole sampling.

Error due to blasting

Ore movement can result in the predicted ore being displaced so that the material eventually mined is different from that which was planned. The degree of dilution and ore loss that this causes is dependent on the lateral displacement of the ore block boundaries, and the vertical heave resulting in mixing across horizontal mining levels. Heave is not an issue at Escondida since the ore is blasted and mined on a single mining bench. It was decided to model two scenarios, one where the lateral blast movement was negligible and one where the movement was 3 m in both the east-west and north-south directions, this being the movements observed on site for a number of blasts.

Mining selectivity

Perfect mining of any orebody is always impossible due to two factors; the availability (and quality) of data to define boundaries, and the ability of the equipment to dig a defined boundary, which decreases with the production scale of the operation. The effective minimum mineable block size can be expected to relate in some way to these factors and, consequently, in a resource model the point estimates of grade, interpolated from drill hole (quasi-point) data, may be aggregated to a mineable block size.

The degree of mining selectivity represented by a resource block model is defined as the selective mining unit (SMU). This SMU block size may be regarded as the minimum viable size of a mining block, although of course, the average size of the mined blocks may be much bigger. The degree of misclassification that generally occurs along any block boundary during mining is directly related to the production rate and size of the mining equipment. The concept of the SMU block size can assist in understanding the impact of the mining method on the orebody and how well this can be represented by the resource model.

CALIBRATION OF THE CHAIN OF MINING MODELS AND COMPARISON WITH PRODUCTION

The conditional simulation model developed for the Escondida deposit was used to test the impact of various mining selection parameters and the impact of the various expected errors. A series of ten cases was developed using the parameters defined in Table 1 to address misclassification errors likely to arise during mining. These CoM models were then tested against production records and compared to the Escondida resource model.

TABLE 1

Parameters used in the Chain of Mining (CoM) analysis for the various CoM models examined, with results for the reconciliation period.

Case	Blasting movement	Sampling error	SMU (15 m high)	Mt	Grade % CuT	Comment
1	0	Low	16 × 16	109.4	1.875	
2	0	Low	8 × 16	107.5	1.893	
3	0	Low	8 × 8	103.3	1.929	
4	0	High	16 × 16	108.0	1.885	
5	0	High	8 × 16	105.3	1.907	
6	**0**	**High**	**8 × 8**	**99.0**	**1.953**	**closest to mine production data**
7	3	Low	16 × 16	109.4	1.861	
8	3	Low	8 × 16	107.5	1.873	
9	3	High	8 × 16	105.3	1.887	
10	3	High	8 × 8	99.0	1.921	

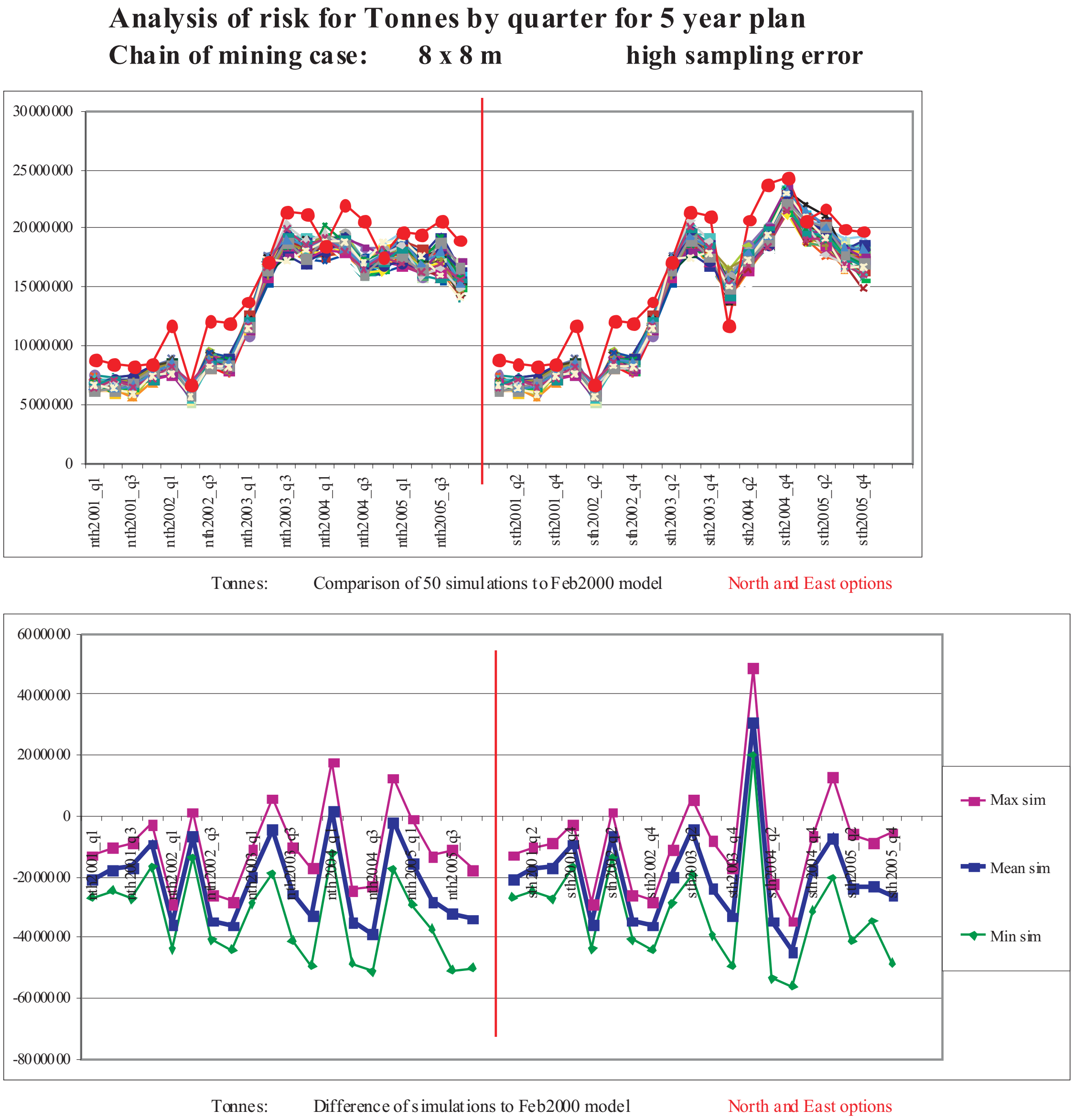


FIG 9 - Risk associated with tonnes in the five year plan by quarters.

Analysis of the results for the different scenarios indicated that Case 6 was the closest to the Production data total of 100.294 Mt at 2.11 per cent CuT. The selected case used no blasting movement, a high sampling error consistent with blasthole samples, and an 8 × 8 m SMU block area. The smaller SMU size provided better selectivity at the cut-off grade, producing a lower tonnage and higher grade than that predicted by the resource model. Case 6 was regarded as the base case. Various models were intersected with each wire-frame defining the mine plan, and the results were aggregated by both quarterly period and major pushback increment. For the Chain of Mining cases, each of the 50 simulations was separately intersected with each wire-frame to provide a risk profile of the chance of not achieving the scheduled tonnes and grade for the period that the wire-frames represented. The tonnages and grades within each simulation realisation were determined for the quarterly and pushback increments for the base case (8 × 8 m SMU with high sampling error). The results are presented in graphical form in Figure 8 to Figure 12. In assessing the relative risk using this graphical data, occurrences below the horizontal line indicate where the expectation of tonnes or grade was not reached, ie periods when the resource model is at risk under the assumed mining scenario.

CONCLUSIONS

The five-year schedule options adequately fit with the *in situ* resource. However, the Chain of Mining case (8 × 8 m SMU, high error) selected to best emulate the production data indicates a significant expected shortfall in tonnes. What had not been evident until this study, and could only be demonstrated using the exhaustive data set provided by a conditional simulation study, is that there was considerable risk of a shortfall in tonnes. This was because the selectivity evident in the actual mining strategy

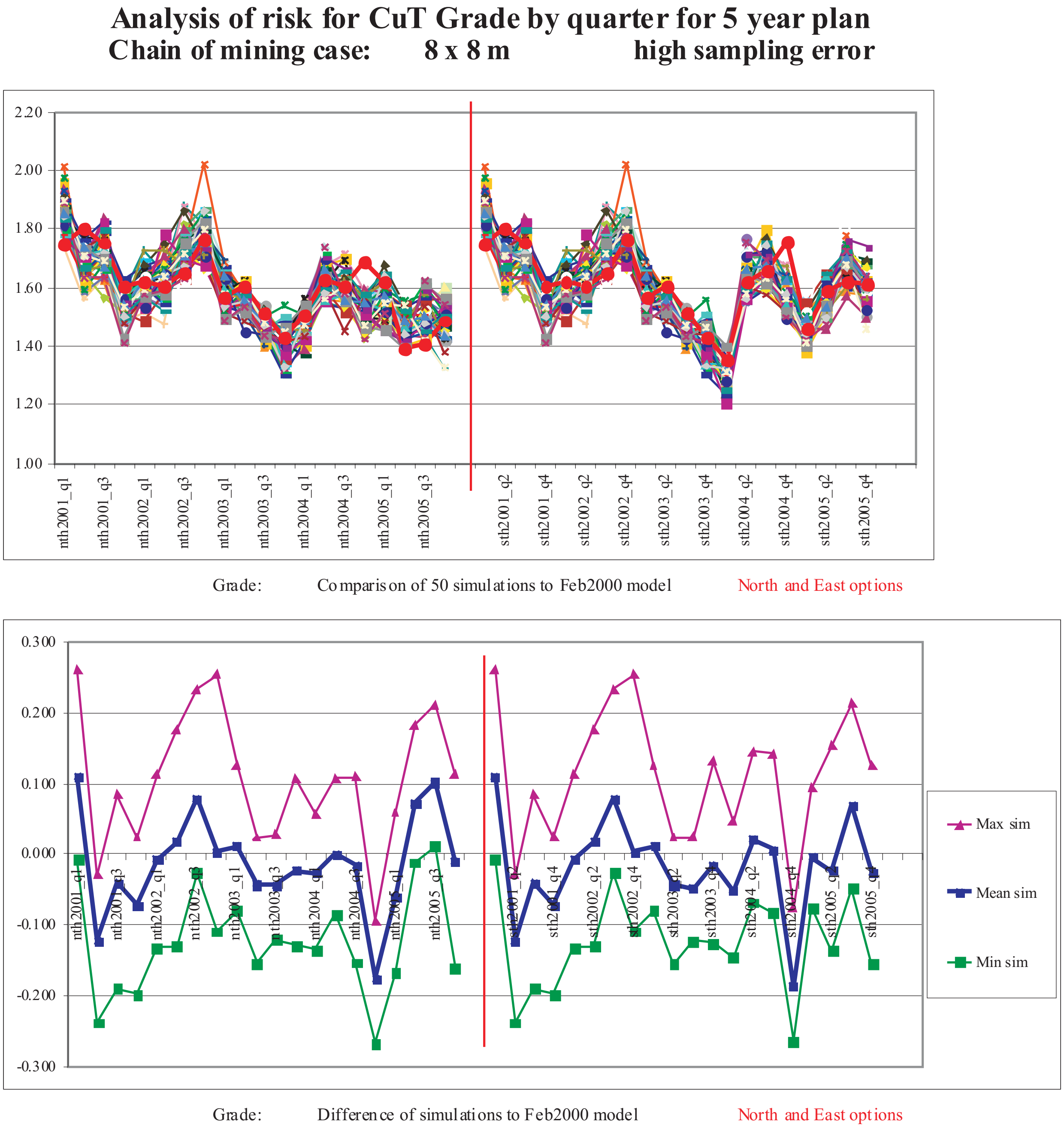
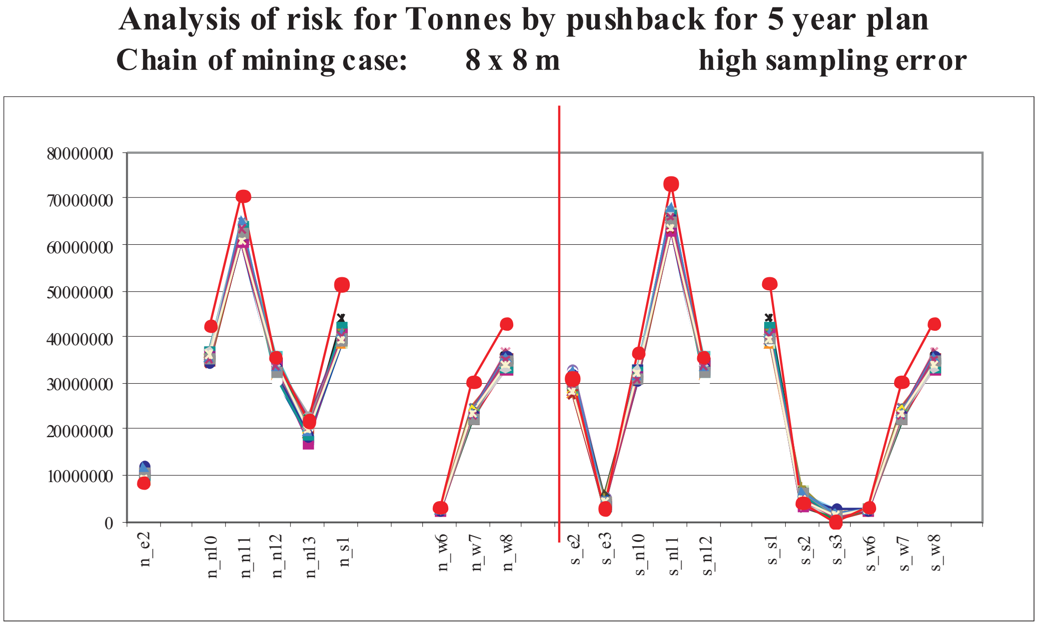
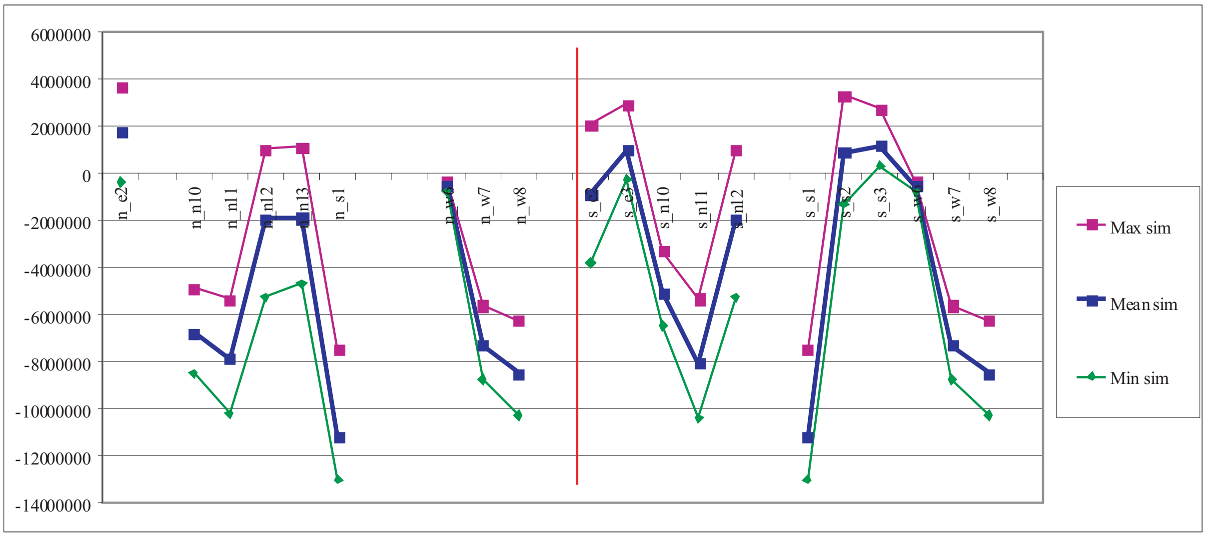


FIG 10 - Risk associated with grade in the five year plan by quarters.



Tonnes: Comparison of 50 simulations to Feb2000 model North and East options



Tonnes: Difference of simulations to Feb2000 model North and East options

FIG 11 - Risk associated with tonnes in the five year plan by pushback.

differed significantly from that inherently assumed in the resource model. High quality grade control practices on site were effectively providing higher selectivity than that assumed in the resource model. This lead to a scenario of 'vanishing tonnes' (David, 1977), a concept demonstrated in this study that is familiar to many large mines. This problem can be related to attempts to improve the head-grade to unrealistic targets applied on a short-term (sometimes daily) basis. Visual grade control and other decisions to remove small parcels of contaminating material in order to maintain a high mill head grade may lead to an artificially small effective mining selectivity that is not related to the SMU block size assumed in the resource modelling.

The quantification of risk using simulation of the Chain of Mining is a technique that can be used to identify a potential shortfall in tonnes or grade for a given mining scenario. Alternative plans can then be developed and tested before the shortfall impacts production. An approach such as the one demonstrated here for Escondida can determine if a plan is realistic and the predicted results will be obtained. Hence, the risk inherent in a given plan can be quantified. Testing alternate mining scenarios, operating practices and policies to determine if they will indeed deliver as intended, therefore, provides considerable advantages to both mine planners and operators.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the input of site personnel into this study. Minera Escondida Limitada and BHP Billiton granted permission to publish the results of this study once its direct commercial relevance was superseded by other ore reserve and mine planning studies.

REFERENCES

Benndorf, J and Dimitrakopoulos, R, 2007. New efficient methods for conditional simulation of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

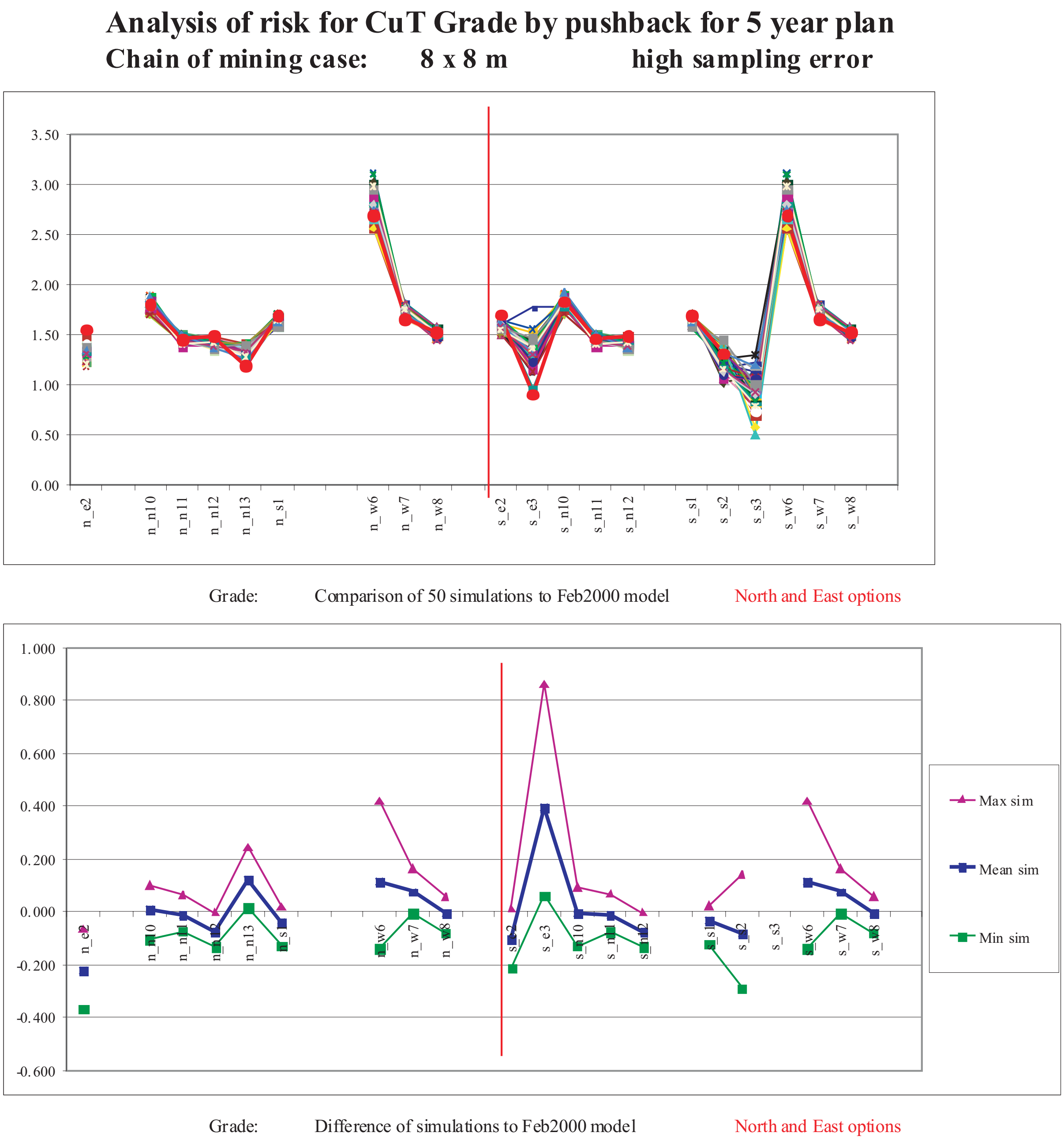


FIG 12 - Risk associated with grade in the five year plan by pushback.

David, M, 1977. *Geostatistical Ore Reserve Estimation,* 364 p (Elsevier: Amsterdam).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 483 p (Oxford University Press: New York).

Gy, P, 1979. *The Sampling of Particulate Materials – Theory and Practice*, 431 p (Elsevier: Amsterdam).

Journel, A G and Huijbregts, C J, 1978. *Mining Geostatistics,* 600 p (Academic Press: London).

Journel, A G and Kyriakidis, P C, 2004. *Evaluation of Mineral Reserves: A Simulation Approach* (Oxford University Press: New York).

Khosrowshahi, S and Shaw, W J, 1997. Conditional simulation for resource characterisation and grade control – principles and practice, in *Proceedings World Gold '97*, pp 275-282 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Nowak, M and Verly, G, 2007. A practical process for geostatistical simulation with emphasis on Gaussian methods, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 69-77 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Shaw, W J and Khosrowshahi, S, 2002. The use of the Chain of Mining method, based on conditional simulation models, to quantify mining risk – a reality check for resource estimates, in *Proceedings Symposium on Quantifying Risk and Error*, pp 111-118 (Geostatistical Association of Australasia).

Shaw, W J, Khosrowshahi, S, Bertinshaw, R G, Weeks, A and Church, P, 2002. Beyond grade control: broken links in the Chain of Value, in *Proceedings Value Tracking Symposium*, pp 85-89 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Integrated Strategy Optimisation for Complex Operations

B King[1]

ABSTRACT

While large mining operations frequently provide enormous value for their shareholders, they also contain enormous challenges for those determining the operating strategies that maximise the net present value (NPV). Experience shows that value could be increased by several percent through additional or 'second order' optimisation of the extraction policies. For operations constantly looking for ways to trim costs and add value, analysis of the optimisation process may help initiate a step change in the project's NPV. Determining the best operating policy is often limited by the analysis time and the availability of skilled engineers to appropriately utilise various planning tools. For example, the size, shape and timing of even a single pushback may have thousands of valid alternatives. For a very small operation, these alternatives may be evaluated to determine which one results in the best project value. Mines with durations of more than five years often have so many valid alternatives that the number of neutrons in the universe appears small in comparison.

Optimisation algorithms implemented in commercial software tools for maximising project value provide guidance for some key parts of the process to determine the best operating policy. It then becomes a matter of how we can answer questions that are not explicitly optimised by the algorithms. These questions may concern the mining and processing capacity that should be installed, the size and timing of a processing expansion, the timing of extracting resource from nearby mines, and the timing of mining a resources from underground rather than from the surface. This paper outlines a framework for optimising many of these policies in large, and often very complex, mineral resources. Examples are presented from experiences at major operations in Australia, Chile, Peru and the USA. It is hoped that this will assist engineers to more responsibly exploit our finite resources.

INTRODUCTION

In mining, several decisions can be made with the guidance of commercially available optimisation algorithms and tools. This paper focuses on how to determine the best choice for a policy that is not optimised by these tools. We do not have to search far in large operations to find strategies that are not optimised but which may add substantial value through appropriately made decisions. While small mining ventures have ridden the tides of metal prices and market conditions for short-term profit and unconstrained resource high grading, the following objectives of a modern mining company are more commonly stated. Firstly, the modern company will aim to act responsibly as a steward of the resources in its care so that they benefit both the countries in which they are found and the world at large which depends on them. Secondly, the modern mining company will aim to create long-term wealth for its own shareholders. These objectives are believed to be in harmony with each other, and both are a vital part of the mining industry. In determining realistic policies, environmental, safety and political constraints must all be considered. There is clearly little point investing time and effort in developing plans that cannot be implemented for failing to obey these constraints.

Ideally, all possible decisions that could influence a mine's value should be considered to achieve designs which will result in the maximum net present value. The number of combinations of these parameters over the life of the mine is overwhelming for any global optimisation technique unless several assumptions are made. Until the entire mine design problems can be solved with one integrated algorithm, smaller components of the process are often worked on sequentially. These sub-problems normally form a sequential mine planning process that can be repeated iteratively. Higher value results are expected as more of the sub-problems are simultaneously considered in a single optimisation. Efficient algorithms are required to ensure the solution times do not explode as the complexity increases. These may be very expensive to look for and, if found, costly to turn into practical tools for the mining industry. For example, the algorithms based on dynamic programming that are used in the COMET software integrate pushback timing, cut-off grade, processing policies and financial analysis. This allows the interaction between the policies to be exploited to maximise the project value. Further presentation of COMET is provided by Wooller (2007, this volume) and King (2004).

Because any 'optimising' tool only works on a limited model of reality, results should be reviewed to check that they are reasonable. When breaking the problem down into components, the resulting designs lose any guarantee of finding the maximum present value; they simply hope to be close to the maximum. While this may be somewhat disappointing to management or investors, it is a sobering reminder of the complexity of mine planning and the need to appropriately resource this vital work. Large operations with multiple orebodies, mining areas and processing alternatives are often so complex that the number of feasible solutions makes the number of neutrons in the universe (10^{128}) seem small. The next part of this paper discusses available tools for solving some of these problems found in large operations. The problems selected are ones that are important to many large operations and that do not have integrated optimisation algorithms to solve them.

GENERIC PROCESS

The question to be answered is 'How can I use the available tools to optimise policies that are not usually optimised by these tools?' Let us assume we have policies A, B and C to optimise, though only policies B and C can be simultaneously optimised with available algorithms. This paper offers the following generic steps for approximating the optimal choice of policy A.

The first step is to choose a tool that uses an objective that (a) accurately reflects your business objectives (maximising NPV is assumed in this paper) and (b) simultaneously optimises as many of the other key policies (B and C) as possible. (Sometimes multiple tools are required to model the process. COMET software has been used for the problems identified in this paper.) The next step is to identify the broad options for policy A. Then, for each policy A alternative, optimise the remaining policies (B and C). The final step is to choose the highest value options for some more detailed analyses if the schedule values are close (within the accuracy of the estimate). The option for policy A that gave the highest value is chosen to optimise this policy. This process often contains a substantial manual component, which is both expensive and time consuming. If this decision is to be evaluated often, then some automation may be justified.

The above process can also be used when several policies must be chosen even though they are not able to be simultaneously optimised with the available technology. Policy A can be a complex policy, or a combination of several policies. If we have independent policies X and Y, both with two options (X_1, X_2 and Y_1, Y_2), policy A can be defined as a set of four policies of X and

1. MAusIMM, Managing Director, Strategy Optimisation Systems Pty Ltd, 24 Harris Street, Wellington Point Qld 4160, Australia.
Email: brett.king@stops.com.au

Y (X_1Y_1, X_1Y_2, X_2Y_1, X_2 Y_2). The number of options can rapidly approach the number of neutrons in the universe again, so some judgement may be needed to consider only reasonably likely options. In addition, while the above approach may be deduced by common sense, the key issue to recognise is that all combinations of policies A, B and C do not have to be searched in order to find the highest value path. Only one path for each policy A alternative needs to be valued. This can have enormous time-saving benefits when applied to many complex operations. The following examples have been used to illustrate how this process may be applied to a number of problems found in large, complex mining projects.

SURFACE TO UNDERGROUND INTERFACE

Many large resources mined from the surface also have a potential resource that can be extracted from underground. Although surface mining methods may be used to extract much of the resource, the highest value for the project should consider both underground and surface options.

There are many factors that impact on the ideal transition from surface to underground operations. Surface issues include cut-off grades, waste stripping, and stockpile generation and reclaim. Underground issues include access to higher grades, dilution, the proportion of resource extracted (due sterilisation associated with the mining method), production costs and capacities, and capital requirements. Combined issues include tailings capacity and closure cost implications. There is currently no algorithm (and therefore software product) for determining the best transition between surface and underground mining that will take into account all of the above issues. Where currently available software tools attempt to answer these questions, only a few of these aspects are considered. However, it is not the objective of this paper to list the limitations of commercially available software tools, many of which can still be profitably used despite these shortcomings. Thus, the question becomes 'How can we use the available tools to optimise the transition between surface and underground mining?'

By applying the generic process suggested above the best transition from surface mining to underground can be evaluated. For the example illustrated in Figure 1 there are three different options to evaluate. Each of the underground mining options has a different design, production schedules, capital requirements, life and of course value. The open pit designs have several pushbacks that extract different portions of the resource. These underground alternatives impact on the opportunity costs for processing surface material, since every day spent processing surface ore could alternatively be spent processing underground ore. The surface policies, such as cut-off grade and ultimate pit limits, are dependent on the value of the remaining underground resource. If the underground is considered without reference to the open pit, Option A (large underground) is chosen. If the open pit is evaluated without the underground impact, Option C (large open pit) is selected.

Without an underground, the open pit cut-off grades will normally drop down close to break even as the last material is mined. With a highly profitable underground that cannot start until the surface operation is complete, open pit cut-off grades will generally increase to bring forward the value from the underground resource. Although specific policies are dependent on the particular project, Figure 2 shows the general pattern of change in cut-off grade for the three cases shown in Figure 1. Each of these schedules was optimised using the COMET software and a dynamic programming algorithm based on successive approximation (Roman, 1973).

The changes in policies are dependent on the constraints, economics and resource mined. Figure 2 shows that the shorter open pit options generally utilise lower cut-off grade strategies that have the result of extracting more value from the earlier open pit phases. As is usually the case with optimised cut-off grade policies, the policies rise as more high-grade material is reached and then generally decline with time. While the cut-off grade policies are interesting, the most important number is the NPV presented in Table 1. The highest value schedule was Option B, with the medium surface and underground designs. Of interest in Table 1 is the mine life, which increases with pit size. The primary reason for this is the lower grade material that was processed in the larger pit options. The underground costs do not justify the removal of all of this material and so larger underground designs have smaller reserves. A second point to note in Table 1 is that all three cases yielded positive NPVs, some were just a little more positive than the others! This should serve as a reminder that a high-value schedule does not necessarily mean that an even higher value schedule is not possible with a little more effort. Further information from the best case (Option B) is presented in Figure 3.

TABLE 1

Surface to underground transition summary results.

Option	A	B	C
Open pit size	Small	Medium	Large
Underground size	Large	Medium	Small
NPV ($M)	2287	2425	2410
Life (years)	21	22	27

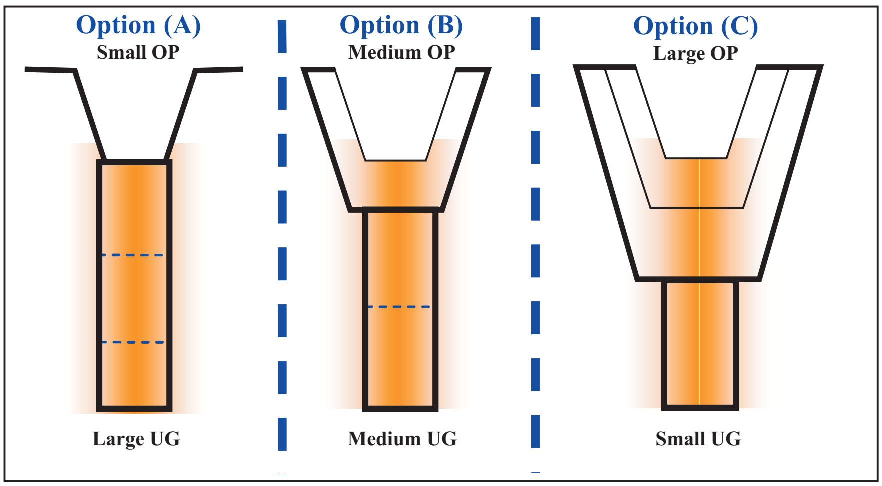


FIG 1 - Surface to underground transition options.

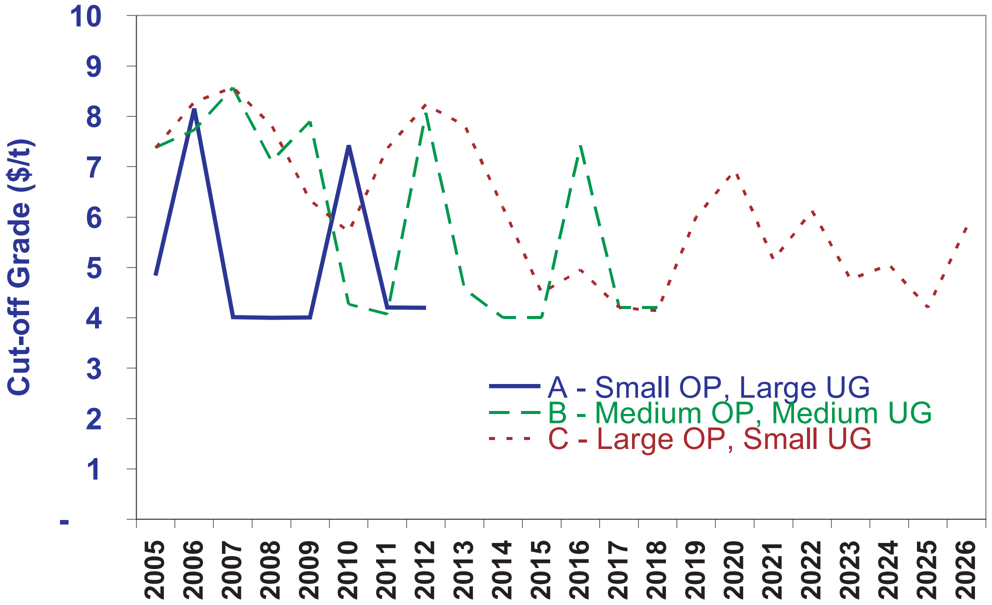


FIG 2 - Cut-off grade policies for surface to underground transition.

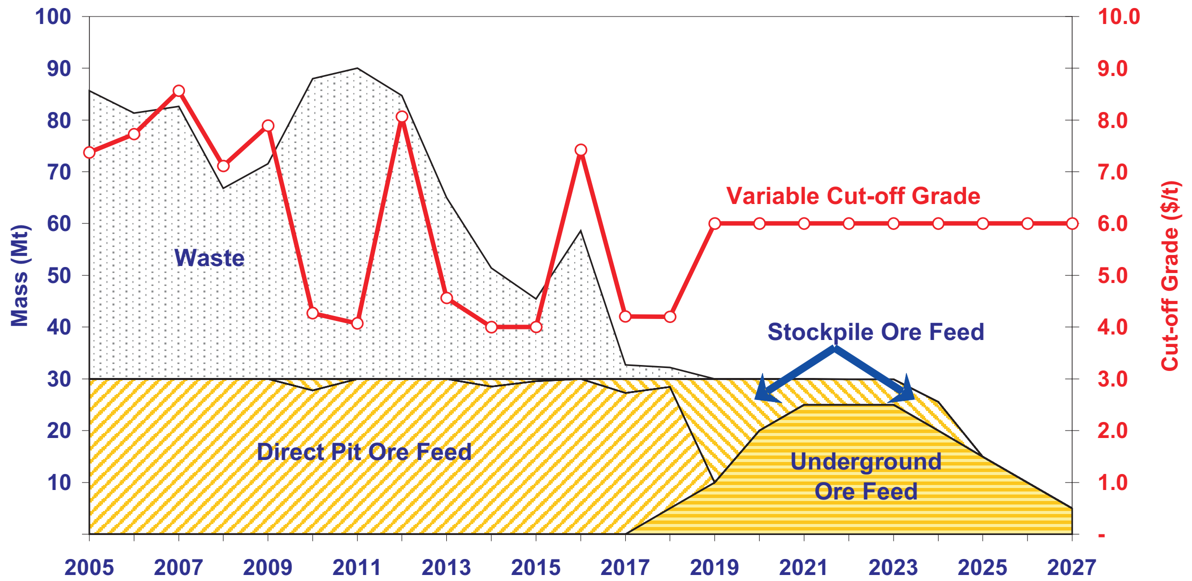


FIG 3 - Highest value integrated surface and underground schedule (Option B, $2425 M) with variable cut-off grade and pushback sequencing.

Figure 3 shows that the low-grade stockpile is reclaimed once the open pit ore runs out and during the underground mining period. The underground ore was higher grade than the stockpile but was not able to be mined at a sufficient rate to use the full mill capacity. Ideally, the mill throughput/recovery would be modelled to add further value. A new set of schedules was therefore undertaken to exploit a time varying grind policy to maximise the project value. For the purpose of this paper, a grind relationship was used in which the mill could process up to ten per cent more material with the loss of five per cent in recovery, or process 20 per cent less material and realise ten per cent higher recoveries. Figure 4 shows the schedule when optimised with a variable throughput/recovery policy, which is optimised simultaneously with the cut-off grade and pushback sequencing. A substantial increase in value (from $2425 M to $2523 M or four per cent) was realised by simultaneously optimising the grind policy (throughput/recovery relationship) with the other policies (cut-off grade and pushback sequencing). An outline of the theory of how to optimise multiple policies like these is presented by King (2001). The same surface to underground transition was found to produce the highest value and all schedules had higher values. The addition of the grind policy to the optimisation is an example of adding complexity as a model of the project is developed. As time is spent analysing and understanding the project value drivers, some areas are obvious candidates for greater model accuracy.

It is important to review the sensitivity of these decisions to price, cost and constraint variation. A low-reserve schedule that has the highest value at a low price may not be best schedule at a higher price (since more reserves can utilise the higher prices). It is also important to recognise the different risk profiles of the resulting schedules. Risk is often a more difficult property to measure; however, there are normally some parameters that reflect this risk.

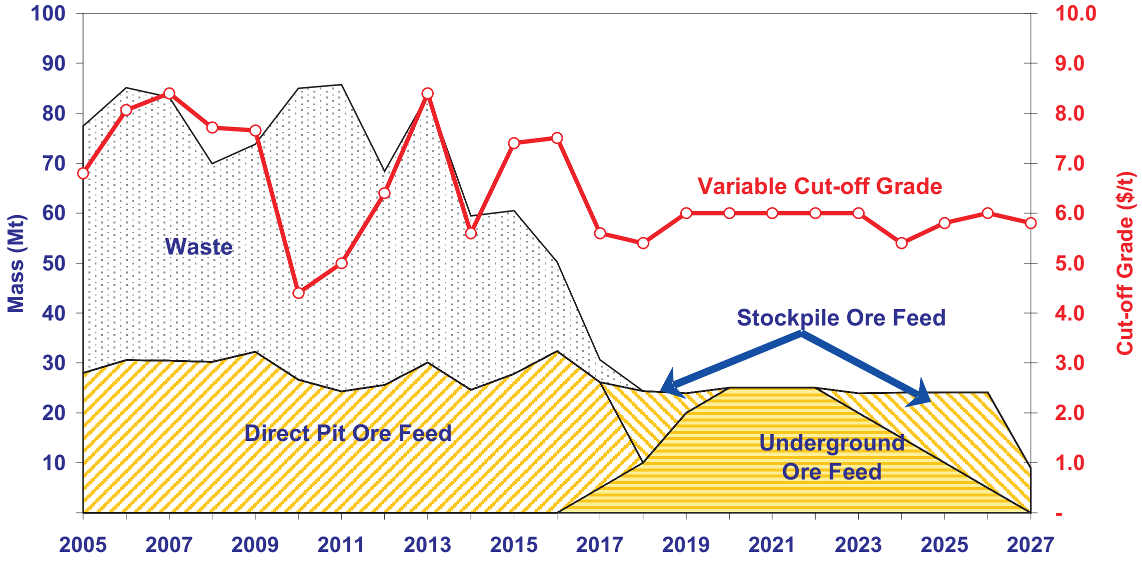


FIG 4 - Best integrated surface and underground schedule (Option B, $2523 M) using variable grind (throughput/recovery), cut-off grade and pushback sequencing.

ADDITIONAL COMPLEX POLICIES

Pushback designs

Designing realistic pushbacks is a fundamental part of planning a large surface mining operation. Many engineers use tools based on the ultimate pit algorithms, such as the Lerchs-Grossmann algorithm (Lerchs and Grossmann, 1965; Whittle, 1988), to provide guidelines for creating intermediate pushbacks. These tools are often run at lower than expected metal prices to determine a nested set of shells. While this approach does provide useful guides, there are a number of issues that arise in large operations that limit the usefulness of these guides. For one, there are a number of factors that are not considered, such as mining and processing capacities; time dependent properties, including prices and costs; operating policies such as cut-off grade; interaction between material mined and processed; and ramp locations and some geotechnical constraints (such as stress unloading). Another issue is that shells may be much smaller or larger than can be practically mined. The above reasons may provide substantial uncertainty of the best shape for intermediate pushbacks. Several options may need to be manually designed and scheduled to find the best designs and maximum value.

The ultimate pit size is also subject to the same assumptions and therefore limitations as described above. For example, the location of the final pushback may need to be confirmed by grouping shells into a realistic width and scheduling with all other policies (such as cut-off grade and stockpiling) optimised.

Mine and process expansion optimisation

Mine and processing expansions may provide the keys to unlock substantial additional project value. These capacities are not automatically optimised by the currently available algorithms and software tools, so we ask ‘What is the optimum mining and processing capacity for the project?’. Although a simple question, the answer can involve a complex combination of policies throughout the business. For example, increasing the flotation capacity would also require increasing the SAG capacity, crushing and grinding capacities, concentrate handling capacities and, quite possibly, the tailings capacity. Once the entire processing system has been upgraded and new cost and recovery functions implemented there may still be negligible increase in value. The reason could well be due to the operation being constrained by the mining equipment. When mining is constrained, cut-off grades drop to breakeven grades, and very marginal material is processed. In order to reveal the full value of a processing expansion it should be coupled with a mining capacity expansion. Although schedule optimisation tools may not directly provide the optimum choice of mining or processing capacity, by scheduling several options an engineer can rapidly determine the optimum choice of both mine equipment fleets and process capacities.

To evaluate all the possible options of truck and shovel fleets alone would be an enormous and unnecessary task. Most of the options are able to be discarded as unlikely to achieve higher value. For example, expanded truck fleets without associated shovel fleets are unlikely to reveal any further value unless the operation was already truck constrained. By applying sensible boundaries to the options and reviewing results as they are generated, options for analysis can be greatly reduced.

CONCLUSIONS

Strategic and long-term plans set the context for shorter term decision-making. Substantial value is realised by ensuring that strategic and long-term planning follow the corporate objectives, normally defined using the net present value.

Many optimisation algorithms have been developed to solve parts of the planning problem. However, there are still important problems that are not able to be automatically optimised with these algorithms. This paper demonstrates that, by using an efficient schedule optimisation tool, many of these policies can be optimised to add substantial value to a project.

ACKNOWLEDGEMENTS

I am grateful for the opportunity and support to spend the last decade researching optimal mine scheduling and developing tools useful for engineers around the globe. Many professional engineers at Rio Tinto Technical Services and BHP Billiton have willingly given time and ideas to make sure that what was being developed would be useful to real-world mining problems. Richard Wooller has been a tremendous sounding board and debating partner on many issues. Thanks also to Brian Baird for his support and help getting these ideas implemented into major operations around the world.

REFERENCES

King, B M, 2001. Optimal mine scheduling policies, Royal School of Mines PhD, London University, United Kingdom.

King, B M, 2004. COMET Optimisation Services, Strategy Optimisation Systems Pty Ltd [online]. <http://www.stops.com.au/Products/comet.htm>.

Lerchs, H and Grossmann, I F, 1965. Optimum design of open-pit mines, *Trans CIM*, LXVIII:17-24.

Roman, R J, 1973. The use of dynamic programming for determining mine-mill production schedules, in *Proceedings Tenth Symposium on Application of Computer Methods in the Mining Industry*, pp 165-169 (South African Institute of Mining and Metallurgy: Johannesburg).

Whittle, J, 1988. Beyond optimization in open pit design, in *Proceedings First Canadian Conference on Computer Applications in the Mineral Industry* (eds: K Fytas *et al*), pp 331-337 (Balkema: Rotterdam).

Wooller, R, 2007. Optimising multiple operating policies for exploiting complex resources – An overview of the COMET scheduler, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 309-316 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Simulation of Orebody Geology with Multiple-Point Geostatistics — Application at Yandi Channel Iron Ore Deposit, WA and Implications for Resource Uncertainty

V Osterholt[1] and R Dimitrakopoulos[2]

ABSTRACT

Development of mineral resources is based on a spatial model of the orebody that is only partly known from exploration drilling and associated geological interpretations. As a result, orebody models generated from the available information are uncertain and require the use of stochastic conditional simulation techniques. Multiple-point methods have been developed for petroleum reservoir modelling enabling reproduction of complex geological geometries for orebodies. This paper considers a multiple-point approach to capture the uncertainty of the lithological model at the Yandi channel iron ore deposit, Western Australia. Performance characteristics of the method for the application are discussed. It is shown that the lithological model uncertainty translates into considerable grade-tonnage uncertainty and variability that is now quantitatively expressed.

INTRODUCTION

Geological controls of physical-chemical properties of ore deposits are important, thus, understanding and modelling the spatial distribution of deposit geology is critical to grade estimation, as well as the modelling of any pertinent attributes of orebodies (eg Sinclair and Blackwell, 2002; King *et al*, 1986). In iron ore deposits, for example, geological domains typically include lithology, weathering, ore and contaminant envelopes. Domains for other physical properties such as density, hardness and lump-fines yield may be required. The traditional approach to model geological domains is the drawing of outlines of the geological units by the geologist, resulting in an over-smoothed subjective interpretation. Automatic interpretations are rare and include solids models that are, however, also inherently smooth. Furthermore, such single 'best-guess' interpretations do not account for uncertainty about the location of boundaries and corresponding volumes, leading to inconsistencies between mine planning and production.

Stochastic simulation techniques address the above type of challenges in modelling the geology of, or the uncertainty about, a deposit. Unlike in the petroleum industry, stochastic simulation of geological units of mineral deposits has been limited in the mining industry due to the above-mentioned traditional practices, despite early efforts (David, 1988). The principle behind stochastic simulation is interpreting the occurrence of a geological unit at a location as the outcome of a discrete random variable. This probabilistic approach honours the fact that the geology at any location cannot be known precisely from drilling data. All available information including data, data statistics/geostatistics, and geological interpretations are included in such an approach to yield the most realistic models. Stochastic simulation methods have been developed and tested on geological models of mineral deposits. Methods mainly consist of sequential indicator simulation or SIS (Goovaerts, 1997) type approaches and the truncated pluri-Gaussian simulation approach or PGS (Le Loc'h and Galli, 1997; Langlais and Doyle, 1993). Various implementations and applications include the modelling of mineralised envelopes with a predecessor to SIS approach (David, 1988), simulating geologic units with nested indicators (Dimitrakopoulos and Dagbert, 1994), generation of ore textures with 'growth' (Richmond and Dimitrakopoulos, 2000), simulation of oxidisation fronts with PGS (Betzhold and Roth, 2000), ore lenses in an underground mine (Srivastava, 2005), uranium roll-fronts (Fontaine and Beucher, 2006) and kimberlite pipes (Deraisme and Field, 2006). Alternative approaches include methods based on Markov transition probabilities (Carle and Fogg, 1996; Li, 2007) and object based methods (eg Seifert and Jensen, 2000).

The main drawback of the above methods is their inability to capture non-linear geological complexities, and it becomes obvious when curvilinear features such as faults, multiple superimposed geological phases, fluvial channels, or irregular magmatic bodies are simulated. The reason for this limit is that conventional methods represent geological complexity in terms of second order (two-point) statistics. Variograms describe the variability of point-pairs separated by a given distance and, although they capture substantial geological information (David, 1988), there is a limit to the information they can convey (Journel and Alabert, 1988; Journel, 2007, this volume). Figure 1 illustrates the limits of variograms in fully characterising geological patterns. Figure 1 shows three geological patterns with different spatial characteristics where the variograms of the three patterns cannot differentiate between the three geological patterns.

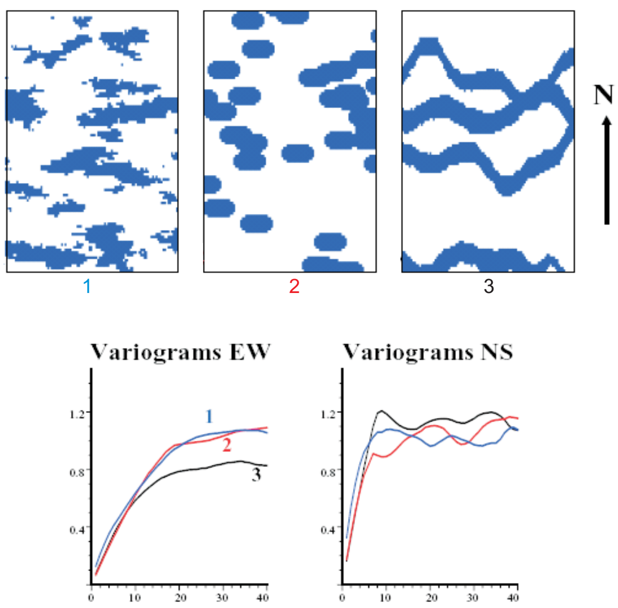


FIG 1 - Vastly different patterns show same variogram (modified from Journel, 2007, this volume).

1. MAusIMM, Resource Geologist, BHP Billiton Iron Ore, 225 St Georges Terrace, Perth WA 6000, Australia. Email: volker.osterholt@bhpbilliton.com

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

In advancing from the above limits, substantial efforts have been made to develop new techniques that account for the so-called high-order spatial statistics. These include the most well established multiple-point (multi-point or MP) approach (Strebelle, 2002; Zhang *et al*, 2006), as well as Markov random field based, high-order statistical approaches (Daly, 2004; Tjelmeland and Eidsvik, 2004) or computer graphic methods that reproduce multiple-point patterns (Arpat and Caers, 2007). These efforts replace the two-point variogram with a training image (or analogue) so as to account for higher order dependencies in geological processes. The training image is a geological analogue of a deposit that describes geometric aspects of rock patterns assumed to be present in the attributes being modelled and reflects the prior geological understanding of a deposit considered.

The multiple-point or MP simulation approach examined herein and adopted for the modelling of the geological units of an iron ore deposit is based on the MP extension of SIS (Guardiano and Srivastava, 1993; Strebelle, 2002; Liu and Journel, 2004), where MP statistics are inferred by scanning a training image (TI). The TI is regarded as a geological analogue, forms part of the geological input, and it should contain the relevant geometric features of the units being simulated. Until recently, the MP simulation approach has mainly been used for modelling of fluvial petroleum reservoirs. It is logical to extend its application to modelling mineral deposits, where the TI can be derived from geological interpretations of the relatively dense exploration or grade control drill hole data, and/or face mappings.

This paper revisits multiple-point simulation as an algorithm for the simulation of the geology for mineral deposits. In the next sections, the MP method is first reviewed and outlined. Subsequently, an application at the Yandi channel iron ore deposit is detailed. Implementation issues, the characteristics of the resulting simulated realisations and the resource uncertainty profile are also discussed. Finally, conclusions from this study are presented.

SIMULATION WITH MULTIPLE-POINT STATISTICS REVISITED

Definitions

Multiple-point or MP statistics consider the *joint* neighbourhood of any number n of points. As indicated above, the variogram can be seen as a MP statistic consisting of only two points; hence, it cannot capture very complex patterns. Using MP statistics sequentially on difference scales, large and complex patterns can be reproduced with a relatively small neighbourhood size n of about 20 to 30. MP statistics can be formulated using the multiple-point data event D with the central value A. The geometric configuration of D is called the template τ_n of size n. Figure 2 shows an example of a data event on a template with n=4.

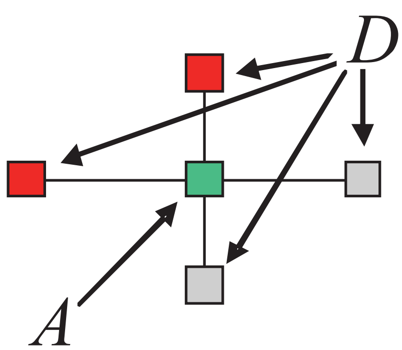


FIG 2 - Naming conventions to define MP statistics.

The size n of the template and its shape can be adjusted to capture any data events informing central value A. As MP statistics characterise spatial relations of closely spaced data, they may not always be calculated directly from drilling data. The method used for this study defines MP statistics on a regular grid, and are inferred from the TI, a regular cell model that serves as a 3D representation of the geological features concerned. The geometries contained in the TI should be consistent with the geological concept and interpretation of the deposit. In practice, this can always be confirmed by a geologist familiar with the deposit.

A conditional simulation algorithm

Consider an attribute S taking K possible discrete states $\{s_k, k=1, \ldots, K\}$, which may code lithological types, metallurgical ore types, grindability units, and so on. Let d_n be a multiple-point data event of n points centred at location $\mathbf{x}$. d_n is associated with the data geometry (the data *template* τ_n) defined by the set of n vectors $\{\mathbf{h}_\alpha, \alpha=1, \ldots, n\}$ and consists of the n data values $s(\mathbf{x}+\mathbf{h}_\alpha) = s(\mathbf{x}_\alpha), \alpha=1, \ldots, n$. While traditional variogram-based simulation methods estimate the corresponding conditional distribution function (*ccdf*) by somehow solving a kriging system consisting on the two-point covariances, the MP *ccdf* is conditioned to single joint MP data events d_n :

$$f(\mathbf{x};s_k \mid d_n) = E\{I(\mathbf{x};s_k) \mid d_n\} = \Pr\{S(\mathbf{x}) = s_k \mid d_n\},\ k=1,\ldots,K \qquad (1)$$

Let A_k denote the binary random variable indicating the occurrence of category s_k at location $\mathbf{x}$:

$$A_k = \begin{cases} 1, \text{ if } S(\mathbf{x}) = s_k \\ 0, \text{ otherwise} \end{cases} \qquad (2)$$

Similarly, let D be a binary random variable indicating the occurrence of data event d_n. Then, the conditional probability of node $\mathbf{x}$ belonging to state s_k is given by the simple indicator kriging (SIK) expression:

$$f(\mathbf{x};s_k \mid d_n) = \Pr\{A_k=1 \mid D=1\} = E\{A_k\} + \lambda[1-E\{D\}] \qquad (3)$$

where, $E\{D\} = \Pr\{D=1\}$ is the probability of the conditioning data event d_n occurring, and $E\{A_k\} = \Pr\{S(\mathbf{x}) = s_k\}$ is the prior probability for the state at $\mathbf{x}$ to be s_k. Solving the simple kriging system for the single weight λ leads to the solution of Equation (3):

$$f(\mathbf{x};s_k \mid d_n) = E\{A_k\} + \frac{E\{A_k D\} - E\{A_k\}E\{D\}}{E\{D\}} = \frac{\Pr\{A_k = 1, D = 1\}}{\Pr\{D = 1\}} \qquad (4)$$

Therefore, given a single global conditioning data event, this solution is identical to Bayes' definition of the conditional probability. However, one might consider decomposing the global event D_J into more simple components whose frequencies are easier to infer. From its definition, it is obvious that D_J can be any one of the 2^J joint outcomes of the J binary data events $A_\alpha = A(\mathbf{x}+\mathbf{h}_\alpha), \alpha=1, \ldots, J$ with $A_\alpha \in \{0,1\}$. Equivalent to the common SIK estimate, the conditional probability of the event $A_0 = 1$ can be written in a more general form as a function of the J conditioning data (Guardiano and Srivastava, 1993):

$$\begin{aligned} &\Pr\left\{A_0 = 1 \mid A_\alpha = s_i; \alpha = 1,\ldots,J; i \in \{1,\ldots,K\}\right\} \\ &= E\{A_0\} + \sum_{\alpha 1=1}^{J} \lambda^{(1)}_{\alpha 1}[A_{\alpha 1} - E\{A_0\}] + \sum_{\alpha 1=1}^{J} \sum_{\alpha 2>\alpha 1}^{J} \lambda^{(2)}_{\alpha 1 \alpha 2}[A_{\alpha 1}A_{\alpha 2} - E\{A_0\}] \\ &+ \sum_{\alpha 1=1}^{J} \sum_{\alpha 2>\alpha 1}^{J} \sum_{\alpha 3>\alpha 2}^{J} \lambda^{(3)}_{\alpha 1\alpha 2\alpha 3}[A_{\alpha 1}A_{\alpha 2}A_{\alpha 3} - E\{A_0\}] + \ldots + \lambda^{(J)} \\ &\quad \left[\prod_{\alpha=1}^{J} A_\alpha - E\left\{\prod_{\alpha=1}^{J} A_\alpha\right\}\right] \end{aligned} \qquad (5)$$

The 2^J - 1 weights $\lambda_{\alpha j}^{(i)}$ call for an extended system of normal equations similar to a simple kriging system that takes into account the multiple-point covariances between all the possible subsets $D_J^{'} \prod_{\beta \in J'} A_\beta, J' \subseteq \{1, \ldots, J\}$ of the global event D_J. These multiple-point covariances are inferred by scanning the training image for each specific configuration. For the case when all J values a_α are equal to 1, Equation (4) is identical to Bayes' relation for conditional probability. The decomposition of the global event D_J illustrates that the traditionally used two-point statistics lose their exclusive status in an extended simple kriging system.

The numerator and denominator of Equation (4) are inferred by scanning a training image and counting both the number of replicates of the conditioning data event $c(d_n)$, and the number of replicates $c_k(d_n)$, among the c previous ones, with the central value $S(\mathbf{x}) = s_k$. In the Single Normal Equation Simulation or SNESIM algorithm (Strebelle, 2002), these frequencies are stored in a search tree enabling fast retrieval. The required conditional probability is then approximated by:

$$f(\mathbf{x};s_k \mid d_n) = \Pr\{A_k{=}1 \mid D{=}1\} \approx \frac{c_k(d_n)}{c(d_n)} \tag{6}$$

To simulate an unknown location **x**, the available conditioning data forming the data event d_n is retained. The proportions for building the *ccdf* (Equation 6) are retrieved from the search tree by searching the retained data event and reading the related frequencies.

The SESIM algorithm and the options provided in the implementation have been covered elsewhere (Strebelle, 2002; Remy, 2004; Liu, 2006) and will not be repeated here in detail. An overview of the general steps of the simulation is given below:

1. Scan the training image and store occurrences of all data events *D*. This may be seen as building a database of jigsaw puzzle pieces of different shapes (*D*) and their central values (*A*) from the TI.
2. Define a random path and visit nodes one by one.
3. Simulate each node by:
   - retrieving all data events (jigsaw puzzle pieces) fitting the surrounding data and previously simulated nodes,
   - derive the local probability distribution from stored frequencies of central values; the probability of finding a certain lithology at the node given the surrounding data event *D* is given by Bayes relation for conditional probability, and
   - pick randomly from the distribution and add simulated node to the grid.
4. Start again at Step 1 for the next realisation, as may be needed.

CASE STUDY

Geology of the Yandi channel iron ore deposit

A number of operations in the Pilbara region of Western Australia produce iron ore from clastic channel iron ore deposits (CID) formed in the Tertiary. These deposits contribute a significant portion of the overall production from the region. Their formation in a fluvial environment with variable sources and deposition of the material as well as post-depositional alteration resulted in very large high quality but complex iron orebodies. The CID consists of an incised fluvial channel filled with detrital pisolite ore that is affected by variable clay content. Ore qualities depend on lithological domains that are modelled using sectional interpretations and grade cut-offs. Defining and modelling boundaries to low-grade overburden and to internal high-aluminous areas cause problems in the current resource estimation, assessment and modelling practices.

Figure 3 shows a schematic cross-section through the CID showing the various lithologies. The erosional surface of the incised channel is covered by the BCC. From bottom to top, LGC, GVL, GVU, WCH and ECC sequentially fill the channel. ALL covers the whole channel sequence including the surrounding WW bedrock. The GVU and the GVL are the only units that currently fall within economic mining parameters. The WCH is a high SiO_2 waste unit with a gradational uncertain boundary to the GVU below. These two ore bearing lithologies and the transitional WCH are encapsulated by high Al_2O_3 waste (WAS), which consists of various clay-rich low-grade strata in both the hanging wall and the footwall (ALL, ECC, LGC, BCC and WW).

The study area is located at Junction Central deposit of the Yandi CID (Figure 4) and consists of the so-called Hairpin model area. The existing Hairpin resource orebody model is rotated by 45°. To accommodate for this rotation, this case study was performed in a local grid with north oriented to 285°. All results are presented in this rotated grid.

The study area has been drilled out in various campaigns to nominal spacing of 100 m by 50 m. This data and the knowledge of absence of CID outside the drilled area are used to interpret the deposit. To introduce the knowledge about undrilled areas into the simulations, the areas around the drilled CID was 'infilled' using 50 m by 50 m spaced data points with WAS code assigned (Figure 5).

Deriving a training image

The training image (TI) has to contain the relevant geological patterns of the simulation domain. In the context of the Yandi CID, this means that the TI has to characterise the shape of the channel and of the internal boundaries within the study area. The geological model of the mined out initial mining area (IMA) is the best available source for this information:

1. the model is based on relatively dense exploration drilling on a 50 m × 50 m grid, and
2. it consists of a straight section of the CID thus having a constant channel axis azimuth.

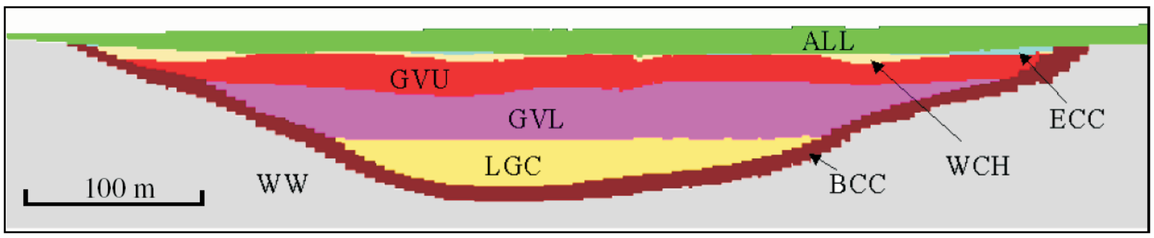


FIG 3 - Schematic cross-section through CID showing the various lithologies. Lithologies: ALL – alluvium, ECC – eastern clay conglomerate, WCH – weathered channel, GVU – goethite-vitreous upper, GVL – goethite-vitreous lower, LGC – limonite-goethite channel, BCC – basal clay conglomerate, WW – Weeli Wolli formation.

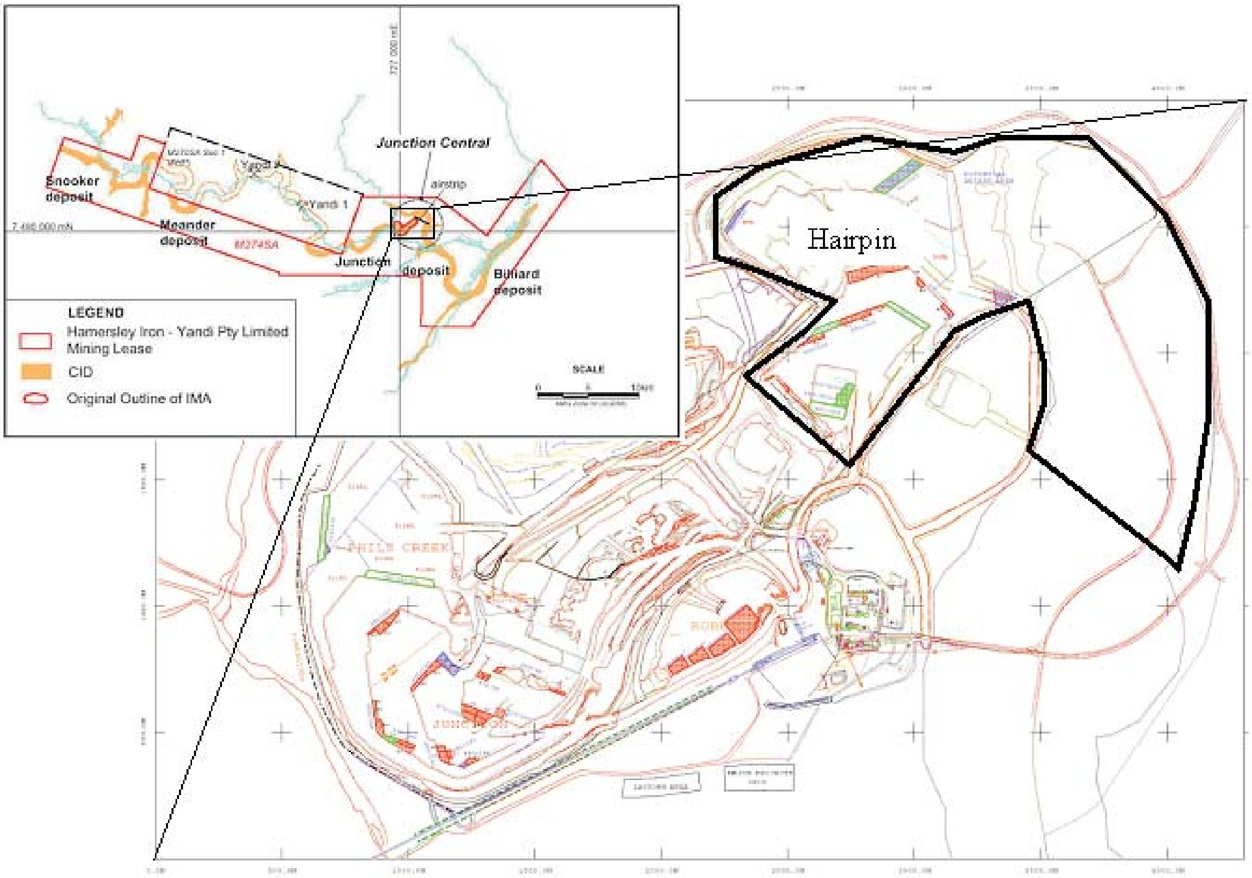


FIG 4 - Location map of the study area.

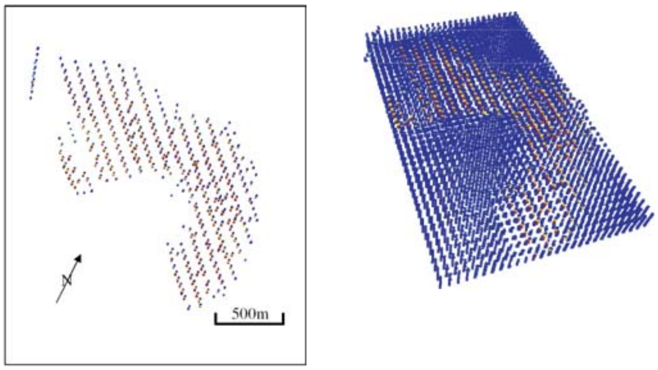


FIG 5 - Drill hole data set (left) and infilled grid data (right).

The TI was generated as a regular geological block model of the IMA prospect into 10 m × 10 m × 1.25 m blocks. This resulted in 80 × 80 × 125 or 800 000 blocks in total. Four slices of the training image are depicted in Figure 6 and show the main direction of the channel (EW) and the slight undulation of the channel axis. The boundaries between the various units are smooth, reflecting the wireframe model upon which the TI was based.

As such, ensuring that the training image is consistent with the available data within the simulation domain is a measure needed to assess the validity and limits of the TI. Here, the variograms and cross-variograms of the geological categories are used for this validation. Two data sets will be compared with the TI:

1. the data at IMA that was used for constructing the geological model; this shows the differences of two-point statistics occurring between exhaustive 3D data and sparse drill hole data, and

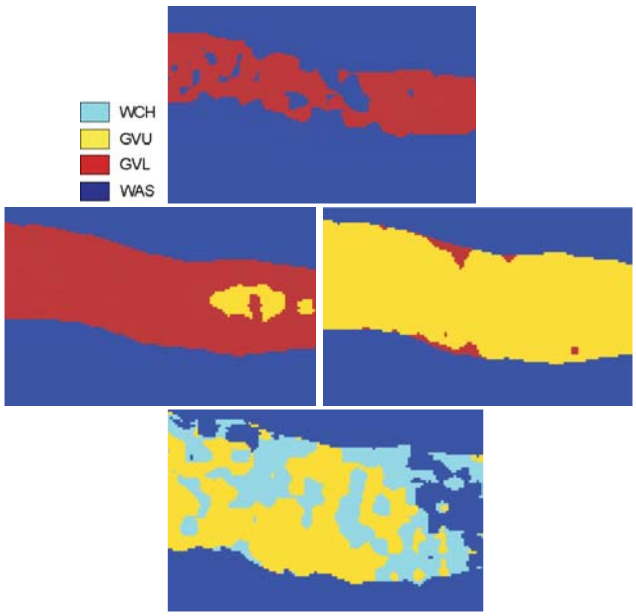


FIG 6 - Bench sections (channel bottom to the top) of the lithological interpretation at Yandi IMA used as training image.

2. the data available in the simulation domain (HPIN) then serves the validation of the TI for use within that domain.

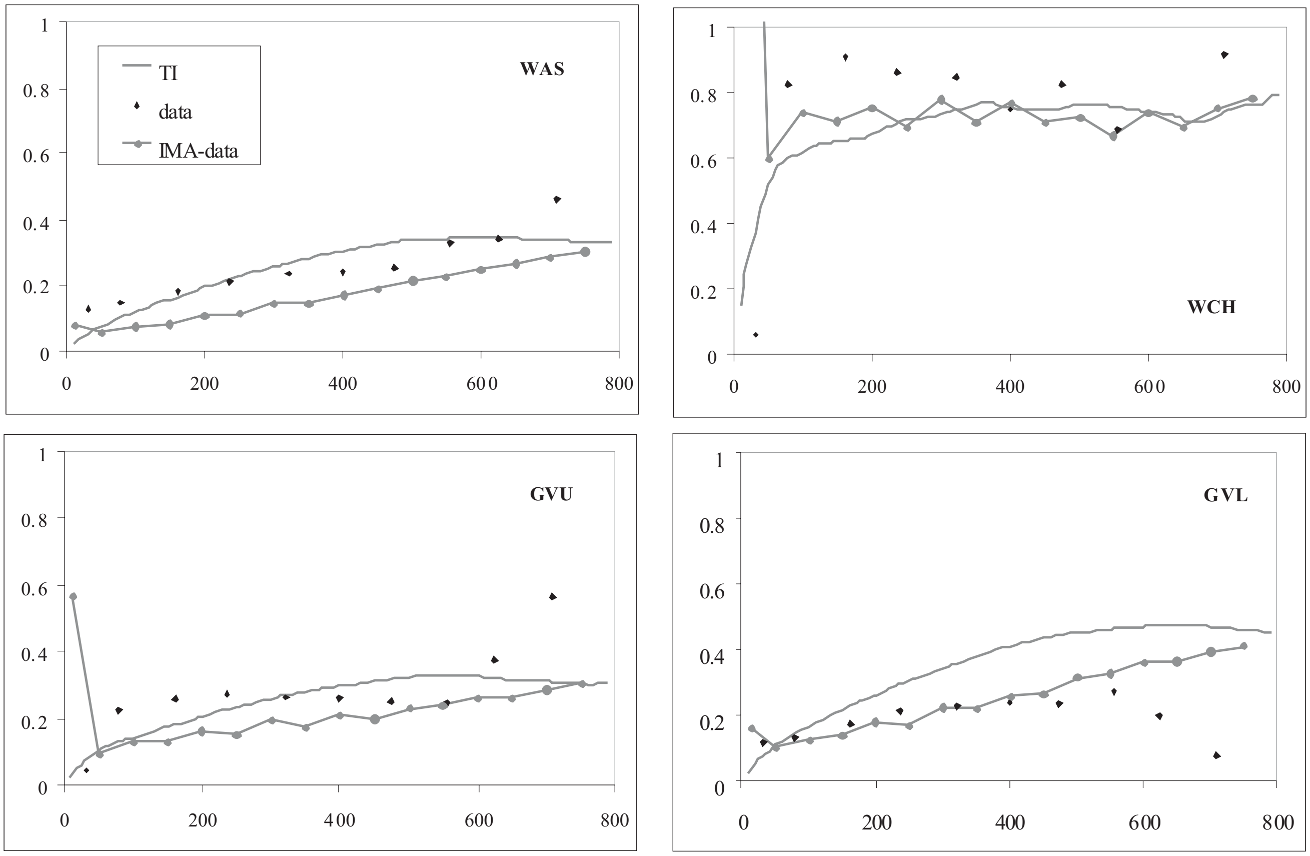


FIG 7 - Variograms of the four categories parallel to the channel axis for the TI, the data (at HPIN) and the data at the IMA, ie the area of the TI. The x-axis shows the lag in metres, the gamma values are given on the ordinate.

Note that this procedure checks the change of the two-point statistics between the data in the TI-domain (IMA) and the simulation domain (HPIN). The statistics of the TI help to evaluate this change: If the differences between the TI-statistics and the HPIN-statistics are grossly larger than those between the TI and the IMA statistics, one will have to consider the reasons for and consequences of these differences.

Overall, the variograms of the four categories perform well in this validation (Figure 7). For unit WAS (please refer to the geological unit abbreviations in the previous section), the HPIN variogram coincides more closely with the TI than the IMA. The WCH variogram of the HPIN data shows larger values at lags up to 350 m than both IMA data variograms and TI. However, these differences are relatively small. The GVU variograms follow a very similar structure; only at short lags do the HPIN variograms have slightly larger values. For the GVL, both data variograms have almost the same values but they are smaller than the TI variogram, suggesting stronger continuity.

Simulation results

To assess the geological uncertainty 20 realisations were generated. Each realisation of the 3.6 m nodes took 7.5 minutes on a 2.4 GHz personal computer, making the process very practical in terms of computational requirements.

Figures 8 and 9 show bench 490RL and a cross-section of the channel, respectively; each figure includes two realisations along with the interpreted deterministic model (wireframe). The bench view shows that the overall shape of the channel has been well reproduced. The incised shape of the channel was generated on a large scale and the stratigraphic sequence has been reproduced. The continuation of the tributary in the north-east was not generated due to very widely spaced drilling in the area. The

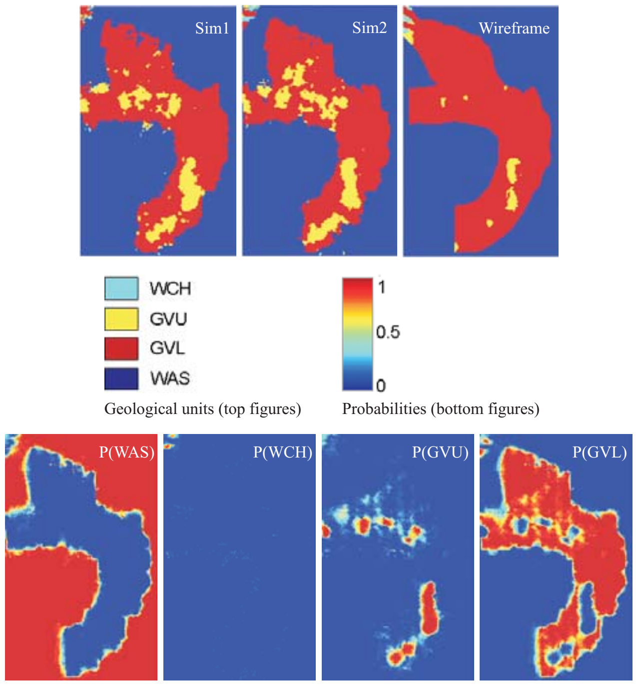


FIG 8 - Two simulations and wireframe interpretation (top); probability maps (bottom) for bench 490mRL – units WAS, WCH, GVU and GVL.

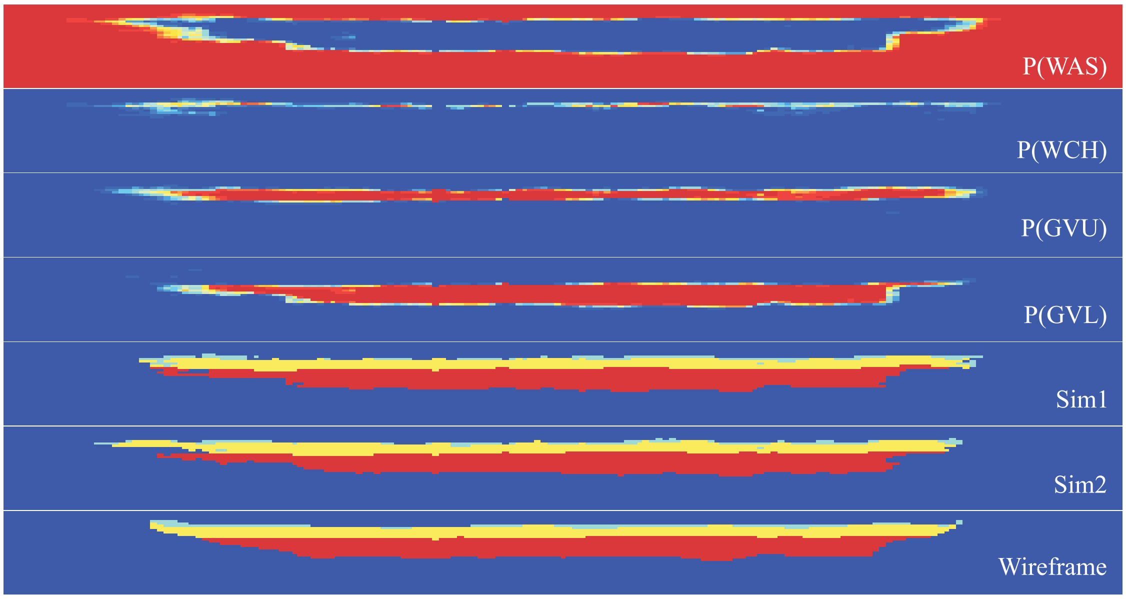


FIG 9 - Two simulations and interpretation and indicator probability for an E-W cross-section (see Figure 8 for colour coding).

proportion of GVL in this bench is higher in the simulations than in the interpreted model however, globally, proportions were reproduced. Boundaries in the simulations are less smoothed for both the GVL-GVU and the GVL-WAS contacts. In some areas, channel material was generated in small pods outside the continuous channel. The cross-section view supports these observations. However, on the channel margins, holes and saw-tooth shaped contacts are inconsistent with the depositional environment of the deposit.

Bench 490RL (Figure 8) cross-cuts the boundary of GVL and GVU. The boundary is undulating and shows an increased irregularity in comparison with the wireframe model. Furthermore, the overall proportion of GVU in bench 490RL is larger in the realisations than in the HIY model. On average probability for unit GVU (P(GVU)), the locations of the lowermost parts of the GVU are related to the wireframe model. However, there are areas in the northern part of the channel where the realisations contain GVU, while the wireframe model consists mainly of GVL. At the southern end of the channel, the GVU patches in the realisations have an increased extension compared to the wireframe model. The outline of the GVU to the surrounding WAS in the realisations is very fuzzy, overall, compared with the wireframe model. This higher disorder occurs on two scales:

1. On a very fine scale of a few blocks, the outline is strongly undulating.
2. On a larger scale of about 15 - 25 blocks, the undulations are less extreme. However, they are still present and not consistent with the TI.

In the cross-section in Figure 9, the shapes of the channel margins are not well reproduced. Instead of an expected rather smooth outline as in the wireframe model, the appearance is sharply stepped (left margin of Sim1 and Sim2). The top part of the channel is very fringy. All the sections depicted in Figure 9 show saw-tooth shaped features at the channel margins, indicating slight problems of the algorithm to reproduce the patterns of the channel margins.

Reproduction of two point statistics

The validation takes the major direction of continuity, EW or along the channel axis, into account: Figure 10 shows the experimental variograms of the data (black diamonds), of the TI (dark grey line), and of the 20 simulations (bright grey lines). The consistency of the data and the TI was described earlier. Two interesting aspects are compared here: (a) simulations versus TI; and (b) simulations versus data.

The WAS variograms are well reproduced in the main direction (EW), but the experimental data variograms suggest less continuity of lags up to 350 m, although this difference is not excessive. For WCH, the variogram reproduction is mediocre and suggests more continuity of the simulations compared to the data. The simulations deviate for lags larger than 50 m and reach the sill of the TI-variogram only at a lag of about 450 m. GVU and GVL variograms are well reproduced and correspond to the experimental data variograms. Cross-variogram reproduction for WAS/GVL and GVU/GVL is good regarding the TI, however there is inconsistency with the data.

Volumetric differences with deterministic wireframes and uncertainty in grade tonnage curves

The intersection of stochastic realisations and estimated grades allows an assessment of uncertainty due to uncertain geological boundaries. For example, Al_2O_3 is chosen here to show the differences between simulated geology and conventional wireframing, because Al_2O_3 is not a well understood variable in the resource model of the deposit. Grade-tonnage curves below Al_2O_3 cut-offs are generated to reflect ore cut-offs. Blocks were selected only within the limits of the ultimate pit as optimised for the deposit at Harpin and below the WCH/GVU boundary that serves as the hanging wall ore limit. The grades used in the comparisons are estimated conventionally (ordinary kriging) and within each of the 20 simulated lithology models.

A two per cent Al_2O_3 cut-off was applied to the Yandi Hairpin block grades to generate a product of about 1.35 per cent Al_2O_3. Figure 11 shows the grade – tonnage curve of Al_2O_3 for the resource within the ultimate pit limits and the uncertainty profile for Al_2O_3 grade and resource tonnage. The two figures compare results based on the simulated lithology models (solid lines) and the deterministic (wireframe) lithology model (dashed line). The grade uncertainty appears relatively small. However, the resource tonnage indicated by simulations is on average 12 Mt (nine per cent), smaller than the tonnage indicated by the best-guess wireframe model. The simulations allow for estimating a tonnage confidence interval. With 70 per cent confidence, the final pit at

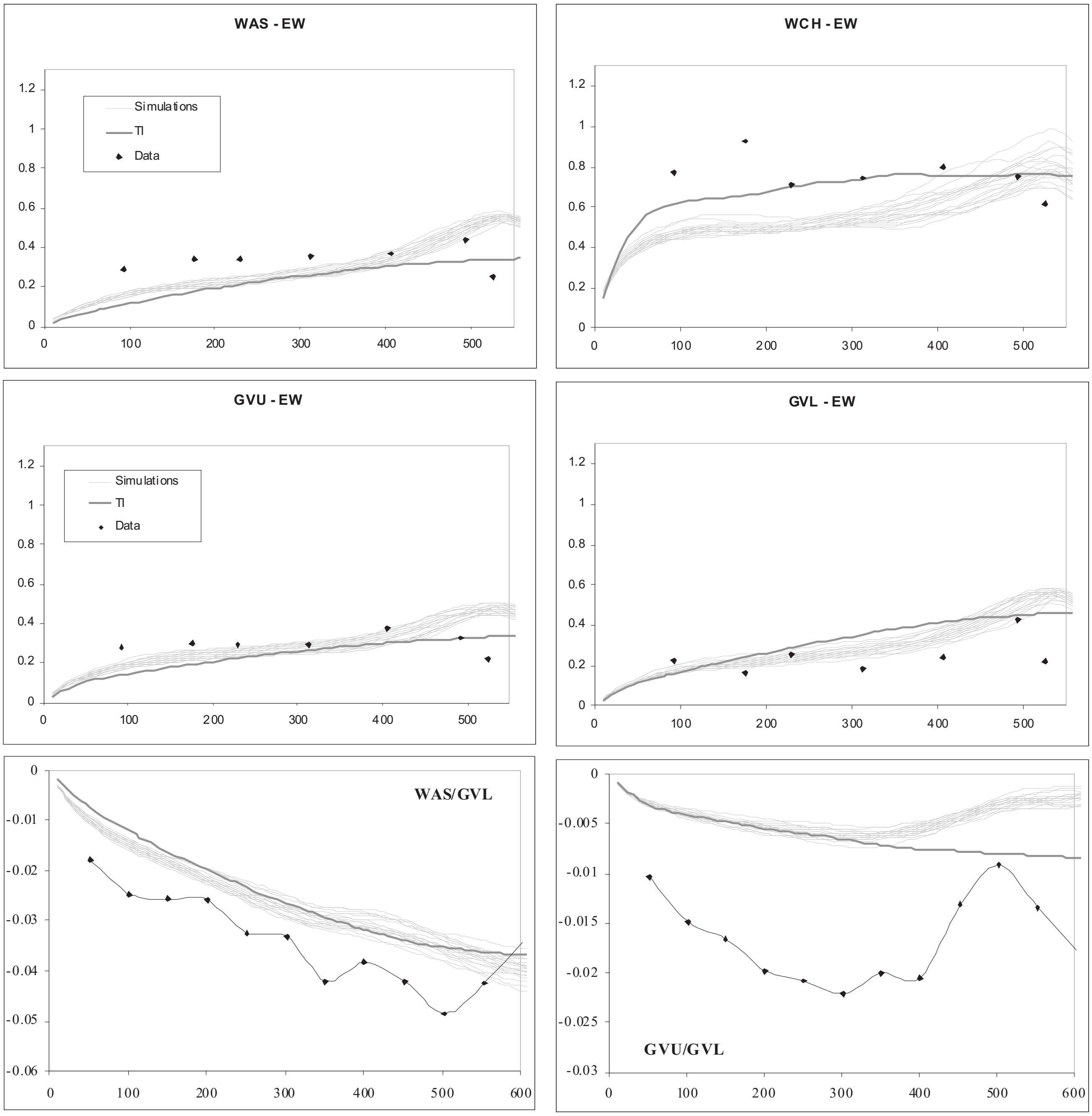


FIG 10 - Variogram and cross-variogram reproduction of simulations versus TI and data for various units.

the Hairpin deposit contains 95 - 97 Mt of ore within Al_2O_3 specifications. This shows that the contribution of the geological uncertainty to the overall grade uncertainty is considerable.

CONCLUSIONS

Multiple-point simulation provides a practical and powerful option to assess uncertainty in the geologic units of mineral deposits. The application of the MP method at Yandi utilises geometric information from a mined-out area. The generated realisations are easily comparable to the existing geological model and reproduce general channel shapes and the rotation of the channel axis. Geometries borrowed from the mined-out area are, in general, well reproduced. The position of boundaries in between drill holes changes from realisation to realisation, thus reflecting the uncertainty about the boundaries' exact shape. On the margins of the channel, the generated patterns are not always geologically meaningful. The MP method can incorporate information from dense drill hole data as available in typical mining applications.

The visual validation showed inconsistencies of the algorithm, reproducing patterns at the margins of the channel. In bench views, the outline of the GVL, the GVU, and the WCH undulates on a scale of 15 - 25 blocks. Additionally, the simulations show a strong, short-scale fuzziness for the GVU and the WCH. This visual impression is underpinned by the larger perimeter-to-volume ratio of the realisations compared to the TI. In the cross-sections, the major critical observation is that the erosional contact to the Weeli-Wolli formation is not consistent with observations in the pit nor with geological knowledge originating from modern geomorphologic analogues. Two sources for these issues with pattern reproduction have to be considered, ie the TI and the algorithm.

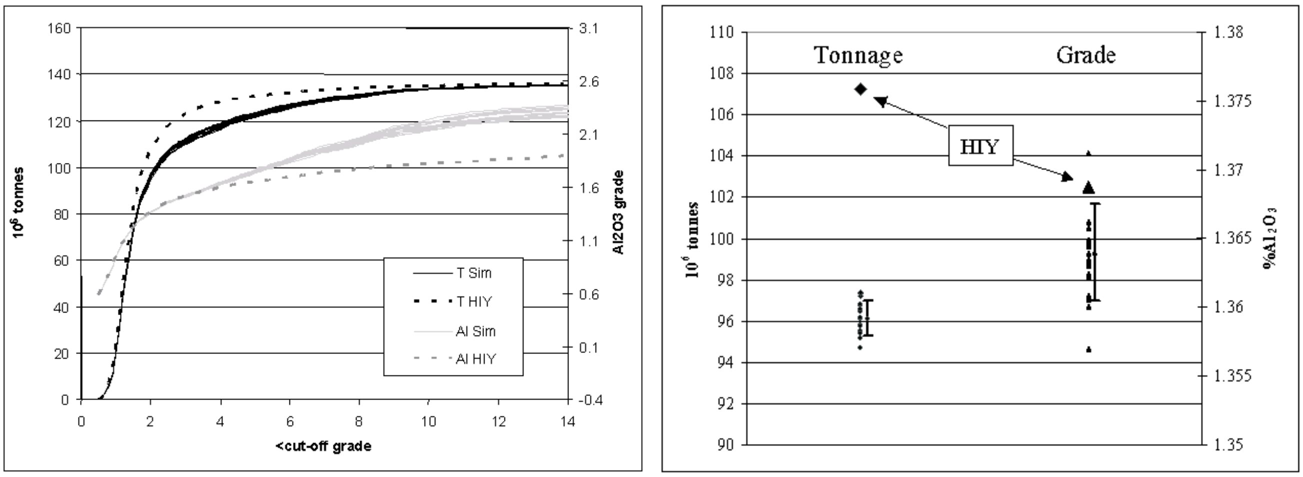


FIG 11 - Al_2O_3 grade-tonnage curves between WCH and the ultimate pit limits (left) and uncertainty profile at two per cent Al_2O_3 (right) where bars depict the mean of the simulation ±1σ . Note that the grade variability (<1 per cent) is not significant. (T – tonnage, Al – Al_2O_3, Sim – simulations; HIY – wireframe.)

It was shown that the TI and the data in the simulation domain are not fully consistent with respect to two-point statistics. The extent to which this influences the quality of reproduced patterns is difficult to assess. Using a set of different training images can provide further insight.

Resource grade and tonnage uncertainty due to uncertain lithological boundaries was assessed by combining probabilistic realisations of the geology with a standard grade estimation technique. At an alumina cut-off of two per cent, the ore tonnage based on the simulated geology ranges from 94.5 to 97.5 Mt (wireframe model: 107 Mt) with bulk alumina grades below the cut-off ranging insignificantly between 1.357 per cent and 1.37 per cent (interpreted model: 1.37 per cent). Using grade simulation instead of grade estimation techniques would add realistic grade variability to this model and allow the assessment of total grade tonnage uncertainty.

Potential areas of application are in areas of little geological understanding or definition of boundaries by drilling. At Yandi, internal clayey high-aluminous waste that cannot be defined with the 50 - 100 m spaced resource evaluation drilling and simulation could create value by better defining grade tonnage curve with regard to contaminants. Training images could be constructed from geological interpretation and data gained in previously mined areas of the deposit.

ACKNOWLEDGEMENTS

Special thanks to Michael Wlasenko and Jim Farquhar from Rio Tinto Iron Ore for their support of the case study at Yandi.

REFERENCES

Arpat, G B and Caers, J, 2007. Conditional simulation with patterns, *Mathematical Geology*, 39(2).

Betzhold, J and Roth, C, 2000. Characterising the mineral variability of a Chilean copper deposit using pluri-Gaussian simulations, *Journal of the SAIMM*, March/April, pp 111-120.

Carle, S F and Fogg, G E, 1996. Transition probability-based indicator geostatistics, *Mathematical Geology*, 28(4):5453-5476.

Daly, C, 2004. High order models using entropy, Markov random fields and sequential simulation, in *Geostatistics Banff 2004* (eds: O Leuangthong and C Deutsch) Vol 2, pp 215-224 (Springer: Dordrecht).

David, M, 1988. Handbook of Applied Geostatistical Ore Reserve Estimation (Elsevier: The Netherlands).

Deraisme, J and Field, M, 2006. Geostatistical simulations of kimberlite orebodies and application to sampling optimisation, in *Proceedings Sixth International Mining Geology Conference*, pp 193-203 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopoulos, R and Dagbert, M, 1993. Sequential modeling of relative indicator variables: Dealing with multiple lithological types, in *Geostatistics Troia '92* (ed: A Soares) Vol 2, pp 413-424 (Kluwer Academic Publishers).

Fontaine, L and Beucher, H, 2006. Simulation of the Muyumkum uranium roll front deposit by using truncated plurigaussian method, in *Proceedings Sixth International Mining Geology Conference*, pp 205-215 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Goovaerts, P, 1997. Geostatistics for Natural Resource Evaluation (Oxford: New York).

Guardiano, F and Srivastava, R M, 1993. Multivariate geostatistics: Beyond bivariate moments, in *Geostatistics Troia '92* (ed: A Soares) Vol 1, pp 133-144 (Kluwer Academic Publishers: Dordrecht).

Journel, A G, 2007. Roadblocks to the evaluation of ore reserves — The Simulation overpass and putting more geology into numerical models of deposits, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 29-32 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Journel, A G and Alabert, F A, 1988. Focusing on spatial connectivity of extreme-valued attributes: Stochastic indicator models of reservoir heterogeneities. SPE paper 18324.

King, H F, McMahon, D W, Bujtor G J and Scott, A K, 1986. Geology in the understanding of ore reserve estimation: An Australian viewpoint, in *Ore Reserve Estimation, Applied Mining Geology* (ed: D E Ranta) Vol 3, pp 55-68 (Society for Mining, Metallurgy, and Exploration: Littleton).

Langlais, V and Doyle, J, 1993. Comparison of several methods of lithofacies simulation on the fluvial gypsy sandstone of Oklahoma, in *Geostatistics Troia '92* (ed: A Soares) Vol 1, pp 299-310 (Kluwer Academic Publishers: Dordrecht).

Le Loc'h, G and Galli, A, 1997. Truncated Plurigaussian method: Theoretical and practical points of view, in *Geostatistics Wollongong '96*, (eds: E Y Baafi and N A Schofield) Vol 1, pp 211-222 (Kluwer Academic Publishers: Dordrecht).

Li, W, 2007. A fixed-path Markov algorithm for conditional simulation of discrete spatial variables, *Mathematical Geology*, 39(2).

Liu, Y, 2006. Using the Snesim program for multiple-point statistical simulation, *Computers and Geosciences*, 32(10):1544-1563.

Liu, Y and Journel, A, 2004. Improving sequential simulation with a structured path guided by information content, *Mathematical Geology*, 36(8):945-964.

Remy, N, 2004. S-GEMS – Stanford Geostatistical Earth Modeling Software: User's Manual. Stanford University [online]. Available from: <http://sgems.sourceforge.net/doc/sgems_manual.pdf> [Accessed: 13 March 2006].

Richmond, A J and Dimitrakopoulos, R, 2000. Evolution of a simulation: Implications for implementation, in *Geostatistics Cape Town 2000* (eds: W J Kleingeld and D G Krige) Vol 1, pp 135-144 (Geostatistical Association of South Africa: Johannesburg).

Seifert, D and J L Jensen, 2000: Object and pixel-based reservoir modelling of a braded and fluvial reservoir, *Mathematical Geology*, Vol 32, pp 581-603.

Sinclair, A J and Blackwell, G H, 2002. *Applied Mineral Inventory Estimation*, 381 p (Cambridge University Press, Cambridge).

Srivastava, R M, 2005. Probabilistic modeling of ore lens geometry: An alternative to deterministic wireframes, *Mathematical Geology*, 37(5):513-544.

Strebelle, S, 2002. Conditional simulation of complex geological structures using multiple-point statistics, *Mathematical Geology*, 34(1):1-21.

Tjelmeland, H and Eidsvik, J, 2004. Directional Metropolis: Hastings updates for posteriors with non linear likelihood, in *Geostatistics Banff 2004* (eds: O Leuangthong and C Deutsch) Vol 1, pp 95-104 (Springer: Dordrecht).

Zhang, T, Switzer, P and Journel, A, 2006. Filter-based classification of training image patterns for spatial simulation, *Mathematical Geology*, 38(1):63-80.

New Efficient Methods for Conditional Simulation of Large Orebodies

J Benndorf[1] and R Dimitrakopoulos[2]

ABSTRACT

The application of conditional simulation techniques for modelling orebodies requires efficient algorithms, particularly due to the large number of grid nodes required, often in the order of tens of millions. In this paper, two new efficient conditional simulation methods are reviewed: the generalised sequential Gaussian simulation (GSGS) and the direct block simulation (DBSIM). Both methods gain computational efficiency by simulating groups of nodes simultaneously, using a local neighbourhood as the conditioning data set. The relationship between the group and local neighbourhood sizes used is found to be important to both the accuracy of results and processing efficiency, and it is assessed numerically through a measure of the loss of accuracy.

Practical aspects of the GSGS are demonstrated and assessed in a case study at a porphyry copper deposit. Computational efficiency is demonstrated in the case study involving orebody models with up to 14 000 000 grid nodes, where the method is up to 20 times faster than the well-established sequential Gaussian simulation. At the same time, GSGS maintains a high level of accuracy. The practical aspects of DBSIM are demonstrated in simulating the same copper deposit in a comparable way to GSGS. In the case study, the computational efficiency of DBSIM is marginally better than GSGS; however, there are two major improvements. First, the application of DBSIM results in a substantial reduction of storage requirements and leads to improved data management. Second, the validation of the reproduction of variogram models is performed at the block support scale, which leads to a substantially more efficient variogram validation process than at the point support scale. Both methods, GSGS and DBSIM, provide efficient and reliable tools for practitioners to assess geological uncertainty in large mining applications.

INTRODUCTION

Conditional simulation techniques are being applied more often in the mining industry, realising the value of information these techniques can generate along the chain of mining (Dimitrakopoulos, in press). However, applications in mining present their own challenges, including the size of simulations, computational efficiency and data management in a range of applications from resource/reserve classification to mine design, production scheduling and production reconciliations, and financial analysis. Large orebody models, frequently discretised by up to 10^8 grid nodes, need to be generated (Omre, Sølna and Tjelmeland, 1993; Godoy, 2003). Using conventional conditional simulation techniques, such as sequential Gaussian simulation (Isaaks, 1990), the simulation process can be substantially time demanding. In addition, data management becomes an issue when large size simulated realisations are needed. The application of conditional simulation would be enhanced if practical and computationally efficient methods were available, as already noted in the technical literature (Ravenscroft, 1994; Godoy, 2003).

There are several conditional simulation methods available (Goovaerts, 1997; Chilès and Delfiner, 1999). A frequently used method is the sequential simulation (Scheuer and Stoller, 1962; Journel, 1994), which is based on the decomposition of the multivariate probability density function of a stationary random function, Z(**x**), **x** $\in$ R^d, into a product of univariate conditional probability density functions (Rosenblatt, 1952). When Z(**x**) is Gaussian, the method is termed sequential Gaussian simulation or SGS (Isaaks, 1990), which is a frequently used method due to its relative computational efficiency. Dimitrakopoulos and Luo (2004) suggest the generalisation of this method, termed generalised sequential Gaussian simulation or GSGS, to enhance computational efficiency. The generalisation is founded upon the observation that adjacent nodes share a common neighbourhood (Figure 1), and therefore the GSGS simulates groups of clustered nodes simultaneously instead of node-by-node. The use of groups of nodes amounts to the decomposition of the multivariate probability density function of Z(**x**) into groups of products of univariate conditional probability density functions. This group decomposition is general and includes as 'end member' cases the SGS, where each group has one node only, and the LU simulation method (Davis, 1987), where all nodes to be simulated are in one group. A major extension of the GSGS is the direct block simulation, or DBSIM, presented by Godoy (2003). DBSIM generates realisations directly on a block support to substantially reduce storage requirements. The method is based on averaging internal nodes of one group during the simulation process. The latter process represents a joint point-block LU-type approach. Both GSGS and DBSIM can be extended to the efficient joint simulation of multi-element orebodies using minimum/maximum autocorrelation factors (Desbarats and Dimitrakopoulos, 2000; Dimitrakopoulos and Fonseca, 2003). Further discussion of multivariable joint simulation is presented in this volume by Boucher and Dimitrakopoulos (2007, this volume).

This paper first reviews the theoretical background of GSGS and DBSIM. Then, using GSGS as an example, practical aspects of efficient conditional simulation methods are linked to accuracy in terms of the neighbourhood sizes used and how they are assessed. Subsequently, computational efficiency is demonstrated in an application of the method to a porphyry copper deposit. The application of DBSIM at the same deposit and a comparison with GSGS conclude the paper.

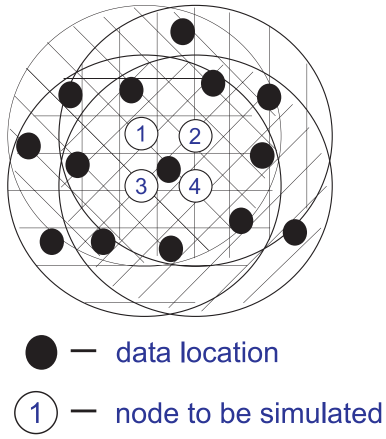


FIG 1 - Shared neighbourhood of group-nodes (Dimitrakopoulos and Luo, 2004).

1. GAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: jorg.benndorf@mcgill.ca

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

EFFICIENT GENERATION OF CONDITIONAL SIMULATION

Following the geostatistical terminology, a geological attribute under consideration is conceptualised as a random function $Z(\mathbf{x_i})$. Consider the stationary random function $Z(\mathbf{x_i})$, $\mathbf{x_i} \in \mathbf{R}^d$, indexed on a discrete grid D_N of N grid nodes at location $\mathbf{x}_i$, i=1,…,N, and a set of conditioning data $\mathbf{d}_n=\{d(\mathbf{x}_\alpha), \alpha=1, \ldots n\}$ representing exploration data. In addition, consider the set including conditioning data and previously simulated nodes Λ_i for each location $\mathbf{x}_i$ such that, $\Lambda_0 = \{\mathbf{d}_n\}$ and $\Lambda_i = \{\Lambda_{i-1} \cup Z(\mathbf{x}_i)\}$, for example $\Lambda_1 = \{\mathbf{d}_n, Z(\mathbf{x}_1)\}$. Following this notation, the conditional simulation on D_N is based on sampling from the N-variate distribution conditioned on the data set Λ_0:

$$F(\mathbf{x}_1,...,\mathbf{x}_N; z_1,...,z_N|\Lambda_0) = P(Z(\mathbf{x}_1) \le z_1,...,Z(\mathbf{x}_N) \le z_N|\Lambda_0) \quad (1)$$

The sequential conditional simulation is based on the decomposition of the multivariate probability density function into a product of univariate conditional distribution functions (Rosenblatt, 1952; Scheuer and Stoller, 1962; Journel, 1994).

$$\begin{aligned} &f(\mathbf{x}_1,...,\mathbf{x}_N; z_1,...z_N|\Lambda_0) = \\ &f(\mathbf{x}_1; z_1|\Lambda_0)\cdot f(\mathbf{x}_2; z_2|\Lambda_1)\cdot...\cdot f(\mathbf{x}_N; z_N|\Lambda_{N-1}) \end{aligned} \quad (2)$$

The decomposition, described in Equation 2, is general and well established in the general field of simulation (Law and Kelton, 1999).

Generalised sequential Gaussian simulation

As mentioned in the introduction, grids D_N that are to be simulated have, in practice, overlapping neighbourhoods between adjacent grid nodes. It is therefore reasonable to consider the use of groups of nodes simultaneously instead of node-by-node as in the common simulation process. This sequential Gaussian conditional simulation of groups of nodes is described in Dimitrakopoulos and Luo (2004) and briefly outlined here.

The simulation starts with the partitioning of the simulation grid D_N into k groups of n_j, j=1,…,k clustered nodes and define N_j as number of nodes in the first j groups $N_j = \sum_{i=1}^{j} \nu_i$; j = 1, ... , k; $N = N_k$; j=1,…,k; $N=N_k$. Then, the decomposition of the conditional density in Equation 2 into conditional densities for k groups becomes:

$$\begin{aligned} &f(x_1,...,\mathbf{x}_N; z_1,...,z_N|\ \Lambda_0) = \\ &\prod_{i=1}^{N_1} f(\mathbf{x}_i; z_i|\Lambda_{i-1})\cdot...\cdot \prod_{i=N_{k-1}+1}^{N_k} f(\mathbf{x}_i; z_1|\ \Lambda_{i-1}) \end{aligned} \quad (3)$$

In the implementation of Equation 3 the exhaustive neighbourhood Λ_{i-1} is replaced by a local neighbourhood λ_{i-1}, resulting in Equation 4:

$$\begin{aligned} &f(\mathbf{x}_1,...,\mathbf{x}_N; z_1,...,z_N|\ \Lambda_0) \approx \\ &\prod_{i=1}^{N_1} f(\mathbf{x}_i; z_i|\lambda_{i-1})\cdot...\cdot \prod_{i=N_{k-1}+1}^{N_k} f(\mathbf{x}_i; z_1|\ \lambda_{i-1}) \end{aligned} \quad (4)$$

where:

λ_{i-1} denotes the local conditioning data set, including sample data and previously simulated nodes

The nodes of group j are generated using Cholesky decomposition (Davis, 1987) of the conditional covariance matrix of one group into an upper **U** and lower triangular **L** matrix, and are computed by the following operation:

$$Z(\mathbf{x}_i^{Nj}|\ \lambda_{i-1}) = \mathbf{m}_j + \mathbf{C}_{j\lambda_{j-1}}\mathbf{C}^{-1}_{\lambda_{j-1}\lambda_{j-1}}(\mathbf{Z}_{\lambda_{j-1}} - \mathbf{m}_{\lambda_{j-1}}) + \mathbf{L}\mathbf{w}_j \quad (5)$$

where:

$\mathbf{m}_j$ and $\mathbf{m}_{\lambda j-1}$ are the vectors of prior means of group $Z(\mathbf{x_i}^{Nj})$ and the set of data in λ_{j-1}

$\mathbf{C}^{-1}_{\lambda j-1\lambda j-1}$ denotes the inverse of the prior covariance matrix of conditioning data

$\mathbf{Z}_{\lambda j-1}$ denotes the vector of the conditioning data set λ_{j-1}

$\mathbf{C}_{j\lambda j-1}$ is the prior covariance between $Z(\mathbf{x_i}^{Nj})$ λ_{j-1}

$\mathbf{w}_j$ is a vector of identically and independently distributed N(0,1) random numbers

It is obvious that, if the number of nodes in one group ν is equal to one, the algorithm is identical to SGS. And if the number of nodes in one group is equal to the whole grid size, the algorithm is identical to LU-decomposition. The implementation of the algorithm includes the following major steps:

1. define a path visiting each group j of the grid and a path visiting each node in a group,
2. define the local neighbourhood of the current group,
3. calculate the conditional mean vector and conditional covariance matrix,
4. generate the simulated values of one group using Equation 5,
5. add the simulated data values of the current group to the conditioning data set, and
6. loop through Steps 2 to 5 until all groups are simulated.

Direct block simulation

A natural extension of the GSGS algorithm is the direct block simulation detailed in Godoy (2003) and briefly reviewed here. When simulating large grids, values simulated need to be retained as conditioning information. This generates increased memory requirements, issues of data management and, in general, leads in practice to performance decline. A new simulation algorithm is developed to simulate directly at the block support scale based on GSGS, whereby the group of nodes discretises a block.

Consider a normal score transformation of the random function $Y(\mathbf{x}_i)$ to $Z(\mathbf{x}_i)$. The regularised random function over a block support $Z_v(\mathbf{x}_j)$ with $\mathbf{x}_j \in R^d$, can be expressed as a linear average of Z( ) over the volume V, centred at the block centre $\mathbf{x}_j$, and approximated by averaging the ν internal nodes from a group: $Z_v(\mathbf{x}_j) = \frac{1}{V}\int_{\mathbf{x}\in v} Z(\mathbf{u})d\mathbf{u} \approx \frac{1}{\nu}\sum_{i=1}^{\nu} Z(\mathbf{x}_i)$. Since the objective is to simulate block values $y_v(\mathbf{x}_j)$ in data space and not Gaussian space $z_v(\mathbf{x}_j)$, after simulation a back-transformation from the Gaussian space into the data space needs to be performed. However, since the normal score transformation was done using point values, there is no back transformation for blocks of type $y_v(\mathbf{x}_j)=\Phi^{-1}_v(z_v(\mathbf{x}_j))$ available, unless restricting distribution assumptions are made. A solution to this problem is given by the approximation $y_v(\mathbf{x}_j) \approx \frac{1}{\nu}\sum_{i=1}^{\nu}\Phi^{-1}(z(\mathbf{x}_i|\ \lambda_{j-1}))$, which is an averaging of all back-transformed internal nodes $y(\mathbf{x}_i| \lambda_{j-1})$ for i=1, …, ν of one group. To derive these values, the group $Z(\mathbf{x}_i^{Nj})=(Z(\mathbf{x}_i),i=1,\ldots,\nu)$ is first simulated, which corresponds to simulating the ν internal nodes discretising the block. After simulation of the internal nodes of a group and back-transforming these, the simulated block value is calculated as the average of the point values in Gaussian space and in data space, and subsequently point values are discarded. The simulated Gaussian block value is then added to the conditioning data set, and the block value in data space is added to the results.

Conditioning data come in two types: point values Λ_i and block values included in the new subset $\Lambda^V{}_i$. With this definition, and considering the screen effect approximation, the GSGS formulation in Equation 4 can be rewritten in terms of point and block conditioning:

$$f(\mathbf{x}_1,\ldots,\mathbf{x}_N;z_1,\ldots z_N| \Lambda_0) \approx$$
$$\prod_{i=1}^{N_1} f(\mathbf{x}_i;z_i| \lambda_{i-1})\cdot \prod_{i=N_1+1}^{N_2} f(\mathbf{x}_i;z_i| \lambda_{i-1}\cup\lambda_1^{\nu})\cdot\ldots\cdot \qquad (6)$$
$$\prod_{i=N_{k-1}+1}^{N_k} f(\mathbf{x}_i;z_i| \lambda_{i-1}\cup\lambda_{k-1}^{\nu})$$

To integrate the block support conditioning data, the algorithm is developed in terms of a joint-simulation. The second variable relates to the block value sequentially derived throughout the simulation process. The parameters of the successive conditional Gaussian distributions are obtained by solving a joint simulation system (Myers, 1989), identical to joint LU-simulation. The simulation of the internal nodes of each block is similar to GSGS. The only difference is the inclusion of conditioning data of different support scale, namely point values and block values. The implementation of the direct block simulation algorithm proceeds as follows:

1. define a random path visiting each of the blocks to be simulated;
2. normalise data;
3. for each block, generate the simulated values in Gaussian space of the internal nodes discretising the block;
4. derive the simulated block value by averaging values of simulated nodes in one group in Gaussian space and calculate the block value in data space;
5. discard values of internal nodes and add the simulated block value in Gaussian space to the conditioning data set; keep the block value in data space as the result; and
6. loop through Steps 3 to 5 until all blocks are simulated.

A major practical advantage of the algorithm above is the decrease in memory allocation due to the discarding of the internal points. Furthermore, the method takes advantage of the GSGS formalism and is thus a fast algorithm. Note that the method does not call for a block transformation function, which is often based on a global change-of-support model. Note also that the variogram validation at a block support scale is substantially more efficient than at point support.

PRACTICAL ASPECTS OF GSGS

Computational costs of GSGS, implemented according to Equation 5, may be assessed in terms of the number of floating point operations (flops) required. Dimitrakopoulos and Luo (2004) show computational costs of GSGS to be:

$$O\left(\frac{N}{\nu}(\nu_{max}{}^3+\nu^3)\right) \qquad (7)$$

where:

O denotes the number of flops (‘in the order of’)

N is the number of grid nodes

ν is number of nodes in one group

ν_{max} is the maximum size of the local neighbourhood, including sample data and previously simulated nodes

The grid size has a linear influence on the runtime behaviour of the algorithm. Critical parameters in terms of efficiency are group size and local neighbourhood size, as they influence the runtime behaviour to the power of three. Considering a grid of N = 1 000 000 nodes, the number of flops required as a function of the group size ν and local neighbourhood size ν_{max}, is shown in Figure 2. For a fixed local neighbourhood size ν_{max}, minimum computational costs occur when $\nu \approx 0.8\ \nu_{max}$. Considering a fixed group size ν, increasing the size of the neighbourhood drastically increases the runtime (number of flops). On the other hand, a smaller neighbourhood size causes a larger difference between the simulated value conditioned to the local neighbourhood and the ‘ideal’ value conditioned to all available information. This difference is the loss of accuracy due to the use of a finite neighbourhood (screen effect approximation) and can be quantified using the measure ‘relative screen effect approximation loss’ (Dimitrakopoulos and Luo, 2004), which is discussed in the next section.

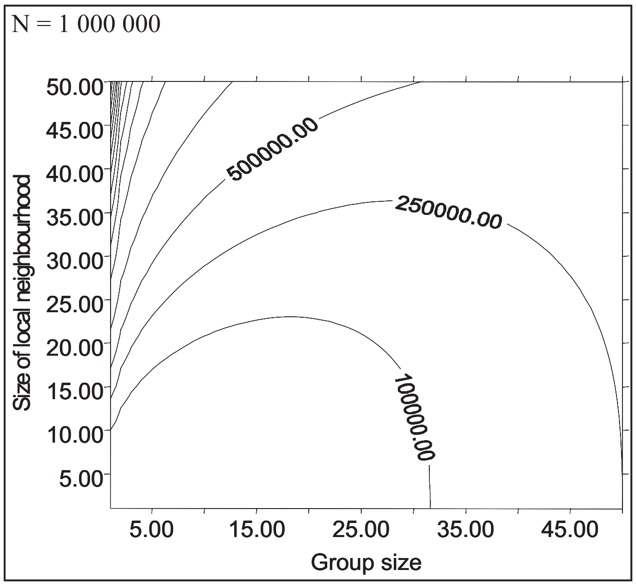


FIG 2 - Theoretical runtime behaviour of the GSGS algorithm as a function of the local neighbourhood size and the group size, for a grid size N of 1 000 000 nodes.

Successful application of GSGS requires an understanding of the interaction between group size ν and local neighbourhood size ν_{max} and their effect on accuracy and computational efficiency. As a convention in the following paragraphs, GSGS with group size ν will be denoted as group configuration GSGS $i \times j \times k$, where i denotes the number of nodes in X direction, and j and k in Y and Z directions respectively.

Group size, neighbourhood size and accuracy: theory and practice

To assess the effects of group size and neighbourhood size, the relative screen effect approximation loss (RSEAL) may be defined by the half of the expected value of the squared difference between simulated values $Z(\mathbf{x}_i)$ conditioned on a local neighbourhood λ_{i-1} and conditioned on all values Λ_{i-1}, standardised by the mean. That is:

$$\rho_R\left(Z(\mathbf{x}_i|\lambda_{i-1}\Lambda_{i-1}\right) = \frac{[E\{Z(\mathbf{x}_i|\lambda_{i-1}) - Z(\mathbf{x}_i|\Lambda_{i-1})\}^2]}{2E\{Z(\mathbf{x}_i)|\Lambda_{i-1}\}} \quad (8)$$

The RSEAL depends on the local neighbourhood size ν_{max} and on the group size ν. To understand the interaction between those two parameters and the accuracy of the result, a relatively simple study can be carried out, as described here. This experimental determination of the RSEAL is based on Equation 8 and includes the following two steps:

1. for the given dataset a base-case simulation is generated using an exhaustive neighbourhood Λ_{i-1}, resulting in a grid containing values $Z(\mathbf{x}_i|\Lambda_{i-1})$; and
2. simulations are subsequently generated, using an incrementally decreased local neighbourhood of size λ_{i-1} and the same random seed, resulting in a grid containing the values $Z(\mathbf{x}_i|\lambda_{i-1})$.

A node-by-node comparison of the generated simulation with the base case, in combination with the application of Equation 8, gives the RSEAL.

For illustration purposes, a test data set containing 100 data is used. The study field represents the southwest area of the Walker-Lake data set (Isaaks and Srivastava, 1989). Simulations are performed on a 2D grid of 7600 nodes, using the inferred covariance structure of the data. Group configurations under investigation are 2 × 2, 4 × 4, 8 × 8 and 16 × 16. Figure 3 summarises the results.

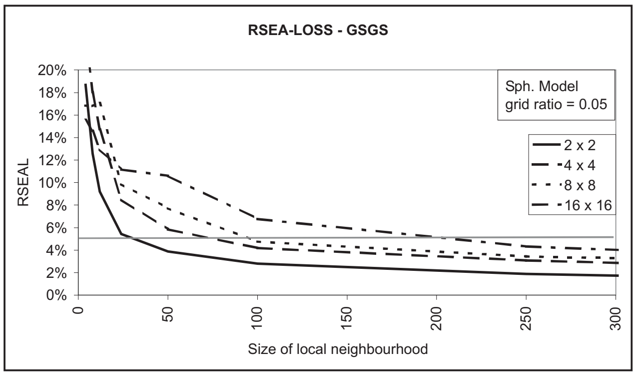


FIG 3 - Relative screen effect approximation loss (RSEAL) considering different GSGS group configurations.

Results show a higher loss of accuracy for larger group sizes than for smaller groups, when considering a fixed local neighbourhood size. A larger local neighbourhood size has to be chosen for larger groups to maintain a certain level of accuracy. By drawing a horizontal line at an acceptable loss of accuracy, eg five per cent, the appropriate local neighbourhood size can be obtained, as shown in Figure 3. Generally, when only a small local neighbourhood is used, internal nodes for large groups no longer share a common neighbourhood. As well, adjacent groups only have a few neighbourhood data in common, which can cause non-continuous transitions between adjacent groups, experienced as artefacts.

An approach as described above provides a general and relatively simple way to obtain an understanding of the effects of neighbourhood sizes on accuracy.

Group size, neighbourhood size and computational efficiency: theory and practice

To understand the relationship between group size, neighbourhood size and computational efficiency, the theoretical runtime behaviour of the GSGS algorithm will be analysed in more detail, and practical aspects will be stressed.

Recall that Figure 2 plots contour lines of the computational costs of GSGS as a function of group size and local neighbourhood size, as in Equation 7. The plot is characterised by very dense contour lines at a group size of one. Thus, considering a fixed local neighbourhood size, an increasing group size substantially decreases computational costs up to a certain point. Following the contour lines, it can be seen that, even if the neighbourhood size has to be increased by a few data when increasing the group size, there is still a reduction of computational costs. The theoretical runtime analysis of an algorithm considers the most expensive computations to be simulated, in the case of GSGS the solution of Equation 5, which has a linear relationship with the grid size N. The theoretical analysis does not consider that there are more operations in the algorithm that are linear with problem size, including handling of the irregular shape of the orebody or the neighbourhood search. Larger group sizes will drastically reduce search time, since it is done simultaneously for all nodes in a group. Then, the algorithm may in practice perform much faster (and does as demonstrated next) than Equation 7 indicates, while still maintaining a high level of accuracy.

An application of GSGS to a porphyry copper deposit aims to demonstrate the practical aspects of the technique. Key questions under investigation, in addition to reproduction of data, statistics and variogram, are the computational costs and performance using different group sizes. The deposit accounts for 185 drill holes in total and 1407 composites of five metres length are taken from these drill holes. After inferring declustered sample statistics and variography, simulated orebody models are generated. To study the effect of different group sizes as a function of grid size, the deposit is discretised by different density grids, as specified in Table 1. The six resulting orebody model sizes range from 72 900 to 14 201 000 nodes.

TABLE 1

Orebody model definitions.

Orebody model name	Model size	X- spacing	Y- spacing	Z- spacing
Model 1	72 900	10 m	10 m	5 m
Model 2	291 600	5 m	5 m	5 m
Model 3	1 821 500	2 m	2 m	5 m
Model 4	3 590 300	2 m	1 m	5 m
Model 5	7 100 600	1 m	1 m	5 m
Model 6	14 201 000	1 m	0.5 m	5 m

Figure 4 shows exemplarily a plan view of orebody realisations for different group sizes applied to orebody model three (discretisation: 2 m × 2 m × 5 m). A visual inspection suggests that the algorithm performs well for all group sizes, and no artefacts can be detected in the realisations. Figure 5 shows the excellent reproduction of histogram and variogram models in normal space using GSGS 2 × 2 × 2. All other group configurations performed equally well on all considered orebody models.

To compare the runtime of GSGS for different group sizes, one realisation was generated for all orebody models, as specified in Table 1, using GSGS with group configurations 1 × 1 × 1, 2 × 2 × 1, 2 × 2 × 2, 3 × 3 × 2 and 4 × 4 × 2. Suitable neighbourhoods were used for different GSGS group sizes, based on the accuracy of results derived in the previous section. Table 2

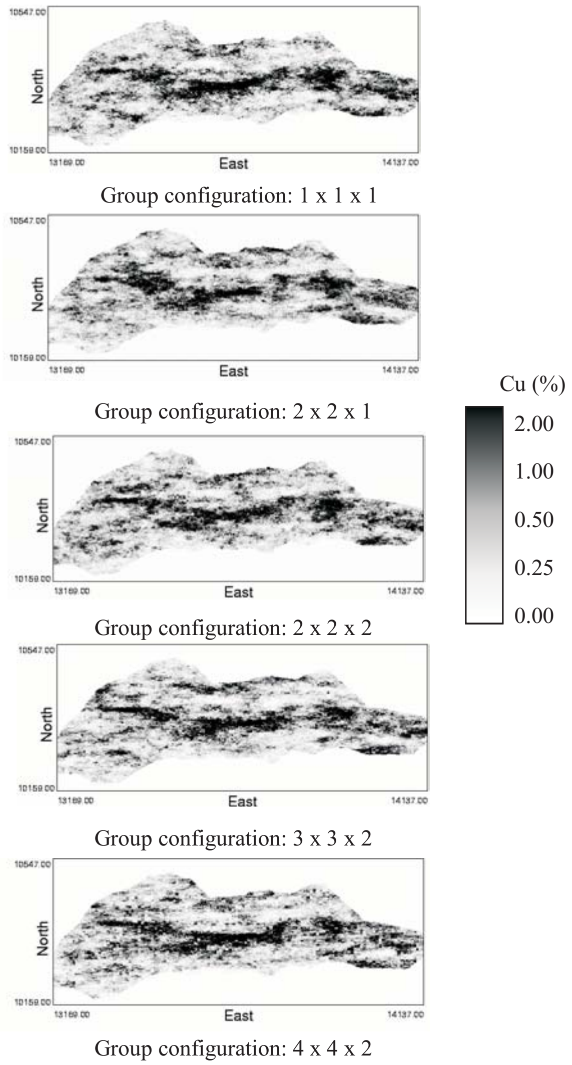


FIG 4 - Plan view of realisations of GSGS using different group sizes applied to model three.

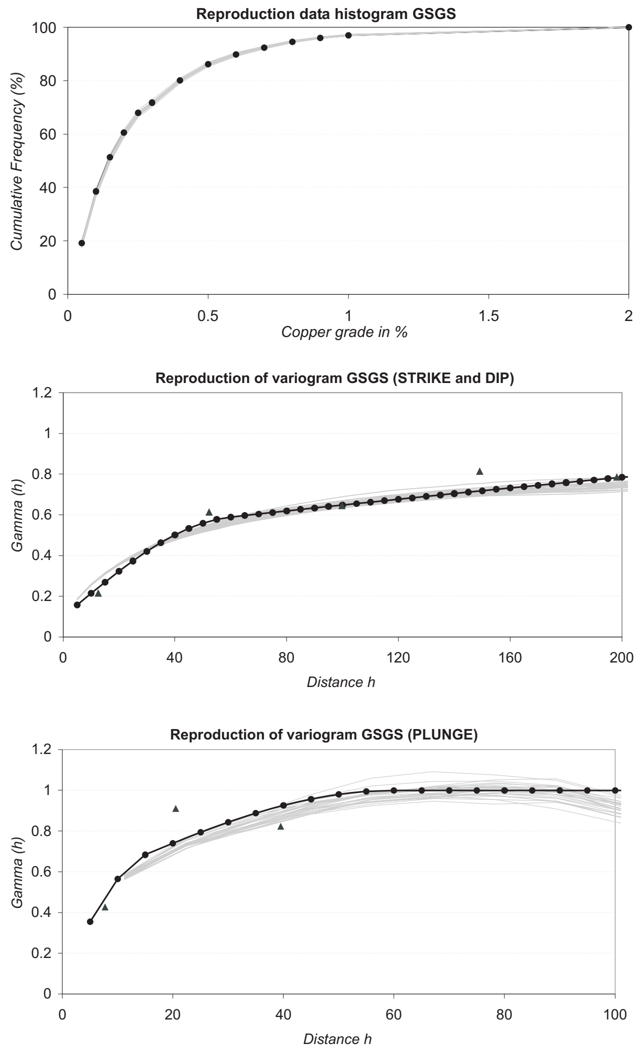


FIG 5 - Reproduction of data histogram, variogram model and reproduction of experimental variogram in normal space for the directions of anisotropy.

TABLE 2

Neighbourhood sizes used for different GSGS group configurations.

	GSGS $1 \times 1 \times 1$	GSGS $2 \times 2 \times 1$	GSGS $2 \times 2 \times 2$	GSGS $3 \times 3 \times 2$	GSGS $4 \times 4 \times 2$
Number of data and previously simulated nodes	20	30	45	60	90

TABLE 3

Runtime of GSGS using different group sizes relative to SGS applied to different large orebody models.

Group size	$1 \times 1 \times 1$	$2 \times 2 \times 1$	$2 \times 2 \times 2$	$3 \times 3 \times 2$	$4 \times 4 \times 2$
Model	**Runtime of GSGS relative to SGS**				
Orebody model 1	100%	53.5%	64.8%	71.8%	142.3%
Orebody model 2	100%	33.1%	39.2%	42.1%	73.9%
Orebody model 3	100%	12.8%	10.8%	9.6%	20.1%
Orebody model 4	100%	19.8%	12.1%	8.3%	14.7%
Orebody model 5	100%	13.8%	4.8%	4.5%	6.0%
Orebody model 6	100%	23.2%	9.8%	4.3%	4.6%

summarises the neighbourhoods used. Figure 6 and Table 3 shows the computing times for each considered orebody model size. To make the comparison general, in Figure 6 runtimes are standardised to GSGS 1 × 1 × 1 applied to model six. Figure 6 concludes that when simulating small orebody models, say less than one million nodes, there is limited benefit of using GSGS considering any of the group sizes. In this case, the runtime of the algorithm can be reduced, by up to about 30 per cent compared with SGS, using small groups. When simulating large orebody models containing several millions of nodes, the runtime can be reduced substantially, up to 20 times in the case of GSGS 3 × 3 × 2. Results demonstrate that GSGS can substantially reduce the computational costs, especially when simulating relatively large orebody models. Experiments with GSGS show that small groups, such as 2 × 2 × 2 to 3 × 3 × 2 nodes, perform best and balance accuracy with efficiency.

ASPECTS OF DBSIM AND COMPARISON

To demonstrate practical aspects of the direct block simulation algorithm, the data from the porphyry copper deposit described in the previous section is used to generate ten realisations of the orebody. Block dimensions are chosen to be 10 m × 10 m × 5 m and are discretised by $10 \times 10 \times 1$ internal nodes. The

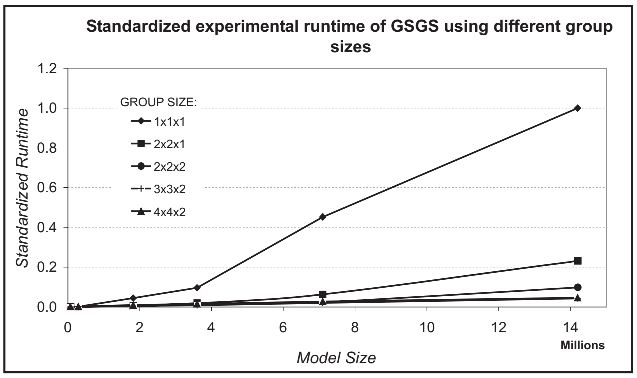


FIG 6 - Standardised experimental runtime of GSGS using different group sizes applied to different large orebody models.

neighbourhood used includes six previously simulated blocks and 12 sample data. Figure 7 represents the reproduction of point-histogram and regularised variogram for DBSIM. Both aspects indicate a good reproduction of data statistics.

To compare DBSIM with GSGS, for instance, in terms of reproduction of sample statistics and the benefit in terms of storage requirements when simulating direct block values, the following is performed. Ten realisations were generated using GSGS 2 × 2 × 2 on a grid using a discretisation of 1 m × 1 m × 5 m. The GSGS neighbourhood was chosen according to Table 2. Realisations were re-blocked to a block size 10 m × 10 m × 5 m, to comply with the block size used for the DBSIM generated realisations. Figure 8 compares realisation number one of Cu per cent in the deposit for both methods. The results are indistinguishable and both methods are 'artefact free'.

The computing time for DBSIM was 20 hours and ten minutes compared with 21 hours and 40 minutes in case of GSGS without reblocking (Pentium 4, 2 GHz processor). The difference can be explained by differences in implementation details and the faster neighbourhood search in the case of DBSIM, since only a few blocks need to be considered instead of a number of point data. The difference in the storage requirements of result files is substantial: 36 Mbytes in case of DBSIM and 3.65 Gbytes in case of GSGS, reflecting the block discretisation. In addition, the validation of the variogram on block support requires, on average, 33 000 pairs to be calculated, on a point support about 3 300 000.

The above results demonstrate that a simulation done directly at block support scale, as realised through DBSIM, meets industrial requirements for the above-discussed reasons. It is more computationally efficient than point-by-point methods and delivers reliable results. Note that issues on DBSIM neighbourhoods are different from GSGS, and generally DBSIM is insensitive to the size used. Experience shows that a neighbourhood with about six blocks and about twice as much sample data is sufficient for excellent simulation results (Godoy, 2003).

CONCLUSIONS

The application of conditional simulation techniques in mining generally requires efficient algorithms for large size applications. In this paper, two new efficient and practical methods for large applications are reviewed: the generalised sequential Gaussian simulation, and the direct block simulation.

Using GSGS as an example, practical issues pertinent to computational efficiency and accuracy were studied. Accuracy of results is predominantly affected by the size of the local neighbourhood. The relative screen effect approximation (RSEAL) is a measure that quantifies this accuracy and assists the selection of suitable neighbourhood sizes for different group sizes. The results presented herein on the size relationships are reasonably general. Results suggest that, when using larger group sizes, larger neighbourhoods sizes need to be considered to maintain the desired level of accuracy.

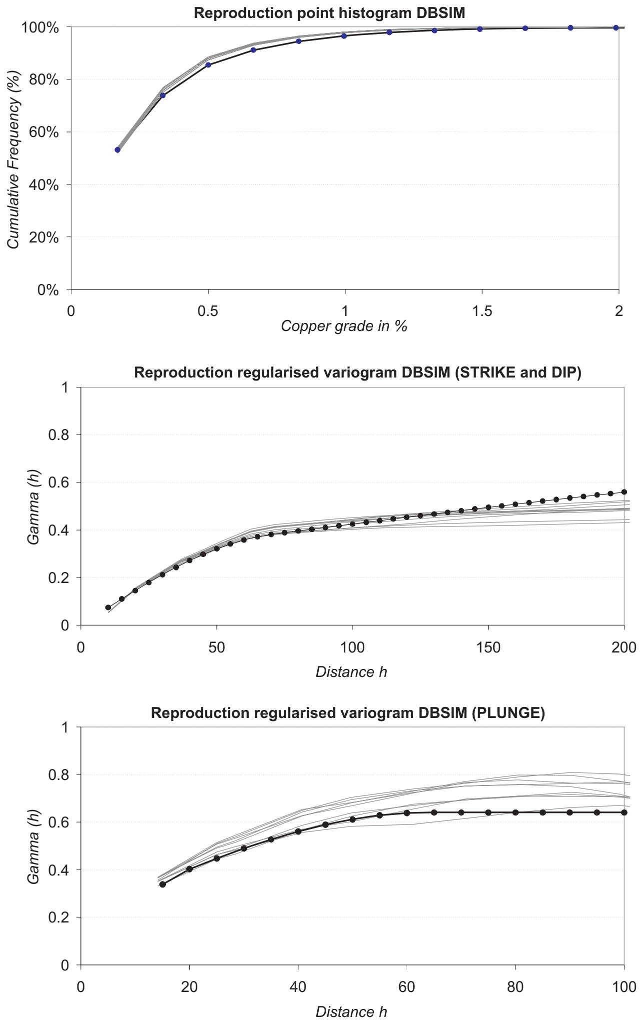


FIG 7 - Reproduction of point histogram and regularised variogram model for DBSIM applied to a copper deposit.

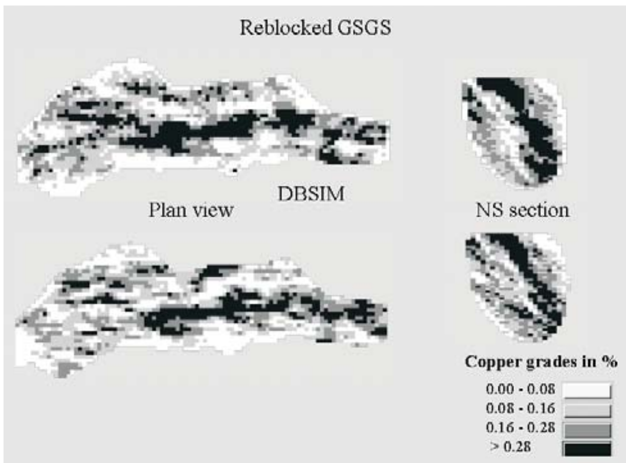


FIG 8 - Plan view and NS section of realisations generated by DBSIM and reblocked GSGS.

The application of GSGS to a porphyry copper deposit demonstrated the efficiency of the method. While maintaining a given level of accuracy, GSGS can improve computational efficiency substantially, being up to 20 times faster.

A comparison of GSGS and DBSIM using the same deposit shows that both algorithms are fast, due to the fact that both are based on the group decomposition of the multi-variate probability density function. The application of DBSIM results in a substantial reduction of storage requirements and leads to improved data management. Both GSGS and DBSIM provide efficient and reliable tools for practitioners to assess geological uncertainty in large mining applications.

REFERENCES

Boucher, A and Dimitrakopoulos, R, 2007. A new efficient joint simulation framework and application in a multivariable deposit, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 345-354 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Chilès, J-P and Delfiner, P, 1999. *Geostatistics, Modelling Spatial Uncertainty,* 695 p (John Wiley and Sons: New York).

Davis, M D, 1987. Production of conditional simulations via the LU triangular decomposition of the covariance matrix, *Mathematical Geology*, 19(2):91-98.

Desbarats, A J and Dimitrakopoulos, R, 2000. Geostatisical simulation of regionalized pore-size distributions using min/max autocorrelation factors, *Mathematical Geology*, 32(8):919-942.

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R and Fonseca, M B, 2003. Assessing risk in grade-tonnage curves in a complex copper deposit, northern Brazil, based on an efficient joint simulation of multiple correlated variables, in *Proceedings APCOM 2003, 31st International Symposium on the Application of Computers and Operations Research in the Minerals Industries*, May, Cape Town, South Africa.

Dimitrakopoulos, R and Luo, X, 2004. Generalized sequential Gaussian simulation on group size n and screen – effect approximations for large field simulations, *Mathematical Geology*, 36(5):567-591.

Godoy, M, 2003. A new minimum risk, strategic open pit mine planning and long-term production-scheduling framework, PhD thesis (unpublished), 256 p, W H Bryan Mining Geology Research Centre, The University of Queensland, Brisbane.

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 483 p (Oxford University Press: New York).

Isaaks, E H, 1990. The application of Monte Carlo methods to the analysis of spatially correlated data, PhD thesis (unpublished), 213 p, Department of Applied Earth Sciences, Stanford University, Stanford, California.

Isaaks, E H and Srivastava, R M, 1989. *An Introduction to Applied Geostatistics*, 561 p (Oxford University Press: New York).

Journel, A G, 1994. Modelling uncertainty: some conceptual thoughts, in *Geostatistics for the Next Century* (ed: R Dimitrakopoulos) pp 30-43 (Kluwer Academic Publishers: Dordrecht).

Law, A M and Kelton, W D, 1999. *Simulation Modelling and Analysis*, 760 p (McGraw-Hill Higher Education: Singapore).

Myers, D E, 1989. Vector conditional simulation, in *Proceedings Third International Geostatistical Congress* (ed: M Armstrong), pp 283-293 (D Reidel Publishing Company: Dordrecht).

Omre, H, Sølna, K and Tjelmeland, H, 1993. Simulation of random functions on large lattices, in *Proceedings Geostatistics Troia '92* (ed: A Soares), pp 179-199 (Kluwer Academic Publishers: Dordrecht).

Ravenscroft, P J, 1994. Conditional simulation for mining, in *Geostatistical Simulations* (eds: M Armstrong and P A Dowd), pp 79-87 (Kluwer Academic Publishers: Dordrecht).

Rosenblatt, M, 1952. Remarks on multivariate transformation, *Annals of Mathematical Statistics*, 23:470-472.

Scheuer, E M and Stoller, D S, 1962. On the generation of normal random vectors, *Technotronics*, 4:278-281.

A Practical Process for Geostatistical Simulation with Emphasis on Gaussian Methods

M Nowak[1] and G Verly[2]

ABSTRACT

The theory of geostatistical simulation is relatively well documented but not its practice, which can be problematic since simulation is not as straight forward as linear estimation. As a result, costly mistakes can be made that sometimes go undetected. In this paper, a process for simulation is introduced with the objective of reducing the likelihood of such mistakes. The context is sequential Gaussian simulation within the mining industry. However, a significant part of the process can be applied in other simulation approaches.

Each aspect of the process is described with some steps receiving greater attention than the others, notably the definition of the simulation objectives, bootstrapping, trend reproduction, post-simulation checks and adjustments, worst/best scenario choice, and risk assessment of the simulated model.

INTRODUCTION

Although the simulation methodology is well documented, a practical process leading to valid and representative realisations of *in situ* grades is rarely a focus of attention within the geostatistical community. To a practitioner, this can lead to the frustration of applying a methodology that may produce poor results. There is a need for simulation procedures that are systematic, robust and easy to follow. This paper presents a practical description of a simulation process with the emphasis on sequential Gaussian simulation (Figures 1 and 2). The intent is to have the simulation process as part of a geological process that Placer Dome has recently defined with the objective of guiding practitioners through exploration, resource and reserve estimation, and reconciliation.

Sequential Gaussian simulation (SGS) starts by defining the univariate distribution of values, eg assay grades, performing a normal score transform of the original values to a standard normal distribution, and assuming multi-normality of the normal scores. The multi-normal assumption ensures that the conditional distribution at a given location is normal with mean and variance provided by simple kriging. Simulation of normal scores at grid node locations is done sequentially, most often with simple kriging using the normal score variogram and a zero mean (Isaaks, 1991; Deutsch and Journel, 1998; Goovaerts, 1997). Once all normal scores are simulated, they are back-transformed to original grade values.

As shown in Figures 1 and 2, the designed simulation process is more complex than just normal score transformation, variogram modelling, simulation and back-transformation. A number of calibration, validation, and adjustment steps have been added to account for trends, and improve the reproduction of variograms and distributions. To reproduce the uncertainty on the grade distribution, a resampling procedure called bootstrap is described that accounts for spatially correlated data. Finally, a risk assessment of the issues that affect the outcome of the simulation is suggested.

The remainder of this paper is a chronological presentation of the steps of the process as shown in Figures 1 and 2. The emphasis of the paper is on the process in general and not on the details. A detailed discussion of some of the steps can be found in Nowak and Verly (2004).

DEFINE OBJECTIVES (I)

Although deceptively simple this step is quite often overlooked. Clearly defined objectives have two advantages. First, they help to design a procedure that will address the issues of specific interest to an exploration or mine manager. Second, they may reduce the number of necessary steps, thus reducing the time spent on the simulation. For example:

- Bootstrapping is a necessary step when the objective of simulation is to assess the difference between an optimistic and pessimistic scenario. However, it may not be necessary for a study of weekly fluctuations in sulfur content.
- A scope study may require simulating only a representative portion of the orebody.
- When future open pit grade control is considered, there is no need for simulating a dense grid of values. For example, a simulation of blast hole values on a 5 × 5 m grid spacing, followed by block kriging based on the simulated values, will reduce significantly the number of simulated nodes.
- In an active operation, kriging and not simulation may be the best solution for grade control if the profit is a linear function of the grade (Verly, 2005).

DEFINE ZONES (II)

Two zones or envelopes should be defined: a simulation zone, and an exploratory data analysis (EDA)/validation zone. The simulation zone covers the area of interest for the purpose of a study, ie the area that will be simulated. Areas where reasonable simulation results cannot be achieved should be excluded or at least flagged as such.

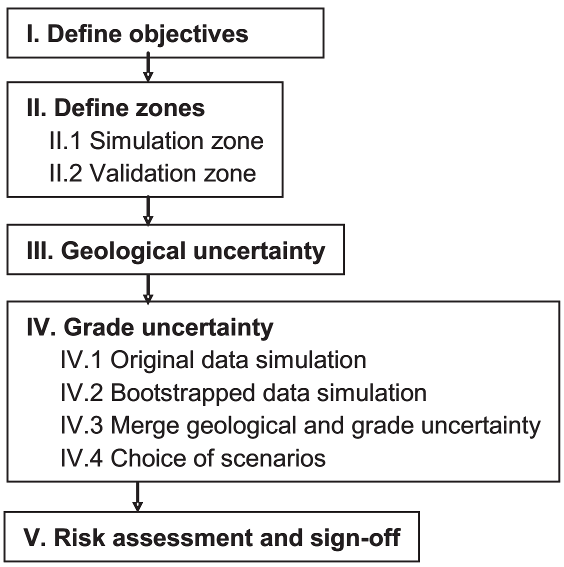


FIG 1 - High-level simulation process.

1. SRK Consulting, 2200 – 1066 West Hastings Street, Vancouver BC V6E 3X2, Canada. Email: mnowak@srk.com

2. AMEC, 400 – 111 Dunsmuir Street, Vancouver BC V6B 5W3, Canada. Email: georges.verly@amec.com

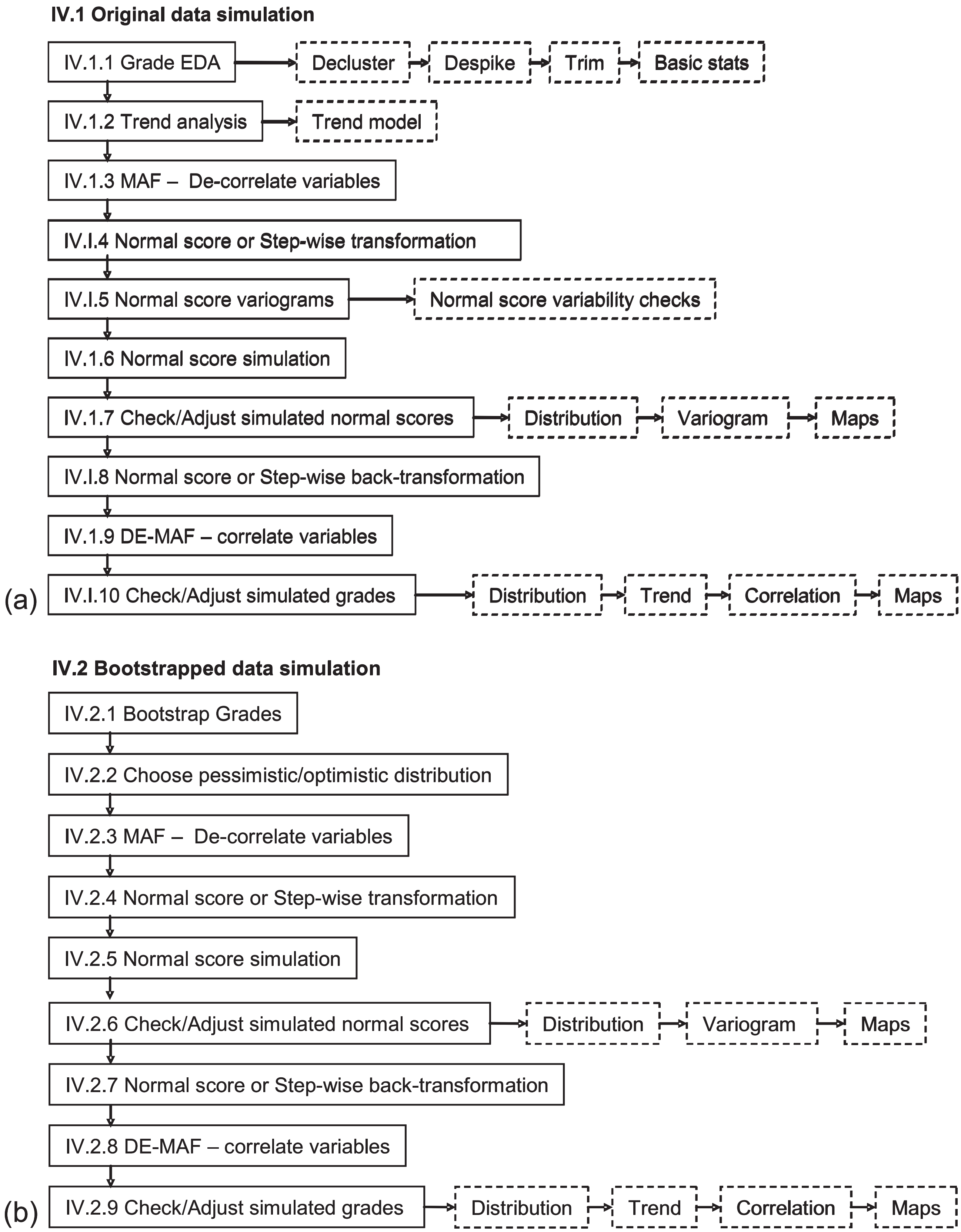


FIG 2 - Process for simulation of (a) original data; and (b) bootstrapped data.

The EDA/validation zone serves a dual purpose: calibration and validation. The calibration is geared towards a proper assessment of the simulation parameters such as trimming values, grade distributions, and variograms in a series of geological domains. The simulated results are then validated against the original calibrated statistics. If the validation zone is large, eg identical to the simulation zone, then extensive areas may not be properly sampled and the statistics of the simulated values will be different from the calibrated statistics. This could make the validation exercise very difficult, or even worse, could lead to erroneous conclusions and unnecessary modifications to simulated values (Figure 2a, Step IV.1.7).

GEOLOGICAL UNCERTAINTY (III)

Jackson *et al* (2003) give a striking example of geological uncertainty where three reasonable geological interpretations by different geologists result in significantly different ore tonnages. Geological uncertainty, however, is seldom included in a simulation study. Instead, a single geological model is developed from the drill hole data, which artificially restricts the space of uncertainty.

Specific geological scenarios, dependent on the objectives of the study, should be used in the simulation process (Srivastava, 2005). Three simulation methods that have been used in the mining industry for geological uncertainty are the indicator simulation (Alabert and Massonat, 1990; Deutsch and Journel, 1998), the plurigaussian technique (Armstrong *et al*, 2003; Skvortsova *et al*, 2000), and a probability field based approach (Srivastava, 2005). Multi-point statistics (Strebelle, 2002; Journel, 2007, this volume; Osterholt and Dimitrakopoulos, 2007, this volume) and the potential field method (Chilès *et al*, 2007, this volume) look also very promising.

The geological uncertainty simulation process is not considered in this paper. This process, however, has some similarities to the grade uncertainty process, such as EDA statistics (Figure 2, IV.1.1), trend analysis (IV.1.2), bootstrap (IV.2.1), checks and adjustment (IV.1.10), and scenario choice (IV.3).

GRADE UNCERTAINTY (IV)

Original data simulation (IV.1)

This section corresponds to Figure 2a.

Grade EDA (IV.1.1)

One objective of simulation is to reproduce the grade distribution. The input grade distribution must therefore be estimated properly, which entails declustering (Isaaks and Srivastava, 1989), trimming of high values, despiking (Verly, 1984 and 1985), and limiting the work within the EDA envelope mentioned earlier.

Sample declustering is needed to get an unbiased estimate of the grade distribution. Two of the popular methods are the cell and the polygonal declustering methods. Whatever the method, it should be used with care. The declustering cell size should be realistic; the polygonal declustering weights should not be too large on the fringe of the EDA zone.

Trimming is needed to avoid spreading high-grade values. The choice of the trimming values is similar to the one used for estimation. There should be some explanation if the trimming values chosen for simulation are significantly different from those chosen for estimation.

Finally, if there is a significant amount of identical values, despiking is needed to ensure a proper normal score transform and/or to avoid an artificial noise in the normal score variogram. The method simply involves ordering identical assays according to the surrounding average grade. Note that despiking is probably not necessary if residuals after trend removal are simulated instead of original grade values.

Trend analysis (IV.1.2)

Trends are not always well reproduced in sequential Gaussian simulation, owing to the stationarity assumption necessary for the normal score transform and the strict multinormal assumption usually assumed for the normal scores (normal scores multinormally distributed with mean 0 and covariance $C(h)$).

One simple way to deal with this problem is to filter the trend, simulate the residuals, and add the trend after simulation (Deutsch, 2002). Unfortunately, this process may produce simulated grade values that are negative. An obvious way out is to reset the negative values to zero, but this may result in significant bias and poor reproduction of the trends. A second solution consists in defining the local prior means to be used by SK with a correction factor for all kriging variances (Goovaerts, 1997). A third solution is given by Leuangthong and Deutsch (2004) who suggest a step-wise normal score transform, which is discussed further in Step IV.1.4. A fourth solution consists in a post-simulation trend adjustment (Nowak and Verly, 2004) that is further discussed in Step IV.1.10.

In most circumstances, it is useful to analyse the trend by producing average grade profiles along various directions (eg elevation, easting, northing). If necessary, a 3D estimate of the trend should then be obtained, for example by ordinary kriging with a relatively high nugget, and used in Step IV.1.4 or Step IV.1.10.

MAF – Decorrelate variables (IV.1.3)

In multi-element deposits correlation between the elements must be taken into account. It is relatively easy to co-simulate two correlated variables (Verly, 1993). Difficulties, however, increase significantly with more variables and the minimum/maximum autocorrelation factor method (MAF) is a practical and simple solution. The MAF approach was developed by Switzer and Green (1984), used by Desbarats and Dimitrakopoulos (2000) to simulate pore-size distribution within samples and by Dimitrakopoulos and Fonseca (2003) and Boucher and Dimitrakopoulos (2007, this volume) in a mining context. The method amounts to a principal component approach that accounts for some global spatial statistics. According to Desbarats and Dimitrakopoulos (2000), MAF appears to produce factors that are reasonably non-correlated for all lag distances, which is better than other methods suggested in the past, such as a classical principal component analysis. If there is a combination of trend and multiple variables, a reasonable procedure is to de-correlate first, then to perform the trend analysis.

Normal score or step-wise transformation (IV.1.4)

Transformation of the data to normal score value is quite straightforward with two possible options: single normal score transform or step-wise normal score transform.

If the trend is not an issue, or if a post-simulation trend adjustment is made (Step IV.1.10), then a single normal score transform is performed per geology domain. This transform is a table that associates each grade value with a standard normal score value such that cumulative frequencies of both values are identical. The transform tables are first obtained per geology domain within the EDA envelope using the declustered grade distributions. Data outside the EDA envelope are not used to build the tables, but are transformed to normal scores using these tables for the simulation.

If the trend is an issue, the step-wise transform suggested by Leuangthong and Deutsch (2004) is promising (Figure 3). The method consists of defining, for a given geology domain within the EDA envelope, the trend and residuals followed by a normal score transform of the residuals conditional to the trend. This method is very promising because the normal score transform is not global, but conditional to the trend. The method ensures that there is no trend in the normal score space, and that a proper normal score variogram is used. Finally, the method greatly reduces the number of negative grade values after the step-wise back-transform.

This method can be modified to a transformation of the original values conditional to the trend instead of residuals conditional to the trend. This modification would ensure that there are no negative grades after back-transformation.

Normal score variograms (IV.1.5)

The normal score variogram is generally less noisy and easier to fit than the original grade variogram. Srivastava and Parker (1989) suggest the correlogram as a better choice than the traditional variogram for skewed distribution. The correlogram works also very well with normal scores.

By construction, the normal score conditioning values are standard normal within the zone of interest Z (eg one geology domain within the EDA envelope), which means that the dispersion variance of the normal scores within Z is 1.0, ie:

$$D^2(0|Z) = \bar{\gamma}(Z,Z) = 1$$

where:

$\bar{\gamma}(Z,Z)$ is the average normal score variogram value within Z

The normal score variogram fit should be consistent with the above equality, which means that the variogram sill is larger than one if the zone Z is not very large, as it can be in the case of local grade control.

In practice, the variogram is often fitted first with a sill of one. The value of $\bar{\gamma}(Z,Z)$ should then be computed. If the $\bar{\gamma}(Z,Z)$ value is within five per cent of one, a simple rescaling of the variogram values is reasonable, otherwise a variogram adjustment (sill and range) is suggested (Figure 4).

If two variables are simulated, the normal score correlation is an indication of the sill of their cross-correlogram.

Normal score simulation (IV.1.6)

The simulation, and its speed, may be influenced by a number of parameters. The number of realisations needed depends on how many are sufficient to characterise the uncertainty being addressed (Deutsch and Journel, 1998). In practice, good results can be obtained with 50 realisations, although sometimes this number may be reduced to 20 due to the size of the individual realisations. The sample search criteria depend on the study (Dimitrakopoulos and Luo, 2004). The authors, however, have obtained good results by retaining 16 closest values (actual or simulated), ie two values per octant, to simulate a node. If necessary, locally defined anisotropy directions of a variogram model should be considered. As shown by a handful of articles there are definite benefits to this local anisotropy approach (Sinclair and Giroux, 1984; Srivastava, 2005).

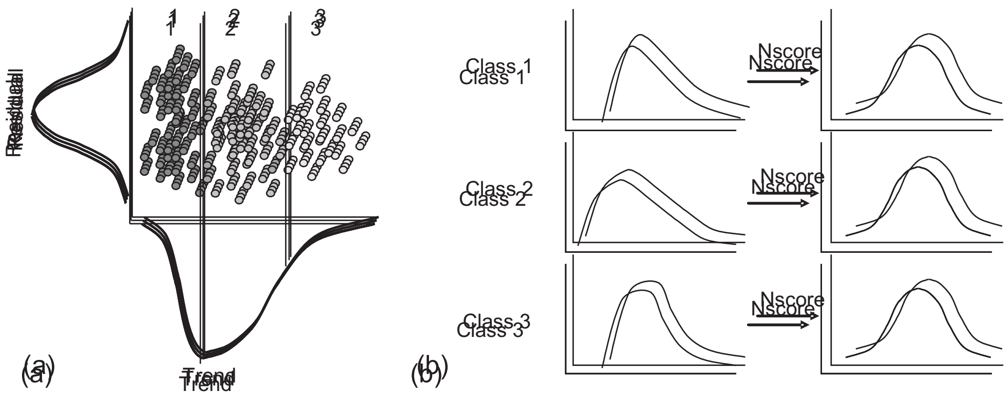


FIG 3 - Normal score transform of residuals conditioned to trend component. (a) Residuals are partitioned into classes based on trend component. (b) Residuals from each class are standard normal score transformed.

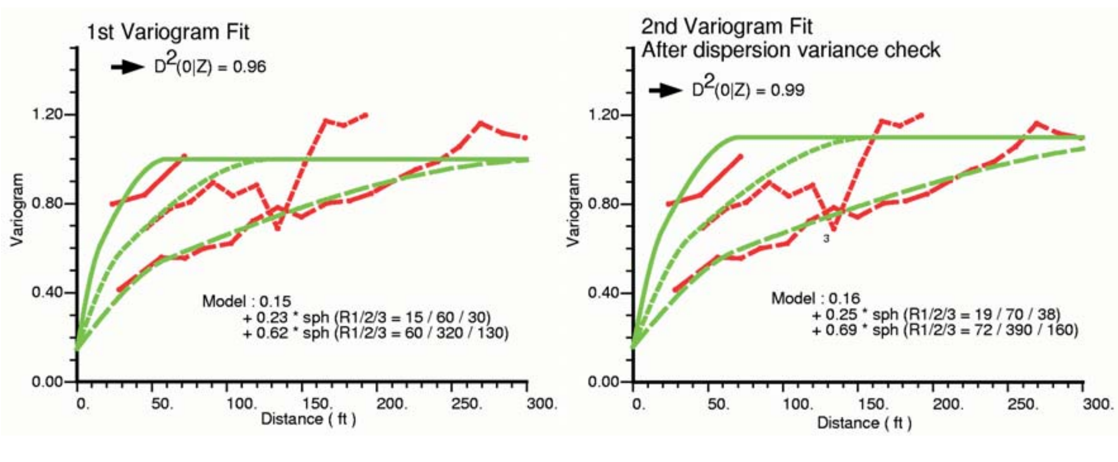


FIG 4 - Example of variogram models before and after normal score variability check. (a) Variogram model with total sill of 1.0 results in a dispersion variance within the EDA zone of 0.96. (b) Modified model with total sill of 1.10 results in a dispersion variance of 0.99.

If possible, a multi-grid approach can be used. In this approach, the simulation starts on a very large grid and ends, after a few passes, with the required grid size. An advantage to this approach is lower execution time and lower computer memory requirement. Moreover, very good reproduction of long ranges of continuity can be accomplished (Verly, 1993). Finally, if several realisations can be loaded in memory, using the same multi-grid random path can drastically reduce the computing time. Indeed, at a given grid node, the same steps are used for all realisations with only one different random number used to draw a value from the conditional distribution (Verly, 1993). Note the same random path is not recommended if it is not a multi-grid random path.

Check/adjust simulated normal scores (IV.1.7)

Post-simulation checks are necessary to ensure a reasonable reproduction of data distribution and spatial correlation. Both the histograms and the variograms of the simulated normal score values should be checked against the original normal score histograms and variograms obtained in Steps IV.1.4 and IV.1.5. Checks should be completed per geology domain and within the EDA validation zone, ie the same zone that has been used to get the simulation parameters such as declustered grade distribution, the normal score transform, and the normal score variograms. All these checks should be done with hard boundaries between geological domains. Finally, all simulated values should be considered for the checks to account for the statistical differences, in particular in average and variance, due to fluctuations between realisations.

The checks may reveal that the simulated normal scores are not standard normal, that the input variograms are poorly reproduced, or that two co-simulated normal score values do not have the proper correlation.

If the variance of the simulated values is lower than 1.0 it is highly probable that dispersion variance $D^2(0|Z)$ is too low and that the sill of the variogram model must be increased (Step IV.1.5). If the correlation between two co-simulated normal score values is poorly reproduced, the sill of the input cross-correlogram should be reviewed and eventually modified (increasing the sill will increase the correlation).

If the average of the simulated normal scores is different from 0.0, or their distribution is not normal, it is possible that there is a mismatch between the EDA and the validation zones even if the two zones are physically identical. For example, there could be fringes or extensive areas that are not sampled. Under those circumstances, a modification of the validation zone is needed to reduce the impact of the unsampled areas. A properly defined validation zone may represent the area relatively close to the conditioning data, for example extending not further than a search radius that was used for polygonal declustering. Another possible cause is an improper declustering, which means that the original normal score distribution has not been correctly defined. For example, the cell declustering size may be inappropriate or the polygonal declustering may be incomplete due to some search radius restriction.

Ultimately, if the source of the differences is not well known, and the original distribution is considered accurate, the simulated values can be progressively adjusted with a correction that increases with increasing distance from the conditioning data (Xu and Journel, 1994; Nowak and Verly, 2004).

Normal score or step-wise back-transformation (IV.1.8)

This step is straightforward and does not require any particular attention. Further checks of the back-transformed distributions could be considered.

De-MAF – correlate variables (IV.1.9)

This step is straightforward and is required only if the original variables have been decorrelated using the MAF approach (Step IV.1.3). Further checks related to the correlated distributions could be considered.

Check/adjust simulated grades (IV.1.10)

In Step IV.1.2, the trend has been analysed along various directions and a 3D trend model has eventually been produced. The simulated average should be compared against the trend along the same directions to assess the need for some adjustment (Nowak and Verly, 2004). If the step-wise normal score transform has been used in Step IV.1.4, trends should be very well reproduced in the simulation. If a single normal score transform has been used, some trend adjustment may be necessary.

A reasonable approach to adjust for the trend has been suggested by Nowak and Verly (2004). The approach consists in a gradual adjustment from a maximum correction (simulated average reset to the trend value) far away from the conditioning data to no correction at data locations. The approach is simple, flexible and guarantees that the trend is reproduced far away from data locations. Moreover, the coefficients of variation of the simulated values before and after adjustment have been noted to be quite similar.

All checks and possible adjustments made in normal score space (Step IV.1.7) are necessary but not sufficient to disregard the checks on the simulated values after back-transformation and trend adjustment. Comparisons between the simulated values and the original data should be made per geology domain within the validation envelope. Histograms, probability plots, scatterplots and visual checks of maps of simulated values are useful tools. Care should be given to ensure that the simulated mean grade in a geological domain is similar to the average estimated grade in that domain. If they are different, the simulated/estimated grades may have to be adjusted either by modifying some parameters, such as trimming values, and resimulating/reestimating, or by further adjustment of the simulated/estimated values to the required average. If it is a requirement that the distribution of simulated values is very similar to the data distribution, a correction can be made that increases progressively with the increased distance of simulated values from the data locations (Xu and Journel, 1994).

Bootstrapped data simulation (IV.2)

The previous section IV.1 describes a simulation that assumes that the distribution of *in situ* grades is known from the declustered grade histogram. The additional risk associated with an imperfect knowledge of the grade distribution is described in this section.

Except for the bootstrapping (Step IV.2.1 below), the simulation from bootstrapped data is in many respects simpler than the simulation of the original data. Indeed, many steps have already been computed such as grade EDA, trend analysis, and variable decorrelation. Some steps are not needed such as the normal score variograms (same variograms are used) or the various checks (checks are only done using the original dataset). Some steps are exactly the same, such as simulation or the various adjustments. The same adjustments that were made for the original data simulation are also made for the bootstrapped data simulation.

Bootstrap grades (IV.2.1)

Using a bootstrapping methodology, statistical fluctuations can be investigated by sampling from the original distribution. A typical bootstrap procedure consists of creating a series of possible datasets by drawing randomly with replacement as many values, with the attached declustering weights, as there are in the original distribution. The fluctuations between the various datasets are then investigated.

When there are many sample values, such as in mining, the classical bootstrap approach results in datasets that are very similar to each other. This similarity would be perfectly correct if the sample values were uncorrelated, but this is not the case in a typical mining situation.

Spatial correlation can be addressed by drawing fewer values from the original distribution (Srivastava, pers comm). Indeed, the variance of the mean grade is:

$$Var1(\text{Mean}) = \frac{1}{N^2}\sum\sum c_{ij}$$

where:

N is the number of samples in the original dataset

C_{ij} is the covariance for the distance between sample i and j, and can be deduced from the variogram

If P values are drawn randomly from the original dataset, the variance of the mean is:

$$Var2(\text{Mean}) = \frac{1}{P}Var(\text{Data})$$

where:

Var(Data) is the variance of the original data set

The required fluctuation for the mean is achieved if P is chosen such that $Var2$(Mean)=$Var1$(Mean), ie:

$$P = \frac{Var(\text{Data})}{Var1(\text{Mean})}$$

Note that this formula could be refined to account for declustering weights. The bootstrapping may be done on data from all geological domains or on data from one domain at a time. If the former is used, the choice of optimistic (high average) and pessimistic (low average) distributions is more difficult, because the distributions from one or two domains may influence the results. The authors feel that bootstrapping per domain is a better solution. Under those circumstances, a pessimistic/optimistic declustered distribution can be truly pessimistic/optimistic in all domains. Of course, care should be given when choosing the bootstrapped distributions for simulating the grades. The distributions should not be overly pessimistic or optimistic.

The impact of the bootstrap on the input mean grade uncertainty can be very significant as shown in Figure 5. In this figure, the sample grade distribution has a mean of 0.39. Classical bootstrap indicates that this distribution mean grade can vary between 0.36 and 0.42. Spatial dependence bootstrap indicates that the mean varies between 0.30 and 0.48.

Choose pessimistic/optimistic distribution (IV.2.2)

Prior to the final choice of the optimistic and pessimistic distributions, it may be useful to have some insight on the potential impact of that choice on the simulated values. Applying a cut-off grade on the bootstrapped distribution corrected for change of support may provide such insight.

The choice of the declustered optimistic/pessimistic distributions is related to the objectives of the simulation. If there are several variables, then the choice can be based on the most significant variable. The distributions are chosen for each geology domain and later are combined for further processing (Step IV.2.4).

MAF – Decorrelate variables (IV.2.3)

This step is straightforward. Decorrelation of variable values corresponding to the original grade distribution has been described in Step IV.1.3. The same decorrelation formula is used to get the decorrelated values of the bootstrapped distribution values.

Normal score or step-wise transformation (IV.2.4)

Once a bootstrapped distribution is chosen, it is used first to generate a single or step-wise normal score transform as per Step IV.1.4 (Figure 6a). The bootstrapped distribution and its transform are then used to convert the original grade values to normal score values (Figure 6b). The cumulative frequencies of the original sample grades are deduced from the bootstrapped distribution, then used to get the corresponding normal score values. Note that the resulting normal score values are not standard normal. For example, in the case of an optimistic bootstrapped distribution as in Figure 6, the average of the normal score values is less than zero. The inverse of the bootstrapped distribution normal score transform is used for back-transformation of the simulated normal score values.

Normal score simulation (IV.2.5)

This step is straightforward. The only difference from Step IV.1.6 is the conditioning normal score values that depend on the bootstrapped distribution used for the normal score transform. Note that the conditioning sample locations are the same, the sample original values are the same, but their normal score values are different.

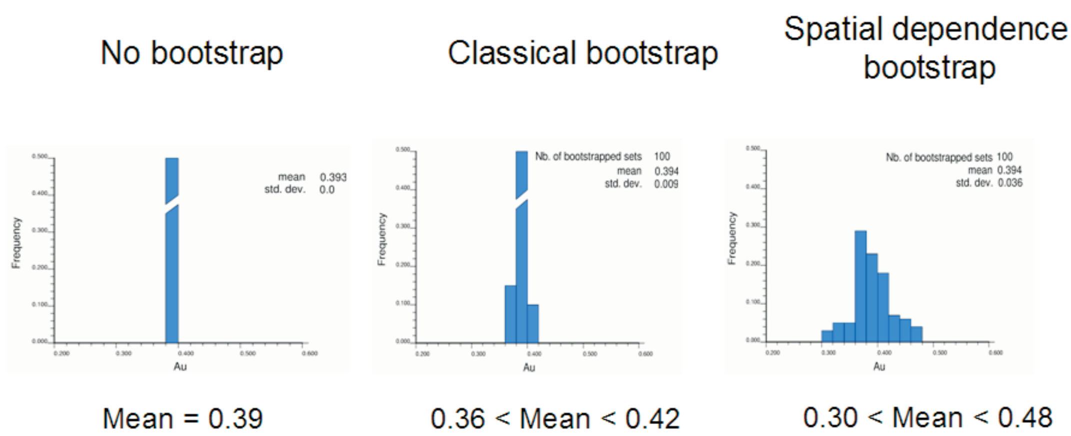


FIG 5 - Uncertainty on the mean grade of a distribution, obtained by bootstrapping.

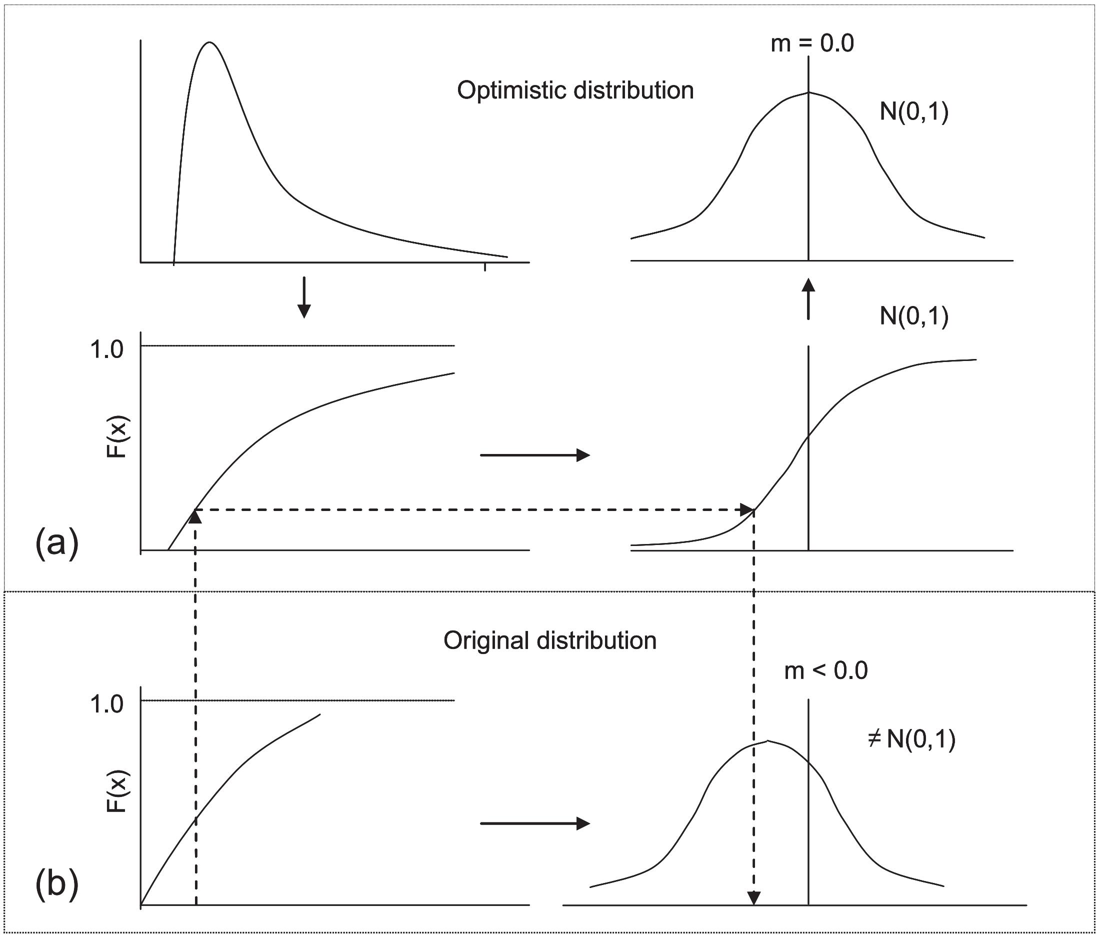


FIG 6 - (a) Standard normal score transform based on an optimistic bootstrapped distribution. (b) Original grade distribution converted to normal scores using the bootstrapped distribution and its normal score transform.

The same random seed could be used here to ensure that the differences observed are due only to the bootstrapped distribution used for the normal score transform. If this is deemed unreasonable the number of realisations should be increased.

Check/adjust simulated normal scores (IV.2.6)

All the checks have already been done in Step IV.1.7. Whatever adjustments were made on the original normal score simulation, they are also made on the bootstrapped normal score simulation.

Normal score or step-wise back-transformation (IV.2.7)

This step is identical to Step IV.1.8 except that the back-transformation is based on the bootstrapped distribution.

De-MAF – correlate variables (IV.2.8)

This step is identical to Step IV.1.9.

Check/adjust simulated grades (IV.2.9)

Most of the checks have already been done in Step IV.1.10. Whatever adjustments were made on the original data simulation, they are also made on the bootstrapped data simulation. This includes any trend adjustment.

Additional checks could be made though no particular surprises should be expected if the same random seed has been used for the original and bootstrapped data simulation.

Merge geological and grade uncertainty (IV.3)

This step is not an issue if Step III has been skipped and a deterministic geology model has been used in Steps IV.1 and IV.2. If, however, several geological realisations have been simulated in Step III, two passes through Steps IV.1 may be necessary. The first pass is a careful calibration and validation of the grade simulation using one 'median' realisation of the geology. The second pass is a series of 'blanket' simulations per geology domain that are then attached onto the appropriate geology as per the realisations obtained in Step III.

Note that only one pass of 'blanket' grade simulations is necessary for the bootstrapped grade simulation.

Choice of scenarios (IV.4)

Ideally, all realisations should be processed accordingly to the specified objectives. Unfortunately, flexible software is still often lacking to efficiently process multiple realisations. The choice of what realisations to process is then critical and should depend on the objective of the simulation that has been defined in Step I.

If bootstrapping has been part of the procedure, an important choice of scenarios has already been done in Step IV.2.2. In this Step IV.4, a series of realisations is available and the choice of which ones to retain is generally based on the value of some quantity within an area of specific interest.

If the simulation objective is the fluctuation in size of an ultimate pit, a minimum of three realisations could be retained: 'worst', 'median', and 'best' scenarios. The quantity on which the choice is based could be an SMU average grade above a

TABLE 1

Example of simulated model risk assessment.

Issue	Impact	Likelihood	Consequence	Risk
Some assays are biased low	Simulated grades are too low	D	B	Medium
Geological model is deterministic and based on relatively little data	The model does not properly reflect the controls on mineralisation; geological uncertainty not accounted for	B	B	High
Bootstrapping incorrectly describes uncertainties on grade distribution	Space of uncertainty on grade too narrow	D	D	Low
Trends are not properly defined at some distance from the data	Improper trend reproduction far away from data	C	C	Medium

given cut-off grade or some NPV value. The area of interest could be the area between a very optimistic and a very pessimistic pit shell, ie an area that excludes the mineralisation core that will be mined anyway. The area could be further divided into octants. Within each octant one of the worst and one of the best realisations are chosen, then merged into one very pessimistic and one very optimistic hybrid realisation.

If the simulation objective is to illustrate the change of support, then a 'hybrid' realisation may be again the best solution. In this case, one realisation is picked per geology domain such that the simulated average and coefficient of variation are as close as possible to the original data corresponding statistics. The chosen realisations are then merged into one 'hybrid' realisation that reproduces very closely the original statistics per geology domain.

If the mining process is already defined (eg ultimum pit shape, scheduling), then all realisations should be considered to assess the risk by looking at the process response to the different realisation results (Dimitrakopoulos *et al*, 2002). Though this can generally be done with minimum programming/scripting, it does not indicate if the process is optimum or not. To get the optimum process, new techniques have to be designed that process all realisations, such as some described in this volume (Dimitrakopoulos, Martinez and Ramazan, 2007; Grieco and Dimitrakopoulos, 2007; Ramazan and Dimitrakopoulos, 2007; Menabde *et al*, 2007).

RISK ASSESSMENT AND SIGN-OFF (V)

The sign-off step serves two objectives:

1. formalised transfer between different individuals, and
2. risk assessment of the simulation.

The first objective helps to establish common ground between different stakeholders who should discuss and understand the simulation results, notably their limitations.

The second objective helps to put in perspective the simulation results. In the course of a simulation study a practitioner may come across a number of issues that affect the outcome of the study. In addition, he/she is forced to make a number of decisions that may have a significant impact on the simulation results. These decisions and issues should be explicitly stated and the associated risk for the company assessed. An example of risk assessment is presented in Table 1. A number of issues/events and their impact on the simulation model are given. The event likelihood and consequence are rated from A to E corresponding to 'almost certain' to 'very rare' for the likelihood, and 'very high' to 'insignificant' for the consequence. A 'low' to 'high' risk is deduced from the likelihood/consequence combination. The risk column is then used to decide if more work is needed on the simulation model, or if the corresponding issue(s) must be part of subsequent risk assessments made at the reserve estimation and financial decision stages.

CONCLUSIONS

In this paper, a process for simulation with emphasis on sequential Gaussian simulation is presented. The subprocess corresponding to Gaussian simulation contains many more steps than the usual normal score transformation, variogram modelling, simulation and back-transformation. A significant portion of this subprocess may also be used for other simulation methods. The authors believe that using similar processes in the mineral industry would avoid costly mistakes.

Some aspects of the simulation process are extremely important. Properly defined objectives of the study enable a correct design of the simulation parameters, which in turn can lower the time spent and the costs of the simulation. Although trends, in some cases, may not have to be defined, grade bootstrapping should be considered in most situations. Frequent checking of the results is emphasised. A dispersion variance per geology domain should be computed and if different from 1.0, the modelled variogram sill should be readjusted. Comparisons of simulated values with the conditioning data should be conducted both in normal score space and after back-transformation. The simulation study should be followed by a risk assessment of the important issues noted during the study.

ACKNOWLEDGEMENT

The authors wish to thank Placer Dome Inc for permission to publish this paper.

REFERENCES

Alabert, F and Massonat, G J, 1990. Heterogeneity in a complex turbidic reservoir: Stochastic modeling of facies and petrophysical variability, in *Proceedings 65th Annual Technical Conference and Exhibition,* pp 775-790 (Society of Petroleum Engineers, SPE Paper 20604).

Armstrong, M, Galli, A G, Le Loch, G, Geffroy, F and Eschard, R, 2003. *Plurigaussian Simulations in Geosciences*, 149 p (Springer Verlag).

Boucher, A and Dimitrakopoulos, R, 2007. A new efficient joint simulation framework and application in a multivariable deposit, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 345-354 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Chilès, J-P, Aug, C, Guillen, A and Lees, T, 2007. Modelling the geometry of geological units and its uncertainty in 3D from structural data — The potential-field method, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 355-362 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Desbarats, A J and Dimitrakopoulos, R, 2000. Geostatistical simulation of regionalised pore-size distributions using min/max autocorrelation factors, *Mathematical Geology*, 32(8):919-942.

Deutsch, C V, 2002. *Geostatistical Reservoir Modeling*, 376 p (Oxford University Press: New York).

Deutsch, C V and Journel A G, 1998. *GSLIB: Geostatistical Software Library and User's Guide*, 380 p (Oxford University Press: New York).

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-88.

Dimitrakopoulos, R and Fonseca, M B, 2003. Assessing risk in grade-tonnage curves in a complex copper deposit, northern Brazil, based on an efficient joint simulation of multiple correlated variables, in *Proceedings 28th International Symposium on Computer Applications in the Minerals Industries* (ed: F A Camisani-Calzolari), pp 373-382 (The South African Institute of Mining and Metallurgy: Johannesburg).

Dimitrakopoulos, R and Luo, X, 2004. Generalised sequential Gaussian simulation on group size v and screen effect approximations for large field simulations, *Mathematical Geology*, 36(5):919-942.

Dimitrakopoulos, R, Martinez, L and Ramazan, S, 2007. Optimising open pit design with simulated orebodies and Whittle Four-X — A maximum upside/minimum downside approach, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 201-206 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 467 p (Oxford University Press: New York).

Grieco, N and Dimitrakopoulos, R, 2007. Grade uncertainty in stope design — Improving the optimisation process, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 167-174 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Isaaks, E H, 1991. Application of Monte Carlo methods to the analysis of spatially correlated data, PhD thesis (unpublished), Stanford University, Palo Alto.

Isaaks, E H and Srivastava, R M, 1989. *Introduction to Applied Geostatistics*, 561 p (Oxford University Press: New York).

Jackson, S, Fredericksen, D, Stewart, M, Vann, J, Burke, A, Dugdale, J and Bertoli, O, 2003. Geological and grade risk at the Golden Gift and Magdala gold deposits Stawell, Victoria, Australia, in *Proceedings Fifth International Mining Geology Conference,* pp 207-213 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Journel, A G, 2007. Roadblocks to the evaluation of ore reserves — The simulation overpass and putting more geology into numerical models of deposits, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 29-32 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Leuangthong, O and Deutsch, C V, 2004. Transformation of residuals to avoid artifacts in geostatistical modelling with a trend, *Mathematical Geology*, 36(3):287-305.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Nowak, M and Verly, G, 2004. The practice of sequential Gaussian simulation, in *Proceedings Geostatistics Banff '04: Seventh International Geostatistics Congress.*

Osterholt, V and Dimitrakopoulos, R, 2007. Simulation of orebody geology with multiple-point geostatistics — Application at Yandi Channel iron ore deposit, WA and implications for resource uncertainty, in *Orebody Modelling and Strategic Mine Planning*, second edition, pp 51-59 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Sinclair, A J and Giroux, G H, 1984. Geological controls of semi-variograms in precious metal deposits, in *Geostatistics for Natural Resources Characterization* (eds: G Verly, M David, A G Journel and A Maréchal), 2:965-978 (NATO Advanced Study Institutes series, series C).

Skvortsova, T, Armstrong, M, Beucher, H, Forkes, J, Thwaites, A and Turner, R, 2000. Applying plurigaussian simulations to a granite-hosted orebody, in *Geostats 2000 Cape Town, Proceedings of the Sixth International Geostatistics Congress,* (eds: W J Kleingeld and D G Krige) Cape Town, South Africa, April.

Srivastava, R M, 2005. Probabilistic modelling of ore lens geometry: an alternative to deterministic wireframes, *Mathematical Geology,* 37(5):513-544.

Srivastava, R M and Parker, H M, 1989. Robust measures of spatial continuity, in *Geostatistics: Proceedings Third International Geostatistics Congress,* (ed: M Armstrong) pp 1:295-308 (Kluwer Academic Publisher: Dordrecht).

Strebelle, S, 2002. Conditional simulation of complex geological structures using multiple-point statistics, *Mathematical Geology*, 34(1):1-22.

Switzer, P and Green A, 1984. Min/Max autocorrelation factors for multivariate imaging, Technical report No 6 (unpublished), 14 p (Department of Statistics, Stanford University).

Verly, G, 1984. The block distribution given a point multivariate normal distribution, in *Geostatistics for Natural Resources Characterization: Proceedings Second International Geostatistics Congress* (ed: G Verly *et al*), pp 1:495-515 (Reidel Publishing Company: Dordrecht).

Verly, G, 1985. Multigaussian kriging – a complete case study, in *Proceedings 19th International Symposium on Application of Computers in the Mineral Industry* (ed: R V Ramani), pp 183-298 (The Society of Mining, Metallurgy, and Exploration Inc: Littleton).

Verly, G, 1993. Sequential Gaussian cosimulation: a simulation method integrating several types of information, in *Geostatistics Troia '92: Proceedings Fourth International Geostatistics Congress* (ed: A Soares), pp 1:543-554 (Kluwer Academic Publisher: Dordrecht).

Verly, G, 2005. Grade control classification of ore and waste: a critical review of estimation and simulation based procedures, *Mathematical Geology,* 37(5):451-475.

Xu, W and Journel, A G, 1994. Posterior identification of histograms conditional to local data, Stanford Center for Reservoir Forecasting Report No 7, School of Earth Sciences, Stanford University.

Conditional Simulation by Successive Residuals — Updating of Existing Orebody Realisations

A Jewbali[1] and R Dimitrakopoulos[2]

ABSTRACT

Conditional simulation by successive residuals (CSSR) is a new simulation method based on the column decomposition of the covariance matrix, which leads to the expression of the simulation process in terms of successive conditional covariance matrices. The method follows successive steps where, at each step, random variables in a group are simulated using a lower-upper decomposition of a covariance matrix of updated conditional covariance residuals. The updating process does not require the solution of large systems of equations – a limitation of other simulation methods – thus it is more efficient. A practical consequence of CSSR is the fast updating of existing simulations when additional data becomes available. An implementation of CSSR, using data from a stockwork gold deposit, demonstrates the approach. In addition, simulated realisations both before and after the update are benchmarked against a known sequential Gaussian simulation implementation. The fast updating is found to improve computational efficiency by 65 - 77 per cent.

INTRODUCTION

Stochastic simulations of Gaussian random fields have been used for risk analysis and management in various aspects of orebody modelling and mine planning (Ravenscroft, 1992; Dowd, 1994; Dimitrakopoulos, Farrelly and Godoy, 2002; Godoy and Dimitrakopoulos, 2007, this volume; Ramazan and Dimitrakopoulos, 2007, this volume; Dimitrakopoulos, in press). These approaches require the generation of multiple realisations of random fields conceptually representing the attribute of interest and, if the mineral deposit is large, the simulation may involve tens of millions of nodes. As exploration or mining progress, additional information, termed 'future data', becomes available (for example, through infill drilling or exploration near the mine). Incorporating the newly acquired data into the orebody modelling, risk assessment or optimisation process requires re-simulating the orebody with the new information. For large simulations, this constitutes rerunning simulations that require a substantial computational effort and time (Dimitrakopoulos and Luo, 2004). The ability to provide mechanisms for fast updating of existing realisations would contribute to the practical use of simulation technologies, particularly their integration into new optimisation formulations and mine production scheduling.

A conditional simulation approach based on successive residuals (Vargaz-Guzman and Dimitrakopoulos, 2002; Vargaz-Guzman and Dimitrakopoulos, 2003), which can update existing simulations when new data becomes available, is presented in this paper. The approach is founded on a new, column partitioning of the lower-upper (LU) decomposition of the covariance matrix C of data and grid node locations to be simulated. The approach overcomes the size limitations of the LU method in Davis (1987). It is useful to recall that the LU method will generate a realisation z of a spatial random field $Z(x)$, $x \in R^n$, at a set of grid node locations conditional to the available data from z = Lw, where w is a vector of white noise and L is generated from the decomposition $L = CU^{-1}$. The size of matrix L poses the well-known limitation of the method to only being able to generate realisations up to a few thousand grid nodes. Simulating with all data z_d available and following the matrix form of kriging, the realisation z is a simple estimate plus a random component, such that $\mathbf{z} = \mathbf{A}_{21}\mathbf{L}_{11}^{-1}\mathbf{z}_d + \mathbf{L}_{22}\mathbf{w}$, where the partitioning $\mathbf{L} = \begin{bmatrix} \mathbf{L}_{11} & 0 \\ \mathbf{A}_{21} & \mathbf{L}_{22} \end{bmatrix}$ is used. The L_{11} matrix is derived from the LU decomposition of the data covariance matrix, and A_{21} and L_{22} matrices from the partitioned L matrix in the decomposition of the covariance C shown above.

The method discussed here provides an alternative formulation that is able to overcome the limitations of the LU decomposition. The new method is based on the column decomposition of the covariance matrix using conditional covariance matrices. Conditional simulation by successive residuals (CSSR) is a method that can simulate, in successive steps, a small group of nodes using the LU decomposition of a matrix of updated conditional covariance of residuals. The simulated nodes are then used to update residuals, a step that eliminates the solution of large systems of equations. The successive process amounts to the separation of influences from different data sources, allowing recalculation of only those sources that introduce new information when updating. Thus, the process can uniquely facilitate the fast updating of simulated realisations with new data when appropriate, without having to repeat the complete simulation process.

In the following sections, the conditional simulation by successive residuals is first explained and then its implementation is discussed. Subsequently, a case study from a stockwork gold deposit explains the practical aspects of the updating of simulated deposit realisations with CSSR. A brief discussion of performance issues and conclusions follow.

CSSR: EXPLAINING WITH AN EXAMPLE

The CSSR method is explained here, using the example shown in Figure 1. In this example, six grid nodes are to be simulated, conditional to four data points. CSSR divides the data and grid nodes into groups. Hence Figure 1 shows the data divided into two groups, P and S, and the grid nodes into two groups, V and M. Next, the covariance matrix C containing the covariances between the data and grid nodes is generated, and it is:

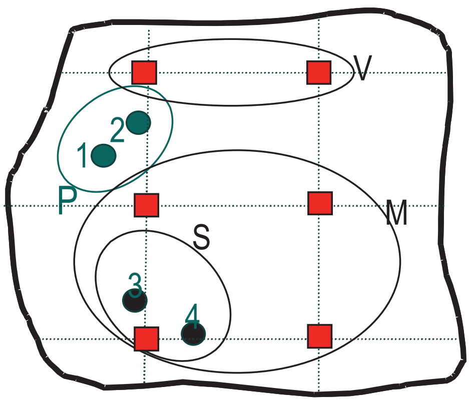


FIG 1 - Area to be simulated containing six grid nodes (squares) and four data points (circles).

1. GAusIMM, Resource Estimation Geologist, Rio Tinto Iron Ore, GPO Box A42, Perth WA 6837, Australia. Email: arja.jewbali@riotinto.com

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

$$\mathbf{C} = \begin{bmatrix} \mathbf{C_{pp}} & \mathbf{C_{ps}} & \mathbf{C_{pv}} & \mathbf{C_{pm}} \\ \mathbf{C_{sp}} & \mathbf{C_{ss}} & \mathbf{C_{sv}} & \mathbf{C_{sm}} \\ \mathbf{C_{vp}} & \mathbf{C_{vs}} & \mathbf{C_{vv}} & \mathbf{C_{vm}} \\ \mathbf{C_{mp}} & \mathbf{C_{ms}} & \mathbf{C_{mv}} & \mathbf{C_{mm}} \end{bmatrix} \tag{1}$$

where the terms C_{ij} contain the covariances between the nodes in group i and those in group j, such as groups P, S, V and M in the present example. C is subsequently split into a lower and upper triangular matrix such that $C = LU = LL^T$, and it is followed by the column-wise decomposition of L in which each element in a column is expressed as a function of its diagonal element (Vargaz-Guzman and Dimitrakopoulos, 2002). This is:

$$\mathbf{L} = \begin{bmatrix} \mathbf{L_{pp}} & & & \\ \mathbf{C_{sp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{L_{ss}} & & \\ \mathbf{C_{vp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{E^{[2]}_{vs}}[\mathbf{E^{[2]}_{ss}}]\mathbf{L_{ss}} & \mathbf{L_{vv}} & \\ \mathbf{C_{mp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{E^{[2]}_{ms}}[\mathbf{E^{[2]}_{ss}}]\mathbf{L_{ss}} & \mathbf{E^{[3]}_{mv}}[\mathbf{E^{[3]}_{vv}}]\mathbf{L_{vv}} & \mathbf{L_{mm}} \end{bmatrix} \tag{2}$$

where $\mathbf{E^{[2]}_{js}} = \mathbf{C_{js}} - \mathbf{C_{jp}}[\mathbf{C_{pp}}]^{-1}\mathbf{C_{ps}}$ and $\mathbf{E^{[3]}_{jv}} = \mathbf{E^{[2]}_{jv}} - \mathbf{E^{[2]}_{js}}[\mathbf{E^{[2]}_{ss}}]^{-1}\mathbf{E^{[2]}_{sv}}$ for j = s,v,m. To generate simulated values z, L is multiplied by a vector w of independent N(0,1) random numbers, and it is:

$$\begin{bmatrix} \mathbf{L_{pp}} & & & \\ \mathbf{C_{sp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{L_{ss}} & & \\ \mathbf{C_{vp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{E^{[2]}_{vs}}[\mathbf{E^{[2]}_{ss}}]\mathbf{L_{ss}} & \mathbf{L_{vv}} & \\ \mathbf{C_{mp}}[\mathbf{C_{pp}}]^{-1}\mathbf{L_{pp}} & \mathbf{E^{[2]}_{ms}}[\mathbf{E^{[2]}_{ss}}]\mathbf{L_{ss}} & \mathbf{E^{[3]}_{mv}}[\mathbf{E^{[3]}_{vv}}]\mathbf{L_{vv}} & \mathbf{L_{mm}} \end{bmatrix} \begin{bmatrix} \mathbf{w_p} \\ \mathbf{w_s} \\ \mathbf{w_v} \\ \mathbf{w_m} \end{bmatrix} = \begin{bmatrix} \mathbf{z_p} \\ \mathbf{z_s} \\ \mathbf{z_v} \\ \mathbf{z_m} \end{bmatrix} \tag{3}$$

The column-wise decomposition of the L matrix facilitates updating when new data is received, because it splits influences from the various groups of data and grid nodes. For instance, consider the second group of grid nodes M to be simulated after simulation of nodes in V; this corresponds to the last row in Equation 3, and is:

$$\mathbf{C_{mp}}\mathbf{C^{-1}_{pp}}\mathbf{L_{pp}}\mathbf{w_p} + \mathbf{E^{[2]}_{ms}}[\mathbf{E^{[2]}_{ss}}]^{-1}\mathbf{L_{ss}}\mathbf{w_s} + \mathbf{E^{[3]}_{mv}}[\mathbf{E^{[3]}_{vv}}]^{-1}\mathbf{L_{vv}}\mathbf{w_v} + \mathbf{L_{mm}}\mathbf{w_m} = \mathbf{z_m} \tag{4}$$

The first two components in Equation 4, $\mathbf{C_{mp}}\mathbf{C^{-1}_{pp}}\mathbf{L_{pp}}\mathbf{w_p}$ and $\mathbf{E^{[2]}_{ms}}[\mathbf{E^{[2]}_{ss}}]^{-1}\mathbf{L_{ss}}\mathbf{w_s}$, describe the influence of the data in groups P and S on the simulated grid nodes in group M, while the third component, $\mathbf{E^{[3]}_{mv}}[\mathbf{E^{[3]}_{vv}}]^{-1}\mathbf{L_{vv}}\mathbf{w_v}$, contributes the influence of the previously simulated values in group V. If future data becomes available in, say, group V, then the simulated grid nodes in group M will be updated by only recalculating $\mathbf{E^{[3]}_{mv}}[\mathbf{E^{[3]}_{vv}}]^{-1}\mathbf{L_{vv}}\mathbf{w_v}$. The CSSR approach requires that the locations of the future data that will become available are known and included in the set of grid nodes to be simulated (in the present example this is group V). In addition, Equation 3 considers that the structure of spatial correlation remains unchanged when future data is included.

AN IMPLEMENTATION

The implementation of CSSR considered here divides the grid nodes to be simulated into groups using a local search neighbourhood. This is reasonable considering that data that is available far from the nodes being simulated will have negligible influence on the values being simulated. A further discussion of group sizes may be found in Benndorf and Dimitrakopoulos (2007, this volume) and Dimitrakopoulos (in press). The steps followed in the implementation are as follows:

1. divide the grid nodes into groups;
2. randomly select a group and simulate only the future data locations within the group; repeat this for each of the groups;
3. define a random path that visits each group once and define a group to start with;
4. at a group, find the data, the future data locations already simulated, and any other simulated nodes within the neighbourhood;
5. simulate the group using partitions of the covariance matrix into two columns and the equivalent of Equation 3 given in Equation 5:

$$\begin{bmatrix} \mathbf{L_{11}} & \\ \mathbf{C_{21}}\mathbf{C^{-1}_{11}}\mathbf{L_{11}} & \mathbf{L_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{w_1} \\ \mathbf{w_2} \end{bmatrix} = \begin{bmatrix} \mathbf{z_1} \\ \mathbf{z_2} \end{bmatrix} \tag{5}$$

where:

z_1	contains the conditioning data and the already simulated future data locations/nodes within the search neighbourhood
z_2	contains the remainder of the grid nodes being simulated within the group
w_1 and w_2	are vectors of independent N(0,1) random numbers. For updating purposes, the lower triangular component of C^{-1}_{11} and $L_{22}w_2$ are retained

6. move to the next group; and
7. continue with Steps 2, 3 and 4 until all groups are simulated.

The steps for the updating are as follows:

1. consider the group to be updated and retrieve C^{-1}_{11} and $L_{22}w_2$,
2. generate C_{21} and update z_2, and
3. move to the next group and continue with Steps 1 and 2 until all groups have been updated.

As noted earlier, a practical advantage of the updating algorithm is that the simulations do not have to be regenerated when future data becomes available. At the same time, by simulating clusters of grid nodes at each step, as in the generalised sequential Gaussian simulation or GSGS (Dimitrakopoulos and Luo, 2004), the above implementation adds computational efficiency. The updating capabilities (outlined above), however, have increased storage requirements compared with methods that do not perform updating, such as the GSGS.

A CASE STUDY AT A STOCKWORK GOLD DEPOSIT

Deposit, data and characteristics

The data from a stockwork gold deposit is used here to explain CSSR and the implementation above. In the deposit, most of the mineralisation occurs in a quartz diorite intrusion, with gold in narrow quartz/calcite/pyrite veins. There are 29 vertical drill holes, at a spacing of about 25 m, in a 200 m × 200 m section of the deposit. From the available 5 m composites, 496 are used to generate the first set of realisations in CSSR (first dataset). Later, an additional 18 inclined drill holes are used to update the realisations from the first dataset, leading to the conditioning of the simulated deposit models with 763 of the 5 m composites in total (second dataset). To remove any bias from clustering, both datasets have been declustered, using cell dimensions determined from plotting means against block sizes. The declustered statistics for the 5 m composites for both sets of data are shown in Table 1 and Figure 2. The spatial distribution of gold for the 5 m composites is shown in Figure 3.

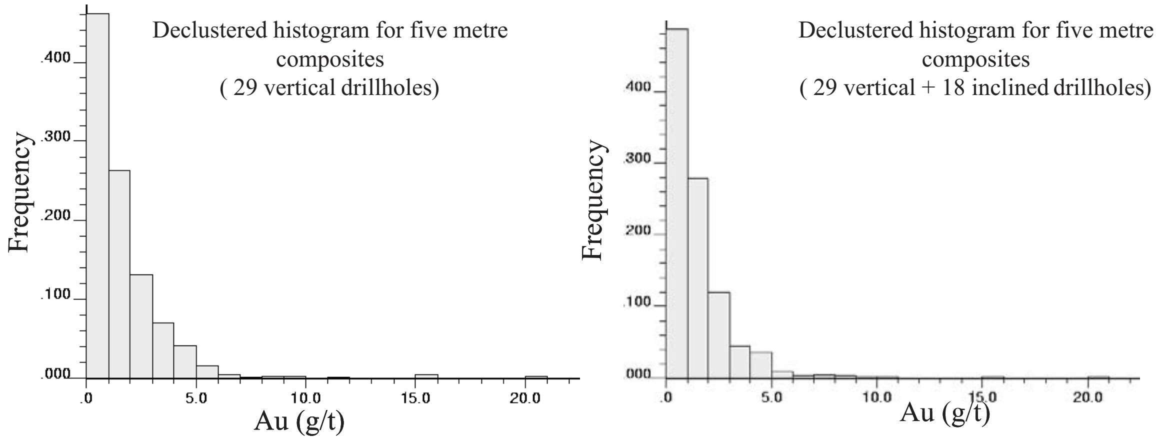


FIG 2 - Declustered data histogram of both sets of data.

TABLE 1

Declustered statistics for both sets of data.

Au (g/t)-29 vertical drill holes		Au (g/t)-29 vertical + 18 inclined drill holes	
No of data	496	No of data	763
Mean	1.75	Mean	1.66
Median	1.07	Median	1.04
Standard deviation	4.20	Sandard deviation	3.82
Coefficient of variation	2.40	Coefficient of variation	2.30
Maximum	85.13	Maximum	85.13
Minimum	0.0	Minimum	0.0

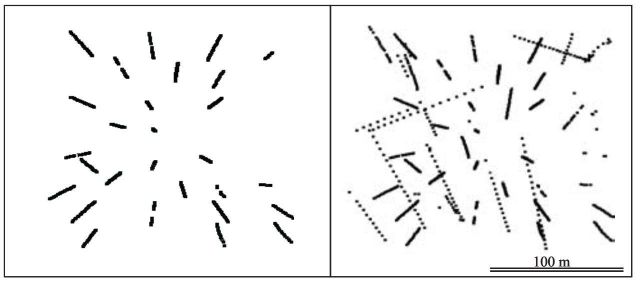


FIG 3 - Locations of the 29 vertical drill holes (left) and the 29 vertical + 18 inclined drill holes (right).

Both sets of data are transformed to normal scores, and variography is performed on each set. Figure 4 shows the experimental and model variograms of both datasets. The figure suggests that the model in the vertical direction is defined very well with just the original 29 drill holes and that addition of the 18 inclined drill holes does not alter the model. On the other hand, the variogram horizontally is not equally clear, and addition of the 18 inclined drill holes does not lead to any substantial differences. Note that the addition of the 18 drill hole data does not change the variogram structure.

Conditional simulation and updating

Conditional simulation is performed using CSSR with data from the first 29 vertical drill holes in the first instance. Without loss of generality, ten realisations are generated here within a study area of 200 m × 200 m × 100 m and a grid spacing of 4 m × 4 m × 4 m, leading to 65 000 nodes. The group size used is 2 × 2 × 2 nodes. To evaluate how the simulation of groups of nodes coupled with the use of future data may affect the realisations, the results from CSSR are benchmarked against results obtained from sequential Gaussian simulation (SGS) (Deutsch and Journel, 1998) using the same 29 vertical drill holes and corresponding composites. For the benchmarking, the histograms and variograms are compared, and the simulations generated from the two different methods are compared visually.

Figure 5 shows two randomly selected realisations produced by each method, CSSR and SGS, based on data from the 29 vertical drill holes. There are no visual differences in terms of structures between the realisations from the two methods. The histograms and variograms for the realisations generated by CSSR and SGS are shown in Figure 6 and Figure 7. Both

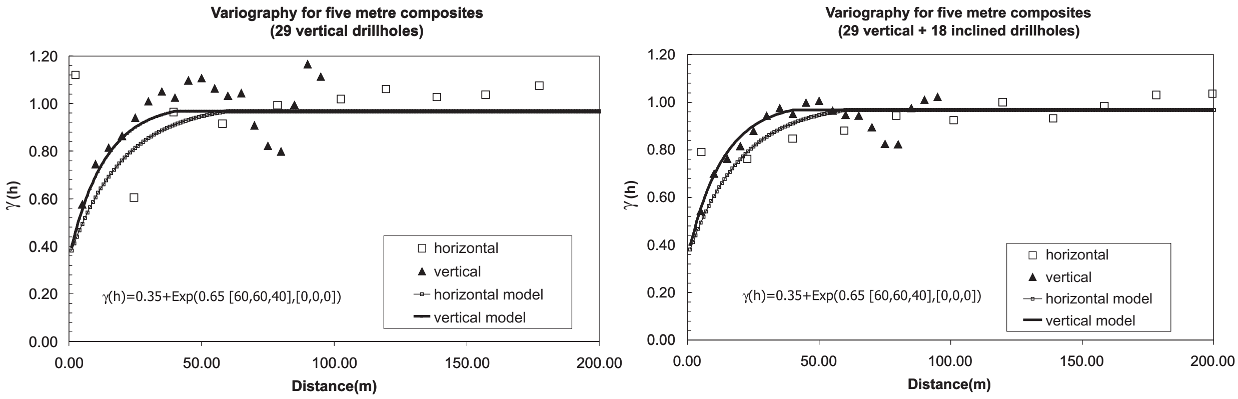


FIG 4 - Experimental and model variograms of both datasets (normal scores).

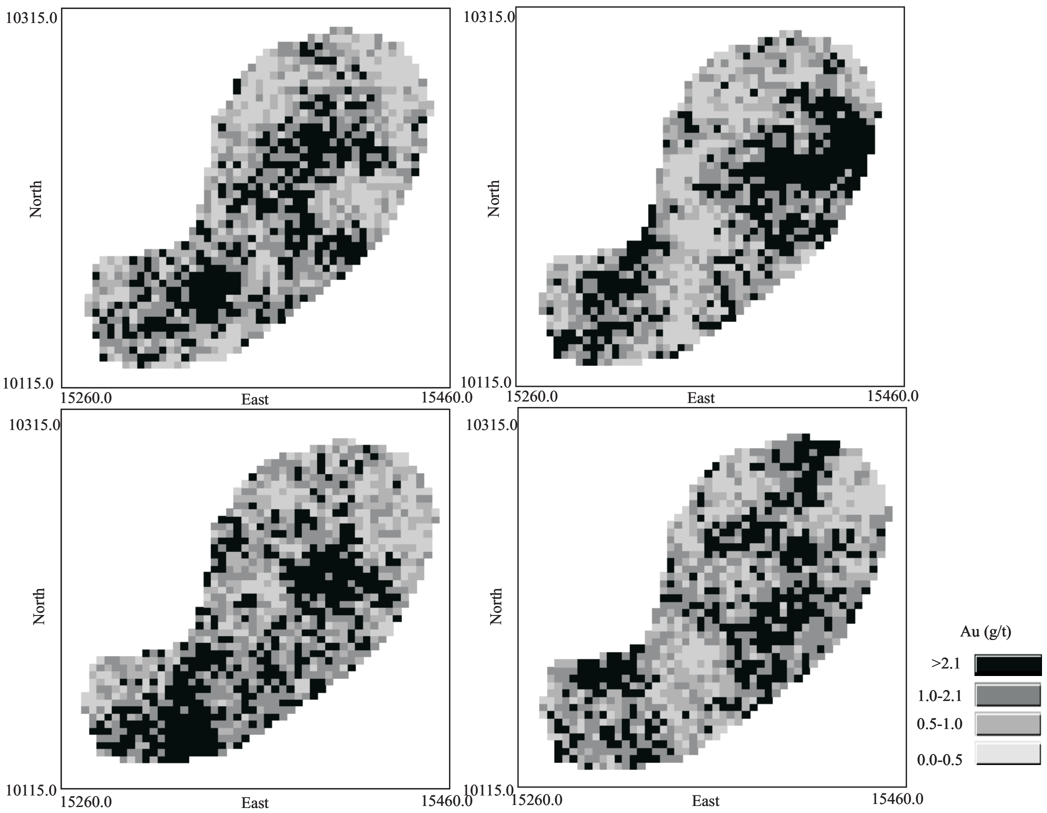


FIG 5 - Randomly selected Au realisations of a horizontal section of the deposit. The top two realisations were produced by CSSR (pre-updating) and the bottom two by SGS.

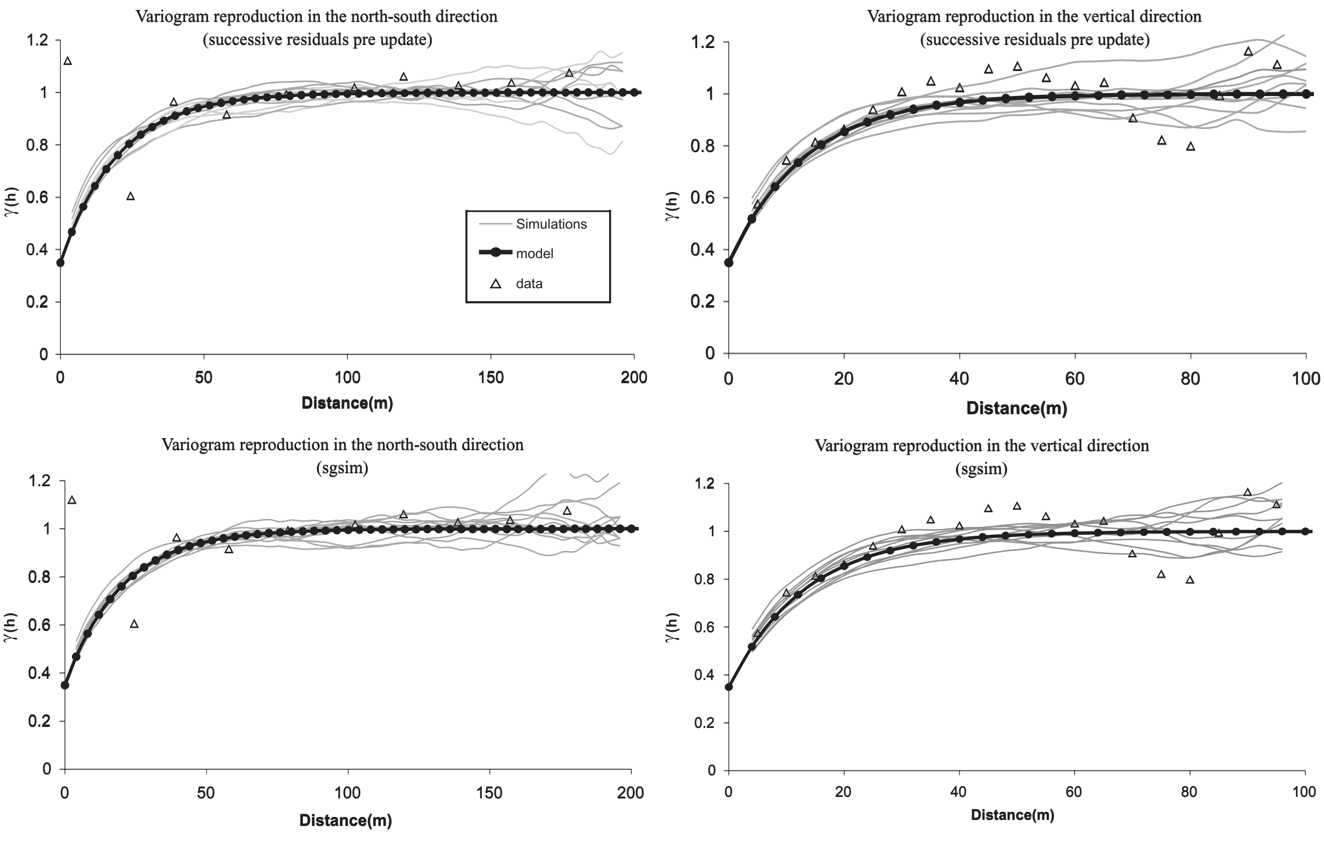


FIG 6 - Variograms of the CSSR realisations in the normal score space prior to updating (top) and variograms of normal score SGS realisations (bottom), for the 29 vertical drill holes (first dataset).

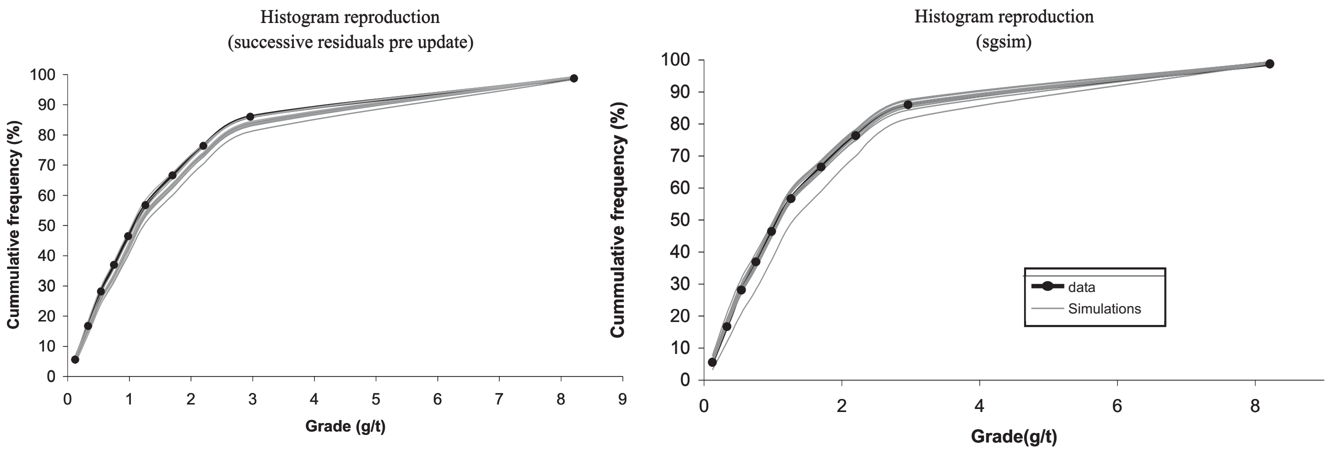


FIG 7 - Reproduction of histograms in the data space (29 vertical drill holes).

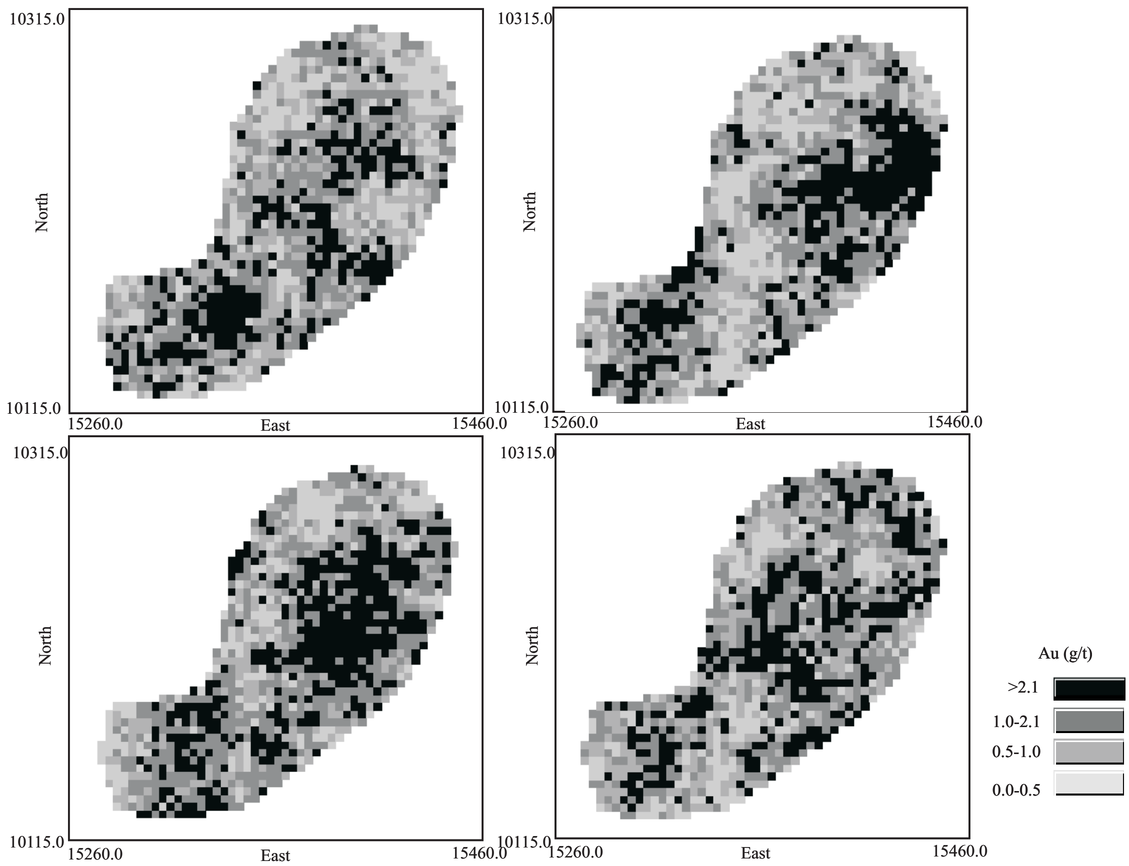


FIG 8 - Selected Au realisations of the same horizontal section of the deposit as in Figure 5. The top realisations are produced by CSSR after updating with new data and the bottom two by SGS and all data available.

methods appear to reproduce the data histogram and variogram well, and no particular distinction in the results between these two methods can be made in general.

In the next part of this case study, the realisations generated by CSSR are updated with the data from the 18 inclined drill holes. To facilitate the current example, this data is moved to the closest nodes of the grid used. In a similar way to the case above, the results from updating are then benchmarked against simulations generated using SGS and the second dataset (which includes data derived from both the 29 vertical and the 18 inclined drill holes).

Figure 8 shows the two updated realisations from CSSR and two realisations generated using SGS for the above case. The comparison of Figure 5 and Figure 8 shows that the updated simulations, which use the new data, are different from the realisations generated from the first dataset, due to the new data in the updating. Figure 9 and Figure 10 present the histograms and variograms after updating. As in the comparison for the initial dataset, no distinction can be made between the two methods based on the variograms and histograms.

COMMENTS ON PERFORMANCE

To facilitate a further understanding of practical issues of CSSR and updating existing realisations in terms of computing time related to the updating, the 5 m composites from the 29 vertical drill holes were used to simulate fields containing 68 000,

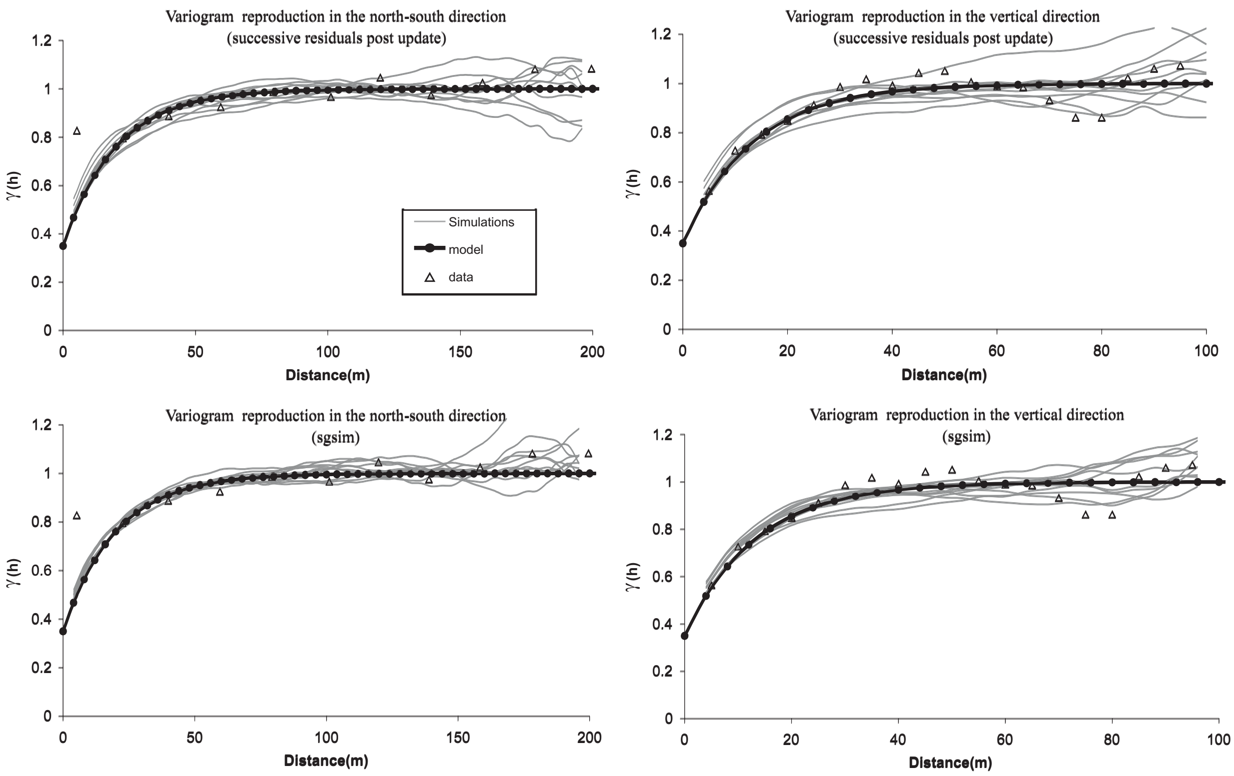


FIG 9 - Variograms of the realisations in the normal score space post-update, compared with experimental variograms of normal score data (29 vertical + 18 inclined drill holes) and models.

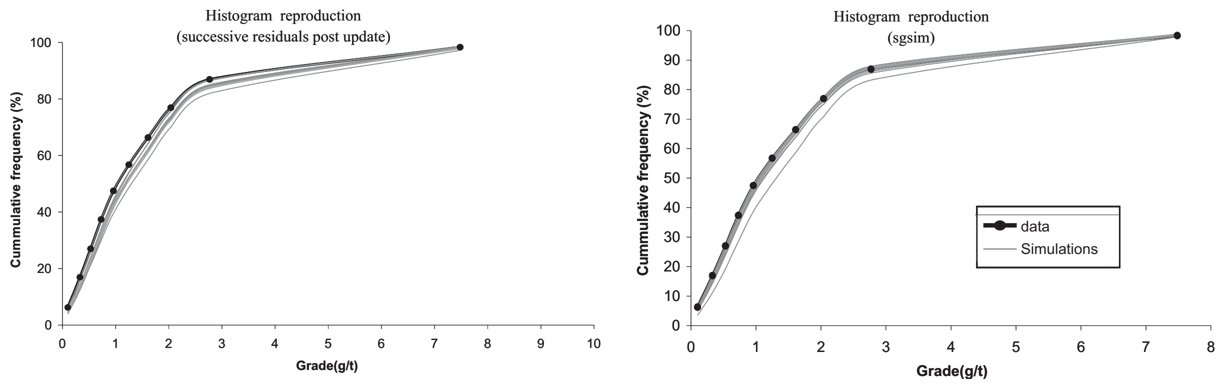


FIG 10 - Reproduction of histograms in the data space post-update (29 vertical drill holes + 18 inclined drill holes).

500 000 and 4 000 000 nodes with group sizes of 2 × 2 × 2, 3 × 3 × 2 and 4 × 4 × 2 nodes. For the larger group sizes, a larger search neighbourhood was used to maintain accuracy and prevent artefacts from occurring (Benndorf and Dimitrakopoulos, 2007, this volume). The generated realisations were then updated using the 5 m composites from the 18 inclined drill holes. The time required for the updating is compared with the time required for a rerun of the simulations. Note that in the first instance, for the first set of simulations, the future data locations within the neighbourhood are actually previously simulated nodes. During the second run when updating, the future data locations within the neighbourhood become legitimate data values. The results, summarised in Table 2, show that the update times are in the order of 65 - 77 per cent of the rerun times, depending on the size of the field being simulated. Generally the larger the field, the smaller the savings from updating. The savings from updating are a balance between the computational cost of the covariance matrix recalculation and the computational cost of the data search and other operations.

SUMMARY AND CONCLUSIONS

A new simulation method, termed conditional simulation by successive residuals has been presented and examined in this paper. The method enables the efficient updating of existing simulated realisations, a characteristic of particular interest to the simulation of orebodies. CSSR is based on the column-wise decomposition of the covariance matrix. This decomposition amounts to implementation of the simulation process with successive conditional covariance matrices. In each successive step of the simulation, random variables in a group are simulated with an LU decomposition of a covariance matrix of updated residuals of conditional covariances. The fast-updating aspect of CSSR is implemented in this study sequentially, a process that is found to perform well. Application of the approach to a stockwork gold deposit supports this assessment and shows the effect of the updating process on the realisations generated. A comparison of the CSSR realisations with the realisations of the deposit generated by the well-known sequential Gaussian

TABLE 2

Relative update times for different field and group sizes †.

		Group size (number of nodes)		
Field size	**Node spacings**	**2 × 2 × 2 = 8**	**3 × 3 × 2 = 18**	**4 × 4 × 2 = 32**
68 000	4 × 4 × 4 m	72%	69%	65%
500 000	4 × 4 × 4 m	72%	76%	75%
4 000 000	4 × 4 × 4 m	76%	77%	77%

† Relative update time = time required for updating divided by time required for rerunning the simulations.

simulation (SGS) shows the end results from these two methods to be indistinguishable. The methods were compared initially using a dataset composed of 29 drill holes and, subsequently, using an updated dataset containing an additional 18 drill holes. The performance studies have shown that the computing times for updating are in the order of 65 - 77 per cent of the rerun times, depending on the size of the field being simulated.

ACKNOWLEDGEMENTS

The work in this paper is part of ARC Grant #LP0211446 to Roussos Dimitrakopoulos, and is also funded by Anglo Gold Ashanti, BHP Billiton, Rio Tinto and Xstrata.

REFERENCES

Benndorf, J and Dimitrakopoulos, R, 2007. New efficient methods for conditional simulation of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Davis, M D, 1987. Production of conditional simulations via the LU triangular decomposition of the covariance matrix, *Mathematical Geology*, 19(2):91-98.

Deutsch, C V and Journel, A G, 1998. *GSLIB Geostatistical Software Library and User's Guide,* second edition, 368 p (Oxford University Press: New York).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R, Farrelly, C and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A87.

Dimitrakopoulos, R and Luo, X, 2004. Generalized sequential Gaussian simulation on group size v and screen-effect approximations for large field simulations, *Mathematical Geology*, 36(5):567-591.

Dowd, P A, 1994. Risk assessment in reserve estimation and open pit planning, *Trans Inst Min Metall*, Section A, Mining Technology, 103:A148-A154.

Godoy, M and Dimitrakopoulos, R, 2007. A multi-stage approach to profitable risk management for strategic planning in open pit mines, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 337-343 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hartman, H L *et al* (editors), 1992. *SME Mining Engineering Handbook, 1, 2,* 2260 p (Society for Mining, Metallurgy and Exploration: Littleton).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ravenscroft, P J, 1992. Risk analysis for mine scheduling by conditional simulation, *Trans Inst Min Metall*, 101:A104-A108.

Vargaz-Guzman, J A and Dimitrakopoulos, R, 2002. Conditional simulation of random fields by successive residuals, *Mathematical Geology*, 34(5):507-611.

Vargaz-Guzman, J A and Dimitrakopoulos, R, 2003. Successive nonparametric estimation of conditional distributions, *Mathematical Geology*, 35(1):30-52.

Fractal-Based Fault Simulations Using a Geological Analogue — Quantification of Fault Risk at Wyong, NSW, Australia

J Scott[1], R Dimitrakopoulos[2], S Li[3] and K Bartlett[4]

ABSTRACT

The modelling of fault populations and quantification of fault risk is a challenge for earth science and engineering applications, including minerals and coal mining, tunnel construction, forecasting of petroleum production, and selection of subterranean repositories for the disposal of toxic waste. This paper discusses a new advance in the use of stochastic fault simulation methods for the quantification of fault risk. The fractal properties of a fully known fault population are used as an analogue of the properties of an undiscovered fault population. The approach is elucidated through the quantification of fault risk in a prospective coalfield at Wyong, New South Wales, Australia, and incorporates spatial patterns of available 'hard' and 'soft' geological data. The method does not find faults unequivocally; rather the output is a map of fault probability. Simulations are found to be consistent with the available information and are statistically and spatially reasonable in geological terms. Significantly, the analogue approach provides a robust, quantified assessment of fault risk using limited exploration information.

INTRODUCTION

Faults may have severe impacts in the mining industry. Unexpected faulting can cause dilution and ore losses in underground metal mines, shut downs and delays to production in underground coal mines with consequent severe financial losses, geotechnical hazards that impact upon safety, and so on. Examples of the adverse effects of faults in mining are known. For example, as recently as 2001, Longannet Deep Mine, Scotland, incurred production losses in the order of 250 000 tonnes of coal as a result of unexpected faulting (BBC News, 2001). In addition to mining, fault risk quantification is an important part of studies in a variety of earth science and engineering projects including petroleum reservoir engineering, groundwater, tunnel construction and the selection of subterranean repositories for the disposal of toxic waste, where fault risk may render a project infeasible.

To quantify the uncertainty in geological structures, mathematical modelling frameworks using stochastic fault simulation have been developed to take advantage of the fractal characteristics exhibited by fault populations (Dimitrakopoulos and Li, 2000). These methods do not identify faults unequivocally; rather their strength lies in using all available geological interpretations and exploration data to generate a series of possible fault population realisations that are then used to quantify the risk of faulting in a terrain of interest. Such approaches are necessary because detecting subterranean fault surfaces directly is difficult and uncertain, even when using modern remote sensing technologies.

Simulation methods for fault populations based upon various approaches including fractals have been developed in the modelling of petroleum reservoirs (eg Gauthier and Lake, 1993; Munthe, More and Holden, 1993; Chilès *et al*, 2000; Mostad and Gjerde, 2000; Holden *et al*, 2003) and have been linked to fluid flow to forecast overall production performance. In the simulation of fault populations in mining environments, differences in data and engineering needs present an opportunity for tailored fault simulation algorithms that can incorporate qualitative geological interpretations. Such an algorithm is presented in Dimitrakopoulos and Li (2000) and applications of it in Li and Dimitrakopoulos (2002). This algorithm is based on fractal fault size distributions and length-throw statistical relations, combined with a probability field approach to 'thinning' a Poisson process so as to locate fault centres. The method has been extensively tested, including back-analysis in a mined-out part of an underground longwall coal mine (Dimitrakopoulos and Li, 2001), showing excellent performance in mapping locations of high fault risk as well as documenting that geological fault maps tend to seriously under estimate fault risk. The performance of the method relies upon hard fault data from which robust estimates of the fractal characteristics are obtained to determine the number, size and proportion of undiscovered faults. Such hard data are traditionally acquired at significant cost from sources such as high-resolution 3D seismic surveys or dense drilling, and are not always available. An alternative in the absence of hard fault data is to use suitable geological analogues, a practice adopted in the petroleum industry to infer spatial statistics in petroleum reservoirs (Walcott and Chopra, 1991; Chilès *et al*, 2000).

A coaliferous prospect at Wyong, New South Wales (NSW), Australia, presents a not uncommon example of developing a longwall underground mine where fault information is very limited. The orientations, sizes and locations of unexposed faults are not known within the prospect due to a variety of commonly encountered factors, including the cost of data acquisition, technical limitations and access restrictions. As a result, input parameters for the fault simulation must be obtained elsewhere. In this study, a novel approach to coalfield fault simulation is presented where a well-known and geologically analogous fault population in a nearby mined-out coal seam provides the fractal properties used to make a robust, quantified assessment of fault risk from limited exploration data within the area of interest.

In the following sections, relevant aspects of fractal theory are described, the fractal fault simulation algorithm with analogues is outlined, and issues of hard data, soft data and geological analogues discussed. Then, a novel application at the Wyong coalfield, NSW, Australia, is presented, including the mapping of fault risk over the study area. Issues concerning the integration of 'soft' data are discussed, including their use and effects. Finally, conclusions and recommendations are presented.

FRACTAL FAULT SIMULATION WITH GEOLOGIC ANALOGUES

Some aspects of fractal theory

The fractal properties of fault populations have been recognised since the 1980s (eg King, 1983; Turcotte, 1986; Childs, Walsh and Watterson, 1990) and have been investigated in numerous studies (eg Marret, Ortega and Kelsey, 1999; Berkowitz and Hadad, 1997). In general, the theory suggests that various fault parameters are invariant with respect to scale or are

1. Roche Mining Pty Limited, PO Box 8221, Woolloongabba Qld 4102, Australia. Email: Justin.Scott@roche.com.au

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

3. CRCMining, The University of Queensland, 2436 Moggill Road, Pinjarra Hills Qld 4069, Australia. Email: s.li@crcmining.com.au

4. Coal Operations Australia Limited, Level 39, Grosvenor Place, 225 George Street, Sydney NSW 2000, Australia.

'self-similar', providing a model that can be used for predictive purposes. In fractal theory, fault size distributions (throw or length) can be described by a power-law (fractal) model over a wide range of fault size such that:

$$\log(N_s) = \alpha - \beta \log(S) \tag{1}$$

where:

N_s is the cumulative number of faults of size greater than or equal to fault size S

S is either length L or throw T

α is a function of the fault density and when α is high, the fault density is high

β is the fractal dimension of the fault population that defines the relative number of large and small faults; when β is high, the number of small faults is high relative to the number of large faults

Techniques to obtain the fractal dimension β are discussed elsewhere (eg Main *et al*, 1999).

The fractal fault simulation process

The fractal fault simulation method outlined herein follows four steps.

In the first step, using a given set of available data, the fault simulation process begins with the inference of fault statistics and fractal models, which are then used to define relative numbers of larger-throw to smaller-throw faults, expected lengths of faults of a given throw and the total number of faults expected within a study area.

In the second step, the spatial density patterns of known faults are mapped, and their underlying spatial continuity quantified with variograms that are subsequently used in the simulation in the fourth step. The process of mapping fault densities and quantifying the underlying spatial patterns of faults tests the reliability of fault interpretations made from multiple sources of soft data, whilst at the same time it constrains the locations of simulated faults in a manner consistent with the underlying spatial patterns of the available data.

In the third step, soft data are numerically coded. This coding is important as it allows all available geological data not used elsewhere to be synthesised. In addition, it quantifies the geological understanding of expert local geologists. Furthermore, it provides a mechanism for updating simulation results as exploration continues.

In the fourth step, fault populations are simulated with the algorithm outlined below (Dimitrakopoulos and Li, 2000; Dimitrakopoulos, in press):

1. Within a study area A, define a random path to be followed in visiting locations x to be considered as centres of fault traces. There are N locations $\{x_i, i=1,\ldots,N\}$ to be visited. The N locations exclude the known fault centres.
2. Generate a realisation of an auto-correlated probability field $\{p(x_i), i=1,\ldots,N\}$ reproducing the uniform marginal cumulative distribution function and the variogram $\gamma_p(h)$ corresponding to the variogram $\gamma_x(h)$ of the uniform transform of the fault densities, $\lambda(x_i)$, in the study area A. Integrate soft data when generating $p(x)$.
3. Estimate at the first location x_i the density of an inhomogeneous Poisson process $\lambda(x_i)$ using a planar Epanecnikov kernel estimator.
4. Use the probability value $p(x_i)$ at location x_i to thin a Poisson point process from:

$$1 - p(x_i) < \lambda(x_i) / \lambda^* \tag{2}$$

where:

λ^* is the density of a corresponding homogenous Poisson point process and $\lambda(x_i) \leq \lambda^*$

If the above constraint is met, a fault centre exists at x_i, if not, the next node on the random path is visited until the constraint is met.

5. Randomly select a maximum fault throw from the fractal model of the fault size distribution in Equation 1.
6. Grow the fault in opposite directions from the centre of a fault trace by sampling randomly from the fault strike distribution and using a distance step and directional tolerance at each step until the fault length reaches the length sampled from the length versus maximum throw power-law model.
7. Repeat points three to six until the total number of faults satisfies the fractal fault size distribution in Equation 1.

The algorithm outlined above is, in practice, used to simulate a large number of realisations (or equally likely scenarios) of the undiscovered fault population. Fault realisations are consistent with the statistical characteristics of the fault data available, spatial characteristics of local data and soft information incorporated. An advantage of the algorithm is its capacity to incorporate both 'hard' and 'soft' data, and thus utilise geological understanding as well as meet specific engineering requirements of the project. It should be noted that the combination of simulated fault realisations can be used to generate probability maps over a study area for faults of sizes of interest.

To aid understanding, the terms 'hard' data, 'drill hole' data and 'soft' data are further defined. The definitions and examples are articulated here with the case study that follows in mind but without loss of generality for the method presented.

'Hard' data refers to the most reliable and complete set of fault data available. Hard data encompass faults mapped during mining, and the fault locations and maximum fault throws considered to be known. 'Drill hole' data refers to faulting detected in drill holes. These data are regarded as equally reliable as hard data, but are incomplete. This is because, although fault locations are known, no fault throw, strike or length information is quantifiable. The term 'soft data' refers to faulting or faulting trends interpreted from indirect observations such as the ones discussed next. Soft data are further recognised as 'linear' and 'trend' soft data. So-termed 'linear' soft data encompass fault interpretations from sources such as, for example, 2D-seismic survey lines, air photos, drainage analysis, aeromagnetics, and structure contours. The locations and orientations of the interpreted faults are regarded as uncertain and no fault throws or lengths are quantifiable. So-termed 'trend' soft data encompass background-type information delimited by a polyregion, which is used to identify regions of higher and lower fault susceptibility. Examples are mapped stress directions, roof conditions, seam splits and volcanics; interpretations of volcanics based on aeromagnetics; interpretations of circular features based on aerial photography; and expert local geologists' interpretations of fault trends and structurally anomalous regions. 'Soft' data are regarded as both less reliable and less complete than hard data or drill hole data.

Using geological analogues

It is common to find that available hard data are sparsely located. To circumvent a paucity of available hard data, fractal fault population statistics may be inferred from completely known and geologically analogous fault populations. The fractal dimension and size distribution models obtained can then be used as input to the fault simulations of the undiscovered fault population. Simulations are then consistent with the statistical characteristics of the geological analogue, spatial characteristics of local data and the soft information incorporated.

Geological analogues have been used previously in the modelling of atypical oil reservoirs (eg Research Intelligence, 2004; Ruf and Aigner, 2004; Cronin and Kidd, 1998) and detailed outcrop observations of natural analogues have already been incorporated into stochastic models and simulations of fracture populations in petroleum and geothermal reservoirs (Chilès *et al*, 2000). Natural analogues are attractive alternatives for the study of fault populations both by academics and by industry. Outcrop analogue investigations contribute to the understanding of the architecture and behaviour of subsurface hydrocarbon reservoirs (Ruf and Aigner, 2004). Representative outcrops of reservoir rocks, or information culled from open-file sources describing similar reservoir contexts, can reduce uncertainties and increase confidence in geological models (GeoScience Ltd, 2004).

In the application of geological analogues to fault risk assessments, the key question is: Are there geological reasons to expect a similar frequency of faulting in both the study area and the analogue area? Where it can be shown that two areas exist within the same structural domain and where geological controls on fault development (such as layer thickness and the location of basement structures) are consistent across both areas, a similar frequency of faulting may be expected.

To be able to implement fractal models based on a geological analogue in fault simulations of an unknown population, there are two prerequisites:

1. the expected number of unexposed faults, determined from the fractal model of the analogue, must be scaled to the size of the study area; and
2. drill hole data or soft data must be available within the study area to help constrain the locations of simulated faults.

CASE STUDY AT WYONG

The case study presented in this section pertains to work conducted as part of pre-feasibility studies evaluating a coaliferous prospect of Coal Operations Australia Limited (COAL) at Wyong, NSW. The Wyong area, located within the north-eastern margin of the Sydney Basin, contains the last significant quantity of undeveloped, export-quality thermal coal resources in the Newcastle Coalfield, which in 2001 - 2002 saw production of 20.4 Mt of raw coal. A number of collieries currently mine coal seams to the immediate north of the study area. Of particular importance to the evaluation of a potential underground operation at Wyong is the presence of faults and their possible effects on longwall operations and related planning. Related key issues are the very limited access to the study area for the collection of subsurface information, and the need to assess fault risk from limited and incomplete hard information within the study area. This made Wyong a particularly suitable case for the use of geological analogues in assessing fault statistics.

Geological setting of the Wyong area

The study area (Figure 1) falls within the southern part of the Newcastle Coalfield, in the north-eastern part of the Sydney Basin, NSW, Australia. The coal resources are contained within the upper part of the Permian Newcastle Coal Measures. The Lochinvar anticline and Hunter thrust provide regional geological structural bounds to the west and north of the study area respectively (Herbert, 2002). Southern and eastern structural bounds are not well defined; however there is no evidence that such a structural bound occurs within or between the prospect and historic coal mines to the northeast. Locally, the lease area is geologically continuous, separated only by three narrow conglomerate channels that form seam-splitting bodies. Two of these channels define the borders of the prospect. Local faulting is predominantly normal. Some reverse and thrust structures are known. The character of the faulting differs between the northwest and the northeast trending orientations, with northwest trending faults typically of smaller throw and more numerous than the northeast trending structures. Previous work conducted by COAL concluded that: 'the density of northwest-southeast trending faults and dykes exposed in mine workings to the northeast is expected to be repeated through the project area' (BHP Billiton Internal Report, 2002).

For this study, faults are grouped into two populations based upon orientation. One included northwest trending structures and the other northeast trending structures. In the interest of brevity, only the results for the northwest trending population are shown herein.

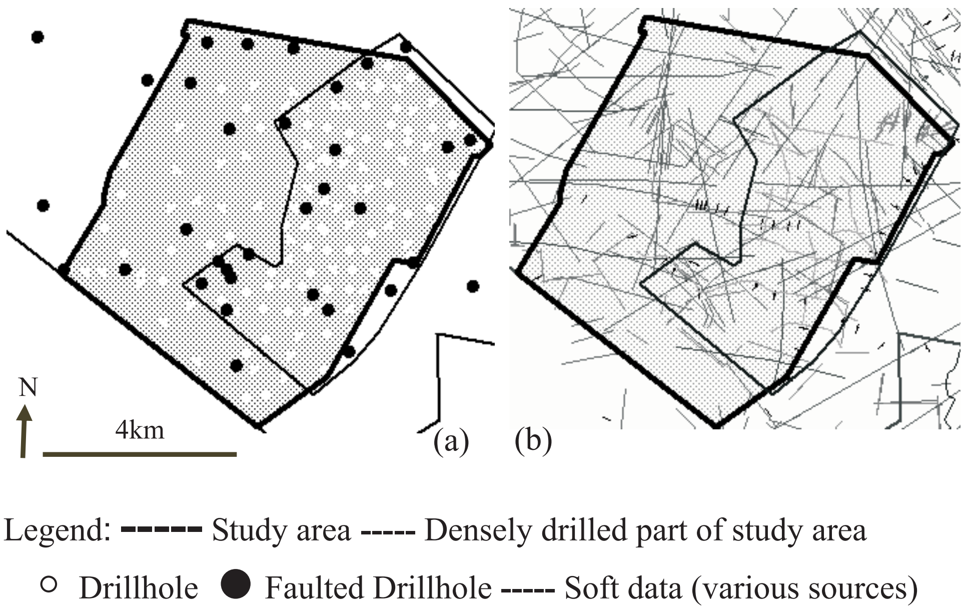


FIG 1 - (a) Available drill hole and faulted drill hole data within the study area. (b) Available linear soft data within the study area. Interpretations of faults and lineaments based upon aeromagnetics, aerial photography, drainage patterns, recent and reprocessed 2D-seismic and structure contours.

Available data and the geological analogue

As previously mentioned, the limited number of hard data available in the current study is typical of the early stages of minerals exploration and also of projects where access or cost restrictions apply. In place of hard data within the prospective area, data analysis for fractal properties is conducted on an analogue fault population. The fault population used as an analogue was established in conjunction with expert local geologists based on regional and local structural characteristics. The data available on the geologically analogous fault population used in this study are acquired from mapped faults in mined-out historic coal mines approximately 9 km northeast of the study area. The dataset is composed of 1159 normal faults and includes measurements of fault locations, throw, length and orientation. Normal faults are typically hinged, with dips ranging from 55° to 75° and maximum throws generally ranging from 0.5 m to 5.0 m (occasionally up to 15 m) in their central section and zero at their extremities. While uncommon, low-angle thrust and high-angle reverse faults do occur along both northeast and northwest trends. Figure 2 shows the spatial distribution and orientation of fault traces encountered during historic mine workings. Note that in the analogue used a heterogeneous spatial distribution of fault traces and centres is evident, as is the clustering of faults within discrete locations. The use of a suitable geological analogue provides, in the absence of hard data in the study area, the best possible alternative in understanding fault statistics and is consistent with the known characteristics of local fault populations.

In the study area, significant drilling data are also available to record fault intersections. Figure 1 shows:

1. the available drilling data, highlighting fault-intersecting drill holes; and
2. the sources of soft data available within the study area.

Soft data available from eighteen sources are split into groups of linear and trend data, as described in a preceding section. Lithology at Wyong could not be correlated with faulting and is not used in this study. Expert local geologists incorporated stress directions and amplitudes into fault trend interpretations.

Fault population statistics and fractal models

The northwest trending fault population included faults oriented between 270° and 360°, with a mean orientation of 317° and a standard deviation of 12°. Figure 3 shows the fault size distribution obtained from the analogue fault dataset. It is well defined by a single fractal model over an order of magnitude from 1.0 m to 14.0 m maximum throw. The fractal model is described by the equation shown in Table 1 in which T is the fault throw; Nt is the cumulative number greater than or equal to a given throw; α is the model intercept and β is the fractal dimension. A β value of 1.98 is within the range reported in the technical literature (eg Cowie and Scholz, 1992). Scaling the fractal model of the analogue to the size of the study area, the number of northwest trending faults within the study area expected to have a throw greater than or equal to 4.0 m is 30. Scaling was necessary as the study area covers about 60 km^2, and the historic coal mines 31 km^2.

TABLE 1

Power-law equation describing the fractal model of fault size distribution.

Fractal Model   **Log $N_t = -\beta$ log T + α**
Log N_t = – 1.98 log T + 2.67

The length-throw relationship is shown in Figure 4. The scatter is most likely to be a consequence of sampling limitations. To calculate a practical and realistic fractal model of the fault throw-length relationship, a subset of the available data was used. Available data were ranked according to fault throw and also ranked according to fault length. Faults were included in the subset if:

1. the throw rank approached the length rank, and
2. the maximum throw of the fault was greater than or equal to 1 m.

The fractal model of the throw-length relationship is described by the equation shown in Table 2, in which T is the fault throw; N is the model slope; L is fault length and C is the model intercept.

FIG 2 - Fault traces of faults detected during historic mine workings.

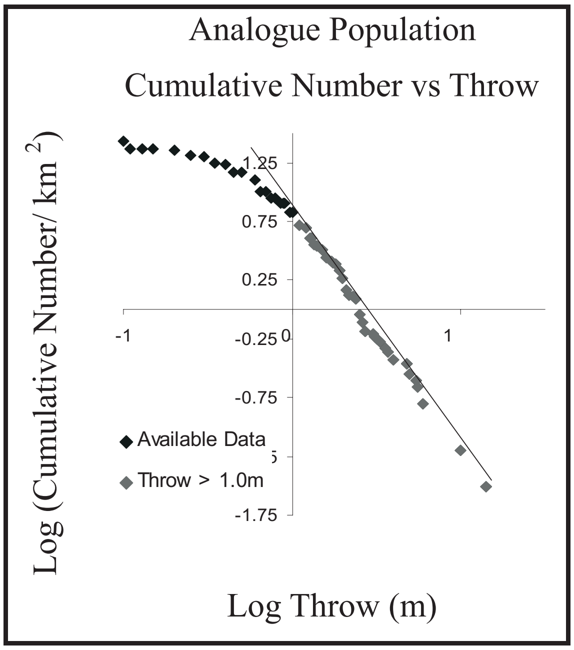


FIG 3 - Fractal model of fault size distribution of geological analogue.

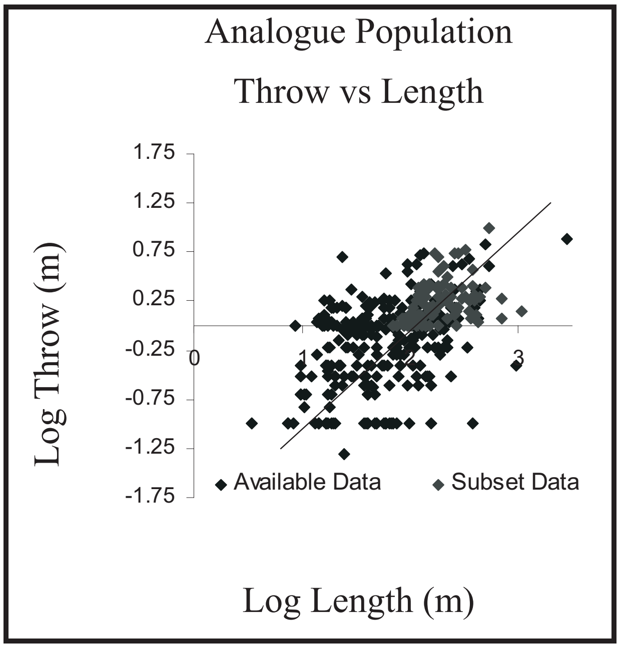


FIG 4 - Fractal model of the throw-length relationship of geological analogue.

TABLE 2

Power-law equation describing the fractal model of the throw-length relationship.

Fractal Model Log T_{max} = N log (L) - c
Log (T_{max}) = 1.0 log (L) - 2.05

Continuity of spatial patterns and incorporation of soft data

Mapping and quantifying underlying fault spatial patterns is necessary to constrain the locations of simulated faults and, in turn, produce realistic fault simulations. The underlying spatial correlation of fault locations within the study area can be inferred from the density of fault-intersecting drill holes mapped over the study area. Spatial patterns are modelled using variograms and quantify the continuity in the spatial patterns of known faults; they are used in generating the probability field in step three of the fault simulation method described earlier. Table 3 shows the variogram model used in simulations of the northwest trending fault population at Wyong.

Numerating soft geological information

Soft geological information must be numerically coded for it to be integrated into the fault simulation algorithm. In this process, all available drilling and soft data are synthesised into a prior probability map. The reliability of available soft data sources is tested by comparing the underlying spatial patterns of soft data interpretations to the underlying spatial patterns of fault-intersecting drill holes within the study area. The soft data sources are then ranked and weighted in conjunction with expert local geologists. Tables 4 and 5 show the ranking of available soft data. The final step before fault simulation is undertaken is the generation of a prior probability map for fault locations. The study area is divided into 200 m × 200 m grid cells, each with a fault susceptibility determined from the soft and hard data available. The prior fault probability map is used as input in step two of the simulation algorithm described earlier.

TABLE 3

Variogram model describing continuity of fault centres.

	Model type	Direction	Sill	Range (m)	Anisotropy ratio
	Nugget		0		
Structure 1	Spherical	Southeast (135)	0.0755	2210	0.87
Structure 2	Spherical	Southeast (135)	0.0293	2710	0.86

TABLE 4

Ranking of reliability of linear soft data within the study area by consideration of discussions with expert local geologists and the comparison of fault spatial patterns from soft data with those of hard data and drill hole data (1 – most reliable, 7 – least reliable).

'Linear' soft data ranked by reliability	Rank
	Northwest population
2D Seismic (Trial)	1
2D Seismic (Jilliby Ck)	2
Seam split mapping	3
Reprocessed seismic	4
Aeromagnetics	5
Aerial photography	6
Drainage analysis	7
Regional mapping	7

TABLE 5

Ranking of reliability of trend soft data by consideration of quantity and type of available data used to define the trend (1 – most reliable, 7 – least reliable).

'Trend' soft data ranked by reliability	Rank
	Northwest population
Geologists' low risk zones	1
Geologists' high risk zones	1
Field mapping	1
Mapped volcanics	4
Structural anomalies	5
Volcanic plugs (aeromagnetics)	5
Circular features (air photos)	7

Conditional simulation of faults

Fault simulations are generated over 60 km^2 within the limits of the study area. Fifty fault realisations are generated in this study and are used to quantify fault probabilities. Simulation results are validated and the available data, power-law models of fault size distribution, fault throw-length relationships and spatial correlations are reproduced so as to comply with all data available, including the geological analogue used. The validations of the fault simulations are not presented here. Figure 5 shows one result of using the fractal model of the geological analogue for the fault simulation of the undiscovered fault population. The simulated faults have a minimum throw of 2.0 m (a), and 4.0 m (b), and the fault population appears geologically reasonable, with smaller faults clustered about larger faults and distinct areas of higher and lower fault density evident. En echelon arrangements of faults, typical of the surrounds of the study area, can also be inferred from Figure 5.

Fault risk is calculated for each cell as the proportion of all realisations in which a fault is generated within that cell. Figure 6 shows the probability map resulting from 50 fault population

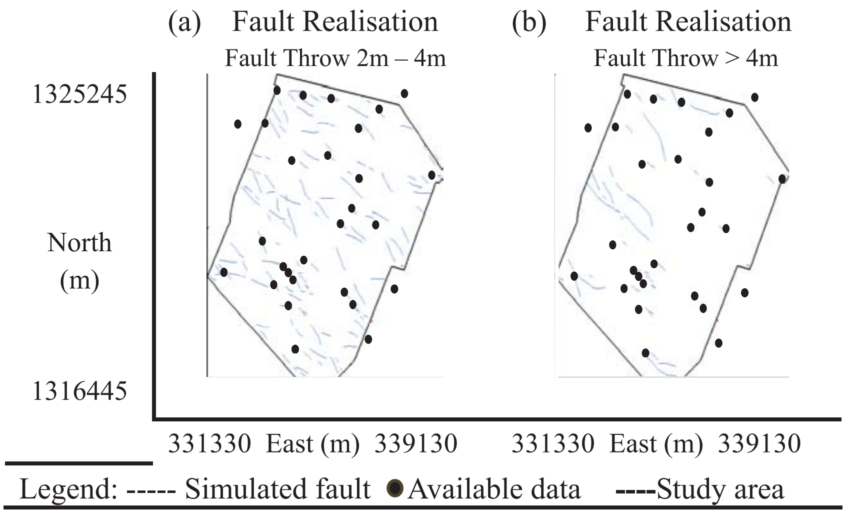


FIG 5 - Maps of fault population simulations within the study area based upon fault characteristics at historic coal mines. Fault size cut-off: (a) 2 m throw (b) 4 m throw.

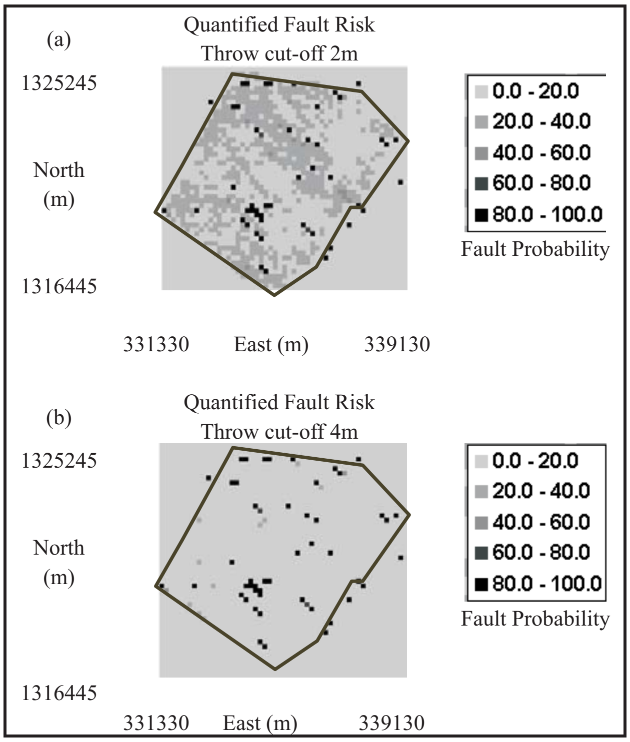


FIG 6 - Maps of fault risk within the study area based upon fault characteristics at historic coal mines. Fault size cut-off: (a) 2 m throw (b) 4 m throw.

realisations at a cut-off of 2.0 m throw (a) and 4.0 m throw (b). Existing faults are shown in black and the fault risk is shown through a grey scale, with higher fault probabilities being darker and lower fault probabilities being lighter. Unshaded parts of the study area have a very low fault probability.

The fault probability map indicates that coal resources are unlikely to be affected by northwest-trending structures. Mine planners are able to use the fault probability map to reduce the risk of encountering structural hazards and impediments by focusing early stages of mining into low-risk areas as well as orienting mine layouts in a way that the effects of faulting are minimised. Further exploration may be targeted to the parts of the study area that are neither classified as high risk nor low risk using the available data.

COMMENTS AND CONCLUSIONS

A new approach to the quantification of geological uncertainty using limited exploration data has been presented in this paper. Fractal-based fault simulations are conducted using fractal characteristics of an analogue fault population and available soft data. A series of simulations are generated, and the probability of faulting for any fault size of interest within the study area represents the corresponding quantified fault risk.

When attempting to characterise geological fault populations, technological limitations, access restrictions and the expense of data collection can all impede data acquisition and understanding of fault uncertainty. In such circumstances, the use of geological analogues to assess fault risk has two main advantages:

1. the quantification of fault risk, rather than the identification of faults *per se*, even beyond the resolution limits of seismic and into areas that are otherwise inaccessible for social, environmental or logistical reasons; and
2. the incorporation of hard data, drill hole data and soft data into the quantification of fault risk such that the continuity of fault spatial patterns within the study area and/or domain of the geological analogue are incorporated into fault population simulations.

A case study was conducted at Wyong, New South Wales, where sufficient analogue fault information was available from historic mine workings located 9 km to the northeast of the prospect. It is possible to simulate a fault population in an area where minimal data is available using the models of fault orientation, throw and length inferred from a geological analogue. The use of the known fault population as a geological analogue was considered appropriate given the geological continuity that exists between the study area and the historic mine site. The algorithm successfully utilises the analogue fractal models of fault size distribution, and simulations successfully reproduce the spatial correlations of the available data and are constrained by the faulting density and susceptibility trends identified by expert geologists or drilling. The results show that these simulations can be used to assess the probability or risk that an area is faulted. This is determined from the proportion of realisations in which a fault is generated at a given location.

Fault simulations of a northeast trending population are shown. Based on many simulations, fault risk is very low (<20 per cent) over the majority of the study area when a cut-off of 4.0 m fault throw is used and low (mostly <40 per cent) when a cut-off of 2.0 m throw is used. Fault probability maps can be used to display the risk of undetected faults, identify those areas sufficiently explored and those in need of extra investigation, compare risk at different locations, and enable decision-makers to choose an appropriate level of risk. Future work could address the uncertainty associated with fault interpretations from seismic surveys and undertake back-analysis of fault risk quantified using a geological analogue.

ACKNOWLEDGEMENTS

Coal Operations Australia Limited and BHP Billiton are gratefully acknowledged for their support throughout this project.

REFERENCES

BBC News, 2001. Mine jobs go after serious rock fall [online]. Available from: <http://news.bbc.co.uk/1/hi/scotland/1657071.stm> [Accessed: 4 May 2007].

Berkowitz, B and Hadad, A, 1997. Fractal and multi-fractal measures of natural and synthetic fracture populations, *Journal of Geophysical Research*, 102(B6):12205-12218.

BHP Billiton, 2002. Wyong Coal Project – Concept Study Volume 2, Revision 1.4, p 54, internal report.

Childs, C, Walsh, J J and Watterson, J, 1990. A method for estimating the density of fault displacements below the limit of seismic resolution in reservoir formations, in *North Sea Oil and Gas Reservoirs II*, pp 309-318 (Graham and Trotman: London).

Chilès, J-P, Bourgine, B, Castaing, C and Genter, A, 2000. Stochastic modelling and simulation of fracture populations in petroleum and geothermal reservoirs, in *Geostatistics 2000* (eds: W J Kleingeld and D G Krige) Vol 1, pp 413-423, Cape Town, South Africa .

Cowie, P A and Scholz, C H, 1992. Displacement-length scaling relationships for faults: data synthesis and discussion, *Journal of Structural Geology*, 14:1149-1156.

Cronin, B T and Kidd, R B, 1998. Heterogeneity and lithotype distribution in ancient deep-sea canyons: Point Lobos deep-sea canyon as a reservoir analogue, *Sedimentary Geology*, 115:315-349.

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R and Li, S, 2000. Fault simulation of faults and uncertainty assessment in longwall coal mining, in *Proceedings Geostatistics 2000* (eds: W J Kleingeld and D G Krige) pp 692-703, Cape Town, South Africa.

Dimitrakopoulos, R and Li, S, 2001. Quantification of fault uncertainty and risk management in longwall coal mining: back-analysis study at North Goonyella Mine, Queensland, in *Geological Hazards – The Impact to Mining* (eds: Doyle and Moloney) pp 175-182.

Gauthier, B D M and Lake, S D, 1993. Probabilistic modelling of faults below the limit of seismic resolution in Pelican Field, North Sea, offshore United Kingdom, *AAPG Bulletin*, 77(5):761-777.

GeoScience Limited, 2004. Fractured reservoir characterisation [online]. Available from: <http://www.geoscience.co.uk/geofrc/georesanal.html> [Accessed: 4 May 2007].

Herbert, C, 2002. Personal communication, September.

Holden, L, Mostad, P, Nielsen, B F, Gjerde, J, Townsend, C and Ottesen, S, 2003. Stochastic structural modeling, *Mathematical Geology*, 35(8):899-914.

King, G, 1983. The accommodation of large strains in the upper lithosphere of the Earth and other solids by self-similar fault systems: the geometrical origin of b-value, *Pure and Applied Geophysics*, 121(5-6):761-815.

Li, S and Dimitrakopoulos, R, 2002. Quantification and assessment of fault uncertainty and risk using stochastic conditional simulations, *Journal of Coal Science and Engineering (China)*, 8(2):1-11.

Main, I G, Leonard, T, Papasouliotis, O, Hatton, C G and Meredith, P G, 1999. One slope or two? Detecting statistically significant breaks of slope in geophysical data, with application to fracture scaling relationships, *Geophysical Research Letters*, 26(18):2801-2804.

Marret, R, Ortega, O J and Kelsey, C M, 1999. Extent of power-law scaling for natural fractures in rock, *Geology*, 27(9):799-802.

Mostad, P and Gjerde, J, 2000. Multifractal fault simulation, in *Proceedings Geostatistics 2000* (eds: W J Kleingeld and D G Krige) pp 358-368, Cape Town, South Africa.

Munthe, K L, More, H and Holden, L, 1993. Sub-seismic faults in reservoir description and simulation, SPE paper No 26500.

Research Intelligence, 2004. Visualising sandstone reservoirs [online]. Available from: <http://www.liv.ac.uk/researchintelligence/issue19/sandstonereservoirs.html> [Accessed: 4 May 2007].

Ruf, M and Aigner, T, 2004. Facies and poroperm characteristics of a carbonate shoal (Muschelkalk, South German Basin): a reservoir analogue investigation, *Journal of Petroleum Geology*, 27(3):215-239.

Turcotte, D, 1986. A fractal model of crustal deformation, *Tectonophysics*, 132:361-369.

Walcott, D S and Chopra, A K, 1991. Investigating infill drilling performance and reservoir continuity using geostatistics: in *Proceedings Third International Reservoir Characterization Technical Conference* (ed: B Linville) pp 297-326 (PennWell Books: Oklahoma).

The Use of Conditional Simulation to Assess Process Risk Associated with Grade Variability at the Corridor Sands Detrital Ilmenite Deposit, Mozambique

M Abzalov[1] and P Mazzoni[2]

ABSTRACT

The Corridor Sands deposits represent the largest known economic resource of titanium dioxide minerals. The West Block of Deposit 1 alone contains a measured and indicated resource of 1.7 billion tonnes at 4.14 per cent ilmenite. Total resources in the project are inferred to be about 16 billion tonnes containing five per cent total heavy minerals (THM) of which about half is expected to be ilmenite.

A geological model for the West Block was established to describe the geological variability of the mineralised sand complex, and to provide a framework for the resource modelling. Six geological domains were recognised from distinct colour, grain size, silt content and mineralogy differences. The delineation of domain boundaries in the geological model was used to constrain the variography and grade interpolation used to derive the resource model. While the resource model for West Block carries a high degree of confidence, it is recognised that the drilling density is such that there will be uncertainty in the model on the predictability of local grade variations (on a daily or weekly production basis).

A conditional simulation study was conducted to examine the possible risk at the front end of the plant for local grade variability to exceed the primary concentrator (PCP) tolerance limits. The study focused on silt and THM grade in Domain 1 as the two variables of greatest concern to the PCP. The work demonstrated that for a selective mining unit (SMU) size of 10 m × 10 m × 12 m, there will be no issues with the PCP ability to handle silt variability in ROM at the designed maximum tolerance limit of 25 per cent silt. At a lower plant feed tolerance of 20 per cent maximum silt then about 1 in 3 SMU of Domain 1 ROM could be expected to exceed this. In-pit blending with ore from domains with lower silt content would be required to control PCP feed composition. For Domain 1 THM, the simulations show that the optimal THM grade range of six per cent to 15 per cent will be regularly exceeded. The PCP feed rate can be slowed to accommodate these grade 'surges' even if in-pit blending options were not available.

INTRODUCTION

This paper documents the application of conditional simulation at the Corridor Sands heavy mineral sand deposit located in the south-eastern Mozambique (Figure 1).

A distinct feature of the deposit is the presence of an abundant <45 µm 'silt fraction' thought to represent fine weathering products of the original mineralised sand. The primary concentrating plant (PCP) is designed to run continuously at up to 25 wt per cent 'silt' and 15 wt per cent THM grades. It can cope with 'silt' grades above 25 wt per cent however this can lead to loss of process efficiency and additional process cost such as excessive flocculant consumption. 'Silt' grade of 3 m drill samples can occasionally exceed 25 wt per cent which suggests that average silt grade of small volumes of ore, such as selective mining units (SMU), can exceed the PCP tolerance limits.

To assess the risk of delivering ore with 'silt' or THM grades exceeding the PCP tolerance level, the spatial distributions of these variables have been modelled using the sequential Gaussian simulation (SGS) (Goovaerts, 1997) algorithm implemented within the ISATIS software (Bleines *et al*, 2001).

The SGS method has been primarily applied to confirm that the PCP as designed is capable of dealing with short-range grade fluctuations in the resource. Several sizes of the SMUs have been tested in this study to assess dependence of the recovered grade on the mining selectivity. A secondary outcome of the work was a comparison of the conditional simulation model with the ordinary kriging estimates as an independent validation of the global resource estimation.

PROJECT BACKGROUND

The Corridor Sands Project is based on the very large deposits of ilmenite bearing heavy mineral sands near the town of Chibuto in southern Mozambique (Figure 1). The deposits are about 190 km north of the capital city Maputo and between 20 and 60 km inland from the Indian Ocean. They collectively represent the largest known resource of ilmenite. Deposit 1 alone contains measured and indicated resources of 2.7 billion tonnes at four per cent ilmenite. Total resources are in the order of 16.5 billion tonnes at five per cent THM of which about half is expected to be ilmenite. The deposits were discovered in 1997 during exploration of Pleistocene dune sands along the east coast of Africa. Exploration subsequently focused on the apparently largest and highest grade Deposit 1.

Three drilling campaigns were completed. Aircore drilling on 1 km spaced N-S traverses in 1998 established inferred resources at Deposit 1. Aircore drilling on 250 m × 125 m, WNW oriented grids during 1999 - 2000 established measured and indicated resources for the East Block and West Block of Deposit 1. Aircore and triple tube diamond drilling during 2001 - 2002 established proven and probable reserves at West Block sufficient for the first 25 years of mining. The initial mining area was drilled on 100 m × 100 m centres and some detailed 25 m and 50 m grids and crosses were drilled to assist with the variography. About 1200 holes for 80 000 m have been completed at Corridor to date of which approximately 55 000 m has been into West Block. A bankable feasibility study (BFS) of the deposit was completed by WMC Resources in 2002.

The Project envisages the establishment of a fully integrated heavy mineral sands mining, mineral processing and beneficiation operation together with its associated infrastructure, including an export facility for shipment of final products. An open pit mine is planned as a conventional truck and shovel operation delivering ore from free digging faces to a two-stage mineral processing plant. A fleet of 100 t trucks will be used for the first five years of production then 200 t trucks for the remaining mine life. The PCP will utilise trommels and desliming cyclones to remove the oversize and silt (<45 µm) fraction. Heavy minerals are recovered from the remaining sand by wet gravity spirals. The magnetite is stripped off magnetically to produce a heavy mineral concentrate (HMC). The valuable heavy minerals, ilmenite, zircon, rutile and leucoxene are then separated in the mineral separation plant (MSP). A smelting complex located adjacent to the mining and mineral processing operations will upgrade the ilmenite to a titanium dioxide slag containing about 85 per cent titanium dioxide, together with a high purity foundry iron product. Sale of slag to pigment producers and iron to foundries will provide the bulk of the project revenue. A layout of resources and planned infrastructure at Deposit 1 is shown in Figure 1.

1. FAusIMM, Exploration Manager – New Opportunities (Asia Region), Rio Tinto Exploration Pty Ltd, PO Box 175, Belmont WA 6984, Australia. Email: marat.abzalov@riotinto.com

2. FAusIMM, Consultant, 191 Great Eastern Highway, Belmont WA 6104, Australia. Email: pma27066@bigpond.net.au

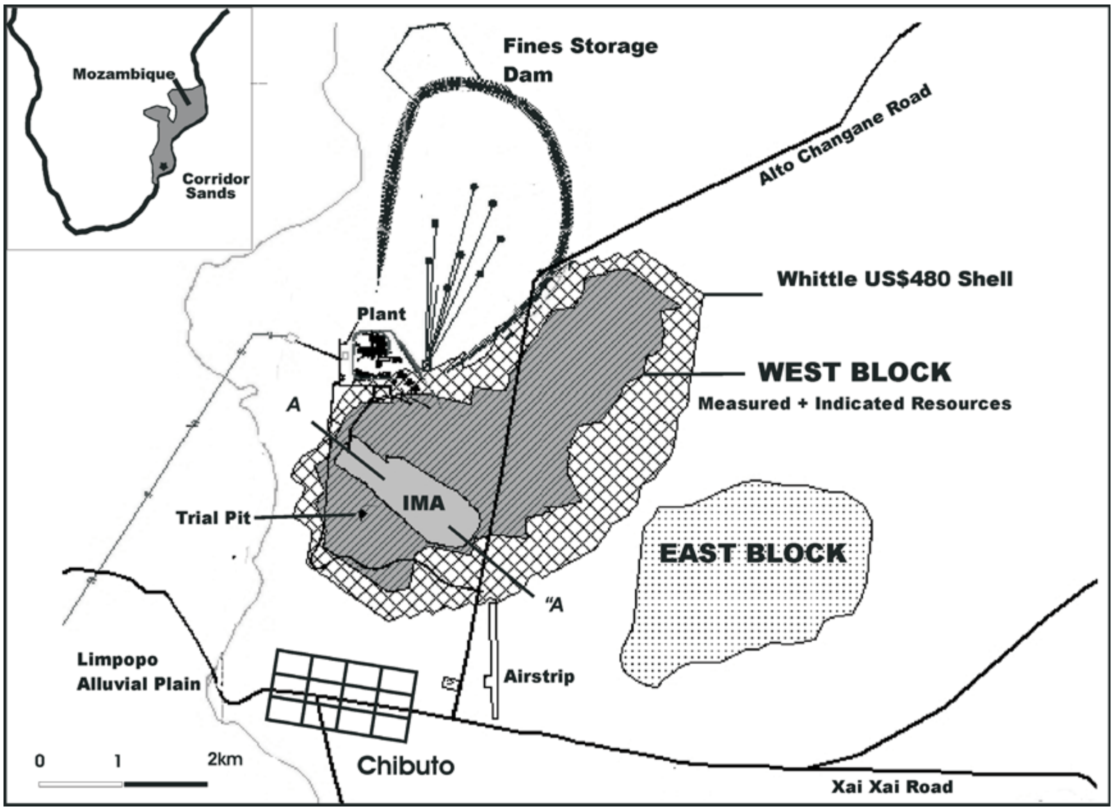


FIG 1 - Corridor Sands Deposit 1 – resources and planned infrastructure.

GEOLOGY

West Block structure

Six geological Domains were recognised during geological logging of drill holes, mapping of the trial mining pit, silt and THM grade interpretation. These six domains show distinct colour and grain size differences as well as demonstrably different mineralogy. They were numbered 1 to 6 from surface down. Subsequently, Domains 1A, 2A, 4A and 5B were found to have consistently distinct silt and THM contents and spatial distribution sufficient to warrant subdividing them out of the original domains. Both the geophysical wire line logging data and the mineralogical data support the definition and identification of the individual domains. The domains are illustrated in Figure 2.

The domains have been shown to be laterally continuous and can be correlated from section to section along the strike of West Block. In gross morphology the individual sand bodies that represent each of the domains are sheet like bodies with lens, prism (wedge) or ribbon geometries. In cross section Domains 1 to 5 make up a wedge of variably mineralised stratigraphy, which thickens south eastwards to over 140 m. They apparently accumulated over a NE striking SE facing bank in the underlying Domain 6. The contacts between the domains are gently undulating rather than planar and irregular trough and fill like contacts are visible in the trial pit mined for metallurgical bulk samples. The domains are essentially stratigraphic units representing a superimposition of different depositional facies and post depositional pedogenic weathering processes, thus they can be regarded as distinct geological units. Contacts between the domains are unconformable or low angle unconformities each representing either a hiatus in deposition or the erosion of the underlying domain prior to the deposition of the overlying sequence. Sharp contacts and textural differences between the stratigraphic units are clearly visible in the trial pit.

The depositional breaks are sometimes accompanied by evidence of soil forming process including induration. Contacts, where seen in the pit, are usually sharp but often appear gradational in aircore holes. In diamond core, contacts can appear gradational, inter layered or sharp and sometimes are accompanied by local colour mottling.

The domain boundaries from the geological model were critical in constraining the variography used to derive the resource model. Similarly the domain data allowed more robust estimates for the valuable heavy minerals because the geology constrains the distribution patterns for crude ilmenite, zircon and rutile in West Block.

West Block stratigraphy

Domain 1A represents a distinct silt-depleted zone which mostly appears to drape over Domain 1 following the current topographic surface. It is loose and unconsolidated. Domain 1A is interpreted as a modification of Domain 1 related to the current land surface and pedogenic development since deposition of Domain 1.

The main visual distinguishing feature of Domain 1 is its bright red colour and high silt content. The origin and depositional environment of this unit is interpreted to be aeolian but original bedding is not obviously preserved. Domain 1 extends over all of the West Block deposit as a gently undulating blanket and is a major host of the ilmenite mineralisation. The thickness of this unit lies mostly in the range of 30 m to 40 m.

Domain 2 and 2A are variants of the same sandy wedge that separates the more silt-rich Domains 1 and 3. It extends over most of West Block with the pinch out position running parallel to but several hundred metres inside the NW edge of West Block. Domain 3 is a very distinctive unit with a dark red colour and a high to very high silt content. It is also the most competent unit and in the trial pit can be seen to be variably indurated.

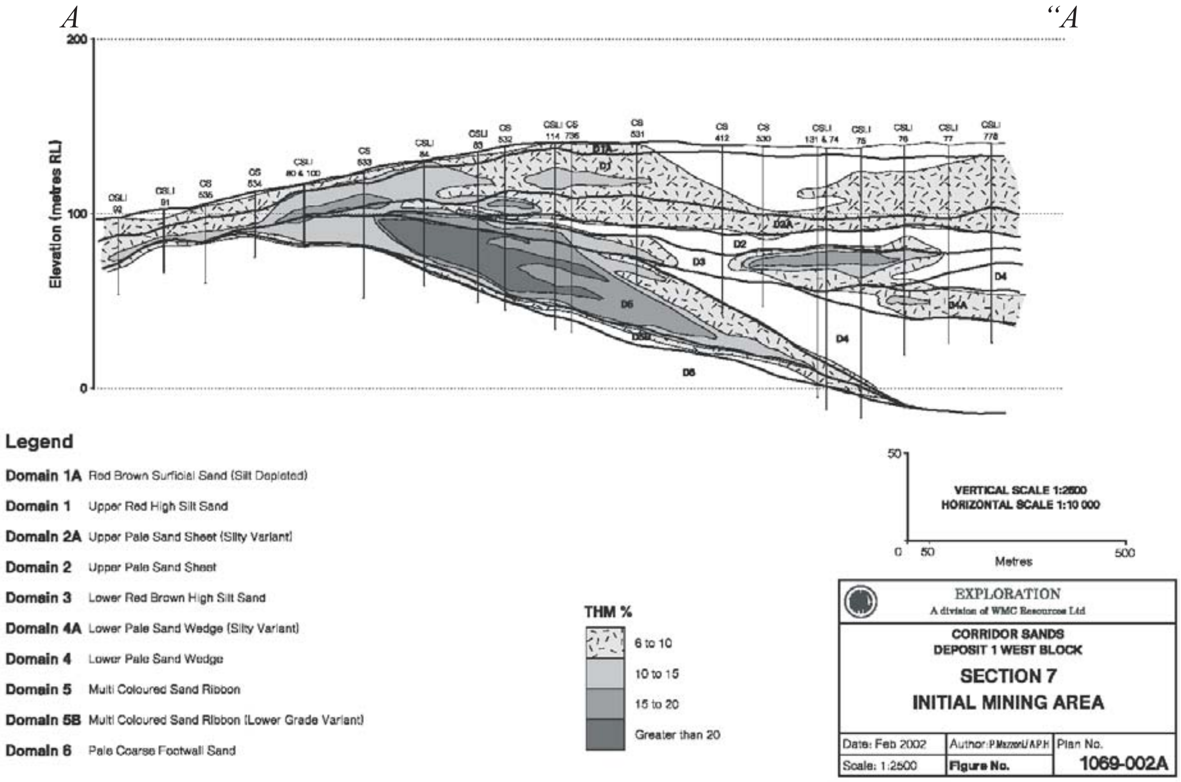


FIG 2 - Geological section – West Block initial mining area.

Domain 4A is the more silt rich and higher THM grade variant of 4. Together they form a south eastwards opening wedge of essentially lower grade sandy material separating high silt Domain 3 from the underlying Domain 5.

Domain 5 is a largely sandy unit although the silt content tends to increase northwards. It usually contains abundant heavy minerals and grades of up to 40 per cent THM over 3 m intervals have been intersected in drilling. One of the most striking features of Domain 5 in the pit is the presence of abundant black manganiferous rhizoconcretions (rhizoliths). These are made up of normal Domain 5 sand, which has been cemented by a mixture of manganese oxide and witherite. The overall geometry of Domain 5 is that of a flat ribbon with an almost sigmoidal or lozenge-shaped cross-section on some sections. It runs north eastwards along the full length of West Block dipping at about four degrees to the southeast. This Domain could be interpreted as a littoral sand facies and its general geometry and relationship to underlying Domain 6 supports this. Domain 5B has essentially been distinguished as a variant of 5 and usually underlies Domain 5 with a gradational THM grade decrease or occurs as a lateral grade transition.

The upper contact of Domain 6 coincides with the base of mineralisation, or more correctly, with a transition to low or very low grades of THM (<2 per cent). Low silt content and a yellow to orange colour is typical of this domain. In general it is coarser grained than the overlying units and includes some pebble bands.

West Block mineralogy

The mineralogy and chemistry can be considered in terms of mineralised sand comprising silt (<45 µm), oversize material (>1 mm), light sand, and THM. The heavy minerals can then be subdivided into magnetic fractions. The mineralogy of the 'crude ilmenite', and that of the 'non-magnetic' fractions, which contain the rutile and zircon, are the important aspects for the recovery processes.

The heavy mineral component comprises varying proportions of magnetite, ilmenite, altered ilmenite, haematite, goethite, leucoxene, chromite, rutile, anatase, epidote, pyroxene, amphibole, andalusite, staurolite, zircon, sphene, monazite, garnet and kyanite. The valuable heavy minerals (ilmenite, rutile, leucoxene and zircon) are generally finer grained than the other heavy minerals and are finer grained than the host sand. Magnetite, ilmenite, altered ilmenite, and chromite make up the bulk of the 'magnetic' and 'crude ilmenite' fractions. Rutile, zircon and andalusite are essentially confined to the 'non-magnetic' fraction. The remaining heavy minerals make up the bulk of the 'magnetic-others' fraction.

STUDY SUMMARY

Methodology

Sequential Gaussian simulation (SGS) is a Gaussian based method of conditional simulation (Chilès and Delfiner, 1999; Goovaerts, 1997). This method uses data transformed to a Gaussian distribution with a zero mean and a unit variance (ie Gaussian anamorphosis) which is then used to simulate spatial distribution of the variable of interest. Simulated realisation is achieved by defining a random path through the grid nodes including the conditioning data, which has been migrated to the nearest grid nodes and considered as hard data. A sequential neighbourhood of the target node is established which includes hard data (original data) and already simulated nodes used to calculate a local conditioning distribution and derive a simulated value at the target node. The simulated value is determined as:

$$Z_s = Z_K + \sigma_K U$$

where:

Z_S is the SGS simulated value

Z_K is the simple kriging estimate

σ_K is the standard deviation of the kriging estimate

U is a random normal function

As the SGS method assumes multiGaussian property of the studied random variable and its diffusive distribution model, these assumptions need to be tested prior to application of the modelling methodology. Border effect can be tested by calculating the ratios between cross-variograms of the indicators and indicator variograms (Abzalov and Humphrey, 2002, 2003). MultiGaussianity can be tested by calculating variograms of indicators calculated for the chosen data percentiles and comparing them with indicator variograms calculated for the same percentiles of the Gaussian transformed data (Goovaerts, 1997).

Implementation

Data analysis and processing

All data used in this study have been obtained from air-core holes drilled on 100 m × 100 m centres through the IMA area and locally on 25 m crosses. All holes has been sampled at regular 3 m intervals and assayed for 'silt' and THM contents. The study database includes 1246 'silt' assays and 1244 THM assays (Figure 3).

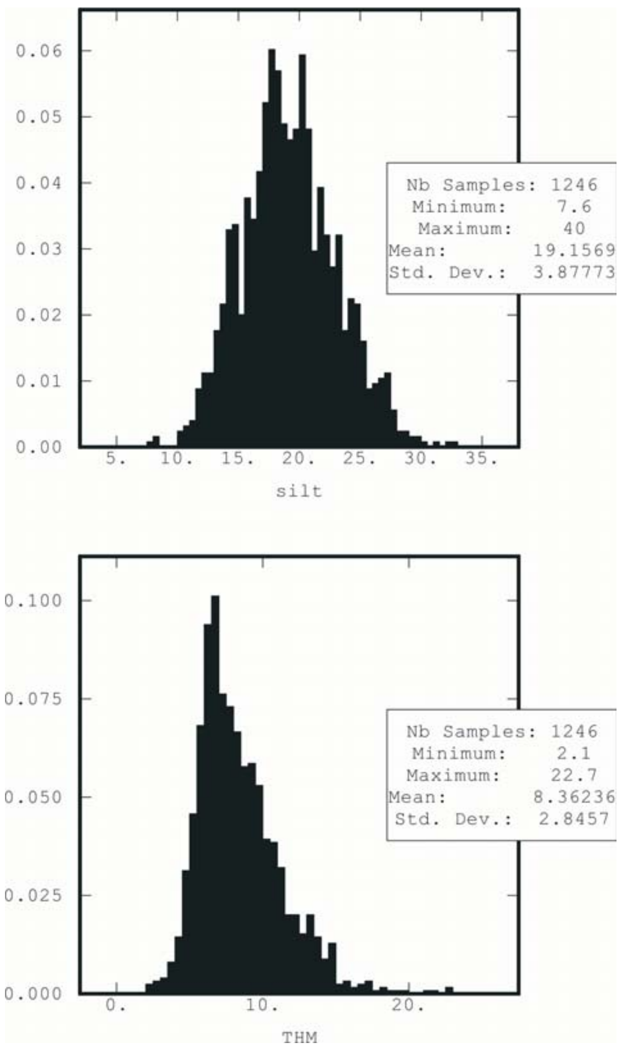


FIG 3 - Histograms (non-declustered data) of the THM and 'silt' grades of the 3 m drill hole samples collected from the IMA, Domain1 + 1A.

Data, prior to their Gaussian transformations, has been declustered to remove bias associated with clustering of the holes around high-grade areas. A cell declustering method (Goovaerts, 1997) implemented in the Isatis software has been applied in the present study. The optimal declustering results have been obtained using 150 × 150 × 3 m moving 'window'. Statistical distribution characteristics of the raw and declustered assays are summarised in the Table 1. A normal score transformation model has been numerically derived by applying the Hermite polynomials expansion technique. A frequency inversion method (Bleines *et al*, 2001) was utilised for Gaussian transformations of the raw data.

TABLE 1

Comparison of the declustered and non-declustered (raw) assays. Initial mining area, Domain 1 + 1A selection.

		Raw data	Declustered data
THM	Mean	8.36	7.89
	St Dev	2.85	2.52
'Silt'	Mean	19.16	18.82
	St Dev	3.9	3.9

Grade continuity study (variography)

Grade continuity has been analysed by calculating variograms of the 'silt' and THM grades and their transformed values. Data transformations included calculation of the grade indicator values and Gaussian transformations. Directional variograms of the Gaussian variables and their models are presented in Figures 4 and 5. These variograms (Figure 4) show a noticeable anisotropy with a major anisotropy axis oriented at 100°SE. Indicator variography, which is routinely used by authors to enhance the grade distribution patterns, accords well with the findings of the normally transformed data variography.

Simulation parameters

Sequential type of the search neighbourhood has been utilised for application of the SGS methodology. The search parameters are as follows: NX = 70, NY = 70, NZ = 1, where NZ, NY and NZ are the numbers of grid points extension of the search in the three axes of the grid. Maximum number of data nodes has been limited to 35, maximum number of simulated nodes is 27.

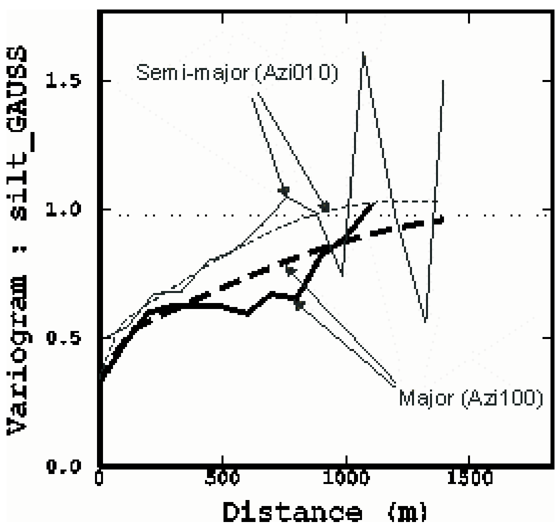


FIG 4 - Experimental semi-variogram (solid lines) and fitted models (dashed lines) of the normally transformed 'silt' values (SILT_GAUSS) calculated along the major and semi-major anisotropy axes. Three-metre samples, Domain 1 + 1A, IMA.

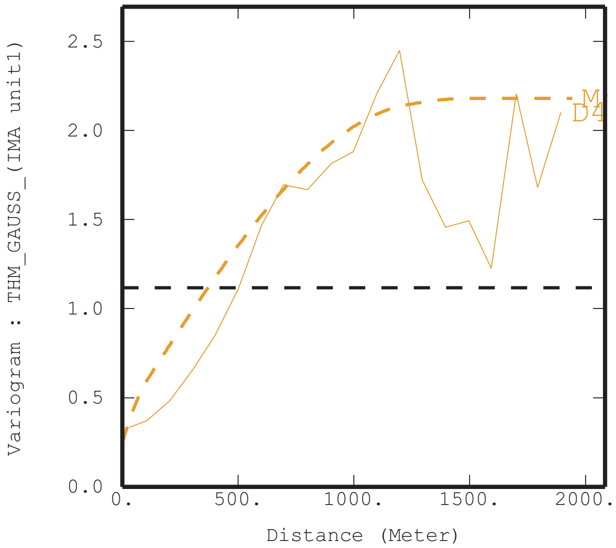


FIG 5 - Experimental semi-variogram (D1) and fitted model (M1) of the normally transformed THM values (THM_GAUSS) calculated along the semi-major anisotropy axis. Three metre samples, Domain 1 + 1A, IMA.

Initially the grades have been simulated to 5 × 5 × 3 (m) blocks which later have been combined to a larger blocks, 10 × 10 × 12, 25 × 25 × 12 and 125 × 62.5 × 3 (m) representing the different SMU sizes.

The simulated 'silt' and THM values of the 125 × 62.5 × 3 (m) blocks have been compared with their kriged block grades obtained by ordinary kriging (OK).

RESULTS AND DISCUSSION

A range of SMU sizes (5 × 5 × 3, 10 × 10 × 12, 25 × 25 × 12 and 125 × 62.5 × 3 (m)) were tested to assess the effect of mining selectivity on recovered grade and assess the risk of delivering ore with high silt levels.

Comparison of OK and SGS grade estimates

Comparison of the average simulated 'silt' and THM grades and their kriged values is shown in Table 2 and presented as scattergrams in Figures 6 and 7.

Global THM and 'silt' grades for Domain 1 + 1A in the IMA area, estimated by OK method and independently modelled by SGS method, are statistically insignificant. Differences in the mean grades obtained by the two methods (OK and SGS) were 0.1 wt per cent of SILT (ie 0.53 per cent of the kriged mean) and 0.22 wt per cent THM (ie 2.27 per cent of the kriged mean).

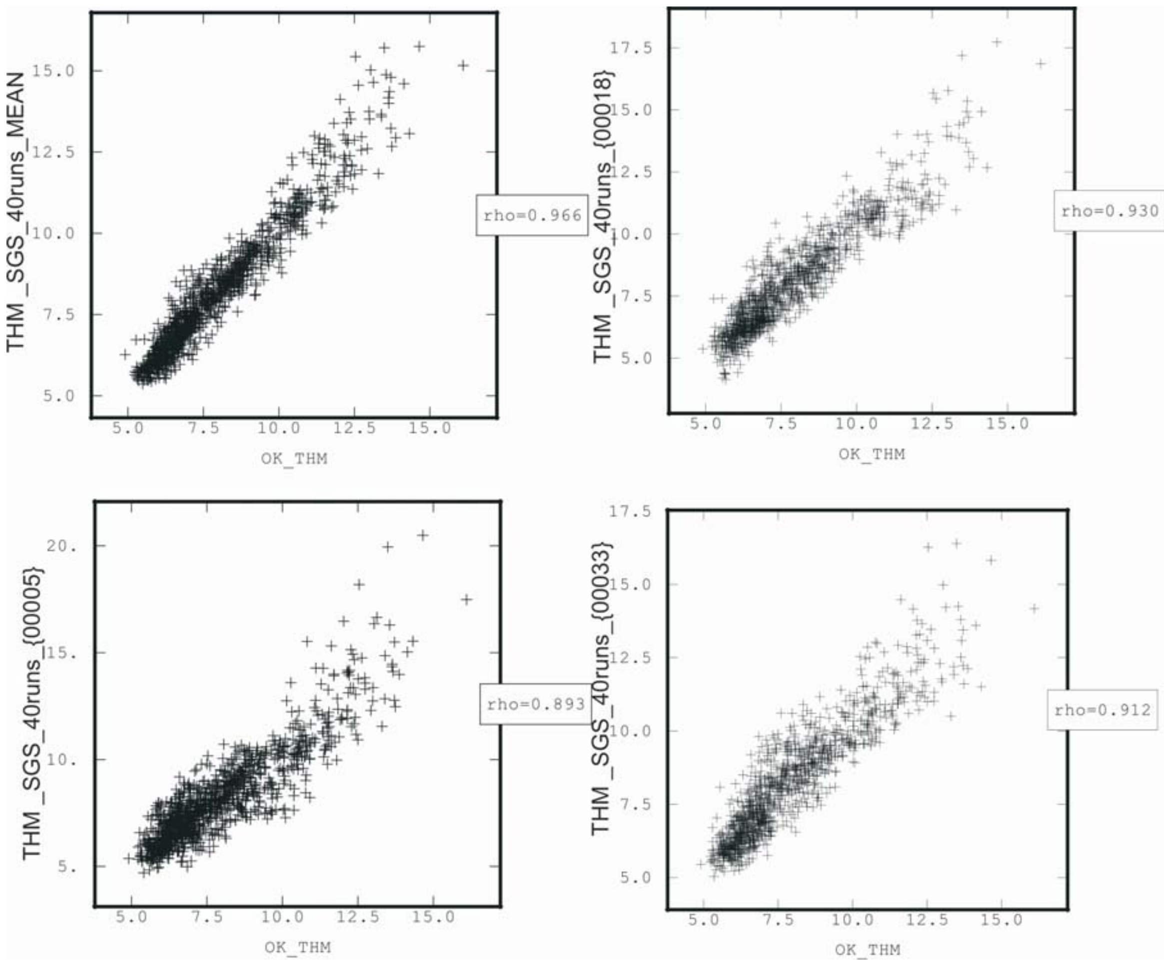


FIG 6 - Scatter-diagrams comparing THM block grades estimated by ordinary kriging (OK_THM) with their grades obtained by conditional simulation (SGS model). MEAN = average grade of the 40 equiprobable realisations, {00018} = 18th realisation representing 25th percentile of the ccdf, {00005} = 5th realisation representing 50th percentile of ccdf and {00033} = 33rd realisation representing 75th percentile of the ccdf.

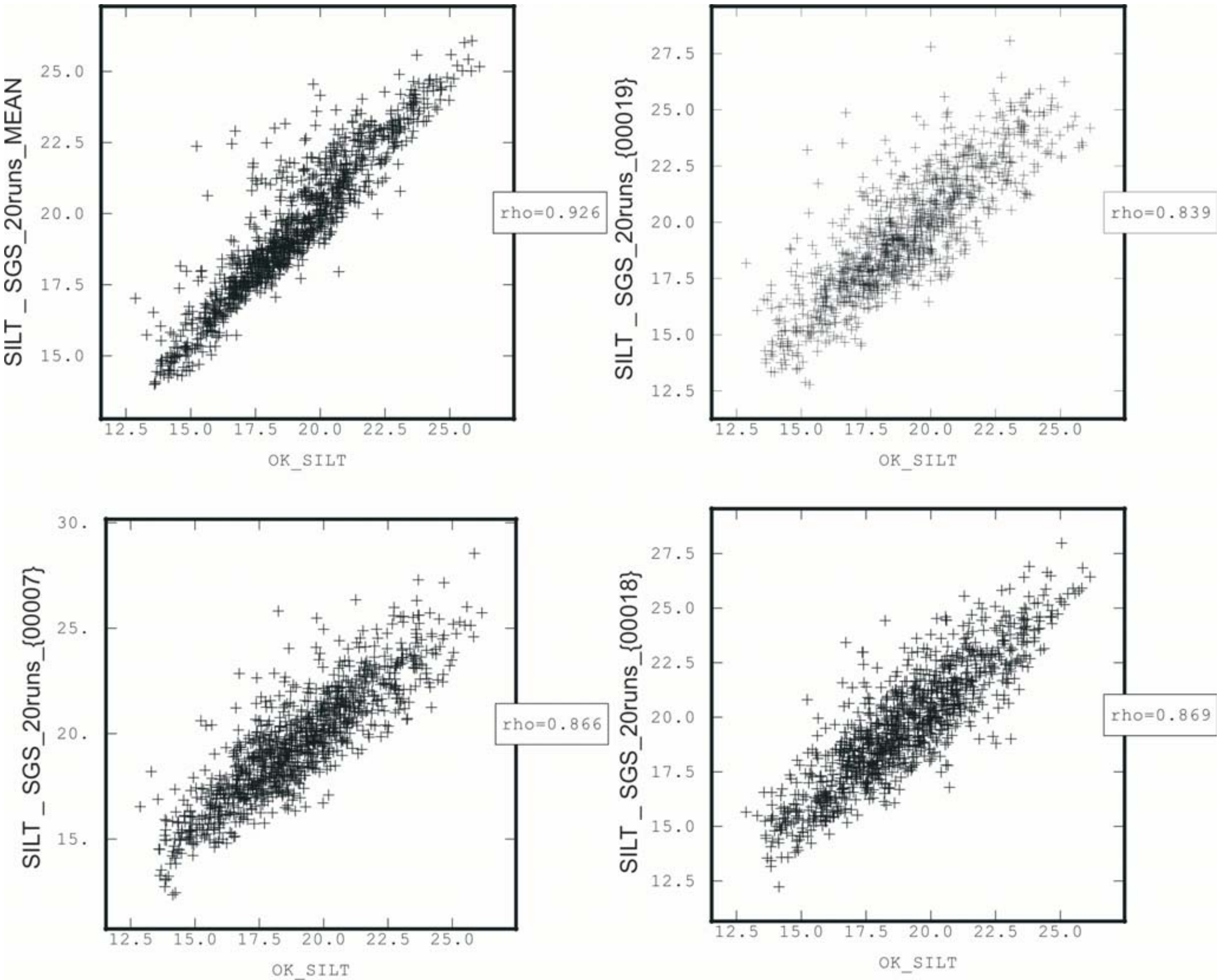


FIG 7 - Scatter-diagrams comparing 'silt' block grades estimated by ordinary kriging (OK_SILT) with their grades obtained by conditional simulation (SGS model). MEAN = average grade of the 20 equiprobable realisations, {00019} = 19th realisation representing 25th percentile of the ccdf, {00007} = 7th realisation representing 50th percentile of ccdf and {00018} = 18th realisation representing 75th percentile of the ccdf.

TABLE 2

Comparison of the OK estimates with SGS model. Domain 1 + 1A, IMA, Corridor Sands.

	SILT ± 2σ	THM ± 2σ
OK	18.97 ± 5.12	7.93 ± 3.84
SGS	19.07 ± 5.38	8.15 ± 3.94
Variation	-0.10	-0.22
% of OK estimate	-0.53	-2.77
Correlation coefficient	0.93	0.97

Recoverable resource estimations

The resources recoverable at the given SMU sizes have been simulated and presented as grade-tonnage diagrams (Figures 8 and 9). These results suggests that recoverable 'silt' grade seems to be sensitive to the chosen size of SMU (Figure 9). In particular, if Unit 1 + 1a (IMA area) were mined using 10 × 10 × 12 m SMU sizes, five per cent of the mined ore blocks would have 'silt' grade exceeding 23 wt per cent.

Spatial distribution of the THM and 'silt' values is presented on bench plans showing grade distribution by simulated 5 × 5 × 6 m blocks (Figures 10 and 11). The simulated plans shows a significant heterogeneity of the 'silt' distribution. THM values are distributed more compactly than 'silt' (Figures 10 and 11) These differences in the spatial distribution patterns accord well with the simulated grade-tonnage relationships of the THM grades (Figure 8) which are less sensitive to changing the SMU size than 'silt' grade (Figure 9).

Risk of exceeding plant tolerance thresholds

The multiple realisations of the SGS model have been used to construct a probability model estimating the likelihood of SMU grades being below 6 wt per cent THM or exceeding the plant tolerance limits for 'silt'). Results of the probabilistic estimation of the grade ranges are summarised in Figure 12.

Conditional simulation study suggests that risk of delivery high-'silt' (>25 per cent) ore from the Domain 1 + 1A (IMA area) is negligible if 10 × 10 × 12 m minimum mining blocks are used. On the other hand, risk of exceeding 'silt' tolerance limits rapidly increases if the actual PCP tolerance is lower than 25 wt per cent 'silt'. Thus, approximately one third of the total 10 × 10 × 12 m blocks are characterised by a very high probability (0.75) of exceeding 20 wt per cent 'silt' grade (Figure 12).

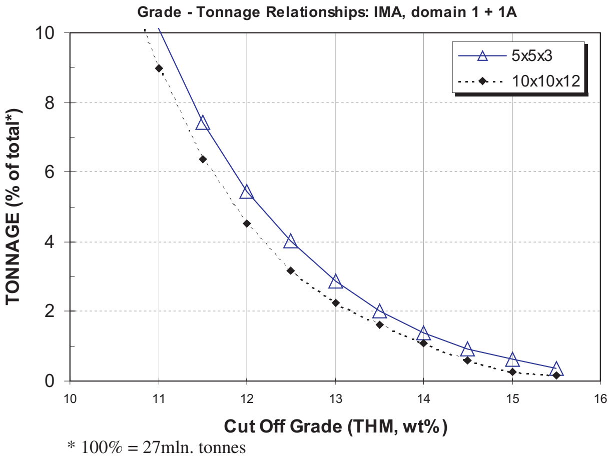


FIG 8 - THM grade-tonnage curves, calculated for the different SMUs.

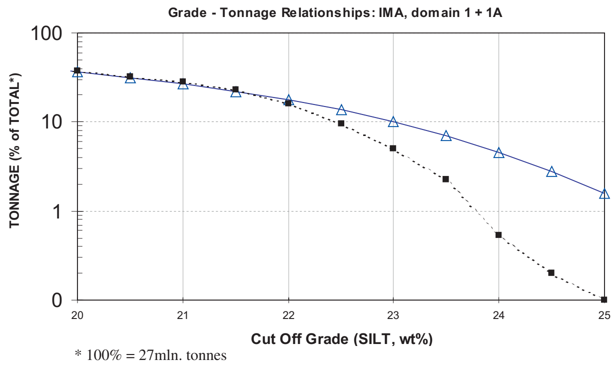


FIG 9 - SILT grade-tonnage curves calculated for the different SMUs. Dashed line 5 × 5 × 3 m blocks; solid line 10 × 10 × 12 m SMU.

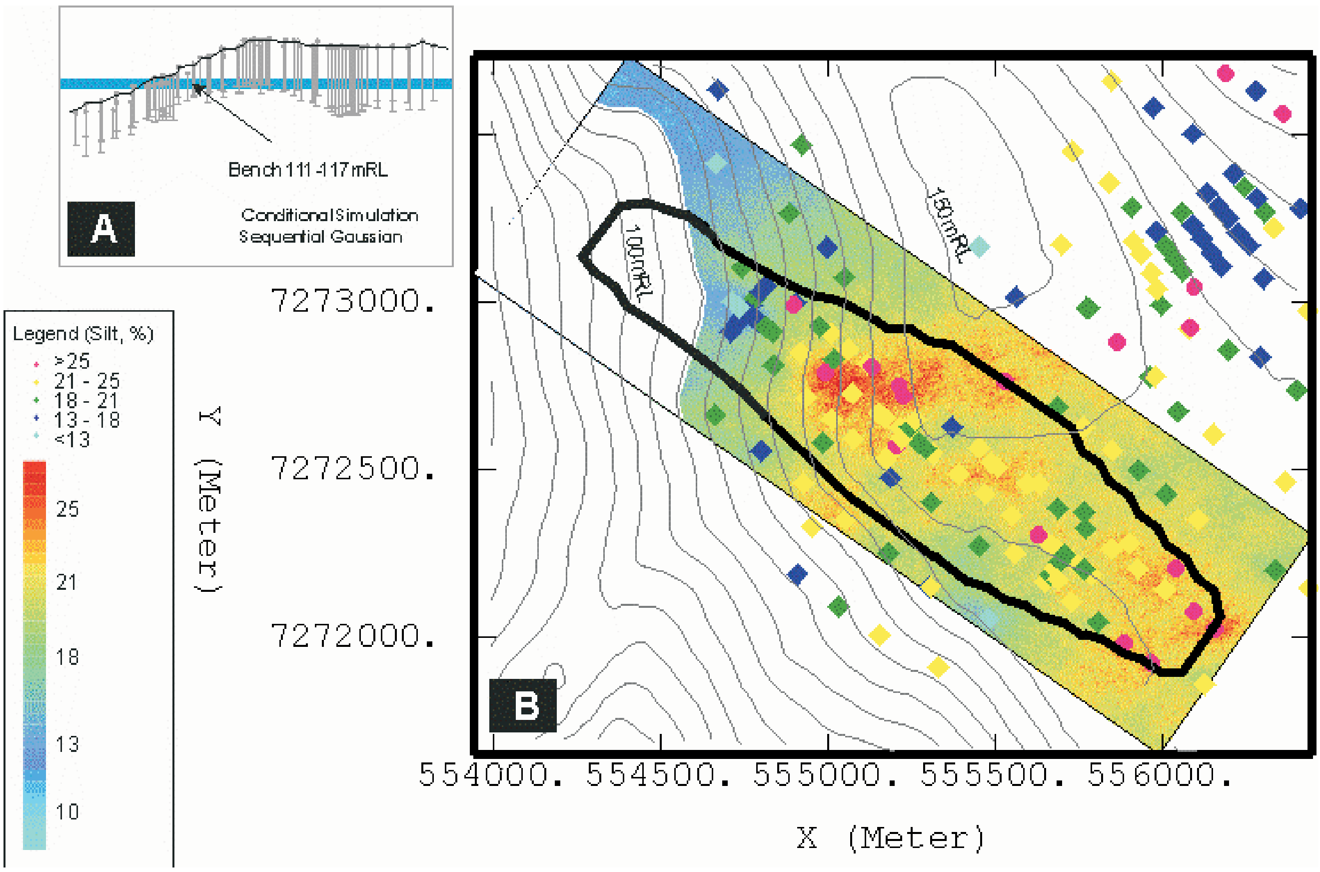


FIG 10 - 'Silt' distribution, IMA area, bench 111-117 m RL. Average 'silt' values of the 5 × 5 × 6 m blocks as modelled by SGS method. A – bench location, B – simulated grade values (back-ground) and drill hole data (symbols).

Study of the THM distribution shows that approximately nine per cent of the 10 × 10 × 12 m blocks can be below 6 wt per cent THM. Risk of exceeding 15 per cent THM grade in the ore parcels is small, as conditional simulation results shows that less than one per cent of SMU will contain high-THM grades (>15 per cent) (Figure 8). However, compact distribution of the high-THM mineralisation (Figure 11) suggests that the PCP feed rate will need to be slowed to accommodate these grade 'surges', particularly if in-pit blending options are not available.

SUMMARY AND CONCLUSIONS

Differences of the mean grades obtained by OK and SGS methods are 0.1 wt per cent of 'silt' (ie 0.53 per cent of kriged mean) and 0.22 wt per cent THM (ie 2.27 per cent of the kriged mean). Similarity of the global means and also the strong correlation between the block grades obtained by OK methodology and the SGS technique support the validity of the OK model.

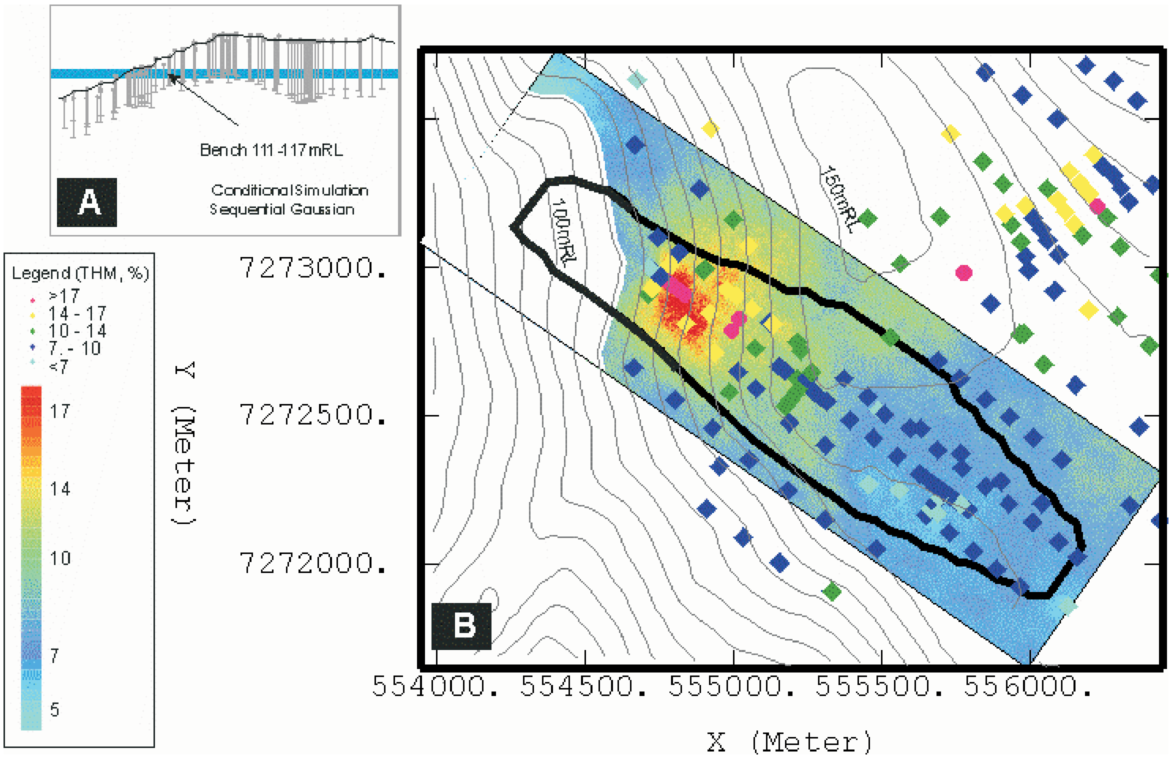


FIG 11 - THM distribution, IMA area, bench 111-117 m RL. Average THM values of the 5 × 5 × 6 m blocks as modelled by SGS method. A – bench location, B – simulated grade values (background) and drill hole data (symbols).

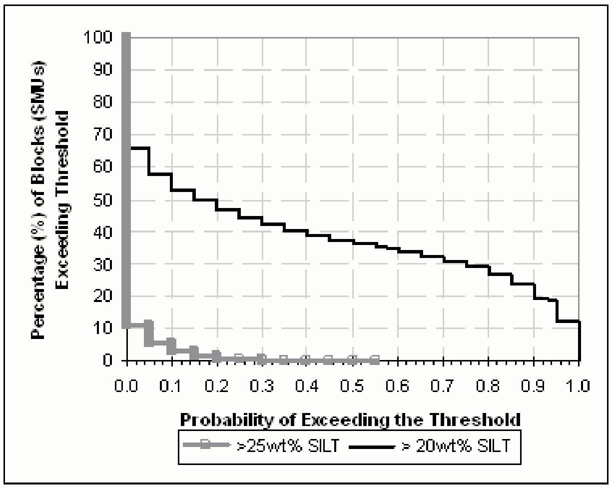


FIG 12 - Percentage of blocks versus probability of exceeding the threshold diagram, showing risk of exceeding 25 wt per cent and 20 wt per cent 'silt' values for 10 × 10 × 12 m blocks (SMUs) of IMA Domain 1 + 1A.

The conditional simulation suggests that the risk of delivering high-'silt' (>25 per cent) ore from the Domain 1 + 1A (IMA area) is negligible if 10 × 10 × 12 m minimum mining blocks are considered.

The risk of exceeding 'silt' tolerance limits rapidly increases if the actual tolerance is lower than 25 wt per cent 'silt'. Approximately one third of the total 10 × 10 × 12 m blocks are characterised by 75 per cent probability of exceeding a 20 wt per cent 'silt' threshold.

The simulated grade distribution plans reveal significant short-range variability and discontinuity in the high-'silt' zones.

ACKNOWLEDGEMENTS

The authors express their sincere gratitude to the geologists of the Corridor Sands project for geological contributions and M Humphreys (SRK Consulting) who assisted with sequential Gaussian simulation. Permission by WMC Resources to publish this paper is gratefully acknowledged.

REFERENCES

Abzalov, M Z and Humphreys, M, 2002. Geostatistically assisted domaining of structurally complex mineralisation: method and case studies, in *Proceedings The AusIMM 2002 Annual Conference: 150 Years of Mining*, pp 345-350 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Abzalov, M Z and Humphreys, M, 2003. Resource estimation of structurally complex and discontinuous mineralisation using non-linear geostatistics: case study of a mesothermal gold deposit in northern Canada, *Exploration and Mining Geology*, 11(1-4):19-29.

Bleines, C, Deraisme, J, Geffroy, F, Perseval, S, Rambert, F, Renard, D and Touffait, Y, 2001. *ISATIS Software Manual,* 531 p (Geovariances and Ecole des Mines de Paris).

Chilès, J-P and Delfiner, P, 1999. *Geostatistics: Modelling Spatial Uncertainty,* 695 p (John Wiley and Sons Inc: New York).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation,* 483 p (Oxford University Press: New York).

Risk Management Through the Use of 2D Conditional Co-Simulation at an Underground Gold Mine in Western Australia

M Dusci[1], D R Guibal[2], J S Donaldson[3] and A G W Voortman[4]

ABSTRACT

Geological and resource variability and uncertainty is a fundamental source of risk, often having the greatest economic impact on a mining project. Grade variability should be quantified to enable optimisation of underground mine design and associated financial decisions. The management of risk associated with resource uncertainty at the Argo underground gold deposit, through the implementation of 2D conditional co-simulation, has led to better informed mine planning decisions.

The Argo meso-thermal lode gold deposit is located in the Archaean Yilgarn Block of Western Australia owned by Gold Fields Ltd. The orebody is positioned within a large, structurally complex shear system in the Kambalda-St Ives structural corridor, below a 60 m thick sequence of Tertiary sediments. Production history comprises five open pit mining stages and the deposit is currently being mined from underground. Resource estimation of the Argo deposit integrates two different estimation techniques to reflect orebody uncertainty and differing drill densities; a 3D ordinary kriged (OK) estimation has been utilised in areas of greater drill densities and geological confidence (the upper part of the deposit). For the deeper portions of the mine, where the drilling density makes it difficult to use OK for block sizes appropriate to mining, a 2D conditional co-simulation is used for modelling horizontal orebody thickness and gold accumulation. This is based on the assumption that there is no mining selectivity across structure, which is reasonable as the horizontal thickness is generally less than 20 m. The simulation method used is the Gaussian-based Turning Bands method, where variograms and cross-variograms of thickness and accumulation are reproduced, giving an accurate picture of their variability at deposit scale. A total of 100 realisations are calculated at a 2.5 m × 2.5 m spacing. These results are then regrouped into 10 m × 15 m mining units, used for mine planning.

The 2D conditional co-simulation has been integrated into the mine planning stage with incorporation of mining parameters into the simulation. This has enabled the simulation to reflect the probability of achieving 'stope evaluation cut-off grades' as a result of grade uncertainty. The simulated model forms a fundamental part of optimising the underground mine design and managing risk at the Argo gold deposit.

INTRODUCTION

The Argo gold deposit is located 25 km southeast of Kambalda within the Archaean Yilgarn Block of Western Australia and is owned by St Ives Gold Mining Company, a wholly owned subsidiary of Gold Fields Ltd of South Africa. The orebody was discovered by WMC in 1991 from aircore drilling an airborne magnetics target over the Condenser Dolerite Unit and is positioned within a large structurally complex shear system in the Kambalda-St Ives structural corridor (Figure 1). Numerous mineralised surfaces have been mined from open pit and underground. Mining at Argo is currently underground from four main surfaces and current reserves are over 4.6 Mt at 5.7 g/t Au for 845 koz, with a total mineral inventory in July 2004 of more than 8 Mt at 6.4 g/t for 1.7 Moz.

Production history at Argo comprises five open pit mining stages since 1994, terminated at the end of 2003. Underground development commenced in July 2002 and is presently in operation. Access to the four main underground ore surfaces is by a decline starting from within the open pit approximately 100 m below the surface. The underground phase is planned to operate for seven years, mining a total reserve of 2.7 Mt at 7.1 g/t Au for more than 620 koz. Based on a gold price of $550/oz the underground mine has a NPV of $61 M.

As part of continued exploration of the significant gold field, a large underground drilling program in excess of 19 000 m was undertaken during 2003 and 2004. It probed additional ore surfaces in the footwall. This program has delineated in excess of 175 koz in Indicated Resource and 200 koz in Inferred Resource at a 3.5 g/t Au cut-off. More than 100 koz is projected as further down-dip potential. There is significant potential to increase the reserve and resource with increased underground exploration.

The rationale for using a form of co-simulating grade and thickness of mineralisation is that these parameters form the basis for underground mine design and therefore greatly affect the risks related to economic extraction. To make appropriate design decisions, the mine planner has to be aware of the impact of this risk, both positive and negative, on the outcome. Managing these risks should be based on the understanding that they reflect a potential upside as well as downside, which is a fundamentally different approach.

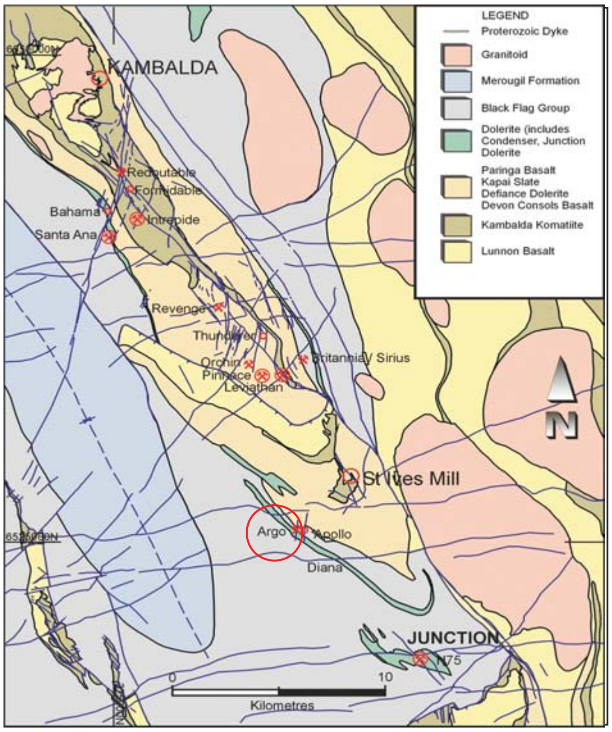


FIG 1 - Location plan showing the Argo deposit within the highly mineralised Kambalda-St Ives structural corridor, close to the St Ives Mill.

1. Mineral Resource Manager, Gold Fields Venezuela, Choco 10, Venezuela. Email: mdusci@echexploration.com.ve

2. FAusIMM(CP), Corporate Consultant – Geostatistics and Resources, SRK Consulting, PO Box 943, West Perth WA 6872, Australia. Email: dguibal@srk.com.au

3. MAusIMM, Resource Geologist, St Ives Gold Mining Company Pty Ltd, PO Box 359, Kambalda WA 6444, Australia. Email: john.donaldson@goldfields.com.au

4. FAusIMM(CP), Senior Resource Geologist, BHP Billiton Limited, 152 - 158 St Georges Terrace, Perth WA 6000, Australia. Email: llmp@iprimus.com.au

GEOLOGY, STRUCTURE AND MINERALISATION

The Argo deposit is an Archaean meso-thermal lode gold deposit, positioned on the western limb of the Kambalda-St Ives antiform 25 km to the south-east of Kambalda. The Condensor Dolerite, a 500 m thick subvertical to SW dipping differentiated sill, hosts the mineralisation at Argo. The Condenser Dolerite is stratigraphically equivalent to the Golden Mile Dolerite and has intruded along the contact between the Paringa Basalt and Black Flag Beds. The most differentiated section of the dolerite is the most important host for mineralisation.

Gold mineralisation at Argo is predominantly confined to the Argo shear (A1 mineralised surface, Figure 2). The Argo shear is north striking and west dipping, extending to more than 800 m down-dip. The shear system extends over a 1 km strike and attenuates at the contact with the Paringa Basalt to the north, and the Black Flags Beds to the south. Two east-west trending subvertical Proterozoic dolerite dykes cross-cut the system. The south end of the deposit is covered by a sequence of Tertiary sediments up to 60 m thick.

Different types of gold mineralisation are evident at Argo, with the majority of metal sourced from primary shear-quartz lode hosted mineralisation. Mineralisation is also hosted in a high-grade paleo-placer hosted deposit at the base of the Tertiary cover sequence, and supergene mineralisation within the Tertiary sediments and Archaean regolith.

Gold mineralisation at Argo resulted from complex interaction between structural and host-rock controls. The development of multiple shear structures in the Argo deposit increased permeability and localised hydrothermal fluid flow through the Condenser Dolerite. Rheology and iron chemistry enabled fluid-wallrock redox reactions to occur, which played an important role in localising mineralisation (Gressier and Kolkert, 1995).

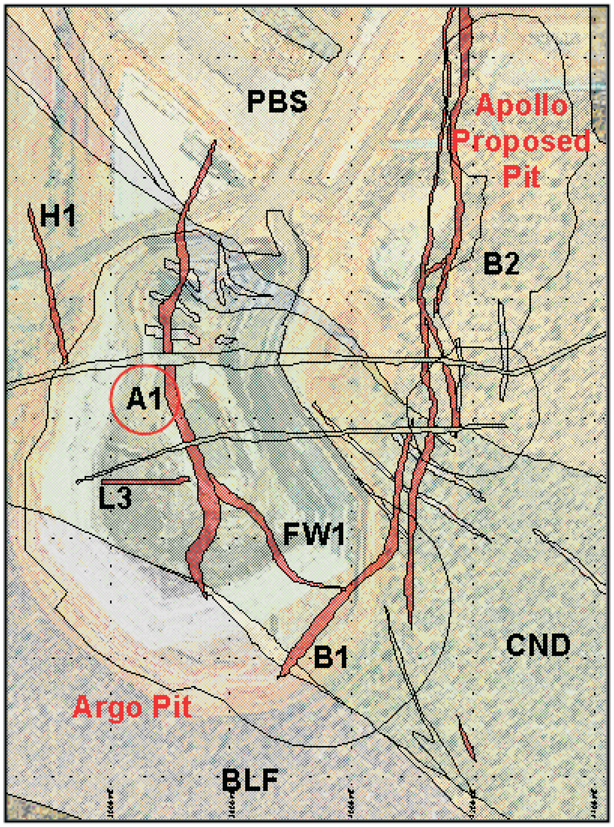


FIG 2 - Plan map of the Argo deposit and location of the A1 Shear (PBS – Paringa Basalt, CBN – Condenser Dolerite, BLF – Black Flag Beds).

The variable geometry of the Argo shear zone is a principal control influencing deformation mechanisms and development of gold mineralisation. In areas where the Argo shear is shallowly dipping, the combination of high fluid pressure and low stress causes brittle failure, characterised by the formation of dilational vein sets, breccia zones and pervasive silica alteration. In areas where the Argo shear is steeply dipping, the combination of low fluid pressures and high normal stress results in ductile shear failure characterised by intensely developed shear and mylonitic fabrics with minor extensional veins (Gressier and Kolkert, 1995).

Mineralisation within the Argo shear is typically associated with quartz-chlorite-biotite-albite-sulfide alteration of the dolerite host. Mylonites, quartz vein and breccia lodes occur and mylonites formed subparallel to the shear margin. This fabric developed from rapid ductile deformation resulting in re-crystallisation of mineral grains. Where the dip of the shear flattens, pervasive silica alteration and en-echelon shear veins overprint pre-existing mylonitic fabric.

The Argo shear is accompanied by a great number of mineralised satellite structures. These can be divided into two main structural domains: within the hanging wall of the main A1 ore surface, structures are characterised by a variable strike with a relatively steep dip, dominated by mylonite; the footwall structures consist of listric flat-lying structures (Figure 3). The footwall structures include the Apollo shear, which bounds the eastern margin of the mineralised system.

MANAGEMENT OF RISK IN RESOURCE ESTIMATION AND MINE PLANNING

Risk modelling approach

The most important factor in making financial decisions is the understanding of risk and return. Risk can be described as the combination of likelihood and magnitude of a particular event occurring. This comes from the imperfect knowledge of the outcome, such as is the case with resource estimation. Investment decisions in the mining industry are continually being made without full awareness of the impact of risk, both positive and negative, on the outcome of projects.

Managing risk requires a fundamental change in thinking to move away from the more traditional approaches of 'building conservatism' into decisions to quantify project risk. The

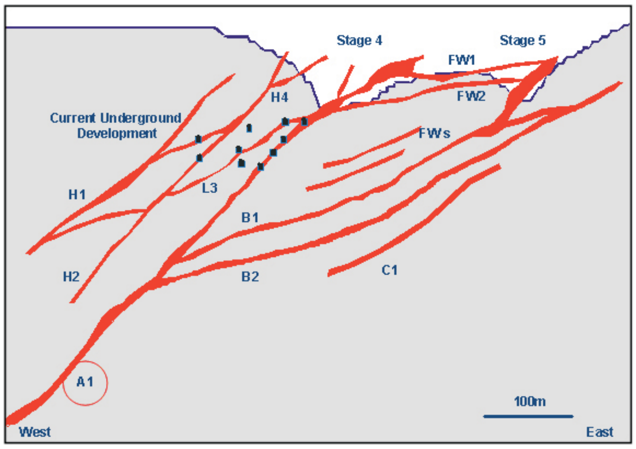


FIG 3 - Oblique schematic cross-section of the Argo mineralised system consisting of a complex array of both steep and shallow dipping structures. The main mineralised structure is the A1 ore surface. Current development is contained between the footwall of the H1 and hanging wall of the A1 ore surfaces.

understanding of risk reflects a potential upside, not just downside. It applies to both resource estimation and mine planning.

Currently at Argo, a 3D ordinary kriging (OK) model is used in regions of the A1 ore surface where there is increased geological understanding and sample data. A strong understanding of grade distribution across the lode is required in this region to enable detailed mine planning.

In the down-dip extension of the A1 ore surface, drill spacing is broader (typically 60 m × 60 m or greater) and this makes it very difficult to use linear estimation techniques like OK. In effect, mine planning is based on 10 m (NS) × 15 m (RL) blocks and 3D (or even 2D) OK of such blocks from scarce data gives very smooth and conditionally biased estimates, leading to biased resources and reserves. In addition, the risk associated with OK or other classical linear estimates (like Inverse Distance) is difficult to quantify.

The most appropriate solution to the over-smoothing of OK and to the measure of the risk inherent in the estimation is to use the now well-known technique of conditional simulation. The Argo structure shows a relatively low thickness (less than 20 m in general) and it is reasonable to assume that no mining selectivity across the lode will be undertaken. Consequently, a 2D modelling approach, which ignores grade variability across the lode, is seen as applicable.

The 2D simulation

The 2D co-simulation utilises two correlated variables: thickness (‘tonnes’) and accumulation (‘metal’, or more precisely product of thickness by grade) as the modelled variables. Grade is not directly simulated as it is not an ‘additive’ variable. It is calculated by the ratio of simulated accumulation and simulated thickness. The weighting of the variables by bulk density should also be considered for a 2D approach. Several options are possible for defining these two variables. Because the deposit is steeply dipping and shows relatively small variations in dip, it was decided to use horizontal thickness and accumulation. Description of the variables is as follows:

- Horizontal thickness (HZTK) – thickness of the A1 ore surface calculated perpendicular to the vertical longitudinal plan on which the 2D simulation is performed.
- Accumulation (ACCUM) – calculated as the product of HZTK by the full-length composite gold grade across the A1 ore surface defined by the hanging wall and footwall geological contact. This is represented as a gram*metre intersection.
- Density – variable bulk densities were not used in the modelling, because:
  - The density of mineralisation has a limited range from 2.64 gm/cm^3 to 3.26 gm/cm^3 based on point support. The variance of this data is 0.09 (gm/cm^3)^2, which is significantly reduced by compositing the data across the lode.
  - Au and density showed a very poor correlation with a correlation coefficient of 0.12. It was therefore concluded that density would have little influence in the simulation process, and the evaluation of the gold grade in particular.

The properties of conditional simulations are well known. They reproduce the statistical characteristics (histograms and correlations) of the variables as well as their spatial correlations (as measured by variograms and cross-variograms) and they honour the data.

A detailed conditional simulation study was first completed in 2003. This was followed in April 2004 by a second study using the results of an infill drilling program.

The introduction of risk management through the use of 2D conditional co-simulation for grade uncertainty at Argo Underground has been driven by the philosophy that ‘if you can’t measure it; you can’t manage it’.

REALISATION OF THE CONDITIONAL SIMULATION

The data

There are 410 drill hole intersections within the A1 ore structure. Their location is given in Figure 4 and unweighted statistics are shown in Table 1. Clearly the drilling density decreases sharply in depth, hence the need for declustering the data: a 50 m × 50 m declustering cell is used and the corresponding statistics are shown in parenthesis in Table 1. There are significant differences, suggesting that the distribution of the variables is not very homogeneous. Nevertheless, as indicated by the variation coefficient, the level of variability is not very high for a gold deposit.

There are indeed non-stationary features in the spatial distribution of HZTH and ACCUM, as indicated by the graph of the average of the variables per 50 m slices shown in Figures 5 and 6. The top of the orebody has elevated thickness and accumulation, linked to a high concentration of Au in the south. Note that these elevated values coincide with a higher data density.

Correlations between HZTH and ACCUM (0.57) and ACCUM and Au (0.71) are significant, but moderate, while HZTH and Au are uncorrelated (0.03). This is a very interesting result, which suggests a possible simple model for the joint estimation of HZTH and ACCUM, the residual model (Rivoirard, 1994).

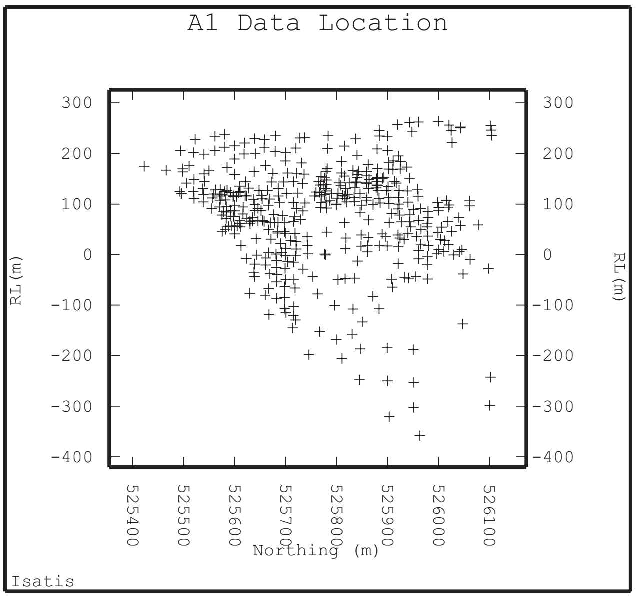


FIG 4 - Location of the data.

TABLE 1

Elementary statistics of the data (de-clustered in parenthesis; for units, see text).

	No data	Min	Max	Mean	Variance
ACCUM	410	0.03	448.30	57.74 (53.61)	5118.29 (4470.73)
HZTH	410	1.10	45.20	11.15 (11.38)	57.69 (71.67)
AU	410	0.01	43.11	4.93 (4.59)	30.99 (29.01)

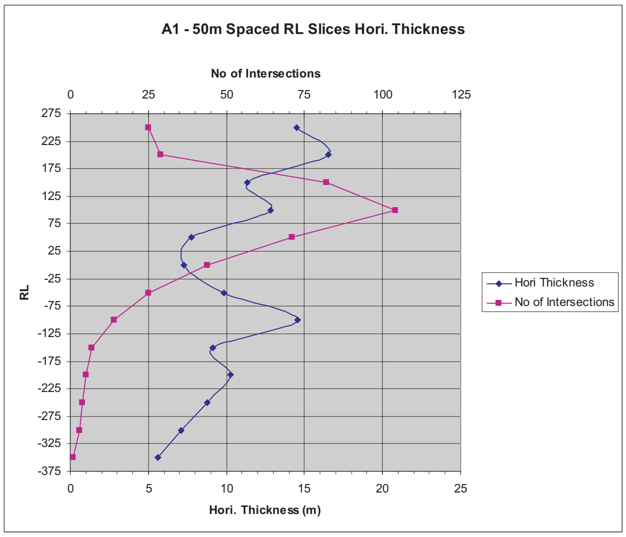


FIG 5 - Vertical variation of average horizontal thickness per 50 m slices.

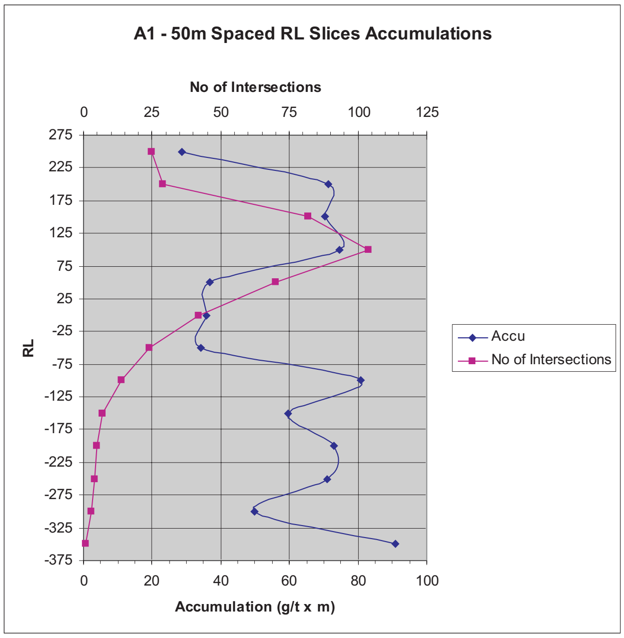


FIG 6 - Vertical variation of average accumulation per 50 m slices.

The conditional simulation method

Despite the local departures from stationarity, and after testing alternative methods, it was decided to use a standard Gaussian simulation method; the Turning Bands method. In effect, the conditioning is relied upon to reflect the local higher thicknesses and accumulations.

Consequently, both HZTH and ACCUM are transformed into Gaussian variables gHZTH and gACCUM, with mean 0 and variance 1, by a process known as Gaussian anamorphosis, which models the declustered histogram through a series of 50 orthogonal polynomials. As already mentioned, the use of the declustered histograms is essential to get a representative picture of the distribution of both variables.

The variograms of the Gaussian variables

The experimental variograms are well structured, with a major axis plunging 45° north, which agrees with the geological trend. The cross-variogram between gHZTH and gACCUM is very similar to the two direct variograms. They are all shown in Figure 7, with the original model fitted to them, using the linear model of coregionalisation. This model is a combination of a nugget effect and two spherical models with anisotropic ranges (55 m and 140 m in the direction plunging 45°, and 30 m and 100 m in the perpendicular direction).

For simulation purposes, the original model is transformed so that the sills are adjusted to the declustered variances of the Gaussian values, ie one.

Realisation of the conditional simulation and validation

A point conditional simulation was performed on a 2.5 m × 2.5 m grid; 100 different realisations were calculated using 800 Turning Bands. The choice of the number of realisations is a compromise between a requirement to correctly sample the space of uncertainty and the need for a manageable set of results. As far as the number of Turning Bands is concerned, a fairly large number was selected so that the 2D space was well covered. The number 800 is not predestined as any number over 100 or 200 is likely to have produced a representative simulation. The data conditioning was based on kriging with a kriging neighbourhood chosen after systematic empirical tests (investigating parameters like kriging efficiency, slope of regression, etc). The resulting neighbourhood is characterised by an ellipsoid of 310 m by 210 m plunging 45° towards the north, and an octant search with an optimum of 24 data. The Gaussian values simulated are back-transformed using the Gaussian anamorphosis models. Finally, after back-transformation, the average gold grades at the points are calculated dividing the simulated ACCUM by the simulated HZTH.

Validation of the simulations was performed at various levels:

- Statistics of the individual point simulations – Not surprisingly, taking into account the fact that 50 896 points were simulated from only 410 data, there are significant variations from one realisation to another, with the average HZTH varying from 9.6 m to 12 m, the average ACCUM varying from 43 g/t*m to 62 g/t*m and the average Au grade varying from 3.85 g/t to 5.6 g/t.

  The histograms are consistent with the original histograms (which is not surprising as they are built directly from the anamorphosis model) and the correlations between HZTH and ACCUM are well reproduced.
- Variograms and cross-variograms – These have to be checked on the Gaussian simulations (before back-transformation). Again there are fairly large variations from realisation to realisation, but, in general, the variograms obtained show similar shapes and ranges to the simulated model. The largest range is the most variable due to ergodicity issues (the simulated field size is fairly small). An example is given in Figure 8.
- Conditioning – This is a matter of visually looking at the individual realisations; conditioning plays the expected role: the high-grade zones correspond to high-grade data. Also, as expected, where the data density is high (top of the structure), there is much less variability from realisation to realisation than down dip, where the lack of data means large fluctuation (and thus higher risk).

Post processing of the simulation for mine planning

As indicated, mine planning is based on 10 m × 15 m mining units (SMU). To simulate the behaviour of such units, the point

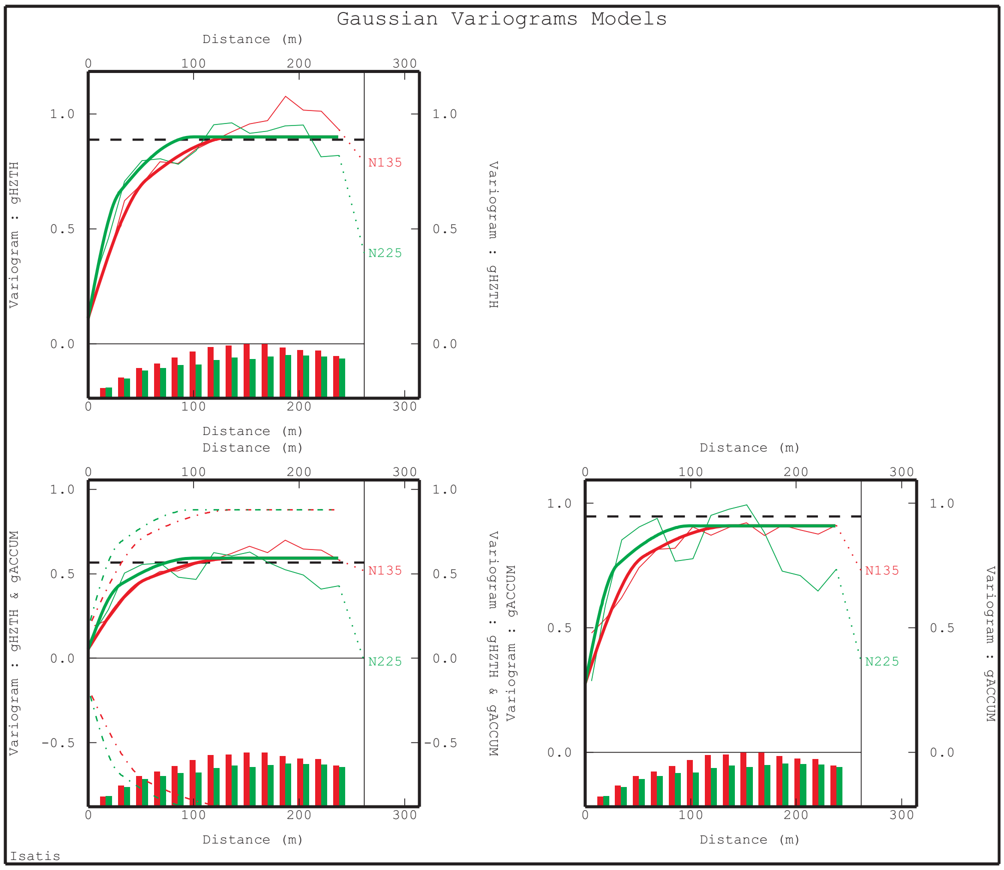


FIG 7 - Gaussian models for variograms and cross-variograms.

simulated values are averaged into 10 m × 15 m blocks for each realisation. As usual, the average grade of the SMU is obtained by dividing the simulated SMU accumulation by the simulated thickness.

The effect of this averaging is to reduce the variability between simulations. As an example, Figure 9 shows two realisations of the simulated Au grade.

It is instructive to compare these individual simulations to the average of the 100 realisations, which generates estimates of the conditional mean of both HZTH and ACCUM. (Figure 10 gives the corresponding Au values). The latter is close to what would be obtained by OK; the smoothing effect is quite striking.

The simulation results have multiple uses:

- Building confidence intervals – For any given SMU, from the 100 realisations, it is easy to associate confidence intervals to the grade, simply by ranking the realisations in increasing order and finding the quantiles corresponding to given probabilities. The results can be used to help classify the resource (by grouping several SMU into larger blocks associated to production periods).
- Risk analysis – It is possible to calculate for each individual realisation a mean characteristic (for instance, the average grade over a cut-off). After ranking the results in increasing order, it is easy to find the probability for this characteristic to be below a given threshold, thus getting a handle on the risk incurred in a project. Figure 11 shows the risk curve associated to the global mean grade (at a 0.0 g/t Au cut-off): from it we can state that there is a 20 per cent chance that the mean grade is below 4.6 g/t Au and a 15 per cent chance that it is higher than 5.2 g/t Au.

At Argo, the simulations were actually integrated into the planning process.

USE OF SIMULATION IN UNDERGROUND MINE PLANNING

The implementation of a conditional simulated model into underground mine optimisation presented a number of challenges due to the time-consuming manual methodology of performing an underground mine design. Completing multiple underground mine designs on various scenarios, as reflected by a range of simulations, is not practical to implement as a routine tool in an operational environment. This is not the case for open pit optimisation with the utilisation of optimisation software such as Whittle, which enables multiple scenarios to be evaluated.

Underground mine design shapes are defined by a 'stope evaluation cut-off grade'; the economic cut-off grade of a selective mining unit incorporating mine planning parameters. The stope evaluation cut-off grades will vary throughout an underground mine, due to the variable economic costs associated with mining different ore parcels (eg mining method, trucking distance, capital development and backfill methods). Dilution and ore recovery need to be incorporated into the *in situ* block estimate grades of the orebody to determine the mining grades.

The challenge in implementing 2D conditional co-simulation into the mine planning process at Argo resulted from the differences between mining grades and *in situ* block simulated grades. Confidence intervals provided an understanding and measure of risk for *in situ* block simulated grade variability; however, the model did not quantify the probabilities of achieving the stope evaluation cut-off grades as required by mine planning. Stope shapes could not be defined based on the probabilities to achieve *in situ* block estimate cut-off grades. Mine planning factors needed to be incorporated into the simulated model to reflect mining grades before the simulation could be fully utilised in the management of risk at Argo Underground.

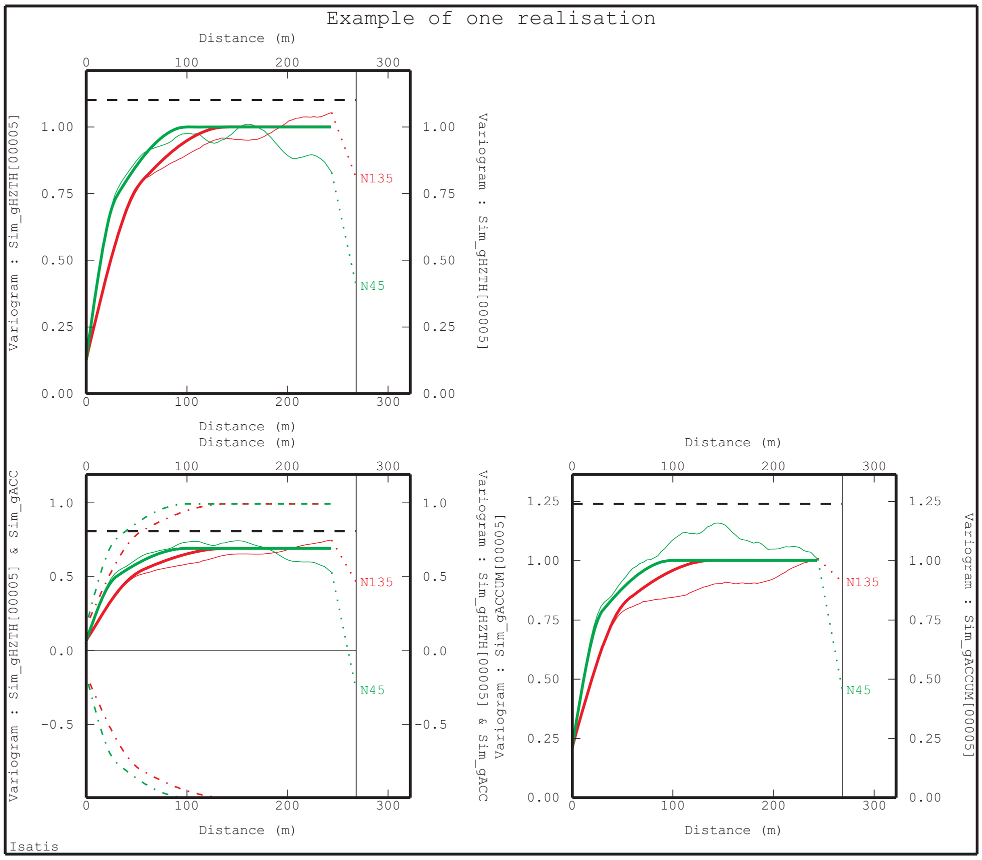


FIG 8 - Variograms and cross-variograms obtained from one realisation of the simulation.

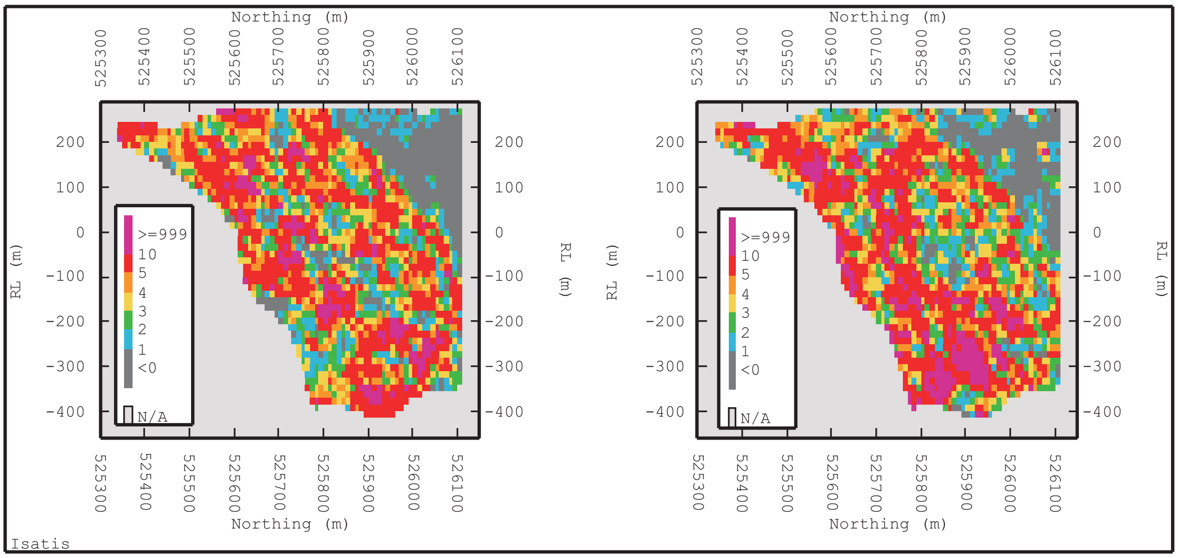


FIG 9 - Example of two realisations of the simulated Au grades of 10 m × 15 m mining units.

Ore recovery and dilution mine planning factors

The main factor affecting ore recovery of the A1 orebody is the requirement for footwall pillars (Figure 12). This is to ensure the tight filling of the hanging wall contact necessary for geotechnical support during stoping of the orebody. The proportion of recovery is dependent on the horizontal thickness and the dip of the ore surface as defined in Table 2.

Stope ore dilution of 1.4 m of the true thickness is added to the width of mineralisation at 0 g/t to account for stope over-break. This dilution is necessary to determine the mining grade of a stope.

Simulating mining grades

The mining parameters have been added to each of 100 simulations based on the mining assumptions, as shown in Figure 13. This has enabled the calculation of a simulated mining grade for each selective mining unit, rather than using *in situ* simulated grades for each simulation. The confidence intervals based on the simulated mining grade from the 100 simulations were subsequently used in the mine planning process to define the probability of achieving the stope evaluation cut-off grade (Figures 14 and 15). This has enabled mine planning to quantify

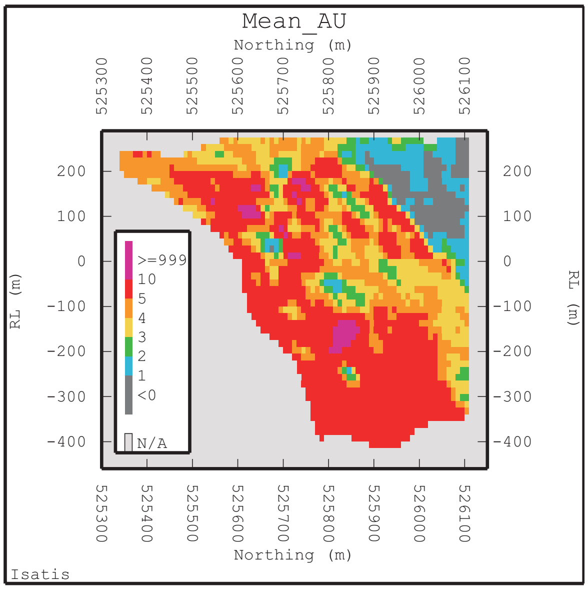


FIG 10 - Estimation of Au grade for 10 m × 15 m mining units (obtained by averaging the 100 realisations of the simulation).

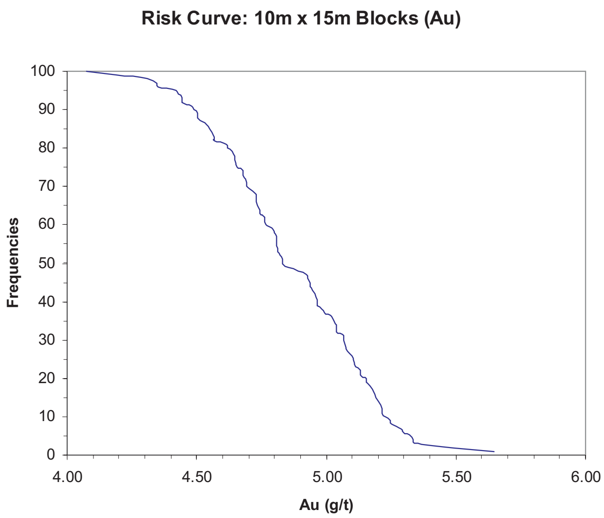


FIG 11 - Risk curve for Au grade of the global structure (at cut-off 0).

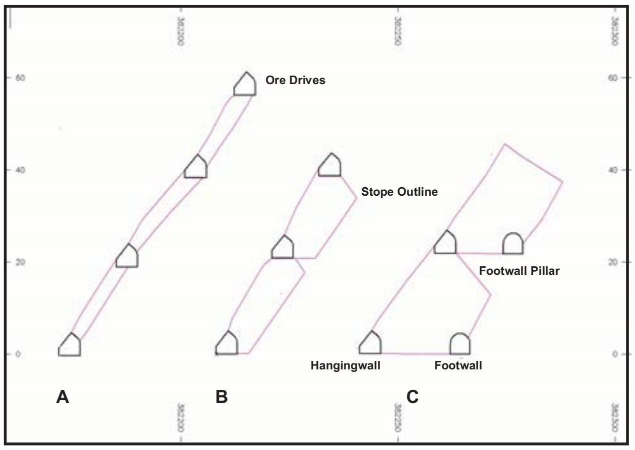


FIG 12 - Cross-section view of the various thicknesses of the A1 ore surface, showing the stope shape and ore drive profiles. The footwall pillar size is dependent on the thickness of the ore surface.

TABLE 2

Ore recovery factors associated with footwall pillars due to variable widths and dips of the A1 ore surface. The ore recovery factor can be assigned to each block based on the simulated horizontal thickness and the assigned dip for each 100 simulations.

Horizontal thickness	Dip			
	>60°	50° to 60°	40° to 50°	<40°
7.5 to 12.5 m	0.985	0.99	0.933	1
12.5 to 17.5 m	0.922	0.935	0.95	0.973
17.5 to 22.5 m	0.844	0.866	0.894	0.93
22.5 to 27.5 m	0.759	0.79	0.83	0.88

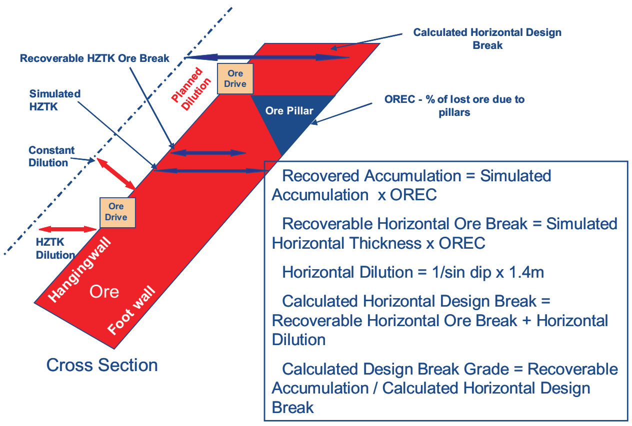


FIG 13 - Schematic cross-section of the A1 ore surface showing the calculation of the mine parameter variables added to each of the 100 simulations, where HZTK is horizontal thickness, and OREC is equal to the percentage of ore loss due to footwall pillars. Simulated mining grades can be calculated for each 100 simulations. Probabilities of the mining stope being above the economic stope cut-off grade can be determined and stope shapes modified accordingly.

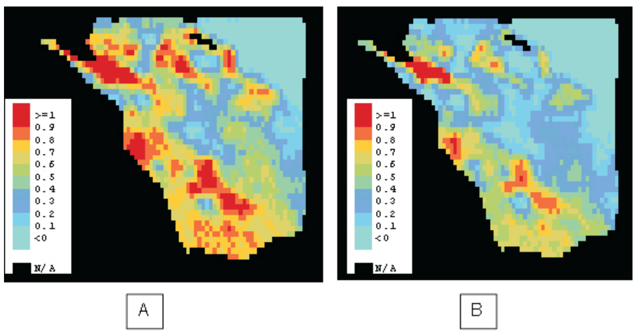


FIG 14 - Long-section probability maps showing variability of mining grades looking west: (A) at a 3.5 g/t Au stope evaluation cut-off grade and (B) at a 4.8 g/t Au stope evaluation cut-off grade.

mining grade uncertainty and risk associated with any underground mine stope, and to define stope shapes based on the probability of achieving the stope evaluation cut-off grade.

CONCLUSIONS

Conditional co-simulation is a very powerful tool for measuring first and then managing resource variability and risk. The present paper shows its applicability to an underground gold deposit, where it has helped optimise the mine design and planning.

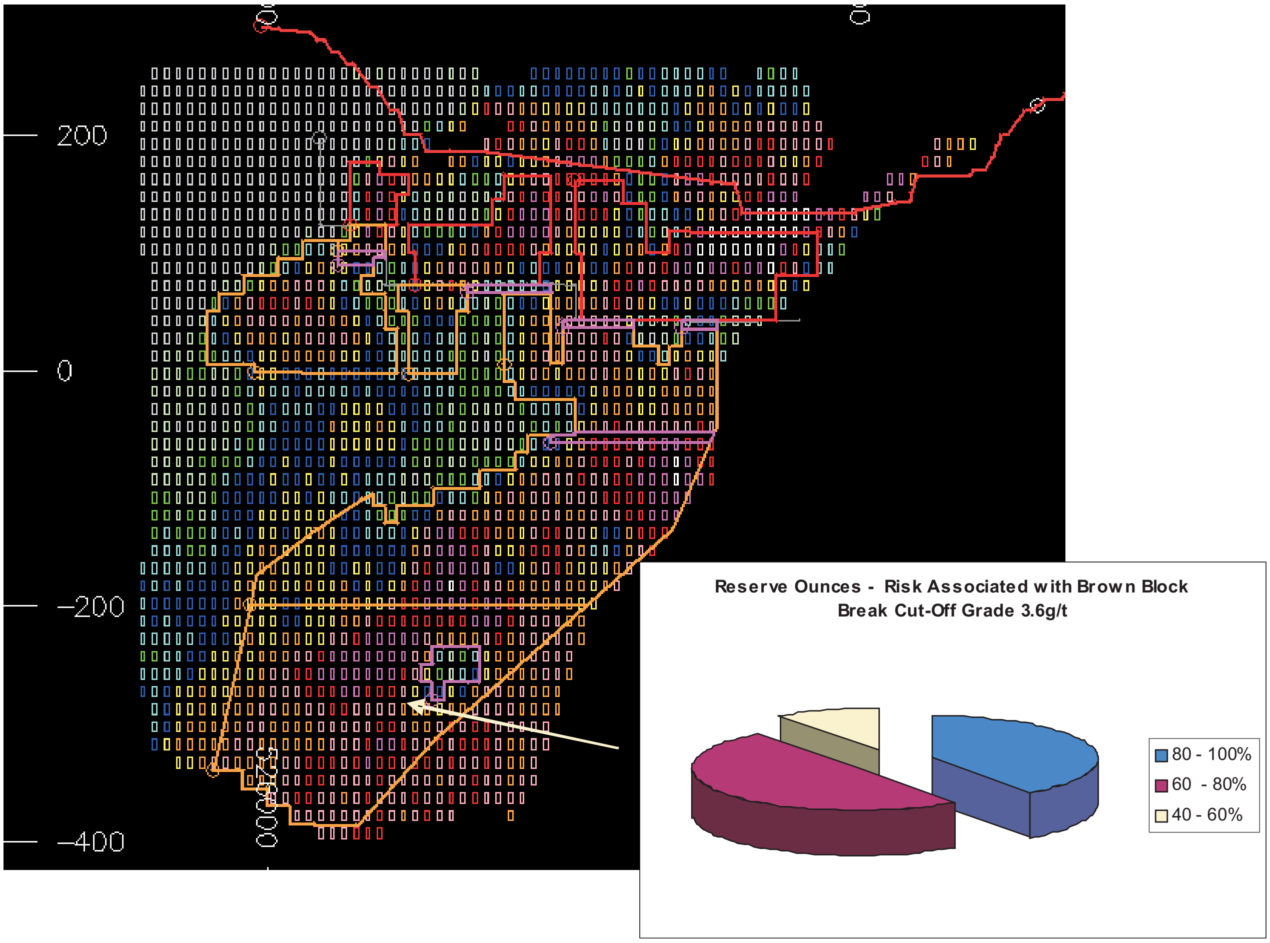


FIG 15 - Probability map of the A1 ore surface looking east, at a 3.6 g/t Au stope evaluation cut-off grade. The probability of the lower mining panel achieving a mining grade above 3.6 g/t Au based on different percentage bins is shown in the lower right-hand image. This has proved an invaluable tool in quantifying mining risk as a result of grade uncertainty.

ACKNOWLEDGEMENTS

The authors would like to thank Rob Smith and the Argo mine planning team for their technical assistance and the numerous iterations that were involved in developing a simulated model that could be utilised in the mine planning process. Jennifer Gressier and Alex Trueman's contributions and editing are greatly appreciated.

REFERENCES

Gemuts, I and Theron, A, 1975. The Archaean between Coolgardie and Norseman – stratigraphy and mineralisation, in *Economic Geology of Australia and Papua New Guinea* (ed: C L Knight) Vol 1, pp 66-74 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Characterization* (Oxford University Press).

Gressier, J M and Kolkert, R, 1995. Structural controls on gold mineralisation of the Argo Deposit, Western Australia, St Ives Gold Mine, in *Gold Fields In-House Structural Symposium 2002*, unpublished internal note, Gold Fields.

Grieco, N and Dimitrakopoulos, R, 2007. Grade uncertainty in stope design — Improving the optimisation process, in *Orebody Modelling and Strategic Mine Planning,* second edition (ed: R Dimitrakopoulos), pp 167-174 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Krige, D G, 1997. A practical analysis of the effects of spatial structure, and of data available and accessed on conditional biases in ordinary kriging, in *International Geostatistical Conference: Geostatistics* (eds: E Y Baafi and N A Schofield) Vol 2, pp 799-810 (Kluwer Academic Publishers).

Osborne, J, Vos, M and Ellery, S, 1998. Argo ore resource, December, Technical report, St Ives Gold Exploration, unpublished internal report, WMC.

Ovanic, J, 1998. Economic optimization of stope geometry, PhD thesis, 209 p, Michigan Technological University, Houghton.

Rivoirard, J, 1994. *Introduction to Disjunctive Kriging and Non-linear Geostatistics*, 180 p (Clarendon Press: Oxford).

Smillie, R W, 1996. Structural and lithological controls on gold localisation at the Argo deposit, St Ives, Western Australia, Report 3683, in SIGM technical report R506, unpublished internal report, WMC Resources Limited.

Stone, W E, 1997. KNO Geology induction handbook, Technical report No 245, Version 2.0 (first edition), WMC Resources Limited.

Sugden, T J and Hein, K A A, 1995. Introduction to the stratigraphy of the Kambalda-St Ives area, in workshop notes, technical report No 229, version 1, unpublished internal report, WMC.

Pseudoflow, New Life for Lerchs-Grossmann Pit Optimisation

D C W Muir[1]

ABSTRACT

The Lerchs-Grossmann (LG) algorithm (1965) has been used for over 30 years for the optimum design of open pit mines. This has been combined with variable grade cut-off and discounted cash flow (DCF) to optimise the net present value (NPV) of the cash flow for the life of the mine. The LG algorithm is a unique, efficient method for solving a special case of an integer linear programming or network flow problem. More general network flow methods implemented in the 1970s were only practical for small problems. The efficiency and effectiveness of the LG method made it the industry standard. During the 1990s and recently newer algorithms for Network Flow have been developed (eg *push- relabel, pseudoflow*) theoretically more efficient than the LG method. Hochbaum generalised the LG algorithm to a *pseudoflow* network model. Two methods, lowest and highest label, theoretically more efficient than the *push-relabel* and other network flow methods, were developed.

Lerchs and Grossmann gave no efficient method of selecting constraints. Some early implementations were inefficient due to data structure and constraint selection. Other implementations were more efficient but the actual details were proprietary to the developers. Recently some of the newer network flow algorithms have been implemented (eg *push-relabel*). This paper will examine an implementation of the *pseudoflow* algorithm incorporating the Hochbaum highest and lowest label methods as well as a relatively efficient generic LG method. In addition a new, and several old, strategies for developing a nested sequence of optimum stage pits are examined. These strategies combined with DCF, Fundamental Tree or other scheduling techniques provide an efficient method of optimising the NPV of a mine.

INTRODUCTION

This paper discusses variations of the classical Lerchs-Grossmann (LG) algorithm (1965) for open pit mine design. The implementation of four methods based on the LG algorithm and their performance on several actual mineral block models are discussed. In addition, several strategies for developing a nested sequence of optimum stage pits are examined. The Optimum Mine Design methods discussed are:

- Lerchs-Grossmann algorithm (Lerchs and Grossmann, 1965);
- Lipekewich and Borgman LG subset algorithm (Lipekewich and Borgman, 1969);
- Hochbaum Lowest Label Pseudoflow algorithm (Hochbaum, 2001); and
- Hochbaum Highest Label Pseudoflow algorithm (Hochbaum, 2002).

The development of a surface mining venture involves expenditures of millions of dollars. An optimum ultimate pit, intermediate stage pits and long-term production scheduling are used to maximise the net present value (NPV) of the venture. These planning methods focus on the sequencing of materials to be mined under technical, economical and environmental constraints. Other considerations such as the uncertainty in the data and the inherent risk in the venture will not be covered in this paper. The papers by Dimitrakopoulos, Farrelly and Godoy (2002); Godoy and Dimitrakopoulos (2004) and Dimitrakopoulos and Ramazan (2004) give some insight into those considerations.

The LG 3D pit design algorithm has been used for over 30 years for open pit mine design. It is well known and has been implemented in commercial software (eg Muir, Whittle and MaxiPit). It wasn't until the 1990s that other efficient network flow algorithms were developed (eg *push-relabel*, Goldfarb and Chen, 1997; *pseudoflow*, Hochbaum and Chen, 2000). These algorithms could theoretically solve the pit optimisation problem more efficiently and some have been implemented commercially (eg MineMax uses *push-relabel*). Hochbaum (1997, 2001, 2002) has extended the recent results from network flow algorithms to the LG algorithm. These *pseudoflow* algorithms are based on the LG algorithm and incorporate lowest label and highest label methods.

The performance of the four algorithms will be compared on three different mineral properties. One is an actual gold mine and two are prospective mines. The first block model is 220 × 119 by 38 benches (994 480 blocks). The second is 450 × 142 by 71 benches (4 536 900 blocks) and the third is 200 × 160 by 55 benches (1 760 000 blocks). These models and subsets are sufficiently large enough for performance data as a function of the number of blocks and arcs. The largest model has over four million blocks and 320 million arcs. These models have quite different grade distributions and slope constraints, which are reflected in the actual performance of the algorithm. Some other performance data on an LG algorithm and other network flow algorithms is given in Hochbaum (1996) and Hochbaum and Chen (2000).

In the following sections, a brief summary of the LG algorithm and the *pseudoflow* labelling methods will be given. Subsequently, the performance of the various methods will be compared. Next, the utility of the newer methods is applied to the fast generation of a sequence of nested pits, optimal for the volume mined. These included pits form a starting point for NPV (Lane, 1988; Wharton, 1996; Hanson, 1997), or fundamental tree scheduling techniques (Ramazan, 2001, 2007, this volume). Lastly, the conclusions of this study follow.

DEFINITIONS

A weighted directed graph $G=(V,M,A)$ is a set of vertices V with Mass M (positive or negative) and directed arcs A. An arc $a=[p,q]$ is a directed edge (p,q) joining two vertices in V. The weight $w(v)$ of a node v in V is called its mass $m_v= w(v)$. The number of vertices in G is denoted by $n=|V|$ and the number of arcs is denoted by $m=|A|$.

A closed subgraph $G_c=(V_c,M_c,A_c)$ is a subset of G such that all arcs originating in V_c also terminate in V_c and is called a closure of the graph. A partial closure is a subset G_p in G for which some but not all arcs originating in G_p also terminate in G_p. A maximum closure G_m in G is a closed subgraph of G that has maximum weight (the total weight of the vertices is maximal).

A rooted tree is an undirected acyclic connected graph T with a designated node as root. All other nodes are usually depicted as suspended below the root node. A subtree T_v of T denotes the subtree suspended from node v that contains all the descendants of v in T. An immediate descendant of a node v is denoted as ch(v), a *child* of v, and the unique immediate ancestor of v is denoted by p(v), the *parent* of v. A node in a rooted tree T is said to be at *level l* in T if it is at a distance of l edges from the root.

A tree T embedded in G is a set of vertices V_T in V such that an arc in T is also an arc in G. Given a rooted tree T embedded in G, T_v is the subtree suspended from node v and $T_{(v,p(v))}$ is the tree suspended from the edge $(v,p(v))$. $T_v = T_{(v,p(v))}$ where $[v,p(v)]$ or $[p(v),v]$ is an arc in G. A node v in an embedded subtree T_v in G

1. Muir and Associates Computer Consultants Inc, 5531 4th Avenue, Delta BC V4M 1H2, Canada. Email: dmuir@aebc.com

is said to support the mass M_v (the sum of the weights of all nodes in T_v). The edge $e=(v,\mathrm{p}(v))$ is also said to support the mass M_v. The mass M_v supported by a node v is a dynamic value that depends on the structure of the subtree T_v suspended from v.

We define an extended graph G^X as the graph G augmented with a dummy root node x_0 and arcs going from x_0 to all nodes of G. The tree T_{x0} linking x_0 to each vertex in G is a spanning tree in G^X. Given an embedded spanning tree T_r rooted at x_0 in G^X a child v of x_0 in T_r defines an embedded subtree T_v of G. This subtree is referred to as a *branch* of G. and that child v of x_0 is the root of the branch T_v. The branches of the spanning tree T_r in G^X define a forest in G.

An arc in an embedded spanning tree T_r rooted at x_0 in G^X either points toward the root (upward) or away from the root (downward). A downward arc is called a *p-edge* (plus edge) and an upward arc is called an *m-edge* (minus edge). A downward arc (*p-edge*) is called *strong* if it supports a mass that is strictly positive. An upward arc (*m-edge*) is called *strong* if it supports a mass that is zero or negative. Arcs that are not strong are called *weak*. A branch T_v of T_r suspended from node v is called *strong* if the arc $[x_0,v]$ connecting v to the root x_0 is *strong*, otherwise it is called *weak*. All nodes of a strong branch are called *strong* and all nodes of a weak branch are called *weak*.

An embedded spanning tree T_r of G^X is *normalised* if the only strong edges are connected to the dummy root x_0. The spanning tree T_{x0} is an example of a *normalised* tree.

For open pit optimisation the slope constraints in mining (the arcs of the graph) are not enumerated directly. They are defined by a *support pattern* as a recursive precedence relationship. A *support pattern* is a set of dependent blocks (usually minimal) that must be removed before a support block can be removed. The dependent blocks have their dependent blocks and so on until the surface is reached. The actual support pattern used depends on the shape of the blocks, the slope angle and slope angle accuracy required. The support pattern used by Case 2 has six levels and Case 3 has eight levels, both have 81 arcs per node. Multiple support patterns (possibly asymmetric) could be used throughout the deposit.

LERCHS-GROSSMANN ALGORITHM

The reader is referred to Lerchs and Grossmann (1965); Zhao and Kim (1992) and Hochbaum (1996, 1997, 2001, 2002) for a more detailed discussion and proof of the correctness of the LG algorithm. The algorithm will be described here but not proven. This description and implementation has been derived from the above references, private notes and from Muir (1972).

The LG optimisation algorithm finds the maximum closure of a weighted directed graph; in this case the vertices represent the blocks in the model, the weights represent the net profit of the block, and the arcs represent the mining (usually slope) constraints. The algorithm thus solves a very special case of a linear programming or network flow problem. Since the problem is a special subset of the general linear programming problem, it is only to be expected that an algorithm specifically designed to solve such a subset may be more computationally efficient. The basic LG algorithm has been used for over 30 years on many feasibility studies and for many producing mines. Hochbaum (1996, 2001, 2002) has extended the LG algorithm with the concept of *pseudoflow* on a network flow formulation of the problem. This formulation of the problem enhances the basic LG algorithm with a structured strategy for determining the next set of arcs to process.

The algorithm as implemented here systematically develops a sequence of normalised embedded spanning trees from extended closed subgraphs of the graph, which at any stage incorporate more arcs from the subgraph. The sequence of trees developed contain subsets (the strong branches) whose vertices form a maximum closure of the embedded tree and a partial closure of the subgraph and ultimately converge to a maximum closure of the subgraph.

The algorithm can start with any *normalised* embedded spanning tree of a closed extended subgraph G^X of the original graph. In this implementation a dummy vertex x_0 is created and a start spanning tree is used which has this dummy vertex as the root and each vertex of the subgraph G as a branch. Thus it is a normalised embedded spanning tree T_{x0} of the extended subgraph G^X. In most of the remainder of this discussion the subgraph will be assumed to be the entire graph without loss of generality. The maximum closure of a closed subgraph of a graph is contained within the maximum closure of the entire graph.

The algorithm consists of two steps that are repeated until the vertices of the *strong* branches form a maximum closure of the weighted directed graph G. There is a *merger* step and a *pruning* step (*normalisation*). The process thus starts with a super optimal set (the initial strong branches of T_{x0} which is the set of all positive vertices) that does not satisfy the constraints and converges to a maximum closed subset that does satisfy the constraints.

At each stage there are several variables associated with each vertex or edge of the normalised tree. These represent the weight of the subtree suspended from an edge or vertex, the type of the edge (p or m) connecting it to its parent (initially the root x_0), and the status of the edge or vertex (*weak* or *strong*). An edge in a normalised subtree is *strong* if and only if it is a *p-edge* and the weight supported by the edge is positive, by construction and 'property 3' (Lerchs and Grossmann, 1965). Other variables are counters and housekeeping variables. The tree data structure itself is fairly complex and is represented as sets of linked lists, a data structure designed to optimise tree traversals and re-combinations. In a normalised tree all vertices of branches connected to the dummy root x_0 are either all *strong* or all *weak*. In the *pseudoflow* methods priority queues are used to determine the order of processing strong trees.

Let $a=[s,w]$ be an arc in the original graph linking a *strong* vertex to a *weak* vertex, a so-called weak-over-strong (*wos*) relationship. Note that s and w are in different branches since the embedded spanning tree is *normalised*. Let w be in the *weak* branch T_{rw} and let s be in the *strong* branch T_{rs}.

Let $C_s = [s,...,r_s,x_0]$ be the chain of edges $(v,\mathrm{p}(v))$ in T_{rs} connecting s to the dummy root x_0.

Let $C_w = [w,...,r_w,x_0]$ be the chain of edges $(u,\mathrm{p}(u))$ in T_{rw} connecting w to the dummy root x_0.

Merger procedure

The first or *merger* step generates a new branch T_m.

1a. Link the *weak* vertex w in the *weak* branch T_{rw} to the *strong* vertex s in the *strong* branch T_{rs}.

1b. Reverse each edge $(v,\mathrm{p}(v))$ and its type (p or m) on the path from the *strong* vertex s to the root r_s of T_{rs} (reverse path of the chain C_s). At r_s sever the dummy edge (r_s,x_0) connecting the branch T_{rs} to the dummy root x_0. This forms the new chain $C_m = [r_s,\ldots,s,w,...,r_w,x_0]$ in T_m connecting r_s to the dummy root x_0. Thus the old *strong* branch is now suspended from the edge (s,w) and is a subtree of the merger tree T_m.

1c. Compute the new weight supported by each edge on the chain C_m from r_s to the root $r_m=r_w$ of the merged branch T_m.

1d. Assign the status (*weak* or *strong*) to each vertex in the new branch T_m.

Pruning procedure

The second or *pruning* step *normalises* the merged tree T_m.

This *normalisation* prunes the merged tree T_m by trimming all *strong* edges of subtrees to form new branches rooted to the dummy vertex x_0. This only requires that all *strong* edges on the chain C_m be severed and the supported subtree be re-rooted to the dummy root x_0 to form a new branch. All other edges of the merged tree T_m have not changed so they have the same type and support the same mass. As the original trees T_{rs} and T_{rw} were normalised, none of these other edges are *strong* and do not need to be pruned.

2a. Find the first strong edge, say $e = (x_b,x_a)$ on the chain C_m, but not (r_w,x_0) thus let $C_{sbaw} = [r_s,...,x_b,x_a,..., r_w,x_0]= C_m$.

2b. Sever the edge (x_b,x_a) and form the new branches T_{rb} and T_{ra} rooted at x_0 with chains $C_b = [r_s,...,x_b,x_0]$ and $C_a = [x_a,..., r_a,x_0]$ where r_a=r_w.

2c. Compute the new weight supported by each edge in the chains C_b and C_a.

2d. Assign the status (*weak* or *strong*) to each vertex in the new branches T_{rb} and T_{ra}.

2e. Repeat steps a through d on the remaining chain C_a of the pruned branch T_{ra}.

The procedure terminates when all the constraints (arcs) of the original subgraph have been scanned and no further weak-over-strong conditions exist. The maximum closure of the subgraph is then the set of vertices contained in the *strong* branches. This closure of the subgraph is contained in the maximum closure of the complete graph, and hence these vertices can be recorded as forming part of the maximum closure and deleted. New branches formed from a larger closed subgraph can then augment the remaining branches. The procedure for finding the maximum closure is then repeated for this subgraph. Finally, when the entire graph has been processed, the remaining weak vertices are not in the maximum closure.

Combined merger and pruning procedure

The operations c and d of the second *pruning* step are essentially the same as the corresponding operations in the first *merger* step. In practice, as each edge on the chain C_m is traversed in step 1c, the new weight supported by the edge is computed. If it is a strong edge (x_b,x_a) the edge is severed in step 2b and the new branches T_{rb} and T_{ra} rooted to x_0 are formed. The weight supported by each edge in the severed subtree T_{rb} has already been computed as part of step 1c, hence only step 2d has to be performed on T_{rb}. This procedure is repeated on the next edge on the chain until all edges except (r_w,x_0) have been processed. If the last edge (r_w,x_0) is strong, the remainder of the merged branch is strong and the number of weak branches is reduced by one. If the last edge (r_w,x_0) is weak then the remainder of the merged branch is weak. Note that no new weak branches are formed. The only remaining task is to assign the status of each vertex on the remainder of the merged branch, step 2d. The merger edge (s,w) is an m-edge since it points toward the root x_0. It initially supports a positive mass, the entire old strong branch, and hence is not a *strong* edge. It can be shown that the merger edge remains weak even if parts of the old strong branch are pruned.

The critical parts of this procedure from a computational time viewpoint are:

1. Selecting a weak-over-strong relationship (s,w).
2. Processing the chain C_m linking the root of the strong branch to the root of the weak branch. This involves inverting edge links and type, computing the new weight of the edges, normalising and assigning the status (*weak* or *strong*) to vertices of the normalised branches.

The first operation is dependent on efficient search techniques or processing strategies and the second on efficient tree traversal and processing methods. Both are dependent on a suitable data structure. The original paper by Lerchs and Grossmann (1965) gave no constructive method of processing the branches and no specific method of selecting the order of processing weak over strong arcs. Muir (1972) implemented the LG algorithm with efficient tree data structures and depth first search (DFS) and breath first search (BFS) techniques. This was a large improvement over the Borgman (1968) implementation that used a crude exhaustive search. It is further improvements in the strength of these operations and in the data structure representation that give current implementations their speed and flexibility.

PSEUDOFLOW ALGORITHM

Recent works by Hochbaum (2001, 2002) and Anderson (2001) have adapted the normalised trees of the LG algorithm to a more general network flow model with the concept of *pseudoflow*, similar to *preflow*. A *pseudoflow* on a network satisfies capacity constraints, but may violate flow balance constraints by creating deficits and excesses at nodes. A *preflow* satisfies capacity constraints, but may violate flow balance constraints by creating only excesses at nodes. The *pseudoflow* algorithm solves the maximum flow problem on general networks and works with *pseudoflows* instead of masses. The relationship between the *pseudoflow* algorithm and the push-relabel algorithm is clearer than that between the LG algorithm and the push-relabel algorithm. The LG and *pseudoflow* algorithms work with *sets of nodes* (branches) that can accumulate either excesses or deficits. In the *pseudoflow* formulation of the problem, the mass M_{rs} supported by the root r_s of the strong tree is treated as a *pseudoflow* and is pushed to the weak root r_w and hence to the dummy root x_0 (both source and sink node). The push-relabel algorithm works with *preflows*. The push-relabel algorithm works with nodes (rather than sets of nodes) and the excess at a node is pushed to those nodes closer to the sink as measured by distance labels, relabel updates the value of the label.

The *pseudoflow* algorithm allows for several systematic ways of processing the weak-over-strong vertices. The best of these methods are the lowest label and highest label variants. See the above referenced papers for full details of the *pseudoflow* algorithm. Here we will only discuss how to implement the lowest and highest label methods.

The lowest and highest label methods work with the concept of a distance label. A *distance label* for a node is a non-decreasing function and is non-decreasing with level, within any generated tree. See Hochbaum (2001) for proof that a distance label is non-decreasing with level within a tree and for a weak node v is a lower bound on level (v). This distance label function is similar to the distance labels introduced by Goldberg (1985) and those used in network flow methods such as *push-relabel* (Goldfarb and Chen, 1997).

In the initial normalised embedded spanning tree T_{x0}, all strong nodes are assigned the label 2 and all weak nodes the label 1. To efficiently manage the strong branches, a priority queue with index is created and maintained. A counter keeps track of the number of strong root nodes and an index list pointing to the first strong root node with each label is maintained (initially all 0). Initially all positive nodes are strong and are the roots of their respective branches. All these nodes have label 2 and are placed in the queue. At this stage order is arbitrary, since all strong nodes have label 2 although the actual order determines how branches are processed. A pointer to the first node with label 2 is placed in the index list. When selecting the next strong tree to process the top of the queue (either highest or lowest order) is selected and removed from the queue. When the merger and normalisation processes generate a new strong branch it is inserted into the queue at the top position for that label.

Lowest label method

In the lowest label method a strong branch of lowest root label L is extracted from the priority queue of strong branches. Processing from the root down, nodes of the strong branch with lowest label L are scanned for a weak dependent of label $L-1$ (a *wos* relationship). If a merger arc [s,w] is found the branches are merged and pruned (normal LG merger/pruning) and any resulting strong branches are added to the priority queue of strong branches at the appropriate label of the root. The merger node s has the same label L as the root node of the strong branch and as labels are non-decreasing with level, all nodes on the chain $C_s=[s,\ldots,rs,x_0]$ have label L. Hence, reversing the edges does not violate the non-decreasing distance rule. The label of w is $L-1$ and the label of r_w is 1 since no weak branches are created. If no weak dependent arc [s,w] is found all nodes in the strong branch with label L are increased to $L+1$ and the branch is inserted back onto the priority queue with root label $L+1$ if $L <$ n (number of nodes). If the new label $L+1 >$ n then the process is finished. In practice the *gap certificate of optimality* rule (Anderson, 2001) is used to determine completion. This rule states that if the lowest label strong branch has root label L and there are no nodes with label $L-1$ then the process can terminate.

An earlier lowest label method (LLP) shown in the case studies is a method without priority queues. This uses the same search as the LG algorithm but only merges if the strong node has Label L and the weak node has label $L-1$. This was used as a trial to show that the order of the selection of nodes for next merger edge is very important .The priority queue data structure allows for a much more efficient method of selecting nodes.

The lowest label method is particularly suitable for parametric implementation. The method has features that make it especially easy to adjust to changes in capacities (weights). This is covered in the paper by Hochbaum (2001) and is not currently implemented in the versions discussed here.

Highest label method

The highest label method is similar to the lowest label method except that the priority queue is reversed and the branch with the highest (not lowest) label root is processed (Anderson, 2001). One main difference between the lowest and highest label methods is in determining early termination. In the lowest label method, as soon as a gap in the labels exists, the program can terminate. This is not the case with the highest label method because if no merger is found the method will increase the root label and all subnodes with the same label. This branch will still be of the highest root label and will continue to be processed. Eventually if there are no mergers there will be a label gap between the strong and weak nodes. The process cannot be terminated because other strong branches may exist that have not been processed. Instead, if a strong branch has root label L and there are no nodes with label $L-1$ then that branch is part of the maximum closure and needs no further processing. In this implementation, that means its label is set to n and it is not placed back on the priority queue.

The *pseudoflow* variants do not modify the generic LG algorithm weak-over-strong merger process. However, they do state how and in what order weak-over-strong links should be processed. There is also a requirement to update node labels and maintain the priority queue. These methods also give means of detecting when all weak-over-strong links have been processed without an exhaustive search needed by the original LG algorithm. In this implementation a final LG exhaustive scan of all strong nodes is done to verify that all weak-over-strong arcs have been processed. This usually takes less than a second even for large matrices. All the methods give the same optimal solution although the sequence of mergers and prunings differ for each case.

SUBSET AND PREPASS

For most pit designs the arcs are all directed upwards and reflect the slope constraints on mining. This means that subproblem decomposition such as level-by-level optimisation (Lipekewich and Borgman, 1969) can be done. In general, any closed subset of nodes can be optimised and then augmented with a larger closed subset and the procedure repeated as often as necessary until the entire graph has been processed. This is referred to here as the LG subset and *pseudoflow* subset methods.

Prepass techniques are methods of generating limiting pits for the optimisation. There are several reasons for limiting the number of blocks, depending upon the optimisation method, memory available and computational time required. Chen (1976) describes various prepass methods on the profit matrix to limit the volume to a closed subset containing all profitable blocks to a given level. This reduces the number of blocks that have to be optimised and the time for optimisation to that bench.

The largest pit that needs to be considered can be found by using the set of all positive (profitable) blocks and expanding them and dependent blocks to the surface. This pit is a closed subset of the blocks and since it contains all positive blocks it contains the ultimate pit (a subset of the positive blocks and their dependents that is of maximum value).

These techniques are quite useful for large problems using the generic LG methods. For *pseudoflow* or *push-relabel* methods, unless the problem is very large and memory is a problem, the extra effort for prepass probably would not reduce the overall time. The program implemented here can use a limiting pit and also a starting pit (eg a previous stage pit). Neither starting pits nor limiting pits have been used in the cases studied here. Some programs (eg Whittle) automatically ignore arcs (constraints) and blocks that would never be mined. The *pseudoflow* methods after initialisation never even look at blocks that are not positive or within a positive block's dependence volume.

CASE STUDIES

Case 1: 220 × 119 by 38 bench profit matrix

The first example is a 220 × 119 by 38 bench (994 840 blocks) profit matrix. The blocks are 5 m × 5 m × 5 m and a simple one-up-one-over and second level Knight's move *support pattern* (Lipekewich and Borgman, 1968) is used. This pattern with five blocks on the first level and eight blocks on the second level at offsets 1,2 or 2,1 (a Knight's move in chess) is used to give a crude approximation to a 45° slope with a total of 13 arcs per node. Tables 1 and 2 are test results using the above methods. In Table 2 an interesting result is that with the highest and lowest label algorithms, the total number of mergers and pruning is greatly reduced. For example, for the bench 38 run, the number of mergers and pruning for the LG is 950 175 and 1 703 036, while for the lowest label it is only 329 599 and 459 454. This general reduction in the number of mergers and prunings shows the superiority of the *pseudoflow* selection of mergers. All methods give the same optimum result.

In this example to optimise to level 38, the LG takes 25 min 27 sec (1527 sec) to optimise in one pass. It takes10 min 28 sec (628 sec) with level-by-level optimisation (using a subgraph to a specific start level and incrementing by two levels to the next subgraph). The lowest label method time is 9 min 16 sec (556 sec) and for level-by-level optimisation is 2 min 40 sec (160 sec) without priority queues. Using priority queues the lowest label time is 2 min 6 sec (126 sec) and the highest label time is only 29 seconds. The priority queue lowest and highest label methods are clearly the fastest. In this case the highest label method is also superior to the lowest label methods.

TABLE 1

Optimisation times (seconds) to various pit levels for $220 \times 119 \times 38$ profit matrix.

Bench	LG	LGS	LLP	LLPS	LLPQ	HLPQ
26	285	56	186	91	23	9
28	398	94	247	107	35	13
30	632	130	327	125	54	17
32	878	176	410	145	83	28
34	1157	243	480	152	107	27
36	1387	478	541	157	116	28
38	1527	628	556	160	126	29

Legend:

LG Normal Lerchs-Grossmann

LGS Subset Lerchs-Grossmann

LLP Lowest Label Pseudo Flow (no priority queue)

LLPS Subset Lowest Label Pseudo Flow (no priority queue)

LLPQ Lowest Label Pseudo Flow (priority queue)

HLPQ Highest Label Pseudo Flow (priority queue)

TABLE 2

Statistics for level 38 for 220 x 119 x 38 profit matrix.

	LG	LLPQ	HLPQ
Profit value	57 118 058	57 118 058	57 118 058
Blocks removed	95 228	95 228	95 228
Blocks remaining	830 754	830 754	830 754
Branches relinked	950 175	329 599	420 244
Branches pruned	1 703 036	459 454	638 088
Time (seconds)	1527	126	29

Legend:

LG Normal Lerchs-Grossmann

LLPQ Lowest Label Pseudo Flow (priority queue)

HLPQ Highest Label Pseudo Flow (priority queue)

Case 2: 450×142 by 71 bench profit matrix

The second example is a 450×142 by 71 bench (4 536 900 blocks) profit matrix The blocks are 2 m × 6 m × 6 m and a more accurate and complex six level 53 degree slope *support pattern* with a total of 81 arcs per node is used. Table 3 shows test results using the above methods. The lowest and highest label methods are clearly superior. An interesting result is that for some runs the lowest label method is also faster than the highest label method. The generic LG without subproblem decomposition has quite large run times for the higher levels. In practice either subproblem decomposition and/or prepass techniques would have been used for the LG methods.

Figures 1 and 2 compare the performance of the methods in Cases 1 and 2 against functions of n.

Case 3: 200×160 by 55 bench profit matrix

The third example is a 200×160 by 55 bench (1 760 000 blocks) profit matrix The blocks are 20 m × 20 m × 15 m and a more accurate and complex eight level 45 degree slope *support pattern* with a total of 81 arcs per node is used. This model has good mineralisation close to the surface and extending to depth. This makes it relatively easy to optimise. For this example the

TABLE 3

Optimisation times (seconds) to various pit levels $450 \times 142 \times 71$ profit matrix.

To bench	LG	LGS	LLP	LLPS	LLPQ	HLPQ
31	19	19	71	71	3	3
36	144	97	239	156	11	8
41	1521	1114	1085	556	105	45
46	14 737	8340	2730	1257	384	146
51	26 787	18 257	6621	3622	1369	749
56	35 969	20 857	8086	5272	1042	893
61	42 165	22 192	9560	6714	652	1003
66	47 608	22 837	11 654	7940	519	1053
71	56 843	23 668	12 976	9153	645	1140

Legend:

LG Normal Lerchs-Grossmann

LGS Subset Lerchs-Grossmann

LLP Lowest Label Pseudo Flow (no priority queue)

LLPS Subset Lowest Label Pseudo Flow (no priority queue)

LLPQ Lowest Label Pseudo Flow (priority queue)

HLPQ Highest Label Pseudo Flow (priority queue)

optimisations are done for volume penalty pits instead of pits to specific levels. Table 4 shows test results using the above methods. Here the highest label method is clearly superior to both the LG and the lowest label methods. An interesting result is that for some runs the LG method is also faster than the lowest label method.

STAGE PITS

An optimum ultimate pit and intermediate stage pits are needed for long-term production scheduling. There are various methods of generating stage pits, eg optimum pits to different levels, optimum pits for different mining and milling costs or mineral prices. Milner (1977) discussed programs implemented at Gibraltar Mines for the development of long-range mine plans. An LG 3D pit design program (descendant of Muir, 1972) is used for pit design and sequencing of stage pits. These pits are optimal to a specific bench level. A mine simulation program is then used to produce production schedules and open pit mining equipment requirements. This method of generating stage pits can be easily implemented by saving the intermediate pits generated by the LG level-by-level subset method. Multiple *pseudoflow* runs can also generate them quickly. These stage pits, although optimal to a specific level, have drawbacks. These stage pits are not necessarily optimal for the volume mined.

A better and relatively easy method of generating intermediate stage pits is to apply a (volume) penalty to each block in the model. This is basically equivalent to increasing the mining cost of all blocks. The larger the penalty the smaller the pit. An advantage of this method is that the stage pits thus defined have the property that they are of maximum value for the volume mined. Also, the profit matrix does not have to be modified externally. A sequence of pit optimisations can be generated quickly by applying a sequence of penalty values to the program. Hochbaum (2001) shows that the lowest label *pseudoflow* method is particularly adapted to parametric scaling of profit values. In this study, a range of parametric scaling is not done within the program. Complete individual runs for a range of block penalty values are performed for each method. Internal parametric scaling within the program would be more efficient but would make it more difficult to compare timings and performance.

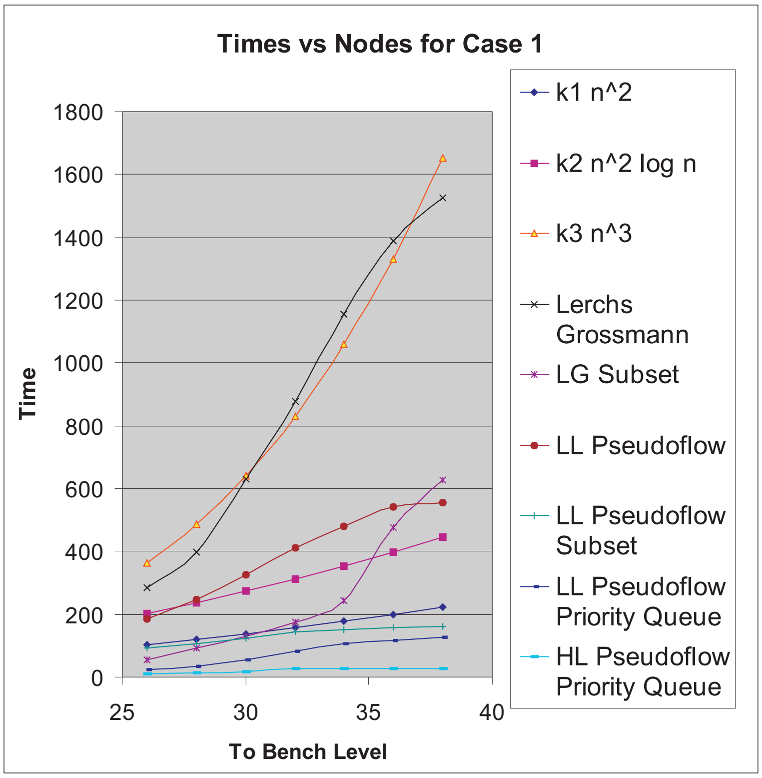


FIG 1 - Optimisation times (seconds) versus nodes for Case 1. Times for various methods to bench levels.

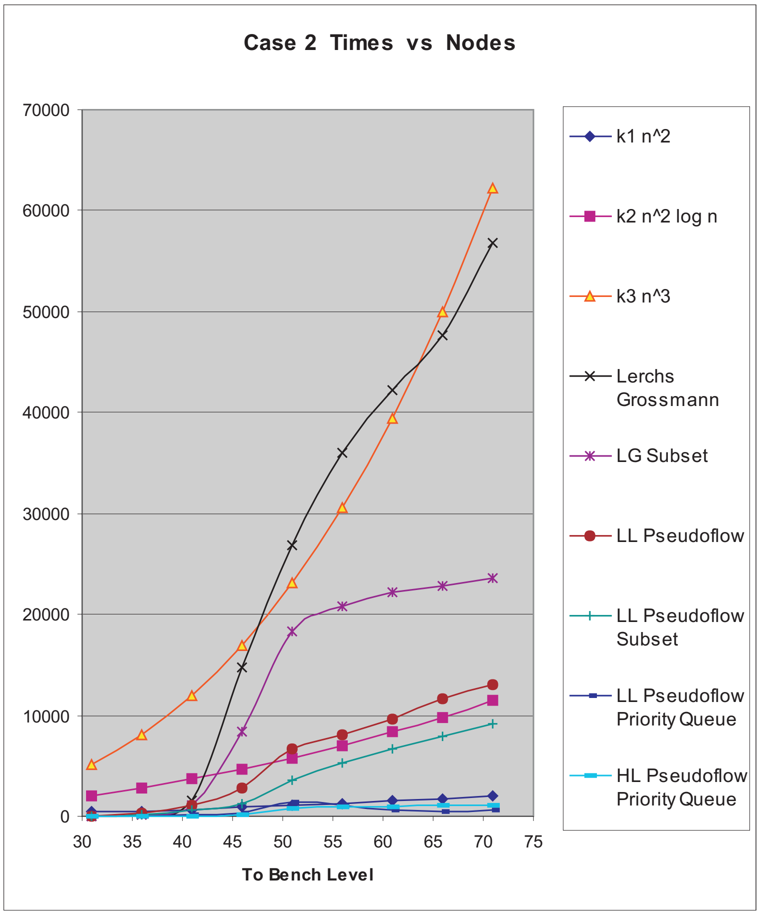


FIG 2 - Optimisation times (seconds) versus nodes for Case 2. Times for various methods to bench levels.

TABLE 4

Optimisation times (seconds) for various penalties for $200 \times 160 \times 55$ profit matrix.

Penalty	LG	LGS	LLPQ	HLPQ
Ultimate (0)	95	66	429	19
500	115	87	301	19
1000	178	112	242	20
1500	290	145	129	19
2000	376	190	105	18
2500	330	217	55	18
3000	498	391	56	27
3500	307	313	20	16
4000	629	483	24	21
4500	1167	572	22	9
5000	265	381	4	3
5500	30	193	3	2
6000	7	99	2	2
6500	4	39	2	2
7000	5	22	1	1
7500	8	12	1	1
8000	3	7	1	1
8500	2	4	1	1
9000	1	3	1	1

Legend:

LG Normal Lerchs-Grossmann

LGS Subset Lerchs-Grossmann

LLPQ Lowest Label Pseudo Flow (priority queue)

HLPQ Highest Label Pseudo Flow (priority queue)

There are also drawbacks to this method. Sometimes there is a pocket of rich ore at depth that is sufficiently rich that even with large volume penalties it would be part of the optimal volume pit. It may not be desirable or even practical to actually mine a narrow cone to such a depth for early stage pits. In a case like this the optimisation can also be limited to a specific bench. The volume mined may then be less for that penalty value but would still be an optimal pit to the actual level and volume mined. Another method is to apply an increasing penalty with depth, but this has the disadvantage that the pit is not now optimal for the volume mined. Any additional constraints limit the solution. See Ramazan (1996) for further discussion of these methods and drawbacks.

The volume penalty stage pits generated are of maximum value for the volume mined and form a sequence of nested pits. Stage pits for one-year, two-year, five-year, etc targets can be selected from this sequence. In practice a range of penalty pits is generated and tonnage and grade tables produced for each pit. The penalty pits that bracket specific targets are determined and a new set of penalty pits with finer penalty increments over the desired ranges can be generated. For Case 3, as shown in Table 4, the highest label times for penalty pits are quite small. A set of nested stage pits with a finer increment could be generated in one set, say 180 pits with penalties ranging from 500 to 9500 in steps of 100 would take less than 30 minutes. A subset of these pits can be selected as a set of stage pits.

An interesting result particularly for the LG and lowest label methods is that penalty pits may take longer to optimise than an ultimate pit. A common practice for the LG methods was to start with the smaller highest penalty pit. Subsequent penalty pit optimisations with smaller penalty would then use the previous penalty pit as a starting pit and the ultimate pit (if already determined) as a limiting pit. That method is another example of subproblem decomposition and can save time for large problems. This has not been done here so that actual timings for full optimisation of penalty pits can be illustrated. These results show that the time for optimisation depends on the distribution of block values as well as the total number of blocks and arcs.

Table 4 displays the performance timings for a range of penalties using four methods. These timings are for the 200 × 160 by 55 bench Case 3 profit matrix.

CONCLUSIONS

The LG generic algorithm is pseudo-polynomial (Hochbaum, 1996), but for the practical cases studied here is more of the order of ($m\ n^2$) as shown in Figures 3 and 4. The highest and lowest label *pseudoflow* priority queue variants are of the order of ($m\ n^2$) or ($m\ n \log n$) if dynamic trees (Sleator and Tarjan, 1983) are used. Here n is the number of blocks (nodes) and m is the number of arcs (slope constraints). In Case 1 we have m=13n and in Case 2 we have m=81n. The last n or log n for the order of the methods is related to the maximum length of the chain C_m for the merger and pruning step. In the cases studied here the maximum lengths <1000 <<n. In Figures 1 and 2, the priority queue methods show this small chain length by having times more of the order n^2. Sleator and Tarjan Dynamic trees were not used in these implementations since the chain C_m lengths were relatively small. This empirical study and theory show that the *pseudoflow* variants become increasingly more efficient than the generic LG algorithm as the number of blocks increases. In practice this means that larger models can now be economically processed.

In Cases 2, 3 and numerous other trial runs (not included here) the lowest label method times vary more than the highest label times. The highest label times are usually shorter, more consistent and normally increase with problem size. The performance of the generic LG methods are more like the lowest label results but are generally slower, but as shown in Case 3 can be faster.

Pseudoflow methods give new life to the Lerchs-Grossmann pit optimisation. The highest label method in particular is consistently faster than the generic LG methods and usually faster than the lowest label method. The increase in speed can be from two to 50 times faster than the LG methods, and theoretically much faster for larger problems.

The highest label method can also be used for the fast generation of intermediate stage pits. These stage pits optimal for the volume mined can form a starting point for NPV or Fundamental Tree scheduling techniques.

The *pseudoflow* methods can also be applied to more general network flow problems. The network flow formulation of Johnson (1968) would be an interesting study. In the 1970s this method was only capable of solving small problems due to memory, processor constraints and available algorithms.

Another interesting study would be the use of the normalised strong trees in the LG and *pseudoflow* optimised stage pits to generate the Fundamental Trees defined by Ramazan (2001) and used by Johnson, Dagdelen and Ramazan (2002).

ACKNOWLEDGEMENTS

The author thanks Dr Dorit Hochbaum for her encouragement and correspondence on the LG and *pseudoflow* algorithms. Also thanks to Dr Roussos Dimitrakopoulos for his encouragement to write this paper and to the referees for additional references, thoughtful and simplifying comments and numerous corrections.

REFERENCES

Anderson, C, 2001. Pseudoflow solvers for the maximum flow problems, UC Berkeley manuscript.

Borgman, L E, 1968. A computer solution to the three-dimensional optimum design of open pit mines, Documentation Manual, University of California, Berkeley (Proprietary to Chapman, Wood and Griswold Ltd, Placer Developments Ltd and Brenda Mines Ltd).

Chen, T, 1976. 3-D pit design with variable wall slope capabilities, in *Proceedings 14th APCOM,* pp 615-618 (Pennsylvania State University: Pennsylvania).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, Springer – SME, 350 p, Dordrecht.

Dimitrakopoulos, R, Farrelly, C and Godoy, M C, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open pit mine design, *Trans Inst Min Metall (Section A),* 111:A82-A89.

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Trans,* 316:106-112.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Trans,* 316:43-50.

Goldberg, A V, 1985. A new max-flow algorithm, Tech report MIT/LCS/TM-291, MIT, Cambridge MA.

Goldfarb, D and Chen, W, 1997. On strongly polynomial dual algorithms for the maximum flow problem, *Special Issue of Mathematical Programming B*, 78(2):159-168.

Hanson, N, 1997. Towards a broader view of cut-off grade in resource estimation and reserve reporting, in *Resource to Reserve Inputs Seminar*, pp 11-16 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hochbaum, D S, 1996. A new-old algorithm for minimal cut on closure graphs, UC Berkeley manuscript, June.

Hochbaum, D S, 1997. The Pseudoflow algorithm: a new algorithm and a new simplex algorithm for the maximum flow problem, UC Berkeley manuscript, April.

Hochbaum, D S, 2001. A new-old algorithm for minimum cut and maximum-flow in closure graphs, *Networks, s*pecial 30th anniversary paper, 37(4):171-193.

Hochbaum, D S, 2002. The Pseudoflow algorithm: a new algorithm for the maximum flow problem, UC Berkeley manuscript, December.

Hochbaum, D S and Chen, A, 2000. Performance analysis and best implementations of old and new algorithms for the open-pit mining problem, *Operations Research*, 48(6):894-914.

Johnson, T B, 1968. Optimum open pit mine production scheduling, PhD thesis, Mining Engineering Department, University of California, Berkeley.

Johnson, T B, Dagdelen, K, Ramazan, S, 2002. Open pit mine scheduling based on fundamental tree algorithm, in *Proceedings APCOM 2002*.

Lane, K F, 1988. *The Economic Definition of Ore* (Mining Journal Books Limited: London).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open pit mines, Joint CORS and ORSA Conference, Montreal, May, in *Transactions CIM*, pp 17-24.

Lipekewich, M and Borgman, L, 1969. Two- and three-dimensional pit design optimization techniques, in *Proceedings 1969 International Symposium on Computer Applications and Operations Research in the Mineral Industry – A Decade of Digital Computing in the Mineral Industry* (ed: A Weiss) (AIME).

Maxipit, 2007. LG Mine Design program by Earthworks. <http://www.earthworks.com.au>.

Milner, T E, 1977. Long-range mine planning at Gibraltar Mines Limited, *The CIM Bulletin*, Oct:889-894.

MineMax, 2007. Push-Relabel Mine Design program, by MineMax Pty Ltd. <http://www.minemax.com>.

Muir, D C, 1972. Open pit design program, Documentation Manual, Proprietary Chapman, Wood and Griswold Ltd.

Muir, D C, 2004. LG and Pseudoflow Open Pit Design Program. Email: dmuir@aebc.com.

Ramazan, S, 1996. A new pushback design algorithm in open pit mining, MSc thesis, Colorado School of Mines, Golden, Colorado.

Ramazan, S, 2001. Open pit mine scheduling based on fundamental tree algorithm, PhD thesis, Colorado School of Mines, Golden, Colorado, p 164.

Ramazan, S, 2007. Large-scale production scheduling with the fundamental tree algorithm — Model, case study and comparisons, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 121-127 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Sleator, D D and Tarjan, R E, 1983. A data structure for dynamic trees, *Journal of Computer and System Sciences*, 24:362-391.

Wharton, C L, 1996. Optimization of cut-off grades for increased profitability, in *Proceedings Surface Mining 1996*, pp 101-104 (South African Institute of Mining and Metallurgy: Johannesburg).

Whittle Programming Pty Ltd, 2007. LG Mine Design Program. <http://www.whittle.com.au>.

Zhao, Y and Kim, Y C, 1992. A new optimum pit design algorithm, in *Proceedings APCOM 1992*, pp 423-434 (The University of Arizona).

Large-Scale Production Scheduling with the Fundamental Tree Algorithm — Model, Case Study and Comparisons

S Ramazan[1]

ABSTRACT

Mathematical programming models are theoretically well suited to optimising long-term production scheduling of open pit mine designs; however, in most cases to date they have not been able to solve the scheduling problem because it is too large in size, especially with respect to the number of integer variables required. Because of this size problem, scheduling is done using larger blocks, usually formed by aggregating many mining blocks on the same bench, which may vary from tens to thousands in number. However, the simple traditional way of re-blocking neighbouring blocks into larger blocks causes difficulties in establishing the mine slope requirement and generating an optimal schedule in terms of maximising the total profit from the mine.

The fundamental tree algorithm, based on linear programming (LP), has been developed to properly aggregate the blocks and reduce the problem size in formulating a mixed integer programming (MIP) model for optimising the production scheduling. This paper presents the fundamental tree algorithm and discusses how it substantially reduces the number of binary variables required in formulating the production scheduling problem as an MIP model and the number of constraints within the MIP. A case study on a multi-variable large copper mine with dual ore processors shows that the proposed method significantly increases the total expected discounted profit from the operation. The application of MIP for optimisation in large open pit mines is often considered to be impossible; the proposed algorithm makes it possible by reducing the problem size significantly.

INTRODUCTION

It is common practice in open pit design for cutbacks to serve as a guide in the scheduling process after the ultimate pit limits are determined. Detailed descriptions of the methods for finding the ultimate pit limits are provided in Hochbaum and Chen (2000). Some of the commonly used cutback design methods are discussed in Ramazan (1996), Seymour (1995) and Whittle (1988). Traditionally, a set of volumes of material that has the specific attributes suitable for the annual production is identified as a feasible solution to be mined each year within these cutbacks. The current scheduling practice is mostly finding more than one feasible solution and choosing the best one among these feasible solutions. There is no method available in open pit mine planning and production scheduling to generate the optimum solution in maximising net present value (NPV) for any given mine data.

Although linear programming (LP) type mathematical models are commonly accepted to be powerful tools in the optimisation of production scheduling in open pit mining, there is no LP or mixed integer programming (MIP) model that can be used to solve production scheduling of any type of large open pit mine. The large open pit mines require a large MIP model, which creates difficulties in solving mathematical formulations even with today's supercomputing systems, which have multiple parallel processors. For example, if there are 5000 mining blocks in a small cutback to be scheduled over three years, it will require 15 000 binary variables to generate the MIP formulation. Getting a solution for an MIP model of this size is still very challenging, even for a small cutback, and impossible for most real data sets.

There have been several attempts to develop and apply MIP/LP type models in optimising annual production scheduling in mining operations. Johnson (1968) developed an LP scheduling model and applied Dantzig-Wolfe (1960) decomposition principles to decompose the model and apply the maximum network flow (maxflow) algorithm developed by Johnson (1968). However, this LP approach uses linear variables and leads to the mining of fractional blocks. Dagdelen (1985) used the Lagrangian decomposition method to decompose and solve a large MIP problem. The drawback of the approach is that the Lagrangian method might not always converge to an optimum solution if the Lagrange multipliers cannot result in a feasible solution. Gershon (1983) presented an LP approach together with MIP models for optimising mine scheduling. The author suggests that the models for optimising production scheduling of open pit mines require too many binary variables and cannot be solved. Alternative efficient methods for long-term production scheduling are presented in Ramazan and Dimitrakopoulos (2004). However, the reductions associated with these may not be sufficient for some large open pit mines, although they are very effective for many cases. Tolwinski (1998) proposes a method that combines the blocks on the same bench, termed 'atoms', and generates a production schedule using dynamic programming. However, combining blocks into atoms may significantly disturb any possibility of achieving the optimal solution, depending on the size of the atoms, which is not mentioned. The Milawa algorithm discussed in Whittle (2000) considers a few benches at each cutback as a variable and uses a search technique called the 'step and stride' algorithm, discussed in Wharton (2000), to identify the regions of high value, rather than identifying individual mining blocks. This is a heuristic approach and doesn't guarantee an optimal solution. Godoy and Dimitrakopoulos (2004) applied a simulated annealing optimisation method for scheduling a large gold mine. Although the method seems promising, it doesn't explicitly include grade blending constraints in its model, limiting its application for mines with blending problems. Dimitrakopoulos and Ramazan (2004) developed an LP model that considers maximising the chance of meeting grade blending requirements and the feasibility of mining operations providing equipment access to the blocks in the objective function rather than NPV maximisation in the objective function. The LP model application in a laterite nickel-cobalt mine shows successful results in preventing fractional mining of a block over multiple time periods and the resultant production schedule is shown to have a better chance of satisfying the blending constraints. This LP model needs more testing in terms of satisfying the sequencing constraints for open pit mines that have significant depth, or multiple blocks vertically. Similar approaches are presented in Dimitrakopoulos (in press). Topal *et al* (2003) developed a methodology to reduce the number of binary variables in optimising long-term scheduling at LKAB's Kiruna underground mine using aggregation of blocks on the same machine production. However the paper does not provide a method for variable reduction or improved efficiency for open pit mines.

In this paper, the fundamental tree (FT) algorithm is presented. The algorithm combines the ore blocks with their overlying waste blocks and some other ore blocks only when such combination is necessary to support the cost of mining the

1. MAusIMM, Rio Tinto, GPO Box A42, Perth WA 6000, Australia. Email: salih.ramazan@riotinto.com

overlying waste blocks. The method prevents unnecessary aggregation of blocks, minimising errors in achieving slope sequencing, and also keeps the resolution of the original data by not averaging out the values of too many combined blocks. The structure of the algorithm is established in a way that makes the high economic value aggregates feasible to mine earlier than the lower aggregates that has substantial impact in generating higher NPV values. Since this algorithm generates many fundamental trees for a given deposit model, it gives the MIP scheduling optimiser the opportunity to be able to generate optimal results. The algorithm reduces the number of binary variables required for the MIP formulation of long-term production scheduling. Since the FT algorithm uses only linear variables in the mathematical model, it is extremely efficient for large deposits. A set of combined blocks is called a 'fundamental tree' if the combination of blocks has three properties:

1. it can be mined without violating the slope constraints,
2. it has a positive total economic value, and
3. it cannot be partitioned into smaller trees without violating 1 and 2.

The function of these three properties in generating an optimal result is discussed in this paper. The LP formulation is an optimal model in terms of generating the fundamental trees with the defined properties as discussed in this paper. After generating fundamental trees, an MIP model is used to optimise the production scheduling based on the fundamental trees instead of blocks.

THE FUNDAMENTAL TREE ALGORITHM

The FT algorithm is applied to the blocks within a cutback that has to be determined using one of the true optimising methods such as Lerchs and Grossmann's method (1965) as implemented by Whittle (1988), or the maxflow algorithm of Johnson (1968). If the cutback is designed using a heuristic method such as implementations of the floating cone method by Lemieux (1979), as presented in Ramazan (1996) or Wang and Sevim (1993), the LP model formulations would be infeasible due to Equation (3).

Steps of the FT algorithm

The FT algorithm is implemented in seven steps as discussed below:

Step 1. Generate a network for the blocks within a pit. The arcs in the network represent the node (block) precedence relationship within the pit. An arc is set from each positive value node to all the overlying negative value nodes on the upper levels that have to be mined before mining the positive value node.

Step 2. Determine the cone value CV_i for each node i having a positive value within the network. The economic values of all the nodes connected to node i with an arc set from node i are summed and referred to as the cone value.

Step 3. Rank the positive value nodes according to their cone value starting from the highest to lowest and starting from the topmost bench (or level) where at least one positive value node exists and moving toward the bottom bench. On the topmost level where more than one positive value node exists, the node with the highest cone value is ranked as 1, and the second highest cone value node is assigned to rank 2, and so on. Then, the ranking process moves one level down. If there are positive value nodes on that level, the node with the highest cone value is assigned to 1+ (the previous largest rank). Otherwise, a lower level is searched for positive value nodes. If two or more positive value nodes on the same level have the same cone value, the ranks may be assigned randomly, and two nodes should not be assigned to the same rank.

Step 4. Set up the LP formulation as discussed later in this paper, using the ranks in Step 3 as coefficients for the objective function. After the problem formulation is ready, it can be solved using one of the solvers available in the market.

Step 5. Calculate the number of trees generated by the LP model. Initially, it is assumed that whole network is one tree. If the number of trees obtained from the current solution is higher than the previous solution, keep the currently found arcs between nodes and go to Step 6 to generate a new network to be used for iterating the algorithm. If the number of trees obtained is the same as the number of the trees obtained from the previous solution, go to Step 7; and the problem is considered to be at optimal solution. Usually, two or three iterations are required for convergence.

Step 6. A network is formed by keeping only the arcs that are between the nodes within the same tree. The arcs that have no flow are first deleted from the network as discussed later in this paper. However, all the arcs that exist between negative value nodes and positive value nodes within the initial network must also exist within the starting network for the nodes that are in the same tree. So if an arc is deleted between two nodes that belong to the same tree, it is re-added. Go to Step 2.

Step 7. Stop.

In Step 5, some arcs are deleted from the network when there is no flow on them. This arc deletion leaves the network with the positive value nodes that are connected to the overlying negative value nodes and these negative value nodes can be supported by only the connected positive value nodes. If some of the connected nodes can be partitioned without violating the slope requirement and the positiveness of the total economic value, the partition will occur during iteration process. When there is no partitioning in a solution, the algorithm will terminate.

Illustration of the steps of the FT algorithm

The hypothetical two-dimensional block model given in Figure 1 is used to show the steps of the algorithm. The node numbers are written on the bottom-right of each block and the expected economic value from each block is written at the centre.

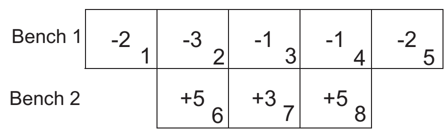


FIG 1 - A hypothetical example block model in two-dimensional section view.

Step 1. The block model is represented with an initial network as shown in Figure 2. A block is called a node in a network form. Since nodes 6, 7 and 8 have positive economic values, the arcs are set from these nodes to the nodes on the upper bench. It is assumed that the blocks are the same size and have to be mined with a 45-degree slope angle in all directions, which is represented by the arcs.

Step 2. If the economic value of node i is V_i, then $CV_6 = V_6+V_1+V_2+V_3 =+5-2-3-1$. That is, $CV_6=-1$. Similarly, $CV_7 =-2$ and $CV_8=+1$.

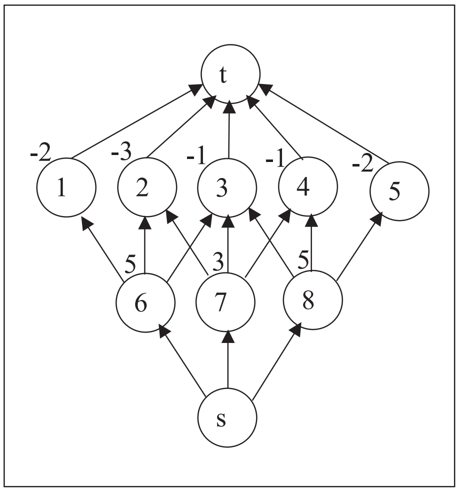


FIG 2 - Initial network for the example block model.

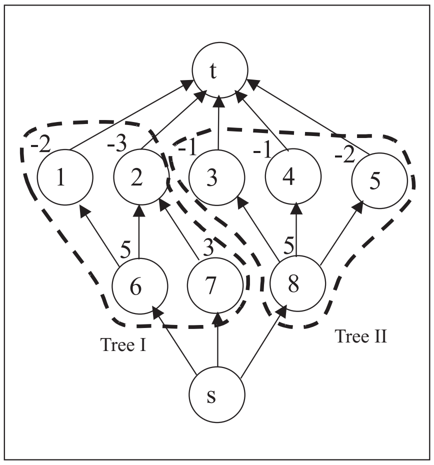


FIG 4 - The network model containing two trees (dashed lines) generated from the initial solution to be used for setting up the iterative LP model.

Step 3. Ranks C_i are assigned to positive value nodes in order of their CV_i value and the levels where nodes are located. Since CV_8 is greater than CV_6 and CV_7, C_8 is set to 1, C_6 is set to 2 and C_7 is 3.

Step 4. The problem solution is given in Figure 3 and the starting network for the next iteration in Figure 4. Figure 4 is generated by deleting the arcs with no flow from the previous network. Note that all the arcs that exist between negative value nodes and positive value nodes within the initial network are kept for the nodes that belong to the same tree. The LP formulation and the figure are explained later in this paper.

Step 5. Two trees are identified from the network in Figure 4. Since there are two trees at the current solution, which is greater than the previous one tree shown in Figure 1, go to Step 6. It should be noted that the tree sequencing is to be done by the MIP model, so the numbering of trees does not refer to the sequencing.

Step 6. A new network is generated as shown in Figure 4 and this is used to make the next iteration. The algorithm now moves to Step 2.

Minimise $2f_{61}+2f_{62}+2f_{63}+3f_{72}+3f_{73}+3f_{74}+f_{83}+f_{84}+f_{85}$

Subject To

$f_{s6}<=5$
$f_{s7}<=3$
$f_{s8}<=5$
$f_{1t}=2.001$
$f_{2t}=3.001$
$f_{3t}=1.001$
$f_{4t}=1.001$
$f_{5t}=2.001$
$f_{61}-f_{1t}=0$
$f_{62}+f_{72}-f_{2t}=0$
$f_{63}+f_{73}+f_{83}-f_{3t}=0$
$f_{74}+f_{84}-f_{4t}=0$
$f_{85}-f_{5t}=0$
$f_{s6}-f_{61}-f_{62}-f_{63}=0$
$f_{s7}-f_{72}-f_{73}-f_{74}=0$
$f_{s8}-f_{83}-f_{84}-f_{85}=0$

Variable	Value
f_{S6}	5.00000
f_{S7}	0.00200
f_{S8}	4.00300
f_{1t}	2.00100
f_{2t}	3.00100
f_{3t}	1.00100
f_{4t}	1.00100
f_{5t}	2.00100
f_{61}	2.00100
f_{62}	2.99900
f_{63}	0.00000
f_{72}	0.00200
f_{73}	0.00000
f_{74}	0.00000
f_{83}	1.00100
f_{84}	1.00100
f_{85}	2.00100

FIG 3 - The proposed LP model formulation at the initial stage and the solution.

Step 2. The cone value CV_6 is now 0 (+5-2-3), CV_7 is 0 and CV_8 is +1.

Step 3. The ranks are determined using CV_i values. C_6 is set to 2, C_7 is set to 3 and C_8 is 1. C_6 and C_7 could also be set as C_6 = 3 and C_7 = 2, which wouldn't make any difference for the optimisation.

Step 4. The iterated LP model and its solution are given in Figure 5 with the current network configuration in Figure 6. Note that the tree having only one positive value node can be excluded from the iterative formulations; it is kept here only for illustration purposes.

Step 5. Two fundamental trees are identified at the current solution from the network in Figure 6. Since the number of trees from the current solution is the same as previous solution, the algorithm moves to Step 7.

Step 7. Stop the algorithm.

The final network generated after solving the iterative LP model contains two trees, which are referred to as fundamental trees. It is clear that these trees have the pre-defined three properties of fundamental trees. First, they can be mined without violating the slope requirement, by first excavating Tree II and then, Tree I. Second, each of the trees has a positive economic value: one has +1 and the other +3 value. Third, these two

Minimise $2f_{61}+2f_{62}+3f_{72}+f_{83}+f_{84}+f_{85}$

Subject To

$f_{s6}<=5$
$f_{s7}<=3$
$f_{s8}<=5$
$f_{1t}=2.001$
$f_{2t}=3.001$
$f_{3t}=1.001$
$f_{4t}=1.001$
$f_{5t}=2.001$
$f_{61}-f_{1t}=0$
$f_{62}-f_{2t}=0$
$f_{63}-f_{3t}=0$
$f_{74}+f_{84}-f_{4t}=0$
$f_{85}-f_{5t}=0$
$f_{s6}-f_{61}-f_{62}-f_{63}=0$
$f_{s7}-f_{74}=0$
$f_{s8}-f_{84}-f_{85}=0$

Variable	Value
f_{S6}	5.00000
f_{S7}	0.00200
f_{S8}	4.00300
f_{1t}	2.00100
f_{2t}	3.00100
f_{3t}	1.00100
f_{4t}	1.00100
f_{5t}	2.00100
f_{61}	2.00100
f_{62}	2.99900
f_{72}	0.00200
f_{83}	1.00100
f_{84}	1.00100
f_{85}	2.00100

FIG 5 - The iterative LP formulation and the solution.

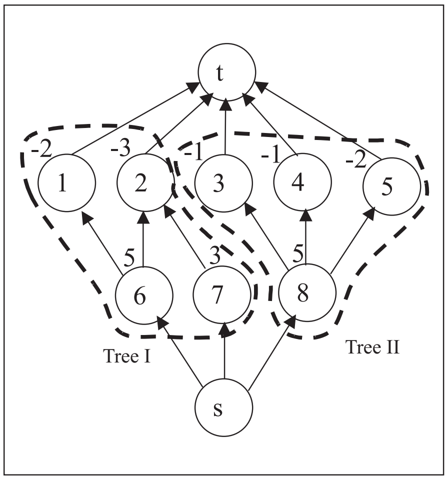


FIG 6 - The final network formed, indicating two fundamental trees.

fundamental trees cannot be divided into sub-trees without violating the first and second properties. Splitting Tree I (as nodes 1 and 6 and nodes 2 and 7) without disturbing the slope would leave at least one of the trees (nodes 2 and 7) without a positive value, violating the second property. This small example illustrates that the LP model successfully finds the fundamental trees. These properties are further discussed in Ramazan (2001).

THE LP FORMULATIONS FOR GENERATING FUNDAMENTAL TREES

This section discusses the LP formulation of the fundamental tree (FT) algorithm as presented in the previous section. The objective function of the model is the minimisation of arc connections in the network weighted by the assigned ranks. The objective function is expressed as:

$$\text{Min} \sum_{p}^{N} \sum_{j}^{W} R_p f_{pj} \tag{1}$$

where:

R_p is the rank for a positive value node p

N is the total number of positive value nodes

$f_{p,j}$ is the flow sent from node p to node j

After the LP model is completed and solved, if there is a flow going through the arc, it is kept in the next network. If there is no flow going through the arc, it is deleted from the network either permanently, or temporarily to identify the connected nodes. Thus, the objective function is the minimisation of the connections between blocks, considering the assigned ranks and the constraints. It should be noted that the intention is not really the absolute minimisation of the existing arc connections. The purpose of this LP model is to find the fundamental trees with the defined properties, rather than to minimise the connections.

In a given bench, the model considers that the highest cone value node, say node p, has the highest chance of being able to support all the negative value nodes on the benches above preventing node p from being mined. Therefore, if the arcs are constructed from the highest cone value node (lowest rank), the number of joint supports for negative value nodes will be minimised, considering the model constraints. This ranking in the objective function, together with the model constraints, will mainly establish the third property of a fundamental tree, which is that it cannot be partitioned into sub-trees. The coefficients also have some role in making the fundamental trees obey slope constraints, although it is not as direct as in Equation (3). Since the arc connections are prioritised from higher cone value nodes, the fundamental trees are generated in a way such that higher value blocks become feasible for mining before the lower value blocks for the MIP scheduling model. This is a desirable condition, although not a pre-requisite for NPV maximisation during the annual production schedule, because this LP model generates enough small trees for MIP to be able to aggregate them in an optimal way.

A positive value node is constrained in a way that it cannot support a higher cost of waste stripping than the expected revenue from this block. The constraint formulation is expressed as below:

$$f_{si} \leq V_p, \text{ for all p's.} \tag{2}$$

where:

f_{sp} is the flow sent from source node s to node p

v_p is the economic value of block p, which is set for only positive value nodes.

The costs of mining negative value blocks are forced to be paid by the ore blocks whose accesses are restricted by the negative value blocks. A small extra value ξ is also forced to be sent to negative value nodes to ensure that the precedence relationship will not be violated by the trees. For example, if there are three blocks a, b and c to be supported by block d, ξ avoids the situation where blocks a and b will be fully supported by block d and block d will not have any more value to make any support for c. By avoiding this situation and requiring extra support by block b if the value of block d is consumed, connected aggregates become feasible in terms of slope. It should also be noted that objective function coefficient also has a role in achieving slope since it will be always better to send flows from the same block until all the values in that block is used.

A very small number, such as 0.001, is given to ξ so that it will not be ignored by the solver. These constraints also ensure that the minimum economic value of a tree is greater than or equal to ξ, which is strictly positive. This is the first pre-defined property of a fundamental tree. Without using the ξ value, if an overlying negative node is fully supported by an underlying positive value node, the total value of negative and positive nodes could be zero, without requiring a joint support. That would not only generate zero value trees, violating the defined property of the FT algorithm for being strictly positive, but also cause violation in the slope requirements. It is by definition that negative value aggregates are not allowed in the model. If one allows a negative value aggregate as a tree, the current block model (mining blocks) is the optimal result in terms of not being able to divide it into smaller trees and that wouldn't have any benefit for reducing binary variables required in MIP optimisation model.

Since the economic values in mining are sufficiently large, thousands in magnitude, to be approximated to integer values, the total of the added ξ values for all the overlying connected nodes should be kept below 1. Otherwise, some trees may violate the last pre-defined property of a fundamental tree; that is, one or more trees may be partitioned into sub-trees having the first two pre-defined properties of fundamental trees if the ξ value is set too high. This constraint formulation is expressed as:

$$f_{jt} = -V_j + \xi, \tag{3}$$

where:

ξ is a small positive decimal number

V_j is the value of the negative value node j

t is the sink node

The flow balance must be constructed around negative value nodes. If the number of positive value nodes from which arcs are set towards the negative value node j is NP_j, then the mass balance constraint around each negative value node is expressed as:

$$\sum_{p=1}^{NP_j} f_{pj} - f_{jt} = 0 \tag{4}$$

The flow balance around each positive value node must be established. If the number of waste blocks overlying positive node p is W_p, the mass balance constraints for the positive value nodes are expressed as:

$$fsp - \sum_{j=1}^{W_p} f_{pj} = 0 \tag{5}$$

The initial LP formulation and solution for the example network model given in Figure 2 are illustrated in Figure 3. Figure 4 is generated by deleting the arcs that are not used by the LP model from the initial network. The iterative LP formulation is generated using the current network of the system as stated earlier in this paper. It should be noted that the number of binary variables required to formulate the MIP model for scheduling this small deposit model is only two (a binary for each fundamental tree) for each period instead of eight (a binary for each block).

After generating the fundamental trees for a given orebody model, the annual production scheduling can be formulated as an MIP model treating each tree as a block having certain attributes. MIP formulations for optimising long-term production scheduling can be found in Ramazan (2001) and Ramazan and Dimitrakopoulos (2004).

A CASE STUDY

The FT algorithm is tested in an application on a large copper mine containing sulfide ore (milling ore), oxide ore (leaching ore), gold and silver in South America. The mine can consider processing 17 000 tonnes of sulfide material per day at the mill and 3.5 million tonnes of oxide ore per year by the leaching process.

The orebody model representing the deposit contained 871 875 blocks with dimensions 20 m by 20 m by 10 m. For this deposit model, four cutbacks are generated using the Whittle 3D program that uses the L-G method (Lerchs and Grossmann, 1965) to find the nested pits. The number of ore and waste blocks, and tonnages of sulfide ore (SO), oxide ore (OO), and waste within each cutback (CB) before haul roads are designed are given in Table 1. In the table, the blocks that have positive values are considered to be ore for finding the fundamental trees, but the ore-waste classification is based on cut-off grade for production scheduling.

TABLE 1

Tonnages for sulfide ore (SO), oxide ore (OO) and waste within each cutback (CB) and number of blocks.

	Tonnage (million tons)				Number of blocks		
CB No	SO	OO	Waste	Total	Ore	Waste	Total
1	3.43	8.56	28.90	40.90	2100	2582	4682
2	9.85	5.78	66.70	82.33	2349	6712	9061
3	19.64	2.26	99.49	121.39	2844	9739	12 583
4	11.68	0.36	104.88	116.92	1457	10 674	12 131
Total	44.60	16.96	299.97	361.53	8750	29 707	38 457

The LP model information and number of fundamental trees found individually for each cutback are given in Table 2. Initially the total number of blocks requiring an integer variable for each scheduling period (last period may be excluded) was 38 457, which is almost impossible to optimise through an MIP model. The FT algorithm reduced this number of blocks down to 5512 fundamental trees. The number of fundamental trees in a deposit depends on the economic value of the ore blocks and the cost of mining the overlying waste material. This dependency appears in Table 2 such that as the deposit becomes deeper, towards cutbacks 3 and 4, the ratio of the fundamental trees to ore blocks decreases.

TABLE 2

LP model and fundamental tree information within cutbacks.

	CB1	CB2	CB3	CB4	Total
LP information for the first iteration					
Constraints	50 884	143 945	337 481	629 595	
Variables	87 722	260 707	637 213	1 222 797	
Objective non-zeros	41 520	125 823	312 315	605 333	
Fundamental tree numbers					
Iteration 1	1883	1644	1624	321	
Iteration 2	1883	1661	1640	328	
Iteration 3		1661	1640	328	5512
The ratio of FTs to ore blocks	0.90	0.71	0.58	0.22	

Although the LP formulation was very large, the solution time was always less than five seconds on a PC with 600 MHz processor for the first iteration. Iterated formulations are much smaller in size and they were solved almost instantly. Since the MIP model for optimising open pit production scheduling needs binary variables, four of the cutbacks are scheduled one at a time to keep the number of binary variables at a low level so that the MIP model could be solved.

Table 3 provides details of the MIP scheduling model and shows that the largest MIP model is set for the third cutback, which contains 4920 binary variables, 13 557 linear variables, and over 41 000 constraints. The problem is stopped when the integer solution reached a 5.5 per cent gap.

TABLE 3

MIP model information for the copper deposit using a 600 MHz PC.

	PB1	PB2	PB3	PB4
Constraints	5719	10 158	41 256	3171
Variables – Linear	5711	10 459	13 557	1335
Variables – Binary	-	3322	4920	328
Variables – Total	5711	13 781	18 477	1663
Objective non-zeros	2735	8433	10 108	996
Percent optimality (%)	100.00	99.3	94.5	100.00
Run time – hr:min:sec	00:00:01	00:04:40	00:36:24	00:00:04

The scheduling results are summarised in Table 4. Since the MIP model considers both mill process and leaching in the optimisation, leaching is performed with full capacity for the first four years of production with a significant contribution to overall profit. The mill is fed with more or less the same ore tonnage, with some variation in grade until the ore is depleted towards the end. The results shown in Table 4 are produced after the designing of haul roads and smoothing of pits necessary for practical operation. Figure 7 shows the plan view of the deposit with access roads at the end of the mine's life.

TABLE 4

Summary results of the annual production schedules (tonnages are in 1000 tons and grades are in per cent).

Years	Sulfide		Oxide				Waste	
	Mill		Leach pad		Stockpile		Dump tons	Total tons
	Tons	Grade	Tons	Grade	Tons	Grade		
1	6258	1.451	3500	1.316	6533	1.316	58 654	68 412
2	6121	1.461	3500	1.268	4929	1.316	67 018	76 639
3	6212	1.492	3500	1.108	4111	1.316	66 395	76 107
4	6036	1.543	3500	1.298	961	1.316	68 573	78 109
5	6134	1.387	2093	1.098			67 414	75 641
6	6325	1.580	487	0.827			67 090	73 902
7	6277	1.951	316	0.889			52 744	59 337
8	1197	2.117	51	0.570			9368	10 616
Total	44 560	1.568	16 947	1.209			457 256	518 763

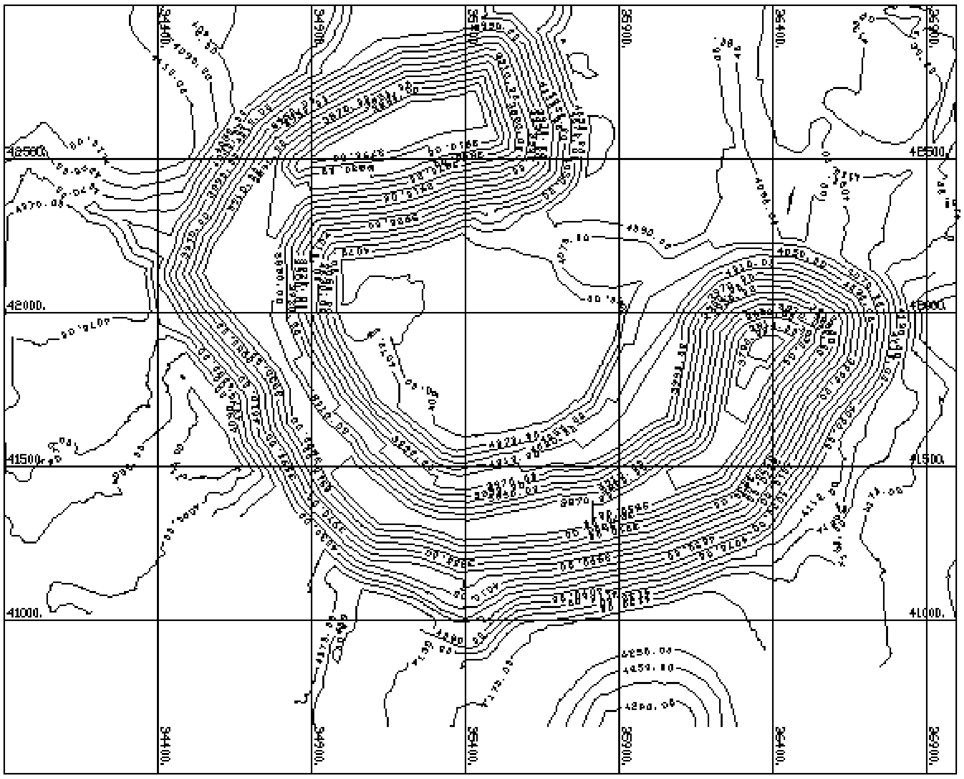


FIG 7 - Plan view of the deposit with access roads at the end of the mine's life.

The mine was also scheduled using MINTEC's M821V18, Earthworks' NPV scheduler19, and Whittle's Milawa mine scheduling programs using the same constraints as the MIP scheduler using the FT algorithm. Detailed descriptions of the scheduling process are presented in Bernabe (2001).

The total undiscounted dollar value of the deposit varies within a narrow range for the different methods. It is around $610.8 million in the NPV scheduler, $612.6 million in the MIP scheduler with FT algorithm, $614.6 million in the Whittle Milawa and $620.2 million in the M821V. The discounted cash flows are calculated at ten per cent rate for each scheduling technique. The discounted total NPV values generated by the three traditional methods are similar to each other, around $400 million. However, the total NPV generated by the MIP scheduler with FT algorithm is $22.2 million more than the Milawa scheduler in the Whittle Four-X program, which resulted in the highest NPV among the three traditional methods, and about $29.5 million more than the NPV scheduler. This refers to about seven per cent higher NPV than the available software package producing the highest NPV. The higher NPV in this specific case study occurs mainly because the MIP model was flexible in terms of being able to consider dual processors at the same time and because the overall scheduling process using the FT algorithm doesn't have the simplifying assumptions of traditional models, such as aggregating the neighbouring blocks on the same bench.

CONCLUSIONS

In this paper, a fundamental tree (FT) algorithm based on a linear programming method using only linear variables has been presented. This method successfully generates fundamental trees, identified by three defined properties. The defined properties prevent the aggregated blocks from losing their optimality for the MIP scheduler. An ore block is aggregated only with the overlying waste blocks that must be extracted before being able to extract the ore block and with another ore block only if it is necessary to support the mining of overlying waste blocks. Because the number of fundamental trees generated by aggregating the blocks is sufficiently large for the MIP model to choose the best combination for an annual schedule, the MIP model can achieve optimality. As well, the MIP-type mathematical models are powerful for solving difficult blending problems. Since the FT algorithm reduces the number of binary requirements and the model size significantly, it is possible to apply the MIP model to produce correct blending requirements in operations requiring some level of accuracy in the blending of grade and ore quality elements, such as iron ore, copper and nickel deposits.

ACKNOWLEDGEMENTS

The author expresses special thanks to Professor Roussos Dimitrakopoulos for editing this paper.

REFERENCES

Bernabe, D, 2001. Comparative analysis of open pit mine scheduling techniques for strategic mine planning of a copper mine in Southern Peru, MSc thesis, Colorado School of Mines, Golden, Colorado, 2001, p 236.

Dagdelen, K, 1985. Optimum multi-period open pit mine production scheduling by Lagrangian parameterisation, PhD thesis, Colorado School of Mines, Golden, Colorado, p 325.

Dantzig, G B and Wolfe, P, 1960. Decomposition principle for linear programs, *Oper Res*, 8(1):101-111.

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions*, 316:106-112.

Gershon, M E, 1983. Optimal mine production scheduling evaluation of large scale mathematical programming approaches, *Int J Mining Engineering*, 1:315-329.

Godoy, M and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Trans*, 316:43-50.

Hochbaum, D S and Chen, A, 2000. Performance analysis and best implementations of old and new algorithms for the open-pit mining problem, *Oper Res*, 48:894-913.

Johnson, T B, 1968. Optimum open pit mine production scheduling, PhD dissertation, University of California, Berkeley, California, p 120.

Lemieux, M, 1979. Moving cone optimizing algorithm, Computer methods for the 80s in the mineral industry (ed: A Weiss), pp 329-345 (SME - AIME).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open-pit mines, *Trans CIM*, LXVII:47-54.

Ramazan, S, 1996. A new push back design algorithm in open pit mining, MSc thesis, Colorado School of Mines, Golden, Colorado, p 138.

Ramazan, S, 2001. Open pit mine scheduling based on fundamental tree algorithm, PhD thesis, Colorado School of Mines, Golden, Colorado, p 164.

Ramazan, S and Dimitrakopoulos, R, 2004. Recent applications of operations research in open pit mining, *SME Transactions*, 316:73-78.

Seymour, F, 1995. Pit limit parameterization from modified 3D Lerchs-Grossmann algorithm, *SME Preprint Number* 95-96.

Tolwinski, B, 1998. Scheduling production for open pit mines, in *Proceedings 27th International APCOM Symposium,* pp 651-662.

Topal, E, Kuchta, M and Newman, A, 2003. Extensions to an efficient optimization model for long-term production planning at LKAB's Kiruna Mine, applications of computers and operations research in minerals industries, in *Proceedings 31st International APCOM Symposium*, pp 289-293.

Wang, Q and Sevim, H, 1993. An alternative to parameterization in finding a series of maximum-metal pits for production planning, in *Proceedings 24th International APCOM Symposium,* pp 168-175.

Wharton, C, 2000. Add value to your mine through improved long term scheduling, in *Proceedings Whittle North American Mine Planning Conference,* Colorado.

Whittle, D, 2000. Proteus environment: sensitivity work made easy, in *Proceedings Whittle North American Mine Planning Conference,* Colorado.

Whittle, J, 1988. Beyond optimisation in open pit design, in *Proceedings Canadian Conference on Computer Applications in the Mineral Industries,* Rotterdam, pp 331-337.

Multi-Mine Better Than Multiple Mines

G Hall[1]

ABSTRACT

It is not uncommon for a number of open cut mines to share infrastructure in the mining value chain. This sharing offers economies of scale and presents additional scheduling options, but also increases the complexity of design and scheduling. How do you best investigate and optimise this type of scenario in order to yield maximum economic benefit?

As a senior developer in the Whittle team, the author has been involved in the creation of a modelling and optimisation system which caters for multiple integrated mines. The objectives of the system design were to provide a process, supported by effective tools, that enables mine planners to maximise the economic benefit of multi-mine operations and to provide a modelling and optimisation environment that allows multiple integrated mines to be planned and scheduled effectively, including adaptation of optimisation engines to the multi-mine situation.

This paper describes the benefit the Whittle Multi-Mine option can bring to such a complex operation and the features that enable that benefit.

INTRODUCTION

It is not uncommon for a number of open cut mines to share infrastructure in the mining value chain. This sharing presents scale economies and presents additional scheduling options, but also increases the complexity of design and scheduling. How do you best investigate and optimise this type of scenario in order to yield maximum economic benefit?

Multiple mines could be treated in Whittle, to a certain extent, before the Whittle Multi-Mine option was introduced. The simplest approach was to model each mine in isolation and then produce a schedule manually. Several authors, Tulp (1997), Whittle (2001) and Desoe (AMDAD), developed techniques that removed some of the restrictions associated with treating multiple mines as a single model within the Whittle environment. None of these processes could entirely remove the restrictions and they all required complex set-up procedures. Within their limits, however, they worked and they all enjoyed success in a restricted number of situations.

The Multi-Mine option allows the flexibility of choice of optimal pit and pushbacks for each mine, independent of the other mines in the model, while still producing a schedule automatically across all mines. Note that there are a few terms that can be used in conflicting ways. To avoid confusion, these terms: 'mine', 'pit', 'shell' and 'operation' are defined in the glossary (see Appendix) along with a more extensive list of terms used in this paper.

MULTIPLE MINES

Background

A multiple mine operation has more than one mine sufficiently close together that they share infrastructure and are planned as a single study. The past approaches to modelling this situation, identified in the introduction, either fail to address the benefits of producing a joint schedule, or limit the definition of the individual mines so that they do not use their optimal pit or pushbacks tailored to that mine.

Planning the mining schedule of a single mine is reasonably well understood. While the process is complex, tools exist to assist the user in creating an optimal open pit shape from a model of an orebody. Tools also exist to assist in planning a mining schedule from the chosen pit. The difficulty in the multiple mine situation in particular, is in defining the best pushbacks and finding the best mining schedule. The Whittle product uses the net present value (NPV) of a mine to drive both the identification of the optimal pit and the creation of the mining schedule.

The optimal pit is found using the Lerchs-Grossmann (LG) algorithm (Lerchs and Grossmann, 1965; Muir, 2007, this volume). The method for determining the optimal pits is the same for a single mine or a set of mines. This, however, is just the start of the solution. The material to be removed from each mine can be extracted in any one of a number of ways, all of which can result in dramatically different NPVs for the mine. The creation of a mining sequence involves defining some useful pushbacks for those mines, then mining those pushbacks in such a way that maximises the potential value of an operation.

The Whittle Multi-Mine solution

The model file of the Multi-Mine option uses a single block model definition, which identifies each mine in the block model separately. You are able to define the pushbacks and choose the optimum pit for each mine separately, then create a schedule that considers all of the mines together. This technique allows each mine to be designed to its full potential because its optimum pit is independent of any other mine. During the scheduling process, however, there are benefits from considering the mines as multiple sources of ore. The scheduler is able to decide when to choose material from the mines such that the value of the operation is maximised.

The key benefits of the Multi-Mine option are that it gives you independence between mines:

- pushbacks can be determined that are ideal for an individual mine,
- the final pit for each mine is separate,
- the order of processing of the mines can be changed easily, and
- mining limits can be tailored for individual mines.

In addition, the material movements in each mine can be tracked separately and extra controls have been added to allow per-mine constraints. By using Whittle to find the theoretical maximum value of the operation, it is possible to cost the decisions that are made along the way as progress is made to a final design for the operation. Sometimes, this means that Whittle presents information that justifies a change in approach because of the increased value that is realisable when that change is implemented.

EXAMPLE CASE STUDY

Example data

The data comes from the 'BlueSky' project and has two mines, called NorthPark (mine 1) and SouthBorder (mine 2). SouthBorder is the standard Marvin mine with three rock types: OX (a surface oxide), MX (a mixed ore) and PM (the primary

1. Whittle Developer Manager, Gemcom Australia Pty Ltd, Suite 36, 574 Whitehorse Road, Mitcham Vic 3132, Australia. Email: geoffh@gemcomsoftware.com

ore). NorthPark has been modified from the original Marvin with the OX and PM rock type tonnages being summed together (called SL) and MX being renamed to RF.

The SouthBorder rock types have their rock type cost adjustment factor (CAF) greater than one to indicate a harder rock than in the original Marvin data.

The slopes of both mines have been modified from the original Marvin and have also been made different from each other.

All rock types have gold and copper elements.

Treat as single mine

Optimal pit

The creation of the optimal pit for each mine proceeds as in the single mine case because the LG algorithm (in a single model file) will treat the mines as independent entities (provided the resultant pits do not touch).

Pushbacks and mine schedule

Before we explore designs for a practical and realistic mine design and schedule, it is useful to look at the operation as a single mine case to provide some indicator values. The initial run introduces the period tonnages involved and forms a framework for developing further refinements.

Key aspects of this run are:

- liberal mining rate (chosen to ensure mining limit is never, or rarely, a limiting factor; in this example 80 million tonnes (Mt) per annum (pa)); and
- conservative processing throughput (chosen to ensure this is the limiting factor and reflects 'reasonable' mine life – 20 Mtpa).

With the above limits and reasonable estimates of the costs required to support the above rates, the Pit by Pit Graph node indicates that the maximum best case NPV that can be achieved is $272 M. This is the pit containing 693 Mt total with a mine life of 24 years (Table 1, line 1).

We have arbitrarily chosen to develop four pushbacks. This is a decision that can be explored further when there are definite costs of starting a new pushback. The more pushbacks you have, the closer you can get to the Whittle 'best case' scenario. When the costs of a pushback are included in the analysis, you can very quickly see when the cost of adding a pushback outweighs the return.

When we add four pushbacks (letting the Pushback Chooser (Whittle, 2004a) decide them for us), the optimal pit is 488 Mt (pit 15) with a value of $186 M over a nearly 19-year mine life (Table 1, line 2).

Three asides

1. The use of geometric values† in defining the revenue factor range generates a good range of pits, giving good starting pits and still keeps the overall number of pits to consider to a minimum.
2. By including the actual cost of establishing pushbacks in an operation, one can determine whether using more pushbacks would improve the value of the operation.
3. The slopes of each mine in an operation could be quite different. Since Whittle has the capability to model these without the use of the Multi-Mine option, they will not be discussed further in this paper.

† 'Geometric values' is a technique for defining revenue factors that produces a greater number of pits at the smaller pit end of the range than at the larger pit end (Whittle, 2004b). It is useful for defining the starter pit and early pushbacks.

Treat as Multi-Mine

Without the Multi-Mine option above, the chosen pushbacks are the same for every mine. The optimal pit is chosen by its pit number and that is also the same for every mine.

Each mine is different, therefore one would expect the ideal pushbacks for each mine to be different. Using the Multi-Mine model we can run the Pushback Chooser separately for each mine. This approach can be used because the Pushback Chooser only uses the relative differences between NPVs in deciding where to put the pushback boundaries. Once we have the pushbacks for each mine, we can explore schedules using input from both mines (each with its own pushbacks) and costings and limits that can be a combination of global and per mine attributes. Note that individual mine constraints are only available with the Multi-Mine option.

The user can now explore the opportunities available to vary the schedule based on the order in which the mines are considered as well as the previous variables associated with pushbacks in a single mine.

At this point, it is useful to note which mines are the biggest contributors. This will help drive the decision as to the order in which we should mine the mines. In this example, the significantly bigger contribution comes from the NorthPark mine, so we will consider it first in the order (Figure 1). Using the Milawa algorithm will improve the NPV if an inappropriate ordering of mines is chosen, but it cannot necessarily find the best NPV.

With each mine having its own (four independent) pushbacks and considering NorthPark first, we end up with a schedule (Figure 2) developing an NPV of $197 M from a combined tonnage of 569 Mt (Table 1, line 3). This is an increase of $11 M with addition of 81 Mt over the previous result, which is a direct result of being able to start with individually optimised mines.

The following steps are not specific to the Multi-Mine option when only global limits are applied, but significant gains in NPV may be available by exploring variations in the processing and mining limits.

Modifying constraints

When analysing the effects of constraints, you should ensure that constraints further back in the process are not causing an adverse impact on downstream processes. For the illustrative purposes of this paper, selling limits are ignored (the last step in the Whittle limits) and we'll deal with the two main limits back up the process stream: the processing capacity, then the mining capacity.

TABLE 1

Summary of results.

Description	NPV ($M)	Tonnage (Mt)
1. First pass – best case	272	693
2. First pass – four pushbacks (as single mine)	186	488
3. Independent pushbacks (four each)	197	569
4. Increased processing capacity	280	"
5. Decreased mining capacity	306	"
6. Milawa NPV	336	"
7. Milawa balanced (untuned)	246	"
8. Milawa balanced (tuned)	328	"

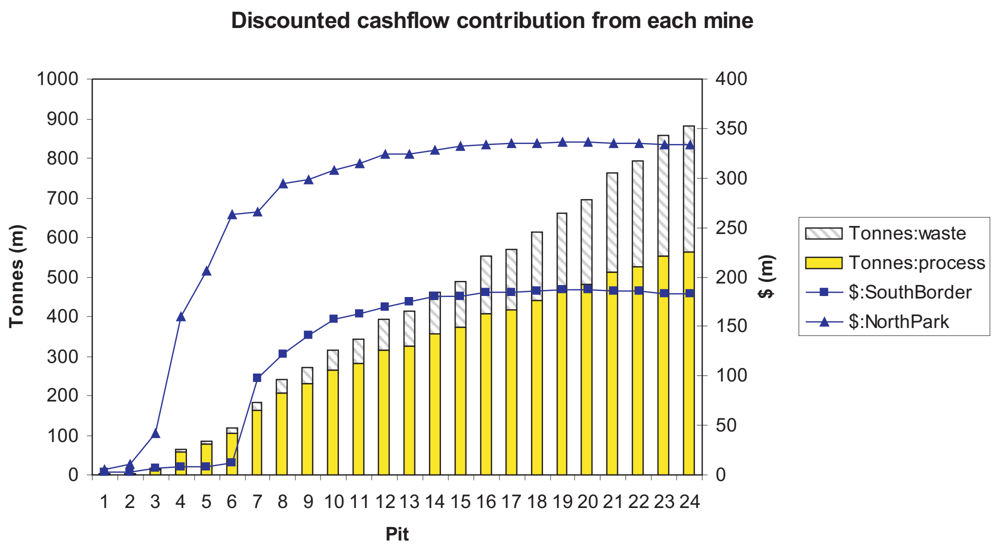


FIG 1 - Cash flow contribution from each mine.

FIG 2 - Two mines, independent pushbacks.

Processing capacity

We have started with a generous mining limit, so the impact of extending the processing capacity can be considered without a tight mining limit confusing the results.

We consider the situation whereby spending an extra $55 M we can add 10 Mtpa to the processing capacity, increasing it to 30 Mtpa‡. The result of this analysis is that we can increase the NPV from the previous analysis to $280 M with the same tonnage (Table 1, line 4). The change is that the mine life is now less than 14 years as compared to over 20 years previously. The increase in NPV is due to being able to earn the money sooner. Note that at this point, mining limits have not been touched and so the mining cost (quoted per tonne) is unchanged.

Mining capacity

From Figure 2 we can see that there are some periods that have mined considerably more material than is required in that period. The pattern is similar after the processing capacity is increased.

Let us consider what would happen if we reduced our mining capacity to 60 Mt, which allows us to save $30 M§. We see that we can add $26 M to the value of the mine, increasing the NPV to $306 M (Table 1, line 5) even though we don't fill the mill in periods five and 12 (by a small amount) (Figure 3).

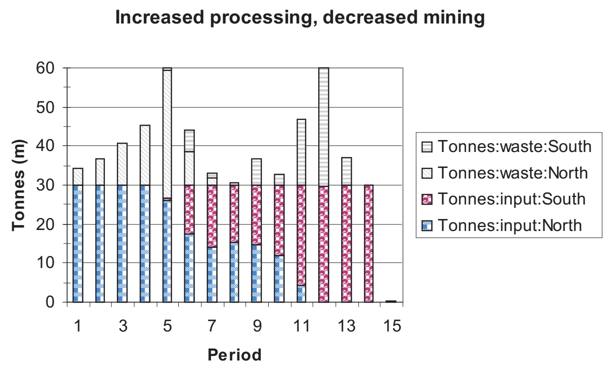


FIG 3 - Increased processing, decreased mining.

Milawa algorithm

The study up until now has only used fixed lead. This has been for several reasons. The fixed lead approach gives results very quickly, which allows us to explore many possible 'what if' scenarios and gives a good feel for the performance of the mine under differing conditions. As we get closer to what we think might be a final solution, we use the Milawa algorithm* to see what extra benefits we can realise out of this mining operation.

The result using Milawa NPV raises the value another $30 M to $336 M (Table 1, line 6). Now the mill is kept full (until the end of the mine). Milawa is now changing the order of processing in the mines to achieve a greater NPV. This becomes more obvious when the mining limit is reduced even further to 50 Mtpa (Figure 4).

The next result, from a Milawa Balanced run, shows that we can balance our mining (and keep the mill filled) at a cost of dropping the value of the operation to $246 M (Table 1, line 7; Figure 5). From Figures 4 and 5, by inspection, it looks like the

‡ In a real study, several scenarios would be considered to explore the benefits of increasing production. Some questions that would need answers are: 'Should we increase production?' 'If we do, by how much?' 'What are the risks involved?'. This example is chosen to illustrate one such scenario.

§ As with the processing capacity, this is an example of a single variation, which in practice would be one of several variations studied.

* The Milawa algorithm is a proprietary algorithm that modifies the selection of material available from every open pushback to produce a schedule that improves the NPV. 'Milawa NPV' focuses on improving NPV; 'Milawa Balanced' focuses on keeping the mining rate balanced.

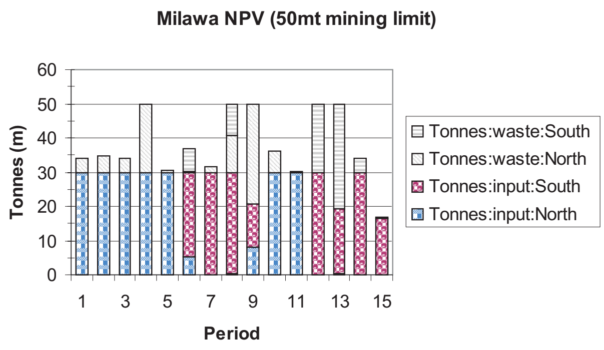


FIG 4 - Milawa NPV.

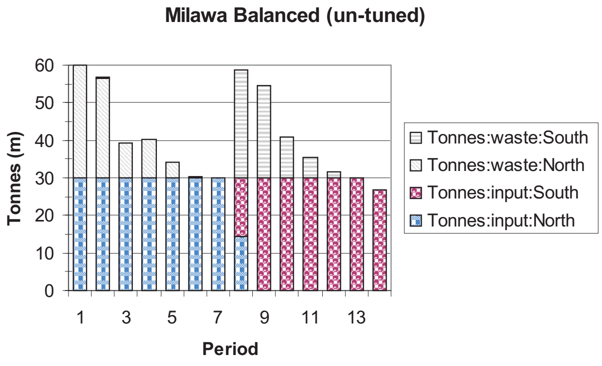


FIG 5 - Milawa Balanced.

increased mining in the early years of the Milawa Balanced solution is contributing to some early costs of mining, which do not occur in the Milawa NPV solution.

We can now consider 'tuning' the mining capacity to improve the Milawa Balanced result. In this example we can achieve a Milawa Balanced schedule (Figure 6) that is a significant improvement over the 'un-tuned' result yielding a value of \$328 M (Table 1, line 8).

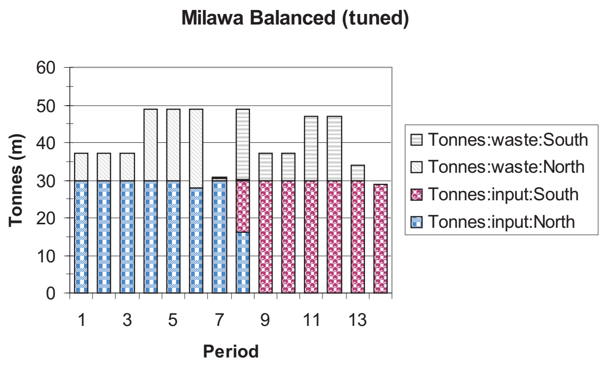


FIG 6 - 'Tuned' Milawa Balanced.

CONCLUSIONS

The optimal pits for individual mines can be determined without the Multi-Mine option in Whittle. The LG algorithm will develop each mine independently.

The basic approach to a multi-mine study is similar to a single mine study with all the single mine features being available in the multi-mine situation.

The differences arise when the key benefits of the Multi-Mine option are used:

- pushbacks can be determined that are ideal for an individual mine,
- the final pit for each mine is separate,
- the order of processing of the mines can be changed easily, and
- mining limits can be tailored for individual mines.

The Multi-Mine option can add significant value to a multiple mine operation.

While this paper discusses a theoretical exercise, the software was put to use in a real project at the Geita gold mine (Joukoff, Purdey and Wharton, 2007, this volume) which demonstrates its practical value.

ACKNOWLEDGEMENTS

The 'BlueSky' project, referred to earlier in this paper, was originally developed as a multi-mine example by Chris Wharton and is based on the 'Marvin' data developed by Norm Hanson for his students at RMIT University and used in the 'Whittle Challenge', a one day add-on to the 'Optimising with Whittle' conference in 1999.

REFERENCES

Joukoff, T, Purdey, D and Wharton, C, 2007. Development and application of Whittle Multi-Mine at Geita Gold Mine, Tanzania, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 185-189 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open-pit mines, Joint CORS and ORSA Conference, Montreal, May 1964, in *Transactions CIM*, pp 17-24.

Muir, D C W, 2007. Pseudoflow, new life for Lerchs-Grossmann pit optimisation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 113-120 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Tulp, T, 1997. Multiple ore body systems, in *Proceedings Optimising with Whittle Conference*, Perth.

Whittle, D, 2001. Multi-pit scheduling in Four-X, Whittle Programming internal document published on user group list server and available on request.

Whittle Programming Pty Ltd, 2004a. Pushback Chooser, *Whittle version 3.2 Help*.

Whittle Programming Pty Ltd, 2004b. Revenue Factors, *Whittle version 3.2 Help*.

APPENDIX: GLOSSARY

Cost adjustment factor (CAF) – This is a term that is used as a multiplicand in a cost calculation. A factor of 1.0 has no effect on the associated cost. The cost to which this CAF applies will always be mentioned in the context of its use.

Discounting – A dollar that we get today is more valuable to us than a dollar that we expect to get next year. When estimating the value of a project, it is common to reduce expected future cash flows by a certain percentage per year, to allow for interest and risk, etc. This process is called discounting.

Mine – A reserve that can be, or is being, mined independently of any other reserve.

Net present value (NPV) – The NPV is the present value of the expected future cash flows minus the present value of the costs.

Operation – The term used in this paper to describe the group of mines that is being studied as a multi-mine scenario.

Pit – One of the possible shapes for a mine. All of the possible shapes produced by Whittle are nested from smallest to largest.

Pushback – A pushback is an intermediate pit outline that is mined to, before mining to another pushback or to the final pit outline.

Revenue factor (RF) – This is the factor by which the revenue for each block is scaled in order to produce one of the nested pits. The factor operates on the element prices.

Shell – The difference between two adjacent pits.

Blasor — Blended Iron Ore Mine Planning Optimisation at Yandi, Western Australia

P Stone[1], G Froyland[2], M Menabde[1], B Law[3], R Pasyar[4] and P H L Monkhouse[5]

ABSTRACT

A new mine planning optimisation software tool called Blasor has been developed and implemented at BHP Billiton's Yandi Joint Venture operation in the Pilbara. Blasor is specifically configured for designing and optimising the long-term pit development plan for the multi-pit blended-ore operation at Yandi. It is used for optimal design of the ultimate pits and the mining phases contained within those pits. In designing the mining phases, Blasor ensures that all market tonnage, grade and impurity constraints are observed whilst maximising the nett discounted cash flow (DCF) of the joint venture operation.

INTRODUCTION

In undertaking a life-of-mine development plan for multi-pit blended-ore mining operations, the mine planner is faced with difficult decisions regarding both the extent of ultimate pits and the design and precedence of the mining phases in each pit. Various commercially available optimisation tools are capable of determining optimal extraction sequences for existing blended-ore pit phase designs – for example NPV Scheduler, Minemax, ECS Maximiser and Whittle Consulting – but planners are usually forced to rely on a mixture of common sense heuristics and personal experience to design the ultimate pit boundary and the mining phase polygons, eg Dincer and Peters (2001) and Noronha and Gripp (2001). A typical pit and mining phase design procedure will require the planner to make arbitrary judgments on in-ground block value – an assumed cut-off grade decision – and then apply a Lerchs-Grossmann algorithm to obtain approximate pit and phase boundaries. These types of approaches become far less tractable when dealing with large multi-pit operations.

The result is that the design of mining phases in blended-ore operations depends largely upon the expertise and experience of the particular mine planner rather than being an objective and repeatable procedure. Once the ultimate pits and mining phases are put in place the flexibility and value attributable to a mining operation over its lifetime is in many ways constrained – no matter what sophistication is applied in optimising panel extraction sequences, the consequences of suboptimal mining phase design can never be overcome.

The mine planning optimisation group within BHP Billiton Technology has developed a mine planning optimisation software tool called Blasor. The concept of Blasor is to use an optimal extraction sequence to design the ultimate pits and mining phases, not the other way around as is the typical approach.

Blasor is specifically designed to optimise the life-of-mine pit development plan for the eleven pits constituting the Yandi Joint Venture operation. It provides Yandi mine planners with a strategic planning tool that can be used throughout the mine life to reconfigure pit development plans as market conditions change. It also enables the operation to rapidly, accurately and optimally value different future market scenarios and/or expansion options using forward pit development plans that are sympathetic to those scenarios and options.

In this paper, we describe the concept and structure of Blasor. The structure of the optimisation problem and the types of constraints applied are outlined before the major design steps are discussed in more detail.

BLASOR IMPLEMENTATION

Blasor has been developed as a PC based (Windows 2000 or XP) integrated stand-alone software package that has the following input/output features:

- Block models are supplied as flat ASCII files.
- Optimisation parameters are entered by the planner through a purpose-built graphical user interface.
- Intermediate data, including all block attributes calculated or assigned by Blasor, can be rapidly viewed in a dedicated 3D visualisation tool.
- Schedule output data, including full tonnage movement and financials, is reported via a number of specialised databases automatically generated by Blasor. A 2D graphical display tool is also provided within the Blasor interface for rapid display of the schedule data on an area and pit-wise basis.

OPTIMISATION PARAMETERS AND SETUP

Blasor's ultimate objectives are to determine the boundaries of the ultimate pits and the best phase designs for those pits so as to maximise the DCF over the life of the operation. In doing so, Blasor uses the commercially available CPLEX mixed-integer linear programming (MILP) optimisation engine from ILOG Inc to determine the optimal extraction sequence contingent upon a number of constraints being strictly observed.

The parameters Blasor uses to constrain the optimisation of the multi-pit development plans are:

- the constraints imposed by practical mining – respecting maximum slopes and mining rates;
- capacity of the downstream supply chain infrastructure; and
- market tonnage, blended ore quality and grade constraints.

A complete list of the constraints applied in the optimisation is given in Table 1.

Other limits to the optimisation model of the real operation are:

- Initial stockpiles are allowed (one for each area). No strategic stockpiling capability is allowed throughout the mine life. Blasor attempts to find an extraction sequence that avoids stockpiling between years.
- No material in the pits is designated as waste *a priori* – the optimiser makes the decision as to how to best blend the material extracted from the pits to make marketable ore. Only blended ore that meets all market grade and quality constraints can have a positive revenue attributed to its extraction.
- Mining and transport costs are attributed to each block – according to their position in the pit different blocks will have different mining and transport costs.

1 BHP Billiton Technology, PO Box 86A, Melbourne Vic 3001, Australia.

2. School of Mathematics, The University of New South Wales, Sydney NSW 2052, Australia. Email: froyland@maths.unsw.edu.au

3. BHP Billiton Project Development Services, PO Box 86A, Melbourne Vic 3001, Australia.

4. MAusIMM, BHP Billiton Iron Ore, PO Box 7122, Perth WA 6850, Australia. Email: Reza.Pasyar@bhpbilliton.com

5. Vice President – Business Strategy for Carbon Steel Materials, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia.

- All material in the pits is allocated a bin number. Material may be assigned to bins on the basis of any combination of grade and impurity dimensions.
- Within each bin of an AGG (an 'AGG' is an aggregation of blocks), the material is assumed to be of homogeneous quality. The optimiser may extract any proportion of an AGG in any year, contingent on other constraints being obeyed.
- The extraction precedence of each AGG is determined by the extraction precedence of its constituent blocks. No part of any AGG may be extracted before all its precedent AGGs have been totally extracted. The rules of precedence are simply that if a block lies above another block (precisely if its centroid lies within the 'cone' transcribed by the maximum slope line for the underlying block), then the overlying block must be extracted before the underlying block.
- Prices for both fines and lump material may be specified to change from year to year.
- All net cash flows are discounted at an appropriate rate.
- The optimisation objective is to find an extraction sequence that obeys all constraints explicitly and results in a maximum nett discounted cash flow.
- The optimisation is global, over the full life-of-mine.

BLASOR OPTIMISATION PROCEDURE

The Blasor optimisation procedure is summarised in Figure 1, illustrating the major steps:

- aggregation of blocks including binning,
- calculation of optimal extraction sequence and ultimate pit limits,
- mining phase design, and
- valuation of the optimal panel extraction sequence.

In the following section, we describe each step of this procedure in more detail.

TABLE 1

Constraints applied in optimisation.

Constraint class	Constraint
Mining	Maximum slope angles enforced at the selective mining unit block size level.
	Maximum mining rate for the operation, each mining area and each pit (variable per annum).
	Earliest start year for pits.
	Smooth mining constraint – large jumps in operation mining rate can only occur after a prescribed duration of near constant mining rate.
	Maximum sinking rate (benches/year).
Transport	Maximum conveying rate for multiple transport paths (variable per annum).
Crushing and screening	Maximum crusher capacity for mining areas and pits (variable per annum).
Market	Target tonnages for fines and lump product individually (variable per annum).
	Maximum and minimum per cent Fe for fines and lump product (variable per annum).
	Maximum and minimum % SiO_2, % Al_2O_3, % P, % Mn and % S for fines and lump product (variable per annum).

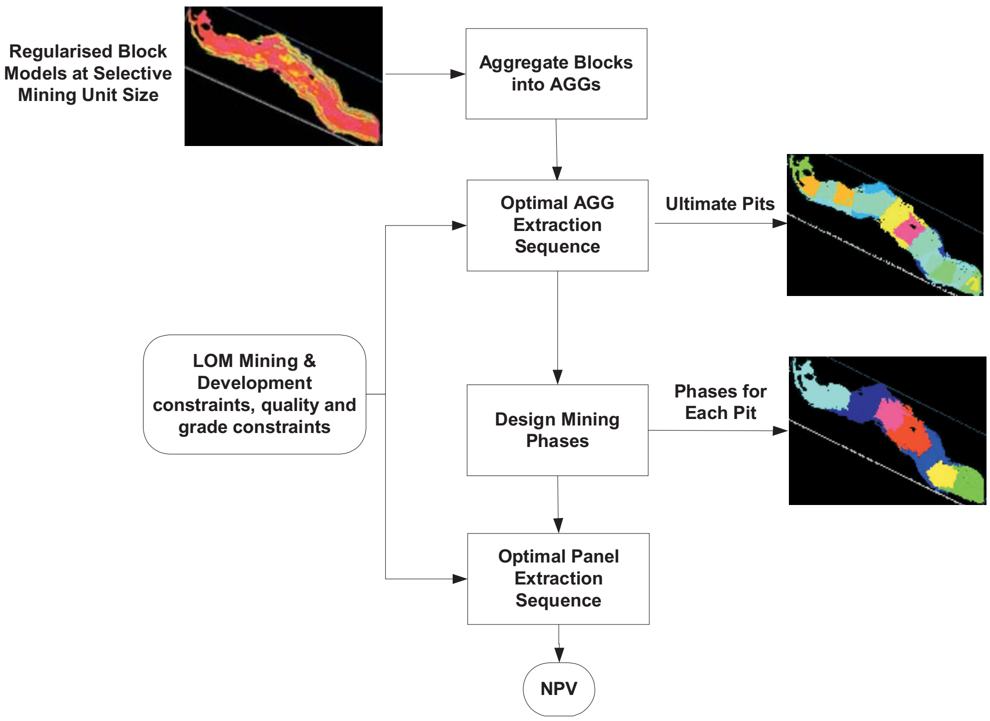


FIG 1 - Blasor pit development optimisation procedure.

Aggregation

For the large block models encountered at Yandi (containing >100 000 blocks), it is necessary to aggregate blocks before they can be tractably scheduled using any linear programming approach. To provide the optimiser with valuable selectivity, binning is used to allow blocks of similar quality to be extracted together by the optimiser. A common method used to aggregate blocks is to re-block the model into a larger block size – this is not the method used in Blasor. The aggregation method used is a proprietary fuzzy clustering algorithm that has the following characteristics, where the term 'AGG' is used to refer to an individual aggregation:

- blocks that are spatially connected and with similar properties are predisposed to belong to the same AGG, and
- the AGG boundaries respect the maximum slope constraints encoded in the selective mining unit block models.

The user may choose to present Blasor with block models that are already cut back to some nominal 'ultimate pit' surface or to allow Blasor to aggregate a larger volume. Each AGG in the larger volume would be presented to the optimiser as a candidate for extraction over the life-of-mine.

After this step, each pit is described by a set of AGGs. Each AGG contains material which is classified in bins. Each bin is allowed to be extracted independently of other bins in the same AGG. A set of AGG precedence rules is also created. These rules, represented as a set of arcs, force the optimiser to extract material in a valid order.

AGG extraction optimisation

This is the vital step in the Blasor design process whereby an optimal AGG extraction sequence is calculated and the blocks in each pit are assigned a period of extraction. The scheduled entities are bins within each AGG and the final AGG extraction sequence will obey all mining, slope precedence, processing and market constraints. The typical size of this optimisation problem for Yandi is:

- 1000 AGGs in total from 11 pits, each AGG containing five bins; and
- 20 time periods over the life-of-mine.

A problem of this size will take between six and ten hours to converge within a 0.5 per cent bound of optimality using the CPLEX MILP engine running on a powerful laptop computer. This optimisation also provides an estimate of the AGG extraction sequence life-of-mine discounted cash flow, which can be used as a benchmark for the DCF of the panel extraction schedule (see below) in assessing the practical optimality of the mining phase design step.

Mining phase design

The mining phase design is performed individually on each pit in the operation. The design procedure uses a proprietary algorithm, which uses the 'period of extraction' block attribute to prioritise the phases within each pit. Some user input is required to assist the algorithm in designing mineable phases – so-called 'rat-holing' can be controlled or overcome through the judicious selection of phase design parameters. Because this step cannot be completely automated, a tool is provided which allows the planner to make practical modifications to the automatically generated mining phases. The interface that allows manual modification of phase designs in Blasor is shown in Figure 2.

Panel extraction optimisation

Having designed the mining phases for each pit, the planner then uses Blasor to generate the panel attributes (where a 'panel' is the intersection of a mining phase and a bench). Panels are represented in the same way as AGGs – via tonnes of all attributes in each bin. The optimal panel extraction sequence is calculated in the same way as for the AGG extraction sequence and uses the same mining, processing and marketing constraints. The final optimal sequence provides the user with a direct estimate of the DCF over the life-of-mine. For the Yandi operation, the optimal panel extraction sequence DCF is usually very close to the optimal AGG extraction sequence DCF, showing that the Blasor phase design process is efficient at preserving the value of the mining operation despite the inevitable compromises that must be made in constructing mineable phases.

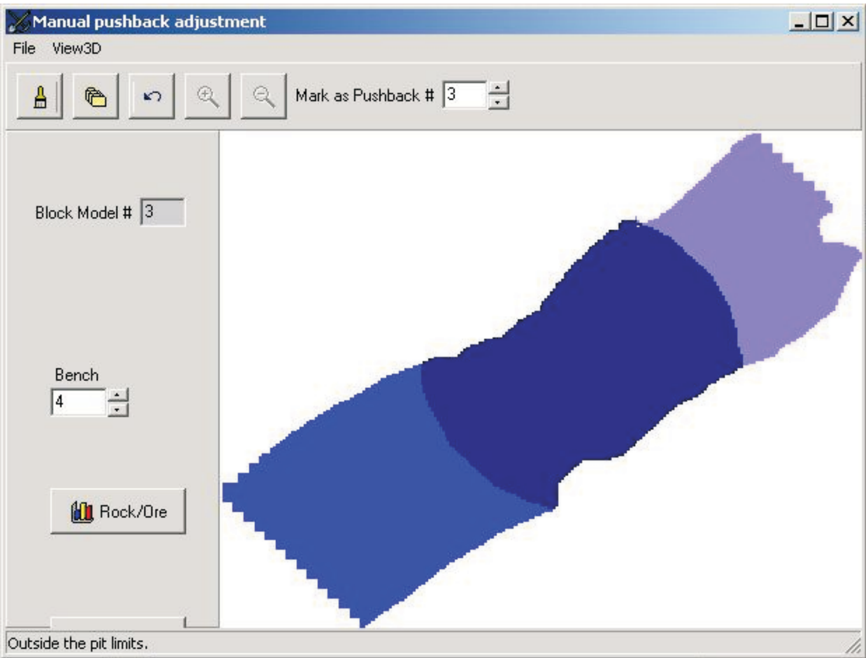


FIG 2 - Interface for the manual phase adjustment tool in Blasor.

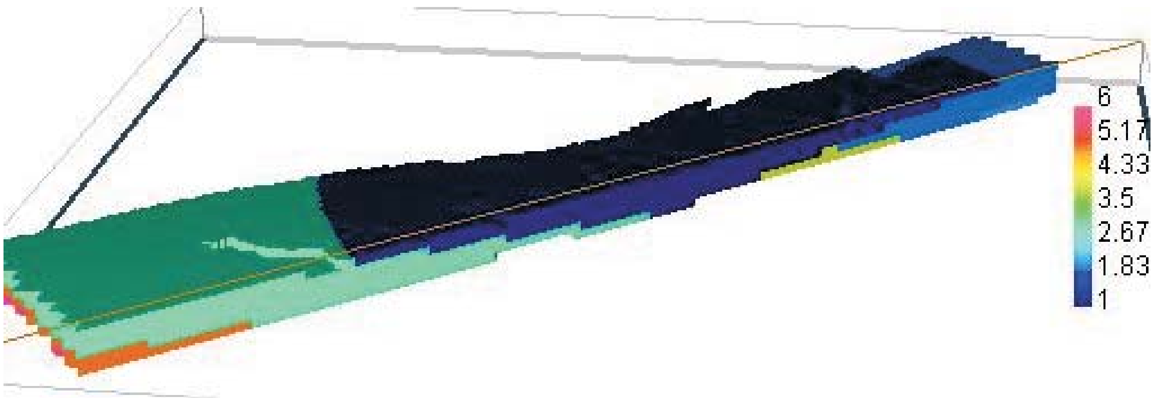


FIG 3 - Optimal period of extraction according to Blasor panel schedule.

The panel extraction optimisation process requires a similar processing time as the AGG extraction sequence optimisation, the final result being an attribution of period of extraction for each block in each pit. An example of the block extraction sequence, illustrated as a colour-coded period of extraction section through the centre line of a single pit, is shown in Figure 3.

CONCLUSION

Blasor provides an efficient and integrated long-term pit development planning and evaluation tool for the Yandi Joint Venture operation. It enables mine planners to design ultimate pits and mining phases that are based upon a globally optimal multi-pit life-of-mine extraction sequence and then to generate an optimal panel extraction sequence from which the practically realisible maximum DCF for the operation can be reliably estimated.

REFERENCES

Dincer, T and Peters, B, 2001. Blending optimum pit mining sequences, in *Proceedings Fourth Biennial Conference: Strategic Mine Planning 2001*, pp 43-53 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Noronha, R A and Gripp, A H, 2001. Ultimate pit selection and design in iron ore, in *Proceedings Fourth Biennial Conference: Strategic Mine Planning 2001*, pp 133-136 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Joint Ore Extraction and In-Pit Dumping Optimisation

M Zuckerberg[1], P Stone[2], R Pasyar[3] and E Mader[4]

ABSTRACT

This paper describes a new software product designed for the net present value optimisation of multi-open pit blended ore operations in which it is desired and/or necessary to dump some or all of the waste rock produced in the course of operations back into the voids created in the process of ore extraction. The software product simultaneously decides over the entire life-of-mine which material to mine in which year, once mined what to do with it, and where to put that material which it has decided to waste, all subject to annual blend and capacity constraints.

INTRODUCTION

Blasor

The strategic mine planning optimisation problem generally addressed in the mathematical literature is concerned with deciding when to schedule the extraction of each block, or aggregate, of ore in an orebody over the life-of-operation whilst respecting all geotechnical slope restrictions and mining and processing capacity constraints (eg Caccetta and Hill, 2003; Gershon, 1983; Tolwinski and Underwood, 1996; Dagdelen, 2007; Menabde *et al*, 2007a, this volume; Ramazan and Dimitrakopoulos, 2007, this volume). There are typically, however, numerous other issues that are involved in the production planning of an actual operation. For example, there are capital decisions that need to be made with regard to the sizing of infrastructure.

Additionally, processing decisions need to be made with regard to the material extracted. More specifically, it needs to be decided which material should be sent to which destination, where destinations may include waste, potentially several stockpiles, and potentially several ore processing routes. In addition, there may also be 'side constraints', such as a constraint that mining rates must be smooth and not vary greatly from year to year, or maximum sinking rate constraints, or minimum exposed ore constraints. Another issue that needs to be considered in a practical plan is that an optimal block extraction sequence may not immediately indicate an optimal phase design.

The Blasor software product, developed by BHP Billiton's Technology division and described in detail in Menabde *et al* (2007b) and Stone *et al* (2007, this volume), seeks to incorporate these and several other issues in its optimisation program.

Blasor-InPitDumping (BlasorIPD)

The standard version of Blasor does not explicitly model waste handling. At some operations, however, the handling of waste is also an integral part of the mine plan. In particular, there are operations where the space available outside of the pits for waste rock dumping may be very limited. In such cases, it eventually becomes necessary to dump waste back into the pit into the voids created by the extractions. This may also be required at some sites due to environmental concerns. Such constraints can add considerable complexity to the task of designing a mine plan. Care obviously needs to be taken to ensure that waste rock is not dumped on top of ore, and thus the mine plan needs to ensure that sufficient space is made available at the bottom of the pits to enable the dumping of waste rock before the external waste dump space is exhausted. However, this requirement may significantly alter the optimal development plan for the orebody.

Blasor-InPitDumping, or BlasorIPD, is a specialised version of Blasor designed to address this waste handling issue. As a specialised version of Blasor, BlasorIPD shares Blasor's general approach to the mine scheduling problem. BlasorIPD, like Blasor, is a mixed integer programming based generalised mine planning tool that utilises the ILOG Cplex solver as its primary engine. The optimisation techniques employed in Blasor were outlined in Stone *et al* (2007, this volume). As indicated, however, the optimisation proposition involved in BlasorIPD is considerably more difficult and requires specialised techniques, as will be discussed at the end of this paper.

Before we describe the BlasorIPD optimisation program the principal steps employed in all Blasor optimisations are overviewed briefly, and are as follows.

Step 1

First Blasor attempts to determine the optimal ultimate pit limits for the blended ore operation taken over all block models that have been input by the user. Blasor uses these limits to constrain the universe of blocks to be considered in constructing the detailed annual schedule.

Step 2

The next optimisation step is phase design. To this effect Blasor first partitions the various block models input by the user into aggregates referred to as 'AGGs'. The aggregation procedure is proprietary to BHP Billiton and will not be described here. The user has a measure of control as to how many such aggregates there will be, though for tractability purposes this number will generally be kept under 1000. Precedence structure among these aggregates is inherited from the precedence relationships that hold for their constituent blocks, and the resulting precedence rules are imposed upon the aggregates. These aggregates are themselves sub-partitioned into smaller aggregates, referred to as 'bins'. A decision to extract an AGG forces the extraction of every bin within the AGG, but the optimiser is still free to make separate processing decisions for each of the constituent bins. The user defines the bins in such a way as to maximise the optimiser's flexibility in processing the material within the aggregates.

Taking an example from an iron ore operation, a typical bin would be the collection of hardcap material in an AGG that has iron grade between 57 and 60 per cent and silica grade less than 1.5 per cent. There will typically be between ten and 20 bins in each AGG. Bin definition is determined by the user, however we are actively researching the possibility of automating the task.

In the standard Blasor formulation, these AGGs and bins are scheduled over the life of mine in such a way as to maximise net present value whilst obeying mining capacity, processing capacity, market capacity and blend constraints. Blasor affords the user a great deal of flexibility to define these constraints, potentially for

1. Principal Scientist, BHP Billiton Technology, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: mark.zuckerberg@bhpbilliton.com

2. Senior Principal Scientist, BHP Billiton Technology, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia.

2. MAusIMM, Principal Mining Engineer, BHP Billiton Carbon Steel Materials, BHP Billiton Limited, PO Box 7122, Cloisters Square WA 6850, Australia. Email: Reza.Pasyar@bhpbilliton.com

3. Senior Mining Engineer, BHP Billiton Carbon Steel Materials, BHP Billiton Limited, PO Box 7122, Cloisters Square WA 6850, Australia.

multiple processing options, as well as to incorporate various capital costs associated with entering new areas or constructing new processing routes. Blasor also contains features to enforce a smooth mining rate, as well as dynamic stockpiling. For further details the reader is referred to Menabde *et al* (2007b).

Step 3

The standard Blasor implementation presents the optimal AGG and bin extraction sequence as a proto-phase design. This proto-design is then converted into an actual phase design through a semi-automated process. Once the phases are designed, Blasor will schedule the resulting 'panels' (a panel is the intersection of a phase with a bench, note that panels are also binned) in order to construct a final schedule. This latter optimisation step also incorporates a feature to optimally enforce maximum sinking rate constraints as well as minimum exposed ore constraints. A BlasorIPD optimisation will follow this same outline, but will additionally make optimal decisions regarding the movement of waste from the pit onto the road network, and from there either back into the pit or to an external waste dump. BlasorIPD will ensure that waste is dumped only into locations within the pit limits that have had their rock previously extracted, and that external waste dump capacities are not exceeded.

OVERVIEW OF BLASORIPD

The fundamental idea utilised in BlasorIPD is that in the same way that we can track the blocks of material with variables that represent when those blocks are extracted, we can similarly track the spaces occupied by those blocks with variables representing when those spaces are filled with waste material. Stated broadly, waste is handled by BlasorIPD according to the following template:

- BlasorIPD chooses to extract an item of material;
- BlasorIPD chooses some proportion of its material to go to waste;
- BlasorIPD chooses a path along a road network upon which to send that waste, incurring the associated cost; and
- the path along the network terminates either at an external waste dump, or back into the space once occupied by some block which BlasorIPD classifies as empty and available for dumping.

A space x can only be classified as available for dumping if all blocks within a user defined radius of x have already been cleared, and if additionally all spaces y within the orebody that lie below x for which the slope angle of the line connecting x and y is greater than the maximum waste repose slope have already been refilled. In accord with the request of mine planners, BlasorIPD has been developed such that a space cannot be made available for dumping if that space sits atop material classified as ore that has not yet been cleared. An alternative implementation may allow dumping to take place on top of ore, thereby sterilising that ore.

BLASORIPD IN DETAIL

BlasorIPD handles the ultimate pit determination (ie 'Step 1') in the same manner as Blasor. AGG and bin design is also performed in the same way as in Blasor, ie blocks are aggregated into 'AGGs' and 'bins' constituting typically between 10 000 and 20 000 separately schedulable units.

Refill AGGs

BlasorIPD introduces the notion of a 'refill AGG'. We noted already that just as we can track the blocks of material with variables that represent when those blocks are extracted, we can similarly track the spaces occupied by those blocks with variables representing when those spaces are filled with waste material. An important difference however, between extraction and refilling is that there is not generally the same demand for precision in locating the refill space as in locating the extraction space. Thus while, at least theoretically, it is desirable to create a vast number of variables to track extractions from a finely partitioned orebody, a coarser partitioning of the space containing the orebody is generally sufficient to capture the various in-pit dumping options.

To this end, a 'refill AGG' is defined as a—typically fairly large—space that may potentially be filled with waste rock. These refill AGGs are constructed by BlasorIPD from the input block models by aggregating the space occupied by blocks in the block model (possibly including air blocks) into disjoint spaces. The specific shape of these refill AGGs is chosen in such a way as to ensure that the refill AGGs may be independently scheduled for refilling, subject to precedence rules, without violating maximum waste repose slope constraints (the maximum slope angle for waste repose is an input provided by the user). For example, say that the space occupied by some constituent block in refill AGG A must be refilled before the space occupied by some constituent block in refill AGG B. The refill AGGs are designed by BlasorIPD in such a way that it will always be the case that there is no constituent block in refill AGG B that must be refilled before the space occupied by any constituent block in refill AGG A. BlasorIPD will thus enforce a precedence rule that refill AGG A must be completely filled before any dumping may take place into refill AGG B, and the AGG volumes/shapes are guaranteed to be such that A can indeed be filled before any dumping is initiated into B.

BlasorIPD's proprietary AGG design method will typically yield a complicated precedence network among these AGGs that will afford a great deal of flexibility in the order in which they may be refilled. The user has a measure of control as to how many refill AGGs there will be, though for reasons of tractability the number should generally be kept under 1000.

Zones

A second additional concept introduced in BlasorIPD is that of 'waste zones'. The principal purpose of these zones is to model the cost of waste movement within the optimisation. BlasorIPD's model of waste movement cost is as follows.

The user defines a road network upon which all waste movement will take place. Nodes on the network are defined by a node number, a road number and a location number on that road. The same node number can be associated with multiple roads (this would indicate an intersection of two roads at that point). The locations along any one road are numbered consecutively from 1. Costs per unit distance forward (ie to the next location number on the same road) and backward are defined for each of the roads within the network, and the external waste dumps are each assigned a location on the network. For each block in the block model the user assigns up to three potential entry points for waste produced in that block to enter the network, along with the associated cost per cubic metre of waste to gain access to the network. It is assumed conversely that these entry points also serve as departure points from which waste material on the road network may be dumped back into the space occupied by that block, and associated costs in dollars per cubic metre are also assigned. We will refer to these entry points as 'block-network links'.

For purposes of tractability however, as in the case of the refill AGGs, we do not track the movement of waste at a block level. Instead we track it at a 'zone' level. For each pit in the operation, the user defines the number of zones into which the pit should be split. The zones are chosen so that two blocks can only belong to a single zone if they both link to the same collection of roads on the network. For each road to which the blocks in a zone link, all blocks within the zone are considered to link to the average of the road locations of the constituent blocks, and at the average of the associated costs. There are thus three distinct types of aggregation performed by BlasorIPD:

- aggregation of blocks into extraction AGGs and bins,
- aggregation of the space occupied by blocks into refill AGGs, and
- aggregation of the space occupied by blocks into waste zones.

These aggregates are defined independently and thus a zone may overlap several extraction AGGs as well as several refill AGGs.

The BlasorIPD waste model

The BlasorIPD waste model is summarised as follows:

- An AGG of material is extracted.
- BlasorIPD chooses some proportion of its material to go to waste.
- The material chosen to go to waste is allocated to the zones overlapping the source AGG proportionally with the overlap.
- For each of the zones that have been allocated source waste from the extracted AGG, BlasorIPD chooses to send that waste to one or more of its predefined network entry points, incurring the associated cost.
- The material moves across the network, incurring the associated costs. Some of the material may terminate at various external waste dumps (subject to the user defined capacities of those dumps), and the rest of the material will pass through a block-network link to terminate at one of the zones within a pit.
- Material designated as terminating at a zone in a pit is allocated proportionally to the refill AGGs that overlap that zone, and at the associated cost. A refill AGG is not considered available for dumping until all extraction AGGs that overlap its user defined radius have been cleared of their original material, and until all of its precedent refill AGGs (ie those that sit 'below' it) have been completely refilled.

Water table constraints

An additional constraint was implemented requiring that blocks sitting below the water table be either refilled by the end of the mine life or never be extracted. As this constraint effectively requires that the entire refill AGG containing a block that sits under the water table to be refilled, we split the refill AGGs at the water table so that no extra refilling will be required to take place. To satisfy this constraint the schedule will typically reclaim material from the external waste dumps in the final year of the mine life and move it through the road network back into the pits.

Phase design and panel scheduling

Phase design takes place in BlasorIPD in the same manner as in Blasor. The extraction AGG schedule serves as a proto-phase design which is then converted into a proper phase design via a semi-automated process. The 'panels' are then defined as the intersection of phases and benches, and they become the new 'AGGs', ie the new aggregates of extraction material. The refill AGGs and the waste zones remain as they were and the optimisation is repeated with panels.

Reporting

The reports are as in Blasor, but with additional fields to record the volume of waste sent to external waste dumps, the volume of material dumped into the pits, and the cost of the waste movements. Additionally there are several reports that track the detailed movement of waste across the network in each period.

Tractability issues

The principal tractability issue we needed to tackle with BlasorIPD was how to handle all of the additional decisions concerning waste movement and refill AGGs without reducing the number of extraction aggregates that can be handled by the product. In particular, to model the refill AGG precedence constraints (the constraints that disallow dumping into a refill AGG before the space below it has been refilled) requires binary integer variables for each refill AGG and each period to track whether or not a refill AGG is available for dumping. In principle this would nearly double the number of required integer variables from the figure required for the standard Blasor implementation for which integer variables are only required to track whether an extraction AGG is available for digging.

The method implemented herein rests primarily on the observation that there are several ways to relax the problem formulation such that while the solution produced will no longer be technically feasible, it will nevertheless typically capture a good deal of the 'structure' of the optimal feasible solution. The specifics of our methods will not be described here, but in general terms, our methods combine cutting planes to tighten the linear programming relaxation with an iterative approach that solves relaxed problems, which are relatively 'easy'. We then use the solutions to the relaxed problems to guide the solution to the original problem.

Practical applications have shown that we can solve life-of-mine problems with on the order of 10 000 schedulable extraction bins (comparable to those solved in the standard implementation of Blasor) and 800 refill AGGs in several hours, and that the iterative techniques applied can maintain the same optimality guarantees.

SUMMARY AND CONCLUSIONS

BlasorIPD is a tool for joint ore extraction and in-pit dumping optimisation and planning; its main aspects may be described as follows:

- BlasorIPD is an extension of BHP Billiton's Blasor software product that adds to Blasor the ability to make waste handling decisions;
- BlasorIPD decides how to move material that it designates as waste onto the road network, along the network, and to its final destination either at an external waste dump (subject to dump capacities) or to an available location inside the pit;
- BlasorIPD optimally determines a joint extraction and waste refill schedule so that space will be made available inside the pits for in-pit dumping as necessary; and
- despite the additional complexity entailed in a BlasorIPD optimisation, BlasorIPD utilises strategies that have led in practice to fast solution times for full sized problems.

REFERENCES

Albach, H, 1967. Long range planning in open pit mining, *Management Science*, 13(10):548-568.

Caccetta, L and Hill, S P, 2003. An application of branch and cut to open pit mine scheduling, *Journal of Global Optimization*, 27:349-365.

Dagdelen, K, 2007. Open pit optimisation — Strategies for improving economics of mining projects through mine planning, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 145-148 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Gershon, M, 1983. Mine scheduling optimization with mixed integer programming, *Mining Engineering*, 35:351-354.

Menabde, M, Froyland, G, Stone, P and Yeates, G, 2007a. Mine schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Menabde, M, Stone, P, Law B and Baird, B, 2007b. Blasor – A generalized strategic mine planning optimization tool, 2007 SME Annual Meeting and Exhibit.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Stone, P, Froyland, G, Menabde, M, Law, B, Pasyar, R and Monkhouse, P, 2007. Blended iron ore mine planning optimisation at Yandi Western Australia, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 133-136 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Tolwinski, B and Underwood, R, 1996. A scheduling algorithm for open pit mines, *IMA Journal of Mathematics Applied in Business and Industry*, 7:247-270.

Optimisation in the Design of Underground Mine Access

M Brazil[1], D Lee[2], J H Rubinstein[2], D A Thomas[1], J F Weng[1] and N C Wormald[3]

ABSTRACT

Efficient methods to model and optimise the design of open cut mines have been known for many years. The design of the infrastructure of underground mines has a similar potential for optimisation and strategic planning. Over the last five years our group has developed two pieces of software to tackle this problem – Underground Network Optimiser (UNO) and Decline Optimisation Tool (DOT). The idea is to connect up a system of declines, ramps, drives and possibly shafts, to minimise capital development and haulage costs over the lifetime of a mine. Constraints that can be handled by the software include: gradient bounds (typically 1:7), turning circle restrictions for navigability, and obstacle avoidance. The latter constraint keeps development at standoff distances from orebodies and ensures it avoids regions that involve high cost, such as faults, voids and other geological features.

The software is not limited to only interconnecting fixed points. It has the useful feature that a group of points can be specified such that the development is required to connect to one member of the group. So for example, if an existing ventilation rise must be accessed at some level, then a group of points along the rise can be selected. Similarly, this gives the opportunity to use variable length cross-cuts from a decline to an orebody. The latter gives important flexibility and can significantly reduce the development and haulage cost of a design.

Finally, the goals for the next phase of development for this project will be discussed, including speeding up the algorithms and allowing for heterogeneous materials, such as aquifers and faults, as additional costs rather than obstacles.

INTRODUCTION

There are several different basic forms for the layout of an underground mine. An underground mine can be viewed as a collection of ramps and drives connecting various points of access at each required level of the orebodies to a surface portal. From this viewpoint the mine can be modelled as a mathematical network in which the nodes correspond to the access points, the junctions and the surface portal and each link corresponds to the centre-line of a ramp or drive. A mine containing a shaft, together with ramps and drives for access and haulage can be modelled in a similar way. Other operational elements such as ore passes fit readily into such a description. If existing mine workings are to be extended to new ore deposits, a similar network can be constructed, connecting into the given structure at a convenient break-out point (or points). In all cases a major challenge for the mine designer is to construct a lowest cost feasible solution incorporating all operational constraints.

Key navigability constraints for mining equipment and haulage trucks include a gradient bound, m, where m is usually between 1/9 and 1/7 for declines and ramps. Also, a minimum turning circle for curved ramps needs to be specified. Typically it will be in the range of 15 m to 30 m, again depending on the equipment to be used in the mine.

1. ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN)†, Department of Electrical and Electronic Engineering, The University of Melbourne, Melbourne Vic 3010, Australia.

2. Department of Mathematics and Statistics, The University of Melbourne, Melbourne Vic 3010, Australia.

3. Department of Combinatorics and Optimisation, University of Waterloo, Waterloo ON N2L 3G1, Canada.

† CUBIN is an affiliated program of National ICT Australia.

In addition, the design should take into account 'no-go' regions that must not be intersected by the ramps or drives. These would usually include a stand-off region around the orebody (to avoid sterilisation of the orebody) and regions of severe faulting or other operational or geological anomalies.

Moreover, future access to prospective new ore zones may be included in the design. As more information becomes available, for example through in-fill drilling, designs may need to be modified. Having efficient software tools makes such updates much simpler and faster than previous approaches.

The Network Research Group, based at The University of Melbourne, has been developing techniques to find solutions to these design problems using new mathematical algorithms and software. In this paper, we will summarise our current methods and outline future plans to make the software faster and more flexible and to deal with extra geological features that are often encountered.

UNDERGROUND NETWORK OPTIMISER – UNO

Our first project involved optimising mine costs by developing a mathematical network model of an underground mine layout, where the links of the network correspond to the basic mine components such as ramps, drives, ore passes and shafts. Although a ramp or drive is generally curved, if it has constant gradient, which is as steep as possible without violating the gradient bound, then its length can be computed from the coordinates of its endpoints alone. This means that in the network model, we can assume each link is a straight-line segment whose length is computed via a suitably defined metric, known as the *gradient metric*. If the link has gradient no greater than the specified maximum value m, then the standard Euclidean length L is used; however, if the link is a straight segment with gradient greater than m, then the standard Euclidean length is replaced by the expression:

$$L = z\sqrt{1 + \frac{1}{m^2}}$$

where:

z is the vertical displacement between the two ends of the link

It can be shown that any feasible path between such endpoints with constant gradient m will have length given by this expression.

The variable (that is, length-dependent) cost C associated with a ramp or drive of length L in metres, is given by a function of the form:

$$C = (D + H_1T + H_2gT)L$$

where:

D, H_1 and H_2 are operational constants

g is the gradient of the ramp or drive

T is the total tonnage of ore to be transported along this section of the mine over the life of the mine

We can view the first term DL as the development cost for this component, the second term H_1TL as the haulage cost if we assumed the section of the mine was horizontal and the final term H_2gTL as the haulage penalty associated with the gradient.

In the case where the ramp or drive has maximum gradient m, the cost function becomes:

$$C = (D + H_1T + H_2Tm)z\sqrt{1+m^{-2}}$$

where:

z is the vertical displacement between the two endpoints

A shaft with fixed surface portal (for simplicity), can be treated in the network model as a variable length vertical line segment with variable cost of the form:

$$C_S = (D_S + H_ST)L$$

where the constant:

D_S is the per metre cost of the shaft of variable length L

H_S is an operational constant associated with the haulage costs

T is again the total tonnage to be hauled up the shaft over the life of the mine

We are also able to deal with cases where there is a choice of locations for the surface portal.

The only significant variable costs associated with ore passes are their development costs, which can be assumed to be proportional to length.

Our mathematical algorithm proceeds to find the least cost connected network of such components, where the cost of the network is the sum of the costs associated with the links of the network, as described above. This is implemented in a software product called Underground Network Optimiser (UNO). Note that the network has to join up the given access points on the orebody to the surface portal. Alternatively, for an extension of an existing mine, the new development may join to one member of a set of possible break-out points in the existing decline system.

Mathematically, there are a number of key issues that need to be resolved to find an efficient algorithm to locate the least cost network. The *topology* of a network is the choice of segments of the network at the different junctions. In terms of the mathematical network, this specifies the pattern of connections in the network, or, equivalently, the network's underlying graph structure. Classically, such networks are called *Steiner trees* (see Hwang, Richards and Winter, 1992, for a good general introduction to this topic). In the network, all access points of the ore zones and the surface portal, or break-out point, are called *terminals* and all additional junctions are referred to as *Steiner points*. At Steiner points there are three incident segments. Links with apparent gradient more than m are realised, in graphical representations of the network, as bent links (zigzags) with each straight line section in the zigzag at maximum gradient m. In the actual mine a bent link will correspond to a curved, possibly helical, drive with constant gradient m. For more details, see Brazil *et al* (2001a).

A primary mathematical difficulty in constructing the optimal network is that the number of possible topologies grows extremely quickly with the number of terminals. So it is essential to have a very efficient method to find the least cost network for a given topology. We then use simulated annealing and genetic algorithm methods to systematically search through the huge number of possible networks.

To find the least cost network with a fixed topology on the links, the idea is to use a descent method, perturbing the locations of the Steiner points. This is not straightforward, since the gradient metric places considerable restrictions on the ways in which Steiner points can move so that the length of the network is reduced. For example, if a link initially has gradient less than m, and after moving the Steiner points at its ends, the link has gradient more than m, then the cost function for the link changes. Making this problem tractable relies on a deep understanding of the geometric structure possible in a minimum Steiner Tree (Brazil *et al*, 2001a). Note that, for a large range of cost functions, the total cost of the network, with fixed topology, is a convex function of the positions of the Steiner vertices. See Brazil *et al* (2005).

The development of UNO was inspired by a case study provided by WMC Limited based on Olympic Dam (Brazil *et al*, 2001b). An example of an application of UNO to another recent case study is shown in Figure 1.

DECLINE OPTIMISATION TOOL – DOT

More recently, in work done based on case studies with Normandy and Newmont Australia Limited, we have developed a Decline Optimisation Tool (DOT) described in Brazil *et al* (2003). We give a quick summary of the key features of DOT.

The mathematical model consists of a surface portal or break-out point and a decline, which is modelled as a concatenation of straight and curved ramps, with variable length cross-cuts attached at points which we again call Steiner vertices. We often assume that the cross-cuts are perpendicular to the decline, although this condition can be varied. Moreover, the cross-cuts can access the orebody at a variable or fixed point on a given level. This extra flexibility can produce considerable savings in tightly constrained designs.

The cost functions associated with the different components of the network are very similar to those given previously. The important constraints are curvature (turning circle) and gradient constraints. The latter are exactly as before; the minimum turning circle (radius of the helical or circular segments) is typically in the range 15 m to 30 m, depending on the haulage equipment to be used in the mine.

Designing such a network so that it has optimal cost is an extremely difficult problem. In order to make the problem tractable, the algorithm focuses sequentially on each section between Steiner points where the initial and final directions of the path are determined in advance. Once a solution method has been developed for this modified problem, one can proceed with a dynamic programming methodology to solve the original problem, visiting the specified points and amalgamating the path entering a point and the one leaving it provided they have the same start and finish directions.

An abstract solution to the problem of finding minimal paths in three-dimensional space, with given start and finish directions and a given minimal turning circle (but no gradient constraint), has been described in Sussman (1995). This solution, however, has the disadvantage of a continually varying gradient, which is an undesirable characteristic. It can be shown that if the additional constraint of an unchanging gradient is put on such a curve, then the shortest possibility is simply a segment of a circular helix. However, if the gradient is both bounded and unchanging, then the shortest path consists of several helical and straight segments joined together smoothly.

The program DOT has several features that produce a good heuristic algorithm for finding low-cost feasible designs. DOT is able to combine several helical segments together with some inclined straight segments or flat circular arcs, where the joins are smooth. By this we mean that at the junction between two curves, the incoming direction of the first matches the outgoing direction of the second. DOT then searches amongst such combinations to try to reduce costs.

DOT generates a three-dimensional image of the optimal decline's centreline and strings of coordinates, which may be loaded into standard mine graphics systems.

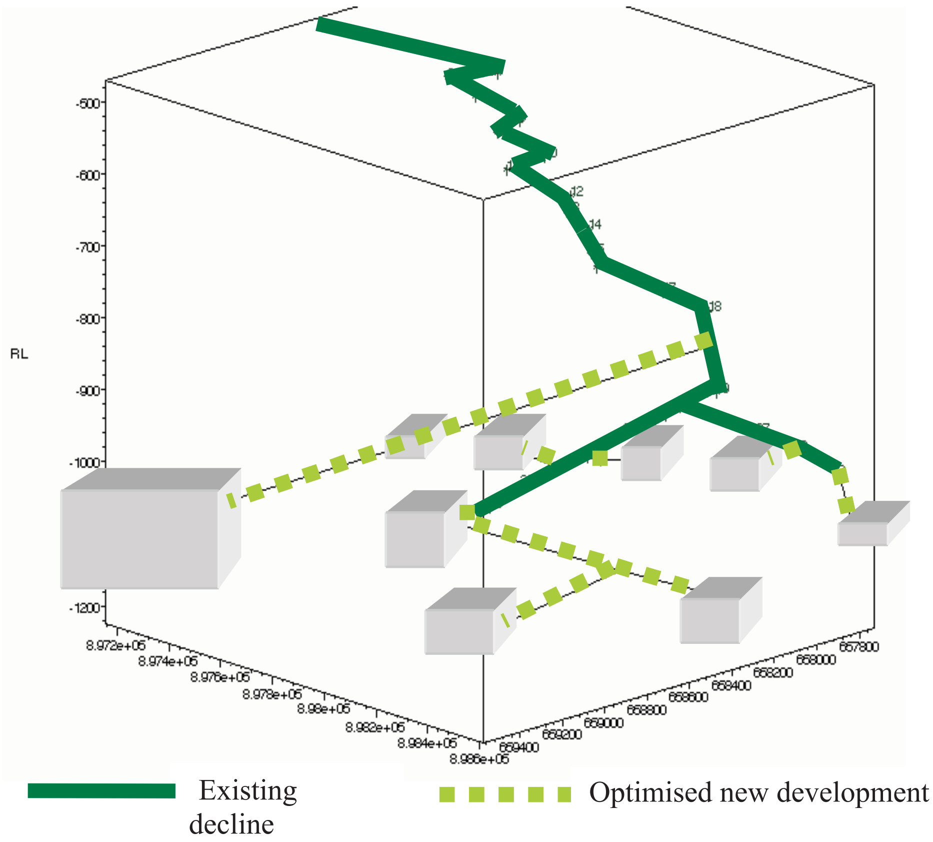


FIG 1 - This figure, from a live case study, illustrates the capacity of UNO to design an optimal network extending an existing mine and accessing new orebodies.

OBSTACLES AND HIGH-COST REGIONS

Obstacle avoidance is implemented by cutting off solutions that pass through barriers, and recomputing using additional prescribed points on the decline where such intersections arise. Standard methods of dynamic programming then enable a sequence of efficient feasible solutions to be joined smoothly at such points and the shortening device in the previous section applied to check if any cost reduction is possible.

At present, highly faulted zones can only be treated as obstacles by our software. In the next phase of the project, to be conducted in conjunction with Newmont Australia Limited, our plan is to treat these regions as feasible regions but ones inducing extra costs. Three different cases are highly fractured material, laminations and aquifers. In the first case, a law of cosines, similar to that for diffraction of light through materials of different density, gives a good method of treating the cost differential for extra reinforcement.

In the second case of laminations, the preferred direction for drives is perpendicular to the planes of faulting, so a cost function needs to be chosen that is direction sensitive. In the final case of aquifers, there is an initial cost to incorporate a pumping facility for each crossing. In all cases, these additional costs need to be incorporated into the software algorithms DOT and UNO.

SPEEDING UP DOT

Ultimately our plan is to incorporate some features of UNO in DOT, so that simple tree networks with multiple branches can be analysed. A major problem to be overcome in this project is to speed up DOT since such a program would involve a large number of computations of low-cost declines. To completely integrate UNO and DOT may be impractical, due to the huge number of steps required in such a program. However, using UNO to determine the overall structure of a low-cost network and then using DOT to design segments of the network, should work very well for even the most complex design problems.

Currently we have been studying how to construct paths that are several segments of helices, flat circular arcs or inclined straight lines, smoothly joined together. Our aim is to completely describe algorithms to find all least-cost paths of this type, joining fixed initial and final terminals, with the initial and final directions also fixed. Then, this can be inserted as a subroutine in DOT. Note that the least-cost solutions for the corresponding problem in the plane (related to vehicle navigation) have been determined in a classical paper of Dubins (1957).

AN APPLICATION OF DOT

This section illustrates the application of DOT to a small design example. The data is based on a recent Newmont investigation into an extension of a gold mine. It describes a mine extension on nine levels with vertical separations between different levels varying between ten and 14 m.

Two snapshots of the DOT-generated decline centreline are shown in Figures 2 and 3. These figures show, respectively, a side view and plan view for the same design. The dots in the figures indicate the access points, at which the decline must meet the cross-cuts. At two of the levels there are alternative access points nominated.

An important capability of DOT is the facility to perform 'what-if' testing of alternative designs. While not part of the original Newmont exercise, Table 1 indicates the cost variation of this design as the turning radius is varied from the original 25 m to span the range 20 m to 30 m over 2.5 m intervals. These values were simply generated by a single parameter change at run time. The cost referred to is the sum of development plus haulage through this segment of the decline.

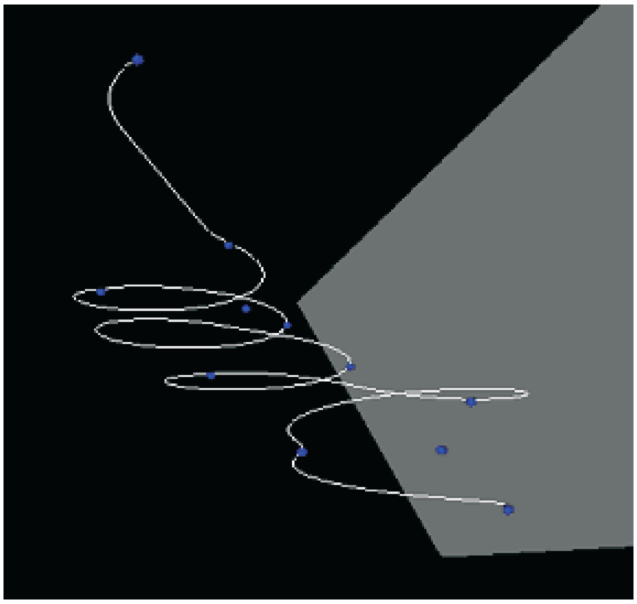


FIG 2 - Snapshot of the DOT-generated decline centreline.

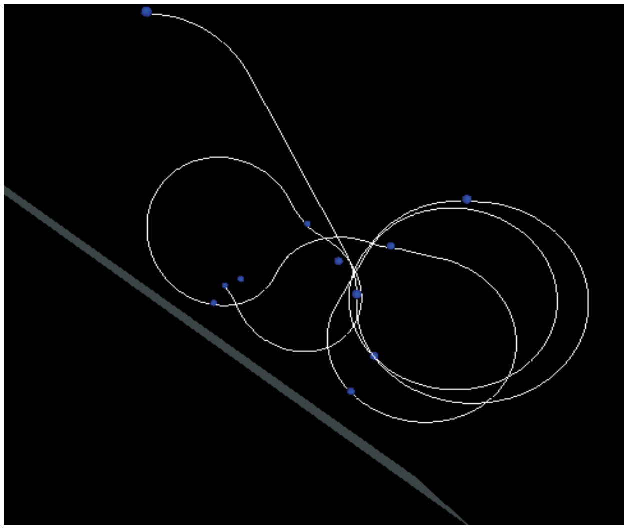


FIG 3 - Plan view of the design corresponding to a 25 m turning radius.

Table 1 illustrates the design is fairly sensitive to the nominated minimum turning radius and the changes are not linear – there may well be mine regimes where there is little change in cost for a larger radius and on the other hand very significant changes near certain critical values. DOT provides a means of testing designs for this sensitivity.

Figure 4 illustrates (in plan view) the design corresponding to a 30 m turning radius; it is qualitatively similar to the 25 m radius design in Figure 3, but significantly more expensive.

The versatility of DOT has been further demonstrated by Carter, Lee and Baarsma (2004), who apply the program to design and cost the infrastructure to serve a nominated tabular orebody mined by the open stope method.

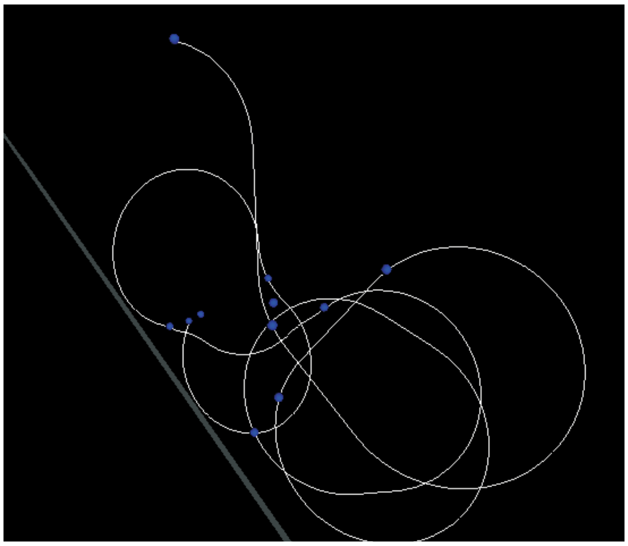


FIG 4 - Plan view of the design corresponding to a 30 m turning radius.

TABLE 1

Cost variation for design.

Minimum turning radius	20	22.5	25	27.5	30
Cost (A$ million)	3.32	3.75	3.93	4.35	4.60

ACKNOWLEDGEMENTS

This research has been supported in part by the Australian Research Council and Newmont Australia Limited.

REFERENCES

Brazil, M, Lee, D H, Rubinstein, J H, Thomas, D A, Weng, J F and Wormald, N C, 2001a. Gradient constrained minimal Steiner trees (I): Fundamentals, *Journal of Global Optimization*, 21:139-155.

Brazil, M, Lee, D H, Rubinstein, J H, Thomas, D A, Weng, J F and Wormald, N C, 2001b. Network optimisation of underground mine design, *Proceedings Australasian Institute of Mining and Metallurgy*, 305:57-65.

Brazil, M, Lee, D H, Van Leuven, M, Rubinstein, J H, Thomas, D A and Wormald, N C, 2003. Optimising declines in underground mines, *Trans Inst Min Metall*, Section A, Mining Technology, 112:A164-A170.

Brazil, M, Thomas, D A, Weng, J F, Rubinstein, J H and Lee, D H, 2005. Cost optimisation for underground mining networks, *Optimization and Engineering*, 6(2):241-256.

Carter, P G, Lee, D H and Baarsma, H, 2004. Optimisation methods for the selection of an underground mining method, in *Proceedings Orebody Modelling and Strategic Mine Planning*, pp 7-12 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dubins, L E, 1957. On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents, *American Journal of Mathematics*, 79:497-516.

Hwang, F K, Richards, D S and Winter, P, 1992. The Steiner tree problem, *Annals of Discrete Mathematics*, volume 53 (Elsevier: Amsterdam).

Sussman, H J, 1995. Shortest three-dimensional paths with prescribed curvature bound, in *Proceedings 34th Conference on Decision and Control*, pp 3306-3312 (IEEE: New Orleans).

Open Pit Optimisation — Strategies for Improving Economics of Mining Projects Through Mine Planning

K Dagdelen[1]

ABSTRACT

The open pit design and scheduling problem is a large-scale optimisation problem that has attracted considerable attention over the last 40 years. The development of the 'know-how' to improve the economics of open pit mining projects through the use of mathematical optimisation techniques goes back to the early 1960s. Unfortunately, until recently, many of these 'optimising algorithms' could not be implemented due to the limited capacity of the computer hardware used in many mining operations. During the last ten years, advancements in the computer hardware technology, along with developments in software technology has allowed open pit mines to have powerful desktop computers that can solve complex optimisation problems on site. One example is the Chuquicamata open pit mine in Chile, which applied optimisation techniques developed in the early 1960s to re-evaluate their cut-off grade strategy. This led to an improvement of US$800 M in the net present value (NPV) of their operations. Newmont Gold Corporation in Nevada, USA has implemented a large-scale Linear Programming Model that was developed in the early 1980s to schedule their entire mine and mill production in the Carlin District, resulting in significant process cost savings. This presentation will outline open pit optimisation techniques that are available today and how they can be used to improve the overall economics of projects that are being planned or are currently in production.

INTRODUCTION

The current practice of planning a hard rock open pit mine begins with a geologic block model (see Figure 1) and involves determination of:

1. whether a given block in the model should be mined or not;
2. if it is to be mined, when it should be mined; and
3. once it is mined, how it should then be processed.

1. Mining Engineering Department, Colorado School of Mines, 1500 Illinois Street, Golden CO 80401, USA. Email: kdagdelen@mines.edu

The answers to each of these questions, when incorporated into the whole orebody block model, define the annual progression of the pit surface and the yearly cash flows that will be coming from the mining operations during the life of the mine. There can be many different solutions to the scheduling problem depending on the decisions made for each of the blocks. The decision as to which blocks should be mined in a given year, and how they should be processed (ie waste, run of mine leach, crushed ore leach or mill ore, etc) defines not only the cash flow for that year, but also impacts the future annual schedules. What is decided today has long-term implications for what can be done in the future, and all of these decisions link together to define the overall economics of a given project. The objective of the planning process for an open pit mine is usually to find optimum annual schedules that will give the highest net present value (NPV) while meeting various production, blending, sequencing and pit slope constraints.

Traditionally, the scheduling problem described above is solved by dividing the problem into subproblems similar to one shown in Figure 2. The solution starts with the assumption of initial production capacities in the mining system and the estimates for the related costs and commodity prices. Once the economic parameters are known, the analysis of the ultimate pit limits of the mine is undertaken to determine what portion of the deposit can economically be mined. Within the ultimate pit limits, pushbacks are further designed so that the deposit is divided into nested pits going from the smallest pit with highest value per tonne of ore to the largest pit with the lowest value per tonne of ore. These pushbacks are designed to include haul road access and act as a guide during the scheduling of yearly production from different benches. The cut-off grade strategy is defined in order to differentiate ore from waste, and further, to determine how the individual blocks should be processed. These steps are repeated in a circular fashion as further improvements are made with respect to the adequacy of the production capacities and the estimated costs.

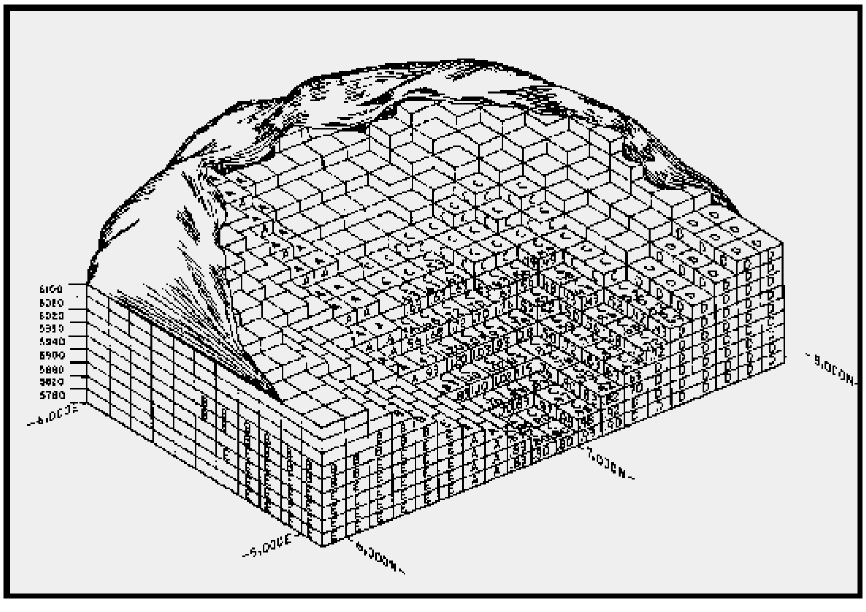


FIG 1 - 3D geologic block model representation of a copper orebody.

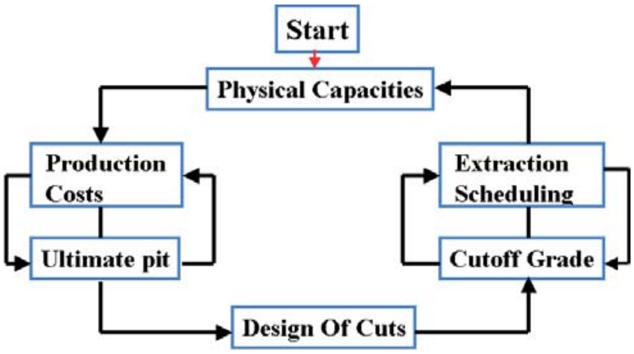


FIG 2 - Steps of traditional planning by circular analysis.

There are many sophisticated software packages in the mining industry to perform ultimate pit limit analysis, design of pushbacks and to determine yearly mine plans and schedules. These computer programs are regularly used by the mining engineers in generating mine plans and schedules that are feasible. These plans are regularly implemented in actual operations without questioning whether they are the best that can be done to obtain the highest returns possible on the capital invested.

The underlying principal for the analysis undertaken during each step tends to be similar for all software packages. The ultimate pit limits, the pushbacks and the cut-off grades are all designed and analysed on the basis of break-even analysis without any consideration given to the time value of money. There are serious shortcomings with these commonly followed practices if the goal of the enterprise is to maximise the NPV of a given project. It is not realistic to believe that plans and schedules obtained on the basis of break-even analysis will give the highest NPV possible for a given project. This paper will discuss why certain mine planning practices result in suboptimal exploitation of resources when NPV is used as the evaluation criteria and it will then provide suggestions and alternative solutions to overcome the shortcomings of current open pit planning and scheduling methods and practices.

ULTIMATE PIT LIMIT DETERMINATION

The final pit limits define what is economically mineable from a given deposit. They identify which blocks should be mined and which ones should be left in the ground. In an effort to identify the blocks to be mined, an economic block model is created from the geologic grade model. This is done using production and process costs and commodity prices at current economic conditions (ie current costs and prices). Then using the economic block values, each positive block is further checked to see whether its value can pay for the removal of overlying waste blocks. The analysis is based on a break-even calculation that checks if undiscounted profits obtained from a given ore block can pay for the undiscounted cost of mining the waste blocks. This analysis is done by using computer programs that utilise either the 'cone mining' method or the Lerchs and Grossmann (LG) algorithm (Lerchs and Grossmann, 1965; Zhao and Kim, 1992; Muir, 2005). The LG algorithm guarantees that the defined pit limits maximise the undiscounted profit, while the cone-mining routine is heuristic and may give suboptimal results.

The decision as to what should be mined within the ultimate pit limits is time dependent and a proper solution needs to take into account the knowledge of when a given block will be mined and how much time is needed to strip the waste. The analysis of pit limits, which maximises the NPV, requires that the time value of money be taken into account when defining which blocks should be mined and which blocks should be left in the ground

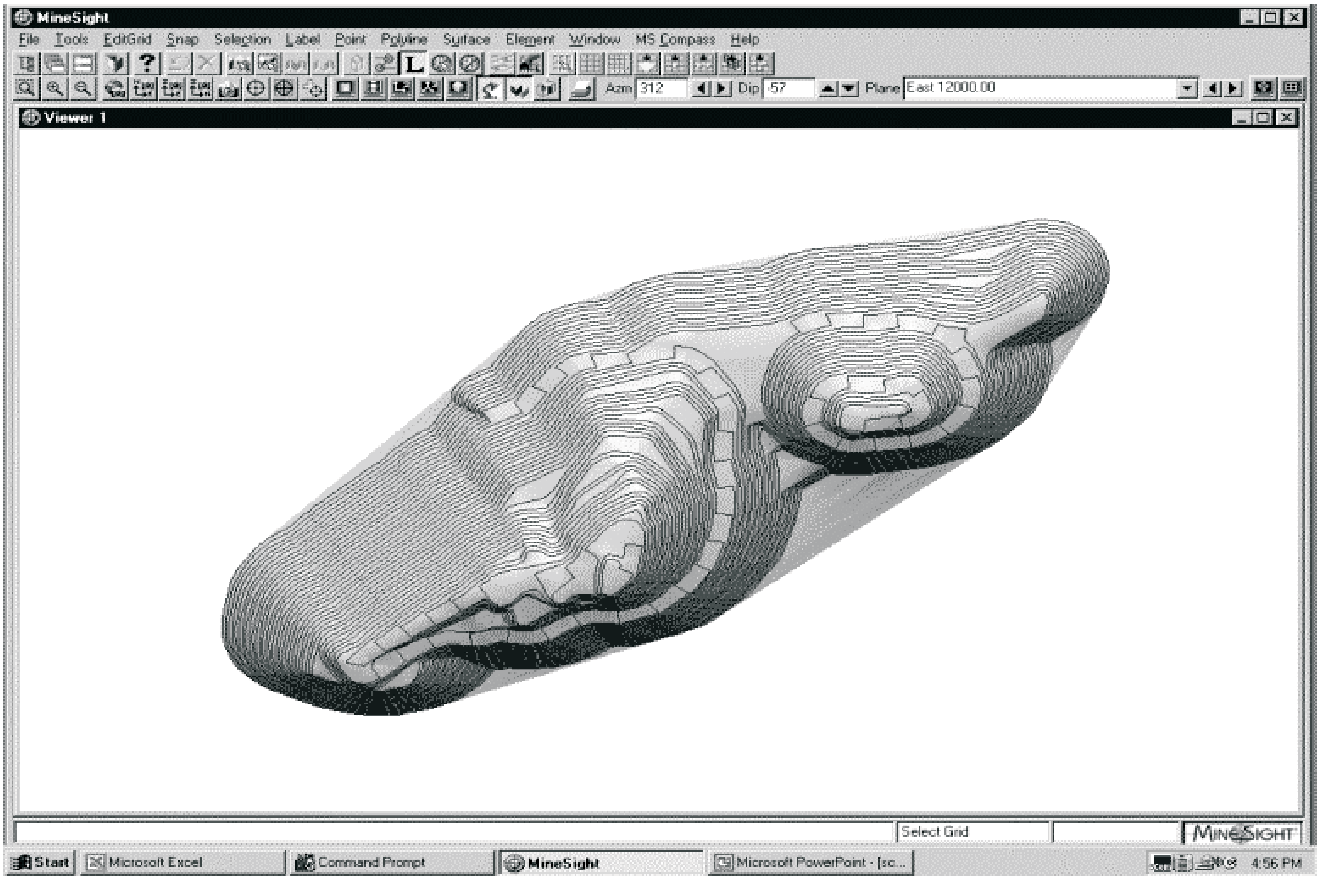


FIG 3 - Ultimate pit limits with designed haul roads.

during the life of the project. The pit limits that maximise the undiscounted profits for a given project will not maximise the NPV of the project.

To overcome this, it is suggested that one carries out a preliminary complete pit design and annual scheduling first. Next, a new economic block model should be generated using time dependent revenues and costs, which are determined by knowing when a given block will be mined and how it will be processed. Using this new economic block model, ultimate pit limits are re-calculated to reflect the effect of the time value of money on the final pit limits. It has been our experience that this new pit is always smaller than the previous one in terms of both contained ore and waste tonnes, and that it gives a higher NPV for the cash flows generated from it. This is due to the effect of discounting on the economic block value calculation, which tends to reduce the values of ore blocks to be mined in the later years of the deposit, while at the same time the waste mining costs to reach these blocks have to be incurred sooner. As such, the ore blocks, which are very marginal in value, drop out from the ultimate pit.

PUSHBACK GENERATION

As part of the planning and scheduling process, the intermediate pits leading to ultimate pit limits are determined to see how the pit surface will evolve through time. The procedure followed in the existing software packages to generate nested pits is to vary commodity price, costs or cut-off grades gradually from a low value to a high value. By changing the commodity price, for example, from a low value to a high value, one can generate a number of pits of increasing size and decreasing average value per tonne of ore contained in the pit. Since the smallest pit contains the highest value ore, the production is scheduled by mining the smallest pit first, followed by the production in larger pits (see Figure 4). The incremental mining from the smallest pit to the largest pit is referred to as 'pushback mining' and there are cases where production is scheduled from more than one pushback simultaneously. Once the nested pits are generated and smoothed and haul roads are added, they are used as pushbacks underlying practical plans, from which yearly schedules are generated.

The nested pit generation also does not take into account the time value of money. They are generated assuming an undiscounted value of the blocks. The pushbacks that will maximise the NPV of a project can be significantly different from the ones found by using existing procedures. It can be shown (Bernabe, 2001) that the nested pit generation from parameterising a single factor, such as the metal price or the production costs or the metal grades, will lead to suboptimum results when more than one process type exists for the ore types in the deposit.

LONG-TERM YEARLY SCHEDULES

Once the pushbacks are generated and designed for haul roads and minimum width requirements, the next step is to come up with yearly progress maps within the pushbacks by dividing the pushback into smaller increments. The yearly progress maps are usually generated by taking into account annual waste and ore mining tonnage requirements for different material types. Ore and waste discrimination is normally done on the basis of break-even cut-off grades. In the simplest case, yearly schedules are determined by mining from the top bench of the smallest pushback towards the bottom bench. Once a given pushback is exhausted, then mining from the top bench of the next pushback starts and continues until the pushback is exhausted. In many cases, this approach does not result in the best yearly schedules that maximise the NPV of the cash flows. Realising this, the newest schedulers in mine planning packages are designed to work with multiple pushbacks simultaneously and the mining activity can be scheduled from three or four pushbacks at the same time. In one scheduling package (Cai and Banfield, 1993) a schedule for a given year is determined by generating plans for all the possible mining scenarios between benches of the pushbacks and choosing the plan that gives the highest profit. This process is repeated for each year, one year at a time, until the whole deposit is mined out. In another scheduling package (Tolwinsky, 1998) possible yearly mine plans between pushbacks are further linked together year by year and analysed with respect to the resulting overall NPV. The overall plan that links together yearly schedules and results in the highest NPV is chosen as the optimum. In another package (Whittle, 1999), yearly ore mining is scheduled within the individual pushbacks in the pushback sequence by mining ore from the benches of the pushbacks without any consideration given to waste tonnages. The schedule obtained by using this process results in fluctuating waste tonnages from one year to another. These fluctuations are smoothed by mining from multiple pushbacks in a given year.

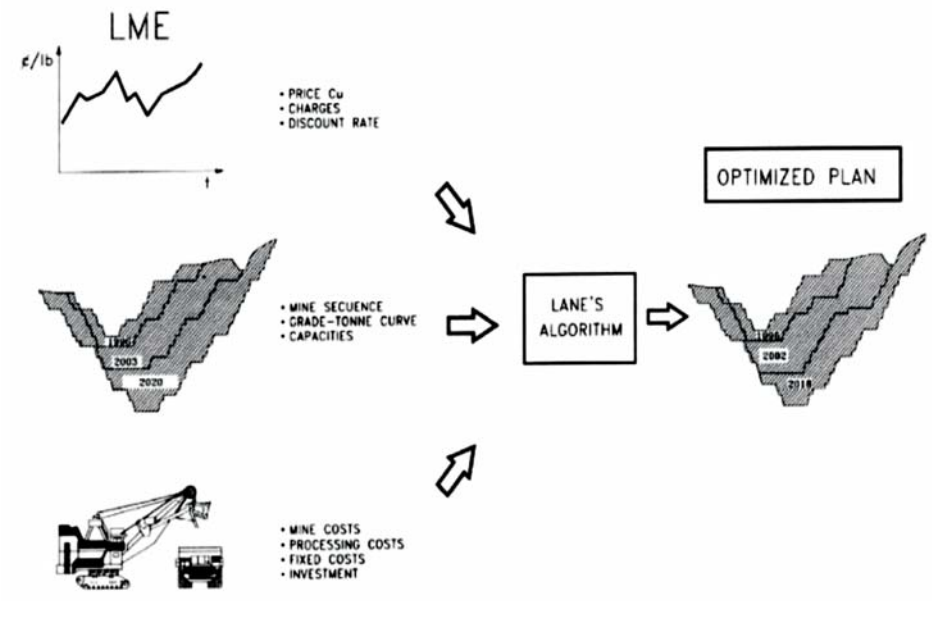


FIG 4 - The nested pits showing pit progress leading to ultimate pit limits (Camus and Jarpa, 1996).

The underlying concept in determining yearly schedules in all of the commercially-available scheduling packages assumes that the previously designed pushbacks will guide the scheduling process and this will result in a distribution of cash flows that will give the highest NPV. Of course this is not the case for many open pit mines, particularly for the ones where the stripping ratio varies significantly from one area of the pit to another, as well as for open pit mines that require the blending of different material types.

CUT-OFF GRADE STRATEGIES

The cut-off grade is the grade that is used to discriminate between ore and waste during scheduling. Most open pit mines are designed and scheduled using cut-off grades that are calculated by using break-even economic analysis. The use of break-even cut-off grades during open pit planning results in schedules that maximise the undiscounted profits (Dagdelen, 1992). The cut-off grade that maximises the NPV of the cash flows is a function not only of economic parameters but also of mining, milling and refinery capacity limitations, as well as the grade distribution within the deposit.

Lane (1964) proposed an algorithm to determine cut-off grades that maximise the NPV of a project subject to mine, mill and refinery capacity constraints. A cut-off grade strategy that results in a higher NPV for a given project starts with high cut-off grades during the initial years of the deposit. As the deposit matures the cut-off grades gradually decline to the break-even cut-off grade, depending upon the grade distribution of the deposit.

Various computer packages have been developed using Lane's algorithm (Lane, 1988; Dagdelen, 1992; Whittle, 1999). Application of these programs in determining the optimum cut-off grade strategy has resulted in significant improvements to the NPV of many projects (Camus and Jarpa, 1996).

FUTURE

The ultimate pit limits cannot be determined without knowing when the individual blocks will be mined. Determination of when a given block will be mined cannot be done without knowing the pushback sequence and the cut-off grade strategy. The pushback sequence and the cut-off grade strategy are themselves a function of when the blocks will be mined in the block model. As such, the optimum solution to this problem deals with many interdependent variables and the problem is currently solved by using heuristic trial and error techniques.

The determination of ultimate pit limits, yearly mine schedules and the cut-off grade strategies for a given open pit mine can be formulated using large-scale LP/IP models (Johnson, 1968; Dagdelen, 1985; Ramazan, 2005). These models include over 100 000 variables and 50 to 100 000 constraints (Akaike and Dagdelen, 1999; Hoerger *et al*, 1999).

The hardware and software technology available to implement the optimisation techniques based on Linear (LP) and Integer Programming (IP) have advanced to a point that we can now solve some of these problems without any difficulty. A good example of a large-scale LP application is Newmont Mining's Carlin operations, involving multiple open pit mines and plants. The implementation of a large scale LP model by the Newmont engineers in actual operations involved over 100 000 variables and close to 30 000 constraints. The model has proved to be successful, resulting in significant improvements in terms of maximising the NPV of these projects (Hoerger *et al*, 1999).

Future developments are needed to deal with uncertainty in geological aspects and commodity prices. Several studies in 2005 looked at these issues including Ramazan and Dimitrakopoulos (2005), Menabde *et al* (2005) and Godoy and Dimitrakopoulos (2005).

CONCLUSIONS

The large-scale open pit operations are looking at ways to improve the economics of their operations using NPV as a criterion. The mine planners of the new millennium are looking beyond the optimisation techniques that traditionally provided the highest undiscounted profits. The available commercial packages are retooling their programs to overcome shortcomings of traditional mine planning techniques in providing NPV-maximised mine plans and schedules. It is a matter of time before the latest operations research-based optimisation tools become commercially available and regularly used. The use of these optimisation tools by mine planners provides great opportunities for increased returns on the large amounts of capital being invested in these projects.

REFERENCES

Akaike, A and Dagdelen, K, 1999. A strategic production scheduling method for an open pit mine, in *Proceedings 28th APCOM*, pp 729-738.

Bernabe, D, 2001. Comparative analysis of open pit mine scheduling techniques for strategic mine planning of a copper mine in southern Peru, MSc thesis, Colorado School of Mines, Golden, Colorado.

Cai, Wen-Long and Banfield, F, 1993. Long range open pit sequencing – a comprehensive approach, in *Proceedings APCOM '93*, Montreal, pp 11-18.

Camus, J P and Jarpa, S G, 1996. Long range planning at Chuquicamata mine, in *Proceedings 26th APCOM Symposium*, pp 237-241.

Dagdelen, K, 1985. Optimum multi-period open pit mine production scheduling, PhD thesis, Colorado School of Mines, Golden, Colorado.

Dagdelen, K, 1992. Cutoff grade optimization, in *Proceedings 23rd APCOM*, pp 157-165, University of Arizona, Tucson.

Godoy, M C and Dimitrakopoulos, R, 2005. A multi-stage approach to profitable risk management for strategic planning in open pit mines, in *Orebody Modelling and Strategic Mine Planning* (ed: R Dimitrakopoulos), pp 311-317 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hoerger, S, Bachmann, J and Criss, K, 1999. Long term mine and process scheduling at Newmont's Nevada Operations, in *Proceedings 28th APCOM*, pp 739-748, Colorado School of Mines, Golden, Colorado.

Johnson, T B, 1968. Optimum open-pit mine production scheduling, PhD dissertation, University of California, Berkeley, Operations Research Department.

Lane, K F, 1964. Choosing the optimum cutoff grade, *Colorado School of Mines Quarterly,* 59:811-829.

Lane, K F, 1988. *Economic Definition of Ore – Cutoff Grades Theory and Practice* (Mining Journal Books Limited: London).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open pit mines, *Canadian Institute of Mining Bulletin*, 58(633):47-54.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2005. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning* (ed: R Dimitrakopoulos), pp 353-357 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Muir, D C W, 2005. Pseudoflow, new life for Lerchs-Grossman pit optimisation, in *Orebody Modelling and Strategic Mine Planning* (ed: R Dimitrakopoulos), pp 97-104 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S, 2005. Large scale production scheduling with the fundamental tree algorithm — Model, case study and comparions, in *Orebody Modelling and Strategic Mine Planning* (ed: R Dimitrakopoulos), pp 105-111 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2005. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning* (ed: R Dimitrakopoulos), pp 359-365 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Tolwinski, B, 1998. Scheduling production for open pit mines, in *Proceedings 27th APCOM*, pp 651-662, London, UK.

Whittle, J, 1999. A decade of open pit mine planning and optimization – the craft of turning algorithms into packages, in *Proceedings 28th APCOM,* pp 15-23, Colorado School of Mines, Golden, Colorado.

Zhao, Y and Kim, Y C, 1992. A new optimum pit limit design algorithm, in *Proceedings 23rd APCOM*, pp 423-434.

Network Linear Programming Optimisation of an Integrated Mining and Metallurgical Complex

E K Chanda[1]

ABSTRACT

Mining companies seek to mine, route and process ore to make the most efficient use of capital equipment during the life of the mine. The situation analysed in this paper relates to optimisation of medium-term production strategy for a group of mines and metallurgical plants. Typical operations under this scenario involve mining of crude ore from shafts and/or open pits; transportation of ore to the milling plants, run-of-mine stockpiles and leach-pads. The concentrate from the mill(s) is sent to the smelters and refineries, from where the finished metal is sent to the markets. If one assumes that the grade of run-of-mine ore varies according to source and that the milling plants are designed to handle different types of ore, plus the fact that mines and plants may separate by considerable distances, optimisation of the production plan becomes imperative. Most of the publications dealing with the subject of mine production planning are limited to mine scheduling optimisation and do not include metallurgical plants. However, the nature of the problem requires the application of a model that incorporates all the elements of the mineral production system.

The methodology outlined in this paper is based on a Network Linear Programming formulation of the production-planning problem for a mining and metallurgical complex. Network LP models are particularly useful in analysing production-distribution type systems such as the one involving a group of mines and metallurgical plants. The problem is formulated using the theory of dual-primal relationships in linear programming. The solution algorithm finds the minimum cost of production and distribution, hence the optimal production and material routing plan for a group of mines and metallurgical plants. The graphs of optimality conditions for each arc in the network could be exploited as a tool for strategic mine planning. The advantages of this formulation are outlined and its application is demonstrated using a hypothetical situation involving an integrated mining and metallurgical complex, specifically six mines, five concentrators, three smelter and two copper refineries.

A computer program called Linear Integer Discrete Optimiser (LINDO) is used to solve the network linear programming model. This program allows the user to quickly input an LP formulation, solve it and perform 'what if' type analyses.

INTRODUCTION

The practical mine planning problem analysed in this paper relates to the optimisation of a medium-term production strategy for a group of mines and metallurgical plants (concentrators, smelters and refineries). Most of the publications dealing with the subject of production planning focus on mine scheduling optimisation and do not include metallurgical plants (Thomas, 2001). However, the nature of the problem requires a model that incorporates all the elements of the production system. Hoerger *et al* (1999) have described a mixed integer/linear programming model for long-term scheduling that includes material tonnage flows between mines, stockpiles and process plants. The resulting Linear Programming (LP) model is very large in terms of the number of variables. The methodology outlined in this paper is based on a network linear programming formulation of the problem of production planning optimisation for a mining and metallurgical complex. Models called network LPs are particularly useful in analysing production-distribution type systems such as the one discussed in this paper.

1. MAusIMM, MTEC Senior Lecturer, Mining Engineering, Curtin University of Technology, WA School of Mines, Locked Bag 22, Kalgoorlie WA 6433, Australia. Email: chandae@wasm.curtin.edu.au

The section entitled 'Linear programming and network techniques' introduces the structure of network LPs, primal-dual relationships and complementary slackness conditions. This review of relevant principles sets the scene for their application to the problem of production planning for a mining and metallurgical complex, presented in the section entitled: 'Network LP formulation of mining and metallurgical production planning problem'. Finally, the section entitled 'A hypothetical mining and metallurgical complex', presents results of LINDO optimisation of the production planning for a typical mining and metallurgical complex.

LINEAR PROGRAMMING AND NETWORK TECHNIQUES

Linear Programming (LP) is a mathematical procedure for determining optimal allocation of scarce resources. LP has been used to solve a variety of practical planning problems in the industry including agricultural, banking, government services, manufacturing and transport problems. Application of this technique in mining dates back over 40 years. Linear programming is the most widely applied operations research technique in the mining industry. Linear programming principles have successfully been used for production scheduling in open pit and underground mining environments, each with their own specific needs (Ricciardone and Chanda, 2001; Chanda, 1990; Saul, 1990; Dagdelen, Topal and Kuchta, 2000; Scheepers and Wellbeloved, 1992; Graham-Taylor, 1992; Ramazan, 2001; Ramazan and Dimitrakopoulos, 2004). The approach adopted in this paper is to combine the concepts of duality in linear programming and network flow to model the production-planning problem as discussed in the introduction. Network LPs are particularly useful in analysing production-distribution type systems. These models have the following advantages:

- they are describable using simple graphical figures (networks),
- they have integer answers and one may find a network LP a useful device for describing and analysing mine-mill production and material routing strategies, and
- they are frequently easier to solve than general linear programs.

In this section, a brief overview of duality in linear programming and the formulation of equivalent network flow (minimal cost) is provided. Though there are a number of techniques for finding the optimal flow through a network, the algorithm in LINDO (Schrage, 1999) is employed because of its simplicity and use as a strategic tool in production planning. This is demonstrated in the section entitled: 'Network formulation of mine production planning problem'. For more details on the theory of network LPs the reader is referred to Ahuja, Thomas and Orlin, 1993 and Bazaraa, Jarvis and Sherari, 1990.

Theoretical background

Each linear programming problem called the primal has a closely related associated linear programming problem called the dual problem (Fulkerson, 1961). The following example illustrates how linear programming duality can be used to analyse production-planning problems in the minerals industry. Consider a copper/cobalt mining operation with six sources of ore (shafts).

Table 1 presents the mine planning data for this operation. It is desired to optimise the mining plan for the month using linear programming. It is assumed that shaft capacities are sufficient to handle the planned mine production. The budget targets production of 20 000 and 200 tonnes of finished copper and cobalt respectively during the period.

TABLE 1

Mine planning data for a copper/cobalt mining operation.

Parameter	Ore source (shaft)					
	1	2	3	4	5	6
Copper grade (%)	2.70	5.00	3.50	4.50	0.90	3.90
Cobalt grade (%)	0.40	0.70	0.07	0.08	0.60	0.20
Unit cost ($/tonne ore)	7.00	5.00	6.00	8.00	4.00	6.50

The primal problem

The LP formulation of this problem is presented as follows:

Let:

x_i = unknown tonnes of ore to be produced from shaft j

The objective is to minimise the total cost of mining. This optimisation criterion will ensure that the mining contribution to profit over the quarter will be maximised. Thus, the objective function is:

$$\text{Minimise}\, Z = 7x_1 + 5x_2 + 6x_3 + 8x_4 + 4x_5 + 6.5x_6$$

Metal production targets are formulated as constraints:

$$\text{(i)}\ 0.027x_1 + 0.050x_2 + 0.035x_3 + 0.045x_4 + 0.009x_5 + 0.039x_6 \geq 20000$$

$$\text{(ii)}\ 0.004x_1 + 0.007x_2 + 0.0007x_3 + 0.0008x_4 + 0.006x_5 + 0.002x_6 \geq 200$$

Non-negativity constraint ensures that production from each shaft is positive:

$$x_j \geq 0 \quad \forall\, j$$

The dual problem

The dual problem to the above primal problem is formulated as follows:

Let:

y_1 = price of copper on the world market ($/tonne)

y_2 = price of cobalt on the world market ($/tonne)

The objective function is to maximise metal sales value in dollars:

$$\text{Maximise}\ \upsilon = 20\,000\, y_1 + 200\, y_2$$

The objective function is subject to the following constraints:

$$\text{(i)}\ \ 0.027y_1 + 0.004y_2 \leq 7.0$$
$$\text{(ii)}\ \ 0.050\, y_1 + 0.007y_2 \leq 5.0$$
$$\text{(iii)}\ 0.035y_1 + 0.0007y_3 \leq 6.0$$
$$\text{(iv)}\ \ 0.045y_1 + 0.0008y_2 \leq 8.0$$
$$\text{(v)}\ \ 0.009y_1 + 0.006\, y_2 \leq 4.0$$
$$\text{(vi)}\ \ 0.039y_1 + 0.002\, y_2 \leq 6.5$$

Non-negativity constraints:

$$y_j \geq 0 \quad \forall\, j$$

Complementary slackness optimality conditions

The necessary and sufficient conditions for a feasible solution of primal and dual to be optimum is they satisfy:

(1) $Y(AX\text{-}B) = 0$

(2) $X(C\text{-}YA) = 0$

where:

X = decision variables in the primal problem (vector)

Y = decision variables in the dual problem (vector)

C = coefficients of the objective function in the primal problem (vector)

B = coefficients of the objective function in the dual problem (vector)

NETWORK LP FORMULATION OF THE MINING AND METALLURGICAL PRODUCTION PLANNING PROBLEM

The above concepts can be applied to production planning for a mining and metallurgical complex. To illustrate the practical application of complementary slackness conditions, the following problem is presented. Consider a simple mining-processing-marketing system as shown in Figure 1. Formulation of an LP model to optimise the production strategy for the system follows.

FIG 1 - Mining-processing-marketing business system.

The following notation is used for the labels in Figure 1 (eg (1,5/4)):

$$(l_{ij}, u_{ij} / c_{ij})$$

where:

l_{ij} = lower bound of material flow through arc (i,j)

u_{ij} = upper bound of material flow through arc (i,j)

c_{ij} = cost per unit flow of material through arc (i,j)

Figure 1 is in fact a network representation of movement of ore from the mine (node one) to the plants (nodes two and three) and marketable product to the market. In certain network formulations, the principle of conservation of flow has to be maintained at all nodes. Closing the circuit from node four to node one with a negative unit cost does this. The objective here is to optimise flow through the network, ie minimise the total cost of the production-distribution system. The out-of-kilter formulation of the primal-dual minimal cost network flow problem for the system is presented as follows.

Let:

X_{ij} = amount of material processed in process (i,j) of the system

The objective function is to minimise total cost of flow through the network.

$$\text{Minimise } Z = \sum_{i,j} c_{i,j}.x_{i,j} \tag{1a}$$

There are three types of constraints in this LP system:

1. Conservation of flow through each node:

$$\sum_{i} x_{i,j} - \sum_{j} x_{j,i} = 0 \quad \forall\ i,\ j \tag{1b}$$

2. Lower bound flow through the arcs:

$$x_{i,j} \geq l_{i,j} \quad \forall\ i \tag{1c}$$

3. Upper bound on flow through the arcs:

$$-x_{i,j} \geq -u_{i,j} \quad \forall\ j \tag{1d}$$

The above formulation is equivalent to the minimal cost flow problem. Equations 1a to 1d are taken over existing arcs only. It is assumed that c_{ij}, l_{ij} and u_{ij} are integral, although this is not a requirement in practice.

For the dual problem, the following dual variables are defined:

π = dual variable for the conservation of flow at each node (node potential)

ϕ= dual variable for the lower bound on flow constraint

ψ= dual variable for the upper bound on flow constraint

For a given set of node potential π, the reduced cost of an arc is defined as:

$$c_{ij}^{\pi} = c_{ij} - \pi_i + \pi_j \tag{2a}$$

The objective function for the dual problem is:

$$\text{Maximise } \upsilon = \sum_{i,j} \phi_{i,j} l_{i,j} - \sum_{i,j} \psi_{i,j} u_{i,j} \tag{2b}$$

The general equation for the dual constraints is as follows:

$$\pi_i - \pi_j + \phi_{i,j} - \psi_{i,j} = c_{i,j} \tag{2c}$$

Non-negativity:

$$\phi_{i,j} \geq 0 \ \ ; \ \ \psi_{i,j} \geq 0 \tag{2d}$$

The complementary slackness conditions for optimality of the OKA formulation are the following:

$$\pi_i \left(\sum_{i,j} x_{i,j} - \sum_{j,i} x_{j,i} \right) = 0 \tag{3a}$$

$$\phi(x_i - l_i) = 0 \tag{3b}$$

$$\psi(u_{i,j} - x_{i,j}) = 0 \tag{3c}$$

and

$$\left[c_{i,j} - (\pi_i - \pi_j + \phi_{i,j} - \psi_{i,j}) \right] x_{i,j} = 0 \tag{3d}$$

As mentioned earlier, any conservation of flow that satisfies the above equations will be optimal. The problem, then, is to search over values of π_i, and conserving x_{ij} until these conditions are satisfied. The complimentary slackness optimality conditions can be stated simply as follows (Ahuja, Thomas and Orlin, 1993):

$$\text{If } x_{ij} = l_{ij}, \text{ then } c_{ij}^{\pi} \geq 0 . \tag{4a}$$

$$\text{If } l_{ij} < x_{ij} < u_{ij}, \text{ then } c_{ij}^{\pi} = 0 . \tag{4b}$$

$$\text{If } x_{ij} = u_{ij}, \text{ then } c_{ij}^{\pi} \leq 0 . \tag{4c}$$

This is the basis for the solution procedure called the out-of-kilter algorithm. The name out-of-kilter reflects the fact that arcs in the network either satisfy the complimentary slackness optimality conditions (are in kilter) or do not (are out-of-kilter). The so-called 'kilter diagram' is a convenient way to represent these conditions (Ahuja, Thomas and Orlin, 1993). As shown in Figure 2, the kilter diagram of an arc (i,j) is the collection of all points (x_{ij}, $c_{ij}\pi$) in the two-dimension plane that satisfy the optimality conditions. For every arc (i,j), the flow $_{xij}$ and reduced cost $c_{ij}\pi$ define a point (x_{ij}, $c_{ij}\pi$) in the two-dimensional plane. If the point lies on the thick lines in Figure 2, it is in-kilter, otherwise out-of-kilter. One can define a kilter number k_{ij} of each arc (i,j) as the magnitude of the change in x_{ij} required making the arc an in-kilter arc while keeping $c_{ij}\pi$ fixed. As expected, the kilter number of any in-kilter arc equals zero. The three-kilter states marked by α (non-profitable), β and γ (profitable) in Figure 2 correspond to arc states satisfying the complimentary optimality conditions (Equations 4a, 4b and 4c). Any arc (processing path) (i,j) for which (x_{ij},c_{ij}) lies on γ, is a profitable arc and is therefore, appropriately at its upper bound, and any arc (i,j) for which (x_{ij}, c_{ij}) lies on α is a non-profitable arc (and is therefore appropriately at its lower bound. From a mining economics point of view, it is preferable for all processing paths to be profitable. This concept is not investigated further here.

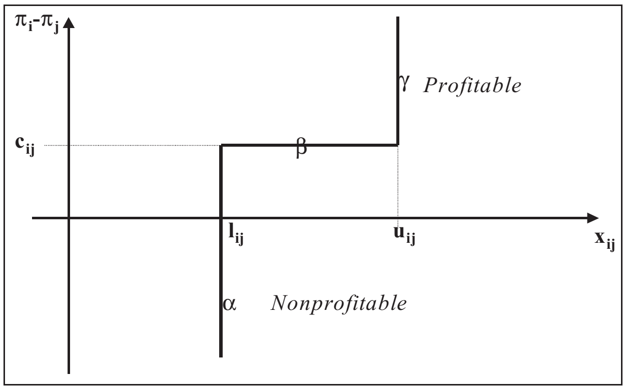


FIG 2 - Graph of optimality conditions for arc (i,j).

A HYPOTHETICAL MINING AND METALLURGICAL COMPLEX

Model development

Suppose that a mining company has a number of ore sources (open pits, underground mines and stockpiles) producing copper ore for delivery to a number of concentrators for downstream processing. The following assumptions are made for the hypothetical copper mining and metallurgical complex (number of facilities):

- underground mines (UG) = 5
- open pits (OPT) = 1

- concentrators (CT) = 5
- smelters (SM) = 3
- refineries (RF) = 2

Each mine has a concentrator located in the vicinity of the mine. The copper concentrate is transported to smelters located in the vicinity of the mine(s). Some of the concentrate is transported to smelters located beyond a radius of more than fifty kilometres from the concentrators. The copper anodes from the smelters are transported to the two refineries for electrowinning of copper. Copper cathodes are shipped to various markets from the refineries. The basic business structure is shown in Figure 3, while Figure 4 shows the network model of the system. The model shown in Figure 4 has the following types of nodes:

1. Mine nodes, representing various ore sources.
2. Plant nodes representing concentrators, smelters and refineries.
3. Intermediate nodes corresponding to each material type processed at a plant. In this application, it is assumed that there is no differentiation in material (ore) types.
4. Market nodes corresponding to each market region. There are two types of arcs in the model:
   - Production arcs – they connect a mine or plant node to an intermediate node. The cost of this arc is the cost of ore mining ($/tonne ore). Production control may place upper and lower bounds on these arcs.
   - Transportation arcs – connect intermediate nodes to plant nodes in accordance with the copper production process. The cost of such an arc corresponds to the cost of transporting the process product from one plant to the other.

The problem is to generate a medium-term (quarterly) production plan for the mining and metallurgical complex with the objective of minimising the total unit cost of production and transportation.

Looking at Figure 4, it is quite clear that there are several combinations of routes along which the material can flow. Each of the routes has a different cost structure, and therefore an opportunity to generate revenue. Clearly, the production and transport plans for the mining and metallurgical complex

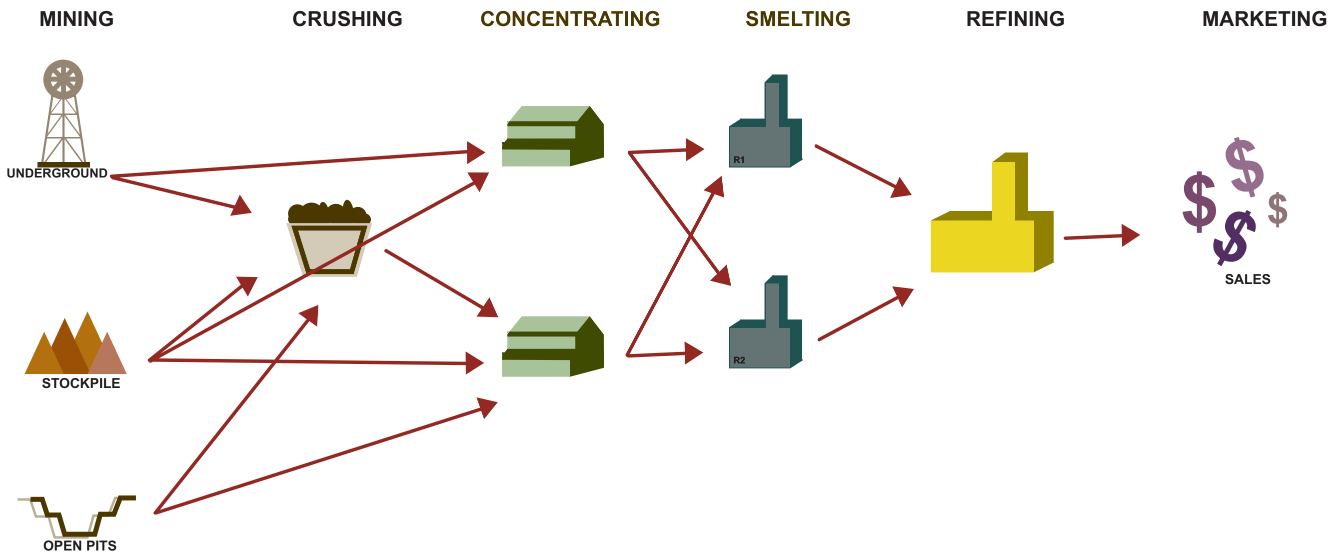


FIG 3 - Basic mining and metallurgical business complex.

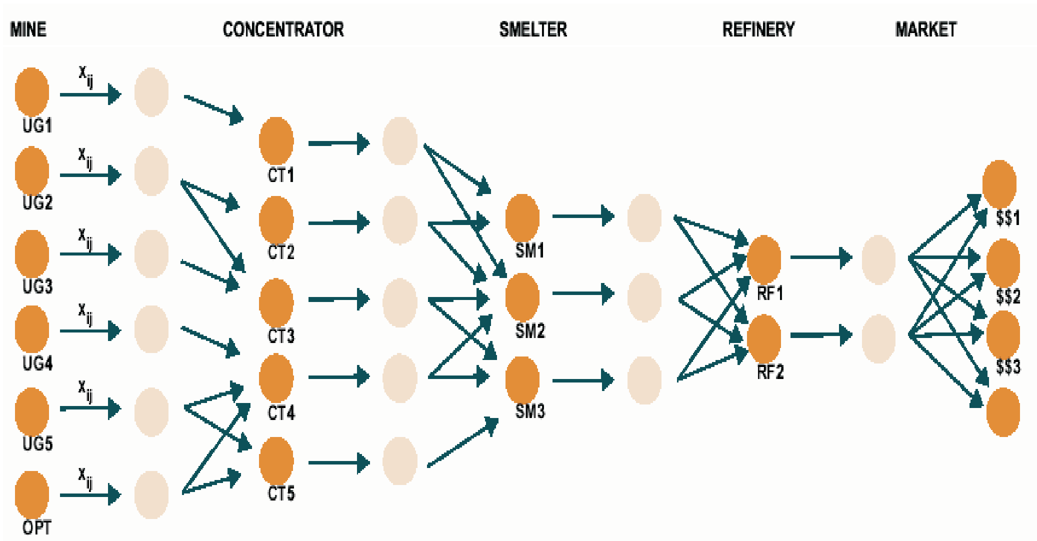


FIG 4 - Network model of the hypothetical mining and metallurgical complex.

correspond in a one-to-one fashion with the feasible flows in this network model. Consequently, a minimum cost flow would yield an optimal production and shipping plan.

Let X_{ij} represent the equivalent tonnage of ore that flows along route (i,j). For each arc in the network the lower bound on material tonnage is l_{ij} and the upper bound is u_{ij}. The unit cost of production/transportation is c_{ij} (\$/tonne ore). Unless otherwise stated, the lower bound on the flow through an arc is assumed zero and the upper bound infinity. The unit costs are made of the following:

- production cost at source i,
- production cost at destination j, and
- transport cost from source i to destination j.

Considering the hypothetical mining and metallurgical complex the following costs in \$/tonne of ore apply:

- mining cost at each ore source,
- ore transport cost from mine to mill,
- milling cost at the concentrator,
- transport for concentrates from the concentrator to the smelter,
- smelting cost,
- transport cost for copper anodes from smelter to refinery,
- refining cost, and
- shipping cost for wire-bars from the refinery to markets.

Table 2 presents the mine production planning criteria for the hypothetical mining and metallurgical complex. Tables 3 and 4 list the cost elements for the production facilities and different routes respectively. For consistence, all costs are expressed in \$/tonne ore equivalent.

Modelling in LINDO

Schrage (1999) describes the application of LINDO (Linear INteractive Discrete Optimiser) software in solving Network Linear Programming models. This software was chosen for this analysis because it is easy to use and is easily understandable to an average mine planner. The essential condition on an LP for it to be a network problem is that it is representable as a network. In this example, there are four levels of nodes and several arcs between nodes.

The following simplifying assumptions are made:

- mining capacity is a major consideration;
- milling capacity is not very critical, as the plants are running at 70 per cent capacity;
- only one type of ore (oxides) is considered, hence simplifying the network;
- unless otherwise specified the lower and upper bound on the arcs equal to zero and infinity respectively;
- two market destinations for finished copper; demand as indicated in the model; and
- material flow as shown in Figure 4, except for the market destinations.

Defining variables in an obvious way, the general LP describing this problem is:

```
! Group Mine/Plant Production Plan – Linear Programming System
! 2nd Quarter 2004
! Analyst: Senior Mining Engineer
! Run: 15/6/04
! Note: coefficients of each variable in the objective function equals
! the sum of production and transport costs
! Objective Function
MIN
2.9XUG1CT1+5.5XUG2CT2+6.1XUG2CT3+3.9XUG3CT3+2.
8XUG4CT4+5.8XUG5CT4+4.7XUG5CT5+3.1XOPTCT4+2.3X
OPTCT5+25.07XCT1SM1+25.04XCT1SM2+30.07XCT2SM1+
30.05XCT2SM2
```

TABLE 2

Production planning criteria.

Mining	UG1	UG2	UG3	UG4	UG5	OPT
Run-of-mine grade (% Cu)	3.6	4.2	3.0	3.3	3.8	5.2
Contained copper (kg Cu/t ore)	36.0	42.0	30.0	33.0	38.0	52.0
Mining capacity (ore tonnes/quarter)	750 000	500 000	1 000 000	575 000	624 000	375 000
Milling	**CT1**	**CT2**	**CT3**	**CT4**	**CT5**	
Mill feed grade (% Cu)	3.6	3.6	3.6	3.6	3.6	
Mill recovery (%)	85.0	87.0	85.0	87.0	84.0	
Copper recovered in mill (kg/t ore)	30.6	31.3	30.6	31.3	30.2	
Smelting	**SM1**	**SM2**	**SM3**			
Smelting loss (kg Cu/t ore)	0.10	0.15	0.20			
Refining	**RF1**	**RF2**				
Refining loss (kg Cu/t ore)	0.09	0.09				

TABLE 3

Production cost for the hypothetical mining-metallurgical complex.

Facility (mine/plant)	UG1	UG2	UG3	UG4	UG5	OPT	CT1	CT2	CT3	CT4	CT5	SM1	SM2	SM3	RF1	RF2
Unit cost (\$/t ore)	2.5	5.0	3.0	2.5	4.5	2.0	25	30	16	22	33	12	15	10	5	7

TABLE 4

Transport cost ($/t ore) matrix for the hypothetical mining-metallurgical complex.

	UG1	UG2	UG3	UG4	UG5	OPT	CT1	CT2	CT3	CT4	CT5	SM1	SM2	SM3	RF1	RF2
UG1	-	-	-	-	-	-	0.4	0.7	1.00	1.3	1.5					
UG2	-	-	-	-	-	-	1.5	0.5	1.5	1.1	1.2					
UG3	-	-	-	-	-	-	1.5	1.0	0.4	0.8	0.9					
UG4	-	-	-	-	-	-	1.2	1.4	1.6	0.3	0.9					
UG5	-	-	-	-	-	-	1.3	1.4	1.2	1.3	0.2					
OPT	-	-	-	-	-	-	1.0	1.0	1.0	1.1	0.3					
CT1							-	-	-	-	-	0. 07	0.04	0.30		
CT2							-	-	-	-	-	0.01	0.05	0.04		
CT3							-	-	-	-	-	0.04	0.01	0.05		
CT4							-	-	-	-	-	0.03	0.06	0.01		
CT5							-	-	-	-	-	0.04	0.05	0.10		
SM1												-	-	-	0.1	0.4
SM2												-	-	-	0.3	0.1
SM3												-	-	-	0.5	0.6
RF1															-	-
RF2															-	-

```
+16.01XCT3SM2+16.05XCT3SM3+22.06XCT4SM2+22.01XC
T4SM3+23.1XCT5SM3+12.1XSM1RF1+12.04XSM1RF2+15.3
XSM2RF1+15.1XSM2RF2+10.5XSM3RF1+10.5XSM3RF2+5.
09XRF1$$1+5.05XRF1$$2+7.9XRF2$$1+7.05XRF2$$2
SUBJECT TO
! Constraints
! Mining Capacity
2)  XUG1CT1=750000
3)  XUG2CT2+XUG2CT3=500000
4)  XUG3CT3=1000000
5)  XUG4CT4=575000
6)  XUG5CT4+XUG5CT5=624000
7)  XOPTCT4+XOPTCT5=375000
!
! Flow balance Constraints
! Concentrators
8)  -XUG1CT1+XCT1SM1+XCT1SM2=0
9)  -XUG2CT2+XCT2SM1+XCT2SM2=0
10) -XUG2CT3-XUG3CT3+XCT3SM2+XCT3SM3=0
11)-XUG4CT4-XUG5CT4-XOPTCT4+XCT4SM2+XCT4SM3=
0
12) -XUG5CT5-XOPTCT5+XCT5SM3=0
! Smelters
13) -XCT1SM1-XCT2SM1+XSM1RF1+XSM1RF2=0
14)-XCT1SM2-XCT2SM2-XCT3SM2-XCT4SM2+XSM2RF1+
XSM2RF2=0
15)-XCT3SM3-XCT4SM3-XCT5SM3+XSM3RF1+XSM3RF2=
0
! Refineries
16) -XSM1RF1-XSM2RF1-XSM3RF1+XRF1$$1+XRF1$$2=0
17)-XSM1RF2-XSM2RF2-XSM3RF2-XSM3RF2+XRF2$$1+X
RF2$$2=0
! Market demand
18) -XRF1$$1-XRF2$$2=-1000000
19) -XRF1$$2-XRF2$$2=-2000000
END
```

There is one constraint for each node that is of the 'sources = uses' form. For example, constraint number three states that the amount transported out, minus the amount transported in, must equal zero.

Table 5 presents the base case optimal production plan. Note that the optimal solution is in terms of equivalent ore tonnes flowing through the network. For example, the mine should haul 750 000 from UG1 to CT1. The minimised cost of production and transport is $156 375 700. For arcs connecting the concentrators and smelters, the amount of concentrate flowing through can easily be calculated from the concentration ratio, which is a function of run of mine and concentrate grades. Similar calculations can be carried out to determine the equivalent tonnes of copper anodes and cathodes flowing through the arcs connecting the smelters and refineries.

Analysis of results and sensitivity analysis

Sensitivity analysis involves the study of the responsiveness of the conclusions of an analysis to changes or errors in input values used to generate a particular solution to the LP network. This is equivalent to answering 'what if' type of questions by interrogating the model. As an example, the impact of reducing the number of refineries to one is considered, ie remove refinery RF2 from the model. This action results in an optimal solution of $139 696 240 (being the minimum cost of production and transport). Of course, the flow of material through the network changes, but the single refinery produces enough copper to satisfy the market. Such types of analysis can be easily performed on any business decision that the company makes, in order to evaluate the impact of the decision on the business.

CONCLUSIONS

The Network LP formulation of the problem of optimising the production planning for a mining and metallurgical complex results in a solution procedure that is easier to solve compared to the general Linear Programming model. There are three types of data required for the Network LP model:

1. for each node (facility) the amount of material available or its capacity;
2. for each arc or route, the cost per unit of material transported along that route; and

TABLE 5

Optimum computer solution.

Variable	Value	Reduced cost
XUG1CT1	750 000	0
XUG2CT2	0	0
XUG2CT3	500 000	0
XUG3CT3	1 000 000	0
XUG4CT4	575 000	0
XUG5CT4	0	0
XUG5CT5	624 000	0
XOPTCT4	375 000	0
XOPTCT5	0	0.29
XCT1SM1	750 000	0
XCT1SM2	0	3.0
XCT2SM1	0	10.4
XCT2SM2	0	13.44
XCT3SM2	74 000	0
XCT3SM3	1 426 000	0
XCT4SM2	0	0.09
XCT4SM3	950 000	0
XCT5SM3	624 000	0
XSM1RF1	0	4.62
XSM1RF2	750 000	0.000000
XSM2RF1	0	4.76
XSM2RF2	74 000	0.0
XSM3RF1	3 000 000	0.0
XSM3RF2	0	3.34
XRF1$$1	1 000 000	0
XRF1$$2	2 000 000	0
XRF2$$1	824 000	0
XRF2$$2	0	13.93

3. the lower and upper bound for the quantity of material along that route.

For the hypothetical mining-metallurgical complex presented here, the base case optimum production plan costs $156 375 700, which is the absolute minimum under the given set of economic and technical data. The material flows through the network of mines and metallurgical plants are thus optimised and satisfy all the capacity, demand and flow constraints.

Computerised modelling and optimisation allows one to investigate various business decisions prior to actual implementation. For example, shutting down refinery RF2 would result in the total cost of production and transportation reducing to $139 696 240 for the quarter, a saving of $16 million compared to operating the two refineries.

REFERENCES

Ahuja, R K, Thomas, L M and Orlin, J B, 1993. *Network Flows – Theory, Algorithms and Applications*, 846 p (Prentice-Hall).

Bazaraa, M S, Jarvis, J J and Sherari, H D, 1990. *Linear Programming and Network Flows*, second edition, 565 p (John Wiley and Sons).

Chanda, E K, 1990. An application of integer programming and simulation to production planning for a stratiform orebody, *Mining Science and Technology,* 11:165-172.

Dagdelen, K, Topal, E and Kuchta, M, 2000. Linear programming applied to scheduling of iron ore production, at the Kiruna Mine, Sweden, in *Proceedings Ninth International Symposium on Mine Planning and Equipment Selection* (eds: G N Panagiotou and T N Mikhalakopoulos), pp 187-192 (Balkema: Rotterdam).

Fulkerson, 1961. An out-of-kilter method for minimal cost flow problems, *SIAM Journal on Applied Mathematics*, 9:18-27.

Graham-Taylor, T, 1992. Production scheduling using linear and integer programming, in *Proceedings 1992 AusIMM Annual Conference*, pp 159-162 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hoerger, S, Bachmann, J, Criss, K and Shortridge, E, 1999. Long term mine and process scheduling at Newmont's Nevada operations, in *Proceedings 28th International Symposium on Computer Applications in the Minerals Industries* (ed: K Dagdelen), pp 740-748 (Society for Mining, Metallurgy, and Exploration: Littleton).

King, B, 2001. Optimal mine scheduling, in *Mineral Resource and Ore Reserve Estimation – The AusIMM Guide to Good Practice* (ed: A C Edwards), pp 451-458 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S, 2001. Open pit mine scheduling based on fundamental tree algorithm, PhD thesis, Colorado School of Mines, Mining Engineering Department.

Ramazan, S and Dimitrakopoulos, R, 2004. Recent applications of operations research and efficient MIP formulations in open pit mining, *SME Transactions*, 316:73-77.

Ricciardone, J and Chanda, E K, 2001. Optimising life of mine production schedules in multiple open pit mining operations: a study of effects of production constraints on NPV, *Mineral Resources Engineering*, 10(3):301-314.

Saul, B, 1990. Optimisation of waste haulage with linear programming, in *Proceedings Mine Planning and Equipment Selection*, (ed: R K Singhal) pp 455-461 (Balkema: Rotterdam).

Scheepers, L and Wellbeloved, D, 1992. Optimisation of integrated mining and metallurgical complexes by means of linear programming and case study, in *Survival Strategies for the Metallurgical Industry* (South African Institute of Mining and Metallurgy: Johannesburg).

Schrage, L, 1999. *Optimisation Modelling with LINGO*, 530 p (LINDO Systems Inc).

Thomas, G S, 2001. Optimisation of mine production scheduling – the state-of-the-art, in *Mineral Resource and Ore Reserve Estimation – The AusIMM Guide to Good Practice* (ed: A C Edwards), pp 441-450 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Application of Conditional Simulations to Capital Decisions for Ni-Sulfide and Ni-Laterite Deposits

O Tavchandjian[1], A Proulx[2] and M Anderson[3]

ABSTRACT

Prior to the acquisition of data from production drilling and grade control sampling, the spatial density of data is usually insufficient to properly address issues related to short-scale variability. Grade interpolation, whether conducted through ordinary kriging or other linear or non-linear regression techniques, usually suffers from significant over-smoothing or conditional bias. Four examples presented in this paper show that conditional simulations provide a viable and powerful alternative in assessing the sensitivity of key variables that are critical to the decisions made prior to moving forward with significant capital expenditures. These variables include the selection of the most appropriate mining method and mining equipment, the optimum cut-off strategy and the short-term variability constraints on process plant feed. The results also demonstrate that conditional simulations can be used to assess the risk associated with many of the technical aspects of the project and its financial performance.

INTRODUCTION

Traditional approaches to mineral deposit appraisal use non-geostatistical and/or geostatistical estimation methods to provide optimal local block grade estimates. When the drilling density is too sparse for the level of detail required for mine planning, these methods fail to properly represent the spatial variability of the estimated grade. Mining decisions made using the resulting smooth estimates may lead to false assumptions about the mineral deposit. Alternative interpolation strategies aimed at reducing the smoothing effect, such as using fewer samples in the search ellipsoid or interpreting geology in a deterministic model, usually result in a grade distribution with significant conditional bias (Krige, 1996).

As with many other metal deposits, the success or failure of a Ni-sulfide or a Ni-laterite mining project is highly dependent on a few key variables, all ultimately related to metal price and metal grade. A proper characterisation of the spatial variability in the grade distribution can lead to more realistic mining and/or processing assumptions and reduced project risk. Project evaluations, which can demonstrate to the company management and investors that the risk is recognised and quantified, and that the implementation plan includes a strategy to manage that risk, have a better chance of advancing to the construction and production stages.

Conditional simulation ('CS') methods aim at reproducing the *in situ* grade variability as opposed to obtaining optimal local estimates. The end results are models of equal probable realisations, which reproduce the input sample data histogram and variogram and are conditioned to local sample point values. A proper characterisation of the spatial variability of the grade provides mining engineers and metallurgists with realistic models for mine planning and the information required to address processing issues related to short-term variability in the feed grade to the processing plant (eg Abzalov and Mazzoni, 2007, this volume; Audet and Ross, 2007, this volume).

1. CVRD Inco, 2060 Flavelle Boulevard, Sheridan Park, Mississauga ON L5K 1Z9, Canada. Email: tavchandjiano@inco.com

2. CVRD Inco, PO Box 5000, Thompson MB R8N 1P3, Canada. Email: aproulx@inco.com

3. Voisey's Bay Nickel Company, Suite 700, Baine Johnston Centre, 10 Fort William Place, St John's NL A1C 1K4, Canada. Email: manderson@inco.com

This paper presents four examples of practical applications of CS in both Ni-sulfide and Ni-laterite deposits conducted by Inco over the past seven years. These examples cover a wide range of projects from the optimisation of open pit and underground mining plans to the risk assessment on the variability of the daily feed grade to both mineral beneficiation and chemical processing facilities. The successful deepening of the Birchtree mine in Northern Manitoba, was dependent on the selection of optimum mining cut-off and production rate together and on a flexible mining schedule. A reliable model was also required to properly assess the risk-weighted benefits of raising the cut-off in an orebody with significant short-scale variability. Similar challenges were faced in the deepening of the Thompson 1D orebody in the same mining camp. In this case, CS were also used early in the evaluation process to compare the economic performance of bulk and selective mining methods. During the feasibility study of an open pit operation at Voisey's Bay in Northern Labrador, the short-scale variability of the feed to the concentrator was identified as an area of risk for the project. In particular, significant variations in the Cu to Ni ratio on a daily, weekly or monthly basis could result in processing recovery losses. The optimisation of the number of concurrent mining faces, the size of stockpiles and the mining sequence was investigated based on CS results in order to minimise the impact of feed variability on milling recovery.

At the Goro Project in New Caledonia, the Ni ore mined from the laterite profile will be processed by a high-pressure acid leaching technology (HPAL). The performance of the HPAL technology is dependent on the chemistry of the feed including Ni and Co content as the two minerals of economic interest but also other major elements such as Mg, Fe and Al oxides because of their impact on acid consumption and Ni-Co recovery. Since the chemistry is highly variable between the various layers of the alteration profile and even within some layers between various size fractions, a proper characterisation of the short-scale variability in both layer geometry and layer composition was recognised as a key factor for a successful feasibility study of this project.

CONDITIONAL SIMULATION METHODOLOGY

Geostatistical CS were developed over 30 years ago in order to perform sensitivity and risk analysis. In a conditional simulation, reproducing certain statistical characteristics of the global population takes precedence over local accuracy. In addition to respecting the histogram, a geostatistical simulation model reproduces the variogram (ie reproduction of spatial correlation) and honours the actual existing data (ie conditioning).

In addition to the original turning bands method (Journel and Huijbregts, 1983), there are now several established methods of carrying out geostatistical simulations including the Sequential methods (Gaussian and Indicator), the LU decomposition algorithm, (Goovaerts 1997; Armstrong and Dowd, 1993; Chiles and Delfiner, 1999), and more recently, generalised sequential simulation (Dimitrakopoulos and Luo, 2004; Benndorf and Dimitrakopoulos, 2007, this volume). It is not the intent of this report to detail the pros and cons of each technique. Comparisons can be found in a number of publications. Gotway and Rutherford (1993) make a comparison of six different simulation methods performed on a variety of datasets. This study revealed the sensitivity of results to particular simulation algorithms and demonstrated some advantages of the conditional Gaussian based algorithms (ie turning bands and Sequential Gaussian) over the other methods. The turning bands approach was selected for this study. Some authors (Deutsch and Journel, 1992; Ravenscroft, 1993) have qualified the method as being computer intensive with built-in limitations, ie number and orientation of bands, number of discretisation points along the bands, rotation of anisotropy axis. Gotway and Rutherford (1993) indicate that most of these problems are related to the improper algorithms used and not to the turning band method itself. The algorithm used in this study is slightly modified from Lantuejoul (1993). The program used for the four projects presented in this paper is not affected by any of the above listed limitations.

The turning band algorithm involves a series of steps including the recognition of different geological domains, the selection of the variables to be simulated, the gaussian transformation of these variables, the non-conditional simulation of these gaussian variables and their linear combination, the conditioning to the actual data by Simple Kriging and their post-processing to reconstitute the original variables. In addition, the simulations presented in this paper benefit from the application of an unfolding algorithm (Datamine, 1997) in order to better simulate the geological controls on grade distribution. At each step, a series of checks is performed and to successfully validate the model.

CALIBRATION AND VALIDATION OF CONDITIONAL SIMULATIONS

In order to validate the conditional simulations, a calibration exercise is undertaken wherever production data are available either in a mined out portion of the same deposit, or in an analogue deposit. The areas selected for these back analyses are usually well drilled and benefit from extensive mapping of underground openings. The record of the mill-credited production may also be used if available. The methodology applied is as follows:

- create an exploration-based data set by removing all infill production drilling;
- perform polygonal, kriging and CS modelling from the exploration data set and assess recoverable resource at various cut-off grades;
- perform polygonal, kriging and CS modelling from the production data set and assess recoverable resource at various cut-off grades;
- compare historical credited production, polygonal, kriging and CS results based on both the exploration dataset and the production dataset; and
- compare actual detailed mapping in the mine openings to spatial patterns produced by all models.

The DATAMINE™ Floating Stope Optimiser (FSO) is used at Inco operations to quickly assess the recoverable resource from all orebody and simulated models at various cut-off grades. The FSO is analogous to the floating cone algorithm used in open pit situations. The FSO does not provide a final mining plan but rather a 'close to finished' product, which requires refinement but provides an effective tool for comparing alternative mining scenarios in a conceptual planning exercise. In order to validate the parameters used in the FSO, a manual exercise of mine planning is performed on one of the simulations. Comparing the FSO runs to manual planning verifies the FSO parameters. All subsequent FSO runs on the simulation models and estimation models use these same parameters.

Examples of successful results indicating the benefit of conditional simulation over traditional interpolation techniques and polygonal method are shown on Figures 1 and 2 for two Ni sulfide deposits. When an exploration dataset is used, the spatial patterns created by CS are more realistic and better represent the anticipated internal dilution. The results show that when little information is available for estimation, the spatial continuity of both the high-grade and low-grade mineralisation is overstated in the polygonal and MIK models. This results in an overestimation in recoverable grade and in an underestimation in recoverable tonnage.

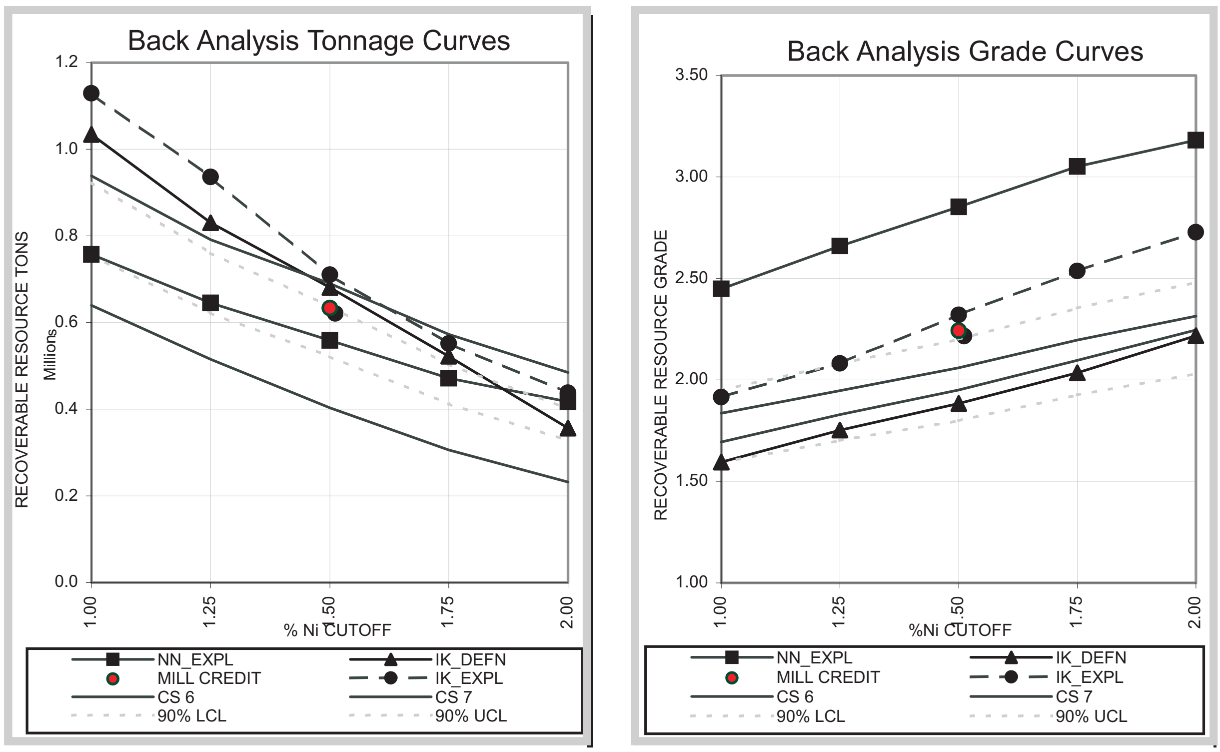


FIG 1 - Comparison of tonnage and grade curves obtained with the various methods with mill credit in the calibration zone. CS 6 and 7 represent the range of 90 per cent of simulated outcomes, while the 90 per cent control limits represent a ±10 per cent difference from the median simulation.

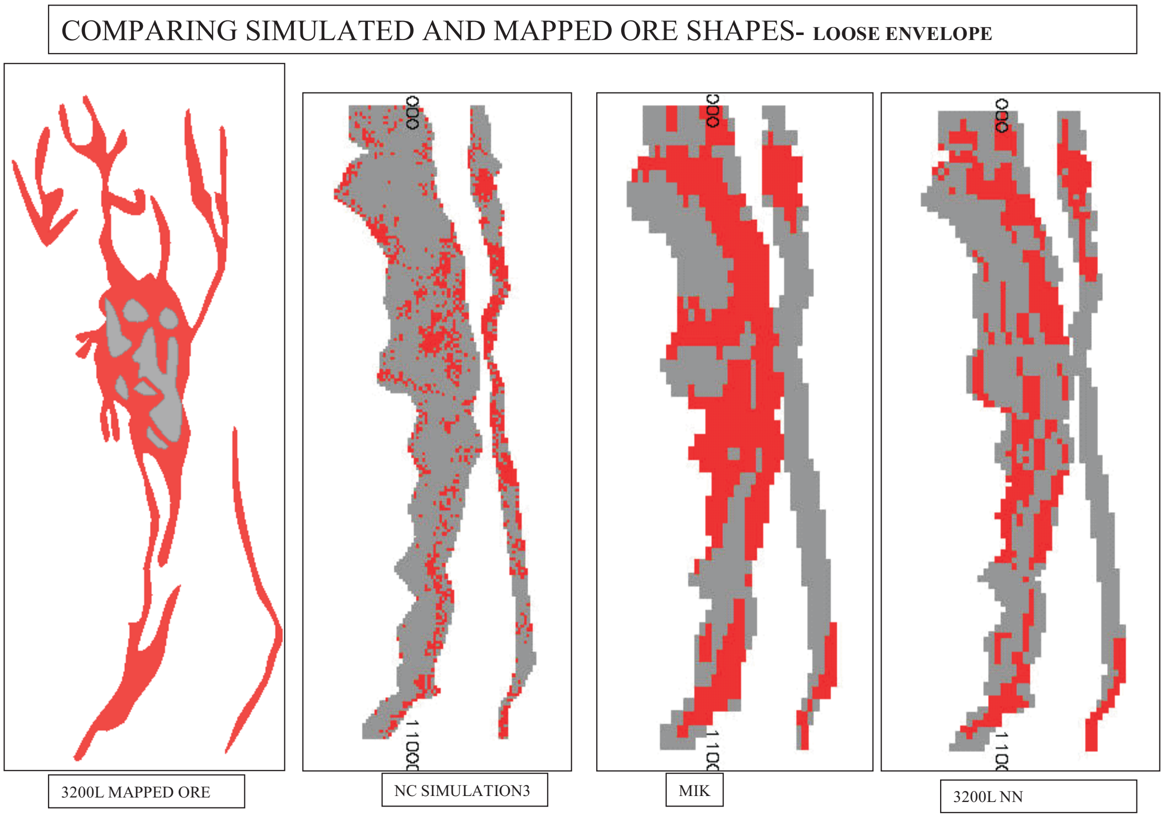


FIG 2 - Compared assessment of actual ore mapping, polygonal (NN), MIK and CS in the calibration zone.

APPLICATION OF CS TO THE SELECTION OF THE OPTIMUM MINING METHOD FOR UNDERGROUND Ni-SULFIDE DEPOSITS

The first application of CS to Ni sulfide deposits is related to prefeasibility studies conducted at two Inco mines in Northern Manitoba, Canada. In both cases, the studies aim at assessing the economic viability of mining deeper in existing deposits and assessing different mine plans and production schedules in the deposit extension based on advanced exploration drilling only. The Birchtree Mine 83 orebody and the Thompson Mine 1D orebody are located in the Thompson Nickel Belt. Orebodies in this belt consist of nickel sulfides with varying amounts of ultra mafic inclusions hosted in Proterozoic-aged metasedimentary units.

Only per cent Ni is simulated in these two cases since it is the only metal of economic significance. Domains of mineralisation are identified using conceptual geological ore genesis models with lithology and structure as the most important features controlling the final emplacement of the mineralisation.

Comparison of mining methods in the 1D lower orebody

Figure 3 summarises the results of a comparative study performed on the lower portion of the 1D deposit between a bulk and a selective mining method. For reference, the results obtained with the polygonal and Multiple Indicator Kriging (MIK) models are also plotted on these graphs.

The series of grade and tonnage curves shown on Figure 3 are obtained by performing a FSO analysis of each simulation and interpolation model with a consistent set of parameters including the minimum stope dimensions, the stope increments, the minimum pillar waste dimensions, the target headgrade and the maximum internal waste allowances. These parameters have been calibrated on a selection of sections and plans against manual interpretation done by experienced mine engineers.

The MIK model provides globally a similar estimate to the CS for the bulk-mining scenario but a significant different estimate for the selective mining scenario. As expected, applying the FSO to a polygonal model also yields significantly different results. The differences obtained from the various methods are related to their different handling of short-scale variability and therefore of internal dilution between mineralised zones. While polygonal techniques clearly overstate the continuity of the high grade mineralisation as expected, the MIK model also underestimate internal dilution when the drill spacing is too large for the level of details required for mine planning as it is the case in the assessment of selective mining. These results imply that making a development decision based on a MIK model only would present a significant risk of incorrectly selecting the most beneficial mining method for the project.

Based on the CS results, mechanised cut and fill mining was selected as the best suited mining method for the deepening for the 1D orebody. A set of sections and plans from selected simulations were further investigated by a team of experienced geologist and mine engineers with production experience in this orebody to perform some sensitivity studies and to optimise the proposed mine plan including ore and rock handling systems, ventilation, etc. The economic and technical parameters were then used as inputs in discounted cash flow analysis. Based on the CS results, a range of ROR and NPV were calculated in order to quantify the risk associated with the base case assumptions together with potential downsides and upsides.

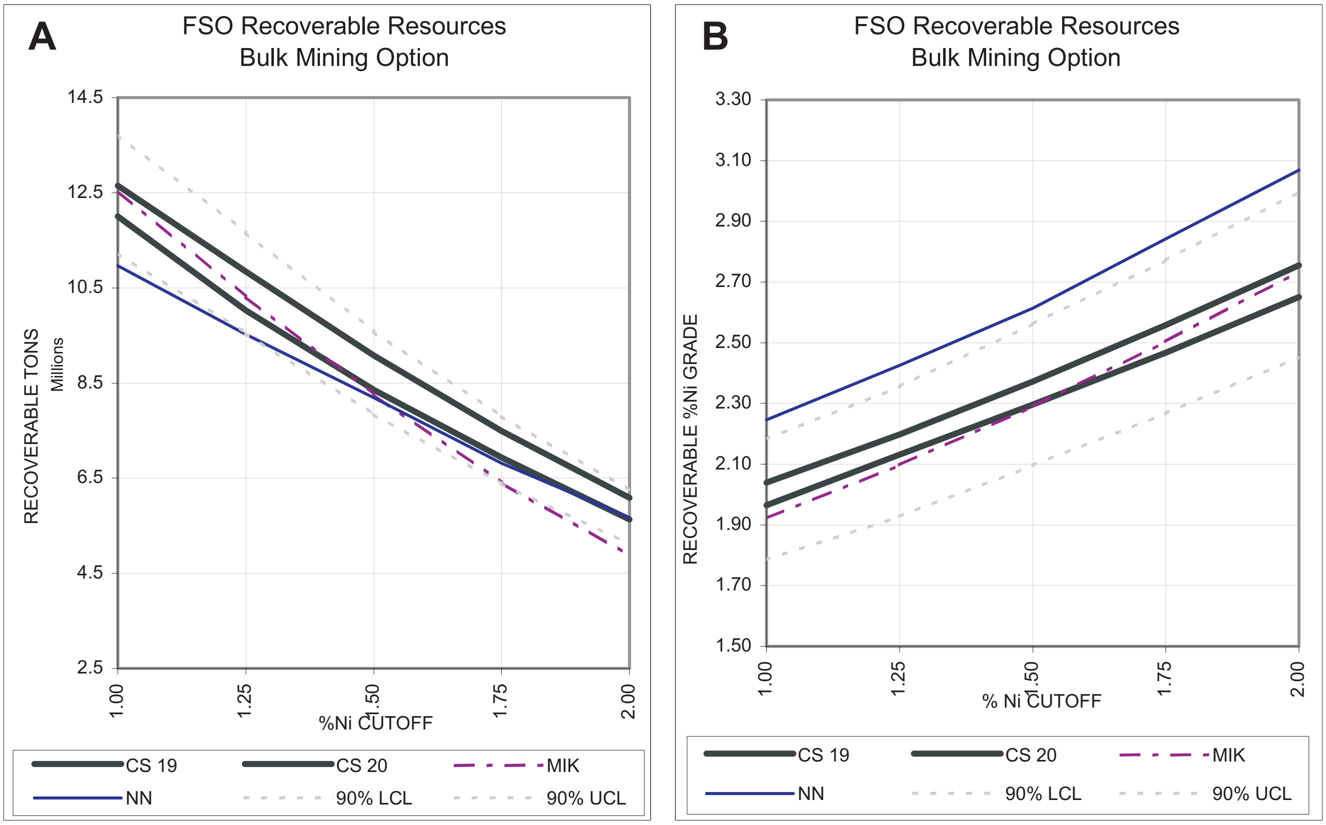


FIG 3 - (a) Comparison of bulk-mining grade-tonnage curves obtained with the various assessment methods. CS 19 and 20 represent the range of 90 per cent of simulated outcomes, while the 90 per cent control limits represent a ±10 per cent difference from the median simulation. (b) Comparison of cut and fill mining grade-tonnage curves obtained with the various assessment methods. CS 19 and 20 represent the range of 90 per cent of simulated outcomes, while the 90 per cent control limits represent a ±10 per cent difference from the median simulation.

Comparison of production profiles in the Birchtree 83 orebody

For the deepening of the Birchtree 83 deposit, the comparison of mining methods provided similar results to the 1D Lower deposit but the high-grading potential was not deemed sufficient and the risk too high to justify a selective mining approach. In this prefeasibility study, the benefit of CS was realised by revisiting the production profile. Due to the large drill hole spacing at depth, the MIK and polygonal models suffer from significant conditional bias. As a result, large areas of high-grade mineralisation are artificially created. A production plan based on these models aim at mining these deep high-grade zones first. The CS produce a radically different model of the deposit showing a much more consistent grade distribution from top to bottom (Figure 4). Based on the CS results, the production profile was modified to both increase the production rate and to mine the orebody both bottom-up and top-down. This orebody has now been in production for two years and operating results have confirmed the validity of the CS results and the bias in the MIK and polygonal estimates.

As in the case of the 1D deposit, mine planners were able to complete a pre-feasibility study including estimates on capital costs, mining rate, mining sequence and production profile based on the CS results. These estimates do not suffer from an under-estimation of the spatial variability in the metal grade distribution and therefore provide more realistic estimates than previous estimates based on polygonal and MIK models.

APPLICATION OF CS TO THE MODELLING OF SHORT-TERM VARIABILITY IN A SULFIDE CONCENTRATOR FEED

The second application of CS in Ni sulfide deposits is an investigation into the optimisation of the mining sequence and the validation of the concentrator design at the Inco Voisey's Bay project in Northern Labrador. The objective is to validate the mining sequence in order to ensure the short-term variability in the composition of the concentrator feed remains within an acceptable range.

Mineral domains and simulation process

The Voisey's Bay concentrator will be supplied for the first 16 years of operation by open pit production from the Ovoid deposit. This deposit is hosted by a troctolite intrusive complex, which is divided into three different domains with variable ratios of massive and disseminated sulfides. In each domain, the massive and disseminated zones were simulated separately. The turning band approach is constrained by a model of linear coregionalisation (Wackernagel, 1998) to maintain the spatial correlations observed in the input data between per cent Ni, per cent Cu, per cent Co, per cent S and per cent Fe.

Short-term variability in the concentrator feed

In order to maximise Ni recovery in the concentrator, it is typically desirable to homogenise the chemistry of the feed on a short-range basis. In particular, the Cu to Ni ratio variability will

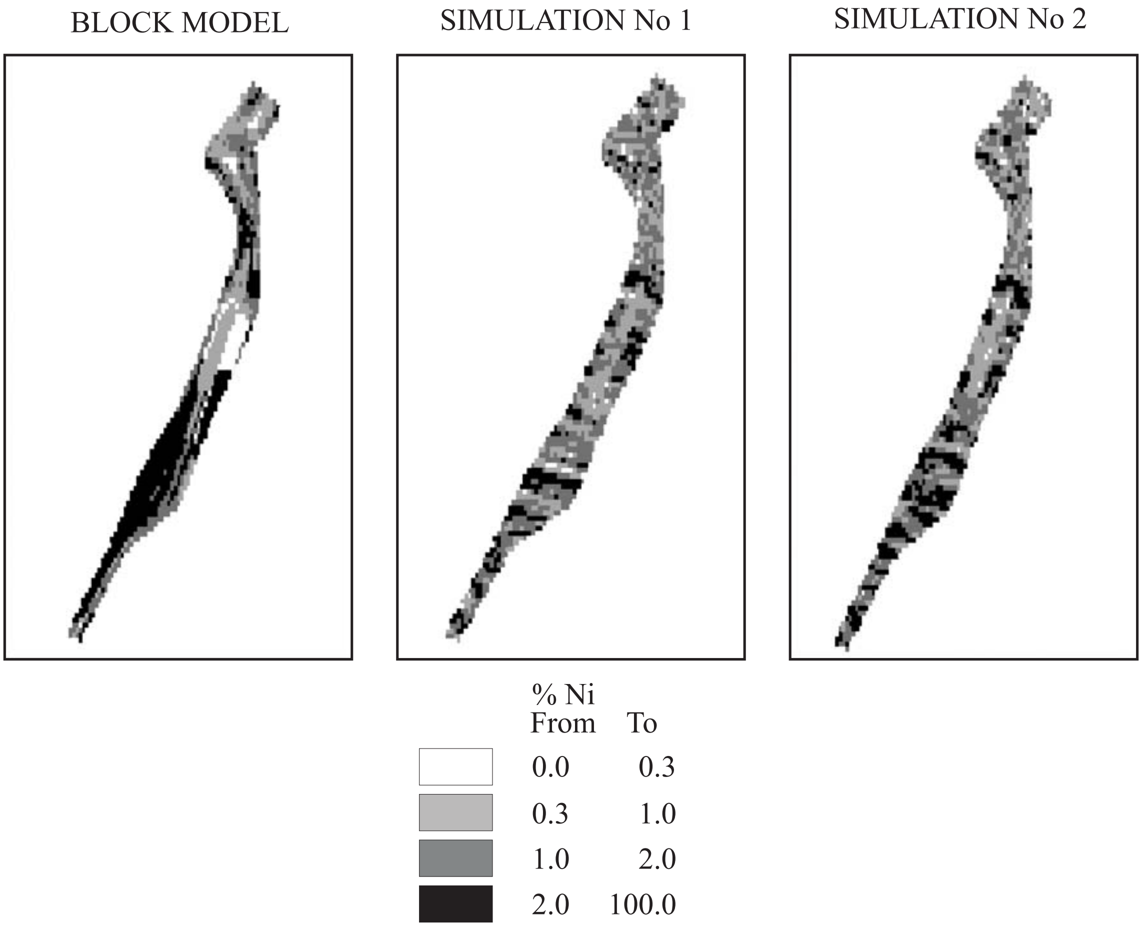


FIG 4 - Comparison of MIK and CS models in the deepening of the Birchtree 83 orebody.

influence the recovery of the two metals in their respective concentrate. Twenty CS realisations were generated over the three domains of the Ovoid deposit and were used to simulate the daily, weekly and monthly variability in the chemistry of the concentrator feed based on the initial mine plan and production profile (Figure 5).

The preliminary results obtained were used to validate and to modify the initial mine plan, including the mining sequence and the number of operating faces in the disseminated and massive sulfides. These simulations also demonstrated the validity of the design of the concentrator and its ability to cope with the anticipated daily variability in mine production. The use of CS also provided a range in the production rate, metal grade and processing recoveries required to conduct Monte-Carlo simulations on the discounted cash flow analysis of the project.

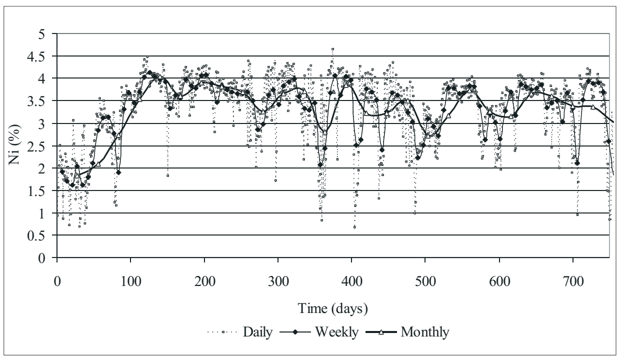


FIG 5 - Short-scale variability in concentrator feed composition at VBN.

APPLICATION OF CS TO THE OPTIMISATION OF THE LIFE OF MINE PLAN FOR A Ni-LATERITE PROJECT

Conditional simulations were extensively used for the Goro Ni-Laterite Project located on the French island of New Caledonia in order to optimise and validate several aspects of the life of mine plan. These aspects included the estimation of bottom ore recovery, grade control as well as the short-term variability in the process plant feed chemistry.

Geological setting and simulation process

The Goro laterite deposit hosts three geological layers of significant economic interest. During the mine planning process, it was recognised that the transition and saprolite layers, when screened at an appropriate size fraction, were entirely representing ore-grading mineralisation. Due to the reduced thickness of these layers, they would likely be mined with one bench. As a result, only 2D simulations were conducted for these two layers. Due to its greater thickness and the presence of a variable portion of the top of the layer grading below the selected cut-off, 3D simulations were required in the yellow laterite layer.

The simulation methodology used for the Goro deposit can be summarised as follows:

1. 2D simulation of the five layers in the profile;
2. 2D simulation of the average chemistry for the yellow laterite, transition and saprolite layers, ie per cent Ni, per cent Co, per cent Fe, per cent SiO_2, per cent MgO, per cent Al_2O_3, per cent Cr_2O_3 and per cent MnO;

3. linear regression of the vertical drift in chemistry based on 2D simulation and unfolded position of node within the simulated yellow laterite layer; and

4. 3D conditional simulation of residuals to the linear regression done in the previous step and creation of a full 3D simulation of chemistry for the yellow laterite layer.

The use of the unfolding process was critical to produce a realistic laterite profile and vertical grade distribution, however, added complexity to the simulation process, since each 2D simulation of the layer profile originated from a new reference system.

Models of linear coregionalisation were used both for the 2D simulations, to maintain the spatial correlation between the physical and chemical properties of the various layers, and for the 3D simulations in the yellow laterite layer to maintain the vertical correlation between the different chemical elements.

Application to mine planning

An initial application to mine planning was to use the 2D simulations to target the areas with the highest probability of combining high-grade nickel, with high mining recovery and large thickness of saprolite. The most favourable area would be preferentially selected as the start up zone for the open pit. The combination of results obtained from layer thickness, saprolite recovery and grade simulations clearly indicated that the southwest extremity of the deposit presents the best economic mineralisation for the first years of production.

The 40 simulations completed for the Goro deposit were rank by increasing variance of bedrock topography, ie bottom of the saprolite layer. Figure 6 shows a cross-section of the interpreted profile for various drill spacing (ie 2 m, 12 m, 24 m and 100 m, respectively) for simulation number 29, selected as the median case for the variance criteria.

A dramatic decrease in the variability of the layer geometry is observed as the drilling density decreases. This variability would impact the recovery and the dilution of the three mineralised layers and also the risk of misclassification of limonitic and saprolitic mineralisation for the purpose of stockpiling. Since the resource model is based on 100 m spaced drill holes, it will suffer from a high level of over-smoothing for layer geometry.

These preliminary findings indicate the value conditional simulations provide in the conceptual mine planning of the deposit ('desktop mining') to assess the impact and the applicability of cut-off grades, bench heights, size of equipment and limonite/saprolite sorting for stockpiling and measure the mining dilution and ore loss factors. Two east-west cross-sections and two north-south cross-sections sliced through these simulations were provided to mine engineers to be used as a basis for planning. These four sections were reproduced using the second worst, the median and the second best simulations ranked according to the variance criteria.

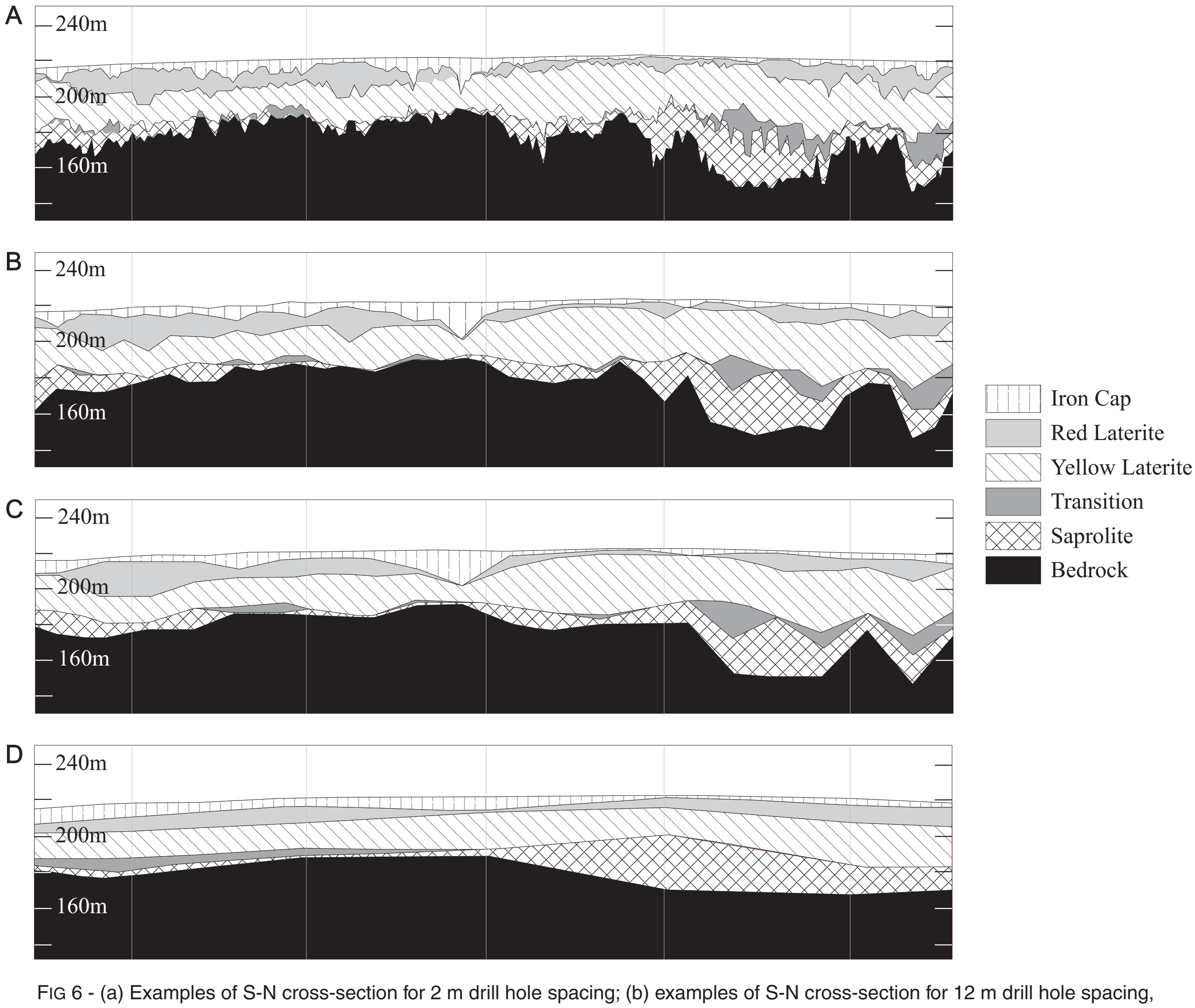


FIG 6 - (a) Examples of S-N cross-section for 2 m drill hole spacing; (b) examples of S-N cross-section for 12 m drill hole spacing, (c) examples of S-N cross-section for 24 m drill hole spacing; (d) examples of S-N cross-section for 100 m drill hole spacing.

Application to grade control

In the yellow laterite layer, a variable Ni cut-off had initially been proposed in order to maintain a consistent limonite/saprolite ratio for the process plant feed on an annual basis. The proposed cut-off ranged between 1.15 per cent Ni and 1.45 per cent Ni. The method suggested in the initial mine plan to define the top of the ore was applied to the simulations assuming a 25 m grade control grid. From simulated drill holes the top of the ore was defined by the first intersection down the hole of two consecutive metres grading above the proposed cut-off. The volume and average grade recovered between the top of the ore and the bottom of the layer were compared in each case with the average of all the simulated nodes, ie assumed reality.

Figure 7 shows the results obtained for Ni cut-off ranging from 1.15 - 1.50 per cent. This figure clearly shows that applying a cut-off grade higher than 1.3 per cent Ni would result in an unrealistic estimation of the production headgrade, and produce in a significant reduction of the recoverable volume. Although this approach can be used to increase the combined production headgrade of limonite and saprolite by lowering the limonite/saprolite ratio, it also results in the significant loss of ore grading material. This ore-grading limonite would be treated as overburden and used as backfill. Alternatives should consider stockpiling to use this mineralisation as incremental ore. Based on these results, a new grade control strategy was adopted to use a cut-off grade no higher than 1.3 per cent Ni in defining the top of the ore in the yellow laterite layer.

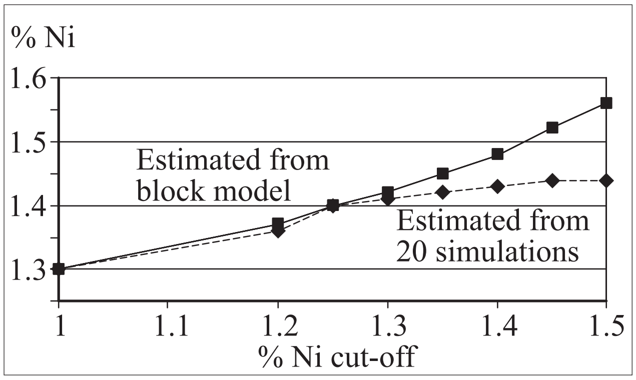


FIG 7 - Comparison of estimated Ni headgrade for various per cent Ni cut-off from block model and simulations.

Application to predicting and managing the autoclave feed variability

Another important application of the 2D simulations at Goro was to estimate the process plant feed variability. In order to meet the planned production, availability of the saprolite mineralisation with the proper chemistry profile is key. Conditional simulations provide a tool to assess the variability in the plant feed at any given scale (eg weekly, monthly, annually). With the proposed mining pushback and 20 simulation realisations, the variability of recoverable metal sent to the preparation plant for each year is assessed.

As expected, the average of the 20 simulations indicate similar results to those obtained from the kriged model for the entire simulated domain. On shorter production periods however, the results presented on Figure 8 indicated that without stockpiling, a potential shortfall in saprolite would exceed ten per cent for 25 per cent of the simulations. The choice of the period to evaluate feed availability is critical as this potential shortfall becomes even more important for a two-month period. The only way to mitigate this potential problem is through stockpiling, blending and through careful scheduling of saprolite production at the mining face. It is critical for mining projects to identify these potential issues at the planning stage rather than having to fix them in an ongoing operation.

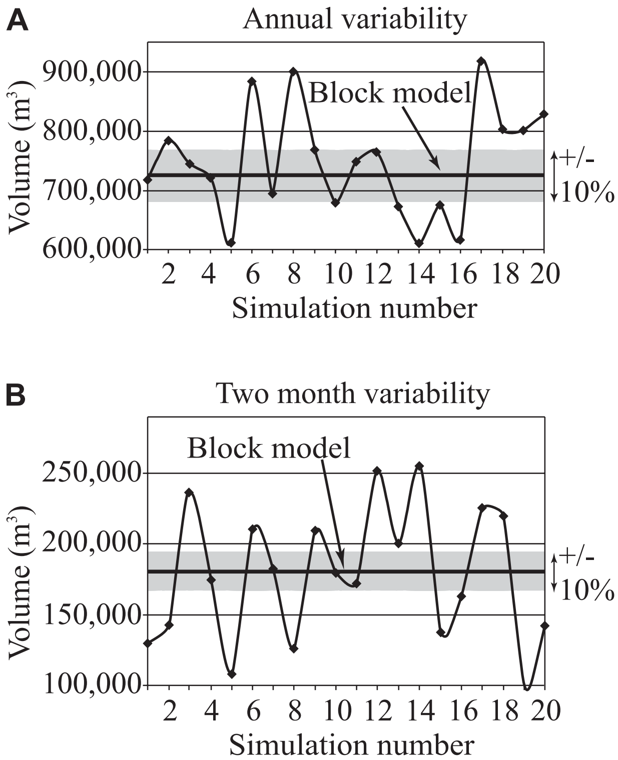


FIG 8 - Availability of saprolite volume from 20 independent simulations for (a) a one year period and (b) a two-month period.

The 2D and 3D CS of the entire profile were also used to generate combined simulation of the mine production on a daily basis using three different scenarios based on four concurrent mining faces on a test area representing approximately one year of production. The test area was selected as being representative of the first eight years of production according to the most current mine plan. The daily feed was used to simulate stockpiles and daily autoclave feed. Results were used to validate the production plan with respect to acid requirements and Ni production targets.

CONCLUSIONS

In the four applications presented in this paper, emphasis is put on demonstrating how the use of conditional simulations has led to the ability to make better business and technical decisions than those from models based on traditional interpolation methods. A proper life-of-mine plan relies on having a good understanding of the grade variability, CS allow the spatial grade variability to be properly characterised.

CS allows practitioners to quantify risk and to perform meaningful sensitivity analyses on project financials. CS used as an additional tool in mineral project assessments enable senior management to better assess the risk associated with mining projects.

In order to gain confidence in the simulation results, seven years of applications of CS at Inco Limited operations have highlighted two fundamental keys to success:

1. to recognise that conditional simulations are only models and to constantly challenge the stationarity and other model assumptions; and
2. whenever possible, to conduct calibration studies based on back analysis of orebodies with production history.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the contribution and support of key Inco personnel to the successful application of conditional simulations to these projects: Alan Aubut, Scott Bishop, Jean-Yves Cloutier, John Kelly, Andre Lauzon and Brian Plamondon. We would also like to thank Lawrence Cochrane for his help in the final edition of this paper.

REFERENCES

Abzalov, M and Mazzoni, P, 2007. The use of conditional simulation to assess process risk associated with grade variability at the corridor sands detrital ilmenite deposit, Mozambique, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 95-102 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Armstrong, M and Dowd, P A (eds), 1993. Geostatistical simulations, *Geostatistical Simulation,* 255 p (Kluwer Academic Publishers).

Audet, M and Ross, A F, 2007. Koniambo lateritic Ni-Co deposits, New Caledonia — A case study from geological modelling to mineral resource classification, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 235-244 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Benndorf, J and Dimitrakopoulos, R, 2007. New efficient methods for conditional simulations of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Chiles, J P and Delfiner, P, 1999. *Geostatistics: Modelling Spatial Uncertainty*, 695 p (J Wiley and Sons).

DATAMINE ™ Ltd, 1997. *User manual for Unfolding Module*, 134 p.

Deutsch, C V and Journel, A G, 1992. *GSLIB: Geostatistical Software Library and User's Guide* (Oxford University Press: New York).

Dimitrakopoulos, R, and Luo X, 2004. Generalised sequential Gaussian simulation on group size n and screen – effect approximations for large field simulations, *Mathematical Geology*, 36(5):567-591.

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 483 p (Oxford University Press: New York).

Gotway, C A and Rutherford, B M, 1993. Stochastic simulation for imaging spatial uncertainty: comparison and evaluation of available algorithms, in *Proceedings Geostatistical Simulation Workshop* (eds: Armstrong and Dowd) pp 1-22 (Kluwer Academic Publishers).

Journel, A G and Hjuibregts, C J, 1983. *Mining Geostatistics*, 600 p (Academic Press: San Diego, CA).

Krige, D G, 1996. A practical analysis of the effects of spatial structure and of data available and accessed, on conditional biases in ordinary kriging, in *Proceedings Geostatistics Wollongong '96* (eds: E Baafi and N Schofield) pp 799-810 (Kluwer Academic Publishers).

Lantuejoul, C, 1993. Non conditional simulation of stationary isotropic multigaussian random functions, in *Proceedings Geostatistical Simulations* (ed: Armstrong and Dowd) pp 147-177 (Kluwer Academic Publishers).

Ravenscroft, P J, 1993. Conditional simulation for mining: practical implementation in industrial environment, in *Proceedings Geostatistical Simulation Workshop* (eds: Armstrong and Dowd) pp 79-88 (Kluwer Academic Publishers).

Wackernagel, H, 1998. *Multivariate Geostatistics*, second edition, 291 p (Springer: Berlin Heidelberg).

Grade Uncertainty in Stope Design — Improving the Optimisation Process

N Grieco[1] and R Dimitrakopoulos[2]

ABSTRACT

Decisions in the mining industry are made in the presence of uncertainty whether it is in the form of technical, financial or environmental risk. In recent years, the main focus of uncertainty has been the mineral resource. Methods for assessing and quantifying grade risk in open pit operations has lead to the ability to forecast problems and improve the design and planning process by integrating this risk. This paper successfully implements these risk-based methods in an underground stoping environment using data from Kidd Creek Mine, Ontario, Canada. Risk is quantified in terms of the uncertainty a conventional stope design has in contained ore tonnes, grade and economic potential. A mathematical formulation optimising the size, location and number of stopes in the presence of uncertainty is introduced and applied. The implementation of different geostatistical simulation methods to the optimisation formulation is discussed briefly and observations made.

INTRODUCTION

Risk is present in all facets of mining be it technical, financial or environmental (Rendu, 2002). When determining the feasibility of a project the uncertainty associated with all sources must be considered and contingencies made. Geological uncertainty is a major component of technical uncertainty, along with mining, and has been isolated as a primary source of risk affecting the viability of projects. This uncertainty is recognised as the key factor responsible for many mining failures (Baker and Giacomo, 1998; Vallee, 1999). Hence, the necessity to quantify geological risk is well appreciated. Modelling geological uncertainty in a mineral resource can be achieved through conditional simulation technologies. The last few years in open pit mining these technologies have been coupled with mine design optimisation methods to assess risk in conventionally generated mine designs and production schedules. The approach allows planners to anticipate fluctuations in key project parameters that would otherwise be impossible (Ravenscroft, 1992; Dowd, 1997; Dimitrakopoulos, Farrelly and Godoy, 2002). These studies have also documented that conventional methods may be misleading in their forecasts as they assume certainty. Recent developments in open pit mining show that direct integration and management of inherent grade risk in mine design and planning have begun (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2007, this volume; Menabde *et al*, 2007, this volume; Froyland *et al*, 2007, this volume; Dimitrakopoulos, in press). The developments provide the opportunity to generate substantially more profitable mine designs; for example, Godoy and Dimitrakopoulos (2004) report a 28 per cent higher NPV from managing geological risk. It is logical to consider how to develop concepts and similar risk-based technologies for underground mining methods.

Optimisation in underground mine design has had less routine application than open pit mines, which is attributed to the diversity of underground mining methods that does not allow the production of general optimisation tools. Related in the technical literature is the work of Ovanic (1998) who considers the economic optimisation of stope geometry, a topic directly linked to the present study; and work on conventional stope optimisers (Thomas and Earl, 1999; Ataee-pour and Baafi, 1999). None of these approaches consider risk and hence assume the inputs are certain. Limited initial work reported, combines simulated orebodies and grade risk models with conventional optimisers (Myers *et al*, 2007, this volume); these however, are limited in their assessment as optimisation formulations are, in general, a non-linear process. Geological risk-based approaches to stope optimisation that directly integrate risk have been recently introduced (Grieco, 2004) and open the possibility to further develop risk-based underground mine design. Current efforts, however, focus on the issue of grade uncertainty. In the longer run these developments need to be fused with geotechnical issues critical to underground mining (Bawden, 2007, this volume).

This paper stems from the need to explore the contribution of geological uncertainty quantification and the direct integration to stope optimisation through a new, risk-based approach to stope design. In the following sections a conventional stope design in a part of Kidd Creek base metal mine, Ontario, Canada, is assessed in terms of copper grade risk, to explore uncertainty in terms of upside potential as well as downside risk. Subsequently, a probabilistic mathematical programming optimisation formulation is outlined and applied. The question of the sensitivity to the geostatistical simulation method is briefly visited. Finally, summary and conclusions follow.

QUANTIFYING GRADE RISK IN CONVENTIONAL STOPE DESIGNS: AN EXAMPLE

Grade risk quantification in a given underground stoping design is similar to that used in the design and production schedule of an open pit mine (Dimitrakopoulos, Farrelly and Godoy, 2002). The quantification process requires two main components:

1. the design of a stoping outline generated using a conventionally estimated orebody model; and
2. a series of simulated realisations of the orebody, quantifying the uncertainty and *in situ* variability.

By putting each realisation through the stoping outline, as if the realisation is the actual orebody being mined, and accounting for potential production from the design, distributions or risk profiles for the pertinent project indicators are generated, thus allowing the quantification of geological uncertainty and risk assessment for the design being considered.

The deposit and study area

Applying the concepts outlined for quantifying the grade risk in a conventional stope design is presented with a case study involving data from Falconbridge Ltd's Kidd Creek Mine. Kidd Creek is a volcanic massive sulfide deposit located in Ontario, Canada and produces about 7000 tonne per day (Roos, 2001) from two major orebodies containing silver, copper, zinc and lead, the main commodities. Production began in 1966 via an open pit mine and has extended into three underground mines reaching depths of over 2000 m and employing various mining methods including sublevel caving, open stoping and sublevel stoping.

1. AMEC Americas Limited, 2020 Winston Park Drive, Oakville ON L6H 6X7, Canada. Email: nikki.grieco@amec.com

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

The focus of this study is a densely drilled area located in the copper concentrated stringer ore 1400 m below the surface in Phase I of Mine No 3. The drill hole configuration consists of 37 drill holes with 1.5 m copper composites in nine vertical fans that are spaced approximately four metres apart. The resulting samples show a high-grade zone in the central region. Statistics outlining the distribution of declustered copper samples is given in Table 1.

Mining in this region is via open stoping methods with stope sizes typically 15 m wide by 20 m long by 40 m high. Blast rings are spaced generally every three metres and have a copper cut-off of three per cent.

Generating estimated and simulated orebody models

Estimation methods are by construction smoothing operations. Conditional simulation methods aim at modelling the *in situ* spatial variability of a given attribute and, unlike the equivalent estimation approaches, reproduce the data histogram and spatial continuity. At Kidd Creek, the study area is first geostatistically estimated, producing 16 236 blocks within the orebody model. Blocks are estimated with a block size of 3.0 m × 3.0 m × 4.5 m, spanning 123 m in the east, extending 51 m in the north and reaching 99 m in the vertical direction. A horizontal section of this estimated model is shown in Figure 1. The same area of the deposit is then geostatistically simulated using the well-established sequential Gaussian simulation method or SGS (Goovaerts, 1997). Forty realisations of the deposit are generated on a 1.5 m × 1.5 m × 1.5 m grid of 19 880 nodes. Figure 2 shows a simulated realisation of copper grades of the same horizontal section as in Figure 1. When comparing the estimated and simulated models in Figure 1 and Figure 2, both reproduce the regions of high-grade mineralisation in the drill hole configuration. The figures also show the typically smooth representation of reality by the estimated model whilst the simulated realisation reflects the likely *in situ* copper variability.

TABLE 1

Declustered data statistics of copper.

Statistic	Declustered data set
Number of samples	2723
Mean	2.43%
Standard deviation	3.17%
Maximum	27.59%
75th percentile	3.00%
Median	1.34%
25th percentile	0.54%
Minimum	0.0%

Risk quantification

In establishing a conventional stope design, a conceptual stoping layout recognising potential development and stoping levels must be first determined. Due to the vertical extent of the orebody models, two potential stoping levels are configured accounting for required drilling and hauling levels (Figure 3). It is assumed that the lower level will be mined and backfilled before the upper level is extracted. Accounting for this stoping layout, a stope outline is produced given the estimated copper grade model using the DATAMINE^{TM} floating stope facility, hence providing a conventional design for which a risk quantification and analysis can be performed. Figure 4 shows a three-dimensional view of the conventional outline generated here incorporating both stoping levels.

For the quantification of copper grade risk in this conventionally generated stope design, first, the simulated copper realisations are re-blocked into mineable rings by averaging the nodes contained within consecutive ring dimensions (15 m × 3 m × 40 m). Then, the conventional design outline is put through each of the orebody realisations and values pertaining to copper grades are recorded. It is subsequently simple to calculate for a set of realisations, such as the 40 here, the ore tonnage, metal, average grade and revenues or any other project indicator, the corresponding histogram of possible outcomes and from that histogram statistics of interest such as the various percentiles and so on. The following discussion refers to the risk profiles of some project indicators.

Figure 5 depicts the risk profiles for the upper and lower stoping outlines providing a means of quantifying copper grade risk in terms of the potential average copper grade the conventional design could contain. The conventional design and approach tend to underestimate the likely contained grade in the lower stoping level, while in the upper level tends to overestimate copper grade.

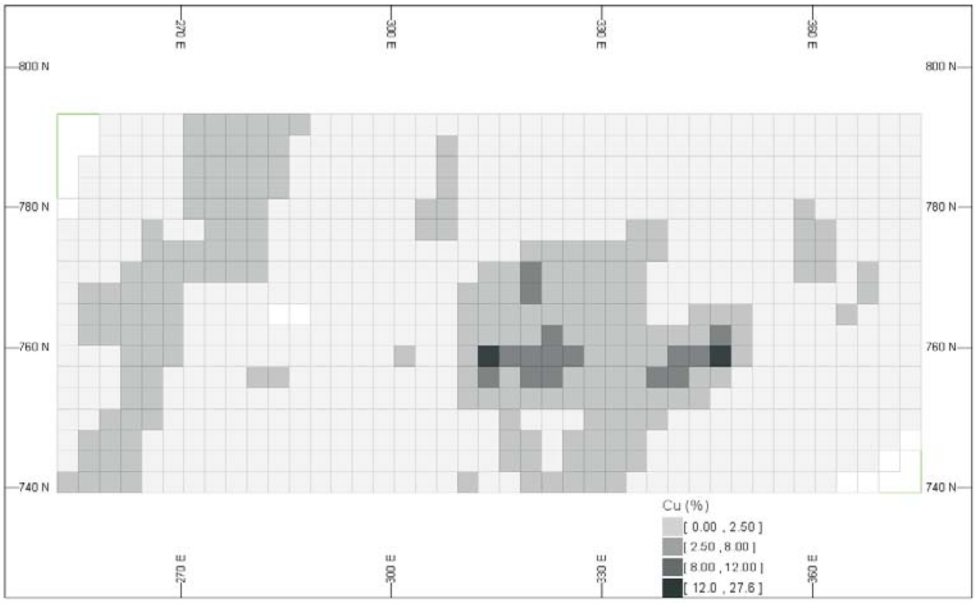


FIG 1 - Horizontal section of the estimated orebody model.

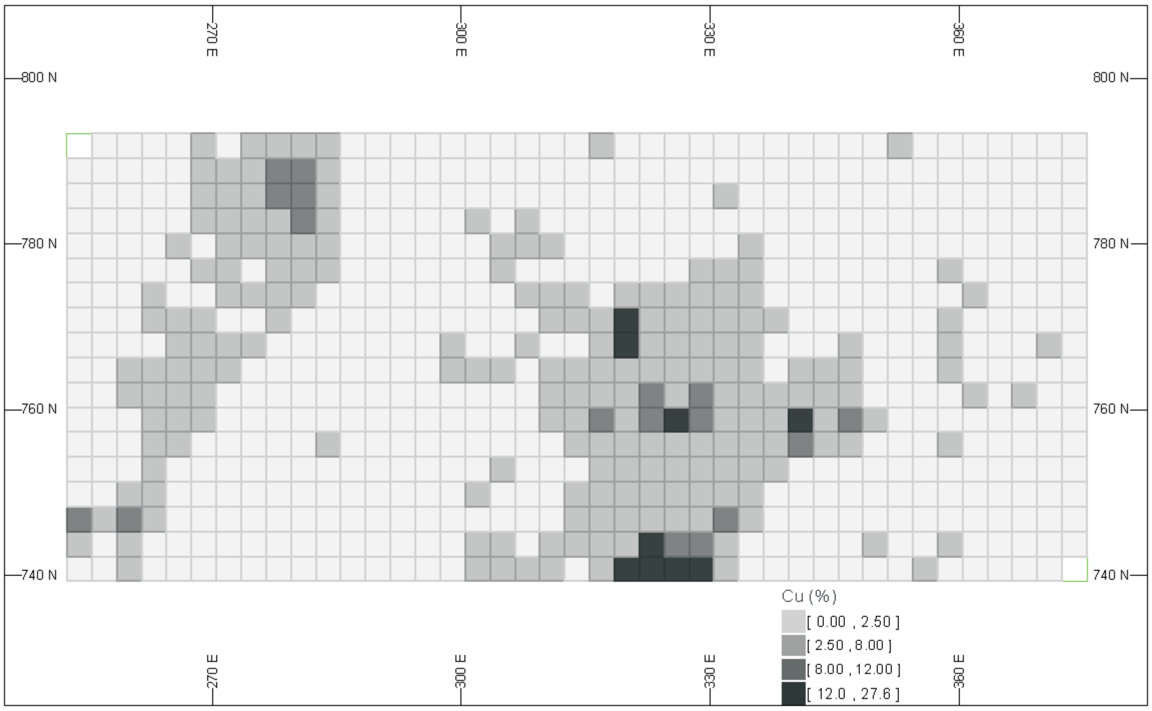


FIG 2 - Horizontal section of a simulated orebody model.

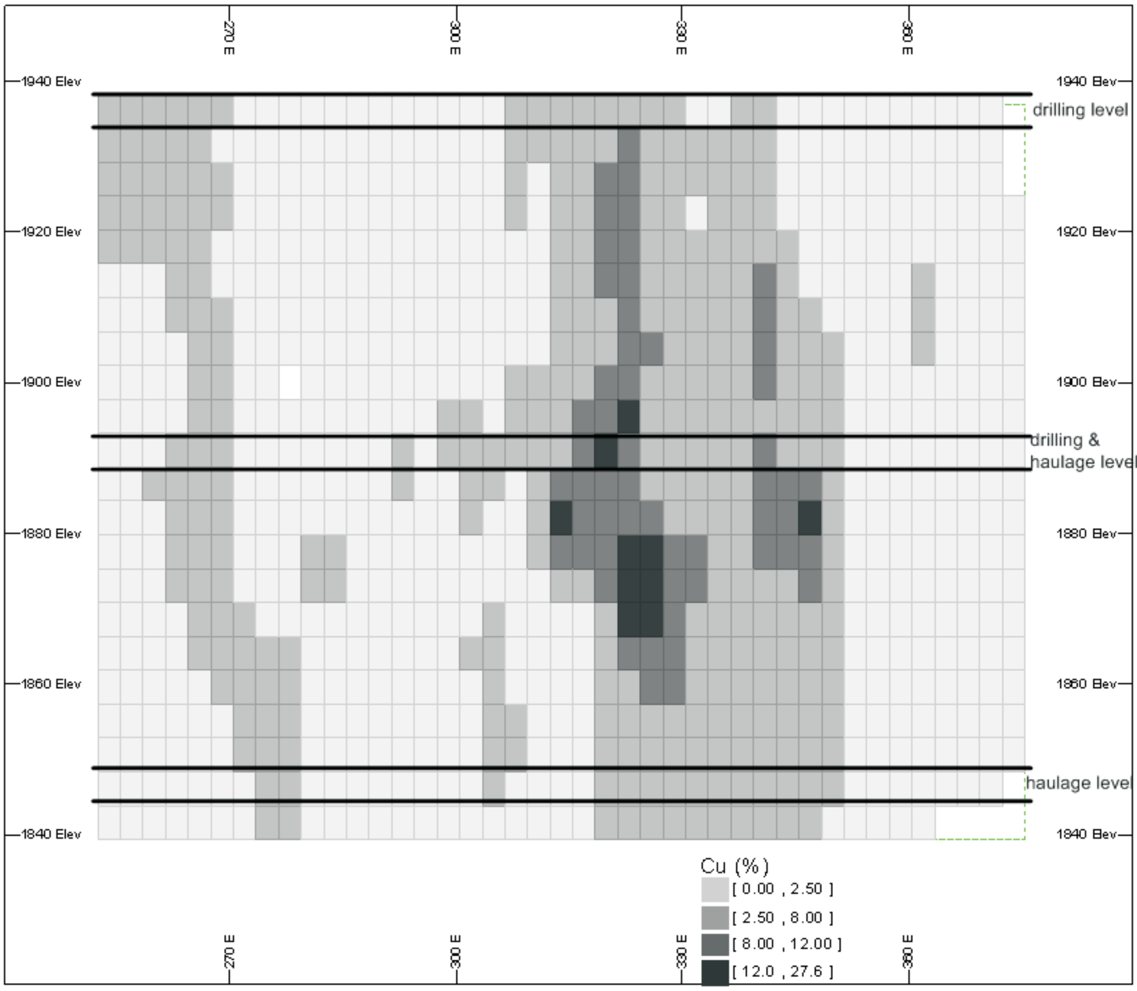


FIG 3 - Stoping layout indicating two stoping levels.

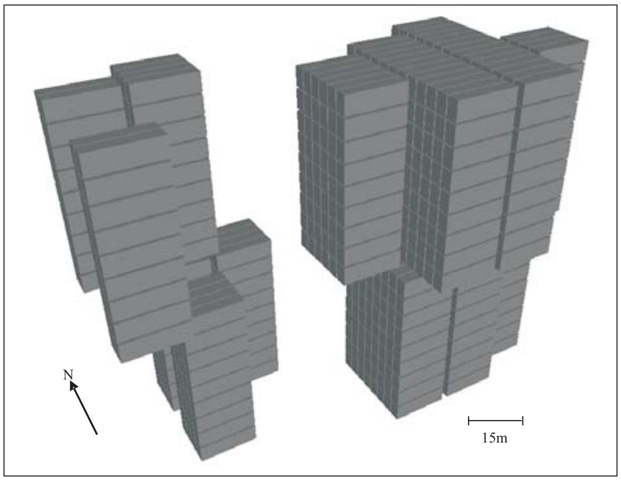


FIG 4 - Conventional stope envelope.

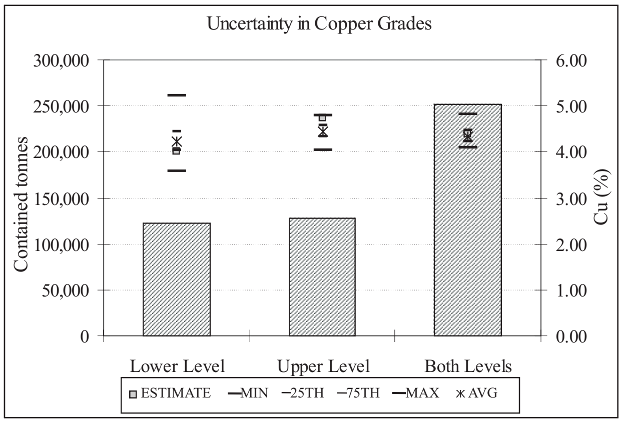


FIG 5 - Quantifying the conventional stope envelope's uncertainty in copper grade.

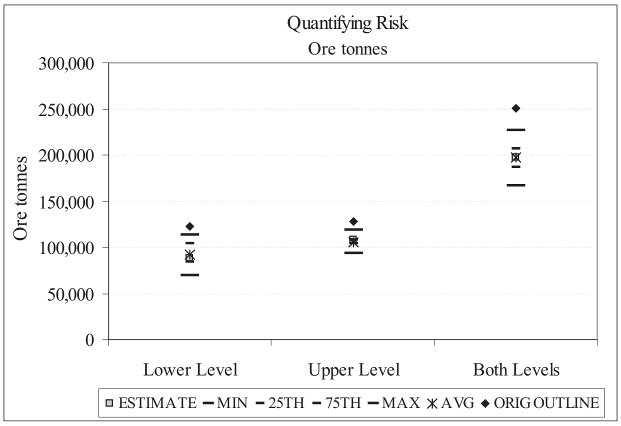


FIG 6 - Quantifying the conventional stope envelope's uncertainty in contained ore tonnes.

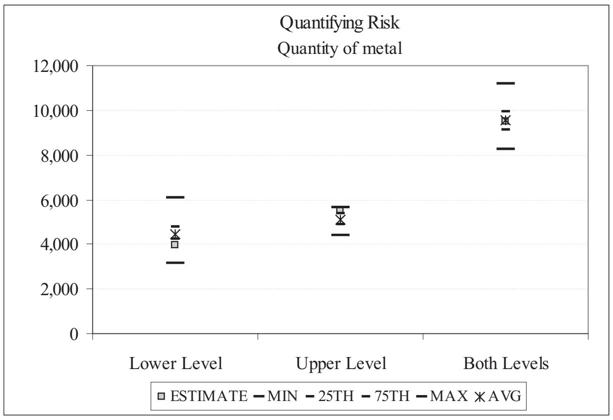


FIG 7 - Quantifying the conventional stope envelope's uncertainty in contained metal.

For analysis purposes only, the rings within the design outline that are less than three per cent copper are removed to uncover how the grade uncertainty within the orebody model effects the amount of ore tonnes, metal and economic potential that could, in reality, be realised. Figures 6, 7 and 8 illustrate the resulting risk profiles of these parameters respectively. Figure 6 also highlights the amount of material within the original design outline before any waste rings are removed (black diamonds). This demonstrates a potential for the conventional outline (both levels) to contain up to 32 per cent waste, significantly affecting the tonnes expected to reach the mill. Both Figures 6 and 7 illustrate a generally small risk the conventional outline presents in the amount of ore and metal tonnes expected from the upper level, as the extreme grade values present a tight distribution in which the expected values fall.

Figure 8 shows the results of an economic evaluation of the stoping levels using values representing the present value before tax. The figure illustrates significant risk in the conventional outline's ability to predict its potential economic value in each level. In addition, the single estimate in the lower level is 17 per cent less than the average predicted economic potential expected, while the estimate in the upper level is 33 per cent above this equivalent average value. Since each level will likely be mined in separate periods, the profit made in the upper level cannot compensate for the potential loss (seven per cent) in the lower level. This potential to incur monetary losses on production could, for example, affect monthly profits expected from this part of the mine.

The conventional stoping design in this specific example is generally straightforward and is found to provide a reasonable assessment of the average economic value of the design. However, several points can be made, including the following:

1. the size of the study area is small and at the same time uncommonly well drilled (nearly three times the density of fans normally expected), thus results are not surprising;
2. if the ability to quantify risk was not available, the assessment would not be possible; and most importantly
3. conventionally, one is unable to foresee the significant upside potential and/or downside risk the conventional design may actually produce (eg Table 2).

In the example presented here, quantifying the risk in terms of economic potential recognises the potential to earn 62 per cent more and the risk of earning 38 per cent less than expected. In dollar terms, this conventional design could be worth as little as 1.8 million dollars or as much as 5.9 million dollars. The above lead to considerations such as:

1. Can grade uncertainty be not only quantified for a design, but also employed during the design process to capture the upside economic potential of the deposit?
2. Can designs be based on a minimum acceptable risk? And generally, can the design process manage grade risk directly and generate benefits?

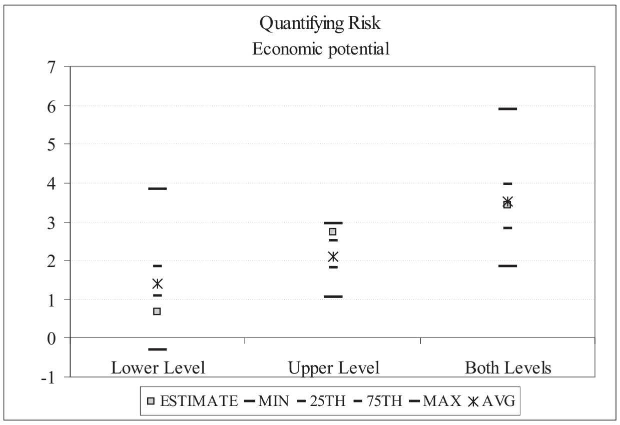


FIG 8 - Quantifying the conventional stope envelope's uncertainty in economic potential.

TABLE 2

Project indicators based on the conventional stope design.

Model	Ore (t)	Metal (t)	Cu (%)	Economic potential ($)	Economic potential % difference
Estimate	196 830	9490	4.82	3 412 999	--
Realisation 3	191 909	8769	4.57	2 285 625	- 33
Realisation 18	167 306	8228	4.92	1 858 484	- 46
Realisation 31	216 513	11 187	5.17	5 905 110	+ 73
Realisation 35	211 592	10 492	4.96	4 820 407	+ 41

In the last decades, major improvements have been made to the time-consuming manual approach to stope design. However, these computer-aided tools are limited in their ability to mathematically optimise the location of designs under uncertainty similarly to the optimisation methods in open pit mine design. With a methodology in place for quantifying grade risk in conventional mine design, the limitations of existing computer planning and optimisation tools force the development of a new optimisation approach based on and integrating grade uncertainty directly into the optimisation process, essentially creating a more versatile computer-aided tool.

GENERATING RISK-BASED DESIGNS

Mathematical programming methods provide a means of optimising an objective function subject to a set of constraints through a mathematical formulation. Such methods allow the development of formulations that integrate grade uncertainty directly into the optimisation process, as well as allow the consideration of a user-selected minimum acceptable risk. In this section, a mathematical programming formulation considering the above to optimise the location of stopes in the presence of grade uncertainty is presented and used at Kidd Creek to produce a risk-based design for comparison and analysis.

The optimisation formulation

A mixed integer programming (MIP) formulation with the aim of locating an optimal stope layout is presented here. This optimal layout is defined by the size, location and number of stopes within an orebody model. Such a model is described as consisting of a series of layers, each of which is composed of a number of rows referred to as panels, where the panels are made up of a series of rings. With multiple simulated orebodies available, each ring can be identified by a probability to be above any cut-off grade and have an average grade, hence introducing grade risk into the process.

The objective function of the formulation focuses on maximising the grade content within a layout in the presence of grade uncertainty, and is:

$$\text{Maximise} \sum_{j=1}^{m} \sum_{i=1}^{n} g_{ij} p_{ij} B_{ij} \quad (1)$$

where:

m is the number of panels within the orebody model

n is the number of rings within a panel

p_{ij} is the probability of ring ij being above a specified cut-off

g_{ij} is the expected grade of ring ij above the cut-off

B_{ij} is a binary variable representing every ring within the model and identifies whether it has been selected ($B_{ij}=1$) or not ($B_{ij}=0$) in the optimal layout

Further to the above, the presence of simulated orebody models allows risk-based designs to be generated for a given minimum level of acceptable risk specified by the planner or decision-maker. The following constraint restricts the total average probability of selected rings within a panel to be greater than or equal to an assigned value representing the minimum acceptable level of risk (PL).

$$\sum_{i=1}^{n} \left(p_{ij} - PL\right) B_{ij} \geq 0 \quad (2)$$

By changing the value of the minimum acceptable level of risk, PL, a number of different risk-based designs can be generated, compared and assessed. Risk profiles can then be generated for the key project indicators by putting each outline through all simulated realisations, in the same procedure that was used to quantify risk in the conventional design of the previous section. A design that best suits the operational requirements can be selected with the risk being quantifiably assessed (Grieco, 2004).

The formulation above is also constrained by limitations on the stope size – both minimum and maximum, which are a direct reflection of the geotechnical restrictions and production requirements of the area. These stope size constraints are based on the number of consecutive rings allowed to form a single stope. The size of the pillars between two primary stopes is also considered. This algorithm determines the minimum number of rings to be left un-mined between stopes and is directly related to the size of the stopes surrounding them. The larger a stope, the larger the pillar is.

Application at Kidd Creek

The MIP formulation for optimising a stope as above is applied to the study area at Kidd Creek mine. Geotechnical requirements in the region restrict a given stope to consist of a minimum of two rings and a maximum of seven. Applying a cut-off grade of three per cent, each ring within the re-blocked orebody model (same configuration as the one used in simulation) is represented by the probability of being above three per cent copper and the average copper grade above this cut-off. A risk-based design with a minimum acceptable level of risk at 80 per cent is generated. Figure 9 illustrates a three-dimensional aspect of the resulting design layout using the simulated model, with dark grey rings representing primary stopes and light grey rings the recoverable pillars. In comparing the size and shape of the conventional design outline (Figure 4) with the new, risk-based design, a notable difference in size is recognised. Introducing the minimum acceptable level of risk has limited the amount of waste (tonnes) contained within the new design as it forces the stopes within a given panel to have an average probability above 80 per cent. This approach grants the planner control over the level of risk permissible within a given design.

The formulation constraints require the stopes and pillars to contain a minimum of two rings and a maximum of seven, providing an optimal combination for obtaining the most metal. The conventional approach produces an envelope of rings for which some combination satisfies the minimum grade and size requirements and further development of a mineable stope layout is needed.

The fluctuation in copper grade within the risk-based design can be predicted by putting the outline through all simulated realisations generated with the SGS method, similarly to the conventional design in a previous section. Figure 10 illustrates the amount of contained material within the primary stopes and recoverable pillars, and the potential grade variation within each. Although grade uncertainty has been accounted for within these

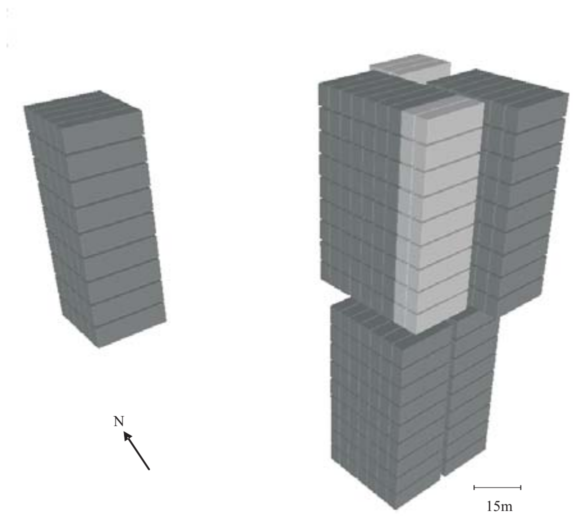


FIG 9 - LP stope design layout based on SGS and 80 per cent acceptable level of risk.

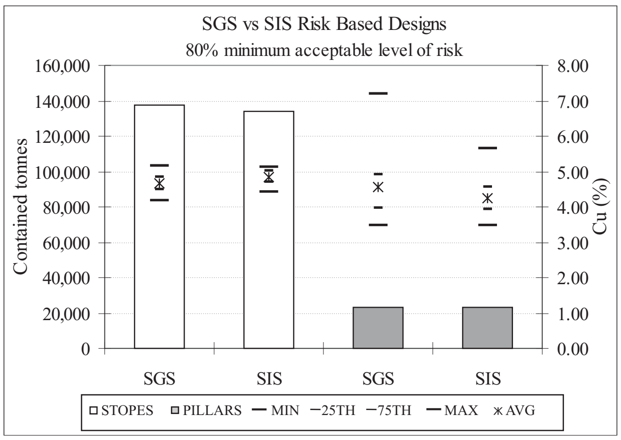


FIG 10 - Primary stoping layout for LP designs based on SGS and SIS orebodies and 80 per cent acceptable level of risk.

designs, the simulated realisations reflect the variability in grade within this area. Additional information shown in Figure 10 is discussed in the next section.

Effects of the simulation method

Conventional estimation approaches used for orebody modelling differ in their formulations as well as orebody models they generate from the same original dataset. Similarly, different implementations of the same method will result in somewhat different representations of the orebody being modelled. The same is also true for simulation methods and the orebody models generated, including the average ring grades and probabilities above the cut-off considered in the stope optimisation approach used here. Thus, it may be of interest to consider how the stope optimisation results may differ, if the orebody used was simulated independently and with a different simulation method. For this study, an alternative method is the sequential indicator simulation method or SIS (Goovaerts, 1997) and was implemented independently from this study at Kidd Creek by Kay (2001). The latter study provides 40 simulated realisations of the same broader domain.

Figure 10 compares the two designs (both with an 80 per cent acceptable level of risk) in terms of the contained tonnage and grade for both the primary stoping and pillar recovery layouts. As expected, these design layouts contain the same amount of tonnes with only slight variations in potential copper grade. The wider risk profile in the pillar recovery layout is not unexpected due to the limited selection of rings remaining for the second pass of the optimiser. The limited extent of pillar recovery can be explained using the same rationale. From the observations made from Figure 10, the difference in simulation method cannot be said to affect the stoping optimising process.

Figures 11 and 12 illustrate, on a given section, the location and size of the relative stoping (dark grey) and pillar (light grey) layouts based on the simulated orebody with the two different methods at an 80 per cent probability above cut-off set as the minimum acceptable risk. The figures reflect how the central high-grade zone evident in the drill holes is consistently reproduced by both simulation techniques, as expected, and hence located by the optimisation process at the specified probability constraint. The lower level stoping layouts are almost identical. In the upper level, the SGS-based layout considers a stope in the north-east part of the study area not included in the layout shown in the figure based on SIS at the same 80 per cent probability. However, if the minimum acceptable level of risk governing these designs is lowered to say, 70 per cent, the same part of the study area is highlighted as the location of a possible stope by the optimisation based on the SIS models. The stoping layout in the upper level based on the SIS orebody models, recognises a larger stope in the sixth panel whose extent is not considered by the layout based on the SGS models.

These minor differences between designs are normal and not significant. Similarly to the various conventionally used estimation methods for orebody modelling leading to variations in stope designs, different simulation methods will perform somewhat differently from each other, as their specific technical specifications and characteristics dictate. For example, SGS is based on one grade variogram whilst SIS requires multiple variograms, each for a series of grade cut-offs (Goovaerts, 1997). The discrepancies arising from different methods are more extensively documented in other areas of application of simulations such as grade control that have long been in practice (Dimitrakopoulos, in press). Independent implementations provide a source of variance for the results, because the detailed specifications of the simulated orebody models and the parameters for their generation are different. These deviations become apparent in the stoping layouts generated.

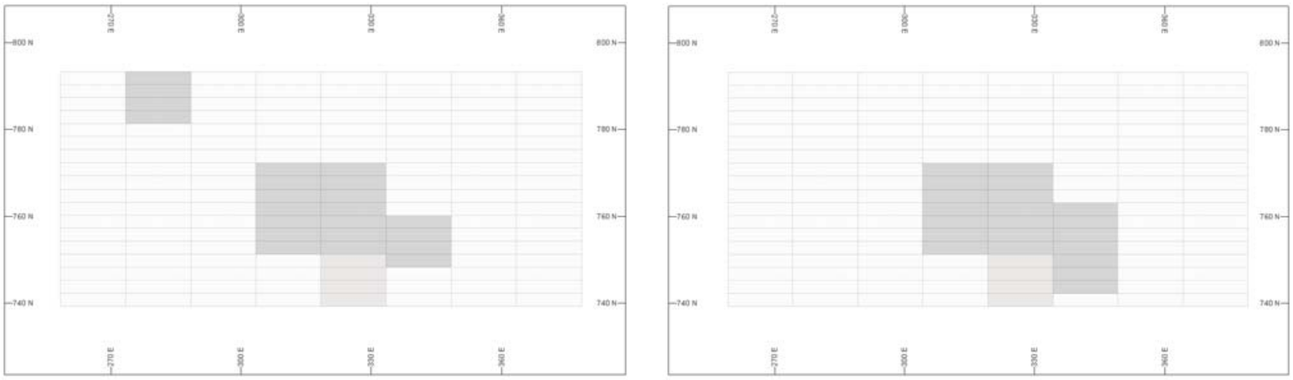


FIG 11 - Horizontal section of the risk-based stope designs in the upper level from the orebodies generated with SGS (left) and SIS (right), for 80 per cent acceptable level of risk.

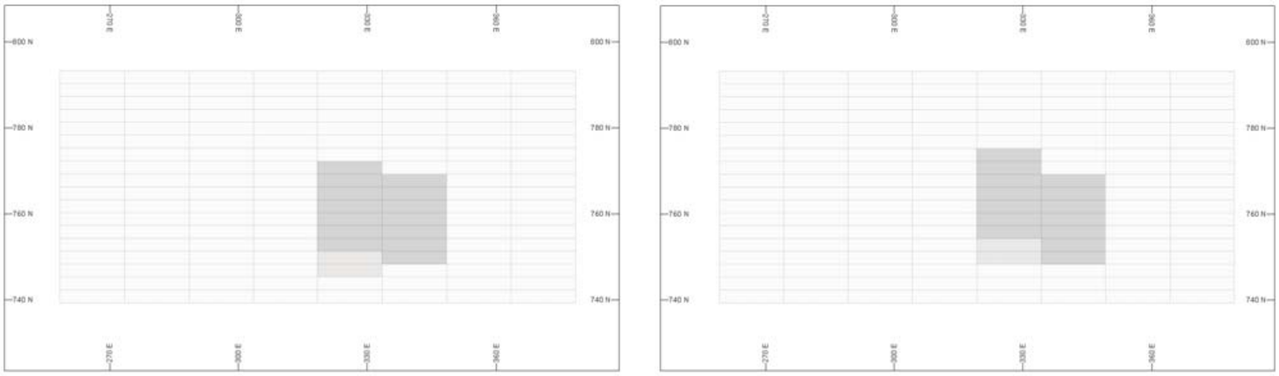


FIG 12 - Horizontal section of the risk-based stope designs in the lower level from the orebodies generated with SGS (left) and SIS (right), for 80 per cent acceptable level of risk.

SUMMARY AND CONCLUSION

This paper extends concepts and technologies used in managing geological risk in open pit mines to underground mining methods. It shows that geostatistical simulation technologies allow grade risk quantification in a stoping design. The example from the Kidd Creek mine, Ontario, Canada illustrates how conventional technologies cannot quantify risk, thus are unable to foresee a significant upside potential and/or downside risk for the conventionally produced designs. The example shows a conventional design could be valued from as little as 1.8 million dollars to as much as 5.9 million dollars. To provide the means of incorporating risk in stope design, geological uncertainty is integrated into the design process through a new mathematical programming formulation that uses risk grades above a cut-off value for rings within a stope, as well as geometric and other traditional constraints. An additional constraint introduced is the minimum acceptable risk allowed in a design. The application shows that the risk-based approach has the ability to generate different designs that meet the pre-specified minimum acceptable risk with a desired risk profile accommodating the selection of designs with preferred upside/downside profiles. Grade uncertainty quantification may be based on different simulation methods. A comparison of orebody models constructed independently with the sequential Gaussian and indicator simulation methods show stope designs with some variation, which is not significant and considered normal when different methods are used.

The work presented here could be further developed. Such developments could include:

1. the formulation of a stope optimisation formulation that replaces the probability of grades above cut-off with the direct use of all available simulated orebodies, and thus integrate more geological information;
2. consider sequencing and thus accommodate risk management and/or geological risk discounting as part of the stope design process; and
3. extend to integrate geotechnical uncertainties starting from over-breaking and under-breaking.

ACKNOWLEDGEMENTS

Special thanks to Paul Roos and Arie Moerman from Falconbridge, Kidd Creek mine who supplied the data and provided support. Thanks also to Mark Noppe and Jörg Benndorf for their constructive comments.

REFERENCES

Ataee-pour, M and Baafi, E Y, 1999. Stope optimisation using the maximum value neighborhood (MVN) concept, in *Proceedings 28th International Symposium on the Application of Computers and Operations Research in the Mineral Industry* (ed: K Dagdelen), pp 493-501 (Colorado School of Mines: Golden).

Baker, C K and Giacomo, S M, 1998. Resource and reserves: their uses and abuses by the equity markets, in *Ore Reserves and Finance*, pp 65-76 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Bawden, W F, 2007. Risk assessment in strategic and tactical geomechanical underground mine design, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 263-271 (The Australasian Institute of Mining and Metallurgy: Melbourne).

DATAMINE™, 1995. *Floating Stope Optimiser User Guide,* Edition 1.2, 20 p (Mineral Industries Computing Limited).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions,* 316:106-112.

Dowd, P A, 1997. Risk in minerals projects: analysis, perception and management, *Trans Inst Min Metall*, Section A, Mining Technology, 106:A9-A18.

Froyland, G, Menabde, M, Stone, P and Hodson, D, 2007. The value of additional drilling to open pit mining projects, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 245-252 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions,* 316:43-50.

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 483 p (Oxford University Press: New York).

Grieco, N J, 2004. Risk analysis of optimal stope design: incorporating grade uncertainty, MPhil thesis, University of Queensland, Brisbane, 204 p.

Kay, M H, 2001. Geostatistical integration of conventional and downhole geophysical data in the metalliferous mine environment, MSc thesis, University of Queensland, Brisbane, 204 p.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Myers, P, Standing, C, Collier, P and Noppè, M, 2007. Assessing underground mining potential at Ernest Henry Mine using conditional simulation and stope optimisation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 191-200 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ovanic, J, 1998. Economic optimization of stope geometry, PhD thesis, Michigan Technological University, Houghton.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ravenscroft, P J, 1992. Risk analysis for mine scheduling by conditional simulation, *Trans Inst Min Metall*, Section A, Mining Technology, 101:A104-A108.

Rendu, J-M, 2002. Geostatistical simulations for risk assessment and decision making: the mining industry perspective, *International Journal of Surface Mining, Reclamation and Environment*, 16:122-133.

Roos, 2001. Underground tour guidebook, Kidd Creek Mine, p 21.

Thomas, G and Earl, A, 1999. The application of second-generation stope optimisation tools in underground cut-off grade analysis, in *Proceedings Strategic Mine Planning*, pp 175-180 (Whittle Programming Pty Ltd: Perth).

Vallee, M, 1999. Resource/reserve inventories: what are the objectives? *CIM Bulletin*, 92(1031):151-155.

Strategic Mine Planning at Murrin-Murrin, Western Australia — Implementing NetVal

R O Jaine[1] and M Laing[2]

ABSTRACT

The long-term strategic net present value (NPV) maximising mine schedules at Minara Resources nickel cobalt laterite operations are determined after a comprehensive series of steps. The optimisations of the 18 individual block models and the long-term mine plans focus on the net value (NetVal) from the contribution of each individual element within each block, with nickel and cobalt being the revenue generating elements and magnesium, aluminium and iron effectively being penalty elements consuming acid and reagents. This results in pit designs and long term schedules that optimise the NetVal of the complete mining and processing system. After completion of the pit designs and schedules, the final bench outlines are provided to geology for ore-waste block classification for mining. To date, these ore blocks have been determined by applying nickel cut-offs without considering the other elements that influence the NetVal of the block.

This paper discusses changing the basis of the ore waste block classification from nickel cut-offs to NetVal cut-offs, outlines the approach taken to determine these new cut-offs and summarises the benefits of adopting this approach including the additional value that can be added to the project.

INTRODUCTION

This paper looks at the current mine planning practice at Murrin-Murrin and at changing the existing practice of ore waste classification using nickel cut-offs to net value cut-offs. The additional value to the business that can be gained by making this change is examined to present the case for making the change.

Mine design and scheduling is carried out using NPV optimisation logic, with the NetVal of material determined as a function of the different ore characteristics, the processing variables and the required financial parameters.

This NetVal logic is used as the basis for over 100 separate pit designs for the life of the mine, which are in turn used as the basis from which to derive various mine schedules that are optimised for different scenarios. Once a mine schedule has been selected, this determines the pits that are to be mined and the sequence in which those pits should be mined to achieve the optimised NetVal for that particular mine schedule/scenario.

Currently, once a pit has been selected for mining the ore-waste block classification is determined using a nickel grade cut-off. While nickel is one of the key drivers of the NetVal of a block it is not the only one and represents a deviation from the optimal mine plan. This deviation presents as an opportunity to add value to the business by changing the basis for the classification of ore.

The challenge has been to implement a system of ore definition based not only on the nickel values of a given block but on a combination of all the factors in the block that contribute to (or detract from) the value of that block, that is, the NetVal of the block. This challenge has included determining the optimum number and ranges of NetVal cut-offs to apply to not only material sent directly to the mill, but also to the material stockpiled in an endeavour to maximise the value of the ore deposits to the business.

1. MAusIMM, Mine Planning Superintendent, Minara Resources Limited, Level 4, 30 The Esplanade, Perth WA 6000, Australia. Email: rjaine@minara.com.au

2. General Manager, OM Manganese Limited, Level 1, 46 Parliament Place, West Perth WA 6005, Australia. Email: mark.laing@ommanganese.com.au

A strategy to optimise the net-value cut-offs and the number of intermediate stockpiles is outlined using MineMax, (NPV maximising mine scheduling software), and a method to determine tonnage equivalent net value cut-offs, that will provide a starting point during the transition of the implementation of ore waste classification based on net value at Murrin-Murrin.

The primary objective is to determine the optimum net value cut-off grades to apply in order to maximise the NPV of the project, and is effectively a practical application of both Lane (1988) and MineMax mine planning software which performs optimisation of multiple resource models and/or mines, simultaneously.

LOCATION AND GEOLOGY

The Murrin-Murrin (MM) nickel cobalt laterite deposits are situated approximately 50 km east of Leonora and 210 km NNE of Kalgoorlie within the north-eastern Yilgarn Craton of Western Australia.

The processing plant and accommodation village is located approximately halfway between Leonora to the west, and Laverton to the east in the northern goldfields of WA as can be seen in Figure 1.

Figure 1 show the three regions of the MM orebodies comprising MM North (MMN), MM South (MMS) and MM East (MME), with MMS and MME some 25 km and 50 km distant from the plant respectively.

The laterite nickel and cobalt ore occurs primarily in three regolith units of serpentinised peridotite within the Archaean Norseman-Wiluna greenstone belt, comprising a ferruginous zone, a smectite zone and a saprolite zone as shown in Figure 2.

The Ferruginous Zone is predominantly waste containing iron oxides and is depleted of nickel and cobalt. Minor amounts of nickel and cobalt mineralisation occur at the base of this zone at the contact with the smectite.

The Smectite Zone is predominantly smectite clay and is generally enriched in nickel and cobalt and depleted in magnesium.

The Saprolite Zone is an altered serpentinised peridotite (metamorphosed ultramafic) dominated by the serpentine group of minerals. The grade of the nickel and cobalt within the saprolite zone is generally gradational and decreasing from the contact with the smectite. The saprolite is also enriched in magnesium.

GEOLOGICAL BLOCK MODELLING

The initial geological block models are created using Vulcan software. Block sizes for the resource models are 25 m × 25 m × 2 m with sub-celling down to 12.5 m × 12.5 m. Block sizes for the grade control models are 12.5 m × 12.5 m × 2 m with sub-celling down to 6.25 m × 6.25 m.

Figure 3 shows the locations of the 18 resource block models with the North model made up of four quadrant block models due to block model size restrictions.

Historically, median indicator kriging was used to estimate the major elements Ni, Mg, Al and Fe and multiple indicator kriging for Co. The remaining minor elements were estimated using inverse distance squared.

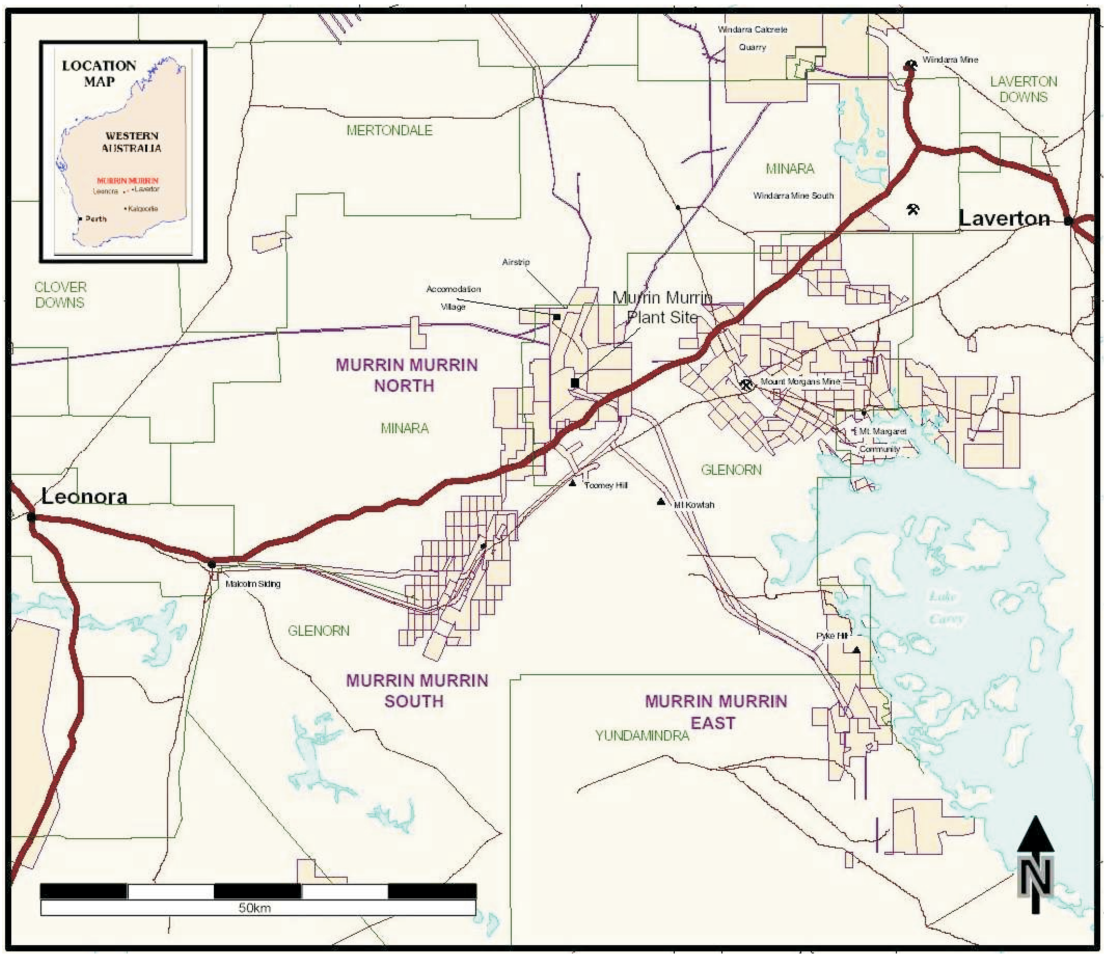


FIG 1 - Murrin-Murrin tenements.

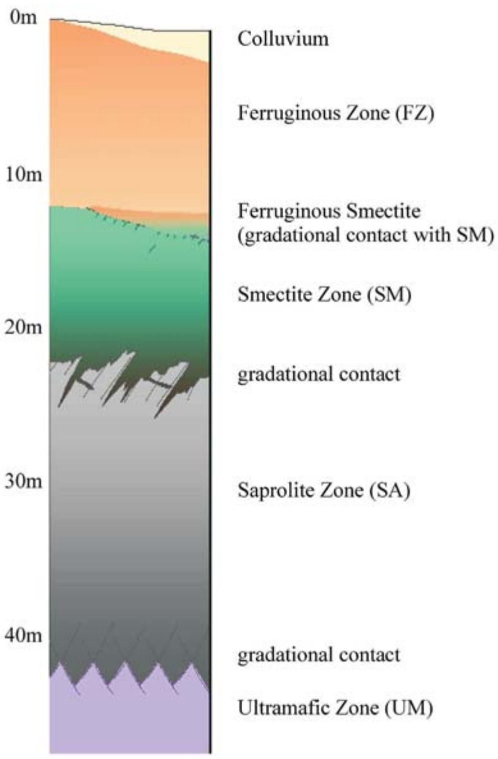


FIG 2 - Murrin-Murrin orebody section.

Recently, the use of sequential gaussian conditional simulation techniques (Isaaks, 1990; Goovaerts, 1997) have been implemented for the major elements Ni, Co, Mg, Al and Fe, and ordinary kriging (Isaaks and Srivastava, 1989) for the minor elements.

The conditional simulation has also simplified the method previously used to estimate the major elements, Ni, Co, Mg, Al and Fe, by eliminating the requirement to wireframe the models prior to estimation. The conditional simulation of each element is determined independently without considering any correlation that may exist between variables.

Currently, MMN is comprised of eight resource and 16 grade control models. MMS and MME are comprised of six and four resource models respectively.

The geological models are then regularised down to the minimum subcell size for each model and imported into a MineSight 3D block model in preparation for pit design and scheduling. This begins the mine planning process summarised in Figure 9 – the mine planning process at Murrin-Murrin.

MINING AND PROCESSING

Mining at MM commenced in February 1998. All mine production to date has been sourced from MMN. The development of MMS is currently underway with ore mining scheduled for October 2004. MME is scheduled to commence mining in January 2006.

The orebody is relatively shallow in occurrence with a maximum depth of approximately 60 m and is mined using a conventional open pit truck and excavator fleet configuration.

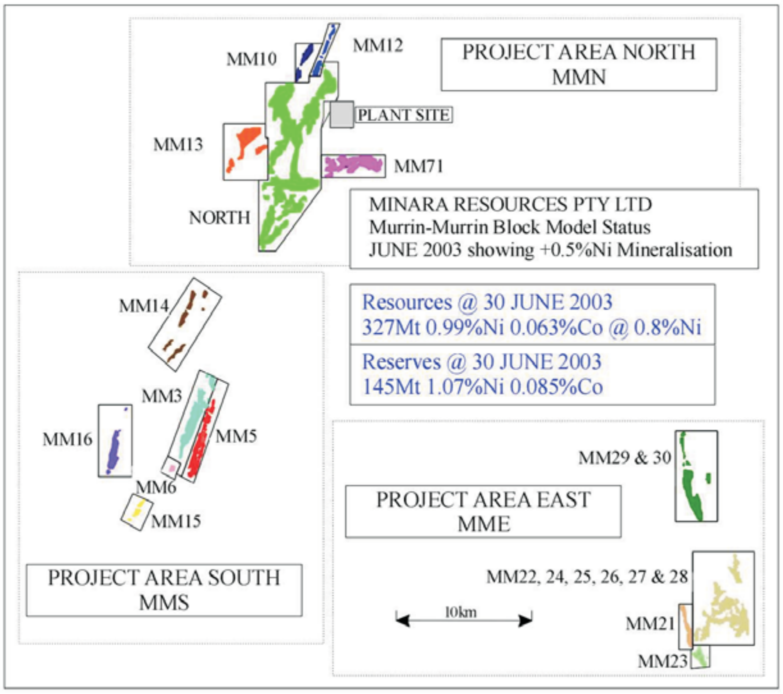


FIG 3 - Geological block models showing nickel mineralisation, resources and reserves.

Mining is carried out on 2 m bench heights with the low bench height necessary to obtain satisfactory selective control during extraction of the high-grade ore. Higher bench heights would result in a reduction in the high-grade ore tonnage for any given bench, pit or orebody due to the volume-variance relationship and would result in either less tonnes of higher grade material or lower average grades.

Current mining rates are of the order of 20 M bank cubic metres (BCM) per annum with an approximate waste/high-grade volumetric strip ratio of 6:1.

To date, a total of over 66 million BCM has been mined, ore has been produced from six of the 18 deposits and a total of 17 pits have been mined out of which five are currently active.

First ore was processed in January 1999, and as at the end of June 2004 over 11 Mt of ore has been processed producing approximately 110 000 t of nickel and 7200 t of cobalt.

The schematic ore-processing path is illustrated in Figure 4. A detailed business model has been developed that models the processing relationships of the various rock-types and forms the basis for calculating the net value of each parcel of each rock type.

BUSINESS MODEL INPUTS

The inputs into the business model have been summarised in Table 1.

Constraints such as production limits or capacities and production requirements are applied to the model as maxima or minima or a combination of the two to reflect the process restrictions and production requirements that any resultant schedules or scenarios must satisfy.

The constraints applied to the business model by period are summarised in Table 2.

Using this information, the ASCII file for each model is processed further using a tool command language (TCL) script to calculate and append additional information to each block into the required MineMax format prior to importation of the blocks into the MineMax model. The data output is in the required format for MineMax to recognise the additional calculated products and their attributes or grades.

The additional information:

- is used to track the material from source to destination using region, deposit and stockpile codes, and to break down the materials into the different ore types;
- allows the tracking of acid consumption requirements (a constraining processing factor) and the production of ammonium sulfate, a saleable by-product of the process, as well as the blend proportions of the ore types in the feed, and to set up constraints and targets in the MineMax models; and
- is used to set up specific constraints and targets in the MineMax models for mine scheduling.

Whittle Consulting Pty Ltd, via a bureau service using their proprietary Multi-Pit/Blending/NPV-Maximising/Mine-Scheduling software are also the recipients of the data at this point. Whittle Consulting independently determines optimised schedules which are used for confirmation and or comparison with those determined inhouse using MineMax.

WHY CHANGE TO NET VALUE?

For any grade-tonnage curve and any Ni per cent cut-off, a tonnage above that cut-off can be determined. From the NVPT-tonnage curve for the same block model a corresponding NVPT (net value per tonne) cut-off can then be back calculated to provide the same tonnage as that using the Ni per cent cut-off.

That is: $\text{Tonnes}_{(>= \text{Ni per cent cut-off})} = \text{Tonnes}_{(>= \text{NVPT cut-off})}$

The NetVal of these same tonnages can be compared to demonstrate the additional value that may be captured by simply changing the basis of blocking out the ore from Ni per cent to NVPT. This process has been applied to the 18/6 pit with the results indicating that changing from a Ni per cent cut-off to an NVPT cut-off would result in a 4.2 per cent increase in NetVal for a 1.0 per cent Ni cut-off, 5.1 per cent for a 1.1 per cent Ni cut-off and a 4.8 per cent increase for both 1.2 and 1.3 per cent Ni cut-offs for the same tonnes. Therefore, by processing the same tonnage using an NVPT cut-off instead of the equivalent Ni per cent cut-off tonnage, the cash flow and total NetVal would be increased by these same proportions.

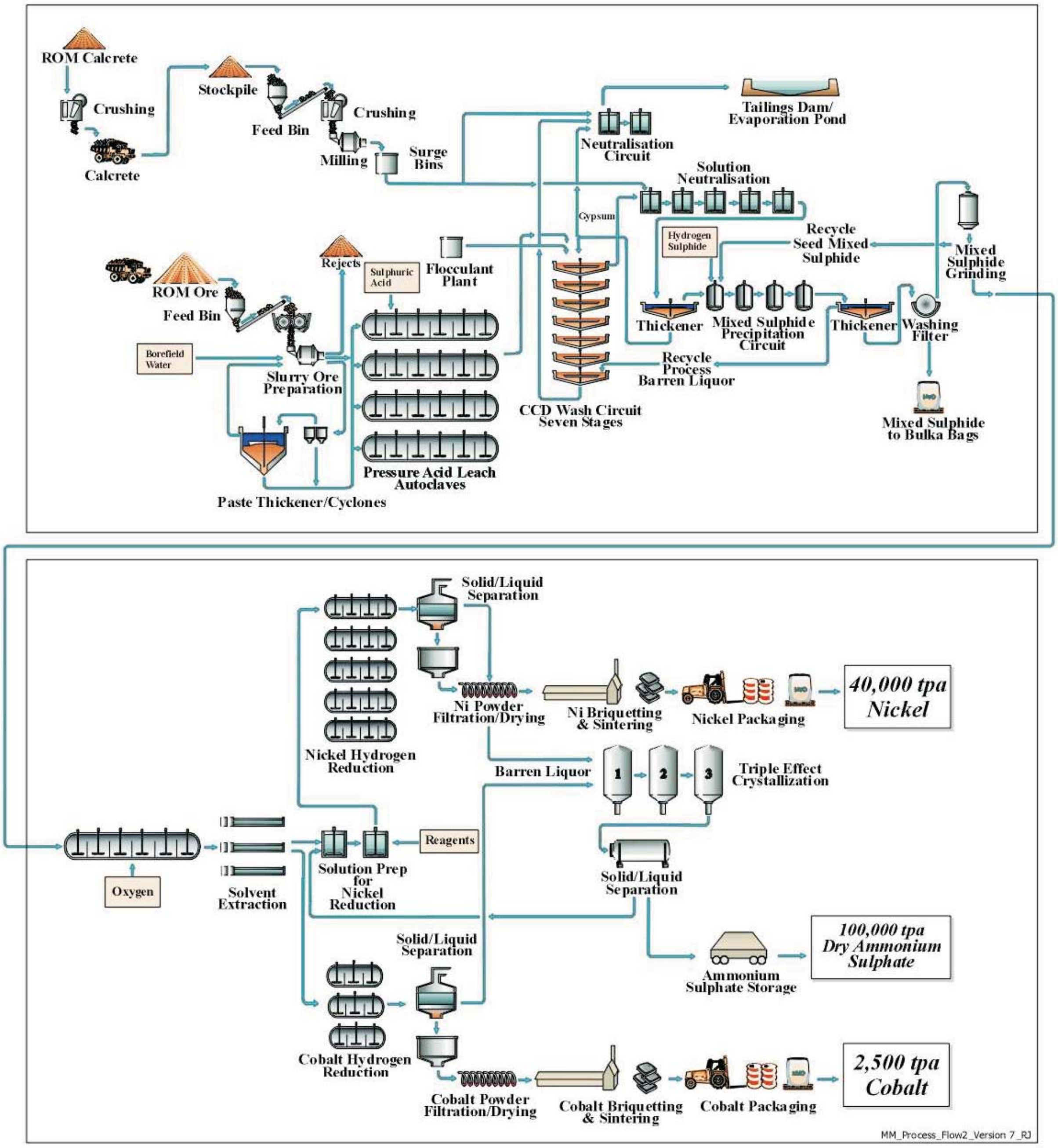


FIG 4 - Murrin-Murrin process and technology flow chart (Anaconda Nickel Limited, 2003).

The increase in NetVal is not constant across the Ni per cent cut-offs, and is due to the different distributions of the other elements within any given Ni per cent band. Figure 5 summarises the increase in cash flow and NetVal by changing from Ni per cent to NVPT cut-offs for any given Ni per cent cut-off.

Figure 6 displays the material above 1.2 per cent Ni for a given bench with Figure 8 displaying the material above the equivalent NVPT (EqNVPT), the cut-off that produces the same tonnes.

The change in ore allocation by changing from a 1.2 per cent Ni cut-off to an EqNVPT cut-off basis can be seen in Figure 7 for pit 18/6, 426 - 428 mRL bench.

For pit 18/6 (Figure 6, 7 and 8), the tonnes above 1.2 per cent Ni equal the tonnes above EqNVPT. Figure 8 shows the material with an NVPT ≥ EqNVPT that would previously have been sent to stockpile, and material with an NVPT < EqNVPT that would previously have been sent to ROM. This reallocation of material based on NetVal is the key mechanism to improve cash flow and add value.

THE MINE PLANNING PROCESS

The mine planning process starts with geological block models created using Vulcan software, that are regularised and exported to MineSight in preparation for pit design. After pit optimisation in Whittle 4X, the optimised pit shells are imported to MineSight for pit design and subsequent block model coding. The next step is the processing of the dumped ASCII block model data using TCL scripts, then importation and re-blocking of this data into the MineMax model. This is followed by the setting up of the scenario parameters, requirements and constraints, and concludes ultimately with the generation of the optimised mine plans and schedules.

TABLE 1

Business model inputs.

Category	Input
Geological block model	Block centroid coordinates and dimensions
	Ni, Co, Mg, Al and Fe grades
	Density (SG)
	Ore type
	Block per cent below surface topography
	Pit name
Deposit	Cl grade
	Haul cost
	Region and deposit name
Processing	Scats reject + beneficiation of nickel and cobalt
	Pressure acid leach density
	Recoveries: mill, counter current decant thickeners, mixed sufide precipitation circuit and refinery
	Rock type particle density
	Acid consumption factors and constants
	Calcrete consumption parameters
	Ammonium sulfate saleable by-product multiples
Economic/ Financial	Commodity prices: nickel, cobalt, ammonium sulfate, sulfur and calcrete
	Exchange rates
	Royalties
Cost components	Capital
	Mining
	Hauling
	ROM handling
	Milling
	Pressure acid leaching
	Calcrete neutralisation
	Acid
	Refining
	Fixed
	Selling and transportation

TABLE 2

Business model constraints.

Description
ROM production requirements (total and by region)
Individual pit and total movements
Number of pits
Ore type blend (saprolite/smectite ratios)
Ni/Co element ratios
Acid consumption
Grades

Stockpiling

Historically, the different saprolite, smectite and ferruginous ore types were stockpiled separately due to their differing milling and metallurgical properties. Table 3 displays the three ore nickel grade ranges. These rock types were further divided into these three nickel-grade ranges of high, medium and low grade with the exception of any >1.0 per cent Ni ferruginous material downgraded to low grade.

Current practice combines all +1.2 per cent Ni material into one blended ROM (run of mine) stockpile, and continues to stockpile medium and low grade material by geology and to downgrade ferruginous material above 1.0 per cent Ni to low grade stockpiles.

This now results in seven destinations in practice, comprising the ROM, the waste dump and five stockpile destinations.

The previous strategy of stockpiling the different grade ranges was to allow for the milling of the higher-grade material as early as possible in the mine life in order to maximise NPV by providing earlier cash flows.

It is clear that applying a Ni per cent cut-off to determine the destination for ROM and stockpile material will result in some material with a higher NetVal being sent to stockpile, some material with a lower NetVal will be sent to the ROM for processing, and that some uneconomic material will be sent to stockpile.

Therefore, to maximise cash flow and NetVal, material classification must be based on NetVal to ensure that the most valuable material is sent direct to process via the ROM, and that no overlap in NetVal ranges occurs between stockpiles.

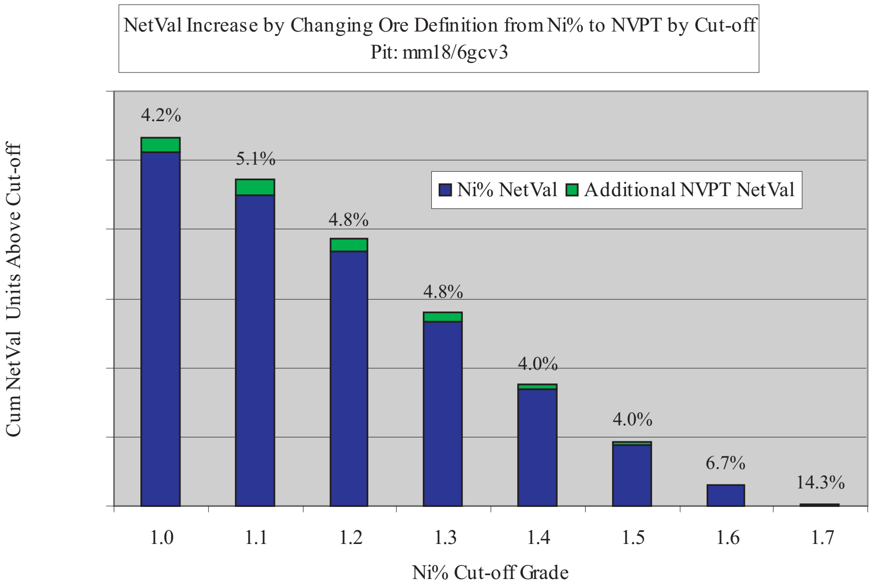


FIG 5 - Potential increase in cash flow + NetVal by changing ore definition from Ni per cent to NVPT.

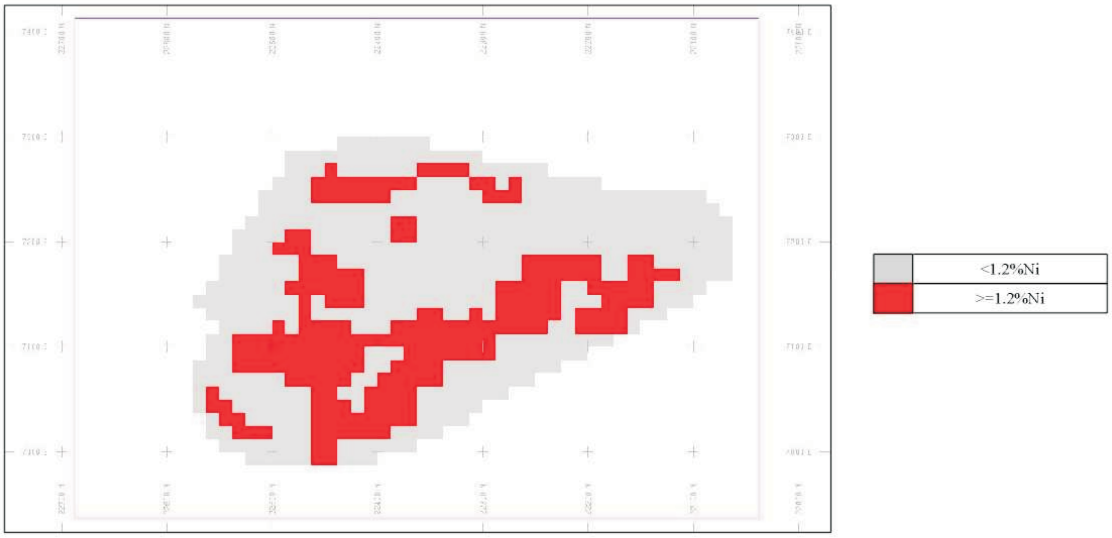


FIG 6 - Material above 1.2 per cent Ni – bench 426 - 428 mRL – pit 18/6.

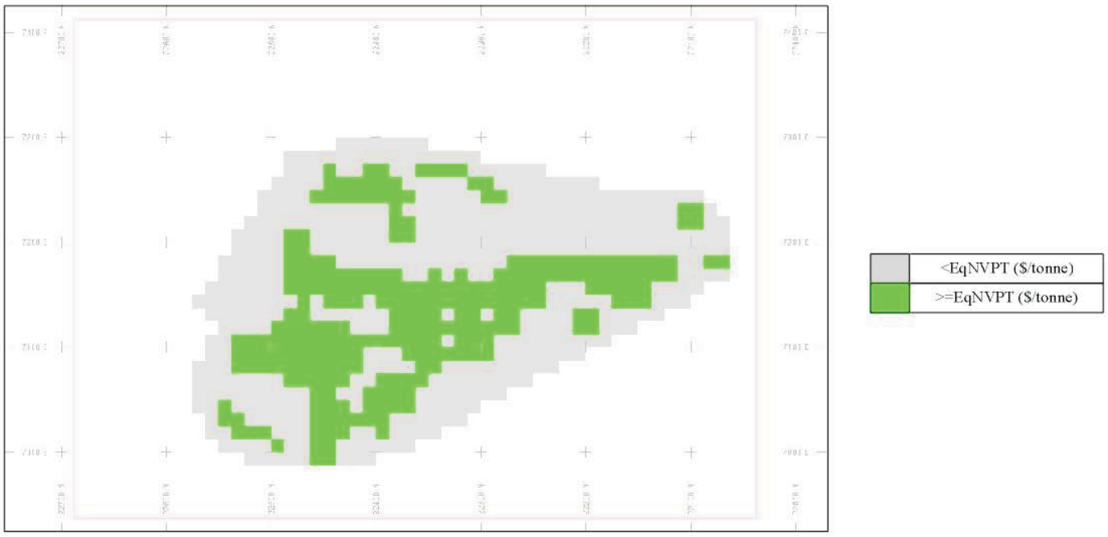


FIG 7 - Material above EqNVPT – bench 426 - 428 mRL – pit 18/6.

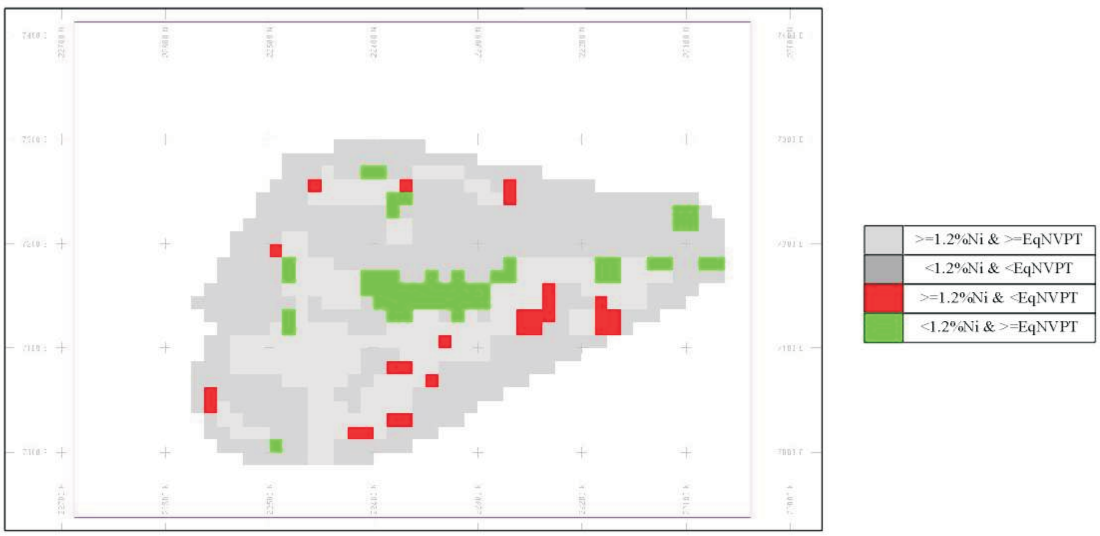


FIG 8 - Misallocated material – bench 426 - 428 mRL – pit 18/6.

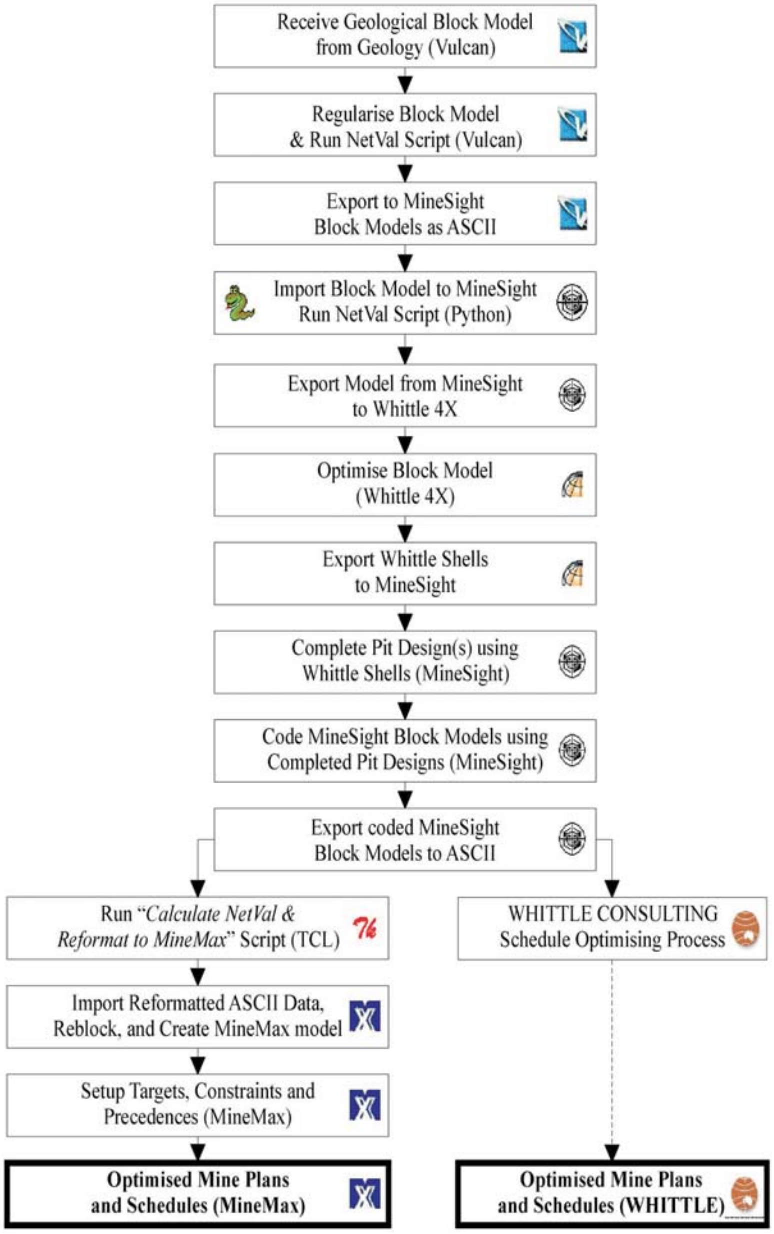


FIG 9 - The mine planning process at Murrin-Murrin.

TABLE 3

Ore classification – nickel grade ranges.

Ni% range	Ore classification
+1.2% Ni	High
1.0 to 1.2% Ni	Medium
0.8 to 1.0% Ni	Low

Blending

Blending of the different ore types reduces the problems associated with the treatment of specific ores that have difficult ore handling properties such as high viscosity or low metallurgical recoveries or that individually exceed limits or constraints.

Historically, the different ore types have been blended from the individual high-grade ROM stockpiles to provide feed to the mill.

Current practice combines saprolite and smectite ROM material into blended stockpiles. This smooths milling, metallurgical and grade characteristics of the individual material types comprising the blended stockpiles, thereby providing a relatively more consistent and homogeneous mill feed when compared to blending from the individual material type stockpiles. This process further ensures fewer excursions beyond plant capacities or limitations.

IDENTIFYING THE NET VALUE POTENTIAL BENEFIT OF STOCKPILES

In order to define the limits of possible benefit to be gained by changing the NVPT cut-offs to stockpiles and the number of stockpiles, the NPV (using MineMax) of the current base case mine schedule is calculated with stockpiling *'turned off'* and another NPV is calculated with a block based stockpiling reclaim option. The difference between the two NPVs is the maximum possible benefit to be realised from stockpiling.

A block based stockpile reclaim is where the schedule optimiser stockpiles every block individually and retains the ability to select that individual block for processing in a future period. In reality this would of course not be possible but provides a useful upper NPV limit for comparison.

Using stockpiles relative to not using stockpiles increased the base case NPV by five per cent. This demonstrates the value adding benefit of using stockpiles. As stockpiles clearly provide significant value to the business, the objective must be in a practical sense to get the greatest benefit with the minimum number of stockpiles as possible.

The next step is to determine the optimum number of stockpiles and the NVPT cut-offs to use for each stockpile.

SELECTING THE NVPT CUT-OFFS

The optimum NVPT cut-offs for stockpiling will be those that maximise the NPV of the project over the life of mine (LOM) subject to practical stockpiling limitations.

The initial NVPT bands chosen for the implementation were those bands that provided the same number of stockpiles as previously using Ni per cent cut-offs, and were also chosen so that the tonnages within each NetVal band was the same as or similar to the tonnages within each Ni per cent band. This is the technique outlined above in the section '*Why Change To Net Value?*'.

Figure 10 shows the relationship between NVPT and Ni per cent cut-offs on an equivalent tonnes basis, for MMN, MMS, MME and the TOTAL respectively.

From this, the corresponding NVPT cut-offs can be read off directly for any Ni per cent cut-off for any region.

SELECTING THE NUMBER OF NVPT CUT-OFFS AND STOCKPILES

The number of stockpiles (or NVPT bands) must take into consideration the proportion of material in the band and how the material 'blocks-out' or 'hangs together' in a geological sense. That is, the material when blocked out should allow for efficient mining.

Other considerations include stockpile management. Every stockpile represents an active dumping face and destination that must be maintained and managed on a daily basis both from an operational and administrative perspective.

While the greater the number of stockpiles, the greater the benefit in NPV contribution terms, it is clear that there is a diminishing returns relationship with every additional stockpile resulting in smaller and smaller increases in NPV.

The key is therefore to select the right number of stockpiles that balances the diminishing returns against the additional stockpile management complexity.

To evaluate different cut-off bands, and or different numbers of stockpiles, each combination requires a new MineMax model to be created. Effectively, the raw data must be reblocked into the selected bands using TCL, and the optimisation rerun utilising a standard scenario common to all runs. This approach allows the impact of the bands to be assessed, as all else is constant.

While the method for determining the NPV using MineMax is beyond the scope of this paper, Minara's MineMax method has been covered more fully elsewhere by Jaine (2003).

DISCUSSION AND FURTHER WORK

This paper discusses a number of related strategies investigated by Minara's Mining Department in their continuing goal to add/maximise value to Minara's Stakeholders and to move toward industry best practice.

Using Lane's definitions, the operation can be considered as mill or processing limited, and that the optimum cut-off grade(s) determined are a mine/mill balanced cut-off grade, that is for the current mining fleet, the cut-off chosen will ensure that the mill/plant remains at full capacity and will be fed the best net value material available from the total material mined.

The NetVal approach is similar in concept to Lane's 'equivalent grade' but rather than assuming the secondary grade elements are equivalent to a fixed ratio of the primary grade, take into account the non-linear relationship between the various elements to derive a measure to enable a direct comparison between material parcels for the purpose of ore waste classification.

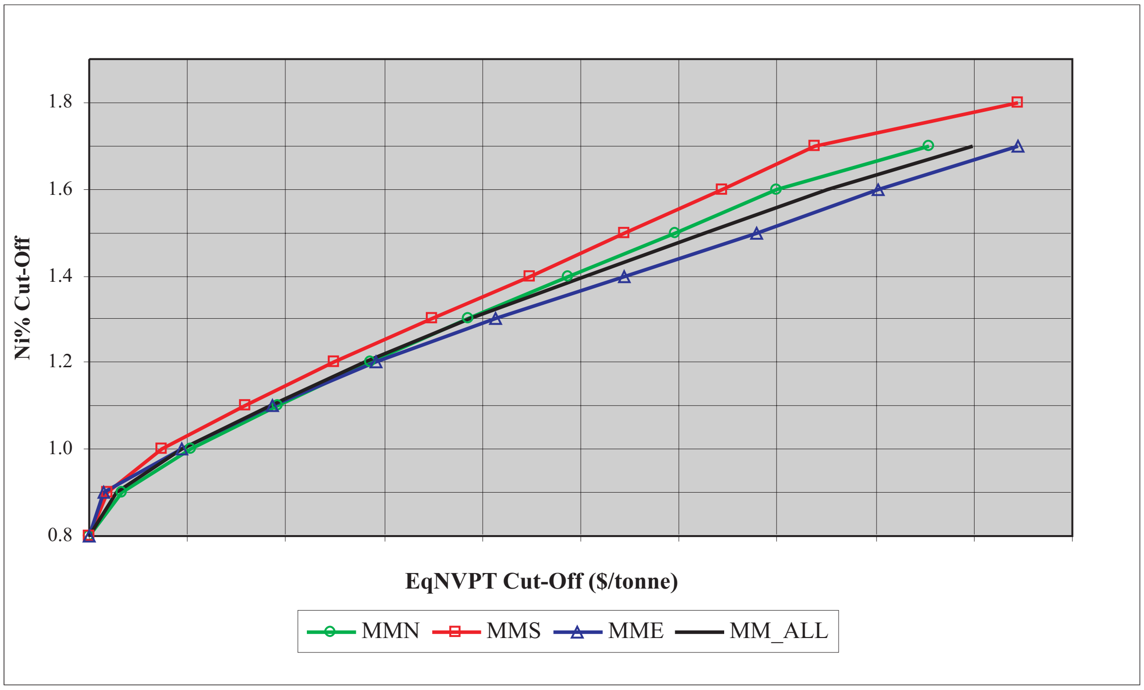


FIG 10 - Equivalent NVPT ($/tonne) and Ni per cent cut-offs, MMN, MMS, MME and TOTALs.

CONCLUSION

The first strategy proposed is simply to change ore definition from a Ni per cent to an NVPT cut-off. Extrapolating the findings above from the analysis of pit 18/6, this has the potential to add an additional 4.2 per cent in Net Value and cash flow to the operation. Applying this same analysis to Minara's *total* reserve base has shown even higher increases in Net Value of up to ten per cent for reserves greater than 1.0 per cent Ni. This strategy targets value rather than nickel grade, accounting for all of the factors that determine or impact on value or cost.

The implementation of NetVal will align the value chain all the way through the individual block model's optimisations, pit designs and LOM schedules, down to the marking up and digging out of the ore blocks on the ground.

In order to manage any change or combination of changes in costs, commodity prices, exchange rates or processing dynamics that impact on the NetVal of any given block, the inputs can be updated and the NVPT cut-offs to apply for block outs can be re-calculated, and the NVPT cut-offs used for the existing stockpiles can be back calculated.

Using the equivalent tonnes approach, any changes that increase (or decrease) the NetVal of a block will also increase (or decrease) the NVPT cut-offs to be applied.

This will allow the application of the current best available processing information and financial/economic views to be incorporated into the mining method and also ensure that material is correctly stockpiled accordingly, with any changes to be made on an 'as required' basis.

Using both MineMax and Whittle Consulting's Multi-Pit/Blending/Alternative-Path-Processing/NPV-Maximising/Mine-Scheduling software packages (solution solve and solution seek algorithms respectively) allows Minara to undertake strategic mine planning using different approaches.

This gives confirmation and greater confidence in the resultant mine plans as the outputs are compared and contrasted. Contrasting differences in the results provides the opportunity for improvements and further developments in the software.

The benefit demonstrated from analysis of pit 18/6 indicates an increase of over four per cent in NetVal for the material greater than 1.0 per cent Ni. This increase in NetVal is greater than ten per cent for the total reserve base. Clearly, Minara is in a position to benefit significantly by simply changing from a Ni per cent cut-off to an NVPT cut-off as soon as it can be implemented, as this will result in a ten per cent increase in NetVal compared to processing the same number of tonnes using a Ni per cent cut-off.

The NVPT cut-offs proposed for the implementation strategy are the equivalent cut-offs (on a tonnage basis) as the Ni per cent cut-offs currently used and will provide a useful starting point for further investigations.

In addition to the ten per cent to be gained from changing to NVPT, stockpiling is also of significant value, and can theoretically contribute up to five per cent of the NPV using the '*block based stockpile reclaim*' function. Using the proposed NVPT cut-offs (equivalent tonnage basis to the Ni per cent cut-offs currently used) captures 44 per cent or 2.2 of the five per cent theoretical maximum.

Further studies will continue as outlined above to investigate the numbers of stockpiles and the NVPT cut-offs for these stockpiles, in an endeavour to identify and extract additional benefit in the ongoing quest to maximise the value of the resource to the business.

ACKNOWLEDGEMENTS

The authors would like to thank the management of Minara Resources Ltd for permission to publish this paper.

REFERENCES

Anaconda Nickel Limited, 2003. Annual report.

Combinatorics, 1999a. MineMax mine scheduling software, User Manual, pp 21-46.

Combinatorics, 1999b. MineMax Scheduler – mine schedule optimisation – White Paper, pp 4-11.

Goovaert, P, 1997. *Geostatistics for Natural Resources Evaluation* (Oxford University Press).

Isaaks, E H, 1990. The application of Monte Carlo methods to the analysis of spatially correlated data, PhD thesis, Stanford University.

Isaaks, E H and Srivastava, R M, 1989. *Introduction to Applied Geostatistics* (Oxford University Press).

Jaine, R O, 2003. Mine planning at Murrin-Murrin – modelling to determine the optimum path, in *Proceedings Twelfth International Symposium on Mine Planning and Equipment Selection*, pp 439-446 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Lane, K, 1988. *The Economic Definition of Ore*, 147 p (Mining Journal Books Ltd: London).

Development and Application of Whittle Multi-Mine at Geita Gold Mine, Tanzania

T Joukoff[1], D Purdey[2] and C Wharton[3]

ABSTRACT

In the past, life of mine scheduling at Geita Gold Mine, Tanzania, has been a largely manual process involving the optimisation and scheduling of each mine as a separate entity. The scheduling has been a time-consuming process undertaken using spreadsheets. Recent advances in the Whittle software have enabled multiple mines to be optimised and scheduled simultaneously, so that the mining sequence that maximises the net present value (NPV) for the entire set of mines as a whole can be determined. This case study presents the results of the development and application of the Whittle Multi-Mine module at Geita Gold Mine. It shows how improvements to the NPV of the life of mine schedule were achieved by using Whittle Multi-Mine as a tool to help guide the preferred order of mining. It highlights the contributions from each of the mines to the overall cash flow of the project and investigates the effect of time on the NPVs from each mine. The cost of deferring production from certain mines has become plainly evident, whilst for others there is little impact. Furthermore, Whittle Multi-Mine has identified areas requiring more focus in terms of the life of mine plan.

INTRODUCTION

Geita Gold Mine is situated in northwest Tanzania, approximately 25 km from the southern shores of Lake Victoria. Historical mining in the area has taken place for many years, with the last major operation being the Geita Underground Mine, which operated from the 1930s through to the 1960s and produced almost 1 Moz of gold. Ongoing small-scale mining continues to this day. The modern Geita mine has been operating since mid-1999, with processing of ore commencing in mid-2000. Towards the end of 2003, 48 Mbcm of material has been mined from three open pits; 14 Mt of ore, grading 3.8 g/t, has been processed and 1.5 Moz recovered. The Life of Mine Plan (2003) indicates a mine life in excess of ten years and entails the mining of ten individual pits, several of which are multi-stage. Total mining is expected to exceed 320 Mbcm, producing more than 80 Mt of high-grade ore and yielding more than 10 Moz of recovered gold. The open pit mines are operated with conventional techniques using excavators and trucks on flitches up to 3.5 m high. Most material requires blasting, ranging from ‘paddock blasting’ in soft laterites and oxides, to hard rock blasting in sulfides.

Pit optimisation at Geita has been an ongoing process, predominantly undertaken using the NPV Scheduler software, however; from early 2003 Whittle software has been used in parallel. Although techniques to evaluate multiple orebodies have existed for some time (July, 1997), each open pit has been optimised and scheduled as a separate entity rather than consideration given to whole of mine optimisation and scheduling. Estimates of the mill throughput likely to be required from each pit were used to guide the pit life and net present value (NPV) calculation. Since the ore delivery rate required was generally not known until the whole mine schedule was finalised using all the pits, this was obviously a flawed process.

1. MAusIMM, Senior Mining Consultant, AMEC Americas Limited, 900 AMEC Place, 801 - 6 Avenue SW, Calgary AB T2P 3W3, Canada. Email: Tamara.Joukoff@amec.com

2. MAusIMM, Senior Mining Engineer, AMEC Americas Limited, 900 AMEC Place, 801 - 6 Avenue SW, Calgary AB T2P 3W3, Canada. Email: dave.purdey@amec.com

3. Principal Consultant, Strategy Optimisation Systems Pty Ltd, 66 Rathmullen Quadrant, Doncaster Vic 3108, Australia. Email: Chris.Wharton@stops.com.au

Once the optimal pit for each mine was decided, pit designs were undertaken, reserves calculated and the entire data set exported to a spreadsheet for manual scheduling. Various guidelines and comparisons between the pits and stages were used to assist with the manual scheduling process, such as strip ratio, profit per tonne milled, cash cost per ounce, profit per ounce and break-even time. This introduced another flaw in the process, where the optimal extraction sequence was not necessarily followed during the manual scheduling process.

It became apparent that this trial and error scheduling method was time-consuming and limited the number of alternate life of mine scenarios that could be evaluated. A need for a technique to optimise the extraction sequence in this multiple mine scenario was identified. Such a tool was available as part of the Whittle suite of mine planning software, but was still in its infant stages, requiring rigorous testing on a real life scenario. This paper describes Whittle Multi-Mine and its application at Geita, but first briefly reviews a technique known as Multiple Ore Body Systems (MOBS) (Tulp, 1997), which has existed for some time now and has been widely applied in situations where multiple orebody deposits exist in proximity. In short, the technique involves agglomerating block models representing each of these deposits into one super model (Figure 1), and optimising and scheduling using Whittle software. The limitations of this method are described next.

To enable the identification of material selected for mining by Whittle from the different deposits, it was necessary to assign unique rock codes that were reflective of the different deposit areas. Furthermore, the rock codes used for Whittle also needed to capture the actual rock type, so that different mining and processing costs could be defined if necessary. This required the assignation of many rock codes and sometimes resulted in the loss of geological definition due to the restriction in the number of codes that could be handled by Whittle software.

Once the optimisations had been completed and the pit shells generated, it was necessary to cut up the super model results file to separate the individual mines, using the polygon intersection functionality in Whittle, so that the results could be exported from Whittle back to a general mining package (GMP). This was because the original coordinates of the individual deposits were lost when they were combined into the super model.

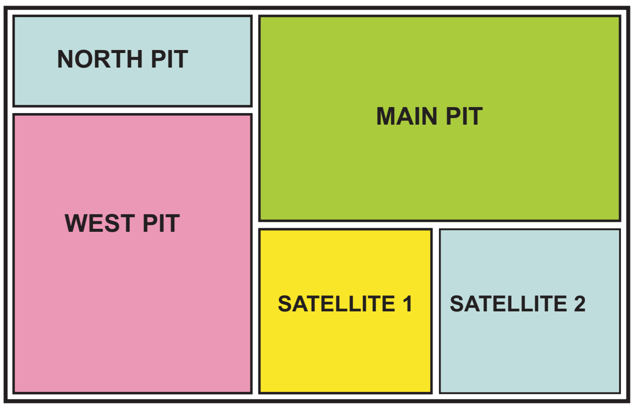


FIG 1 - Example ‘super model’ constructed by merging MOBS in Whittle software.

Issues arose when scheduling MOBS, since it was not possible to control the order that the deposits were mined in without creating complex pit list files with GMPs or by writing scripts with programming packages. Furthermore, it was necessary to ensure that the top surface of all of the models lay on the same Whittle bench level in the combined super model, requiring the user to offset each individual block model so as to create a regular surface over the entire model. This meant that when simulating the mining of a bench in Whittle, the bench was mined from all of the mines in the super model. It was also not possible to have different cut-backs in each mine, nor was it possible to have different final pits per mine. This reduced the effectiveness of the scheduling and did not allow areas of higher value to be deliberately targeted.

For more advanced scheduling using the Whittle Milawa scheduling algorithm, it was necessary to stack groups of pit shells, representing the nested pits derived for each mine, for Milawa to work effectively (Figure 2). This was difficult to set up and comparatively inflexible when evaluating many alternate mining sequences.

Whilst the technique described above generated results that added value to mining operations; it was tedious and much time was spent on manipulating models and data files, thus limiting the amount of time that could be spent on actually evaluating different scheduling sequences and the consequent impact on NPV.

MULTI-MINE

Whittle Multi-Mine provides a much more sophisticated and flexible means of optimising and scheduling in a multiple mine situation, as was proven by its successful application at Geita Gold Mine. The different techniques applied at Geita are described following, using examples (Joukoff and Purdey, 2004) to illustrate the results.

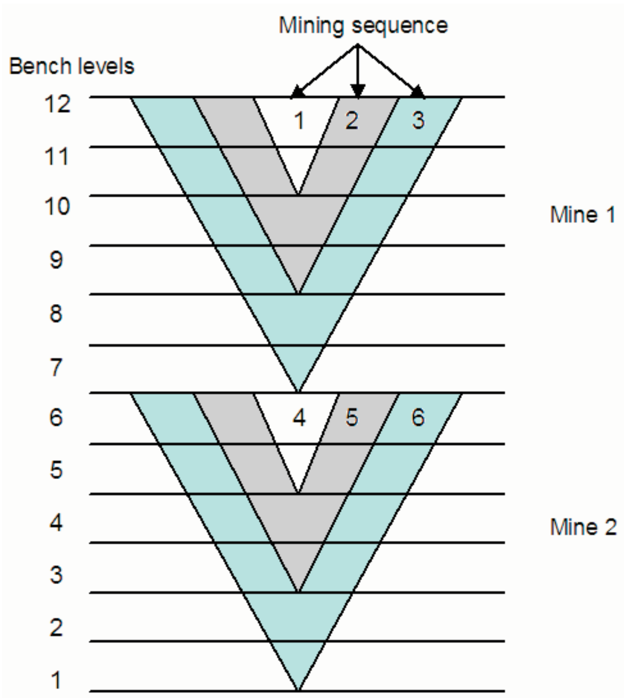


FIG 2 - Stacked pit shells to enable Milawa to operate independently on each mine before the development of Whittle Multi-Mine.

With Whittle Multi-Mine it is no longer necessary to use rock codes to identify material from different deposits. It is now possible for Whittle model files to carry a mine name, so the issue of running out of rock codes is no longer a problem. This allows greater geological detail to be modelled, leading to increased flexibility and detail when modelling costs, recoveries and slopes in Whittle, if desired. Furthermore, because each model can be associated with a mine name, it is possible to view and export results for individual mines. This reduces the amount of time required to be spent on data manipulation and provides more time to deal with strategic issues.

It is possible to optimise all the mines under consideration either simultaneously or individually, because the Whittle model files carry a mine name. The advantage of optimising them together is that the impact of each mine on the combined cash flows of all the mines can be examined and reported.

Scheduling with Multi-Mine is now also much more sophisticated than the MOBS technique previously applied. It is possible to vary the mining rates in different mines and also to control when mining can occur in a particular mine. This functionality proved particularly useful at Geita because some of the mines were remote from the processing plant and ore production from these mines was limited by the long distance haulage capacity (Figure 3). Also, due to Geita's environmental commitment to backfilling completed pits to minimise disturbance caused by the construction of waste dumps, some mines were not able to commence until adjacent mines were completed. Furthermore, either of the Fixed Lead or Milawa scheduling algorithms can be applied as described following.

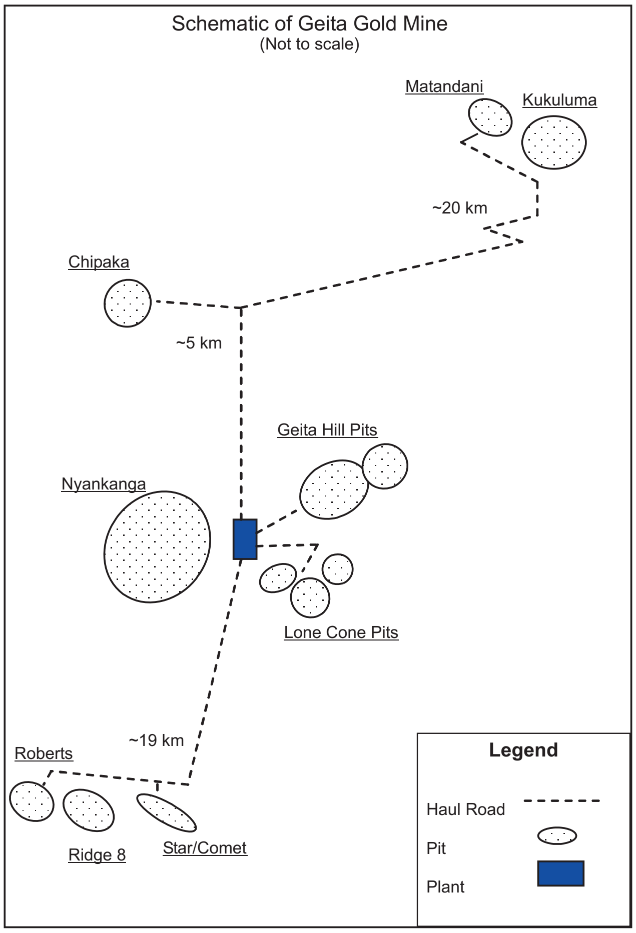


FIG 3 - Schematic map of Geita Gold Mine.

Fixed lead

Fixed lead scheduling can operate with or without precedence controls. By establishing mining precedence rules, different orders of mining the individual mines can be simulated, making it possible to investigate which order maximises the NPV to the company. This technique is particularly applicable in situations where only one mine will operate at a time, such as when the mines are very large and where ore control issues can be handled sufficiently by manipulating the mining sequence within each mine, without the need to blend material from different mines. Each mine may have its own process plant and associated infrastructure but logistically, mining equipment may need to move from one mine to another. The order of mines to which equipment moves can be optimised using this functionality. Alternately, when no particular precedence is required and mining can occur simultaneously in all mines following the same bench lead constraints, fixed lead scheduling can also be applied. These two alternate concepts are illustrated in Figure 4.

Fixed lead scheduling was tried at Geita but with limited effectiveness because the site wanted to be able to mine from many pits simultaneously, rather than mine them sequentially. Although this was possible as described previously, it was not practical in Geita's case because several of the mines were already in production and operating on different bench levels. Furthermore, within the constraints of the existing cut-back designs at Geita, using fixed lead scheduling did not provide an optimal mill feed schedule. Geita needed to be able to draw material from multiple sources to feed the mill, to meet the appropriate oxide/sulfide blend requirements and also to make better use of the available mill capacity. Greater flexibility was required, and to overcome these issues it was necessary to apply the Milawa algorithm.

Milawa

The majority of the Geita scheduling work in Multi-Mine was undertaken using the Milawa scheduling algorithm. This was because Milawa allowed material to be mined from different mines simultaneously, applying different lead and lag constraints to the different mines (as opposed to fixed lead scheduling, which uses the same lead constraint for each mine). There was a requirement at Geita to limit the maximum highwall height between cut-backs to 150 m, for geotechnical reasons. The maximum vertical advance in each mine was also restricted to either 50 m or 100 m per year, depending on the size of the mine. For this reason it was necessary to define different constraints for different mines and this was easily achieved with Multi-Mine.

It would be prudent at this stage to briefly explain the differences between the various Milawa scheduling algorithms. In NPV mode, Milawa will seek to maximise the NPV of the schedule, taking into consideration the number of benches, cut-backs and time periods in the life of the mine (Wharton, 2000). Milawa NPV schedules generally mine just enough waste to uncover the ore required to fill the mill and tend to defer waste stripping as much as possible. Logically, this will lead to increased NPVs. However, this waste deferral may result in insufficient ore availability at some time in the schedule, but only if the cut-backs have not been selected appropriately or if the mining capacity is not well matched to the selected cut-backs.

The Milawa algorithm in balanced mode provides a solution to this problem by producing a schedule that completely utilises all of the available mining and milling capacity where possible. The general effect of such a schedule is to mine more waste than is needed to uncover the ore necessary to feed the mill, hence bringing costs forward and resulting in a reduced NPV. However, both the mill feed schedule and the total mining schedule will be well balanced. A diagrammatic sketch of a Milawa mining sequence is included in Figure 5.

APPLICATION AT GEITA GOLD MINE, TANZANIA

Geita Gold Mining Limited provided a data set representing nine of the mines planned as at November 2003 (Nyankanga, Lone Cone, Geita Hill, Kukuluma, Matandani, Chipaka, Ridge 8, Star/Comet and Roberts). Each model was exported from a general mining package with pre-defined rock types that allowed unique costs and process recoveries to be assigned to each rock type. Although it is possible to model costs in Multi-Mine using a 'Mine' variable, a cost model reflecting the different long distance haulage costs, defined for different rock types, already existed. As well as this, the existing cut-back positions were exported as pit list models, allowing the cut-backs within each mine to be differentiated during subsequent analysis. These pit lists were agglomerated in Whittle to create a results file suitable for use with the Multi-Mine scheduling tools. Some of the required operational constraints have already been described previously in this paper.

Before undertaking any further scheduling in Whittle, a baseline schedule was developed with Multi-Mine that mimicked the existing Life of Mine (LoM) Plan as much as possible. This was so that subsequent NPV calculations for alternate mining sequences would be comparable. An iterative process was used in defining this baseline schedule, using modifications to the min/max lead and max benches constraints to 'force' Multi-Mine to mine in a similar sequence and with similar quantities as defined in the LoM Plan. Concurrent with this work in Multi-Mine was the recalculation of the LoM Plan NPV because this included the effects of many cash outflows that were not applicable in pit optimisation.

Once the Multi-Mine baseline schedule was constructed, the constraints were selectively relaxed to allow Multi-Mine to begin to optimise the schedule. Alternate orders of mining were tested by simply adjusting the preferred order of mining and the mine start and stop times, and the resultant NPV, ore delivery schedule and total mining schedule evaluated.

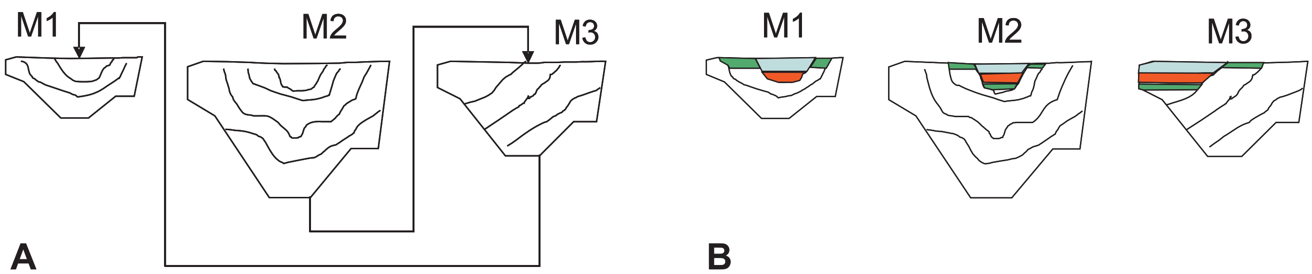


FIG 4 - Diagrammatic representation of two different fixed lead scheduling sequences in Whittle Multi-Mine. (a) Mining precedence applies and equipment moves from one mine to another on completion of each mine (Wharton, 2000). (b) No mining precedence applies and all mining occurs simultaneously in all mines, following specified bench lag constraints (Wharton, 2000).

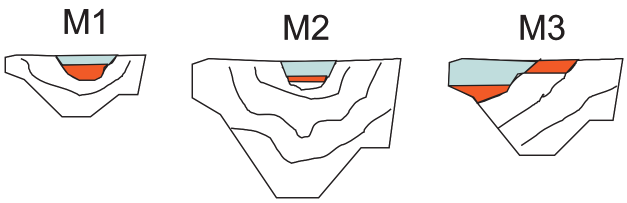


FIG 5 - Diagrammatic sketch of a Milawa mining sequence in Whittle Multi-Mine (Wharton, 2000).

In total, 24 different LoM scheduling scenarios for Geita were considered using the Milawa algorithm in conjunction with Multi-Mine. Comparison of the NPV of each of these schedules with the baseline schedule showed that the NPVs ranged from 87 per cent to 103 per cent of the baseline NPV. Whilst a three per cent improvement in NPV may seem small, in Geita's case it represented an increase in NPV in excess of 1500 times the cost of undertaking the Multi-Mine work. An ore schedule representative of the results generated with Multi-Mine is displayed in Figure 6.

The most significant difference between the Whittle Multi-Mine results and the existing site LoM Plan was that the Milawa algorithm preferred to mine Star/Comet as early as possible, rather than later in the project life as had been previously scheduled. This gave some indication as to the significance of the Star/Comet mine to the overall project NPV. When run unconstrained, Multi-Mine also preferred to mine Matandani in early years, but this was not a favoured option as the waste from Matandani was planned to be backfilled into the Kukuluma mine.

Investigation of the contribution to NPV from each mine for each scheduling scenario helped to determine which mines the overall NPV was most sensitive to. Table 1 contains a representative set of results showing these cash flow contributions for various scenarios. It is clear that for some of the mines changes to the order of mining had little or no effect on their contribution to total NPV, whilst for others the change in contribution to NPV was considerable.

The effect of delaying production from any mine can be seen. The cost of deferring Nyankanga is very evident; the NPV contribution being as much as 67 per cent (Scenario 14) or as little as 46 per cent (Scenario 3). This represents a 21 per cent improvement in cash flow contribution from Nyankanga for Scenario 14 compared with Scenario 3. In fact, in Scenario 3 the NPV from Nyankanga approaches that of 'worst case' mining. As a further example consider Chipaka mine; if this is mined last (Scenario 14) the NPV contribution erodes to just 0.5 per cent, but if it is mined first (Scenario 17), the NPV contribution can be as much as two per cent. However, when considering the NPV of all of the mines concurrently, delaying Chipaka gives the project a better overall NPV. This clearly demonstrated how the order of mining can have a serious impact on the value of the project.

It was concluded from all of the scenarios that the NPV was relatively insensitive to changes in the order of mining from the Chipaka, Kukuluma and Ridge 8 mines. This suggested that it was not worthwhile to further optimise the timing of these mines. Conversely, there was substantial gain to be made by optimising the mining sequence from Nyankanga, Geita Hill, Matandani and Star/Comet. For this reason, the order of mining from these mines was the focus for the remainder of the scenarios and yielded higher value schedules.

Examination of the bench schedules produced by Whittle Multi-Mine helped to understand how much material was mined from each bench, each cut-back and each mine in each period and hence made it possible to determine whether Multi-Mine was adhering to the required operational constraints. The resultant schedules were both safe and practical. Furthermore, by making comparisons between the benches mined in different scheduling scenarios it was possible to understand where the material was being mined from, and the subsequent contribution of that material to the overall value of the schedule. An example bench schedule is given in Table 2.

CONCLUSIONS

This paper has reviewed the techniques available in Whittle to optimise and schedule multiple orebody models and multiple mines. The application of Whittle Multi-Mine at Geita Gold Mine, Tanzania, has demonstrated how improvements to the NPV of the life of mine schedule were achieved, using Multi-Mine to help optimise the mining sequence. The Milawa algorithm in both NPV and balanced mode was able to guide the order of mining benches from the various cut-backs of the various pits, within the operational constraints at Geita Gold Mine.

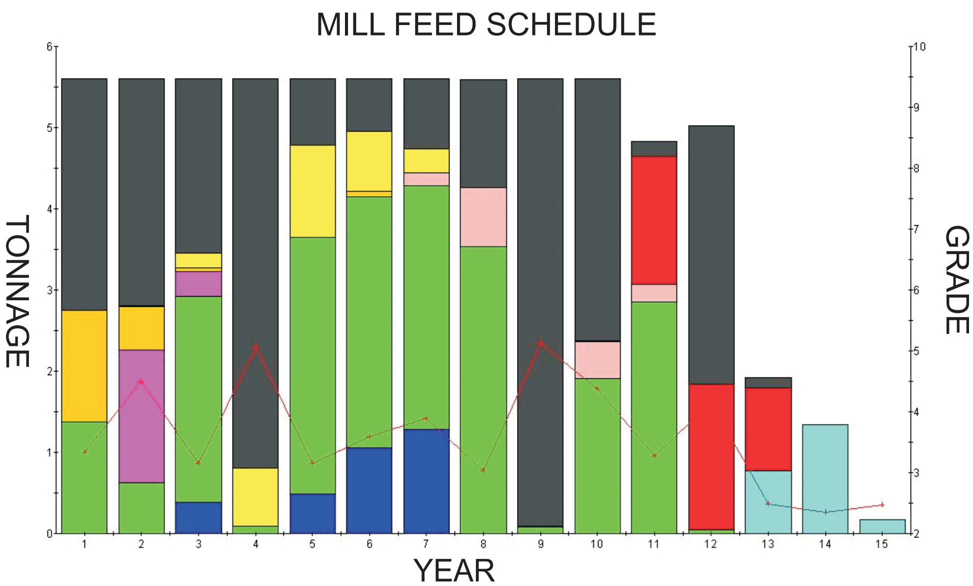


FIG 6 - Representative ore schedule, Geita Gold Mine case study. Different shades represent different mines.

TABLE 1

NPV contributions by pit by scenario, Geita Gold Mine case study.

	Matan’	Chipaka	Geita Hill	Kuk’	Lone Cone	Ridge 8	Roberts	Star Comet	Nyank’	Total
Scenario 2	5%	1%	17%	4%	3%	1%	1%	4%	63%	100%
Scenario 3	5%	2%	28%	5%	4%	1%	3%	6%	46%	100%
Scenario 7	5%	1%	17%	4%	3%	1%	1%	9%	57%	100%
Scenario 14	3%	0%	16%	4%	2%	1%	2%	6%	67%	100%
Scenario 15	3%	1%	18%	4%	3%	1%	1%	8%	60%	100%
Scenario 16	4%	1%	17%	4%	3%	1%	2%	8%	60%	100%
Scenario 17	3%	2%	16%	4%	3%	1%	1%	6%	65%	100%
Scenario 19	4%	1%	18%	4%	4%	1%	2%	8%	59%	100%
Scenario 21	5%	1%	22%	5%	4%	1%	2%	7%	55%	100%
Scenario 23	5%	1%	20%	5%	1%	1%	3%	9%	56%	100%

TABLE 2

Example extract from bench schedule generated using Whittle Multi-Mine.

Pit	Bench	Total	Year 1		Year 2		Year 3	
		Tonnes	Ore (t)	Waste (t)	Ore (t)	Waste (t)	Ore (t)	Waste (t)
Kukuluma	69	1496	-	-	341	1155	-	-
Kukuluma	68	1131	-	-	306	825	-	-
Kukuluma	67	772	-	-	244	527	-	-
Kukuluma	66	453	-	-	194	259	-	-
Kukuluma	65	20	-	-	12	9	-	-
Kukuluma	64	142	-	-	-	-	105	37
Kukuluma	63	56	-	-	-	-	44	12
Kukuluma	62	11	-	-	-	-	9	2
Subtotal								
Lone Cone	66	1266	161	1105	-	-	-	-
Lone Cone	65	1176	201	975	-	-	-	-
Lone Cone	64	1015	154	861	-	-	-	-
Lone Cone	63	901	111	789	-	-	-	-
Lone Cone	62	788	-	-	73	715	-	-
Lone Cone	61	653	-	-	44	609	-	-
Lone Cone	60	532	-	-	37	495	-	-
Lone Cone	59	401	-	-	43	358	-	-
Subtotal								

Many alternate scheduling sequences were very quickly investigated using Whittle Multi-Mine. This process identified which mines demonstrated greater sensitivity to the order in which they were extracted and subsequently stressed the effect of time on the cash flow contribution of these mines to the overall project NPV. It also assisted in highlighting a potential mismatch between the required material movement and the available mining capacity. If the mining capacity is well matched to the selected cut-backs then it will be possible to achieve a balanced schedule together with an improved NPV.

ACKNOWLEDGEMENTS

This paper describes work undertaken by co-author David Purdey whilst employed as chief mining engineer – Geita Gold Mining Limited and is presented with Geita Gold Mining Limited’s permission. The authors would like to thank Geita Gold Mining Limited’s management for their permission to present this paper and also thank the members of the mining department at Geita who contributed to the preparation of the data used in the Multi-Mine analyses.

The opinions expressed in this paper are not necessarily those of Geita Gold Mining Limited.

REFERENCES

Joukoff, T and Purdey, D P, 2004. Improved life of mine scheduling with Gemcom Whittle Multi-Mine at Geita Gold Mine, Tanzania, Gemcom Software International Inc: Vancouver.

Tulp, T, 1997. Multiple ore body systems (MOBS), in *Proceedings Optimising with Whittle,* pp 149-163 (Whittle Programming Pty Ltd: Melbourne).

Wharton, C, 2000. Add value to your mine through improved long term scheduling, in *Proceedings Whittle North American Strategic Mine Planning Conference,* Breckenridge, Colorado.

Wharton, C, 2003. Multi-pit analysis and advanced pit scheduling, Development notes (unpublished), Melbourne.

Assessing Underground Mining Potential at Ernest Henry Mine Using Conditional Simulation and Stope Optimisation

P Myers[1], C Standing[2], P Collier[3] and M Noppé[4]

ABSTRACT

Conditional simulation has been applied at the Ernest Henry copper-gold mine, Queensland, Australia, to quantify resource and reserve risk within the sulfide resource of the so-called Chloe shoot below the planned base of the present open pit mine. High risk areas within the resource were identified to assist with exploration targeting, and a range of possible scenarios (models) of the mineral resource were used as input for a conceptual underground mining study.

Conditional simulation was carried out within a single underground resource domain to determine the resource potential based on a copper equivalent grade. Mixed populations and rotational anisotropy were recognised, so full indicator variogram analysis was undertaken on the copper equivalent data from the exploration drill holes. Final interpreted variogram models for the conditional simulation were based largely on exploration data, but modified to reflect expected short-range continuity modelled from the overlying open pit grade control data. The sequential indicator simulation algorithm was used to simulate a copper equivalent grade into a dense grid of nodes. These were re-blocked into 20 mE by 20 mN by 10 mRL blocks for comparison with the pre-existing resource model, and to report the risk associated with the underground resource model.

Three realisations, representing the median and 95 per cent confidence range for the simulated grade-volume curves, were selected as resource models for stope analysis. This approach was adopted for practical reasons, its limitations thus being recognised. The realisations were re-blocked into a range of alternative stoping geometries, and the resultant mining inventories were reviewed to select the 'optimal' mining method. A conceptual mine plan was determined and modelled financially for each of the three realisations to provide an early insight into the feasibility of the underground project.

INTRODUCTION

The Ernest Henry copper-gold mine is located 35 km north-east of Cloncurry in the Mt Isa-Cloncurry mineral district of north-west Queensland. It is operated by Ernest Henry Mining Pty Ltd (EHM), a wholly owned subsidiary of Xstrata Copper Australia. The orebody is currently mined as an open pit on 16 m benches by conventional load and haul methods using large-scale mining equipment. Approximately 10.4 Mt of ore grading 1.21 per cent Cu and 0.62 g/t Au were processed during the 2002 - 2003 financial year. A deep drilling program initiated in August 2002 has tested the down-plunge extension of the orebody to a vertical depth of over 1 km. The encouraging drilling results have provided a significant increase in the mine's open pit mineral resource to a depth of 560 m, together with the identification of an underground mineral resource.

1. MAusIMM, Principal Consultant Engineer, Snowden Mining Industry Consultants Pty Limited, PO Box 2207, Brisbane Qld 4001, Australia. Email: pmyers@snowden.com.au

2. MAusIMM, Principal Consultant – Resource Division, Snowden Mining Industry Consultants Pty Limited, PO Box 77, West Perth WA 6872, Australia. Email: cstanding@snowdenau.com

3. MAusIMM, Currently: Senior Geologist, Rio Tinto Limited, PO Box 2207, Milton Qld 4046, Australia. Email: perry.collier@riotinto.com Formerly: Senior Mine Geologist, Ernest Henry Mine, Xstrata Copper.

4. MAusIMM(CP), Group General Manager, Snowden Mining Industry Consultants Pty Limited, PO Box 2207, Brisbane Qld 4001, Australia. Email: Mnoppe@snowdengroup.com

The Ernest Henry deposit is a member of the Fe-oxide-(Cu-Au) class of geological deposits. The orebody is hosted within a sequence of moderately SSE-dipping, intensely altered Paleoproterozoic intermediate metavolcanic and metasedimentary rocks, which are concealed by 35 m to 90 m of Phanerozoic cover. Copper (chalcopyrite)-gold mineralisation occurs mainly within the magnetite-biotite-calcite±pyrite matrix of a 250 m by 300 m pipe-like breccia body (Figure 1). The breccia pipe consists of two anastomosing lenses separated by a lower grade, clast-supported breccia zone. Several anastomosing brittle faults affect the macro-geometry of the orebody, and copper grade is proportional to the degree of matrix support in the breccia.

The aim of this study was to generate likely resource models for small block sizes, assuming appropriate selectivity and continuity for modelling underground mining, for input into a conceptual underground mining method and stope design study for EHM. Conditional simulation of a single underground resource domain was used to examine the resource potential based on a copper equivalent grade derived from copper and gold grade data. Due to practical limits, three realisations from the conditional simulation were selected to represent the 95 per cent confidence range and the median of simulated grade-volume outcomes. These three realisations were reblocked to represent resource models and were analysed in a stope analysis software package, Stopesizor (Thomas and Earl, 1999), to generate conceptual 'reserves'. As the stope optimisation analysis is a non-linear process, the results from the stope analysis process no longer represent the above confidence range and median as such. However, they gave EHM an early indication of the project's feasibility potential, while a risk assessment from the simulation results assisted in decisions regarding further exploration and drill targeting.

This paper documents the study undertaken at the Ernest Henry mine. Firstly, data preparation and analysis are outlined. Then, the conditional simulation of the deposit and related intricacies are presented. Subsequently, mine planning and mining inventory are discussed in some detail and, lastly, conclusions are stated.

DATA PREPARATION AND ANALYSIS

All available in-pit grade control RC and resource diamond drill data were used for statistical and variogram analysis. The simulation study focused on the resource below the planned open pit depth of 560 m and used only resource diamond drilling data for grade estimation (Figure 2). In order to simplify the conditional simulation and subsequent stoping analysis, a copper equivalent (CuEq) grade was prepared for the original sample lengths after confirming that the high-grade copper and high-grade gold have similar orientations and ranges of continuity. The copper equivalence formula applied to the input data for the conditional simulation was as follows:

$$\text{CuEq samples} = ((\text{Cu} \times 27.35) + (\text{Au} \times 15.74))/27.35.$$

This CuEq data was length composited to 2 m down hole sample lengths and the hanging wall and footwall domain wireframes were used to flag the composited data. Only data coded as ore zone material (ie from between the hanging wall and footwall) were used for data analysis and conditional simulation.

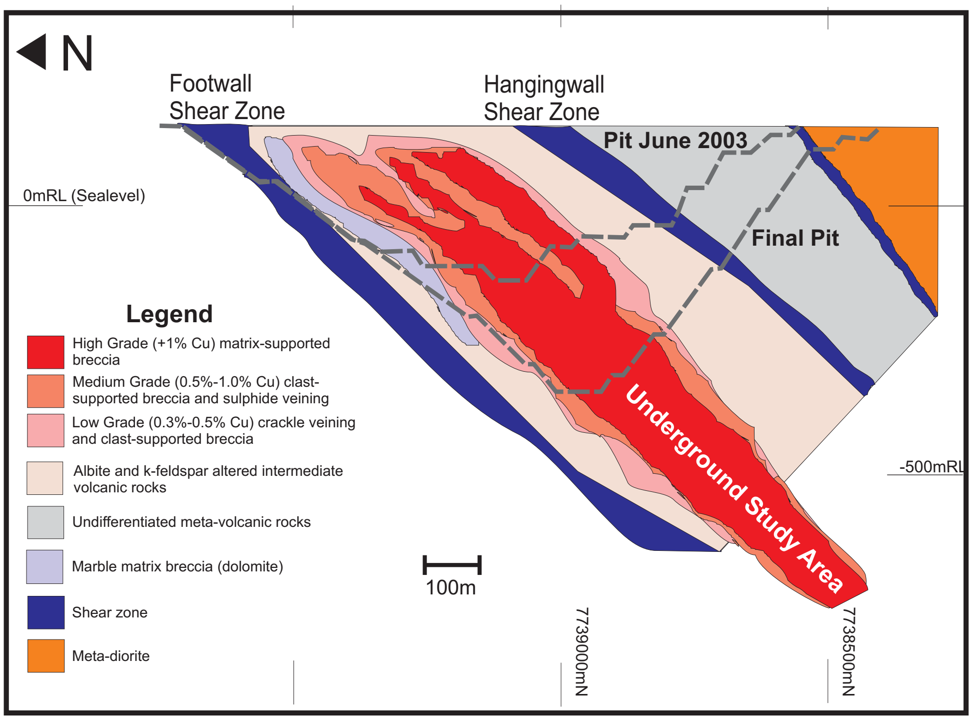


FIG 1 - Schematic N-S geological cross-section of the Ernest Henry copper-gold deposit showing the final pit limits and the underground study area (modified from Collier and Bryant, 2003).

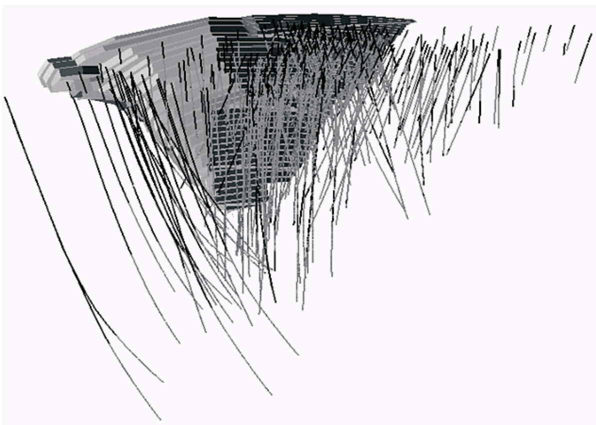


FIG 2 - 3D view of the resource drill hole data and the planned pit looking towards the northwest.

The bulk density (SG) at Ernest Henry has a positive correlation with iron grade and this relationship is incorporated into resource modelling. As this underground study did not incorporate the modelling of iron, a relationship between the CuEq grade and SG was developed. The correlation between CuEq grade and SG is not as good as that between iron and SG. However, there is still a strong positive relationship on average between SG and CuEq. Figure 3 illustrates this with a box and whisker plot based on the middle 50 per cent of the data (ie from the 25th percentile to the 75th percentile of the data). The mean SG was determined for 0.5 per cent CuEq intervals (as listed in Table 1) and these SG values were applied for tonnage determination of the simulated models used in the conceptual underground mining study.

Statistical analysis indicated that the CuEq data had a mixed and positively skewed distribution within the mineralised domain, and preliminary variogram studies undertaken while scoping out this study revealed rotational anisotropy for different grade ranges. Consequently, multiple indicator variography was undertaken using the 2 m composite diamond drill data for CuEq within the ore zone with indicator grades selected at decile intervals.

The direction of maximum continuity for each indicator was interpreted by generating horizontal, across-strike and dip plane variogram fans. The dip plane fan was analysed to determine the direction of maximum continuity, which was down-plunge for the mineralisation in the Chloe shoot. Directions of mineralisation continuity determined from the diamond drill data were reviewed in conjunction with the variogram models and directions obtained from the grade control RC data. Variogram fans based on the diamond drill data indicated a strike direction of 330° to 350° and a dip of -80° to -70° west. The orientation of maximum continuity of mineralisation was generally -39° towards 168° for the lower grade indicators. This rotated to -37° towards 176° for the 60th to 80th percentiles and flattened to -19° towards 167° for the 90th percentile. Variogram fans generated for the grade control RC data (based on the same indicator grades as used for the diamond drill data) confirmed the orientation obtained for the lower grade indicators (10th to 50th percentiles), but a clear rotation in the strike direction to 280° was evident for the higher grade indicators (Figure 4). EHM expects that the mineralisation orientations observed from the grade control data in the upper portion of the Chloe shoot will also occur at depth. Review of the variography based on the diamond drill data indicated that the strike direction for the higher grade indicators was not conclusive although a rotation to ~280° was possible. The orientations obtained from the grade control variogram analysis were therefore applied to the 60th to 90th percentile indicators for the diamond drill data to model the expected continuity of higher grade mineralisation at depth.

Indicator variograms were used to model the spatial variability of the diamond drill data for the down-plunge, down-dip and orthogonal directions. The variogram models for the diamond drill data were reviewed with reference to variogram models developed from the closer-spaced grade control RC data. The grade control variogram models often indicated shorter ranges than had been interpreted from the diamond drilling data, and the final variogram models were modified to incorporate these shorter ranges.

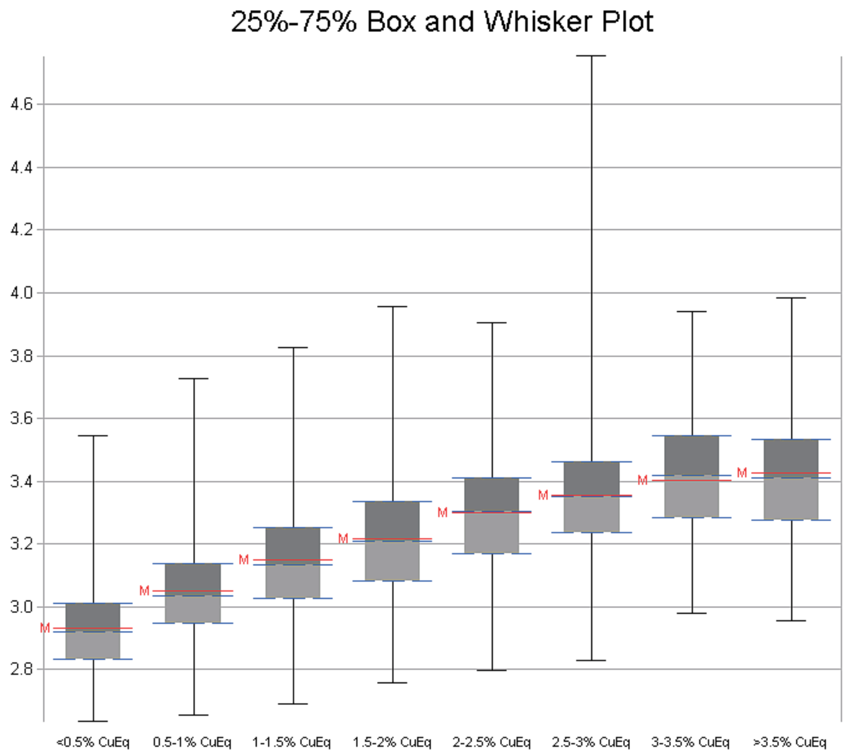


FIG 3 - Box and whisker plot illustrating the relationship between CuEq grade and SG. (Median grade represented by box centroid and mean grade represented by 'M'.)

TABLE 1

Mean SG values based on CuEq grade intervals.

CuEq grade interval	Mean SG (t/m^3)
<0.5%	2.93
0.5 to 1.0%	3.05
1.0 to 1.5%	3.15
1.5 to 2.0%	3.22
2.0 to 2.5%	3.30
2.5 to 3.0%	3.36
3.0 to 3.5%	3.40
>3.5%	3.43

Nugget values are estimated to account for 14 to 20 per cent of the total variance for all indicators, except for the 90th percentile where it increases to 33 per cent. Nested spherical models were interpreted with ranges of 120 m to 980 m in the down-plunge direction, 10 m to 280 m and 35 m to 240 m in the orthogonal directions, with zonal anisotropy present in the down-plunge direction for the 10th and 20th percentile indicator grades. Ranges decreased with increasing indicator grade.

CONDITIONAL SIMULATION

A blank node file for input into the conditional simulation study was prepared using a node spacing of 5 m by 5 m by 5 m. This node spacing was selected to provide sufficient resolution for the conceptual mining study and to provide output in easily managed file sizes.

Examination of the grade control RC data indicated that the minimum dimensions of the high-grade shoots and lenses may be in the order of 20 m to 30 m. The selective mining unit dimensions for conceptual mine planning would be expected to be similar, and the selected node spacing would provide suitable reblocking of grades to these dimensions. In order to model the mixed and skewed grade distribution and the rotational anisotropy, the sequential indicator simulation algorithm (Goovaerts, 1997) was selected for grade simulation. The sequential indicator simulation parameters were defined so that the minimum number of original data used to simulate a point was three, the maximum number of original data used to simulate a point was 30, and the maximum number of previously simulated data used to simulate a point was 30. Indicator grades were defined at the deciles of the data distribution, and variogram parameters defined for each indicator were used in the simulation algorithm. Thirty-five realisations were run.

The sequential indicator simulation (examples of realisations are included in Figure 5) successfully reproduced the patterns of CuEq grade, as recognised from the drilling data, which consisted of trends of high-grade and low-grade mineralisation. Higher grades are indicated by warmer colours, lower grades by cooler colours. The simulation was validated by:

1. visual inspection of the realisations and comparison with the input data,
2. comparison of statistics of the input data and the realisations,
3. using Q-Q plots to compare the distribution of the simulated data with the input data,
4. checking the variograms from the first realisation with the input data, and
5. comparing the average grade of re-blocked realisations to the resource model.

The validation of the simulation indicated that the individual realisations represented the mineralisation grade reasonably well. It was noted that the realisations had a slightly higher grade than the input data for grades below 0.2 per cent CuEq. However, this could be explained by the fact that the input data extended above the simulation area, by over 300 m, into an area of lower grade mineralisation.

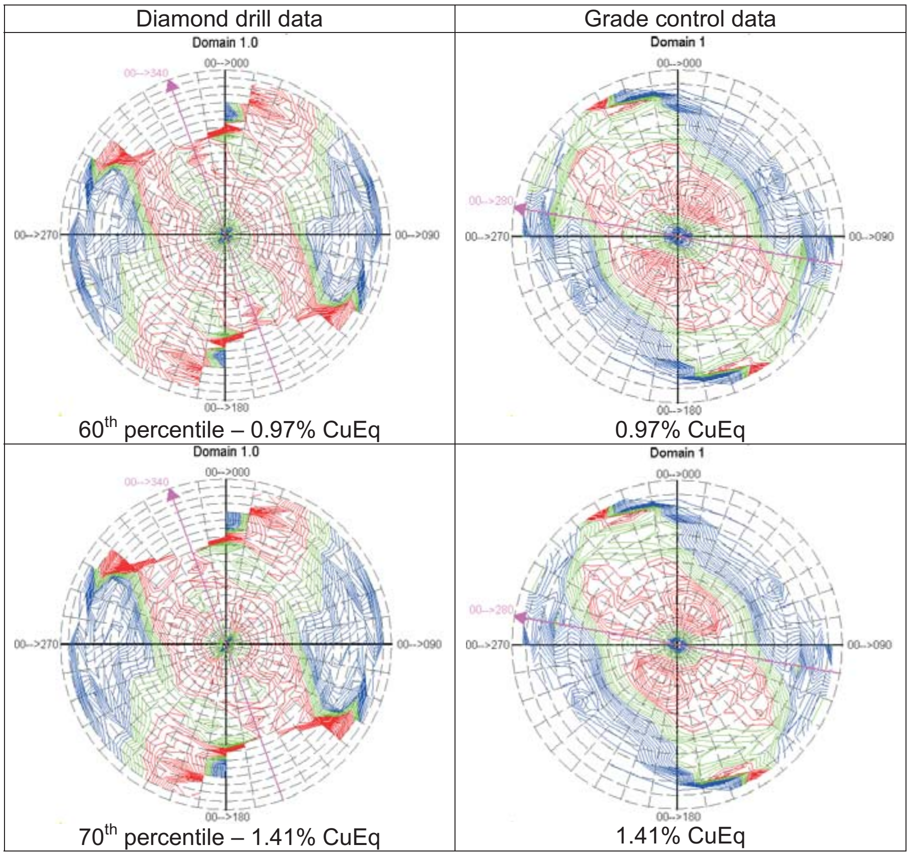


FIG 4 - Horizontal variogram fans based on diamond drill and grade control data. (Note: Gamma values are standardised to the population variance, contour colours represent < 0.66 blue, 0.66 to 1.0 green, > 1.0 red).

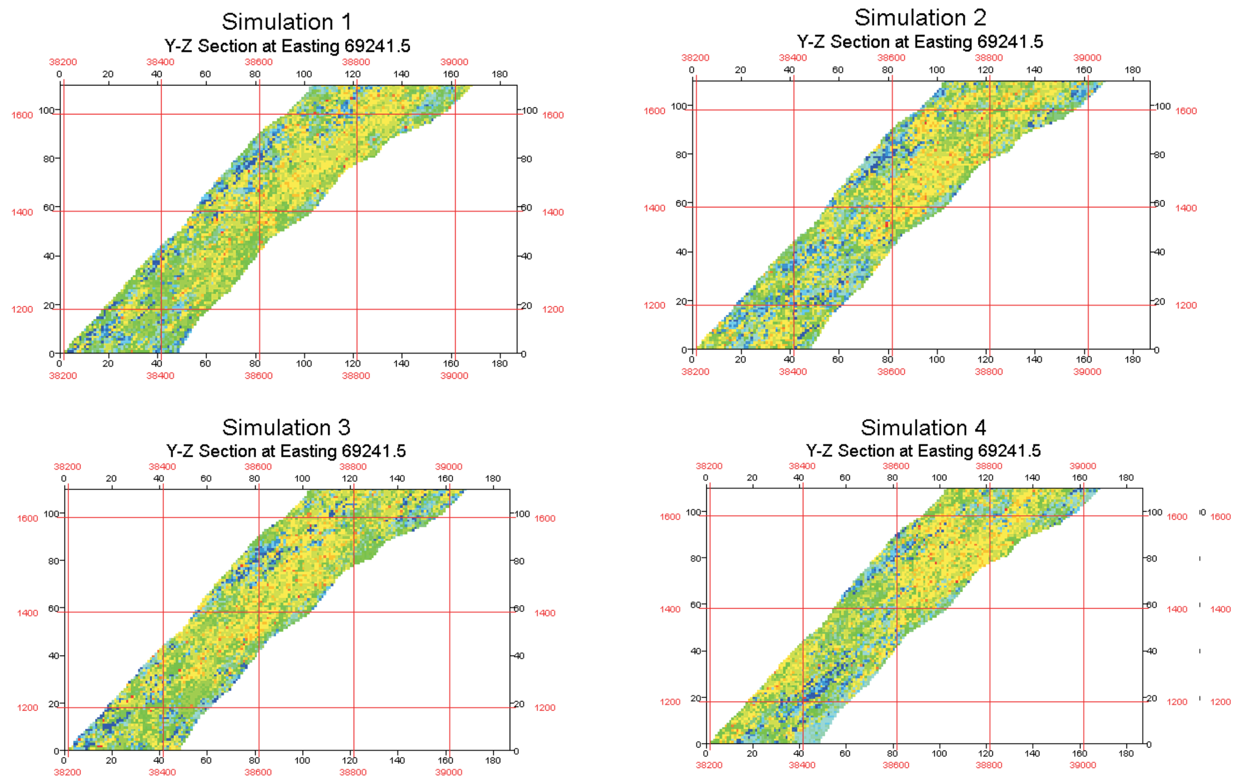


FIG 5 - Long-section view of realisations 1, 2, 3 and 4 for the CuEq 5 m × 5 m × 5 m simulation.

POST-PROCESSING OF THE SIMULATIONS

The grade and volumes, based on the 5 m by 5 m by 5 m node data, were reported at 0.5 per cent CuEq cut-off grade increments from 0.0 per cent to 5.0 per cent CuEq. Grade-volume curves are presented in Figure 6. Preliminary analysis indicated that an underground cut-off grade of ~1.5 per cent to 2.0 per cent CuEq could be anticipated, and so grade-volume curves were examined in this range. The second lowest, the median and the second highest realisations were selected to represent the middle value and the 95 per cent confidence grade-volume limits for input into the mining study. The 5 m by 5 m by 5 m simulated data for the three selected realisations were exported, and the volumes for the 5 m by 5 m by 5 m cube around the simulated sample data were converted to tonnages for grade weighting during re-blocking by application of the SG values listed in Table 1.

Risk analysis on the CuEq grade was performed to assist with exploration targeting with the idea that areas of high risk and high potential would be good targets for future drilling. The simulations were re-blocked into 20 m by 20 m by 10 m blocks, equivalent to the resource model. The risk was quantified as the average deviation of the re-blocked simulation values from the resource block estimate expressed as a percentage value (ie an indication of the possible deviation of the actual value from the estimated value). It was found that the average deviation from the mean was lowest in more heavily sampled areas, as expected, and it also seemed to be lower in high-grade areas of the resource, suggesting greater 'connectivity' between the high grades than may otherwise have been expected (Figure 7).

CONCEPTUAL MINE PLANNING AND MINING INVENTORY

A conceptual mine plan was identified and financially modelled to provide an early insight into the feasibility of the underground project. The setting of the potential underground operation is geotechnically competent, with strong footwall and hanging wall rock masses, though the footwall and hanging wall contacts are characterised by shear zones of minor thickness. Previous studies concluded that the resource could be mined by open stoping methods with stable spans of 40 m by 40 m over the full orebody thickness. EHM's long-term production requirements anticipate mining at a rate of approximately two million tonnes per annum (Mtpa) in addition to open pit production.

The mining method selected was required to provide for bulk tonnage mining production requirements, and to overcome the shortfalls associated with the low to moderate overall grade of the resource. The study investigated the application of various mining methods to the extraction of the orebody and modelled financial outcomes for various sensitivity scenarios.

The conceptual mine planning study employed a stope analysis software package, Stopesizor (Thomas and Earl, 1999), to identify the optimum mining inventories for a range of simulated resource models and simulated mining methods. Stopesizor modifies a geological block model to identify the optimum mining outline for a range of cut-off values (usually grade). This is done by constructing selective mining blocks (SMB), where the SMB represents a practical minimum stoping increment. Each SMB consists of a contiguous group of resource blocks that honour minimum dimension constraints and bearing and plunge angle for each axis. Stopesizor identifies all SMBs where the mean value of the SMB is equal to or greater than the cut-off value. Stopesizor works by identifying the highest value SMB that meets the minimum dimension constraints, then the next highest value SMB, and so on. This process is continued until all possible SMBs with a mean value equal to or higher than the lowest specified cut-off value have been identified. The aggregated SMBs form a mining outline that is optimum for the given cut-off value and SMB dimensions. The optimum mining outline may include dilution that consists of low-value or waste blocks necessary to construct individual SMBs. This form of dilution is generally referred to as planned dilution. As long as the range of cut-off values is specified in decreasing order, a single Stopesizor block model can be produced that represents the optimum mining outlines for a range of cut-off values. Individual mining outlines can be identified by reporting only the resource blocks with a mean SMB value above the required cut-off value. Stopesizor output is in the form of a Stopesizor mining block model (the Stopesizor model) that consists of only those individual resource blocks within the optimum mining outline. Each block retains its initial resource attributes (X, Y, Z, grade, density, domain code, etc) and is assigned the cut-off value at which it was included in the mining outline.

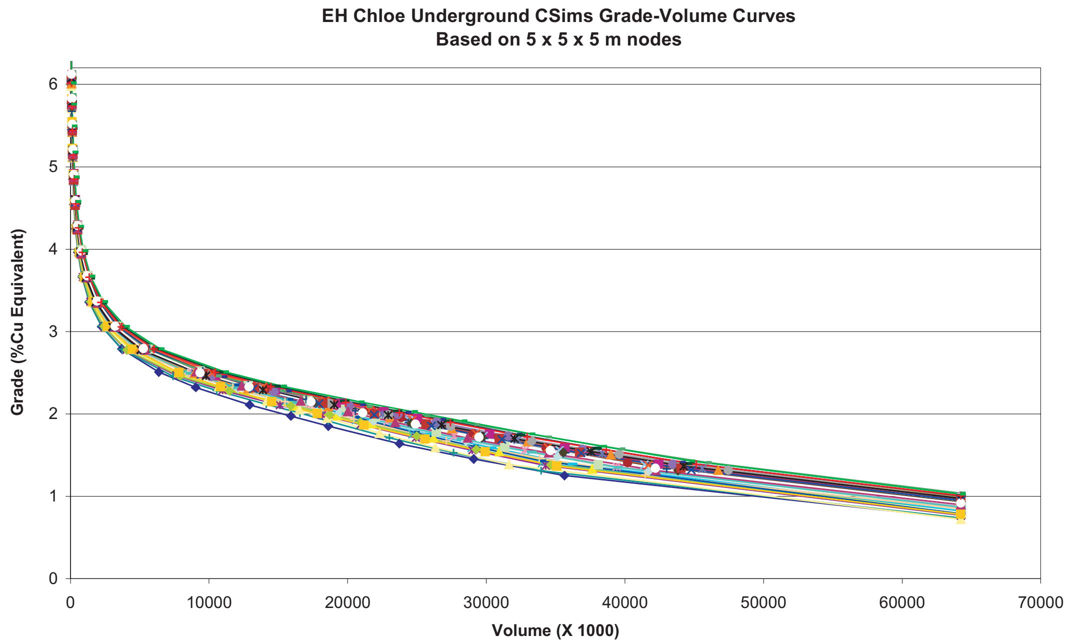


FIG 6 - Grade-volume curves from conditional simulation study.

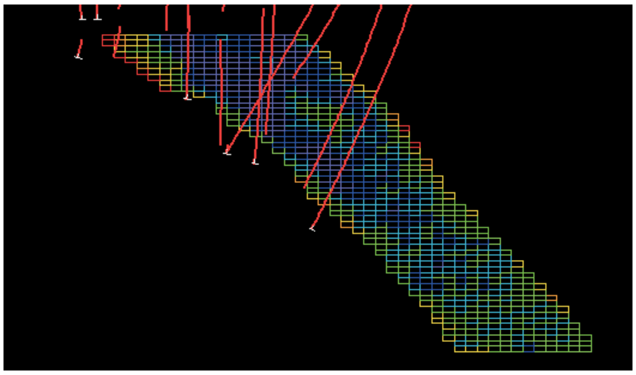


FIG 7 - Section showing quantitative risk map through the resource model showing lower relative risk associated with drilled areas. (Note: Relative colour scale with red representing higher risk areas through to blue representing lower risk areas.)

Stopesizor does not calculate a cut-off grade. Rather, it is a tool that quickly allows the mining inventory to be identified and reported for a range of cut-off grades and the results to be presented in a format that allows the mining inventory to be scheduled and costed. Stopesizor results can then be used to determine the optimum cut-off grade based on NPV or other criteria. Stopesizor can also be used as a guide for detailed mine planning, where mining outlines can be quickly identified at a pre-determined cut-off.

The three simulation outcomes chosen for analysis represented low, median and high tonnage/grade outcomes, corresponding to the 95 per cent confidence range of the simulations. The limits of this approach are discussed in a subsequent section. To gain insight into the similarities and differences between the simulations, a Stopesizor optimisation was performed on each selected simulation model to generate mining inventories for a generic sublevel open stoping case with minimum stope dimensions of 20 mW by 20 mL by 40 mH, where W is the across strike dimension, L is the along strike dimension, and H is height. Table 2 shows the resultant inventories above 2.0 per cent CuEq, a value which was expected to approximate a realistic economic cut-off grade. The inter-level spacing function available within Stopesizor, which anchors stope bases to specified horizons, was not utilised at this time to enable the simulations to be compared on an unconstrained basis. The optimisations of the simulated orebody models were compared for geometry and size, as shown in Figure 8 and Table 2. It was concluded that a large-scale and bulk mining method, rather than a small-scale and selective method, would be an appropriate mining method for each of the simulations. It was also decided that the similarities between each of the optimised mining inventories were such that it would be appropriate (and expedient) for the study to use the high and low simulations as sensitivity variants of the median case, rather than establishing fully independent value estimates for each simulation.

TABLE 2

Simulation inventories at 2.0 per cent CuEq cut-off.

	Simulation mineral inventory		Simulation optimised mining inventory (20 mW × 20 mL × 40 mH minimum stope dimensions)	
Simulation	**kt**	**% CuEq**	**kt**	**% CuEq**
Low	24 444	2.46	6990	2.10
Median	28 924	2.51	10 738	2.14
High	35 910	2.51	9085	2.13

An assessment of bulk mining methods suggested that possible approaches to mining the resource included bulk open stoping with backfill, selective open stoping without backfill (high-grade option), and sublevel caving.

It is noted that the optimised median simulation produced a larger mining inventory below about 2.3 per cent CuEq than the optimised high simulation, as shown in Figure 9.

The asymmetrical translation from mineralisation simulation to mining inventory, and to potential project value or NPV, is an outcome of the nature of the distribution of the grades in the simulated deposit and, in this case, the Stopesizor optimisation process which is non-linear. Non-linear means that the processed percentile resource is no longer symmetrical after optimisation. The asymmetrical translation means that the mining inventories identified do not mirror the 95 per cent confidence range coincident with that of the resource simulations. However, the identified mining inventories were accepted as a suitable range of outcomes for investigation of impact on project value. Dimitrakopoulos *et al* (2002) demonstrated the non-linearity of optimisation processes and a similar outcome when considering grade uncertainty and risk effects in open pit design. In dealing with this issue, mining inventories could be evaluated for each simulation so that a large population is assessed to more fully determine the distribution of mining inventory confidence. Such an approach was beyond the economic scope of the study upon which this paper is based.

Each mining method was represented in Stopesizor using minimum stope dimensions as follows:

- bulk open stoping with backfill – 20 mW by 20 mL by 60 mH, 40 m level spacing;
- selective open stoping without backfill – 20 mW by 20 mL by 20 mH, 20 m level spacing; and
- sublevel caving – 20 mW by 10 mL by 20 mH, 20 m level spacing.

For bulk open stoping and sublevel caving, a cut-off grade of 2.0 per cent CuEq was selected for optimisation as it was expected to approximate the realistic economic cut-off grade. Selective open stoping considered the higher cut-off grade of 2.5 per cent CuEq as a nominal high-grade target. The stope optimisation results are shown in Table 3. These were assessed and conclusions were reached regarding the applicability of each method.

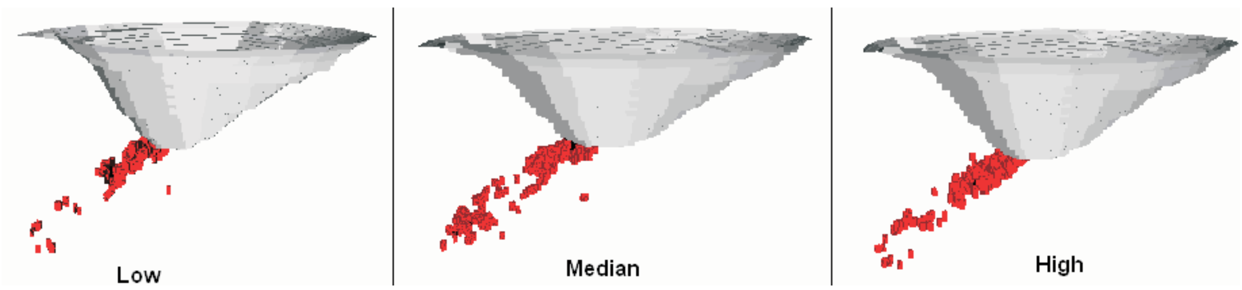


FIG 8 - Low, median and high simulation optimised mining inventories, 2.0 per cent CuEq cut-off.

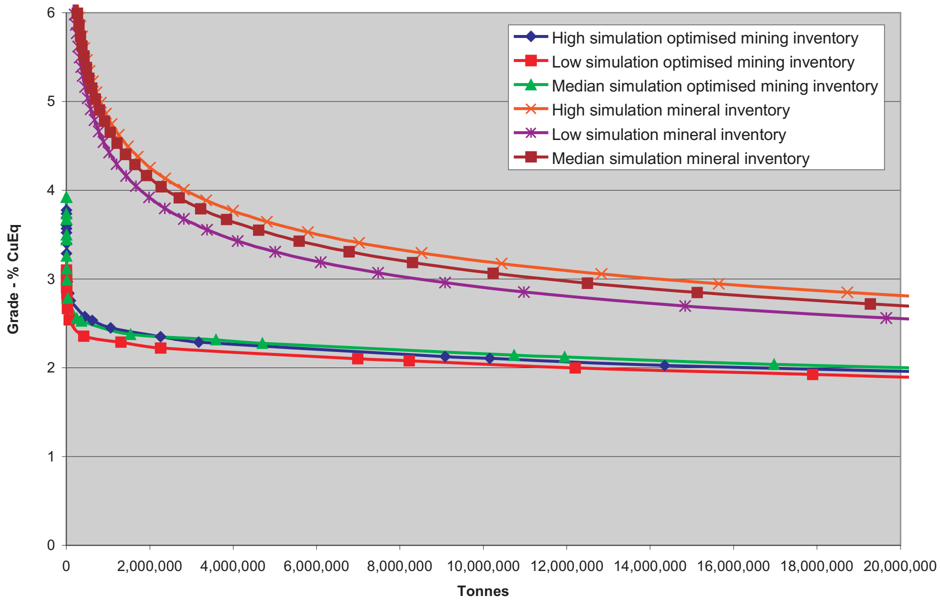


FIG 9 - Simulation mineral inventory and optimised mining inventory grade/tonnage curves.

Bulk open stoping with backfill

Bulk open stoping was considered a candidate mining method because of its potential to maximise the extraction of the resource at a high rate and relatively low cost. The Stopesizor optimisations indicated that significant inventory reductions would occur if excessive minimum stope heights were selected, as shown in Table 3. The table shows the mining inventory for a minimum stope height of 60 m with a 40 m level spacing is considerably less than that for a minimum stope height of 20 m with a 20 m level spacing, at the same cut-off grade. Figure 10 shows the mining inventory for 60 m high stopes with a 40 m level spacing. However, moderate minimum stope heights, as shown in Table 3 for the simulation comparison (40 m) and for the selective open stoping model (20 m), would be more likely to estimate an inventory better suited to maximising the value of the mining project. This mining method was retained for further consideration.

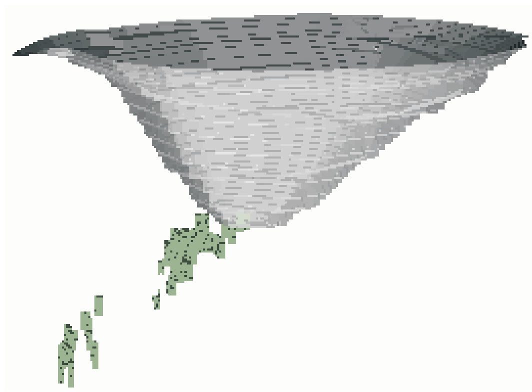


FIG 10 - Bulk open stoping inventory, 20 mW × 20 mL × 60 mH stopes, 2.0 per cent CuEq cut-off.

TABLE 3

Grade/tonnage outcomes for median simulation for selected mining methods.

Method	SMB dimensions W × L × H	Cut-off	Optimised mining inventory above cut-off		
	metres	% CuEq	kt	% CuEq	kt Cu
Bulk open stoping	20 × 20 × 40 (no specified level spacing)	1.0	84 288	1.47	1243
		1.5	39 777	1.80	716
		2.0	10 738	2.14	230
		2.5	246	2.56	6
Bulk open stoping	20 × 20 × 60 (40 m level spacing)	1.0	81 332	1.40	1139
		1.5	26 589	1.77	472
		2.0	4142	2.15	89
		2.5	66	2.50	2
Selective open stoping	20 × 20 × 20 (20 m level spacing)	1.0	77 457	1.53	1186
		1.5	36 600	1.86	682
		2.0	10 423	2.21	231
		2.5	604	2.57	16
Sublevel caving	20 × 10 × 20 (20 m level spacing)	1.0	77 494	1.54	1197
		1.5	37 358	1.88	701
		2.0	11 306	2.23	252
		2.5	981	2.61	26

Selective open stoping

Selective open stoping was considered a candidate mining method because of its potential to produce a high-grade mining inventory without the need for costly backfilling. The Stopesizor optimisation revealed that less than two per cent of metal tonnes were likely to exist in open stopes at a grade above 2.5 per cent CuEq and none were likely to exist above 3.0 per cent CuEq. Figure 11 shows that the optimised mining inventory was scattered and fragmented in its distribution, and that it would suffer increased mining costs and potentially reduced production rates. Consequently, high-grade selective open stoping was rejected as an option for further study.

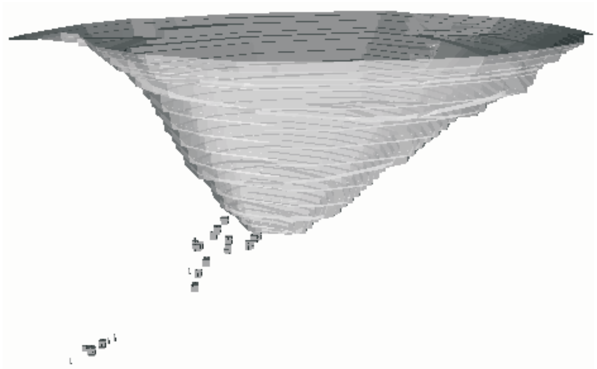


FIG 11 - Selective open stoping inventory, 20 mW × 20 mL × 20 mH stopes, 2.5 per cent CuEq cut-off.

Sublevel caving

Sublevel caving was considered a candidate mining method because of its potential to produce bulk tonnes at a low cost. The Stopesizor optimisation revealed that at a 2.0 per cent CuEq cut-off a mining outline suitable for sublevel caving would suffer significant quantities of included waste, as indicated by the discontinuous distribution of the mining blocks shown in Figure 12. In practice, sublevel caving would require a large proportion of included low-grade or waste material to be mined, reducing the head grade and increasing costs significantly. The added ore dilution and loss inherent in the sublevel caving method would further downgrade the value of any production. Finally, the competent hanging wall rock conditions were likely to present challenges to establishing and maintaining effective caving. Consequently, for the purposes of the study, sublevel caving was rejected as an option. It was recognised, however, that a more detailed assessment of the suitability of sublevel caving using an expanded range of cut-off grades and stope dimensions could have produced a different outcome.

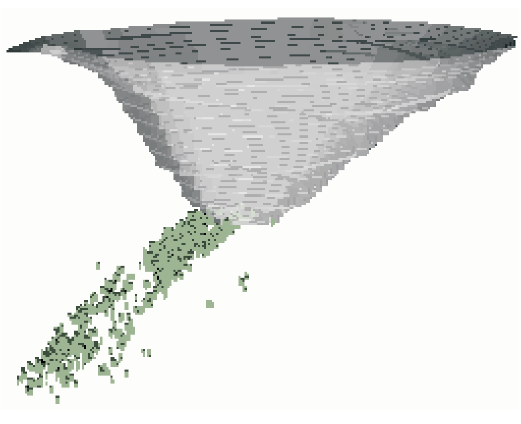


FIG 12 - Sublevel caving inventory, 20 mW × 10 mL × 20 mH stopes, 2.0 per cent CuEq cut-off.

DETAILED EVALUATION OF SELECTED CONFIGURATION

For detailed evaluation, the chosen stoping scenario was one with minimum stope dimensions of 20 mW by 20 mL by 25 mH. The chosen height coincided with standard spacings for large-scale open stoping sublevels as applicable to modern underground mining methods. It sits within the range previously identified as able to provide an acceptable mining inventory. A major level spacing of 50 m was selected as an integer multiple of the minimum stope height and one which would allow for level-to-level stoping if supported by the optimisation, without the need for the development of a dedicated drill sublevel.

For the chosen stope configuration, Stopesizor was used to identify level-by-level mining inventories for the median simulation at a range of cut-off grades to enable production and financial modelling to be undertaken. This in turn enabled an optimum cut-off grade to be identified for the selected method. The mining inventory results are shown in Table 4.

Financial modelling considered the productive capacity and scheduling, capital and operating development requirements, capital and operating mining costs, ore loss and dilution, processing, site and downstream costs, and forecast revenue factors. The financial modelling outcome is shown in Figure 13, which shows project cash flow and net present values as percentages of the maximum derived values for a range of cut-off grades. The results are presented in this way to preserve the commercial confidentiality of the project.

Figure 13 shows that a cut-off grade of 1.5 per cent CuEq provides the best free cash flow, and a cut-off grade of 1.7 per cent CuEq provides the best net present value. The grade mid-way between the two, 1.6 per cent CuEq, was chosen as the base case to represent the optimal cut-off for further analysis. The Stopesizor base case mining inventory determined at a 1.6 per cent CuEq cut-off is shown in Figure 14.

The sensitivity of the project financial outcome was tested against a range of capacity, cost, revenue and resource inputs. The low and high simulations were used to identify the range of project outcomes possible using a realistic range of mineral inventory estimates. The results are expressed in Table 5 as percentages relative to the base case. If required, the full range of realisations could be assessed to enable a probabilistic review of project potential NPVs, as in Dimitrakopoulos *et al* (2002).

As shown in Table 5, the simulations indicate that the resource tonnage at a 1.6 per cent CuEq cut-off could have a range of 28 per cent (+2 per cent and -26 per cent from the median simulation) at 95 per cent confidence, whilst the grade would only vary by some two per cent. The impact of this resource uncertainty on the potential project NPV was between -8 per cent and -40 per cent from the median case, equating to 92 per cent and 60 per cent of the median case NPV. This asymmetrical range of uncertainty probably results from the manner in which the resource model grades are spatially distributed in the simulations. The asymmetrical uncertainty in turn impacts on the final stope geometries and hence the mining inventory. This degree of uncertainty would not be apparent or able to be effectively assessed for project risk without the use of conditional simulation.

Although the limits are recognised and acknowledged above, the use of the 95 per cent resource confidence limit to estimate the range of possible project outcomes provides an interesting dimension in assessing project feasibility. The more commonly used approach is to apply arbitrary dilution and loss factors to a single resource or reserve estimate to identify a range of possible outcomes. The approach used in this study provides a range of mining inventory cases to consider, to which traditional sensitivity assumptions can be applied.

TABLE 4

Stopesizor inventories by level and cut-off grade for 20 mW × 20 mL × 25 mH minimum stope dimensions.

Cut-off	1.30		1.40		1.50		1.60	
Level (50 m)	kt	% CuEq	kt	% CuEq	kt	% CuEq	kt	% CuEq
1	6081	1.69	4988	1.76	3943	1.85	3270	1.91
2	6054	1.73	5071	1.80	4197	1.87	3489	1.94
3	5658	1.81	5022	1.87	4374	1.94	3929	1.98
4	4904	1.74	4155	1.80	3734	1.84	2993	1.92
5	4610	1.64	3644	1.72	2962	1.78	2386	1.84
6	3428	1.67	2876	1.73	2135	1.83	1886	1.87
7	3819	1.68	3165	1.74	2638	1.80	1989	1.88
8	4067	1.69	3306	1.78	2743	1.84	2230	1.91
9	4764	1.79	4152	1.85	3540	1.92	3081	1.98
10	3900	1.69	3280	1.76	2777	1.81	2404	1.86
11	3241	1.60	2484	1.68	1813	1.76	1440	1.82
Total	50 528	1.71	42 142	1.78	34 857	1.85	29 097	1.91
Cut-off	**1.70**		**1.80**		**1.90**		**2.00**	
Level (50 m)	**kt**	**% CuEq**	**kt**	**% CuEq**	**kt**	**% CuEq**	**kt**	**% CuEq**
1	2494	1.99	2106	2.04	1509	2.11	1100	2.17
2	2844	2.00	2237	2.07	1722	2.14	1294	2.20
3	3471	2.03	2816	2.09	2239	2.15	1665	2.22
4	2524	1.97	1844	2.06	1431	2.12	883	2.22
5	1467	1.96	1224	2.00	790	2.08	524	2.15
6	1512	1.93	1145	1.98	646	2.09	437	2.16
7	1487	1.96	1093	2.04	747	2.14	607	2.18
8	1776	1.98	1435	2.03	984	2.12	695	2.18
9	2489	2.06	2087	2.12	1724	2.17	1346	2.24
10	1779	1.93	1352	1.99	896	2.06	541	2.13
11	1058	1.88	712	1.95	406	2.03	192	2.13
Total	22 902	1.98	18 052	2.05	13 093	2.12	9284	2.19
Cut-off	**2.10**		**2.20**		**2.30**		**2.40**	
Level (50 m)	**kt**	**% CuEq**	**kt**	**% CuEq**	**kt**	**% CuEq**	**kt**	**% CuEq**
1	562	2.30	385	2.37	255	2.43	161	2.50
2	804	2.29	532	2.37	320	2.44	168	2.52
3	1235	2.28	855	2.34	484	2.41	202	2.50
4	618	2.29	387	2.37	238	2.46	161	2.50
5	327	2.21	139	2.28	41	2.35	0	0.00
6	268	2.22	131	2.31	82	2.36	0	0.00
7	429	2.24	231	2.33	131	2.41	49	2.47
8	469	2.24	307	2.30	139	2.36	41	2.40
9	1083	2.28	773	2.34	449	2.42	186	2.50
10	362	2.17	55	2.28	10	2.31	0	0.00
11	83	2.24	48	2.30	0	0.00	0	0.00
Total	6242	2.27	3842	2.34	2150	2.42	967	2.50

CONCLUSIONS

Conditional simulation of the underground resource at Ernest Henry mine has been applied successfully to quantify resource and reserve risk within the underground resource. It has been used to provide an early insight into the feasibility of the underground project at a 95 per cent confidence level in the resource. In addition to this, data from the conditional simulation study has been used to evaluate the grade risk associated with different areas of the resource model and to target future drilling.

TABLE 5

Resource-based project outcomes.

Parameter	Units	Base case	High	Low
Resource	T million	29.1	29.8	21.4
+1.6% CuEq	% CuEq	1.91	1.87	1.89
Cash flow	% of Base case	100	91	56
NPV	% of Base case	100	92	60

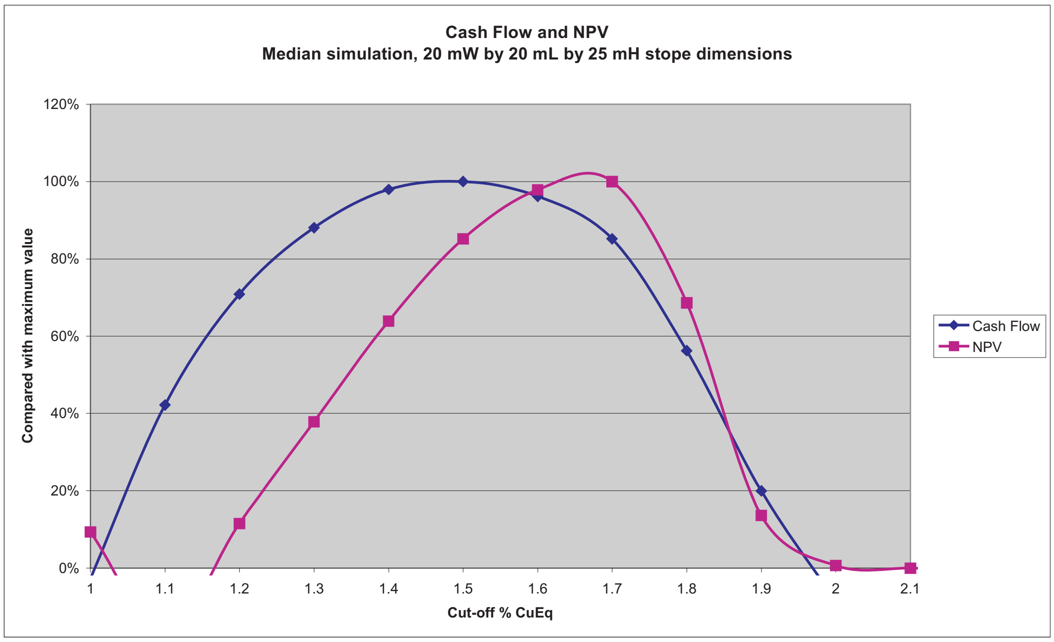


FIG 13 - Cash flow and NPV for median simulation, 20 mW × 20 mL × 25 mH stope dimensions.

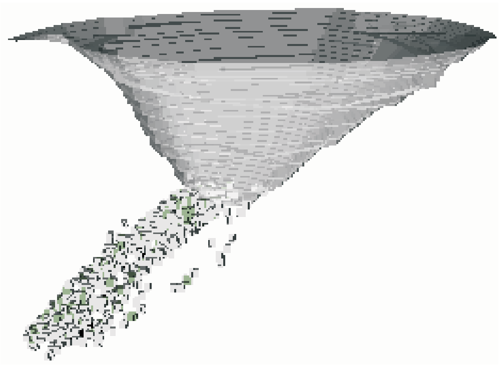


FIG 14 - Base case mining inventory, 20 mW × 20 mL × 25 mH stopes, 1.6 per cent CuEq cut-off.

Information derived from conditional simulation studies can quantify risk profiles associated with resource and reserve delineation, and can be invaluable when evaluating the risk associated with underground resources. Increasing knowledge of the resource, such as information from open-pit mining, can be incorporated into simulation studies. Data from simulation studies provides the range of possible resource outcomes that can be evaluated by conceptual mine planning, particularly if a stope-optimising tool is available to rapidly explore alternative stope geometries and hence potential mining methods. This type of study can be undertaken at major stages in a project, and quantified risk can then be used to assist with key decision points in the project development.

Further reading on stope design under conditions of geological uncertainty and risk includes Grieco (2003), and Grieco and Dimitrakopoulos (2007, this volume). Further reading on conventional stope design approaches includes the work by Ovanic (1998) and Carter *et al* (2004).

ACKNOWLEDGEMENTS

The authors would like to thank Alex Trueman for his contribution to the conditional simulation study, and Vivienne Snowden and Allan Earl for their involvement in this study and review of this paper. We would also like to thank Ernest Henry Mining Pty Ltd and Snowden Mining Industry Consultants for their permission to publish this work and their assistance with the preparation of this paper.

REFERENCES

Carter, P G, Lee, D H and Baarsma, H, 2004. Optimisation methods for the selection of an underground mining method, in *Proceedings International Symposium on Orebody Modelling and Strategic Mine Planning: Uncertainty and Risk Management*, pp 7-12 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Collier, P A and Bryant, J D, 2003. Successful mineral resource development at the Ernest Henry copper-gold mine, NW Queensland, in *Proceedings Fifth International Mining Geology Conference*, pp 73-88 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopolous, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: Grade uncertainty and risk effects in open-pit design, in *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, 483 p (Oxford University Press).

Grieco, N, 2003. Risk analysis of optimal stope design: Incorporating uncertainty, Thesis, Master of Philosophy, The University of Queensland, 204 p.

Grieco, N and Dimitrakopoulos, R, 2007. Grade uncertainty in stope design — Improving the optimisation process, in *Orebody Modelling and Strategic Mine Planning,* second edition (ed: R Dimitrakopoulos), pp 167-174 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ovanic, J, 1998. Economic optimization of stope geometry, PhD thesis, Michigan Technological University, Houghton, 209 p.

Poniewierski, J, MacSporran, G and Sheppard, I, 2003. Optimisation of cut-off grade at Mount Isa Mines Limited’s Enterprise Mine, in *Proceedings Twelfth International Symposium on Mine Planning and Equipment Selection, MPES 2003*, pp 531-538 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Thomas, G S and Earl, A, 1999. The application of second-generation stope optimisation tools in underground cut-off grade analysis, in *Proceedings Optimising with Whittle: Strategic Mine Planning Conference*, pp 175-180 (Whittle Programming Pty Ltd: Melbourne).

Optimising Open Pit Design with Simulated Orebodies and Whittle Four-X — A Maximum Upside/Minimum Downside Approach

R Dimitrakopoulos[1], L Martinez[2] and S Ramazan[3]

ABSTRACT

The management of cash flows and risk during production is a critical part of a surface mining venture as well as an integral part of a strategy in developing new and existing operating mines. Orebody uncertainty is a critical factor in strategic mine planning, the optimisation of mine designs and long-term sequencing. Traditional optimisation approaches are not developed to account for *in situ* grade variability as well as effectively deal with, incorporate and take advantage of geological risk. This paper presents a new approach to mine design based on risk quantification and alternative strategic decision-making criteria. This new approach deals with quantified geological and grade uncertainty in the context of optimal pit design, where designs and long-term production schedules are optimised under uncertainty. The method is founded on the definition of two components. The first component includes the key project performance indicators to be considered, such as the minimum annual ore production, amount of metal produced in given mining periods or discounted cash flows over the life of a mine. The second component includes the decision-making criteria, such as a minimum acceptable project NPV, the minimum acceptable risk in meeting given production targets, and the minimisation of cash flow risk in the short-term, while maximising the potential for profits in the future. An application at an open pit epithermal gold mine presents in a step-by-step fashion the optimisation of its mine design and sequencing under conditions of geological uncertainty.

INTRODUCTION

Open pit mine design and long-term sequencing is an intricate and critically important part of mining ventures. It provides the technical plan to be followed from mine development to mine closure having a profound effect on the economic value of the mine. Mathematical methods provide analytical tools used for optimising open pit mine designs. The most established and frequently used approach is based on the Lerchs-Grossmann three-dimensional graph theory (Lerchs and Grossmann, 1965). This theory is implemented in the Whittle software as the nested Lerchs-Grossmann algorithm (Whittle, 1988, 1999) and remains an efficient and expandable pit optimisation method (Muir, 2007, this volume).

Despite the routine utilisation of mathematical optimisation in mining practice, traditional open pit optimisation is affected by uncertainty in the key input parameters leading to suboptimal net present value (NPV) solutions and deviations from production plans. A critical source of technical risk is geological, including the expected ore grade and tonnes within a given design layout. The importance of geological risk to pit design and mine planning is well acknowledged in the technical literature. For example, Baker and Giacomo (1998) show that out of 48 mining projects in Australasia, nine realised reserves less than 20 per cent of the originally expected, and 13 over 20 per cent more reserves than forecasted. For Canada and the USA, Vallee (2000) refers to a World Bank survey by Buetel Goodman & Co (1990) showing that 73 per cent of mining projects failed due to problems in their ore reserve estimates, and led to a loss of US$1106 million in capital investment.

A study by Dimitrakopoulos, Farrelly and Godoy (2002) tests the performance and limitations of a traditional optimisation approach through the resulting predicted project NPV using an estimated orebody model and its application in Whittle Four-D. Conditionally simulated orebody representations were used to assess grade uncertainty within the pit limits producing results that highlighted a substantial risk associated with the traditional design. This risk assessment indicated a five per cent probability of the traditional design to realise its predicted NPV equating to a value that is 50 per cent less than what the simulated approach provides. In addition, this example shows substantial negative differences in expected quarterly discounted cash flows (DCFs) and a shorter life of mine, considering grade uncertainty within the ultimate pit. This study demonstrates the limitations of traditional technologies, which combine estimated smooth orebody models with complex, non-linear pit optimisation algorithms that assume certainty in their inputs.

Assessing grade risk suggests that there is a probability that a given design may perform better than forecasted; thus, there is an upside potential associated with the orebody considered, similarly to a downside risk where forecasts are not materialised. Seeking mine designs and long-term extraction sequences that have the possibility of capturing the upside potential of the deposit and at the same time minimise any possible downside risk is desirable and now possible. Figure 1 elucidates the concept of 'maximum upside/minimum downside' mine designs based on grade risk. It shows the distribution of DCFs for a pit design that can be generated from simulated orebody models and used to assess the mine design and production sequence. With a defined point of reference such as the minimum acceptable return (MAR) on investment, the distribution that minimises risk or downside and maximises reward or upside leads to selecting a preferred design. Note that in general the MAR is different than the average or median of a distribution.

This paper presents a new approach to developing open pit mine designs that capture the upside potential of the deposit whilst minimising downside risk for key project performance indicators, such as periodical DCFs and amount of ore tonnes and metal production. The methodology employs conditionally simulated orebody models to quantify grade risk and Whittle Four-X with the Milawa NPV scheduler option (Whittle, 1988). The approach complements other advancements moving towards developing optimisation under uncertainty as presented in this volume (eg Godoy and Dimitrakopoulos, 2007, this volume; Froyland *et al*, 2007, this volume; Grieco and Dimitrakopoulos, 2007, this volume; Menabde *et al*, 2007, this volume; Ramazan and Dimitrakopoulos, 2007, this volume).

In the following sections, the approach for maximum upside/minimum downside proposed herein is first detailed and followed by an application at a typical low-grade open pit gold mine. Subsequently, the effect of the gold price on preferred designs is assessed and conclusions follow.

1. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

2. School of Economics and Finance – Faculty of Business, Queensland University of Technology, GPO Box 2434, Brisbane Qld 4001. Email: l.martinez@student.qut.edu.au

3. MAusIMM, Rio Tinto, GPO Box A42, Perth WA 6000, Australia. Email: salih.ramazan@riotinto.com

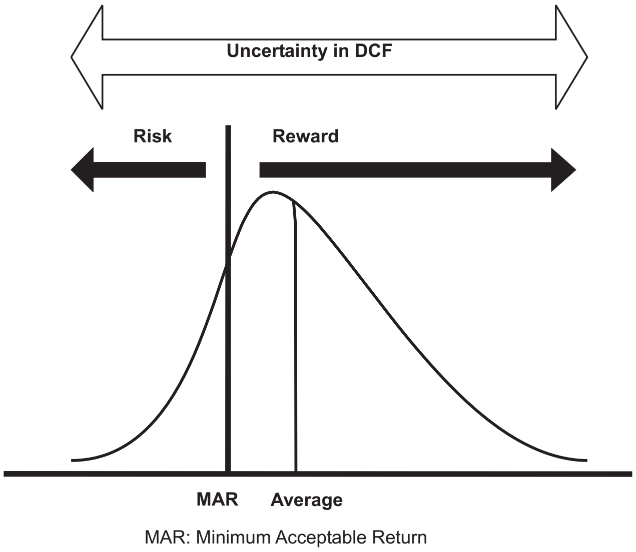


FIG 1 - Uncertainty in a distribution of a key project performance indicator (DCF), reward or upside potential and downside risk with respect to a point of reference such as minimum acceptable return (MAR).

QUANTIFIED RISK AND AN UPSIDE/DOWNSIDE APPROACH TO OPTIMISATION

The maximum upside/minimum downside approach to open pit optimisation suggested here is based on the quantification of geological uncertainty through the generation of a series of equally probable representations of the orebody. Conditional simulation (eg Benndorf and Dimitrakopoulos, 2007, this volume) may be seen as a family of techniques that allow the generation of these orebody representations, all reproducing the *in situ* variability, the available conditioning data and information, the data histogram, and spatial continuity of the orebody. The steps taken by the upside/downside approach are as follows:

1. Conditionally simulate several orebody models using the available data.
2. Implement Whittle Four-X with the Milawa NPV scheduler option to design a pit for each simulated orebody model; each design maximises the discounted net value to be generated from the mine, within operational constraints such as slope angle, mill plant capacity and total mining capacity of available equipment.
3. With the pit limits and sequence of extraction including annual production generated, quantify grade risk in each pit design for the selected key project performance indicators, such as total project NPV, periodical amount of ore material to feed the mill, metal production and cashflows. Risk analysis is performed similarly to Dimitrakopoulos, Farrelly and Godoy (2002). For a given schedule, a set of DCFs is calculated using each of the simulated orebody models for the material in each cut-back or year of production. Similarly to cashflows, distributions can be generated for any other indicator including ore tonnage and metal quantity considering mill capacity and market demand providing the reference levels needed.
4. Discard pit designs that may not meet the key project performance indicators deemed necessary, for example the tonnes of ore that is required at the mill, or cash flows to be met in a given production period and so on.
5. Using the distribution of possible values for any project indicator as found in step three, calculate the upside potential and downside risk for selected project indicators with the remaining designs using a point of reference (eg minimum acceptable return on investment, mill demand, market specifications). Select the designs that meet the preset decision making criteria. A comparison of two designs for a given orebody is shown graphically in Figure 2. Given a value of a project's MAR, the expected DCF above this value provides an assessment of the upside potential whilst the same measure below the MAR is considered the design's downside risk indicator. Different criteria and key project performance indicators lead to selecting a desirable pit design. The discussion on the effect of metal prices on the pit design process above is deferred until a later section.

The approach outlined above provides a process that leads to the selection of a single pit design that captures the upside potential of the orebody and minimises the potential downside risk, given the available data and information integrated into the simulation process. A case study presented next illustrates the practical aspects of the approach.

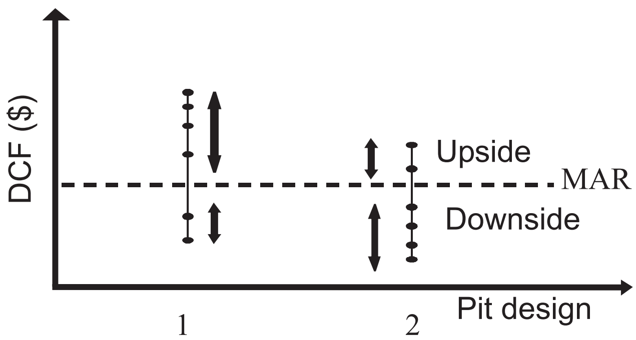


FIG 2 - Upside potential and downside risk for two pit designs for the same orebody.

APPLICATION AT AN EPITHERMAL GOLD DEPOSIT

A typical Australian disseminated low-grade epithermal quartz breccia gold deposit occurring in volcanic rocks and sediments is used to illustrate the approach for the pit optimisation procedure outlined above. The mine produces free milling and refractory ores delivered to a CIL processing plant, with a floatation circuit added for the refractory ore. General information on the deposit and resource as well as the details pertaining to the simulated orebody models are available from previous studies (Farrelly, 2002). Figure 3 shows an east-west section of the orebody with closely located drill holes and their three-metre composite grades. This same cross-section will be shown in figures throughout the study. The first priority of this mine is to meet mill ore demand, particularly during the first year and is deemed in the present case a higher priority than the economic performance of the operation over the life-of-mine.

Optimisation and the development of a mine design for each of the simulated orebody models are generated in all cases using the parameters given in Table 1. In the example presented here, 13 simulated orebody models are used and are sufficient to illustrate the practical aspects of the suggested approach. After designing the ultimate pit limits and generating the corresponding pit shells for each of the 13 simulated orebody models, the mill's demand for one million tonnes of ore per year

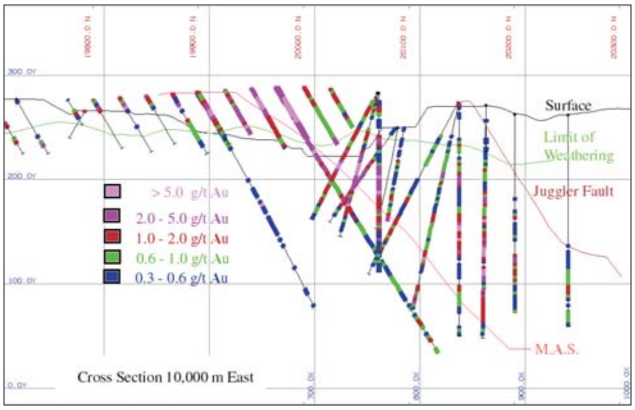


FIG 3 - Cross-section showing geology and closest drill holes from the epithermal gold deposit considered in this study.

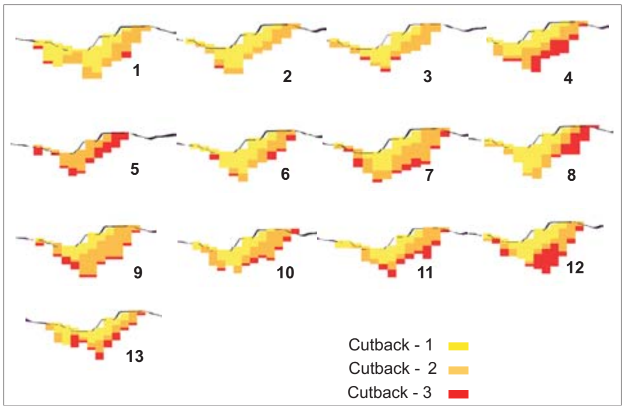


FIG 4 - Cross-sections of 13 pit designs and their cut-backs generated from optimising individual simulated orebody models.

TABLE 1

Technical and economic parameters considered for developing mine designs for the gold deposit under study.

Description of technical and economical parameters	Values
Pit slope	54°
Mining cost	\$1.0 per tonne
Processing cost for oxide ore	\$8.195 per tonne
Processing cost for fresh ore	\$16.86 per tonne
Mill recovery for oxide ore	90%
Mill recovery for fresh ore	84%
Discounted rate	8% per year
Gold price	A\$600/oz

is considered and three cut-backs are generated as an approximate annual schedule using the Milawa-NPV option of the Whittle software. Figure 4 shows cross-sectional views of the 13 designs generated indicating differences in terms of location of cut-backs to be mined periodically and the ultimate pit limits between the designs. Differences in the schedules often result in significant variations in expected cash flow returns.

It is appropriate to note some aspects of the designs generated above. Firstly, an optimal design based on a given simulated orebody model is not, in general, optimal for other conditionally simulated orebody models. Secondly, although the simulated orebody models are equally probable, the corresponding designs are not; there is no reason, for example, why there cannot be fewer designs than simulated orebodies being optimised. Thirdly, the optimisation process is a non-linear function and, therefore, it is not possible to select 'representative' realisations of the orebody to generate 'optimistic', 'average' or 'pessimistic' scenarios. For example, a decile, say 90 per cent, with respect to a potential grade tonnage curve of the resource in the ground will not provide a similar or even predictable decile of any project performance indicator. These aspects of the designs make the selection of a single optimal pit more complex than the traditional pit design approach. The risk analysis discussed next is proposed as a tool that can be used to choose the best design from the available designs. 'Best' is considered here the design that minimises the potential for losses whilst maximises the possibility of better financial performance.

Selecting the pit design: risk analysis on key project performance indicators

For the gold mine considered in this case study, the key project indicators are DCF, periodical ore tonnage and metal content. For a given mine design a distribution of the discounted economic value, total ore tonnage and recoverable metal content for each cut-back is calculated using each of the simulated orebody models. Figure 5 illustrates this process. The distributions of the key project indicators are calculated for the three cut-backs (CB-1, CB-2 and CB-3) with respect to pit design number two, where each simulated orebody model is represented by a single bar in each cut-back of each indicator. This process is repeated for all 13 designs.

Prioritising the importance of the key performance indicators is important for the approach used here. In this case study meeting ore production targets is the more important performance indicator in the first year of operation. However, it is common, for example, that repayments of possible loans and hence the recovery of the initial investment makes DCFs more significant in the first year rather than later years. Figure 6 plots the risk profile of the key project indicator ore tonnage, within the first cut-back for the 13 pit designs. Considering the mill feed requirement of one million tonnes of ore and the requirement that there is a 70 per cent chance of producing at least one million tonnes leads to designs two, four, six, and 12 being retained for further assessment. The remaining designs are excluded from further study, whilst the selected designs will be tested with the second performance indicator of interest, DCF.

Figure 7 shows the DCF project performance indicator for the selected designs within the first, second and third cut-backs. The MARs considered per cut-back are \$12 M, \$2 M and \$1 M during the first, second and third years of operation, respectively, and are shown in Figure 7 as cumulative DCF. If C_t is the MAR value in period t, it is possible to calculate the upside potential, UP_i, and downside risk, DR_i, of design i using the following:

$$UP_i = \sum_j (C_t - V_j^+) P_j \qquad (1)$$

$$DR_i = \sum_j (C_t - V_j^-) P_j \qquad (2)$$

where V_j is the total discounted economic value to be generated for simulated orebody model j; if V_j is greater than C_t then V_j is represented as V^+_j, otherwise, V_j is represented as V^-_j; P_j is the probability from simulated orebody model j. In Equation (1), j refers to the index of the simulated orebody models that have a

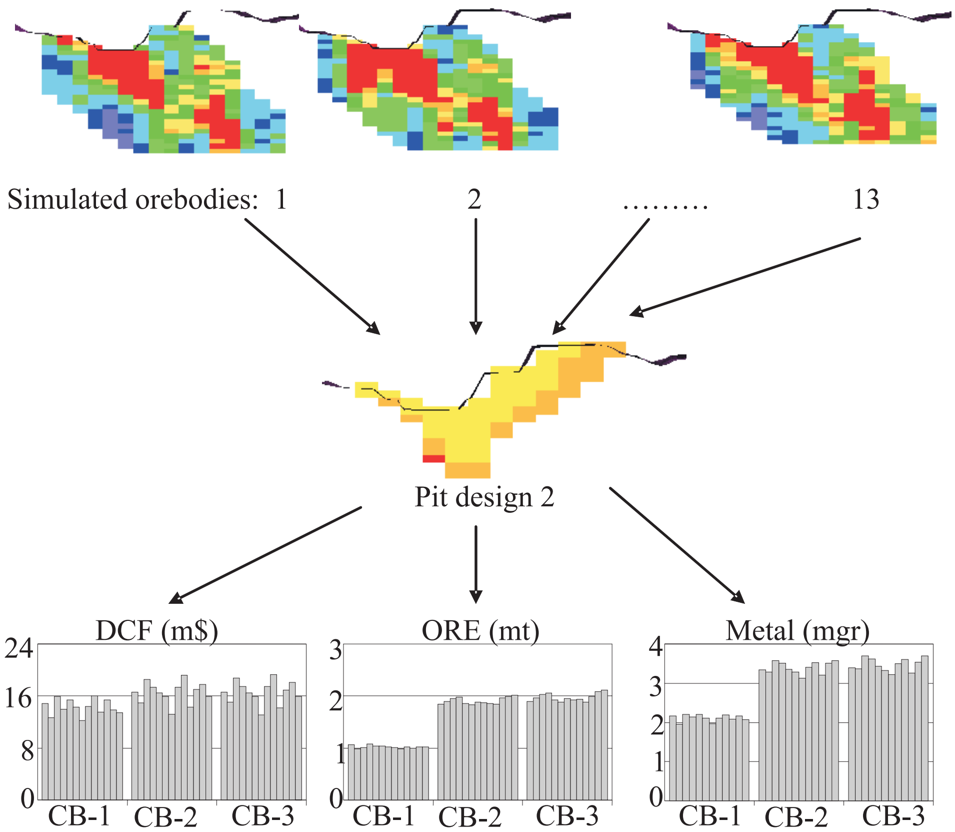


FIG 5 - Illustration of the steps used to quantify risk in a given pit design.

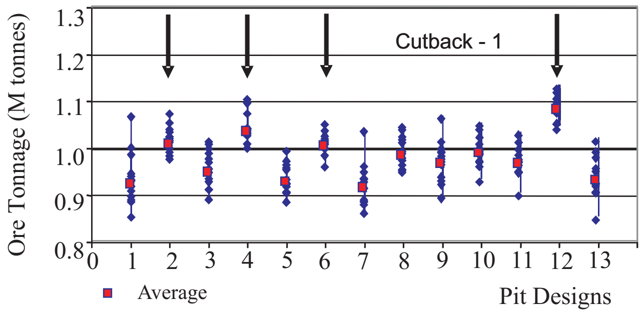


FIG 6 - Comparing risk profiles for ore tonnage within the first cut-back for 13 pit designs generated using simulated orebody models. Arrows on top indicate selected designs with at least a 70 per cent chance of being above one million tonnes of ore.

total discounted economic value greater than C_t during period or cut-back t, and in Equation (2), j is the index of simulated orebody models where $V_j \leq C_t$ during period or cut-back t.

Figure 7 shows the V_j values for each design as cumulated over the production periods, or cut-backs. If cumulated values are used, this case study shows a value of $12 M for C_1, $14 M for C_2 and $15 M for C_3 . Table 2 shows the UP_i and DR_i values for the selected designs within each cut-back to be mined in successive production periods. The table shows that design 12 has a somewhat higher UP within the first cut-back and also shows zero risk for the same cut-back. However, it has the highest DR during the last year of production ($-0.96 M). Designs two and six also have relatively high UP values, zero DR for cut-back one and DR values are better within the second cut-back than those in design 12. Both designs two and six have relatively higher total upside potentials with less risk over their production life than the two others.

It is worth noting for reasons of comparison that the design and sequence generated using a smooth estimated orebody model of this deposit and traditional optimisation approach as reported in Dimitrakopoulos, Farrelly and Godoy (2002) has a 15 per cent chance of not achieving the MAR specified during the first year, eight per cent during the second production period and 31 per cent during the last year.

EFFECT OF PRICE VARIABILITY ON AN OPTIMAL DESIGN

Designs two and six are selected as the best performing mine designs based on the quantified risk of project indicators for a fixed gold price of A$600/oz. To assess the impact of gold price variations on the total upside potential of the project Figure 8 shows the total upside potentials of the selected designs, two and six, when the gold price is increased from $600/oz to $650/oz and to $700/oz. The upside potential of design two decreased significantly when the unit price is increased to $650. This unexpected reduction in UP indicates that design two is sensitive to price variations. This sensitivity is confirmed by the significant increase in the UP value to $5.4 M when the gold price is $700/oz. Design six shows an increasing trend on the total UP value as the price is increased. Although UP values of designs two and six are comparable to each other at $600/oz, the difference becomes significantly large when the price is increased to $700/oz. The increasing difference in the UP values indicate that design six provides the highest upside potential should there be an increase in gold price to $700/oz and would

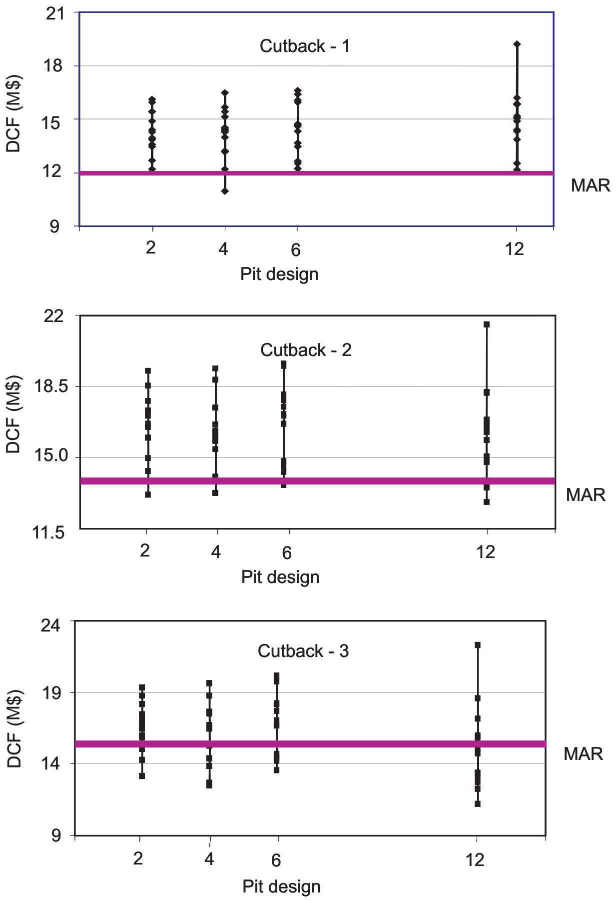


FIG 7 - Comparison of risk profiles for the key project indicator DCF per cut-back for selected pit designs.

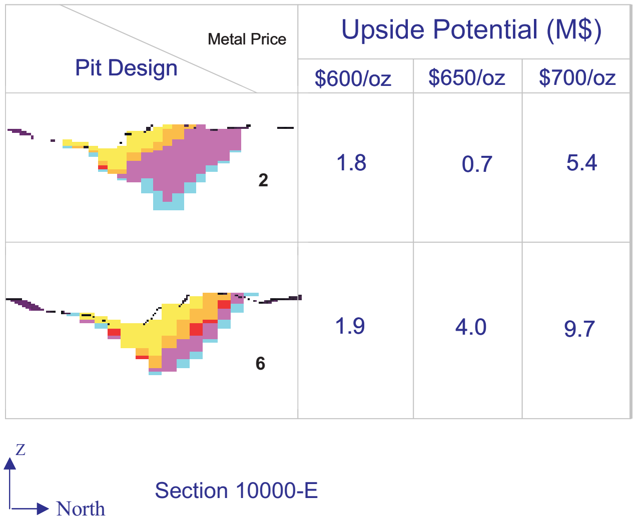


FIG 8 - Upside potential for designs two and six for different gold prices.

FIG 9 - Pit design six and its expansion for different gold prices.

TABLE 2

Upside potential and downside risk values for selected mine designs within each cut-back.

Design	Upside potentials (UP $ M)			Downside risk (DR $ M)		
	CB1	CB2	CB3	CB1	CB2	CB3
2	2.3	2.41	1.8	0.00	-0.08	-0.20
4	1.3	2.1	1.6	-0.78	-0.15	-0.51
6	2.4	2.43	1.9	0.00	-0.02	-0.28
12	2.9	2.4	1.2	0.00	-0.16	-0.96

potentially generate larger revenues. Figure 9 shows the expanding pit design number six for the different gold prices considered.

CONCLUSIONS AND FURTHER WORK

Geological uncertainty has a significant impact on the real value of mining projects. A new approach was proposed for designing open pit mines based on geological uncertainty that combines conditionally simulated orebody models and optimisation with Whittle Four-X. The approach is based on developing designs that capture maximum upside potential whilst minimising downside risk.

The utility of the approach is that it allows the use of traditional and commercially available optimisation tools to address risk issues and produce better designs. However, two main weaknesses may be identified. The approach may be operationally tedious, particularly in the case of larger orebodies, and depends on the ability to efficiently simulate orebody models at the scale required for managing substantial volumes of data. This suggests how it is imperative to consider conditional simulation methods that are truly efficient and can facilitate studies, such as the one here, within weeks rather than months. In this volume, this area of concern is addressed by Benndorf and Dimitrakopoulos (2007, this volume).

An additional issue is that conventional optimisers cannot really provide the optimal upside/downside solution for a set of criteria. The solution provides a single design preferable to those remaining in the group of designs being compared. However, one cannot ensure that the approach will generate the best possible design and mining sequence over the life-of-mine for the criteria used conditional to the understanding of the orebody being considered. The ability to provide truly optimal upside/downside approaches where the upside/downside profile of a mine design is defined by the user requires further development and forms the key reason for research in stochastic mine planning (eg Ramazan and Dimitrakopoulos, 2004, 2007, this volume; Dimitrakopoulos and Ramazan, 2004; Godoy and Dimitrakopoulos, 2004).

Although this study focuses on specific key project indicators, the method presented is general and suitable for any user-defined decision-making process and indicators that may be chosen. The approach can be used in any type deposit and open pit optimisation study.

ACKNOWLEDGEMENT

The work presented herein was part of a research project funded by Anaconda Operations, Anglo Gold, BHP Billiton, Highlands Pacific, MIM Holdings (Xstrata), Pasminco, Rio Tinto and WMC Resources.

REFERENCES

Baker, C K and Giacomo, S M, 1998. Resource and reserves: their uses and abuses by the equity markets, in *Ore Reserves and Finance*, pp 65-76 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Benndorf, J and Dimitrakopoulos, D, 2007. New efficient methods for conditional simulation of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions,* 316:106-112.

Farrelly, C T, 2002. Risk quantification in ore reserve estimation and open pit mine planning: MSc thesis, University of Queensland, Brisbane.

Froyland, G, Menabde, M, Stone, P and Hodson, D, 2007. The value of additional drilling to open pit mining projects, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 245-252 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions,* 316:43-50.

Godoy, M C and Dimitrakopoulos, R, 2007. A multi-stage approach to profitable risk management for strategic planning in open pit mines, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 337-343 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Grieco, N and Dimitrakopoulos, R, 2007. Grade uncertainty in stope design: improving the optimisation process, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 167-174 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Lerchs, H and Grossmann, L, 1965. Optimum design of open-pit mines, *Transactions of CIM,* LXVII:17-24.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Muir, D C W, 2007. Pseudoflow, new life for Lerchs-Grossman pit optimisation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 113-120 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2004. Recent applications of operations research in open pit mining, *SME Transactions,* 316:73-78.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Vallee, M, 2000. Mineral resource + engineering, economic and legal feasibility = ore reserve, *CIM Bulletin*, 93(1038):53-61.

Whittle, J, 1988. Beyond optimisation in open pit design, in *Proceedings Canadian Conference on Computer Applications in the Mineral Industries*, pp 331-337 (Balkema: Rotterdam).

Whittle, J, 1999. A decade of open pit mine planning and optimisation – the craft of turning algorithms into packages, in *Proceedings 28th International Symposium on the Application of Computers and Operations Research in the Mineral Industry* (ed: K Dagdelen), pp 15-24 (Colorado School of Mines: Golden).

Incorporating Grade Uncertainty in the Decision to Expand the Main Pit at the Navachab Gold Mine, Namibia, Through the Use of Stochastic Simulation

M Kent[1], R Peattie[2] and V Chamberlain[3]

ABSTRACT

Incorporating grade uncertainty into an ultimate pit design allows the quantification of the level of risk associated with the decision to expand a project. By generating a number of stochastic realisations of the orebody, the impact of spatial variability in grade and the level of risk therein can be quantified. This is demonstrated at the Navachab gold mine, located 170 km west-northwest of the Namibian capital, Windhoek. Navachab is solely owned by Anglogold Ashanti Namibia Limited (formerly Anglogold Namibia Limited) and produces approximately 80 000 ounces per year, with a current mineral resource of 137 million tonnes at 1.18 g/t for 5.23 million ounces (at a 0.4 g/t cut-off). The mine is currently the focus of a study into the viability of a large cutback to further access the main zone of mineralisation.

Traditional open pit designs do not incorporate grade uncertainty and this is true of Navachab gold mine. The resource is currently based on an orebody model generated by the uniform conditioning (UC) technique. Although UC incorporates change of support based on a selective mining unit, it only gives a single outcome from the optimisation process for a given set of economic parameters. It also gives no indication of the level of risk associated with the mineral resource estimate. Reconciliation of the mineral resource (MR) model also suggests there is a difference in selectivity between the MR model and the grade control (GC) model. To attain a measure of the risk and potential upside associated with the geological model, 100 equi-probable realisations of the orebody were generated using conditional simulation. Each of the realisations was reblocked into blocks equivalent to the current selective mining unit and passed through a pit optimisation program. The various pits were then compared with the base case (UC estimate) of the MR model.

The results of the pit optimisation of the individual realisations showed some risk in the economic pit as defined using the MR model, but also showed some upside potential, particularly at depth. The grade simulations generally compared closely with the MR and GC models with the exception of the high-grade main mineralisation zone, where there appears to be a selectivity difference between the models. As the GC information used in the comparison is largely from blasthole sampling that is notoriously poor and traditionally reports grades at Navachab some ten per cent lower than found at the plant, nothing definite can be concluded from the reconciliations of the MR and simulation models with the GC model.

The exercise has proved useful in highlighting areas for infill drilling, for providing a measure of the grade uncertainty within the Navachab deposit, and for assessing the risk associated with using the MR model in mine design. The ongoing challenge is to find a way of incorporating the risk measures into the mine design process to allow the management of grade risk and not just the quantification of grade risk.

INTRODUCTION

Navachab gold mine is currently the only working gold mine in Namibia. It is solely owned by Anglogold Ashanti Namibia Limited (formerly Anglogold Namibia Limited) and is located 170 km west-northwest of the Namibian capital, Windhoek, (Figure 1). The Navachab gold deposit was discovered in October 1984 as a result of a regional geochemical study. A follow-up drilling campaign and feasibility study in 1987 resulted in construction work beginning in 1988 and the first gold bar being poured, 21 months later, in December 1989. Navachab produces approximately 80 000 ounces per year, with a current resource base of 137 million tonne at 1.18 g/t for 5.23 million ounces (at a 0.4 g/t cut-off).

Historically, the mineral resource (MR) model at Navachab has reported higher grades than those predicted by the grade control (GC) model (Table 1). With the mill reporting approximately seven per cent higher gold grades than the GC model (Figure 2), reconciliation suggests that the MR model is probably closer to reality than the GC model. The poor quality of

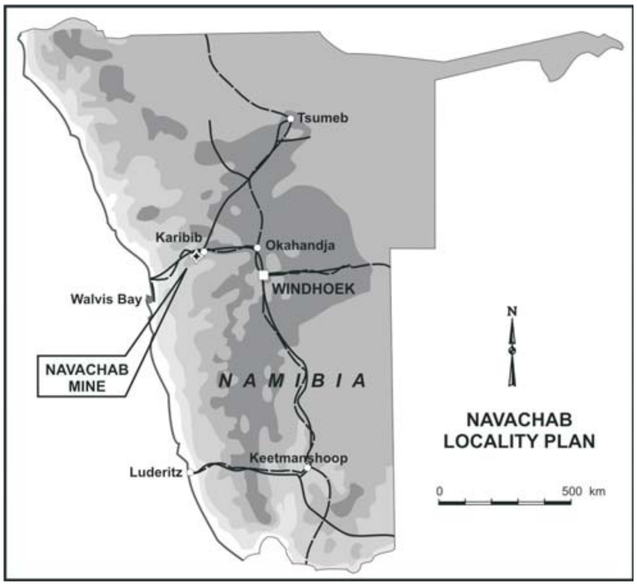


FIG 1 - Location map showing the Navachab gold deposit.

TABLE 1

Historical reconciliations of grade control to the resource.

Mined		GC	Resource	Difference
MC	Tonnes	9 086 759	8 746 783	-3.7%
	Grade	2.07	2.32	12.1%
	Ounces	604 742	652 420	7.9%
HW	Tonnes	15 566 787	14 039 557	-9.8%
	Grade	1.00	1.20	20.0%
	Ounces	500 484	541 659	8.2%
FW	Tonnes	3 707 669	2 997 442	-19.2%
	Grade	1.00	1.14	14.0%
	Ounces	119 204	109 862	-7.8%
Total	Tonnes	28 361 215	25 783 782	-9.1%
	Grade	1.34	1.57	17.1%
	Ounces	1 224 430	1 303 941	6.5%

1. MAusIMM, Senior Resource Geologist, AngloGold Ashanti Limited, Level 13, St Martins Tower, 44 St Georges Terrace, Perth WA 6000, Australia. Email: mkent@anglogoldashanti.com.au

2. Manager Evaluation – South Africa, AngloGold Ashanti Limited, PO Box 62117, Marshalltown 2107, South Africa. Email: rpeattie@anglogoldashanti.com

3. MAusIMM, Manager Mineral Resources and Mine Geology, AngloGold Ashanti Limited, PO Box 62117, Marshalltown 2107, South Africa. Email: vchamberlain@anglogoldashanti.com

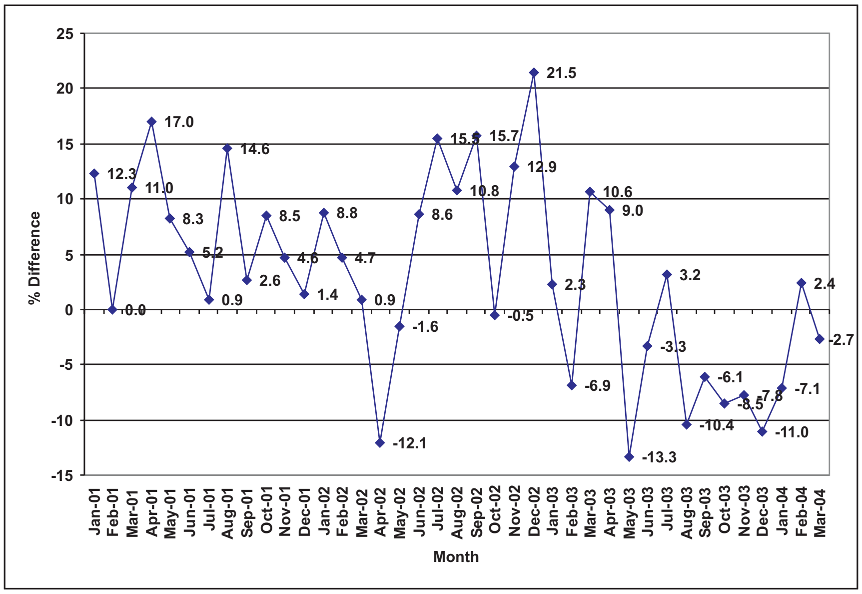


FIG 2 - Grade reconciliation of grade control to mill production. A positive reconciliation indicates that the mill has reported a higher grade than the grade control.

the GC model has made the benchmarking and qualification of the MR model against the GC model in the past very difficult. However, the poor reconciliation of the GC model with the MR model and mill grades has been arrested somewhat since 2003, due to a number of initiatives undertaken at the mine. These initiatives have been to refine the geological modelling of the high-grade main mineralisation zone; remove high-grade top-cuts; switch from blasthole drilling to reverse circulation (RC) drilling; and re-equip the sample preparation laboratory. As a result, the GC model reconciled better with the mill grades in 2003 (Figure 2). However, the GC model then reported three per cent higher grades and 11 per cent more tonnes than the MR model (Table 2). So, while in the past the grade reported by the GC model might have been of questionable quality, reconciliations show that this may no longer be the case. However, sufficient GC data would need to be collected before a detailed reconciliation could be undertaken and the full effect of the changes determined.

TABLE 2

Reconciliations of grade control to the resource model for 2003.

	Resource model	Grade control	% Difference
Tonnes	1 235 572	1 372 046	11
Grade (g/t)	1.85	1.91	3
Gold (kg)	2289	2621	14

A pit expansion study for the Navachab gold mine has been looking into the viability of a large cutback to further access the main mineralisation zone at depth. The large financial risks involved in such an expansion, call for the quantification of all the risks involved. Dowd (1994) and others have stated that one of the biggest risks involved in mining is the risk associated with geological uncertainty. Therefore a measure of the geological uncertainty was sought. Ravenscroft (1992) noted that traditional estimation techniques do not adequately quantify uncertainty and suggests that conditional simulation be used to determine the geological uncertainty. A conditional simulation was therefore modelled at Navachab to better understand the effects of the grade uncertainty on the pit expansion. While recognising the limitations of the current GC data, the resultant realisations could be reconciled against the GC model and the uniform conditioned MR model.

Conditional simulation is the method of drawing equi-probable joint realisations while honouring the available data (Goovaerts, 1997; Chilès and Delfiner, 1999). The resultant realisations are not only conditioned to the available sampling data but also duplicate the histogram and variogram of the sampling data. In this way where the simulations differ, and by how much they differ, gives a measure of the uncertainty in the values. Dimitrakopoulos (1998) proposes a three-step methodology for modelling geological uncertainty using conditional simulation. The three steps are stochastic conditional simulation, followed by implementation of a transfer function, which represents a mining process and is generally non-linear, and finally risk modelling and decision-making.

The general use of simulation in mine optimisation has been limited in the past due to the large models required, the inefficiencies of computers and the general confusion amongst practitioners about how to deal with multiple realisations. The exponential growth in computer power in recent years and the gradual acceptance and understanding of the extent of uncertainty in the mineral resource is steadily moving the mining industry away from a traditional single estimate approach to the mining process. This has necessitated a rethink in how risk is managed in the mining environment (Dimitrakopoulos, Farrelly and Godoy, 2002).

GEOLOGY AND MINERALISATION

The Navachab gold deposit and surrounding gold occurrences are located in the Southern Central Zone of the Pan-African Damara Orogen (Millar, 1983; Moore and Jacob, 1988). The mineralisation occurs on a sedimentary basin edge and is closely associated with northeast-southwest trending thrust zones and associated shear zones.

The mineralisation at Navachab can be divided into three main zones. The main zone is a skarn-type mineralisation occurring within a 35 m thick interbedded calc-silicate and marble unit (MC) at the base of the Okawayo Formation. The mineralisation within the MC zone is located within narrow shoots plunging approximately 25° towards the east. Of secondary importance is the mineralisation in the hanging wall and footwall zones. Mineralisation in the hanging wall zone is contained within sheeted quartz veins hosted by a marble and dolomitic unit (MDM) within the Okawayo Formation. Mineralisation in the footwall zone is contained within sheeted quartz veins hosted by schists and minor marble units of the Spes Bona Formation. The hanging wall zone is situated on the northern side of the MC zone, and the footwall zone on the southern side of the MC zone. In both the hanging wall and footwall zones, sheeted vein sets concentrated in zones parallel to the plunge of mineralisation within the MC zone contain elevated gold grades (Peattie, Badenhorst and Chamberlain, 2004). A schematic section of the geology of the Navachab deposit is presented in Figure 3.

DATA MANAGEMENT AND CONDITIONAL SIMULATION

A systematic process of data verification and preparation was followed, which included both visual and statistical validation of the sampling data, evaluation of the assay and sampling quality control and on-site review. To assist in the reconciliation for this study, the same geological domaining was used for the conditional simulation and the GC model as was used for the MR model. These domains were main (MC), hanging wall ore (HWO), hanging wall waste (HWW), footwall rre (FWO) and footwall waste (FWW). The hanging wall and footwall zones are divided into ore and waste zones by geological indicator kriging. The use of this technique is supported by good reconciliation between the geological indicator kriging and the known geological zoning in the mined-out areas, especially in areas of concentrated mineralisation in the sheeted-quartz vein systems. Similarly, to ensure that there was no tonnage discrepancy between the MR model and the simulation models due to density measurements, densities used in the simulation model were taken from the densities for each block used in the MR model. The calculation and estimation of the density in the MR model is detailed in Peattie, Badenhorst and Chamberlain (2004).

During the data validation process for the 2003 MR model, a significant difference in the mean grades (up to 54 per cent) between the reverse circulation (RC) and diamond drill hole data for the hanging wall zone was noticed (Peattie, Badenhorst and Chamberlain, 2004) (Figure 4). This was traced to a poor intersection angle between the diamond drill holes and the narrow, sheeted vein system that constitutes the ore in the hanging wall zone. Subsequently, the diamond drill hole data was excluded from both the MR and simulation models, as it was felt that its inclusion would introduce bias and lower the overall grade of the zone. A trial area suggested that the removal of the diamond holes would also close the gap between the MR model, GC model and plant grades.

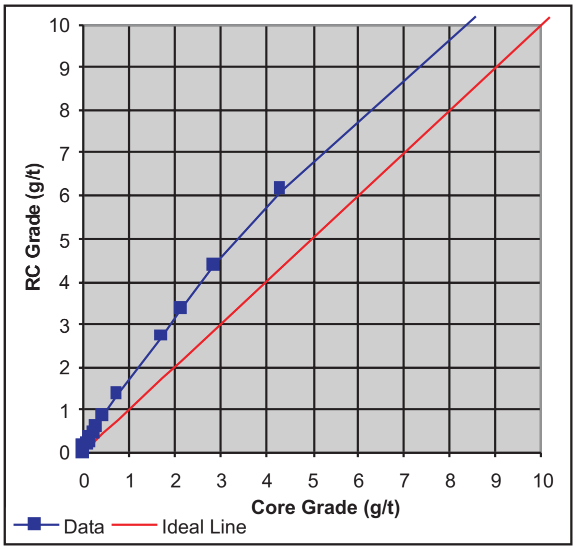


FIG 4 - Q-Q plot of reverse circulation (RC) versus diamond core exploration data for the hanging wall zone, showing that the diamond core data is lower grade than the RC data.

The exploration boreholes, which include both RC and diamond core sampling as well as the GC blasthole sampling, were all composited to 2.5 m lengths, comparable with the ten bench heights. Exploration drill hole spacing is generally 25 m × 25 m, increasing to 25 m × 50 m on the margins of the deposit. Due to high-grade targeting of the exploration drilling, declustering was performed on the exploration data to reduce potential bias caused by over-sampling of higher grade portions of the deposit. A moving window, covering a range of cell sizes,

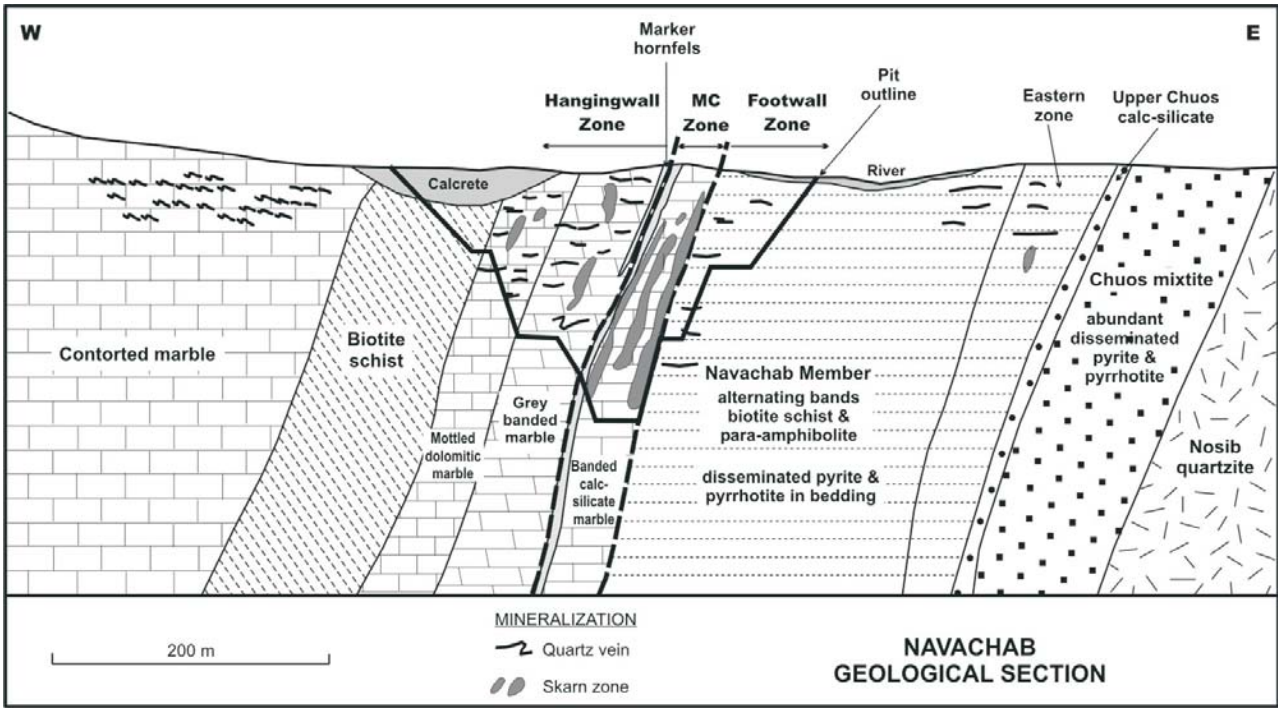


FIG 3 - Schematic geological cross-section of the Navachab gold deposit.

was used to examine which cell size produced the lowest average grade (Isaaks and Srivastava, 1989). The declustered univariate statistics for the five domains are summarised in Table 3. The large number of samples at detection limit also meant that despiking had to be performed, as the normal-score transform required for the conditional simulation must be monotonic. Finally the despiked, declustered grades were transformed to a normal scores distribution, using the declustering weights.

Variograms of the normal scores data were calculated in three directions to account for the spatial variability of the mineralisation. These rotations are shown in Table 4. Directions chosen were consistent with the orientation of the mineralisation. Blasthole GC data was included in the variogram calculation to provide better definition of the short-range structures. Whilst it was recognised that this data was not of the highest quality due to the wide spaced exploration sampling, it was felt that some data was better than no data at all. Lags used for the variogram calculation were consistent with the data spacing of the blastholes. Variograms were modelled using spherical models.

The Navachab block model is 1825 m × 862.5 m × 550 m. Therefore, the 100 realisations required for this exercise would entail a large number of nodes being simulated, a computationally intensive requirement. A small study was therefore run to select the minimum number of nodes realistically required for blocking into the 12.5 m × 12.5 m × 5 m resource block size. The results of this exercise showed that simulating nodes on a grid size of 6.25 m × 3.125 m × 2.5 m, would be sufficient to characterise a resource block. The total number of nodes required to cover the model was therefore 17.73 million.

The validation of the simulation model consisted of checking the reproduction of the input histograms in both normal and real space (Figure 5) and plotting the normal-score variograms from the simulation model against the input variogram (Figure 6). The reproduction of the model was generally excellent with replication of the variograms fair and histograms good.

In addition to the simulation models, a GC model was built from the 2.5 m blasthole samples using ordinary kriging. This model was reconciled against the conditional simulation model over the same volume. The GC model was built on exactly the same block support as the blocked simulation model with the same variogram parameters. However, a shorter search radius was used to avoid negative weights at the extremities of the search. Due to the close spacing of the blastholes, sufficient data to provide a robust estimate was available.

TABLE 3

Declustered univariate statistics for simulation domains.

2.5 m exploration samples					
Domain	MC	HWW*	FWW	HWO*	FWO
Code	1	2	3	4	5
No samples	8121	13 421	20 329	2050	7591
Mean	1.14	0.36	0.29	1.9	1.17
Std Dev	4.02	1.13	0.74	2.61	2.57
CV	3.54	3.1	2.53	1.38	2.2
Maximum	538.3	40.14	17.22	23.08	81.72
Upper quartile	1.17	0.29	0.25	2.08	1.18
Median	0.33	0.11	0.1	1	0.47
Lower quartile	0.1	0.02	0.03	0.51	0.17
Minimum	0.01	0.01	0.005	0.01	0.1

* RC data only.

TABLE 4

Variogram rotations for simulation domains.

Domain Code	MC 1	HWW 2	FWW 3	HWO 4	FWO 5
Bearing Plunge Dip	90 -30 -70	90 0 0	90 0 0	90 -30 0	90 -30 0

VERIFICATION OF SIMULATION AND DISCUSSION OF RESULTS

A straight comparison of tonnes and grade for the GC model, the MR model and the average of the realisations (Table 5) shows that the simulations compare very well with the GC model in the mined-out area at a resource cut-off of 0.4 g/t. On the other hand, the MR model is under on tonnes and over on grade. In the area remaining to be mined, the simulation has considerably more tonnes and grade than the MR model. This suggests that, because of the historical relationship between the MR model and the GC model (Table 1) and the GC model and the mill (Figure 2), there may be considerable upside in the final pit shell determined from the MR model.

The grade-tonnage curves for the simulated realisations and the MR model were compared in the mined-out area with the GC model by domain. The MR model and simulated realisations of the hanging wall and footwall zones generally compared very well with the GC model, supporting the decision to remove the diamond holes from the hanging wall sampling data because of the poor intersection angle of the holes. The simulations compared particularly well with the GC model in the hanging wall. The MR model reconciled less well against the GC model in the hanging wall zone, generating higher grades at cut-off, and better in the footwall zone, reproducing grades and tonnes closer to the GC model than those shown for the simulations (Figures 7 and 8). The grade and tonnage reproduction for the mined-out MC zone was not as convincing as the hanging wall and footwall

TABLE 5

Comparison of material within mined-out and remaining portions of the current design, at a 0.4 g/t cut-off grade.

Mined		Ave sims	Grade control	Resource	Remaining		Ave sims	Resource
MC	Tonnes Grade Ounces	9 699 882 1.90 593 647	9 086 759 2.07 604 742	8 746 783 2.32 652 420	MC	Tonnes Grade Ounces	9 442 925 1.60 482 997	8 188 868 1.90 487 065
HW	Tonnes Grade Ounces	14 528 219 1.02 477 736	15 566 787 1.00 500 484	14 039 557 1.20 541 659	HW	Tonnes Grade Ounces	30 146 506 1.10 1 062 361	24 494 323 1.20 905 637
FW	Tonnes Grade Ounces	3 770 519 1.17 141 560	3 707 669 1.00 119 204	2 997 442 1.14 109 862	FW	Tonnes Grade Ounces	40 312 891 1.20 1 513 390	36 624 230 1.20 1 412 996
Total	**Tonnes Grade Ounces**	**27 998 620 1.35 1 212 943**	**28 361 215 1.34 1 224 430**	**25 783 782 1.57 1 303 941**	**Total**	**Tonnes Grade Ounces**	**79 902 322 1.53 3 058 748**	**69 307 421 1.26 2 805 698**

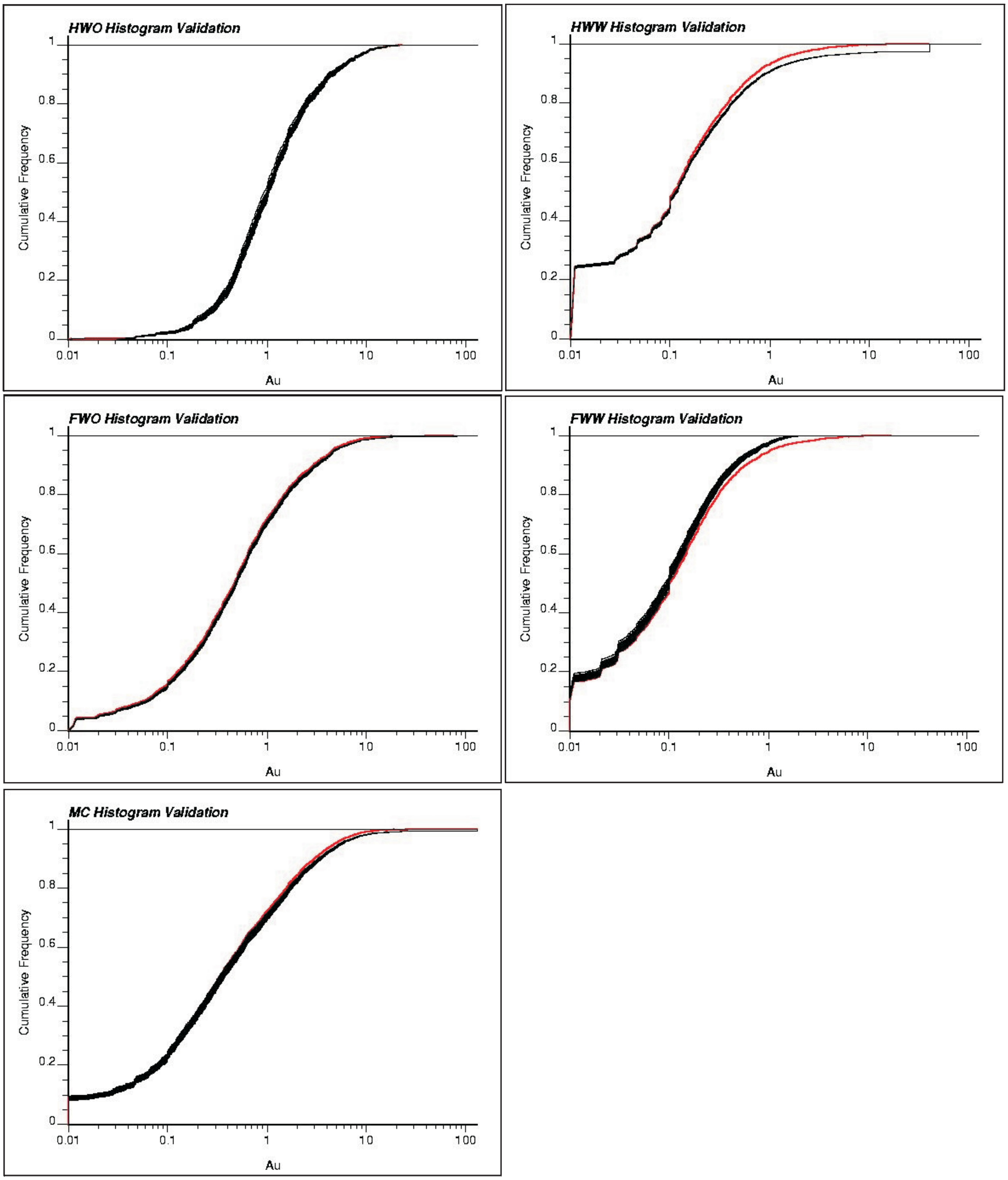


FIG 5 - Statistical validation of the simulated domains.

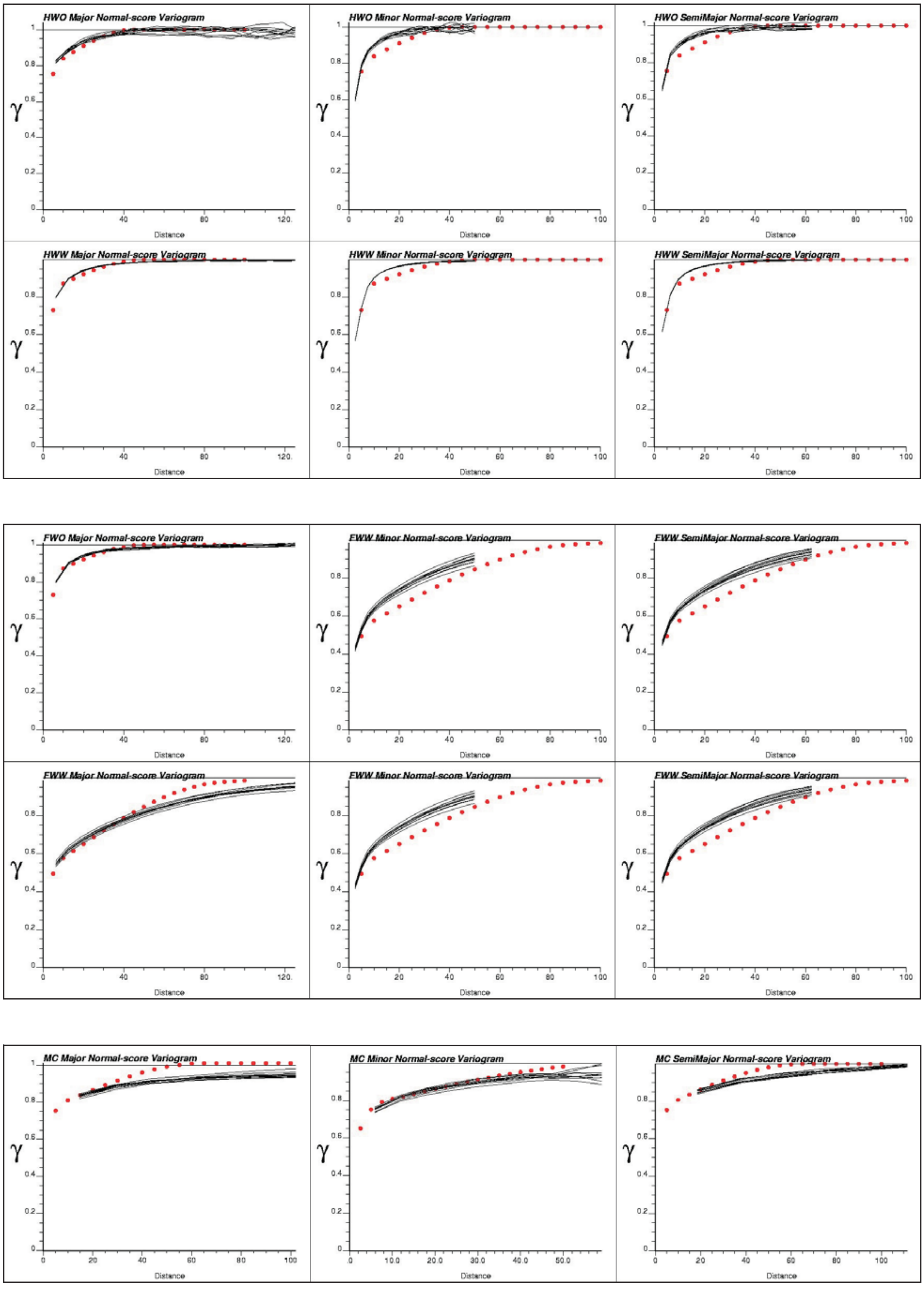


FIG 6 - Spatial validation of the simulated domains.

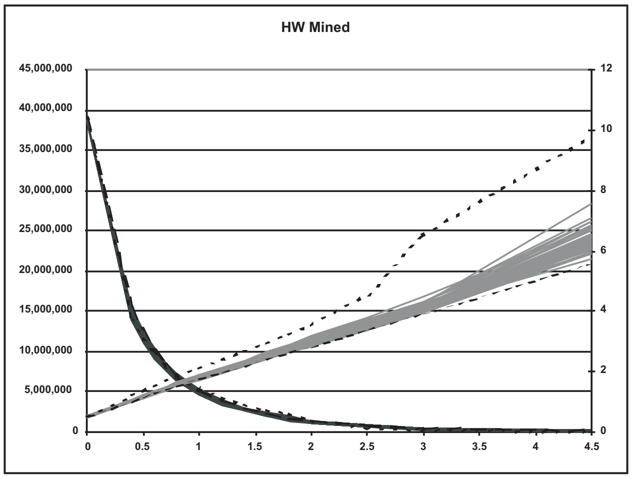


FIG 7- Grade-tonnage curves for the mined-out portion of the hanging wall zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

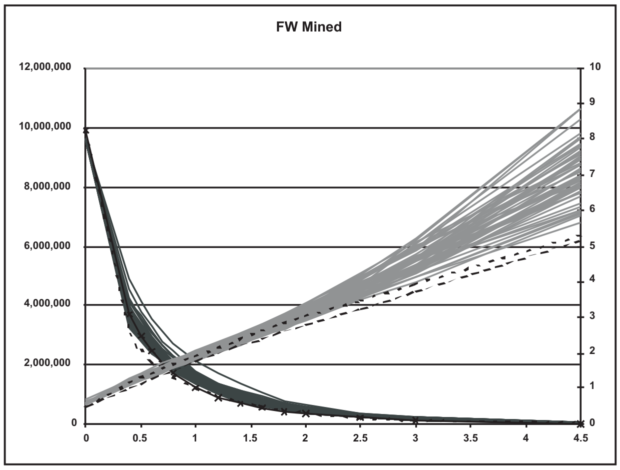


FIG 8 - Grade-tonnage curves for the mined-out portion of the footwall zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

zones for both the simulation and the MR model results, but still fairly good. The MR model is closest at reproducing the GC grade and tonnage, but there appears to be a selectivity problem between the MR model and simulation results that requires further analysis. The MR, GC and simulation results converge around a grade cut-off of 0.5 - 1.5 g/t, which is important for reporting of the mineral resource and ore reserve (Figure 9).

In the unmined areas the simulation and MR models were also compared. The relationship established between the simulations and MR model and GC model in the mined-out areas was used to attempt to predict the potential risk or opportunity in the mineral resource as mining progresses in the future.

In general, the differences between the MR model and simulations are negligible at the cut-offs of interest (Figures 10, 11 and 12). The simulations appear to generate slightly more tonnes in the hanging wall and footwall zones than are produced by the MR model (Figures 10 and 11). In the MC zone the selectivity problem observed in the mined-out areas is repeated, with the simulations producing more tonnes at lower grade in the lower cut-offs and less tonnes at higher grades in the higher cut-offs (Figure 12).

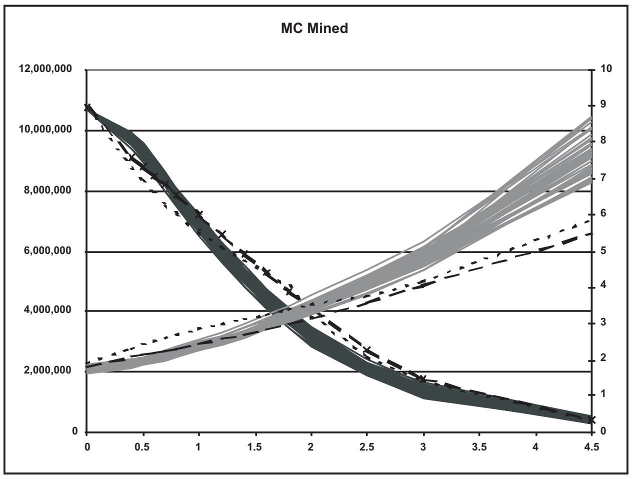


FIG 9 - Grade-tonnage curves for the mined-out portion of the MC zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

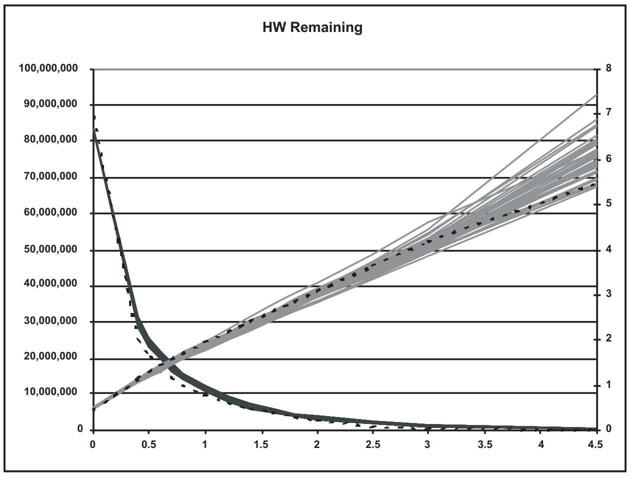


FIG 10 - Grade-tonnage curves for the remaining portion of the hanging wall zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

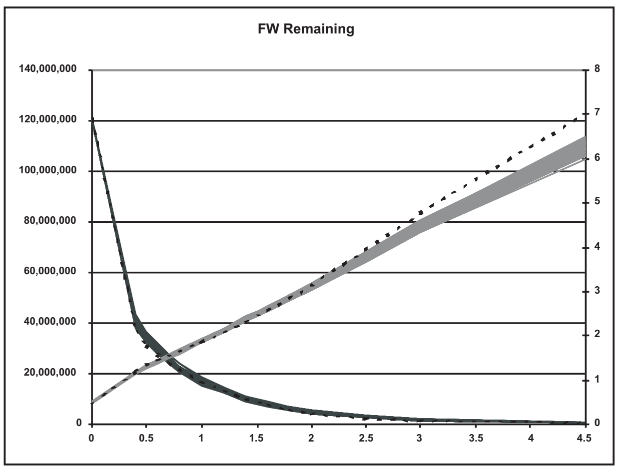


FIG 11 - Grade tonnage curves for the remaining portion of the footwall zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

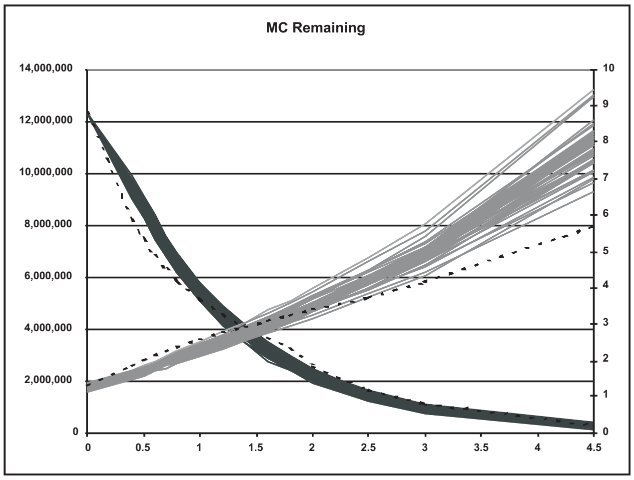


FIG 12 - Grade-tonnage curves for the remaining portion of the MC zone. Legend: simulated tonnes = black lines, simulated grade = grey lines, resource model = dotted lines, grade control model = dashed lines.

On a bench-by-bench basis the main discrepancies are located in the deeper portions of the pit, 1105 - 1145 m/RL, where the simulations show higher tonnes than predicted by the MR model (Figure 13). This suggests a potential upside for the mine and, as it is within the area targeted by the deep expansion, it is a considerable opportunity. The other area that shows opportunity is 915 - 905 m/RL. However, a look at the grade profile for the bench elevations (Figure 14) shows that this result is probably a selectivity issue, with the MR model showing higher grades at lower tonnes.

Historically the Navachab MR model has reconciled fairly well with the mill grades and overestimated the gold content compared with the GC model. However, the comparison with the GC model has been skewed in the past due to problems with the GC process that have now largely been overcome. The effect has not yet been fully realised, however, because of the amount of poor GC data still in the system. Comparison of the simulated realisations with the MR model and GC model can be summarised as follows. In general the simulations correlate well with the GC and MR models. There is a selectivity difference between the simulations and the MR model in the MC zone that might be the result of the information effect. The simulations are predicting more grade and tonnes into the remaining pit than the MR model. Finally, the simulations suggest that there might be a potential upside in the final pit shell compared with the shell created from the MR model.

PIT OPTIMISATION

To quantify the uncertainty in the grade model and its potential effect on the mine design, each of the 100 realisations was optimised to create 100 pit shells (one per realisation), with each pit shell a likely final pit limit based on the equi-probable grade realisation it had been calculated from (Dimitrakopoulos, Farrelly and Godoy, 2002). Optimisation of the simulation realisations was performed using the Lerchs-Grossmann algorithm (Whittle, 1999) and the same optimisation parameters as used for the 2003 Navachab resource model, with a nominal gold price of US$400 per ounce (N$90 per gram). The optimisation parameters are summarised in Table 6.

The optimised simulations produce a range of economic outcomes compared with the single outcome from the traditional mine optimisation. The potential uses of the resultant optimisations are therefore endless. Amongst other things, sensitivities of key project parameters to geological uncertainty can be assessed, potential fluctuations in ore feed to the mill can be anticipated and the effects modelled, and the uncertainty in the final pit limit determined.

In terms of the expansion plans for the mine, the geological uncertainty in the final pit limit was of utmost importance. As a result, the 100 resultant pit optimisation files were collated and

TABLE 6

Pit optimisation parameters.

Slope angle (°)	44
Gold price (N$/g)	90
Exchange rate (N$/US$)	7
Mining cost (N$/t)	6.6
Processing cost (N$/t)	68
Metallurgical recovery (%)	90

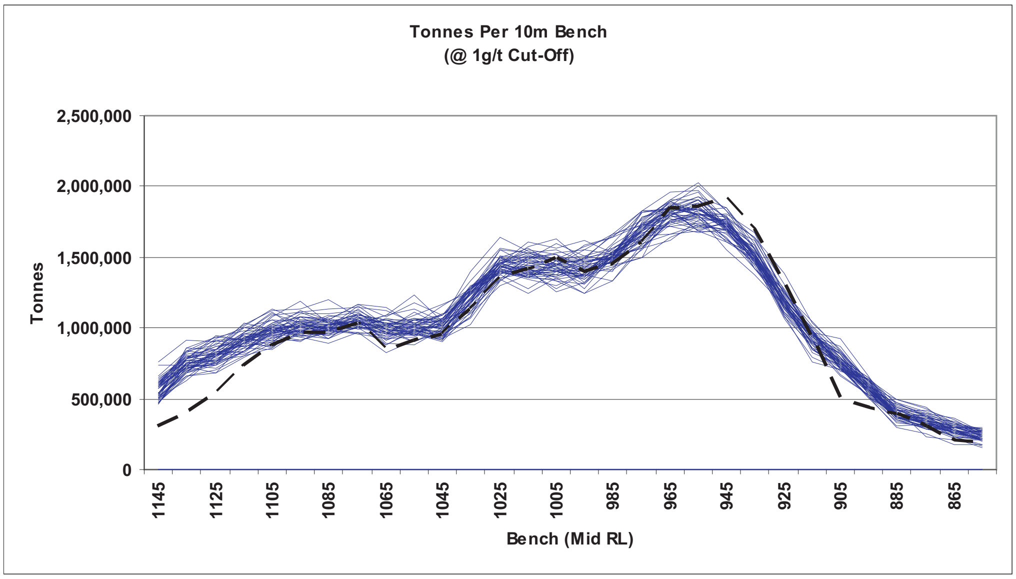


FIG 13 - Tonnes available over 10 m benches above 1 g/t cut-off grade for the remaining portion of the Navachab pit. Legend: simulated tonnes = blue lines, resource model = black lines.

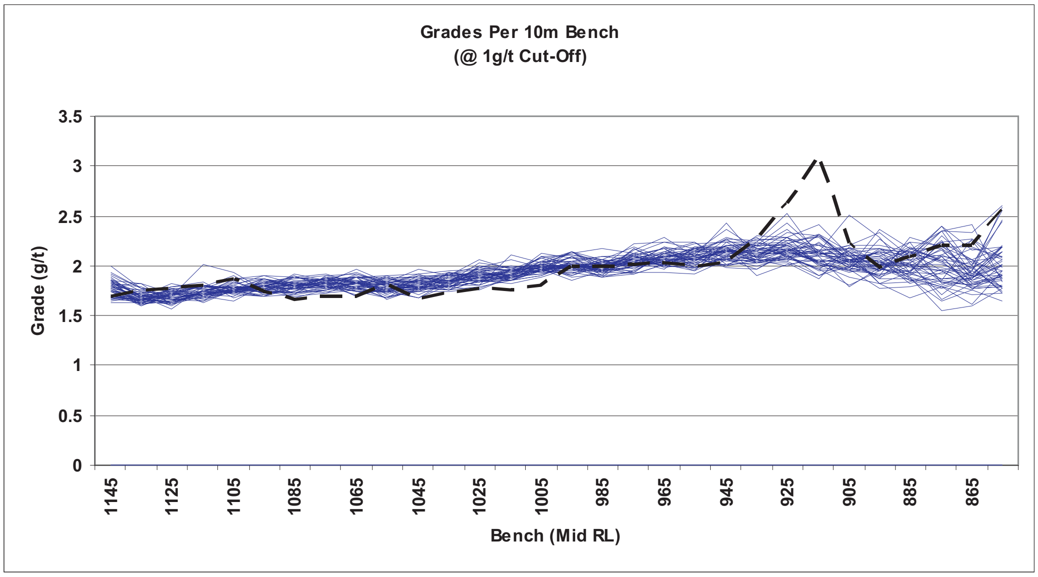


FIG 14 - Grades available over 10 m benches above 1 g/t cut-off grade for the remaining portion of the Navachab pit. Legend: simulated tonnes = blue lines, resource model = black lines.

each block (equivalent to the current selective mining unit) in each realisation was assigned a code depending on whether it was located inside or outside each of the final pit limits. In this way a mining probability map was created, showing the likelihood of each block being mined based on the underlying grade uncertainty (Figures 15 and 16). The resultant mining probability map represents areas of risk and opportunity in the feasibility plan. Blocks of high probability falling outside the current feasibility pit highlight areas of potential loss if excluded, while blocks of low probability in the pre-existing feasibility pit limit highlight areas of potential risk if included. Therefore, in its simplest form, the probability map provides an exploration map for follow-up drilling. It also provides a probabilistic framework that would allow a mine planner to move beyond the traditional single estimated orebody model approach and incorporate geological uncertainty, thereby allowing the risk to be not only quantified but also managed. Ore blocks of low risk and low uncertainty can be mined earlier, and ore blocks of high risk and high uncertainty can be mined later, allowing additional time for exploration and investigation and the generation of a low-risk plan in the short term.

CONCLUSIONS

As part of a pit expansion study at Navachab gold mine, a conditional simulation was used to quantify the uncertainty in the geological model of the deposit. The optimised realisations at Navachab showed that significant 'reserve' potentially exists outside the current final pit shell and that the risk is, therefore, not one of a non-existent mineral resource but rather one of missed opportunity.

The simulation study conducted for Navachab gives confidence that the gold within the pit expansion, as determined by the MR model, can be obtained. The study also gives a good quantification of the grade risks associated with the current pit design at Navachab. The challenge is now to see if this geological model of uncertainty can be incorporated into the current design to better manage the risk, as demonstrated by Ramazan and Dimitrakopoulos (2007, this volume); Godoy and Dimitrakopoulos (2007, this volume) and Jewbali (2006). The uncertainty in ore tonnes mined and milled is of considerable importance to a mining venture. It defines the level of possible fluctuations in ore feed to the mill (Figures 13 and 14), it is

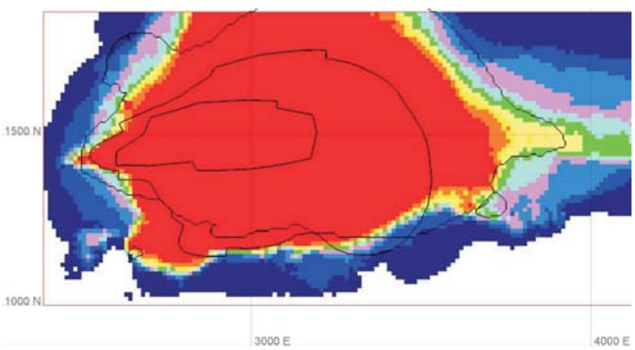


FIG 15 - 1050 m RL plan of the probability of a block being mined. Legend: hot to cold, with red (dark grey) being 100 per cent probability of being mined. Black lines are the outline of current pit, life/limit-of-mine pit and pit expansion feasibility pit.

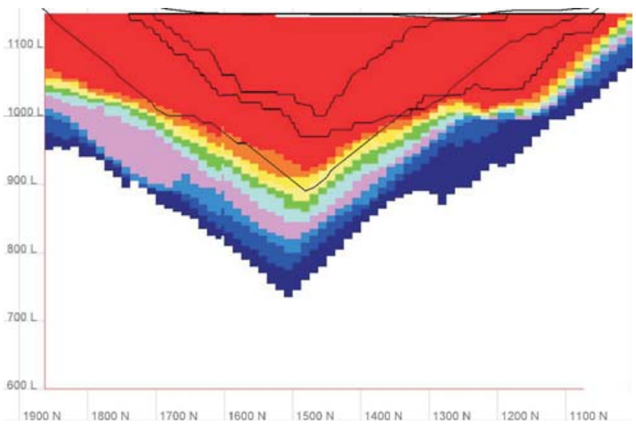


FIG 16 - 2800 east-west section of the probability of a block being mined. Legend: hot to cold, with red (dark grey) being 100 per cent probability of being mined. Black lines are the outline of current pit, life/limit-of-mine pit and pit expansion feasibility pit.

critical for the risk profiling of a mine, and it is central to the attainment of a better financial understanding of the capabilities of the orebody for planning purposes.

The comparison of the simulations and MR model with the GC model noted that the simulation results were similar to the MR model, with the biggest difference existing in the MC zone where there is a difference ascribed to selectivity. The importance of the MC zone to mine feasibility suggests that this selectivity issue needs to be investigated in more detail. Accurate recoverable reserve estimation is critical for the financial well-being of any mining operation. The challenge for the future is to use the information provided by the simulations to incorporate the effect of more and better information into the mineral resource estimate.

ACKNOWLEDGEMENTS

Romulo Sanhueza from AngloGold Ashanti for his assistance in running the Whittle optimisations.

REFERENCES

Chilès, J-P and Delfiner, P, 1999. *Geostatistics, Modeling Spatial Uncertainty*, p 695 (John Wiley and Sons: New York).

Dimitrakopoulos, R, 1998. Conditional simulation algorithms for modelling orebody uncertainty in open pit optimization, *International Journal of Surface Mining, Reclamation and Environment*, 12:173-179.

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Dowd, P, 1994. Risk assessment in reserve estimation and open-pit planning, *Trans Inst Min Metall*, Section A, Mining Technology, 103:A148-A154.

Godoy, M and Dimitrakopoulos, R, 2007. Managing risk and waste mining in long-term production scheduling of open pit mines, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 337-343 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation*, p 483 (Oxford University Press: New York).

Isaaks, E H and Srivastava, R M, 1989. *An Introduction to Applied Geostatistics*, p 561 (Oxford University Press: New York).

Jewbali, A, 2006. Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines, PhD thesis, The University of Queensland, Australia, 280 p.

Millar, R McG, 1983. *The Pan-African Damara Orogen of South West Africa/Namibia*, Geological Society of South Africa, Special Publication, 11:431-515.

Moore, J M and Jacob, R E, 1988. The Navachab sheeted-vein/skarn Au deposit, Namibia, 23:A125, Geological Association Canada/ Mineralogical Association Canada, Quebec.

Peattie, R J, Badenhorst, F P and Chamberlain, V A, 2004. Mineral resource statement for December 2003, unpublished company report, Anglogold Namibia Pty Ltd.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ravenscroft, P J, 1992. Risk analysis for mine scheduling by conditional simulation, *Trans Inst Min Metall (Section A)*, 101:A101-108.

Whittle, J, 1999. A decade of open pit mine planning and optimisation – the craft of turning algorithms into packages, in *Proceedings APCOM '99*, pp 15-24 (Colorado School of Mines: Golden).

Orebody Modelling, Mine Planning, Reserve Evaluation and the Regulatory Environment

J-M Rendu[1]

ABSTRACT

In all major financial markets mining companies are required to publicly report Mineral Reserves and, in most circumstances, Mineral Resources. The objective is to give investors reliable information that can be used to make sound decisions with clear understanding of the risk inherent to investing in the mining sector of the financial market. Regulators have developed rules aimed at ensuring that the information being reported is accurate and relevant. The codes and guidelines that must be followed to model deposits, develop mine plans and demonstrate economic feasibility are nearly, but not exactly, identical in all major financial markets. Generally accepted definitions and guidelines are reviewed. Mining companies must put in place processes which ensure that resources and reserves are estimated and reported as required by regulators and as expected by investors. The role of the Competent Person is analysed. Sources of errors are investigated, with recommendations on how to minimise such errors or mitigate their effect. The importance of specialists other than the Competent Persons is discussed. A manager of resources and reserves should be nominated, whose primary responsibility is to establish, maintain and assess the effectiveness of an adequate internal control structure and procedures for resource and reserve estimation and reporting.

INTRODUCTION

Mineral resources and mineral reserves are calculated and reported for a number of reasons, including technical, financial, legal and regulatory. The methods used by mining companies to model deposits, develop mine plans, evaluate mining properties, and report resources and reserves must take these reasons into account. Processes must be developed and followed to assess uncertainty inherent to estimation of reserves and to define and manage associated risk. In this paper the regulatory environment is discussed, the sources of uncertainty are reviewed, and procedures and controls are proposed which can be used to manage corporate and project risk.

THE REGULATORY ENVIRONMENT

International definitions

Guidelines for the evaluation of mineral deposits go back to pre-Roman times, with the first modern document being arguably *De Re Metallica* published in 1556. Over the last 20 years there was a concerted effort towards development of international standards to define and report mineral resources and mineral reserves. Key dates are listed in Table 1. Australia, with the JORC Code, was at the forefront of this effort, followed by the United States, Canada, South Africa, the United Kingdom and more recently Chile.

The generally accepted framework for classifying Exploration Results, Mineral Resources and Mineral Reserves is illustrated in Figure 1. This figure is an integral part of the JORC Code and other international codes and guidelines. A Mineral Resource must be classified as Measured, Indicated or Inferred, a Mineral Reserve as Proved or Probable. These classifications reflect different levels of geological confidence and different degrees of technical and economic evaluation. A Mineral Resource is estimated mainly on the basis of geoscientific information with some input from other disciplines. A Mineral Reserve, which is a modified subset of a Measured or Indicated Mineral Resource, requires consideration of all factors affecting extraction, including mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors and should in most instances be estimated with input from a range of disciplines. In most countries, resources and reserves must be estimated and classified by a Competent or Qualified Person.

The generally accepted definition of a Mineral Resource is as follows:

> *A 'Mineral Resource' is a concentration or occurrence of material of economic interest in or on the Earth's crust in such form, quality and quantity that there are reasonable prospects for eventual economic extraction. The location, quantity, grade, continuity and other geological characteristics of a Mineral Resource are known, estimated or interpreted from specific geological evidence and knowledge. Mineral Resources are subdivided, in order of increasing geological confidence, into Inferred, Indicated and Measured categories.*

When estimating a mineral resource, particular attention must be given to the following:

- There must be reasonable prospects for eventual economic extraction. What is 'reasonable', and what time frame is implied by the term 'eventual', is subject to interpretation.

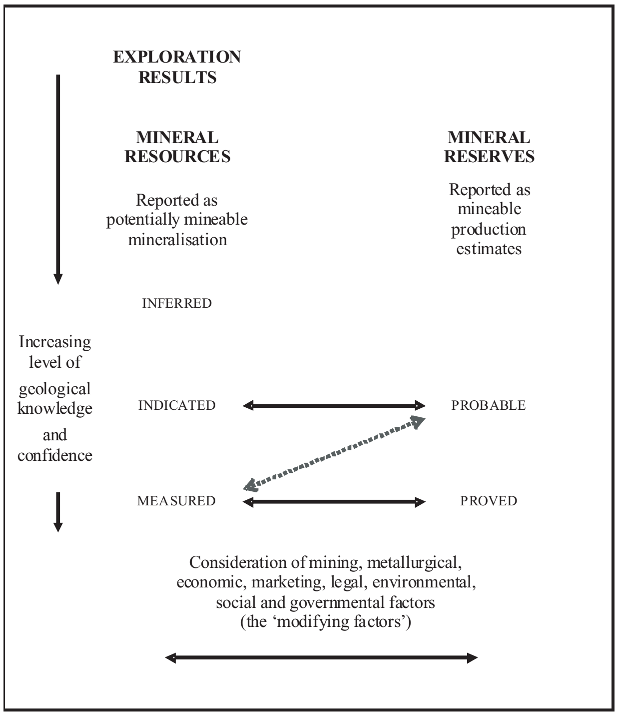


FIG 1 - Relationship between exploration results, mineral resources and mineral reserves.

1. FAusIMM, Mining Consultant, 5451 South Geneva Street, Englewood CO 80111, USA. Email: JMRendu@aol.com

TABLE 1

Development of reporting standards: a time line.

Date	Event
1556	Georgius Agricola publishes *De Re Metallica*.
1909	Herbert Hoover publishes *Principles of Mining*.
1976	US Bureau of Mines and US Geological Survey publish Bulletins 1450-A and 1450-B.
1980	The US Bureau of Mines and US Geological Survey publish Circular 831, 'Principles of a Resource/Reserve Classification for Minerals'.
1983	The US SEC Form S-18 is in effect, later followed by Industry Guide 7.
1989	The 'Australasian Code for Reporting of Identified Mineral Resources and Ore Reserves' (The JORC CODE) is published and included in the ASX rules.
1991	The US SME publishes 'A Guide for Reporting Exploration Information, Resources and Reserves'.
1993	The Mineral Resources/Reserves International Definitions Working Group (The CMMI Group) is formed, with representatives from AusIMM, CIM, SAIMM, IMM and SME.
1994	First meeting of the CMMI Group.
1997	Denver Accord.
1997	Bre-X scandal.
1998 - 1999	Meetings between the CMMI Group and the United Nations ECE.
2000	The South African SAMREC Code is published and accepted by the JSE.
2001	The Canadian Securities Administrators issue NI 43-101: 'Standards of Disclosure for Minerals Projects'. The UK and European 'Reporting Code' is accepted.
2002	CRIRSCO replaces the CMMI Group. Chile joins CRIRSCO.
2003	Chile develops their reporting code, taking into account international guidelines.
2004	The US SME starts a new initiative to resolve resources and reserves reporting issues specific to the US.

- Portions of a mineral deposit that do not have reasonable prospects for eventual economic extraction must not be included in a Mineral Resource.
- The location, quantity, grade, continuity and other geological characteristics of a Mineral Resource must be known, estimated or interpreted from specific geological evidence and knowledge. A mineral resource must be based on data interpretation supported by facts, not speculation.
- The Competent Person must classify resources as Measured, Indicated or Inferred. This classification is subject to interpretation and relevant experience is critical.

For Mineral Reserves the following definition is accepted:

> *A 'Mineral Reserve' is the economically mineable part of a Measured and/or Indicated Mineral Resource. It includes diluting materials and allowances for losses, which may occur when the material is mined. Appropriate assessments, which may include feasibility studies, have been carried out, and include consideration of and modification by realistically assumed mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors. These assessments demonstrate at the time of reporting that extraction could reasonably be justified. Mineral Reserves are subdivided in order of increasing confidence into Probable Mineral Reserves and Proved Mineral Reserves.*

For reserve estimation the key conditions include:

- Only a Measured and/or Indicated Mineral Resource can be converted to a reserve. This requirement is set to reduce the geologic risk involved in developing a deposit. It is meaningful only to the extent that the difference between Measured and/or Indicated and Inferred Mineral Resources is understood.
- Appropriate assessments, which may include feasibility studies, have been carried out. These assessments demonstrate at the time of reporting that extraction could reasonably be justified. Questions remain as to what is 'appropriate' and how to interpret 'at the time of reporting'.
- Realistically assumed mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors – known as 'modifying factors' – must be taken into account. The term 'realistic' is subject to interpretation.
- Evaluation of the modifying factors requires a multidisciplinary approach, with the inherent risk presented by the need for communication between experts from very different disciplines.
- The Competent Person must classify reserves as Proved or Probable. This classification is subjective and relevant experience is critical.

The regulatory environment in major financial markets

An international group, the Combined Reserves International Reporting Standards Committee (CRIRSCO), coordinates the development of standards for the definition and reporting of resources and reserves. Most English-speaking countries that have a large financial market on which mining stocks are traded, have accepted the international standards developed by CRIRSCO. In Australia the industry developed the JORC Code, which is now an integral part of the ASX reporting rules. This code has been in effect for 15 years with the most recent version expected to become effective in 2004. Exploration and mining companies listed in Canada must follow reporting rules specified in NI 43-101. In South Africa the SAMREC code was approved at the end of 2000, which specifies reporting requirements on the Johannesburg Stock Exchange. In Europe the Reporting Code has been accepted by a number of professional societies but has not been incorporated in stock exchange reporting rules. In 2003 Chile joined CRIRSCO and started development of guidelines based on international standards.

For a number of reasons the situation in the United States is still under development. The Society for Mining, Metallurgy, and Exploration, Inc (SME) is a founding member of CRIRSCO and supports the international effort towards standardisation. The SME Guide for Reporting Mineral Resources and Mineral Reserves, first published in 1991, is compatible with JORC, NI 43-101, SAMREC and the Reporting Code. However, this guide is not accepted by the US Securities and Exchange Commission (SEC) and some SEC rules, based on Industry Guide 7, are not compatible with internationally accepted guidelines. In addition, generally accepted US accounting practices (GAAP) are not entirely compatible with rules set by the SEC for the reporting of reserves and resources. Five main issues have been recognised:

- Commodity prices: which price should be used for reserve estimation and reporting?
- Mineral resources: what should be the definition and conditions for publication of mineral resources not included in reserves?
- Economic feasibility: what level of feasibility study should be completed before a reserve can be publicly reported?
- Permitting: which permitting and legal requirements should be satisfied before a reserve is published?
- Industry quality control and self regulation: can we and should we implement a Competent Person requirement in the US legal environment?

To resolve these issues a project was initiated by SME in February 2004 to be completed in March 2005. A group of 15 mining companies and five consulting companies was formed, called the SEC Reserves Working Group, whose mission is:

- to define industry position with respect to issues listed above;
- to reach agreement with the SEC concerning same issues; and
- to update the SME Guide, keeping compatibility with international standards while incorporating requirements specific to the SEC.

DEPOSIT MODELLING AND EVALUATION OF MINERAL RESOURCES

Why should mineral resources be estimated and publicly reported?

Mineral resources are estimated to inform company management of the characteristics and potential economic value of the assets controlled by the company. Evaluation of mineral resources represents the first and most important step in determining whether an exploration property has the potential of being mined economically, and in developing the foundation on which a mining project could be based. Resources are disclosed to help investors develop a reasonable estimate of the fair value of the exploration or mining property and of the company which owns the property.

Role of Competent Person

The Competent Person plays a critical role in controlling and assessing the quality of a resource. He/she takes responsibility for drilling, sampling and assaying methods, drill hole surveying, interpretation of exploration information and development of a geologic model, as well as the choice of a geostatistical method and development of a grade model.

The Competent Person also takes responsibility for development of an appropriate structural and geotechnical model. Once the geologic parameters that define metallurgical recovery and plant throughput have been defined, he/she may be required to develop a metallurgical model. Waste characterisation, including three-dimensional modelling of the acid generating potential of different material types, may also be the responsibility of the Competent Person.

The Competent Person needs to understand the difference between a model that represents the properties of the entire deposit, and a Mineral Resource that represents only that part of the deposit which has reasonable prospects for eventual economic extraction. He/she must assess the reasonableness of assumptions made to identify the prospects for economic extraction. These assumptions include a commodity price usually specified by management, mining assumptions defined by mining engineers, recovery assumptions specified by metallurgists and general assumptions concerning operating and capital costs.

The Competent Person must classify a Resource as Measured, Indicated or Inferred. This classification is mostly based on geologic confidence, including confidence in continuity of mineralisation. Experience and knowledge of accepted practices in similar geologic environments are critical in making this determination.

Errors in resource evaluation and reporting

Sources of errors

Errors made in resource evaluation can be classified as follows:

- data errors;
- data interpretation and modelling errors;
- misclassification of mineralised material as a Mineral Resource; misclassification as Measured, Indicated or Inferred;
- poor or lack of communication between disciplines;
- fraud; and
- reporting errors.

Data errors

In resource evaluation the most significant errors are introduced when data is being collected to characterise the deposit, such as during sampling, sample preparation and assaying. Number and location of sample points must be such that form, quantity and quality of mineralisation can be reasonably assessed.

Sampling errors can result from using inappropriate drilling methods, downhole contamination, excessive use of drilling fluid or excessive inflow of underground water, over or under-sampling of high- or low-grade fines, use of sample bags that are too small or too porous and biased sample splitting methods. Laboratory errors can result from contamination, inappropriate assaying method and poor quality assurance and quality control practices. Inspecting drill sites and following samples from drill hole to laboratory and beyond are part of the duties of the Competent Person. Comparing results obtained using different drilling methods, performing size fraction analyses, reviewing laboratory QA/QC procedures and reports, introducing blank and duplicate samples and analysing the results using appropriate statistical methods are some of the methods that can be used to determine where errors may be introduced. Data transfer errors are reduced by automation but can still occur.

Consistency and appropriateness of drill hole logging methods are necessary to develop a meaningful geologic model. Communication between geologists, mining engineers and metallurgists must be maintained to ensure that all relevant deposit properties are measured and logged. Biases in drill hole locations, such as excessive concentration of holes in known high-grade zones, should be avoided. Surveying errors are commonly encountered even in surface operations. The accuracy of drill hole location and downhole surveys must be ascertained.

Interpretation and modelling errors

A geologic model is needed before a grade model can be developed. During the early stages of exploration a simplified geologic model may be justifiable, but the size of the mineralised zones within which grades are estimated must be reasonable and supportable. If the extent of these zones is underestimated so will be the economic potential of the deposit. If this extent is overestimated the results may be misinterpreted by management and could even be viewed as an attempt to mislead investors.

Zone definition must take into account geologic controls which influenced mineralisation, such as material types, veins, dykes, faults, folds, limits of oxidised, enriched or depleted zones, etc. Zone definition must also take into account the method that will be used for grade modelling. Tightly defined zones are generally best suited if the block values are to be estimated from a small number of samples. Much wider zones may be applicable for methods such as Multiple Indicator Kriging (MIK), which requires a large number of samples to estimate each block.

There are many reliable computer programs that can be used to model deposits. However, programs are only tools and the user's expertise is critical in developing reliable models. Some deposit modellers have only a rudimentary understanding of the basic principles of geostatistics and of the underlying assumptions that must be satisfied to justify the use of a specific modelling method. It is not uncommon to hear managers stating that the model must be accurate 'because geostatistical methods were used'. Overconfidence in poorly understood geostatistical methods is a common source of modelling errors.

Choosing a modelling method requires judgement based on experience. The amount of information available is a critical factor. When only a few holes have been drilled a simpler method should be used, such as Nearest Neighbour (NN), Inverse Distance ($1/D^n$) or even a manual method. When more information is available increasingly complex methods may be applicable such as kriging, indicator kriging, MIK, etc. The choice of a modelling method must take into account the modeller's expertise, sample spacing, complexity of grade distribution and the mining method likely to be used.

One of the most difficult questions concerns the treatment of extreme sample values. Should such values be cut and if yes at which level? Should their distance of influence be restrained and if yes how do we do it? Comparison of results obtained using different methods, such as NN and MIK, helps avoid major modelling errors.

In deposits where high-grade veins were mined by selective underground method and consideration is now given to large-scale open pit mining, the proper use of historical data can be difficult to ascertain. Underground samples tend to be clustered around workings and their location is biased, targeting high-grade veins. Using these samples without declustering and accounting for location biases can result in significant overestimation. Sample locations can also be biased when surface holes are drilled preferentially to target known high-grade zones.

Modelling methods must be chosen, taking into account the mining method likely to be used. For underground mining, geological and grade models must emphasise continuous high-grade zones. For open pit mining all mineralisation of potential economic interest must be modelled, including sub-economic mineralisation likely to be within the pit limits. When modelling disseminated mineralisation, assumptions must be made concerning the level of mining selectivity which will be applicable.

Classification errors

Once a reliable deposit model has been developed, the question remains: what should be reported as a Mineral Resource? By definition, a Mineral Resource is not everything in the ground but only that part of the deposit for which there are reasonable prospects for eventual economic extraction. The Competent Person must decide which criteria should be used to define such prospects. This is likely to require a scoping or conceptual study, based on assumptions concerning mining method, metallurgical recovery, order of magnitude operating and capital costs and commodity prices. Information must be supplied by management, metallurgists and mining engineers. There is no requirement that a metallurgical process has been demonstrated, or that extraction is economic at the time of reporting. However, reasonable and supportable assumptions must be made under which economic extraction would be possible. Identifying these assumptions and the steps that are intended to (or could) be followed to prove or disprove their validity goes a long way towards documenting the reasonableness of a published resource. A cut-off grade must be calculated before a resource is reported. Economic criteria must be considered, including the likelihood that the deposit will be of sufficient value to pay for the capital cost required to develop the project. Other criteria, such as legal or environmental, must be taken into account to confirm that there are reasonable prospects of eventual economic extraction.

The Competent Person is not only faced with the responsibility of defining what is or is not a Resource. He/she must also classify the Resource as Measured, Indicated, or Inferred. This classification takes into account mostly geologic considerations, including variability in grade, drill hole spacing and level of understanding of the geologic environment. There are accepted definitions of what makes a Measured, Indicated, or Inferred Resource, but applying these definitions requires judgement and the Competent Person's experience with the type of mineralisation being studied is critical.

Communication

Poor communication between disciplines can cause significant errors in the estimation and reporting of resources. Resource estimation requires relevant expertise in geology and deposit modelling. It also requires input from other disciplines, including metallurgical and mining engineering. Geologist, modeller, mining engineer and metallurgist must have a common understanding of what is relevant. Papers have been published in which: geology is described in minute details reflecting small-scale controls (from micron size to metre size); a selective mining method is chosen on the assumption of medium-scale continuity in grade (three to ten metres); and the deposit model takes into account only large-scale characteristics (hundreds of metres).

Resource estimation is always an iterative process, not only because new geologic information becomes available – as a result of new drilling or revised interpretation of old data – but also because the requirements formulated by those who use the model change over time. The mining engineer may need to know the impact on tonnage and grade of changes in open pit or underground mining selectivity. Geotechnical requirements change in function of the expected location of pit walls or underground openings. Metallurgical studies may indicate the need to develop a detailed deposit model from which mill throughput and recoveries can be assessed. The need for characterisation of waste dumps and low-grade stockpiles may only be recognised late in the study of a project. New holes may have to be drilled to obtain fresh samples usable for environmental or metallurgical testing.

There is always a risk of poor communication between geologist and other experts, with possible expensive consequences. A typical example is where a metallurgist asks for 'typical high-grade samples' without explanation of what will be done with these samples. Such samples are supplied by the geologist and subsequently used for detailed metallurgical testing and preliminary process design. Only months later is it realised that the typical samples were indeed high-grade, but represented only a small proportion of the total material likely to be processed. New samples must then be collected and new tests performed, resulting in a new metallurgical process being developed after considerable wasted time and expense.

Fraud

Fraud is the extreme case, where resource estimates differ from reality. The best (or worse) example is that of Bre-X and the Busang deposit in Indonesia. The weekly announcements made by Bre-X could have been written in accordance with the JORC Code or any other reporting code. However, the published information was nothing but a figment of someone's imagination. This example illustrates the need to be sceptical, to ask the right questions, to properly audit any estimate, to be wary of situations where confidentiality is used as a reason for not releasing material information and to remember that how information is published – while relevant – is not an indication of the quality of the information.

Bonus systems are often in place, which reward publication of high resource and reserve numbers. Those benefiting from such bonuses may include senior management as well as the Competent Person responsible for resource and reserve estimation. Such bonuses create conflicts of interest. They should be eliminated or, as a minimum, processes and controls should be put in place to ensure that they do not result in fraud.

Reporting errors

Even if resources are estimated properly, additional issues are raised when publicly reporting these resources. Applicable reporting rules, codes and guidelines must be understood and followed. These rules vary between countries, a situation which complicates matters for multinational companies. After publication of resources there is a risk that the reader will misinterpret the meaning of the published information. The most common misinterpretation consists in equating Mineral Resources with Mineral Reserves. It is recommended that when a resource is published, a statement is made that feasibility of economic extraction has not been and may never be demonstrated. Depending on materiality, a mining company may choose to (or have to) explain the reasons why a mineralisation is reported as a Resource but not a Reserve. Questions may be raised by regulators or investors, which the reporting company should be ready to answer. A recommended approach consists in defining in advance and documenting the steps that should be followed to determine whether the Resource can be converted to a Reserve. Such steps may include additional drilling, metallurgical testing or economic evaluations which are scheduled to take place in the foreseeable future.

ISSUES IN MINE PLANNING AND RESERVE EVALUATION

Role of mine planning in reserve evaluation

Once a Mineral Resource has been defined, a number of conditions must be satisfied and studies must be completed before a Mineral Reserve can be declared. First, a sufficiently large part of the Resource must have been classified as Measured or Indicated. Only this material can be reported as a Reserve. Then studies must be completed to demonstrate which part of this resource, if any, can be mined economically. Mining, metallurgical, economic, marketing, legal, environmental, social and governmental modifying factors must be taken into account.

The first modifying factor is 'mining'. A mining engineer must determine which part of the Measured and Indicated Resource can be mined both technically and economically. The chief mining engineer responsible for the project will typically be asked to sign as a Competent Person who takes responsibility for the reported reserve.

Role of Competent Person

The Competent Person is directly responsible for determination of the mining method applicable to the deposit, development of mine plans and mine production schedules, estimation of equipment, staffing and support facilities needed to meet these schedules and determination of mining capital and operating costs. Mining engineers are trained to perform these tasks. However, to evaluate a reserve a Competent Person must depend not only on his/her own expertise as a mining engineer but also on multidisciplinary information received from others. This information can be classified as follows:

- Geologic information: deposit model, possibly including deposit simulations; structural and geotechnical information, hydrological information.
- Metallurgical information: metallurgical properties of the deposit by material type or for each block in the deposit model. These properties may include recovery, concentrate grade, mill throughput, deleterious elements, etc.
- Environmental information: technical, legal and ethical constraints on dewatering, waste characterisation, waste management, stockpiling of low-grade material, noise and air pollution, reclamation, etc. This information will have an impact on mining method, planning, productivity and costs.
- Cost information: in addition to mining operating and capital costs, reserve estimation requires milling, infrastructure, administrative and other operating and capital costs, which must be taken into account to demonstrate the economic feasibility of the project and to optimise mine plans.
- Revenue information: information is needed to determine the value of the product sold. This may include commodity price assumptions, sales agreements, penalties, marketing information, etc.
- Other modifying factors: legal, socio-economic, permitting and governmental factors must be taken into account to the extent that they may constrain the mine's technical and economic operating conditions.
- Evaluation criteria: economic feasibility must be demonstrated before a reserve can be declared. For new projects this is generally interpreted as a positive net present value of future cash flows. An acceptable discount rate must be defined. Assumptions must be made concerning inflation rates and exchange rates. Royalties, taxes and other charges must be included. Each mining company has its own investment criteria which must be taken into account to ensure compatibility between reserves intended to be mined and publicly reported reserves. If different evaluation criteria is used for properties intended to be sold, as opposed to mined by the reporting company, these differences should be justified and documented.

The Competent Person must have not only expertise in mining technology but sufficient relevant experience to identify the need for and the appropriateness of information supplied by others. When writing a technical report, the Competent Person should identify information received from others including the name of those responsible for the information. Supporting documentation should be included. Critical information that the Competent Person uses but does not have the expertise or authority to review should be brought to the attention of the reader.

The Competent Person must not only convert resources to reserves but also classify Mineral Reserves as Proved or Probable. This classification is mostly made on the basis of the Resource classification. A Measured Resource is generally converted to a Proved Reserve, an Indicated Resource to a Probable Reserve. However, the risk associated with economic exploitability of the resource must be taken into account. A Measured Resource may be classified as a Probable Reserve if information concerning some of the modifying factors is considered lacking, for example if processing this material presents particular challenges.

Errors in reserve evaluation and reporting

Sources of errors

The sources of errors in reserve evaluation can be classified as follows:

- data errors, including errors in information received from others;
- interpretation errors, including errors in mine design, production scheduling, equipment selection, mine capital and operating costs;
- errors in economic feasibility study;
- misclassification of a Mineral Resource as a Mineral Reserve; misclassification of a Mineral Reserve as Proved or Probable;
- poor or lack of communication between disciplines;
- fraud; and
- reporting errors.

Data errors

Errors can be found in information used by the Competent Person but received from others. Validating this information demands expertise which may be outside that of the Competent Person. The deposit model falls within this category, as well as hydrological, environmental and metallurgical information, processing plant throughput and recovery, capital and operating costs other than those directly related to mining and other items of a similar nature. For this type of information, the Competent Person must rely on the expertise of others.

Errors can also be found in information for which the Competent Person is directly responsible, and whose assessment is within the Competent Person's expected field of expertise. Geotechnical information, information concerning mine equipment performance and operating conditions, mining costs and staffing requirements, information concerning computer programs applicable to mine design and production scheduling, fall within this category.

Interpretation and design errors

Even if the information available to the Competent Person is accurate and complete, errors can be made in the analysis and interpretation of this information, including choice of mining method and equipment, interpretation of geotechnical parameters (including slope design and design of underground openings), estimation of mine productivity, development of mine plans and production schedules and estimation of mining capital and operating costs. The likelihood of errors can be controlled by ensuring that appropriate information is obtained by sampling, testing and analysis and that the Competent Person has appropriate expertise.

Even in the best situations, errors will be made because of uncertainty concerning controlling parameters. The sources of uncertainty can be classified as follows:

- Spatial uncertainty: The characteristics of the deposit are estimated from limited information. These characteristics vary in a three-dimensional space but are mostly fixed in time.
- Temporal uncertainty: The conditions which will prevail when the mine operates will be different from current and forecast conditions.
- Technical uncertainty: Even if we had perfect spatial and temporal information, the systems we design – including mining, processing and human systems – will perform in ways other than as planned.

Errors in economic feasibility study

Errors in the economic feasibility of a project come first from errors in estimated capital and operating costs, which may include optimistic underestimation or omission of cost elements by oversight. Overestimation of productivity and improper risk assessment are contributing factors. Feasibility study also requires forecasting reasonable and supportable future marketing conditions and commodity prices. Such forecast is somewhat subjective and may be biased. As a minimum, the method used to estimate future prices should be documented and justified. External audits of feasibility studies are highly recommended.

Classification errors

Before reporting a Mineral Reserve, the Competent Person must demonstrate that at the time of reporting extraction could reasonably be justified. Demonstration of reasonableness implies risk assessment. Geostatistical simulation of mineral deposits, computer assisted mine design and Monte-Carlo simulation can be combined to develop optimal mine plans, decreasing risk by assessing the adaptability of various plans to changing operating conditions.

Reserves must be subdivided in order of increasing confidence into Probable and Proved Reserves. This subdivision is based primarily – but not only – on classification of the underlying resource as Indicated or Measured.

Communication

Poor communication between disciplines can cause significant errors in the estimation and reporting of reserves. Reserve estimation requires understanding of all parameters that have a significant impact on the economic feasibility of the project. The Competent Person must understand the assumptions made by others, the meaning of the information received, the strengths and weaknesses associated with this information. He/she must be aware of factors such as legal and permitting issues that may prevent the publication of a Mineral Reserve. There is a constant need for interdisciplinary communications and feedback between experts to minimise the chance that relevant information is not transferred, ignored, or incorrectly interpreted.

Fraud

Fraudulent publication of reserves is uncommon. However, combinations of conflicts of interest, incompetence, poor judgement and unethical behaviour can and does result in inappropriate and even illegal reporting. Processes and controls must be put in place to ensure that such circumstances do not occur. Conflict of interest is most likely to occur when the Competent Person's remuneration, or that of senior management, is directly related to the amount of reserve added to the company. A strong personal motivation to see a project develop may also result in a conflict of interest.

Reporting errors

Even if reserves are estimated properly, additional issues are raised when publicly reporting these reserves. All applicable rules, codes and guidelines must be understood and followed.

OTHER REQUIREMENTS FOR RESOURCE AND RESERVE EVALUATION

In addition to geologists and mining engineers, specialists whose input is critical in the estimation of resources and reserves include:

- senior managers who define corporate objectives;
- project managers;
- metallurgists;
- environmental engineers;
- project and cost engineers who design and cost plant and infrastructure;
- accountants and tax experts;
- economists who forecast future costs, prices and exchange rates, help define investment criteria and confirm the economic value of the project;
- land men;
- lawyers; and
- experts in social sciences, public relations, governmental affairs, political risk assessment, etc.

The need for input from such specialists must be recognised when defining the role and responsibilities of the Competent Person. Unless otherwise specified, when declaring a resource or a reserve the Competent Person accepts responsibility for information received from others, the integrity of which he/she may not be qualified to assess or may not have the authority to question. The Competent Person must identify information received from others and assess the materiality of this information with respect to the resources or reserves being published.

RISK MANAGEMENT, PROCESSES AND CONTROLS

Corporate responsibility

The Sarbanes-Oxley Act of 2002 was voted by the US Congress 'to protect investors by improving the accuracy and reliability of corporate disclosures made pursuant to the securities laws, and for other purposes'.

According to Section 404 of Sarbanes-Oxley, management is responsible for establishing, maintaining and assessing the effectiveness of an adequate internal control structure and procedures for financial reporting. Sarbanes-Oxley specifically addresses accounting and financial reporting practices, but it is commonly interpreted as being inclusive of all aspects of the operations that have a material impact on the financial health of the company. Estimation of mineral resources and mineral reserves clearly falls within this category.

Sarbanes-Oxley was put in place to ensure that basic principles of good management are followed. But good management does not end with following the requirements of Sarbanes-Oxley. Good management can be defined in the context of Total Quality Management (TQM). Implementation of TQM starts with definition of 'quality'. Quality is commonly understood as being a property of a product or service that satisfies the needs or desires of a client. In the context of TQM, the definition of a client includes any individual or entity that benefits from, or suffers the consequences of, completion of the product. One no longer talks about clients, but rather about stakeholders.

Who are the stakeholders in a mining operation? They include:

- Shareholders, who supply the capital needed for the operation and expect a return on their investment.
- Analysts who advise the investing community.
- Banks, who contribute to the supply of financial resources needed by the mining company to operate or expand.
- Employees and their families.
- Users of the final product sold by the mining operation, whether it be coal, gold, concentrate, metal or industrial minerals.
- Suppliers, from whom the mining operation purchases equipment, energy, consumables, supplies or expertise.
- Local communities, including neighbours of the mining operation.
- The local, regional and federal or country governments, who are responsible for the welfare of their citizens and benefit from the taxes levied from the mining company. These governments must plan for new infrastructure, roads, health, education and entertainment, increases in traffic, crime, prostitution, higher demand for water, food, and housing. They also have a fiducial duty to ensure appropriate exploitation of national resources.
- Future generations, which will live with the long-term impact, good or bad, of the mining operation.
- Non-governmental agencies (NGOs) whose mission, self-appointed or otherwise, is to defend the interests of some of the above-listed stakeholders.

Senior management decides how to balance the needs, interests and requirements of the different stakeholders. They must give practical guidelines to those in charge of reserve estimation to ensure that the projects are designed to reach the company's objectives. Maximising shareholder value is often quoted as a company's primary objective. However, a company's objectives must include recognition of responsibilities towards all stakeholders, not only the shareholders.

For reserve estimation, the objective that is easiest to quantify, and for this reason most commonly used, is optimisation of net present value. Most computer programs developed to assist mining engineers in optimising production schedules assume maximisation of net present value as the objective function. Project risk is then measured by the level of uncertainty concerning net present value or related financial indicators.

To ensure that more complex objectives are reached, including objectives that cannot be quantified and to ensure quality assurance in resource and reserve estimation and reporting, appropriate processes and controls must be put in place. This requires nomination of a Manager of Resources and Reserves who understands the corporate objectives and is responsible for establishing, maintaining and assessing the effectiveness of adequate control procedures. Competent Persons are responsible for implementation of these procedures at the site level, be it an operating mine or a project.

Role of corporate manager of resources and reserves

To be effective, the corporate manager of resources and reserves should have a highly developed sense of ethics and a clear understanding of how conflicts of interest can result in resources and reserves being erroneous or – in the worst cases – fraudulent. In addition, he/she must satisfy the following requirements:

- understand the corporate objectives and how they influence estimation and reporting of resources and reserves;
- be aware of all applicable laws, rules and regulations, including those set by governmental and regulatory agencies;
- understand the impact that changes in resources and reserves have on investment decisions;

- understand the relationship between resources and reserves and all aspects of the company's operations, including accounting, taxes, legal, public relations, shareholders communications and governmental affairs; and
- have a good understanding of the technical and economic requirements that must be satisfied to estimate and report resources and reserves, and of the steps to be followed to develop reliable estimates and control risk.

The Manager of Resources and Reserves must establish, maintain and assess the effectiveness of adequate control procedures. He/she must define the processes to be put in place at each site and ensure that appropriate quality assurance and quality control practices are followed. He/she must define the responsibilities not only of the Competent Person, but also of the other specialists who supply information needed by the Competent Person to perform his or her task effectively.

The Manager of Resources and Reserves should assist site management in the recruitment of Competent Persons with appropriate expertise. He/she should make management aware of conflicts of interest occurring when Competent Persons or management are financially rewarded according to the amount of resources or reserves added to the company.

To ensure quality and to mitigate the affect of potential conflicts of interest, both internal and external auditing procedures should be put in place. Whenever possible, the Manager of Resources and Reserves should be required to report to the Audit Committee of the Board of Directors as well as the external auditors at least once a year.

Continuing education plays an important role in ensuring appropriate estimation, reporting and interpretation of resources and reserves. This includes education and training of Competent Persons, as well as education of all stakeholders, including shareholders, analysts, bankers, local and central governments, regulators and NGO representatives. Exchange of information should also be formalised within the company, including exchanges between Competent Persons, management, geologists, mining engineers, metallurgists, project engineers, environmental engineers, professionals responsible for investor and governmental relations, accountants, tax experts, lawyers and other specialists.

Diamond Resources and Reserves — Technical Uncertainties Affecting Their Estimation, Classification and Valuation

W J Kleingeld[1,2] and G D Nicholas[3]

ABSTRACT

The estimation, evaluation and classification of mineral resources and reserves should be part of a holistic, integrated process coordinated by a competent person. The valuation of a company is based on its resources and reserves but must incorporate risks related to the mineral supply, legal tenure and sales forecasts. Resources and reserves are classified according to the competent person's confidence in the sampling data, resource and mineralisation models, and tonnage, grade and revenue estimates. Uncertainty is associated with each of the variables used to calculate the net present value (NPV). In effect therefore, the NPV is a product of compounding uncertainties. Although the estimates of most other mineral commodities are associated with uncertainty due to limited sampling data, diamond evaluation may be associated with even higher uncertainty as more variables are included in the estimation and evaluation processes.

Although considerable progress has been made in the estimation and evaluation of other mineral commodities, such as gold, base metals, and alluvial diamonds, little progress has been made in the area of kimberlite evaluation to consider the correlated effects of resource and reserve uncertainties. No holistic model exists that quantifies the financial impacts of the combination of these resource and reserve uncertainties on the business model or the cost of acquiring additional information versus its financial benefit to mitigate these risks. The application of real options, which evaluate the effects of additional information, production and financial flexibility of mining projects combined with the use of a virtual orebody, may provide a solution to this problem.

INTRODUCTION

The objective of this paper is to highlight that an integrated, holistic approach is essential for a competent person to sign off a company's mineral resources and reserves, specifically considering the impact of technical uncertainties at the sampling, estimation, evaluation, classification, reporting and valuation phases. The problem is that not all companies adequately identify and quantify technical uncertainties within resource models to assess their financial impacts on the business model. Figure 1 depicts the holistic view that a competent person should have regarding the sampling to valuation pipeline.

The primary assets of most mining companies are its mineral resources and reserves. The discovery, evaluation, development and management of mineral reserves are critical to ensure a profitable supply to the market. The basis of this consistent supply is the mineral reserves. The fundamental building blocks of reserves are resources, associated with varying levels of geoscientific confidence and uncertainty. The understanding of this uncertainty is necessary to guide decision makers in the acquisition of or 'walking away' from new projects and assist in the optimal exploitation of reserves.

For mining evaluation purposes, risk can be divided into two categories, viz non-technical risks such as financial, environmental, country, social, political and economic risks, and technical risks. Many financial valuations attempt to quantify technical risks in projects by adapting the discount rate in NPV calculations for cash flow projections. This does not accurately account for overall project risk as it provides a 'blanket' technical risk factor applied to all blocks exploited within the same time period. Technical risk is inherent within sampling, resource models, mine planning, extraction and recovery processes. The risks within these areas can be expressed as a function of variability (the inherent stochastic nature of the deposit) and uncertainty (the assessor's lack of knowledge) from Vose (2002).

Many of the classical approaches to risk utilise Monte Carlo simulations. This method, in which variables associated with a project are drawn at random from a pre-determined distribution, is useful but does not address the effect of spatial correlation between variables that occurs within diamond deposits. This effect tends to compound risk, which if left unquantified and if mitigating strategies are not put into place, could result in reduced profits, higher capital expenditure or eventual project failure. As

1. Group Manager Mineral Resources, De Beers Group, Mendip Court, Bath Road, Wells Somerset BA5 3DG, United Kingdom. Email: wynand.kleingeld@dtc.com

2. Adjunct Professor, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada.

3. Group Geologist Research, De Beers Mineral Resource Management R&D, Mendip Court, Bath Road, Wells Somerset BA5 3DG, United Kingdom. Email: grant.nicholas@dtc.com

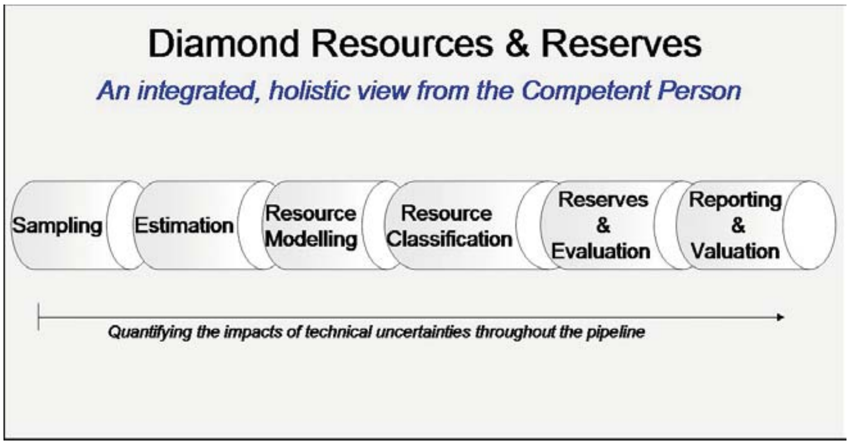


FIG 1 - An integrated, holistic view of the main phases involved in diamond resource and reserve valuation.

demonstrated by Dimitrakopoulos, Farrelly and Godoy (2002), geological uncertainty can have a significant impact on cash flows. Geostatistical simulation techniques have been used more regularly in diamond placer deposits than in kimberlites and have led to techniques such as the Cox process being applied by Kleingeld *et al* (1996). In the case of kimberlites, simulations focused mainly on grade, density and 'ore' thickness uncertainties. These are mostly segmented models and do not form part of an integrated, holistic model that quantifies the financial impact of correlated variables on the business model.

One of the main objectives of risk analysis is to identify, quantify and create an understanding of an adverse event and its associated impact(s) on the business plans. Generally, if higher risk blocks need to be mined within the first ten years of the business model, these necessitate a change in the extraction or recovery models and may incur additional expenses to modify mining methods or treatment plants that were not budgeted for at the time of project valuation. The impact of these unidentified risks may result in an uneconomic project. This paper further elaborates upon the risks associated with the complexities of diamond sampling, estimation and evaluation.

BACKGROUND

Complexities associated with diamond estimation and evaluation

The problem of limited sampling data to predict production forecasts is common to the evaluation of all mineral commodities. What makes diamonds even more difficult is that there are more variables associated with estimation and evaluation that can lead to greater uncertainty regarding the final production and NPV forecasts.

In order to take cognisance of the many valuation complexities associated with diamond deposits, it was necessary to develop specific methodologies for sampling, estimation and evaluation. The particulate nature of diamonds, their size, shape, quality, colour and value are important factors in the accurate estimation and evaluation of diamond deposits. Diamond occurrences in nature are rare and are usually measured in parts per billion, whereas most other mineral commodities are measured in parts per million, parts per thousand or in percentages. Figure 2 ranks the complex nature and difficulties of estimating diamond deposits compared to other mineral commodities as a function of their concentration and homogeneity.

Diamonds are brought to the earth's surface in volcanic host rocks, principally kimberlite. Most of these primary source rocks or kimberlite pipes do not contain diamonds, and those that do are very rarely economic. Approximately 5000 kimberlites have been discovered worldwide, of which only one per cent have been developed into mines. Depending on whether diamonds are contained in kimberlites or placer deposits, they are either free or locked up in the host rock. Though diamond is the hardest natural substance, it is brittle, which makes it susceptible to breakage during its release in either sampling, extraction or treatment.

Geological modelling is an essential first step in the estimation process, as the variability between facies is much higher than the variability within individual facies. Once a geological model has been developed, the required sampling strategy for grade and revenue determination must be defined. This involves establishing sample support size (volume), sample frequency (density) and sample spacing (spatial distribution). The sample size used is a function of the complexity of the orebody and the required level of confidence. During exploitation, selective mining is undertaken locally to 'footprint' the detailed diamond characteristics per geological facies to help in forecasting the

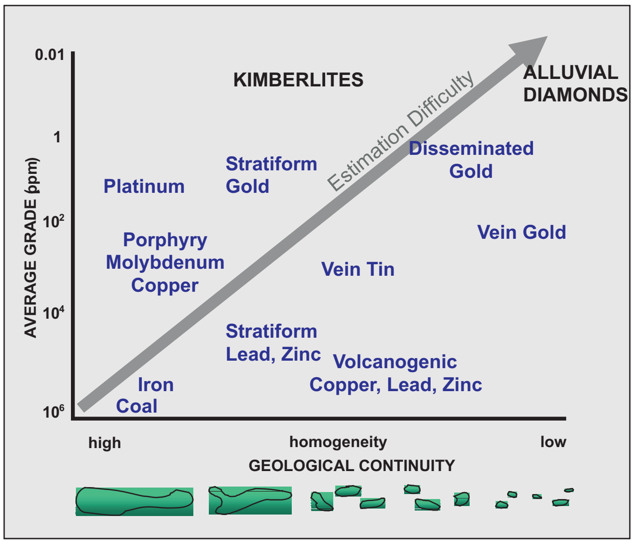


FIG 2 - A ranking of diamonds in relation to other mineral commodities based on sampling and estimation difficulty in relation to grade and geological continuity (after King, MacMahon and Bujtor, 1982).

diamond assortment for planning purposes. Numerous tools are used to produce an estimate for the dollar per carat revenue and the diamond assortment profile such as size frequency distributions (SFD), cumulative pareto-type distributions and extreme value modelling. Further modelling may be necessary to account for diamond breakage in the recovery process, under-recoveries due to plant inefficiencies and differences between bottom cut-off sizes between sample and production plants.

DISCUSSION ON RESOURCE AND RESERVE VALUATION

In order for a competent person to produce a fair valuation of a company's diamond resources and reserves, he/she has to take cognisance of each preceding phase starting at sampling, then the geological model, estimation of grades and other resource models, use of risk analysis to assist with resource classification, application of reserve and modifying factors to derive mine plans in the evaluation phase, and lastly compile the report.

It is becoming common practice in most countries that companies that produce public mining reports and which are listed on one or more stock exchanges, need to produce an annual report fulfilling a list of requirements relating to the classification, reporting and valuation of resources and reserves. In general, answers to the following questions are required:

1. Are there sufficient diamond resources and reserves to support the projections and NPV calculations?
2. Is there sufficient capacity and equipment to mine and recover diamonds according to the production forecasts?
3. What is the political, social, environmental and legal stability of the company in order to efficiently conduct its business?
4. Is there adequate marketing and sales forecasts of its own production and contracted partners?

For the purposes of this paper, only the first question will be assessed in terms of the technical uncertainties associated with diamond resources and reserves to provide accurate production and NPV forecasts. It will be assumed that the latter three questions have been suitably answered. The main problem areas associated with resources and reserves are described below in three sections:

1. estimation and evaluation of resources and reserves,
2. classification and reporting of resources and reserves, and
3. valuation of resources and reserves.

Estimation and evaluation of resources and reserves

One of the main problems regarding the valuation of resources and reserves is that some evaluators do not sufficiently understand the variability of input data into financial models, nor do they adopt a holistic approach for the valuation. Input resource models, such as geology, grade, revenue per carat and density all have associated uncertainties. Each of these models comprises a number of variables, which are uncertain and may be correlated with one or more variables from other models. These resource variables are used to compile mine plans whereupon sensitivity analyses are conducted that do not capture the range of variation associated with the compounding effect of resource uncertainties.

Estimation processes using kriging and/or simulation techniques require accurate input data and an understanding of the uncertainties associated with the modelling of this data. This paper does not delve into estimation techniques but focuses instead on some of the uncertainties associated with the input data into financial calculations, starting with sampling to resource models, viz geological, density, grade, revenue, geotechnical, geohydrological and geometallurgical models, and the impacts of these compounded uncertainties on the mine plans and reserves.

Sampling

It is critical to determine the objective of a sampling campaign. This may appear obvious but all too often a single sampling campaign is drilled to simultaneously yield geological, grade and geometallurgical information with respect to the kimberlite and/or associated waste rocks. The De Beers Mineral Resource Management R&D (MINRAD) Group is currently researching the effect of sampling to reveal geological, grade and geometallurgical characteristics within the kimberlite pipe. Early findings suggest that, in many cases, they require different sampling densities and support sizes to ensure that sampling data is representative of reality. The primary objective of including more samples in a deposit is to reduce uncertainty associated with resource and reserve variables. The natural variability of resource variables within the deposit cannot be reduced by additional sampling; only uncertainty can be reduced. Variability must be managed via the scheduling process in mine plans to allow selective mining of the orebody.

Sampling campaigns must also be designed to reveal information about the target at the scale of mineralisation, which is important for estimation and evaluation purposes. For geostatistical purposes, a sampling campaign must be designed to yield sufficient sampling data at a lateral and vertical frequency that allows a meaningful experimental variogram to be modelled. If micro-, meso- or macro-features have a significant effect on the continuity of mineralisation affecting block estimation, the sampling campaign must be designed to quantify the impact and allow the accurate interpretation of these features. The paradox is that micro-, meso- or macro-features will only be identified through sampling, which will then facilitate the design of a more representative sampling campaign. This may be described as a 'sampling for sampling' process.

Consistent and accurate sampling data are especially important when considering drilling at different densities and/or support sizes that may require de-clustering and/or support size corrections. Different bottom, middle and top cut-off sizes and granulation settings may have been used for the sample estimates compared to the planned mining production scale. Efficient recoveries over time may also be inconsistent. The change of support effect has a significant influence on the estimates (Kleingeld and Lantuejoul, 1992). Drillcore holes are normally a few centimetres in diameter, while most kimberlite mine blocks have dimensions around 50 m × 50 m × 10 m or bigger. Scale corrections must occur to correctly adjust for estimating into larger block volumes from considerably smaller sampled volumes. All these sampling considerations must be included when assessing the uncertainties of the main resource models, described below.

Geological model

The importance of a good geological model forming the foundation for estimation and evaluation modelling, and the use of geostatistics to assist in developing the geological model, has been recognised in the past by numerous practitioners, eg Parker (1977). In developing a geological model for kimberlites, there are two main considerations, viz defining pipe geometry and internal lithologies and their geometries.

The delineation of pipe geometry requires the outer boundaries of the kimberlite pipe to be demarcated in order to distinguish between kimberlite and waste (or country) rock. In practice, delineation of the pipe geometry is very dependent on interpolation between relatively few pierce points from core drilling. The ore/waste contact is usually sharp, while the internal boundaries are often gradational and require interpretation. This high degree of interpolation and interpretation can result in uncertainty around the volume estimates.

The diamond-bearing material within a kimberlite pipe is variable and is the product of different depositional processes and the admixture of country rock fragments and kimberlite-derived constituents. As a result, different kimberlite lithologies can be recognised within the pipes. Lithological boundaries define zones of similar geological and diamond emplacement characteristics. Uncertainty is introduced into the lithological boundaries as it is based on interpolations between only a few intersections from core drilling. An understanding of these lithological zones and the boundaries between them is essential for estimation purposes. This is necessary so that the geostatistician can model a variogram using only samples that fall within the boundaries of the delineated lithology. The authors postulate that where lithological zones are appropriately delineated, uncertainty in grade estimates and/or revenue estimates can be substantially reduced, which will improve the accuracy of the overall estimate.

The definition of lithological zones and the boundaries between them are defined from multiple datasets, including geological, geochemical, geophysical and structural. Each of these have uncertainties. The geological zones must be defined at a scale appropriate to the sampling, evaluation and mining processes.

Density model

Densities may not appear to vary significantly within a particular kimberlite lithology. An exercise using about 6000 samples yielded a 90 per cent probability that the density varied between 2.28 and 2.37. Subtle variations can affect the total tonnages associated with each mine block quite considerably and should be modelled as accurately as possible. It must be reiterated that it is necessary to verify the lithological model before samples are taken to ensure consistency of the density model within a defined zone.

Grade model

The diamond grade model is directly related to the mineralisation model, which in turn is influenced by geological and emplacement models. Diamonds are discrete particles that may be clustered in 'pockets' or may be randomly distributed throughout a particular zone. This complicates effective sampling to recover a representative quantity of diamonds to provide an adequate size frequency distribution (SFD) for estimation purposes.

A few diamond evaluation specialists make use of micro diamonds to provide an indication of the SFD in the limited presence of bigger stones. Large diameter drilling (LDD) sampling usually recovers a high quantity of small diamonds but only a very small quantity of bigger stones and very rarely, if ever, the really big diamonds over ten carats/stone due to limited sampling support size. Conventional large bulk-samples, such as trenching and underground development, are required to recover representative numbers of diamonds over ten carats/stone to confirm the size and value that would be recovered during production. In most instances, the SFD will require a degree of modelling to estimate the quantity of large diamonds considered to be missing either as a result of the sample size or the recovery processes.

Revenue model

The diamond revenue model is also associated with uncertainty and is influenced by the mineralisation, geological and emplacement models. Different lithologies may have different grade and revenue models. The uncertainty associated with diamond revenue modelling is different, and in many ways more complex, than price stochasticity affecting other mineral commodities, such as the gold price. Diamond valuation has four main attributes to consider, rather than only one, in the case of the gold price, for example. The four main attributes are size, colour, model (or shape) and quality. Each of these attributes has an associated variance. The variances are additive and therefore a much larger sample of diamonds is required to estimate the average $US/carat value than to estimate the stone density distribution or the stone size distribution, which together constitute the grade.

LDD drilling (often referred to as mini-bulk sampling) can provide a reasonable indication of the value, provided the orebody is high grade and sufficient diamonds are recovered unbroken. Generally, however, conventional large bulk samples from trenching or underground development are required to obtain a parcel of 3000 to 5000 carats to confirm the value and provide sufficient information about the overall diamond assortment.

Geotechnical and geohydrological models

Geotechnical and geohydrological models are often perceived as subsets of geological models. However, since they influence mine design decisions such as pit slope stabilities and tunnel support, they introduce variables that may not necessarily correlate with those in the geological model. Geotechnical interpretations rely upon interpolations between relatively few sampled measurements, which introduces uncertainty into the model. In most cases, uncertainty increases with depth in the kimberlite as sample data becomes scarcer. There may be some degree of correlation between structural variables quantified in geotechnical studies and geological variables.

Similarly, the geohydrological model is associated with uncertainty based on limited sampling data. It takes geological and structural models into consideration and is likely to be correlated with these models to some degree. The scale at which the geohydrological model is defined normally depends on its potential impact on the mining and business models. Failure to identify and quantify the impacts of significant micro-, meso- or macro-features in the geotechnical or geohydrological models could have detrimental effects on the business model.

Geometallurgical model

The geometallurgical model is a three-dimensional model of an orebody that aims to identify geometallurgical zones that will enable accurate forecasting of the recovery efficiencies associated with variable rock types. Daily production treatment of this variable diamond-bearing material may result in different degrees of liberation, separation and process efficiencies.

For many kimberlite mines, an average recovery factor per geological facies is generated. This limitation is partly due to the high cost of acquiring and processing samples to understand their metallurgical response, and partly due to the challenges associated with understanding the spatial distribution of rock types within the pipe. Uncertainties are associated with each estimate of the geometallurgical properties, such as density, hardness, clay content, etc and are compounded in the final recovery estimate. These uncertainties result in a higher variability of 'ore' characteristics during production treatment than initially forecasted from sampling, and can result in diamond 'lock-up' or poor recovery efficiencies. The De Beers MINRAD Group is currently researching the spatial connectivity between geometallurgical samples to assess the financial impacts of uncertain recovery models on the NPV.

Uncertainty with respect to mine plans

Resource models are not deterministic but are in fact associated with varying degrees of uncertainty and these resource variables may have varying degrees of correlation between them. Some of the key variables have been highlighted in the preceding sections with respect to geology, grade, density and the revenue/carat models. Mine planning optimisation techniques, such as the nested Lerchs-Grossmann (LG) algorithm (Lerchs and Grossmann, 1965), are based on mathematical models that assume inputs into a mine plan are known. This assumption is untrue for the evaluation of most new mineral deposits, but even more so for diamond deposits as there are more variables to consider that are uncertain.

Open pit mine planning is generally more flexible than underground operations. To some extent, open pit operations can adapt their mine designs to accommodate an uncertain resource model, whereas underground mining defines an 'almost' irreversible plan that cannot adapt.

Where conditional simulations have been used to express the uncertainty of resource models, a number of variables such as grade, density and revenue/carat exist for each simulated realisation that may or may not be correlated with each other and are dependent on the geological model. For each simulated realisation, an optimal pit is designed, resulting in the optimal block sequence and schedule based on the maximum contribution per block. But which realisation is representative of reality and which one should the mine plan be based on?

This is not a new problem to the mining world but it is believed that this level of understanding with respect to diamond mining has not been sufficiently understood at the building-blocks level. Davis and Morrison (1999) and Dimitrakopolous, Farrelly and Godoy (2002) developed 'envelope optimisation' methods, focusing mainly on grade and using geostatistical conditional simulations to produce an output envelope of NPV solutions. While these methods are useful in identifying an optimal envelope of possible solutions and highlighting the error in focusing on only one estimated NPV, the mean of all the realisation outputs is not an optimal mine design. This prompts the question, 'what is the mine plan actually optimising and where does it take cognisance of technical uncertainties?'

Godoy and Dimitrakopolous (2004) recognised the shortcomings in the above-mentioned methods and developed a mining transfer optimisation algorithm with objective functions that consider orebody uncertainties in relation to financial, mining and treatment criteria, such as the maximum NPV pit shell, discounted cash flow, etc. Ramazan and Dimitrakopoulos (2007, this volume) and Jewbali (2006) have completed research on a new stochastic integer programming model that considers all the simulated orebody models to find the optimal open pit production scheduling for metal mines such as gold, iron ore, etc. These developments will have to be applied and tested in the diamond industry, considering the main resource variables and their correlated impacts.

Classification and reporting of resources and reserves

Once the sampling, estimation and resource modelling phases are completed, the competent person has to assess all the independent and compounded risks for the project to assist him/her in classifying the resources. Most methods of resource classification can be summarised into two main categories, viz a subjective approach versus a more quantifiable approach. The competent person or evaluator who classifies a diamond resource as inferred, indicated or measured must take cognisance of the following criteria:

1. confidence and continuity of the geological model;
2. representivity and accuracy of sampling data considering change of support calculations;
3. confidence and continuity of the mineralisation model;
4. confidence associated with tonnage, grade and revenue estimates; and
5. testing and reconciliation exercises.

Over and above the criteria mentioned above, the confidence and accuracy of the dilution model; geotechnical and geohydrological models; social, political, legal and environmental considerations; and confidence in the modelling of economic and marketing factors are important for reserve classification.

Whether a subjective or a quantitative approach is pursued to attain the above-mentioned information, both approaches involve a degree of risk analysis to identify and quantify uncertainties associated with resources and reserves. A subjective versus a quantifiable risk analysis approach is discussed below.

Subjective risk analyses

Numerous papers and guide books are available on approaches to subjective risk analyses, for example Vose (2002). A subjective or qualitative project risk assessment normally commences with a risk management plan that assists the risk analyst in identifying the project objectives, principal stakeholders involved and provides a time scale for follow-up risk assessments. Those approaches conventionally use techniques such as probability-impact matrices, risk registers and risk matrices, etc.

The aim of risk registers and potential problem analyses (PPA) is to produce a likelihood of risk occurrence table and magnitude of risk impact tables, which can be used to plot risks in terms of probability and impact. Some risk matrices go one step further and apply weighting factors to the impacts to identify those resource risks that are critical to the project. This provides a guideline to the competent person in terms of the various ranked confidences associated with resource models and their perceived impacts on reserves. They can be applied to project studies within the categories of desktop, pre-feasibility and feasibility phases and related to achieving a required resource classification status. Risk registers and risk matrices are more meaningful and have less bias if they are completed by an inter-disciplinary team rather than a single person.

While these approaches encourage teamwork and provide a good framework for identifying problems and their potential impacts, they are primarily based on subjective opinions. An alternative to this subjective bias is the risk analysis approach adopted by Aspinall *et al* (2002), implemented at a case study on the Montserrat volcano. This approach elicited expert opinion in a more unbiased manner by performing weighted combinations of expert judgments. Weights are determined through a process of calibration and information performance on questions for which the answers are known. Thus, strongly opinionated but not necessarily technically astute people that dominate a team discussion, and who would normally have forced their opinions on the risk matrix, would be downgraded according to their calibrated weighting if their performance was below that of the rest of the team. Less bias to the output of risk matrices or interpretative models will be provided if calibrated weighting methods are used.

However, a degree of subjectivity will always be evident in these models and, as risks compound each other, they will tend to either underestimate or overestimate reality, or unwittingly cancel each other out in the model. This could severely hinder the competent person's judgement. His/her 'understanding' of the risks and their perceived impacts may be considerably diluted by the subjective nature of the risk model. Subjective risk analyses are more suited to situations where only limited data is available or where decisions must be made in a very short time.

Quantitative risk analysis

There are a multitude of statistical techniques available to assist the analyst or project evaluator in the transformation of data to knowledge as a function of time, money and people skills that can influence decision-making. A precursor to quantitative risk analyses is the quality checking of the sampling data, which if ignored, can lead to erroneous results. A few of the key sampling considerations have been highlighted in this paper.

Most risk analysts are familiar with Monte Carlo simulation (MCS), a useful technique that produces a range of possible scenarios that each variable could assume and that weights each possible scenario by the probability of its occurrence. Each probability distribution is sampled in a manner that reproduces the distribution's shape. Part of the problem with some mine planning and financial techniques is that they assume inputs into their models using a worst-case, most likely, and best-case scenario rather than sampling the entire distribution. Grade and revenue sampling yields highly, positively-skewed lognormal distributions. In some cases, the biggest diamonds are found in the last five per cent of the tail of the distribution. The probability associated with this part of the distribution is traditionally very low but the impact on the grade and revenue models can be significantly high. Minimum, most likely and worst-case scenarios may be used to quantify impacts on the business model but may not adequately sample the tails of the distribution and therefore, may not accurately represent the real NPV.

Geostatistical simulations are used to model the uncertainty associated with one or more of the resource attributes, reflected in a number of simulated realisations, Coombes *et al* (2000). Whereas geostatistical techniques take the spatiality of diamonds into consideration, MCS could underestimate the true variability. The objective of quantitative risk analyses (QRA) should be to identify, quantify and assess the impact of the variable(s) that contribute(s) the highest variance to the financial output, such as NPV. This will facilitate decision-making to decide to what extent this risk should be mitigated or not based on its impact on the financial model. Risk assessments should provide an improved quantifiable framework to assist with reserve classification.

An integrated business model based on the concept of a virtual orebody (V-bod) is proposed to quantify the financial impacts of resource uncertainties. A V-bod should be simulated using spatial data for geology, grade, density and revenue/carat, forming a simulated 'reality' model. A mining model is placed on the simulated 'reality' to define the 'real' financial output in terms of NPV/IRR. This simulated 'reality' will then be sampled by a systematic sequence of sampling campaigns generating a series of sampled values. The sampled values are used to generate simulated realisations with a mining model fitted to each realisation to produce an NPV/IRR output, which can be compared to the simulated 'real' NPV/IRR. Variance analysis can be carried out on the business model to identify which variables cause the highest variance in the NPV/IRR. It could be ascertained how many more sampling holes need to be drilled into the V-bod before the variance is sufficiently reduced. It is anticipated that this will allow an improved understanding of resource and reserve uncertainties, their financial impacts and highlight appropriate mitigation strategies.

Role of the competent person

It is the role of the competent person to take cognisance of risk analyses and data provided by various contributing parties, in conjunction with resource and reserve classification code requirements to produce a fair classification of a company's resources and reserves. Representative sampling, continuity and confidence associated with the geological, mineralisation, structural models, etc must be considered by the competent person to allow him/her to make a more informed decision. The final, subjective decision to classify a resource as inferred, indicated or measured lies with the competent person based on all the information available at that point in time.

Annual reports and other documents, such as a competent person's report (CPR), that are released in the public domain, should utilise a team of inter-disciplinary specialists to compile and review the contents. Each of these specialists may be a competent person in his or her own right but ultimately, the team requires a responsible figurehead or chairman to coordinate the integration of individual segments of information into a coherent report to reflect the company's financial value. He/she will rely on information from resource and reserve statements; mine plan schedules and production forecasts; financial indicators relating to tax, interest rates and exchange rates; corporate financial forecasts; legal; and sales and marketing data. The emphasis is on producing a holistic report that has taken cognisance of resource uncertainties and their impact on production plans and revenue forecasts.

Valuation of resources and reserves

Techno-economic probability factors

The basis of valuations are resources and reserves, but according to most stock exchanges, only probable and proven reserves and indicated resources are valued, while inferred resources are not assigned any material value. Mining companies with a proven production track record on existing operations could argue that a proportion of these inferred resources will be upgraded to indicated and eventually measured resource status over a period of time. Exploration projects most likely cannot be treated in this way. Resource classifications on existing operations are generally upgraded through additional sampling. Empirical data for that specific operation should provide evidence for the percentage or factor of resources that are successfully converted to a higher classification and the remaining proportion that are not.

These conversion percentages form the basis of 'techno-economic probability factors' that consider uncertainties associated with inferred resource models and reserve uncertainties associated with mining, treatment, profitability, legal tenure and price. That proportion of inferred resources should be included in the valuation on the basis of empirical data associated with each deposit. The competent person must consider the remaining life of mine (LOM) for each operation depending on what proportion of reserves and/or resources remain. If most of the reserves and indicated resources have been depleted, with the bulk sitting in inferred, the probability factor could be reduced further. For most kimberlite mines, the upper levels are associated with higher resource confidence grading towards less confidence at greater depths. Thus for valuation purposes, reserves are usually depleted first in the LOM production schedule, then indicated resources and finally, inferred resources. Considering the time value of money, resources depleted after the tenth year have little material effect on NPV.

Real options business case

> *In the absence of managerial flexibility, NPV is the only current available valuation measure consistent with a firm's objective of maximising its shareholders' wealth …* (Trigeorgis, 2002).

In light of the arguments presented in this paper regarding uncertain resource models, it is the authors' intention to highlight the main limitations of using conventional discounted cash flow (DCF) and NPV methods for valuing diamond mining projects. One of the problems is the inability of the DCF approach to accurately encapsulate technical variability of mine blocks exploited within the same time period in the global discounting method applied to cash flows per period.

All mine blocks that are exploited in the same time period (i) will be discounted equally by the bottom half of the equation $(1+r)^i$, assigning a global technical discount rate. While it is acknowledged that some merit exists for this discounting using global country and environmental risk factors, the problem is that mine blocks extracted within the same exploitation period may have significantly different technical resource risks, as highlighted in this paper. A contribution estimate (calculated from revenue less the costs) for a single block is the product of many variables, each having uncertainties. The contribution estimate will have a compounded uncertainty that may be significantly different between adjacent blocks mined in the same period. Conventional scheduling methods that aim to provide the best practical NPV, and determine the blocks to be mined within a time period, do not cater for technical uncertainties within individual blocks. Thus, the combination of conventional scheduling methods and DCF calculations could result in large variances between NPV forecasts and actual production values.

In addition to the above-mentioned problem, advocates of real options such as Armstrong and Galli (1997) and Trigeorgis (2002) have proven that flexibility has value. NPV methods do not take managerial flexibility into consideration and tend to underestimate the real value of reserves that have a high degree of uncertainty associated with them. These uncertainties are typically associated with market and financial indicators (interest rates, price, exchange rates, costs, etc) and technical parameters (quantity of mineral, quality of mineral, actual extraction and recoveries, etc). The application of real options could be useful to model the effects of flexibility in three main areas, viz the value of additional information on the resource and reserve estimates, mining and treatment and economic stochasity.

For resource and reserve estimation, what is the cost of acquiring additional sampling data versus its actual financial benefit? In the areas of mining and treatment, what is the cost of maintaining flexibility in mine plans to cater for resource uncertainty versus its actual financial benefit? Should the emphasis on the business model be placed on flexibility with respect to mine plan design, sequencing and scheduling rather than striving for optimisation? In the economic or financial model, what are the effects of price stochasticity and exchange rate fluctuations on the viability of the project? Real options are important in that senior management has flexibility in making strategic decisions and this flexibility creates value, especially when there is uncertainty. This is especially true for marginal projects where the NPV is close to zero but could underestimate its true value. Integrating technical uncertainty gives managers a more realistic value of options, recognising multiple decision pathways and allowing variable risk (Galli and Armstrong, 2002). Real options should not be perceived as a replacement for geostatistical and financial modelling or a stand-alone technique but as an enhancement of DCF/NPV methods to model the input of managerial flexibility in an uncertain world.

CONCLUSIONS

Uncertainties are inherent in each phase of the valuation pipeline, from sampling to estimation, resource modelling, reserves and mine planning, classification and valuation. In some cases, the variables within each resource model are not independent but may be correlated with variables in other models to varying degrees. Uncertainties within mine blocks do not cancel each other out but compound the problem, creating a greater range of uncertainty.

In order to provide a fair valuation, the competent person must consider the technical uncertainties within each phase, focusing on a holistic, integrated approach. This is even more important for diamonds than for other mineral commodities as there are more variables to consider in the estimation and evaluation phases. The paradigm of deterministic resource estimates providing input into mine plans and producing a single NPV estimate can mislead decision-makers, especially when a decision must be made to invest further in a project or 'walk away' from it.

Finally, the use of real options to evaluate mining projects, considering both managerial and financial flexibility and taking cognisance of technical resource and reserve uncertainties, can provide an enhancement to DCF and NPV methods. The application of new valuation methods to calculate upside potential is more important today than ever before as more low-grade, marginal deposits are being discovered. Various research projects have been initiated by the De Beers MINRAD Group to pursue some of the ideas presented in this paper.

ACKNOWLEDGEMENTS

Support and input from the De Beers Mineral Resource Management R&D Group in Wells, UK and our various colleagues in the De Beers Group who have added to the debate and discussion are gratefully acknowledged. The authors would also like to thank Peter Dowd, Roussos Dimitrakopoulos, Harry Parker, David Vose, Alain Galli and Margaret Armstrong for their input.

REFERENCES

Armstrong, M and Galli, A, 1997. Option pricing: a new approach to valuing mining projects, *CIM Bulletin*, 90:37-44.

Aspinall, W P, Loughlin, S C, Michael, F V, Miller, A D, Norton, G E, Rowley, K C, Sparks, R S J and Young, S R, 2002. The Montserrat Volcano observatory: its evolution, organization, role and activities, in *The Eruption of Soufriere Hills Volcano, Monserrat, from 1995 to 1999*, pp 71-91 (The Geological Society of London, Memoirs: London).

Coombes, J, Thomas, G, Glacken, I and Snowden, V, 2000. Conditional simulation – which method, in *Proceedings Mining Geostats Conference 2000* (eds: W J Kleingeld and D Krige) pp 677-691, Cape Town, South Africa.

Davis, B M and Morrison, J, 1999. Enhanced underground design through optimization and risk assessment, in *Proceedings APCOM 1999,* pp 503-510, Colorado, USA.

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Galli, A and Armstrong, M, 2002. Integrating technical uncertainty into real options, paper presented to the SPE ATW, Risk Analysis Applied to Field Development Under Uncertainty, Rio De Janeiro.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions*, 316:43-50.

Jewbali, A, 2006. Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines, PhD thesis, The University of Queensland, Australia, 280 p.

King, H F, MacMahon, D W and Bujtor, G J, 1982. A guide to the understanding of ore reserve estimation, Supplement to *Proc Aust Inst Min Metall*, 281:1-21.

Kleingeld, W J and Lantuejoul, C, 1992. Sampling of orebodies with a highly dispersed mineralisation, in *Proceedings Fourth International Geostatistics Congress* (ed: A Soares), pp 953-964 (Kluwer Academic: Dordrecht).

Kleingeld, W J, Thurston, M L, Prins, C F and Lantuejoul, C, 1996. Conditional simulation of a Cox process: application to diamond deposits, in *Proceedings Fifth International Geostatistics Congress*, pp 683-684, Wollongong, Australia.

Lerchs, H and Grossmann, I F, 1965. Optimum design of open pit mines, *Transactions CIM Bulletin*, 58:47-54.

Parker, H, 1977. Applications of geostatistical methods in exploration program design, paper presented to the National Council for United States-China Trade Technical Change, p 33.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Trigeorgis, L, 2002. *Real Options, Managerial Flexibility and Strategy in Resource Allocation*, pp 23-68 (The MIT Press: London).

Vose, D, 2002. *Risk Analysis, A Quantitative Guide,* pp 6-12 (John Wiley and Sons Ltd: England).

Koniambo Lateritic Ni-Co Deposits, New Caledonia — A Case Study from Geological Modelling to Mineral Resource Classification

M Audet[1] and A F Ross[2]

ABSTRACT

Traditional methods were adapted to estimate and classify the Koniambo nickel-cobalt laterite resources in Nouvelle Calédonie, for use in a feasibility study. As the resultant three-dimensional (3D) model is the basis for establishing the mining reserve and production schedule for a number of processing options, it was important that the model retained information for diverse material types, multi-elements and major oxide grades. Software tools from a number of vendors were used as no single system either suited the requirements at site, or provided full capability for all steps.

The laterite includes typical limonite and garnierite deposits in a highly complex geological succession of altered or weathered facies. The geological interpretation was developed from surface mapping and incorporated structural data from former small-scale open pits. The bulk of geological and assay data came from vertical core drilling on triangular patterns ranging from 320 m to 80 m, supplemented by air-core and reverse circulation drilling on a tight pattern of 40 m. One area selected for trial mining was drilled at 5 m spacings and this provided an opportunity to reconcile local estimates with the prediction from wider-spaced data. The model integrates the concept of geological horizons and geomorphic domaining to create a parent 3D block model. Estimation was conducted in unwrinkled space using ordinary kriging. The non-linear method of uniform conditioning was used to adjust the parent 3D model to reflect the proposed mining selectivity. Conditional simulation was used to assess the risk associated with ore continuity and formed the basis for resource classification.

Based on standards described by the Canadian Institute of Mining, Metallurgy and Petroleum (CIM), a combined Measured and Indicated mineral resource was established at 75.6 million tonnes grading 2.47 per cent nickel and 0.059 per cent cobalt at a 2.0 per cent nickel cut-off. An additional Inferred mineral resource of 83 million tonnes grading 2.5 per cent nickel and 0.07 per cent cobalt was also estimated.

1. Xstrata Nickel, Laval QC H7L 5A7, Canada. Email: maaudet@xstratanickel.ca

2. FAusIMM (CP), Technical Competency Manager, Snowden, PO Box 77, West Perth WA 6872, Australia. Email: aross@snowdengroup.com

* Due to the black and white production of this printed volume, some figures may not appear as distinct as intended. A colour version of the figures can be found on the CD ROM edition of this publication.

INTRODUCTION

Falconbridge Limited and Société Minière du Sud Pacifique (SMSP) are addressing the feasibility of producing 60 000 tonnes of nickel per annum as ferronickel from saprolitic ores developed on the ridges of the Koniambo Ultramafic Massif, Nouvelle Calédonie (Figure 1).

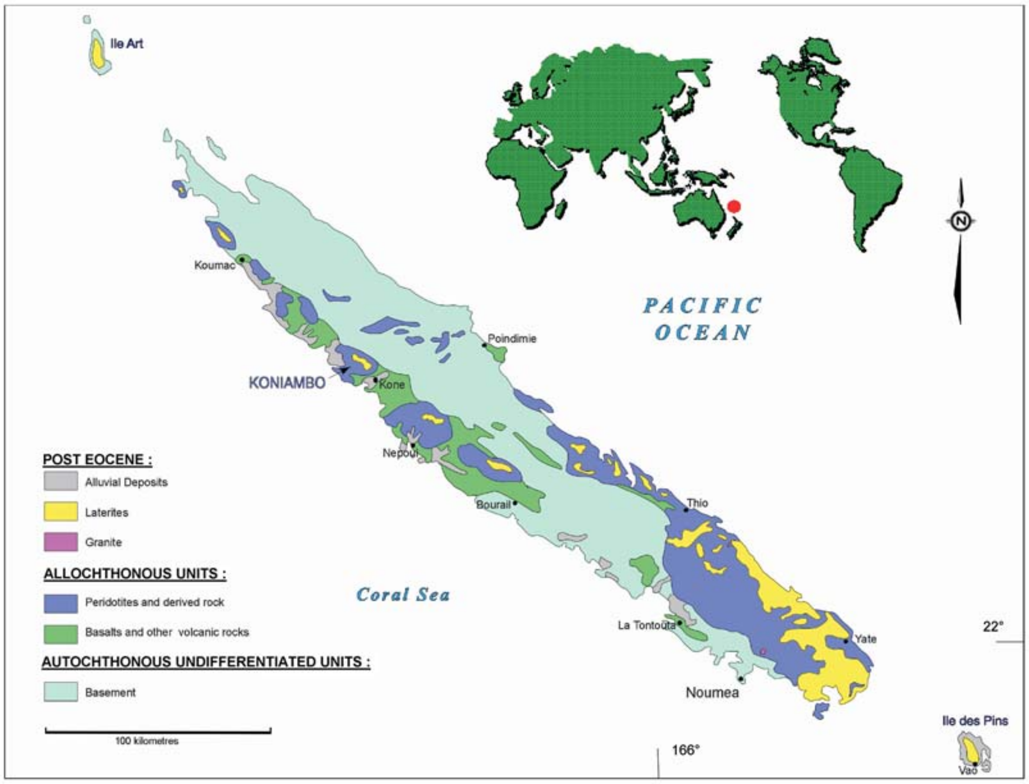


FIG 1 - Location of the Koniambo deposit in Nouvelle Calédonie.

An overview of the process, from developing the geological model to a three-dimensional (3D) resource block model for use in mine planning, and mineral resource classification is presented in this paper.

The Koniambo deposits are typical wet laterites and include a range of magnesium saprolite and limonitic lithologies or facies, and hence diverse chemistry. The plant specifications require that the resource model captures sufficient information to allow the mine planners to address issues such as: blending, stockpiling, dilution and also identification of material types suitable for upgrading. The project required the resource modellers to retain comprehensive facies information for materials handling as well as estimates for Ni, Co, Fe, MgO, SiO_2, Al_2O_3, Cr_2O_3, LOI and density.

Important constraints in the proposed plant are ratios of SiO_2/MgO and Ni/Fe/Co limits, as well as overall Ni grade. A typical feature of the profile is the occurrence of fresh rock boulders that would normally be regarded as a source of dilution. Simple screening of garnierite feed is normally carried out in nearby operations to reject such low-grade harzburgite boulders. One interesting aspect of Koniambo morphology, however, is the recognition of similarly sized, often well-mineralised boulders of dunite (‘hardcore’) that require an alternative beneficiation route, or else this feed would be similarly rejected. These aspects are indicative of the additional information that the resource model sought to capture.

The geological interpretation, assisted by geomorphic and chemical domaining criteria were key aspects of the resource estimates. These were addressed using geological modelling procedures defined by the Falconbridge/SMSP project team. The block model used established methods to address the strong vertical chemical trends, undulating profiles and proposed unit of mining selection. These methods included the unwrinkling transformation, ordinary kriging (OK), and uniform conditioning (UC). conditional simulation (CS) was used to quantify the resource risk and formed the basis of the mineral resource classification.

REGIONAL SETTING AND HISTORY

The micro continent of Nouvelle Calédonie is a patchwork of continental terranes and ophiolites formed during three main periods:

- Permian to late Jurassic: plutonic and volcano-sedimentary terranes were formed in intra-oceanic arc settings; these were obducted onto the ‘pre-Permian’ metamorphic terrane and finally accreted to the east Gondwana margin in late Jurassic times.
- Late Cretaceous and Palaeocene: the continuing break-up of the Gondwana margin resulted in the lifting of oceanic plateaus, which finally reached the Eocene subduction zone of the Loyalty basin.
- Eocene: the subduction was blocked, ophiolitic rocks were obducted in the late Eocene, and several intra-oceanic thrusts occurred. During the Upper Eocene, ophiolitic ultramafic bodies were emplaced by westward thrusting over Eocene basalts and Mesozoic sediments. These peridotitic masses were composed of weakly serpentinised dunites and olivine-rich harzburgites.

Ultramafic formations are exposed over a total area of about 7000 km^2 in Nouvelle Calédonie, with the southern massif alone covering an area of 5500 km^2. A belt of smaller ultramafic massifs extends along the west coast of the island as a series of isolated bodies and the Koniambo Massif is one of these.

Nouvelle Calédonie is the world’s third largest nickel producer after Russia and Canada. Approximately 25 per cent of the world’s known reserves and resources of nickel are found in Nouvelle Calédonie as laterites. The term laterite is commonly used to describe a process of natural weathering that results in the enrichment of trace elements, in this case nickel within the host ultramafic rock.

Lateritisation of the serpentinised harzburgite bodies occurred during the Tertiary period, and the residual laterite profiles are preserved over plateaux/amphitheatres, as elevated terraces and on ridges and spurs.

To date, mining activities on the island have focused on the lower, nickel silicate portion of the profile (ie the magnesian-rich saprolite zone). There has been only limited production from the upper nickel oxide portion of the profile (ie the limonitic zone), and this material is exported mainly to the QNI plant in Queensland, Australia (<2 million wet tonnes/year). Significant limonitic deposits covering extensive areas within the southern massif have yet to be exploited (eg Goro and Prony deposits).

Mining on the Koniambo Massif has occurred intermittently since high-grade ore was extracted in the late 1880s to support the newly established Société Le Nickel (SLN) smelter located first at Thio on the East coast and then relocated to Nouméa in 1930. Since the 1960s, the ability of the Koniambo deposit to support a world-scale operation has awaited the right combination of technological development, favourable political circumstances and market demand.

The Koniambo Massif is a typical ophiolite body that measures 20 km long by 5 km wide, of which approximately 21 km^2 is known to contain significant high-grade nickel laterite mineralisation. The Falconbridge-SMSP project is subdivided into nine sectors; however, in the current study three of these were selected for their large resource potential amenable to pyrometallurgical processing. The mineral resource estimation methods presented in this paper were applied to the three sectors: Bilboquet, Manguen and Centre (Figure 2).

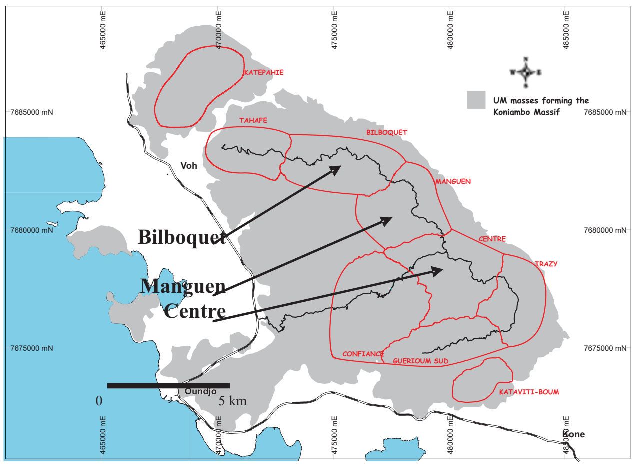


FIG 2 - The Koniambo Massif is subdivided into nine sectors. Three form the resource base for the feasibility study.

The Koniambo massif rises from a narrow coastal plain to 940 m above sea level thereby providing a spectacular mountain backdrop for the Northern Province capital of Koné, located 270 km north-west of Nouméa.

THE WEATHERING PROFILE

Where fully developed, the lateritic profile or succession consists of a thin iron rich ferricrete surface cap overlying limonite, followed at depth by a transition zone leading to a layer of saprolite. The grades of nickel, cobalt, iron, and various oxide constituents vary with depth. The ferricrete and limonite cover occurs mainly over the north-west trending axial ridge of the

Massif, with smaller patches covering several isolated terraces to the west. The limonite cover averages 20 m thick, and ore-grade saprolite is often exposed at surface as a result of erosion.

The weathering process of the harzburgite and dunite rocks at Koniambo consists of a progressive dissolution of magnesia and silica, while other elements such as iron, nickel, cobalt, and aluminium remain in the decomposed lateritic material. With time, the relative concentration of the remnant elements increases. Saprolitic material derived from harzburgite exhibits extensive to complete replacement of primary Fe-Mg silicates (primarily olivine) by Fe oxyhydroxides, nontronite and Fe oxyhydroxides, or amorphous silicates, quartz/chalcedony and Fe oxyhydroxides.

The weathering process commences along joints and fractures that exist within the near surface of the Massif. As the alteration process continues, the weathered product, which progressively replaces the fresh rock until it has completely disappeared, surrounds boulders of jointed/fractured ultramafic material. This describes the initial formation of the saprolite. The overlying limonite is formed after considerable leaching of the silica and magnesia from the saprolite. The alteration profile is thus divided over time into two primary groups consisting of an uppermost limonitic zone composed of remnant iron hydroxide, and a lowermost saprolitic zone in which the silica and magnesia are the main constituents.

A schematic lateritic profile developed over the Koniambo serpentinised harzburgite/dunite is presented in Figure 3.

The Koniambo deposits contain all of the features of classical humid nickel laterites (Brand *et al*, 1996). One important aspect affecting beneficiation is the pervasive alteration of dunite sills. The altered variants are often boulder-like and contain significant nickel grades, in direct contrast to boulders of fresh harzburgite that serve to dilute saprolite ore. The main difference is manifested in the harzburgite, which develops a 'boulder' texture with typical onion-skin and core of fresh material, while the dunite is weathered pervasively without developing onion-skin texture.

CONSTRUCTION OF THE GEOLOGICAL MODEL

The geological interpretation was compiled from surface mapping, including litho-structural information from abandoned pits, as well as data from vertical core and reverse circulation drilling. Borehole patterns are triangular or quincunx, and range in spacing from 320 m to 20 m. One area for trial mining and grade control optimisation was drilled out on patterns of 10 m and 5 m (Manguen M1D).

The distribution and character of the laterite profile at the Koniambo Massif is typically complex at all scales normally considered for resource evaluation. The massif contains five main limonitic areas, and the thickness of limonitic cover can exceed 40 m to 50 m and occasionally reaches 90 m along major structural breaks. These areas include Tahafe, Bilboquet, Centre, Trazy and to a lesser extent a portion of Confiance area (Figure 2). Outside of these specific sectors, the limonitic cover is generally less than 5 m, and in many locations the saprolitic material crops out. The lateritic profile is further complicated by the influence of vertical to subvertical fault systems and joints. These fault systems acted as preferential weathering conduits, whereby the lateritic profile can locally extend several tens of metres deeper than the less weathered surrounding environment. The complex vertical to subvertical fracture systems have resulted in local accumulations of garnierite box-works and sheeted garnierite veins as the expression of high-grade ore.

The geological model sought to preserve the lithologies as alteration-facies within general groupings relative to the idealised complete profile. The model is based on a broad subhorizontal succession of facies with highly variable thicknesses and frequent disruptions due to vertical structures. It was important to the project team that these geological features were honoured in the resultant block model resource estimate.

It is a common impression often gained from the literature that laterite facies occur as horizons, rather like a 'layer-cake'; however, it is the experience at Koniambo that the succession of facies is highly irregular and strongly influenced by structural

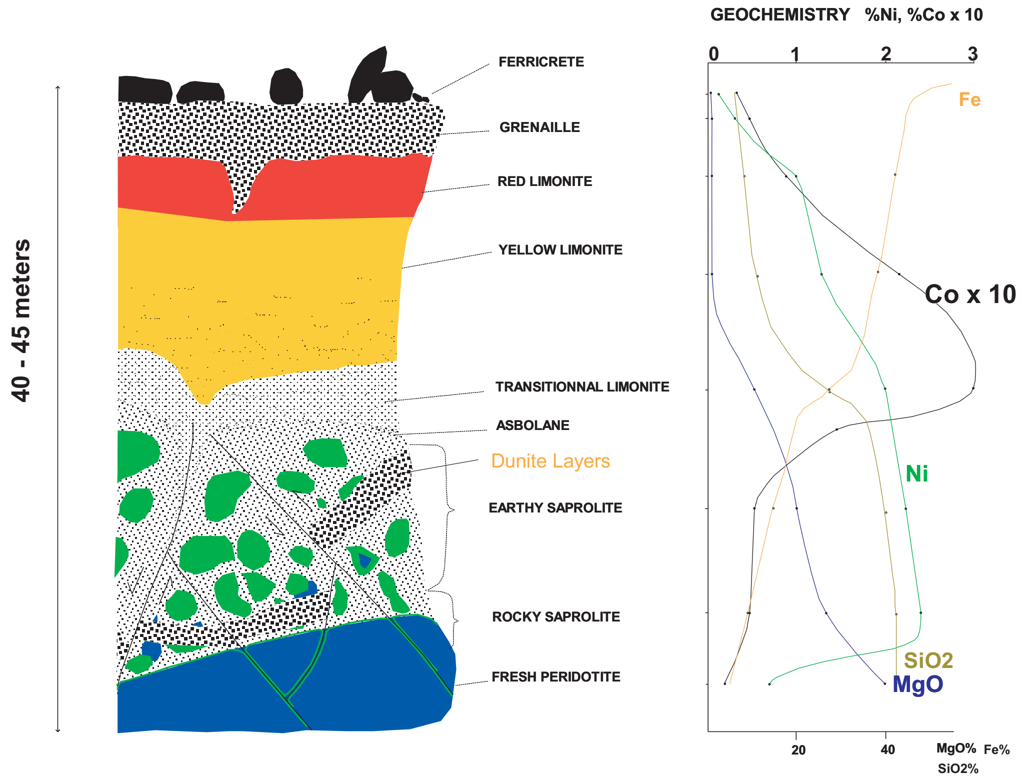


FIG 3 - Schematic profile of lateritic facies and chemical trends.

breaks. The project team intended the geological model to reflect the small-scale facies variability within a given horizon while keeping a broad subhorizontal succession, but also respecting major trends and faults. The comprehension of the structural deformation pattern is of great importance in understanding the ore variability and distribution. Field mapping showed that the succession of geological facies and therefore the chemical composition is highly variable over very short distances.

ALTERATION FACIES, ROCK CODES AND HORIZONS

A series of 'rock codes' is based on visual description of alteration. The original set of codes identified in excess of 50 lithologies to accommodate primary lithotype, degree of serpentinisation, and degree of weathering.

After confirmatory geochemical analysis these rock codes were applied to the smallest logged unit in each borehole (Table 1). Individual units in each borehole were grouped to create facies nomenclature that conformed to the succession model, from limonite at surface to rocky saprolite at depth. The 'rock code' allowed the categorisation of facies into five major groups or 'Horizons' (Limonite, Transition, Upper and Lower Saprolite and Bedrock).

Chemical characterisation and subsequent geological rationalisation permitted the grouping of the original rock codes into four main alteration facies (limonite, earthy, rocky, and fresh rock facies) within each horizon. The four-facies coding was carried through to statistical and variogram analyses and to the estimation phase. The condensed codes also included the identification of dunite protoliths in the Lower Saprolite Horizon.

GEOLOGICAL DOMAINING

The designation of 'domain' was introduced to identify specific areas showing geological similarity.

The drainage and erosion patterns create numerous geological regions with comparable weathering features and lateritic facies. The project team considered that these geomorphic attributes can be used to classify the underlying succession of lateritic facies. The edges of the regions were reasonably interpreted to represent structural breaks. The 'domain' type takes into account the:

- layering succession based on degree of weathering;
- homogeneity and continuity of lateritic assemblage of a given area; and
- geomorphology, structural deformation and faulting.

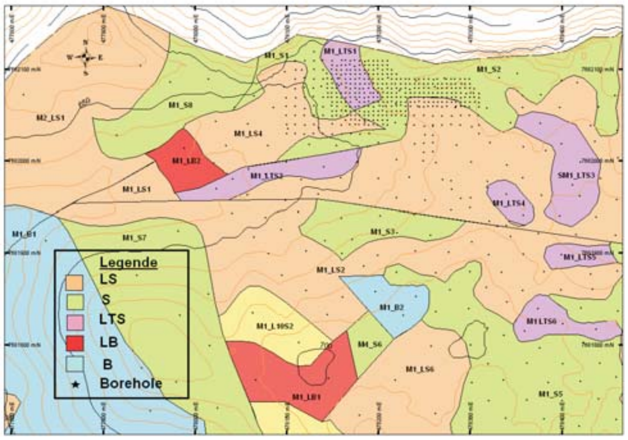


FIG 4 - Domain boundaries may be interpreted as faults from photo-geological evidence and also reflect changes in topography such as ridges and valleys (gullies). Each domain represents a homogeneous grouping of lateritic facies. In this case zone and domain names are indicated; M1_LS3 means the third domain LS in Zone M1, Manguen sect; domain type 'L10S' is not shown in this figure.

Domains were first interpreted from borehole maps and then validated in the field. Mapped domain limits were constrained overall by a global geological envelope that encompassed all plateaux and ridges where the topographic slopes were less than 30°.

Figure 4 shows several geological domains created in the Manguen sector.

Six domain types were inferred for the Koniambo deposit from the combination of lateritic facies. These are:

- LTS: natual succession of Limonite, Transition and Saprolite;
- LS: natual succession of Limonite and Saprolite, no transition material;
- LB: Limonite only developed on basement;
- S: Saprolitic material only;
- L10S: Limonite and Saprolite, Limonite thickness greater than 10 m; and
- B: basement of 'fresh rock'.

A conceptual representation of these domain types is presented in Figure 5.

TABLE 1

Criteria used to classify rock types.

Horizon	Rock code	Criteria 1	Criteria 2
Limonite	100	• Ferricrete (GR), red limonite (LR), yellow limonite (LJ) facies	<5% MgO, >40% Fe
Transition	200	• Transitional limonite (LT)	5 - 15% MgO, 30 - 40% Fe
Upper saprolite	300	• Strongly weathered saprolite, regardless of protore (harzburgite or dunite) • Can include minor horizons of LR, LJ, breccia and some rocky inclusions	15 - 30% MgO, 10 - 25% Fe
Lower saprolite	400	Boulder type • Dominantly harzburgite with several inclusions of 'fresh' material ranging in thickness from 0.10 - 1.0/1.5 m • Onion ring alteration texture • Can include minor horizons of LR, LJ, breccia and earthy material Hardcore type • Dominantly dunitic material • May include harzburgite intervals	>30% MgO, 6 - 10% Fe
Bedrock	500	• Unweathered material, regardless of protore (harzburgite or dunite)	<6% Fe, Ni <0.4%, MgO >SiO_2

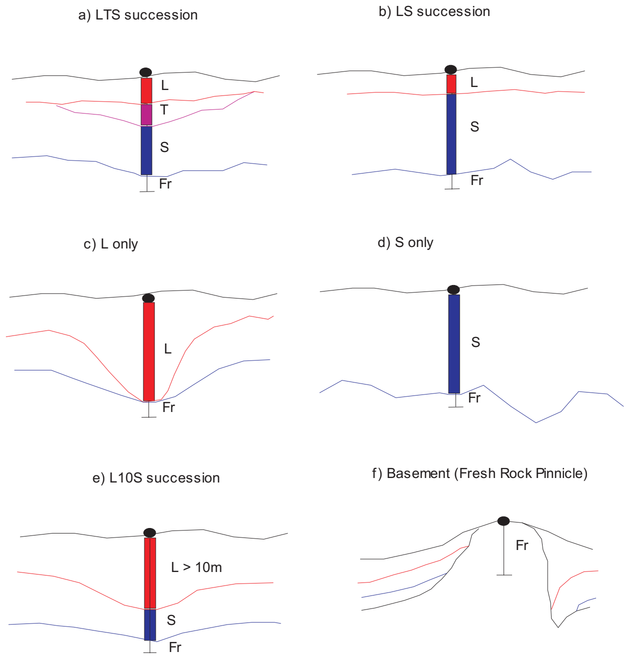


FIG 5 - Conceptual profiles of six possible groupings of lateritic facies, used to define geological domains. L: Limonite, T: Transitional, S: Saprolite, Fr: Bedrock.

GENERATION OF HORIZON SURFACES

From more than one hundred years of experience, miners in Nouvelle Calédonie have learned that the base of saprolite is highly variable, with stepped relief, pinnacles and troughs. Information from boreholes alone may suggest to the inexperienced that the base of saprolite is relatively smooth, thus leading to an unrealistic local interpretation. To overcome this problem, the base of the saprolite was modelled separately in each domain using Laplace gridding methods in Gemcom™ software.

A unique bedrock surface was created for each domain, and when assembled together, these surfaces created steps and troughs at domain boundaries that mimic faults and pinnacles. One possible configuration of the base of the profile is then created.

Surface wireframes of the three overlying horizons (base of Upper Saprolite, base of Transition and the base of the Limonite) were then interpolated using inverse distance squared weighting methods applied to horizon thicknesses. The base of each horizon was generated in sequence by deducting gridded horizon thicknesses from the topographic surface or alternatively, the horizon surface that lay above.

All surfaces were trimmed to the topographic surface. In addition, the base of the limonite horizon model was further controlled by the mapped position of the limonite.

GEOLOGICAL BLOCK MODEL FILL

The rock type block model was constructed by filling blocks of 20 m × 20 m × 2 m dimension between the surface topography and horizon surfaces on a priority basis, leading to the unique assignment of each model block with primary horizon codes. The 50 per cent 'in-out' coding rule was applied such that a minimum volume of 50 per cent was required to assign a horizon code to the block model prototype.

An example of the horizon rock type model prototype is shown in Figure 6.

The horizon surfaces became redundant following the coding of the rock type model. A new set of horizon surfaces was required to control the next step: unwrinkling of composites and discretised points within each block.

UNWRINKLING TECHNIQUE

It was decided early on that unwrinkling of the Koniambo laterite deposits was an appropriate method to improve grade connectivity and interpolation during the estimation process. In undulating terrains, interpolation efficiency is compromised in normal coordinate space due to the variability of horizon boundaries, wide spacing of boreholes and plethora of structural domains (Murphy *et al*, 2002). Unwrinkling is a Gemcom™ term to describe a method of unfolding whereby only the Z-coordinate of spatially located data is moved, to effect a flattening of a geological horizon (Hammer, 2000). In the case of the Koniambo study, the transformation was applied to both composites and discretised points within each block.

The inputs required for unwrinkling of points are: a pair of bounding surfaces defining each horizon, a constant thickness parameter for each horizon and a mid-level elevation to define the new transform. Figure 7 shows the general concept behind the unwrinkling. After grade estimation in unwrinkled space, the estimates were back transformed to normal coordinated space ('rewrinkling'). In the case of Koniambo the median thickness of the horizon was used to define the Z dimension of unwrinkled space.

A new set of horizon surfaces was required to properly honour the coded rock type model. This was achieved in the following way. The coded block models were interrogated to identify the blocks at the top and bottom of each horizon. Regularised

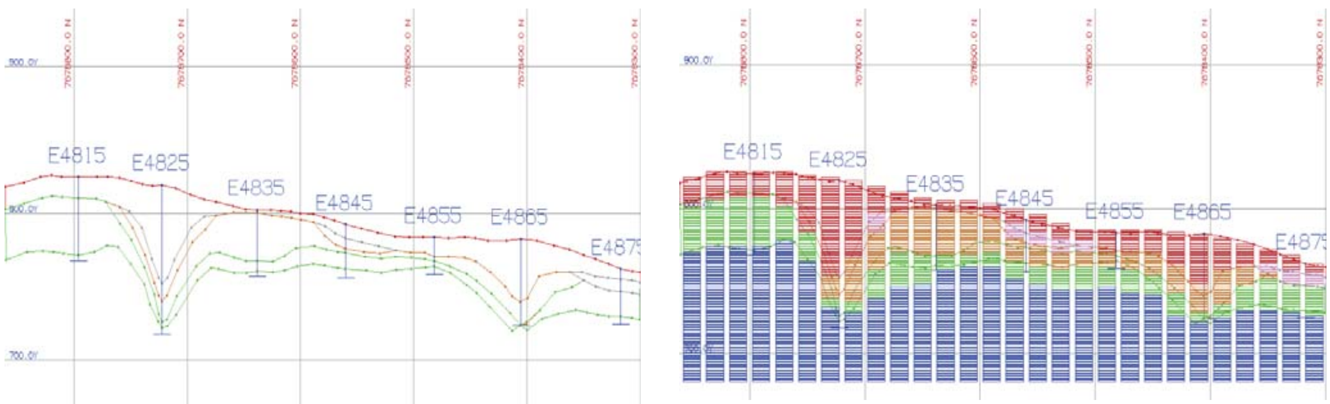


FIG 6 - Interpretation of horizons and ensuing block model for a section at Centre sector. Red: Limonite; Cyan: Transition; Brown: Upper Saprolite; Green: Lower Saprolite and Blue: Bedrock.

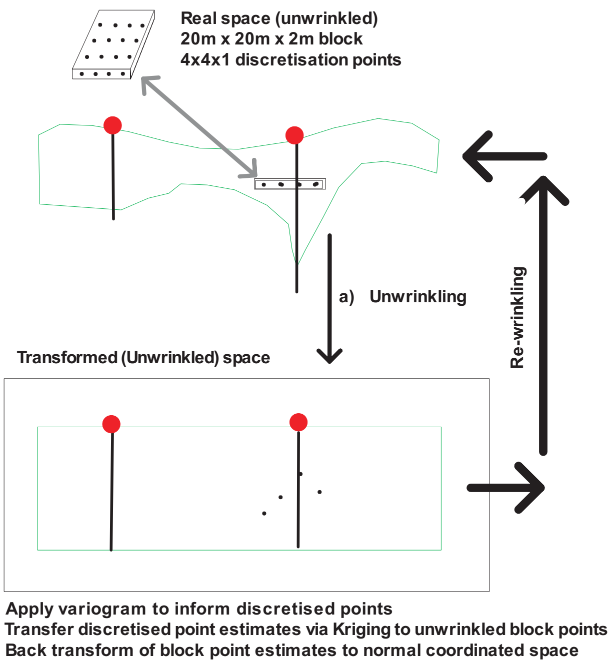


FIG 7 - Use of the unwrinkling transformation is presented in four steps; a) regular discretised points and composites are transformed into unwrinkled space, b) close-spaced nodes are estimated by kriging of unwrinkled composites, c) node estimates are transferred to discretised points, d) grades of discretised points are mapped and averaged to block estimates in normal coordinated space.

surface elevation grid (SEG) models were generated for each horizon contact and these became the basis for the triangulation of horizon surfaces used for controlling the unwrinkling of parent blocks, discretised points and composites.

The discretised points in the parent block model were mapped to an equivalent array of points in transformed space. The point estimates were then independently kriged in transformed space using Datamine software. The back transformation was performed using Datamine software because of efficiency reasons, notably the ability to effect the back transformation of all nine estimated variables in one pass.

COMPOSITING

A composite interval of 1 m was used for the Laterite and Transition Horizons, as the nominal sampling interval was generally 1 m. The situation in the saprolite was more complex, as the sample lengths are highly variable, with logging and sampling conducted on intervals ranging from 0.10 m to 1 m. The distribution characteristics of sample intervals were analysed, and a nominal compositing interval of 0.5 m was found to be appropriate for the Upper and Lower Saprolite Horizons.

The compositing of grade took both density weighting and length into account. Each original rock code was assigned a density value based upon a lookup table from a suite of approximately 2000 determinations.

STATISTICAL ANALYSIS AND VARIOGRAPHY

The project team required the mine planning block model to account for Ni, Co, Fe, MgO, Al_2O_3, SiO_2, Cr_2O_3, LOI, density and facies proportions. The geographic distribution of data across three sectors and numerous drainage 'divides', together with the nine variables provided a challenge for the presentation of routine exploratory statistical analyses. In order to investigate the regional trends, the domains were grouped into drainage zones,

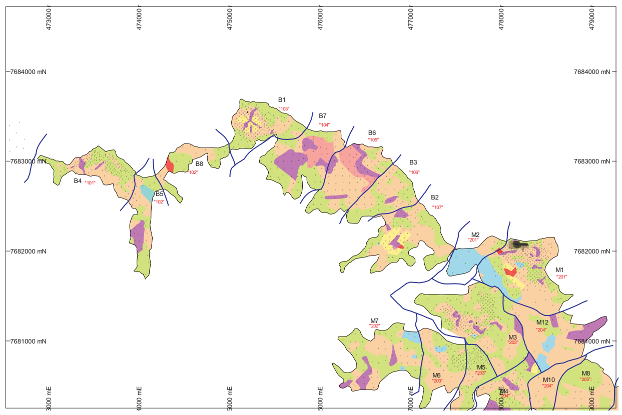


FIG 8 - The distribution of superzones is shown for the Bilboquet sector and the adjacent Manguen sector. Each superzone (labelled 101 to 107) is formed from several domains thus giving sufficient mass of drill data to permit variography. The distribution of superzones in Manguen and Centre sectors is not shown here.

and zones grouped into 'Superzones' (Figure 8). Superzones are groups of zones and domains that generally represent ridges or plateaux, and had sufficient statistical mass to allow variography.

It was found that simple box plots provided a useful medium to convey the statistical character of the deposits and to confirm the partitioning of the lithologies into facies and horizon groupings. The average grades and population distribution characteristics were plotted for each Superzone, along with the number of data points that comprise each dataset. An example of composite grade 'box-and-whisker' plots is presented in Figure 9.

Vertical grade trends and contact permissions to control grade estimates were investigated using graphical contact analysis methods. The intent here was the identification of sudden and significant grade changes across contacts that required the use of hard boundaries for estimation. Grade trends within horizons were found to be related to distances from contacts and this relationship was preserved in the estimates by transforming data coordinates to unwrinkled space.

Element and oxide correlations were investigated during the statistical analysis. It was found that Ni is not strongly correlated with any of the major oxides or elements. Conversely there are moderate to strong correlations within the group Fe, Al_2O_3, Cr_2O_3, and Co (Iron Group). Similarly there are moderate correlations between MgO and SiO_2 (Silica Group).

Variography was used to describe grade continuity for elements and oxides within horizons, facies and by Superzone for each sector. Semivariogram models for one of Fe, Al_2O_3 and Cr_2O_3 were applied to the Iron Group so that correlations could be preserved in the block estimates. Variograms for either MgO or SiO_2 were used for MgO and SiO_2 (Silica Group), as the ratio of these constituents has processing implications. Density was assigned the same semivariogram model as the Iron group. Ni and LOI were modelled independently of any other element.

Continuity of unwrinkled composites was examined using Snowden's Supervisor™ software. By generating and contouring gamma (variability) values at 10° increments in the horizontal plane, and referring to the ranges and sill values in the individual semivariograms, the directions of maximum and intermediate grade continuity were selected. Traditional semivariograms were calculated for the three orthogonal directions, and spherical models were fitted where possible. Log semivariograms were occasionally used for LOI SiO_2 and MgO for some facies and sectors. Where log semivariograms were used, the nugget and sill parameters were converted to traditional spherical models for estimation purposes.

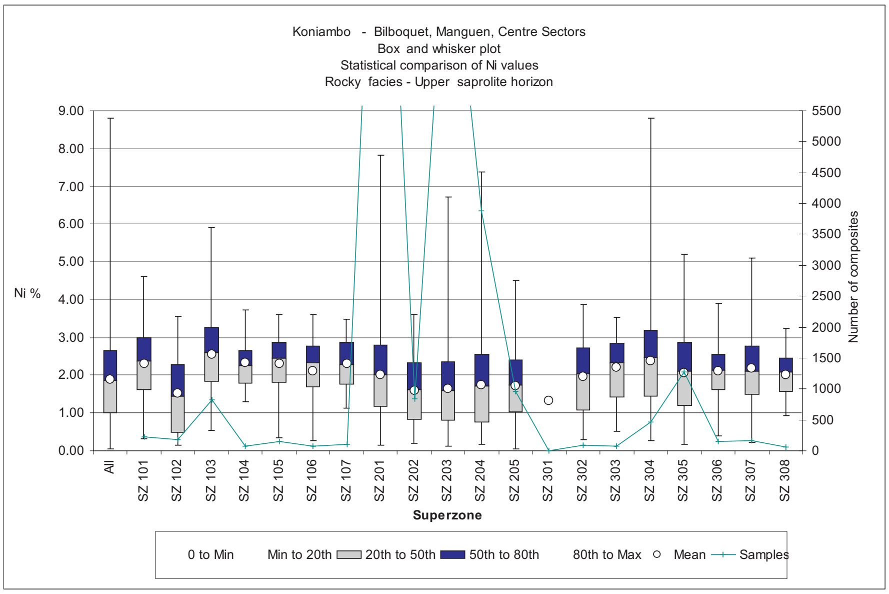


FIG 9 - Box plot of the distribution of nickel (Ni) for the earthy facies within the superzones. Nickel mean shows little variation from the Northwest (SZ101) to the Southeast (SZ 308). The distribution for the entire population is shown in the first column. Box plots for Ni, Fe, MgO, Al_2O_3, SiO_2, Cr_2O_3 and LOI were generated for each facies within each horizon.

ESTIMATION METHOD AND PARAMETERS

The relative variability of the data, expressed by the coefficient of variation (CV) parameter is generally less than 1.0 across limonite, earthy and rocky facies for Ni, Fe, Al_2O_3, and Cr_2O_3. CV values were generally higher for MgO and SiO_2 in the limonite facies of the Laterite Horizon. For the MgO and Co there are restricted areas where CVs are high, notably for MgO in the Transition Horizon. Overall, it was found that the variability patterns exhibited for most grades were generally within the range of expectations for nickel laterites.

The degree of mixing of the individual data populations and their degree of skewness and variability were assessed to identify a suitable estimation method. It was concluded that ordinary kriging (OK) methods would be appropriate in view of the generally low levels of skewness and relative variability.

The optimum kriging configuration was assessed by a study of kriging efficiency ratio (KER) and regression slope (R) (Krige, 1996). The degree of conditional bias imparted by a number of kriging configurations (interpolation ranges and sample constraints) was tested on a block size of 5 m × 5 m × 2 m. This block size represents the volume attributed to points within a 4 × 4 × 1 discretisation of a parent block measuring 20 m × 20 m × 2 m.

The kriging neighbourhood study found that a search strategy of 250 m × 250 m × 8 m and sample constraints of ten minimum and 50 maximum were appropriate. The minimum number of samples was subsequently reduced to five to ensure that most model blocks were informed in areas of sparse drilling. To arrive at this conclusion, the ordinary kriging weights and other parameters from a number of trace blocks were reviewed and assembled from run-file outputs to enable calculation of KER and R. The outputs that were used in calculating KER and R were kriging variance ('KV'), Lagrange multiplier ('LM'), and block variance ('BV').

RECOVERABLE RESOURCE ESTIMATION

It is planned that the Koniambo deposits will be mined by open pit, and it is expected that Selective Mining Unit ('SMU') dimensions may be in the order of 5 m × 5 m × 4 m (or smaller, down to 5 m × 5 m × 2 m). The bench height is expected to be variable and is anticipated to be between 2 m and 4 m in ore.

Ideally, the block size in a resource model should reflect the SMU on which the mine plan will be developed. As described, the parent resource block models for Koniambo were constructed based on panels having dimensions of 20 m × 20 m × 2 m dimensions. Unfortunately, this panel size does not reflect the degree of selectivity envisaged for mining operations at Koniambo. Instead the selection of the minimum parent panel dimension was influenced by: the prevailing borehole spacing, the graphical representation of the horizons, software limitations and computing capacity at site, and the need to avoid the introduction of conditional biases that would have arisen with estimation into a smaller block dimension.

In order to simulate the effect of higher selectivity on the parent panel model and allow a prediction of recoverable resources, a 'change of support' was applied. According to volume-variance relationships, the size of each sample (or block) has a significant impact on the resource estimation, and a relationship exists between this size (or 'support') and the distribution of values. It is generally observed that, as the support of the data increases, the maximum values decrease. Furthermore, averaging values together over larger areas (or panels) has the effect of reducing the variance of the data and of making the distribution more symmetric. Consequently, reducing the support of the data leads to an increase in the standard deviation, the coefficient of variation, and the difference between the mean and the median, while the mean remains unaffected.

A critical aspect of feasibility studies is to evaluate the support of the sample data set and the support intended for the final resource estimates. Since these are different for Koniambo, corrections were applied to the parent 20 m × 20 m × 2 m panel size support. In order to adjust the data distribution such that the variance is reduced without changing the mean value, the UC mathematical method was applied to the Koniambo Ni estimates. The fact that Ni is generally not correlated with Fe, Co, density and the secondary constituent oxides meant that more onerous methods involving co-simulations of the other variables were not contemplated.

UC is a technique provided by Isatis™ that uses a change of support concept to provide local tonnage-grade estimates for a given SMU or block size within large panels (Guibal, 1987). The available literature however is silent on the minimum panel to block ratio required to provide reliable recoverable resources using UC, ie how large should the panel to block ratio really be?

At Koniambo the panel size was a practical choice to honour the geological interpretation and thus be acceptable to the project sponsors, and it is not necessarily the optimum volume required by mathematical theory. The reliability of the SMU estimates however was tested in one area selected for a mining trial and bulk sample (Manguen M1D). Here there is close-spaced drilling at intervals of 5 m × 5 m. It was possible to reconcile the SMU estimates that were derived from 20 m panels and 80 m spaced boreholes with 5 m block estimates from the 5 m grade control boreholes.

MINERAL RESOURCE CLASSIFICATION

Historical practice in Nouvelle Calédonie has been to use an 80 m borehole grid to delineate reserves. Reserves are often estimated by polygonal methods, or two-dimensional kriging methods, without first estimating *in situ* global resources from first principles and application of economic parameters to design optimal pits. Ore is shipped to existing treatment plants. There have been few requirements to identify resources beforehand and to prepare mine plans in support of major capital investment. Thus the familiar stages of mining project feasibility studies have not been a feature of the Nouvelle Calédonie mining industry. There are very few examples of public reporting of resources and reserves from Nouvelle Calédonie laterite projects that allow comparison with similar projects in other countries.

At Koniambo, the mineral resources are classified according to the Canadian National Instrument NI 43-101. NI 43-101 uses mineral resource classifications (Inferred, Indicated and Measured) as described by the Canadian Institute of Mining, Metallurgy and Petroleum (CIM). The classification schemes outlined under these guidelines are not prescriptive and may be based solely on drill spacing pattern and the experience of the resource estimator.

The method adopted for resource classification for the Koniambo deposit is more stringent than required by CIM and is based on risk associated mainly with thickness of ore intercepts at a given cut-off. This approach is viewed as an improvement over the conventional approach as the production risk is also taken into account.

For saprolitic nickel laterite deposits operated at relatively high cut-off grades, grade continuity is very difficult to demonstrate prior to mining. Typically, final mine plans will be based on infill drilling to a 10 m grid. For this type of deposit, the assessment criteria can be less rigorous at the feasibility stage. It is enough to accurately know the probability distribution of ore and waste tonnage and ore grade within specific blocks of ground. It is expected that production areas will be subject to final detailed infill drilling and that there is a choice of mining areas available to the operator.

CS was used to assess the level of risk associated with the variability of three parameters: nickel grade, nickel metal content and thickness of ore. CS is a Monte Carlo type of simulation approach developed for modelling risk in spacially distributed attributes. The intention is to generate equally probable realisations of the *in situ* orebody grade and material type variability (Dimitrakopolous *et al*, 2002). It was found that nickel grade and nickel metal content were the least variable compared with ore intercepts. Consequently, the resource classification scheme was developed from the risk associated with variability of thickness of ore intercepts, and adjusted in localised areas where geological information was conflicting.

It is not the purpose of this paper to present the detailed method used to calculate the confidence limits. This is described in Murphy *et al*, 2004. However, the following steps summarise the process:

- CS on close-spaced nodes: Using the borehole input data and indicator variography models, Snowden computed 100 simulations of ore-thickness and nickel grade using the GSLIB, SISIM programs for sequential indicator simulation (Deutsch and Journel, 1998). The simulations were validated by comparing the input statistics and variography with the simulation outputs. The data histogram and variogram reproduction was found acceptable for both attributes.
- Reblocking: Simulated values were averaged over all nodes present in panels of 100 m × 100 m. Panels should be large enough to be statistically independent and to have a normal distribution of values. It was established that panel dimensions of 100 m × 100 m should be the minimum necessary to allow independence from simulated nodes located in adjacent panel. Each panel had an associated set of values that formed a frequency distribution (100 averages).
- Calculation of 90 per cent confidence level: For samples of size n from a large population, relative 90 per cent confidence limits can be estimated by the product of the standard error of the mean and the standard normal deviate or z-value (1.645) of the confidence limit of interest. The relative errors computed on a panel-by-panel basis do not accommodate the fact that multiple panels will be mined during any mine production period. Because a production schedule is not yet available to allow reblocking or aggregation of panels to reflect actual production periods, it was assumed that a number of panels, n, of similar character (grade and/or depth and/or thickness) would be mined in a given production period. Further, the panels were assumed to be large enough to assume independence between panels.
- For the purposes of this study, the size of the sample n is derived from the ratio of the production period tonnage to the tonnage within each reblocked panel. This formula assumes independence of realisations for each of the panels constituting a production period. The conditional distributions for panels are usually quasi-normal (bell shaped). From normal distribution theory, the 90 per cent confidence limits can be estimated by: ±1.645 (sp), where sp is the standard deviation for simulated panel average values.

Derivation of relative risk

It was assumed that N panels of a similar nature will be mined in a given time period. The 90 per cent confidence limits can then be estimated by: ±1.645 (sp)/√N whereby N = (ore tonnage in time period)/(mean ore tonnage in panel).

For resource classification purposes, the 90 per cent relative confidence limits are of interest: ±1.645 (sp)/[(√N)(Mp)]. Mp is the mean value of all simulations for the panel. The confidence limits are a fraction expressed as a percentage. Panels are identified that have relative confidence limits of less than 15 per cent. This is done separately for quarterly (625 000 t) and annual (2 500 000 t) periods.

Therefore:

$$90\%\ \text{C.L.} = \frac{1.645\,\text{s}}{(m)\sqrt{\text{N}}} = \frac{1.645\,\text{CV}}{\sqrt{\text{N}}}$$

where:

C.L. confidence limits

N (tonnage of ore mined in time period)/(tonnage within ‘grid’)

CV coefficient of variation

Examples of risk maps from the processed simulations are shown in the figures.

Figures 10 and 11 show the relative accuracies at 90 per cent confidence level defined for annual and quarterly increments. These maps were used to identify the limits for Measured and Indicated Resources at 2.0 per cent Ni cut-of-grade, subject to the following criteria.

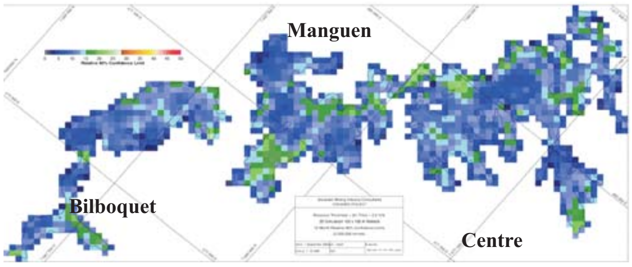


FIG 10 - Confidence in estimated thickness of ore for annual extraction periods. Colours indicate the relative accuracy at 90 per cent confidence level on a yearly basis (2 500 000 T) for panels of 100 m × 100 m. Black and blue panels indicate relative accuracies less than 15 per cent. These areas represent high confidence, low risk in achieving the planned extraction rate. Drill spacing is generally 80 m or less.

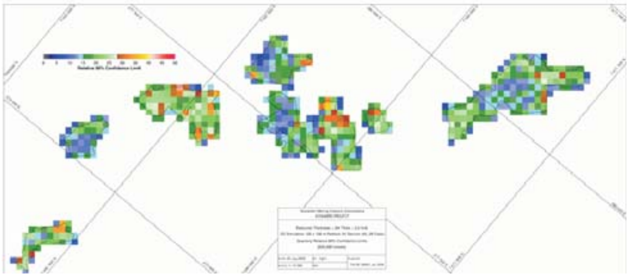


FIG 11 - Confidence in estimated thickness of ore for quarterly extraction periods. Colours indicate the relative accuracy at 90 per cent confidence level on a quarterly basis (625 000 T) for panels of 100 m × 100 m, within the areas of drill spacings of 56 m and less. The areas of high confidence, low risk in achieving the quarterly production schedule are reduced. The map highlights areas requiring infill drilling.

Measured Resources

Measured Resources occur only within areas where drilling was performed on spacings of 56 m and less and where conditional simulation shows that these resources have been estimated within ± 20 per cent relative accuracy on tonnage and ± ten per cent relative accuracy on nickel grade at 90 per cent confidence on an equivalent quarterly production basis (0.65 Mtpa increments).

Indicated Resources

Indicated Resources occur within areas where conditional simulation shows that these resources have been estimated within ± 20 per cent relative accuracy on tonnage and ± ten per cent relative accuracy on nickel grade at 90 per cent confidence on an equivalent annual production basis (2.5 Mtpa increments).

Inferred Resources

Inferred Resources occur elsewhere but within 90 m of a borehole.

Figure 12 shows the outline defined for Measured Resources within the area drilled at a spacing of 56 m and less. It is expected that the risk attached to estimates of ore thickness is influenced by the short-scale variability of mineralised facies and distribution of fresh rock pinnacles. This is indeed indicated in Figure 13 where drilling is generally close spaced and would be expected to confer a high confidence in the interpretation, yet the risk is variable. The figure shows that variability in confidence levels for ore tonnages is not solely related to borehole spacing but also, and more importantly, related to local variability of the weathering profile.

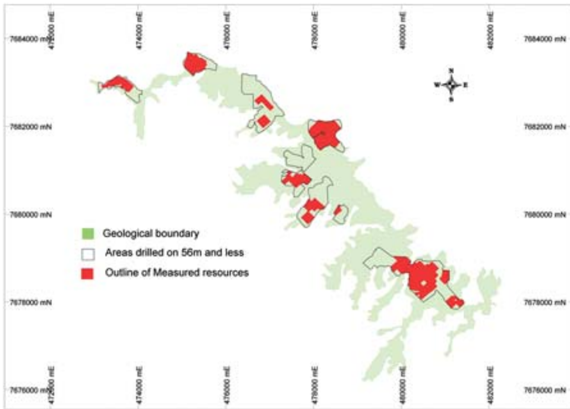


FIG 12 - Outline of Measured Resource in areas where drilling was performed on spacing of 56 m and less.

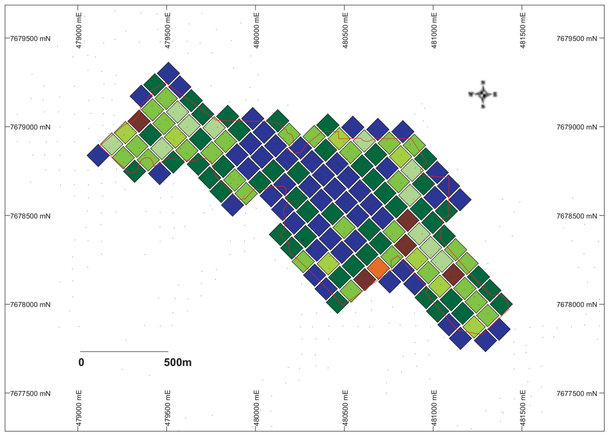


FIG 13 - Impact of local variability on confidence levels. Boreholes (black crosses) are shown in relation to relative accuracy at 90 per cent confidence levels. The basis is a quarterly risk map (625 000 T) for thickness of ore for panels of 100 m × 100 m for the Centre sector. Variability in confidence levels depends on borehole spacing and local variability of the profile (ie frequency of fresh rock pinnacles). For the same borehole density the relative accuracy at 90 per cent confidence level can be highly variable (refer to Figure 11 for colour legend).

MINERAL RESOURCE SUMMARY

Based on the estimation and classification methods described in this paper, a combined Measured and Indicated mineral resource of 75.6 million tonnes grading 2.47 per cent nickel and 0.059 per cent cobalt at a 2.0 per cent nickel cut of grade is estimated within three sectors at Koniambo. An additional Inferred mineral resource of 83 million tonnes grading 2.5 per cent nickel and 0.07 per cent cobalt was also outlined.

Figure 14 shows outlines for Measured, Indicated and Inferred Resources for the Bilboquet, Manguen and Centre sectors of the Koniambo deposit.

CONCLUSIONS

The scale and complexity of the laterite resources, together with site-specific requirements, meant that software tools from a number of vendors were harnessed to construct the resource estimates and long-term mine planning model at Koniambo. Complex 3D geological and block models were required to allow a number of processing options to be considered.

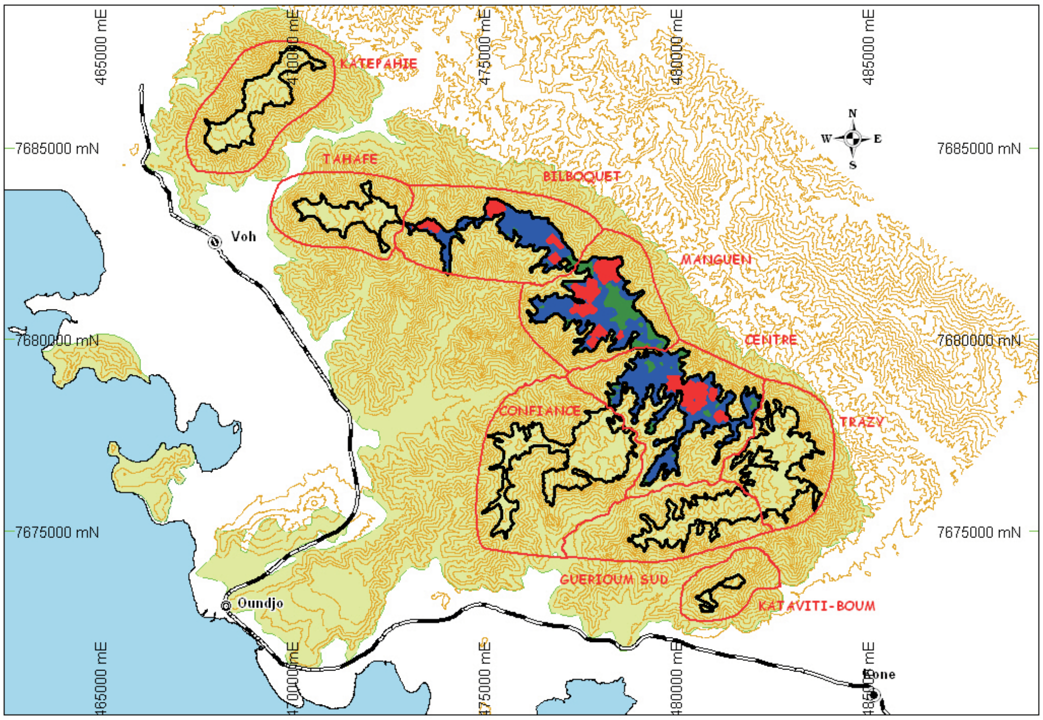


FIG 14 - Mineral resource classification outlines for Bilboquet, Manguen and Centre sectors. Red: Measured, blue: Indicated, and green: Inferred. Mineral resources for all other sectors are considered Inferred.

Considerable effort was applied to the generation of the geological model to best represent the succession of weathered horizons yet still honour the highly irregular bottom contact with the fresh rock. The unwrinkling transformation was then used to unfold data to improve grade connectivity in the kriging of nine variables, followed by rewrinkling of discretised point estimates to normal space.

Resource block parent models for the Bilboquet, Manguen and Centre sectors were built using panels with dimensions of 20 m × 20 m × 2 m. In order to estimate the effect of higher selectivity on the parent model, a non-linear UC method was applied to volumes of 5 m × 5 m × 4 m.

Mineral resources for the Koniambo project are reported using a 2.0 per cent Ni cut-off grade, which represents the intended operating cut-off. The mineral resource was classified using guidelines consistent with the CIM definitions referred to in National Instrument 43-101 into Measured, Indicated and Inferred Resources.

ACKNOWLEDGEMENTS

We would like to thank all Falconbridge/SMSP/Snowden team members for their continuous support and hard work over a long period since drilling commenced in 1998. Special thanks go to Tony Douglas from Snowden Mining Industry Consultants for his skill and enduring patience. Dr D F Bongarçon is thanked for his contributions to aspects of Change of Support. Many thanks go to Dr Harry Parker for his review and contribution as principal auditor.

REFERENCES

Audet, M A and Kohlsmith, R, 1998. Koniambo Project, resource estimation and test drilling program, Pn 950, May-June, Falconbridge internal report.

Audet, M A and Kohlsmith, R, 2003. Koniambo Project, geology and resources estimations, October, Falconbridge internal report.

Brand, N W, Butt, C R M and Elias, M, 1996. Classification and features of nickel laterites, *AGSO J Aust Geol Geophys*, 17(4):81-88.

Canadian Institute of Mining, Metallurgy and Petroleum, 2000. CIM Standards on Mineral Resources and Reserves – Definitions and Guidelines.

Canadian Securities Administrators, 2002. National Instrument 43-101 Standards of Disclosure for Mineral Projects.

Deutsch, C V and Journel, A, 1998. *GSLIB Geostatistical Software and User's Guide*, Second edition (Oxford Press: New York).

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimization: grade uncertainty and risk effects in open pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Gemcom Software International Inc, 1998. *Gemcom for Windows User Manual*, volumes 1 to 3.

Guibal, D, 1987. Recoverable reserves estimation at an Australian gold project, in *Geostatistical Case Studies* (eds: G Matheron and M Armstrong), pp 149-168 (D Reidel Publishing Co: Dordrecht).

Hammer, R, 2000. Gemcom Desktop Edition (Release 4.1) Developer's notes.

Isaaks, E H and Srivastava, R M, 1989. *Applied Geostatistics*, 552 p (Oxford University Press).

Krige, D G, 1996. A practical analysis of the effects of spatial structure and of data available and accessed, on conditional biases in ordinary kriging, in *Proceedings Geostatistics Wollongong '96* (eds: E Y Baafi and N A Schofield), Vol 2, pp 799-810.

Murphy, M, Bloom, L M and Mueller, U A, 2002. Geostatistical optimisation of mineral resource sampling costs for a Western Australian nickel deposit, in *Proceedings IAMG 2002: Eighth Annual Conference of the International Association for Mathematical Geology* (eds: U Bayer *et al*), pp 209-214, Terra Nostra, Berlin.

Murphy, M, Parker, H, Ross, A and Audet, M, 2004. Ore-thickness and nickel grade resource confidence at the Koniambo nickel laterite deposit in New Caledonia – A conditional simulation voyage of discovery, presented at the Seventh International Geostatistical Congress, Banff, Canada.

Parker, H, 2002. Second phase independent audit, Mineral Resources Development Inc (MRDI), May, Falconbridge Nouvelle Calédonie SAS internal report.

Snowden Mining Industry Consultants, 2003. Koniambo Project feasibility study resource estimate, Falconbridge internal report.

The Value of Additional Drilling to Open Pit Mining Projects

G Froyland[1], M Menabde[2], P Stone[3] and D Hodson[4]

ABSTRACT

The value of a mining project is based upon a quantitative model of material of value in the ground, a block model of the deposit, and a schedule for extracting this material including relevant revenues and costs. The schedule usually attempts to maximise the net present value (NPV) of the project over the life of the mine. Frequently, a block model is the result of a smooth interpolation, such as kriging, of data collected from holes drilled throughout the orebody. More drill holes will lead to greater certainty in the contents of block models and from these 'more accurate' block models, schedules of greater ultimate value may be realised. We discuss how conditional simulations can assist with rigorously valuing the trade-off between the cost of extra drilling and the schedules of greater value that may be constructed from the resultant block models of greater accuracy.

INTRODUCTION

In today's competitive world the push to extract ever more value from mining projects continues to increase. Initiatives to decrease costs and increase revenue are being pursued. One of the most attractive options is the application of optimisation tools to schedule the mining operation with the explicit objective of maximising the net present value (NPV) over the life of the operation. At present such tools are applied on a short-term basis to cut costs of daily operations through efficiencies, and on a long-term or life of mine basis to maximise NPV. In the latter case, NPV is increased through:

1. Delaying or eliminating waste stripping.
2. More efficient routing of ore through the network of trucks, crushers, conveyors and beneficiation plants.
3. More efficient resource use through better blending and cut-off grade decisions. The promise is that the resulting plan will deliver pure value increases for little or no cost.

The value of all of this number crunching depends upon the reliability of the input data. The valuation of a mine project depends critically upon the accuracy of the geological block model[†]. On the one hand, we will never know precisely what material is deep in the ground until we have excavated that material. On the other hand, we must make plans for the future with the best information available to us at the present time. While realising that information is not perfect, having a plan is better than having no plan; this much is generally accepted as reasonable.

However, what if a planner were given the option of obtaining more information with which to construct his or her plan? In this paper, additional information will take the form of block models with increased accuracy, but the same principles may be applied to other forms of information. Intuition suggests that if one's block model were more accurate, then one could construct a mine plan of greater value by exploiting this additional knowledge (via a different mining sequence or cut-off grade policy, for example). But how much would one be prepared to pay for this additional knowledge? Clearly, the cost of the additional data should be less than the expected increment in value that can be obtained with this new data, otherwise the planner would construct a mine plan with the data already available. This is common sense – the real problem is how to quantify, and value in a rigorous way, the increment in project value that a mine planner can expect from this additional information. If we can do this, then we will have valued the option of obtaining additional information and have put ourselves in a position of making a decision on quantifiable grounds.

We begin with some background on the numerical construction of block models from drill hole data and the process of kriging. We then formalise what is meant by optimising NPV using a kriged block model as the geological input. For optimisation and valuation purposes the mining schedule is modelled as a mixed integer linear program (MILP); see Johnson (1968), Caccetta and Hill (2003) and Ramazan and Dimitrakopoulos (2004) for prior related work and surveys. We introduce the option of undertaking an additional drilling program and briefly explain why this may or may not increase NPV. Conditional simulations are introduced as a way of quantifying uncertainty and we discuss how to optimise with multiple conditional simulations. We detail a formalism that clarifies the notion of additional knowledge and describe a method of determining the maximum value that one should pay for any additional drilling program. All of the introduced concepts and numerical calculations are illustrated throughout via an example of a simple open pit mine.

ESTIMATED GEOLOGICAL BLOCK MODELS AND KRIGING

The information in a block model is gathered from a series of drill holes. Typically, many long, narrow holes are drilled into the ground in the vicinity of the orebody, and their cores are extracted and analysed for mineral concentrations. For simplicity, in the sequel we will assume that the only relevant information contained in the block model is the total tonnage of each block, and the concentration in per cent of mass of a single metal element. Thus, one knows precisely[‡] the density of the rock in the drill hole core and the concentration of the element (the grade) along the core. The drill hole cores provide a sparse set of data from which we must construct a full three-dimensional model of rock tonnage and percentage by mass of the metal element in each block. This construction is commonly performed using a process known as kriging. The kriged estimate of the block model is derived as a local linear interpolation of the measured drill hole grades. If one assumes that the linear correlation of the grades of pairs of blocks depends only on the distance between the blocks and the direction in 3D from one block to the other, then the kriged estimate of blocks grades is the best linear estimator of the block grades ('best' in the sense of minimum variance); see Cressie (1991) for further details.

1. School of Mathematics, The University of New South Wales, Sydney NSW 2052, Australia. Email: froyland@maths.unsw.edu.au

2. BHP Billiton Technology, PO Box 86A, Melbourne Vic 3001, Australia. Email: merab.menabde@bhpbilliton.com

3. BHP Billiton Technology, PO Box 86A, Melbourne Vic 3001, Australia. Email: peter.m.stone@bhpbilliton.com

4. BHP Billiton Project Development Services, PO Box 86A, Melbourne Vic 3001, Australia. Email: dave.hodson@bhpbilliton.com

† Clearly the project value also depends critically upon fundamental inputs such as the product sales price and the market volume. We do not treat these dimensions in this paper, but they may be considered and quantified in an analogous way.

‡ To within error bounds typical of lab analyses.

LONG-TERM PRODUCTION SCHEDULING WITH ESTIMATED (KRIGED) BLOCK MODELS

We now describe how one creates an NPV optimal life-of-mine schedule using a single estimated block model as input data. To simplify the notion of the value of an open pit mine, we shall make several assumptions.

Assumptions for scheduling process

1. The infrastructure is fixed throughout the life of the mine. For example, process plant capacities and mining capacities are fixed[§]. By using additional binary variables to encode a small finite number of possibilities, it is relatively straightforward to include the variation of infrastructure in an optimisation. For example, what size process plant is optimal; when should the plant be expanded or shut down; when should truck fleet sizes be altered to change mining capacity? For clarity we do not include these additional variables in the problem formulation.
2. The selling price of the product is known perfectly into the future. The price and market volume limits (if relevant) may fluctuate over time, but in a completely predictable manner. This is of course not reality; more realistic considerations of price and volume are additional complications that should be modelled properly and subjected to a rigorous analysis that is beyond the scope of this paper.
3. Grade control is assumed to be perfect. That is, once a block has been blasted, its contents are precisely known. This means that a block with concentration below a cut-off grade will never be sent to product and a block with concentration above a cut-off grade will never be sent to the waste dump. This is not realistic; errors in grade control do occur and may be significant. These errors should be modelled as best they can with the available data and incorporated into the valuation model. For simplicity, we do not consider this issue here.

The objective

Our objective is to maximise the net present value (NPV) of the project. Suppose that a project has annual cash flows c1, c2,…,cT. The NPV of the project is:

$$NPV = \sum_{t=1}^{T} \frac{c_t}{(1 + r/100)^t},$$

where:

r is the discount rate

Our mining project will receive a cash flow from every block that is excavated. We assume that at any given time each block can take on one of two values:

$$value = \begin{cases} -\text{ Mining Cost}, & \text{if the block is waste,} \\ -\text{ Mining Cost} - \text{Processing Cost} & \text{if the block is processed.} \\ +\text{ Sales Price} \times \text{Metal Tonnes}, & \end{cases}$$

We assume that there are N blocks under consideration in our block model. Thus there are N possible cash flows denoted *vi* for *i*=1,…,N. We will apply our discount rate on an annual basis, so all blocks taken in the same year receive the same discount rate. Using the formula above, we arrive at:

§ Truck fleet sizes are varied to maintain a constant mining capacity allowing for changes in haul distance with depth. The cost of these truck fleet size variations are not considered.

$$NPV = \sum_{t=1}^{T} \frac{\sum_{i=1}^{N} \chi_{i,t} v_i}{(1 + r/100)^t}, \tag{1}$$

where:

$\chi_{i,t}$ is a 0,1 variable which takes the value 1 if block i is excavated in period t and 0 otherwise

The binary numbers $\chi_{i,t}$ encode the order in which blocks are taken over the life of the mine. We call this collection of binary variables a mining schedule.

Mining and processing limits

An operation can generally only mine and process certain tonnages each year, depending on the capital invested in the mining and processing capacities. Let M denote the maximum amount that can be mined in one year in tonnes and let P denote the maximum amount that can be processed in one year in tonnes. If r_i and o_i denote the amount of rock (ore and waste) and ore (feed tonnes to a process plant) contained in block *i*, then we can set upper limits on mining and processing rates as follows:

$$\sum_{i=1}^{N} \chi_{i,t} r_i \leq M, \quad \textit{for all } t = 1,\ldots,T \tag{2}$$

$$\sum_{i=1}^{N} \chi_{i,t} o_i \leq P, \quad \textit{for all } t = 1,\ldots,T \tag{3}$$

Wall slope considerations

The blocks should be removed in such a way that at the end of each year, the slopes formed by the blocks remaining in the pit are lower than safe upper limits prescribed by geotechnical studies. In reality, these pit slope limits are observed every day; however, as we only track which blocks are taken in which year, and not when a block is taken within a particular year, we only consider slopes at the end of each year. This tracking is accomplished by:

$$\chi_{i,t} \leq \sum_{s=1}^{t} \chi_{j,s}, \quad t = 1,\ldots,T. \tag{4}$$

whenever slope conditions insist that block j must be removed prior to the removal of block i.

Optimising NPV

Our formulation of this deterministic optimisation problem is not new; see, for example, Caccetta and Hill (2003). The objective and constraints on mining and processing limits are all linear, so that in principle we may employ a mixed integer linear program engine to solve our problem. In practice, there are usually too many blocks and periods for such a formulation to be solved in a reasonable amount of time. The results that we will describe in this paper have been constructed using aggregations of blocks as units to be scheduled. It is standard practice in these sorts of problems that blocks be aggregated into larger units; see Ramazan (2007, this volume) for example. These aggregations are built in such a way as to attempt to minimise the effect of the loss of resolution. The algorithm used is proprietary information and cannot be elaborated upon in this forum. Certainly, there is no loss in accuracy of slopes with the aggregations that we use. We have used the optimiser CPLEX9.0 to perform the optimisations.

An example pit

We will illustrate the concepts in this paper with a single product base metal mine. Our input data is in the form of a kriged block model and 25 conditionally simulated block models. The real discount rate used is r=10 per cent. A metal price is given (assumed known and fixed), and fixed mining and processing rates are given (30 million tonnes/annum and five million tonnes/annum respectively). A cut-off grade has been preselected and applied to the block models to generate a value for each block. It is possible, and desirable, to perform the current analysis with variable optimised cut-off grades and variable optimised mining and processing rates, incorporating capital costs, but for simplicity we have not included such considerations. The block models have around 30 000 blocks; for the optimisation process, the blocks were aggregated into larger units in a way that preserves slopes and minimises errors in accuracy. Figure 1 displays a representation of block value for a vertical slice through our example pit. The blocks are grey shaded so that light grey represents the lowest value and dark grey represents the highest value. Figure 1 shows block values for the kriged block model.

REALISING OPTIMISED NPV AND PERFECT BLOCK MODELS

The previous section makes things sound as though the problem of producing a long-term schedule to maximise project NPV is all sewn up, apart from a few approximations with aggregating blocks. In fact, a major assumption is that the block model actually reflects reality in the ground. If the block model contains errors (and it most certainly will) then what have we optimised? We've produced a schedule that maximises project NPV for an incorrect block model. Wherever reality deviates from our block model, our computed NPV will differ from the NPV that will ultimately be realised from the project. It is clear that the closer the block model is to reality, the closer the optimised NPV will be to a value that can be realised. It also seems intuitive that the realised NPV will be greater if one has a more accurate block model to base one's optimisations on. Obtaining a more accurate block model usually involves further drilling to create drill hole data with a finer resolution. Extra drilling costs money, and how can one balance this additional cost against this vague idea that realised NPV increases with more accurate block models? We now embark upon proving and quantifying this intuition that extra knowledge has a real value.

CONDITIONAL SIMULATIONS AND BLOCK MODEL UNCERTAINTY

We will use the notion of conditional simulations to model the uncertainty in our block model. A conditional simulation (eg Goovaerts, 1997; Dimitrakopoulos, in press) is a stochastically generated block model that is consistent with the drill hole data and their spatial continuity. Consistency with the drill hole data primarily means two things:

1. Each conditional simulation's block attributes (mass, grade, etc) for blocks wholly contained in the drill hole cores are identical to those block attributes measured in the drill hole cores.

2. Each conditional simulation is generated so that its block model would generate a variogram identical to one constructed from the drill hole data. The construction process guarantees that the first order and second order statistics of each conditional simulation agrees with the first and second order statistics of the drill hole data (eg The grade-tonnage curves of each conditional simulation are identical to the grade-tonnage curves of the drill hole data).

Existing computer software (Deutsch and Journel, 1997; Remy, 2004) and newer specialised algorithms (Godoy, 2003; Boucher and Dimitrakopoulos, 2007, this volume) can produce as many conditional simulations possible; that is, different equally probable block models of a deposit, all consistent with the drill hole data. Why should this be done? Our intention is to think of each of these conditional simulations as an 'alternate reality'. We recognise that our drill hole data will always be incomplete and there will always be uncertainty about the contents of blocks that have not been drilled. By creating multiple random block models we build up a probability distribution on the space of block models. For example, if we generated 25 conditional simulations then block i would have 25 different grades assigned to it (one for each simulation), and 25 different net values $v_{i,k}$, k=1,…,25. If block i lay along a drill hole core, then the $v_{i,k}$, k=1,…,25 would all equal the net value computed from the measured grade in the core sample. However, if block i lay away from a drill hole, then the $v_{i,k}$, k=1,…,25 could all take on different values.

Figure 2 displays a representation of block values for a vertical slice through our example pit. As in Figure 1 the blocks are grey shaded so that light grey represents the lowest value and dark grey represents the highest value. Figure 2 shows the values constructed from one of the 25 conditional simulations that we produced. Notice that the kriged block model in Figure 1 has a very smooth value or grade distribution, while the conditionally simulated block model in Figure 2 has a much more heterogeneous distribution of value (and therefore grade).

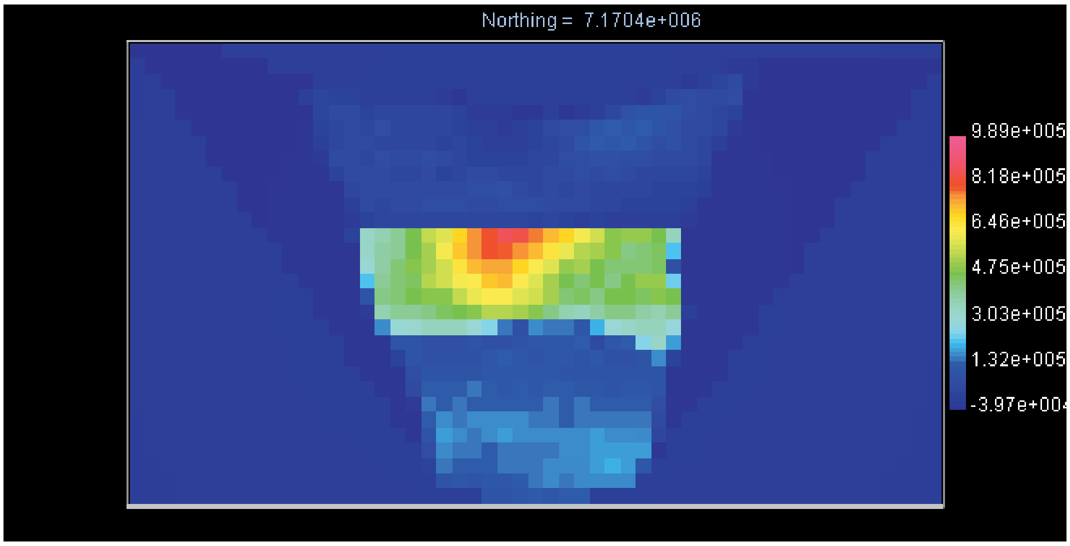


FIG 1 - Kriged block values for a vertical slice through our example pit.

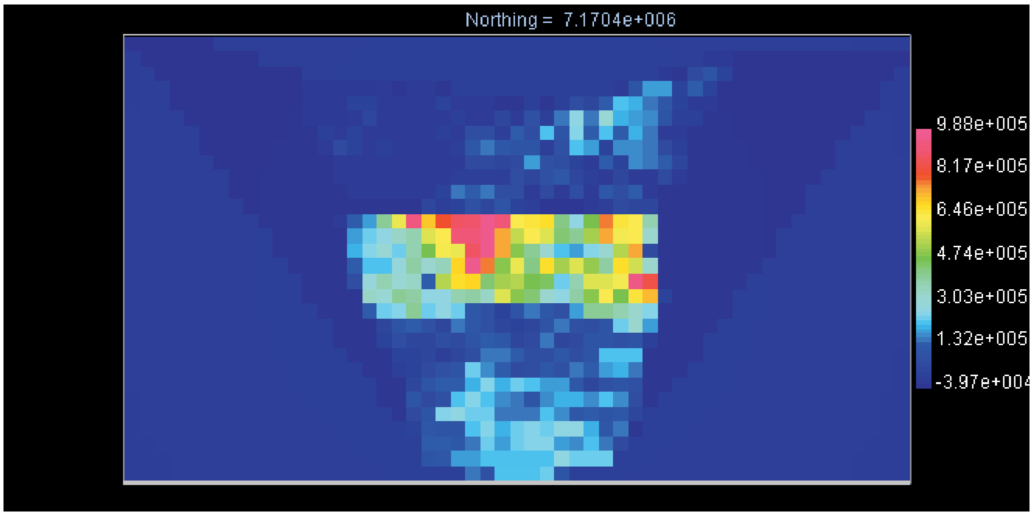


FIG 2 - Conditional simulation block values for a vertical slice through our example pit.

PROJECT VALUATION WITH CONDITIONAL SIMULATIONS

The underlying idea that each conditional simulation represents an alternate equally likely reality of what is actually in the ground rests upon two assumptions. These are that the drill hole data and the derived variogram are:

1. completely true (reality will always agree with the drill hole data and obey the derived variogram), and
2. represent complete information (there is no further information available right now beyond the derived variogram that may help to focus our random sampling further).

If one accepts this idea of alternate realities, which reality should one optimise, if any? Our goal is to determine a schedule s=$\{\chi_{i,t}\}_{i=1,\dots,N}^{t=1,\dots,T}$ that performs well on all or most possible realities. We argue that if one is interested only in maximising NPV (without trying to control risk or uncertainty) then the appropriate thing to do is to find a schedule that achieves the greatest expected NPV. To be precise, let NPV*(k,s)* denote the NPV obtained when the block values in the k^th conditional simulation is used to evaluate using the schedule *s*. Formally:

$$NPV(k,s)=\sum_{t=1}^{T}\frac{\sum_{i=1}^{N}\chi_{i,t}v_{i,k}}{(1+r/100)^t}. \tag{5}$$

Define the expected NPV for a schedule *s* as:

$$\mathrm{E}(NPV(s)):=\frac{1}{k}\sum_{k=1}^{k} NPV(k,s). \tag{6}$$

We propose that one should aim to find the schedule *s** such that:

$$\mathrm{E}(\mathrm{NPV}(s^*))\geq\mathrm{E}(\mathrm{NPV}(s)) \text{ for all feasible schedules } s \tag{7}$$

The schedule *s** will be known as the *schedule that maximises expected NPV.* If one had the opportunity to run the mining project K times, each time using the same schedule but calculating the NPVs on the K different realities (different conditional simulations), then the expected NPV is the natural quantity to maximise. In real life, one only gets one chance to dig up the mine, and the expected NPV will never be realised. What will be realised is NPV(*k**,*s**) where *k** represents the real block model, which is probably different to any of the conditional simulations computed. Nevertheless, we maintain that expected NPV is the best quantity to maximise. To emphasise the fact that this expected NPV is computed using only information available at the present time, we denote E(NPV(*s**)) by $\mathrm{NPV}_{\text{present knowledge}}$.

Optimising expected NPV

Since E(NPV(*s*)) is a linear combination of the linear functions NPV(*k,s*), E(NPV(*s*)) is also a linear function of $s=\{\chi_{i,t}\}_{i=1,\dots,N}^{t=1,\dots,T}$ and so we might try to use a mixed integer linear programming engine to maximise expected NPV. Our objective is:

$$\begin{aligned} E(NPV(s)) &= \frac{1}{K}\sum_{k=1}^{K}\sum_{t=1}^{T}\frac{\sum_{i=1}^{N}\chi_{i,t}v_{i,k}}{(1+r/100)^t} \\ &= \sum_{t=1}^{T}\frac{\sum_{i=1}^{N}\chi_{i,t}\left(\frac{1}{K}\sum_{k=1}^{K}v_{i,k}\right)}{(1+r/100)^t}=\sum_{t=1}^{T}\frac{\sum_{i=1}^{N}\chi_{i,t}\overline{v_i}}{(1+r/100)^t}\end{aligned} \tag{8}$$

The term on the far right-hand side indicates that the expected NPV may be calculated using the mean values of each block v_i computed as $\overline{v_i}=(1/K)\sum v_{i,k}$. This seems natural as we are taking an average. Note that we are averaging the dollar value of blocks, and not the grade of blocks. It is important that one uses the individual block grades $g_{i,k}$ (for block i in conditional simulation k) to compute the block values $v_{i,k}$ and then averages the $v_{i,k}$ (do not average the $v_{i,k}$ and then compute an 'average' value).

Equation 8 takes the place of Equation 1 when maximising expected NPV. We now need to find constraints to replace Equations 2 - 4. Equation 4 may remain the same as all conditional simulations have the same slope conditions. Equations 2 and 3 are problematic as the rock ri and ore oi will vary from simulation to simulation. In the optimisation results reported in this paper, we replace the rock and ore tonnages ri and oi in Equations 2 and 3 with their mean values calculated as $\overline{r_i}=(1/K)\sum r_{i,k}$ and $\overline{o_i}=(1/K)\sum o_{i,k}$. This is an approximation that may result in some schedules being infeasible in terms of mining or processing rate for some individual conditional simulations. We believe that the numerical results reported in this paper are relatively insensitive to this approximation.

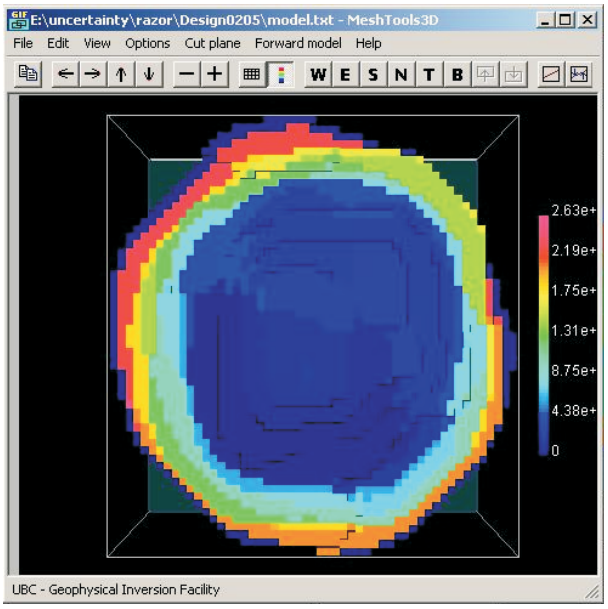


FIG 3 - Plan view of our example pit with blocks coloured according to the schedule obtained by optimising expected NPV.

The schedule obtained by optimising expected NPV is shown in Figure 3. The expected NPV obtained was $761.8 M; a value that is guaranteed to be within 0.2 per cent of the true optimum by our mixed integer linear programming engine CPLEX. Figure 3 is a plan view of our example pit with blocks grey shaded according to their year of excavation; those blocks coloured light grey are excavated first, while those coloured dark grey are excavated last. The white blocks around the edge of the pit are never excavated.

PROJECT VALUATION WITH PERFECT GEOLOGICAL KNOWLEDGE

So far we have been able to compute a schedule s* that maximises the expected NPV of our mining project based on our current knowledge of the orebody. We will now compute the best expected NPV we could achieve if we had complete knowledge of the orebody. Complete knowledge of the orebody is the extreme situation where we drill so much that we know exactly what is in the ground in every block.

Because we know the block model exactly before excavation begins, we can tailor our schedule to that block model. At this stage, we only have the K conditional simulations as possible realities. Complete drilling to resolve exactly what is in the ground is equivalent to knowing exactly which conditional simulation is reality (drawing from our limited selection of K alternate realities). If it turns out that simulation k is reality, we can produce schedule s(k) with the property that:

$$NPV(k, s(k)) \geq NPV(k, s) \qquad \text{for all schedules } s \qquad (9)$$

Let's look at these schedules *s(k)* for our example pit. Figures 4 and 5 show vertical slices through two of the 25 conditional simulations; their simulation numbers are 20 and 8, respectively. The two chosen are the simulations with the highest total block value (#20, Figure 4) and lowest total block value (#8, Figure 5). As before, light grey represents low-value blocks and dark grey represents high-value blocks. Each of these two block models was individually optimised to produce schedules *s*(20) and *s*(8) each satisfying property (9). These schedules are displayed in Figures 6 and 7, whereas before, light grey represents those blocks taken early in the mine life and dark grey represents those blocks taken latest in the mine life. In this example, there are subtle differences between the schedules, but no dramatic difference in how one should excavate the two orebodies.

Returning to our discussion, one must bear in mind that we cannot control which simulation is reality, we only know which one it is. We therefore still need to perform an average. If we know before excavation begins which simulation is reality, then on average we can achieve an NPV of:

$$NPV_{perfect\ knowledge} = (1/K)\sum_{k=1}^{K} NPV(k, s(k)) \qquad (10)$$

where each *s(k)* has the property (9) for k=1,…,K.

$NPV_{\text{perfect knowledge}}$ denotes the expected value of the project if we are able to 'wait-and-see' which conditional simulation is reality before making our schedule (our schedule is based on 'perfect' geological information). For each simulation, we tailor our schedule to that block model, and can have different schedules for different simulations, because we know beforehand which block model is reality. Contrast this to Equation 7 where we had to choose a single schedule upfront. For our example pit, we performed 25 separate optimisations to find the 25 individually optimal schedules *s(k)*. Using Equation 10, we computed that $NPV_{\text{perfect knowledge}}$ = $769.36 M.

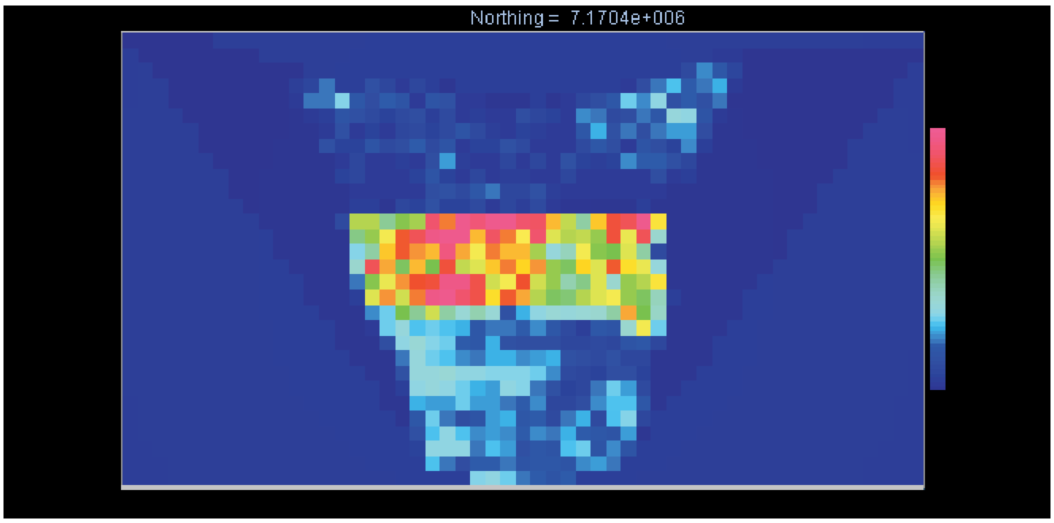


FIG 4 - Conditional simulation block values for a vertical slice through our example pit. This simulation has the highest total block value.

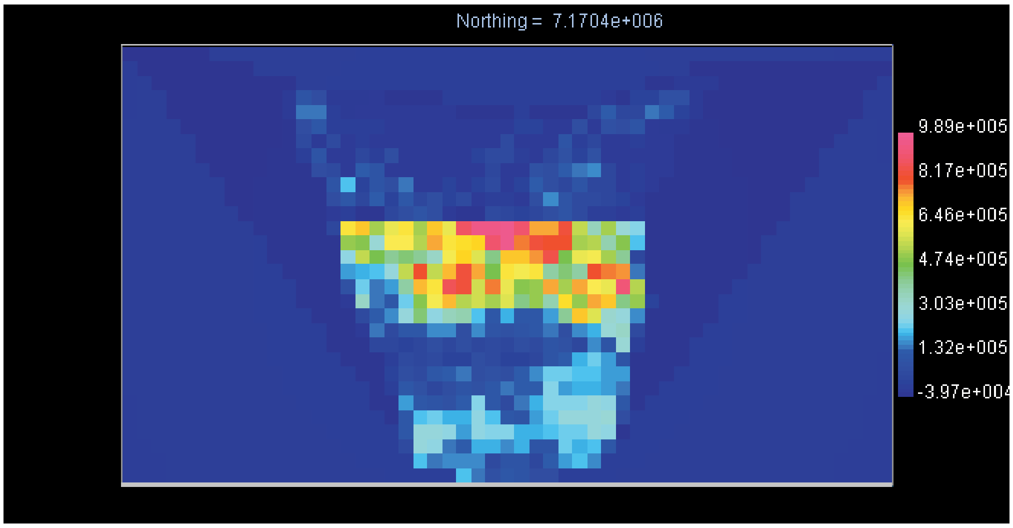


FIG 5 - Conditional simulation block values for a vertical slice through our example pit. This simulation has the lowest total block value.

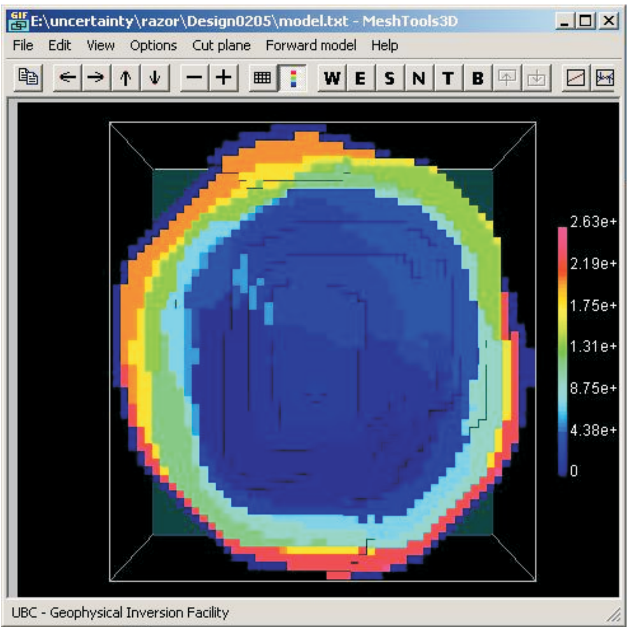


FIG 6 - Plan view of our example pit with blocks coloured according to the schedule obtained by individually optimising the conditional simulation shown in Figure 4.

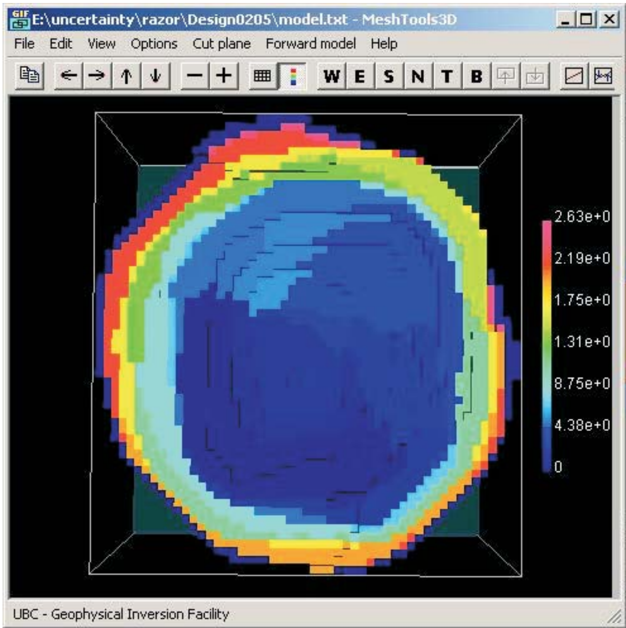


FIG 7 - Plan view of our example pit with blocks coloured according to the schedule obtained by individually optimising the conditional simulation shown in Figure 5.

THE VALUE OF INFILL DRILLING INFORMATION

We now have two NPVs; one representing the best expected NPV achievable with no extra drilling and our present state of knowledge, and the other representing the best expected NPV achievable assuming perfect knowledge of the orebody prior to producing a schedule. These values are $\text{NPV}_{\text{present knowledge}}$ = \$761.8 M and $\text{NPV}_{\text{perfect knowledge}}$ = \$769.36 M respectively. Thus the value of having perfect orebody knowledge prior to scheduling is:

$$\text{VOIDI} := \text{NPV}_{\text{perfect knowledge}} - \text{NPV}_{\text{present knowledge}} \qquad (11)$$

where VOIDI stands for 'Value of Infill Drilling Information'. We will show that VOIDI represents an upper bound for the NPV increment (not including drilling costs) achievable through additional drilling.

It is relatively straightforward to see that VOIDI is always non-negative:

$$\begin{aligned} NPV_{perfect\ knowledge} &= (1/K)\sum_{k=1}^{K} NPV(k, s(k)) \\ &\geq (1/K)\sum_{k=1}^{K} NPV(k, s*) \quad by\ property(9) \\ &= NPV_{present\ knowledge} \end{aligned}$$

How is VOIDI related to the cost of future drilling programs? Any additional drilling will result in the conditional simulations being updated. The spread of block values will generally lessen between simulations because we have more drill holes and we are more certain about the block values. Every extra hole drilled has the potential to add value to the project because we might be able to use that extra information to change our schedule and create greater project NPV. The option to embark on additional drilling can be valued as:

$$\text{Value of Additional Drilling} = (NPV_{\text{additional drilling}} - NPV_{\text{present knowledge}}) - \text{Drilling Cost}$$

At present we can value $NPV_{\text{present knowledge}}$ and Drilling Cost, but we cannot value $NPV_{\text{additional drilling}}$. What we do know is that $NPV_{\text{additional drilling}} \leq NPV_{\text{perfect knowledge}}$. This is because we can never achieve perfect knowledge through additional drilling, and we will never actually realise $NPV_{\text{perfect knowledge}}$. Thus:

$$\text{Value of Additional Drilling} \leq (NPV_{\text{perfect knowledge}} - NPV_{\text{present knowledge}}) - \text{Drilling Costs} = \text{VOIDI} - \text{Drilling Cost.}$$

The conclusion that one can draw from this is that one would never embark on an additional drilling program if the drilling costs exceed VOIDI.

VOIDI FOR OUR EXAMPLE PIT

In the case of our example pit, VOIDI = 769.36 - 761.8 = \$7.56 M. As a fraction of total project value, VOIDI is around one per cent; a very low figure. This indicates that it is probably not worthwhile performing any further drilling on our example resource¤. While we will show in Figure 8 that there is a significant variation in block values between different conditional simulations, and therefore, significant uncertainty in our block model, the NPV-optimal schedules that are tailored to each conditional simulation are not very different. Thus, knowing which block model is reality does not change your decision about how to excavate the pit, and therefore does not generate any additional value for the project. *Additional information only creates value if value-creating decisions are changed in light of the new information.*

Let us review the results of our optimisations in greater detail. Let NPV*(k,s(m))* denote the optimal schedule for simulation m evaluated using simulation *k*, where m=1,…,25, and k=1,…,25. The grey shaded lines in Figure 8 plot the 25 × 25 = 625 NPVs corresponding to NPV*(k,s(m))*, where the y-axis is NPV*(k,s(m))*, and the x-axis is *k*. Thus each vertical column corresponds to a single simulation *k*. It is clear from Figure 8 that *the dominant value differences arise from different simulations, not different schedules*. In fact, relative to variations between simulations, the values are insensitive to schedule differences.

The highlighted red (dark grey) dots are the 25 values of NPV*(k,s(k)),* namely, an optimal schedule for simulation k evaluated with its corresponding simulation. Thus the dark grey dots should appear at the top of the vertical spread of points. The value of $NPV_{\text{perfect knowledge}}$ is the mean value of the dark grey dots.

The highlighted light grey dots are the 25 values of NPV(*k,s**), k=1,…,25. The value $NPV_{\text{present knowledge}}$ is the mean value of the light grey dots. The value of VOIDI is therefore the average difference in value between corresponding light grey and dark grey dots. As the spread for each simulation is relatively small, and the light grey dots are mostly at the upper side of this small spread, the difference between dark grey and light grey is small (the average difference is \$7.56 M).

CONCLUSIONS

We have described a rigorous computational method of determining the largest amount that should be paid for a program of additional infill drilling on an existing resource. This method required the construction of K conditional simulations, each of which was consistent with the existing drill hole data. These K conditional simulations were used to produce K individually optimised schedules *s(k)*. A single maximum expected NPV schedule *s** was also generated via a single optimisation. These K+1 NPVs were then combined to produce VOIDI: $=NPV_{\text{perfect knowledge}} - NPV_{\text{present knowledge}}$. In the case of our example pit, VOIDI clearly demonstrated that it was highly unlikely that any additional drilling would create further project value, saving the company money on extra drilling. The lesson to be learnt here is that high block variability in conditional simulations does not always imply that there is value in further drilling to decrease this variability.

¤ Bear in mind that VOIDI has been calculated under specified conditions of mining rate, processing rate and cut-off grade and is dependent on these parameters.

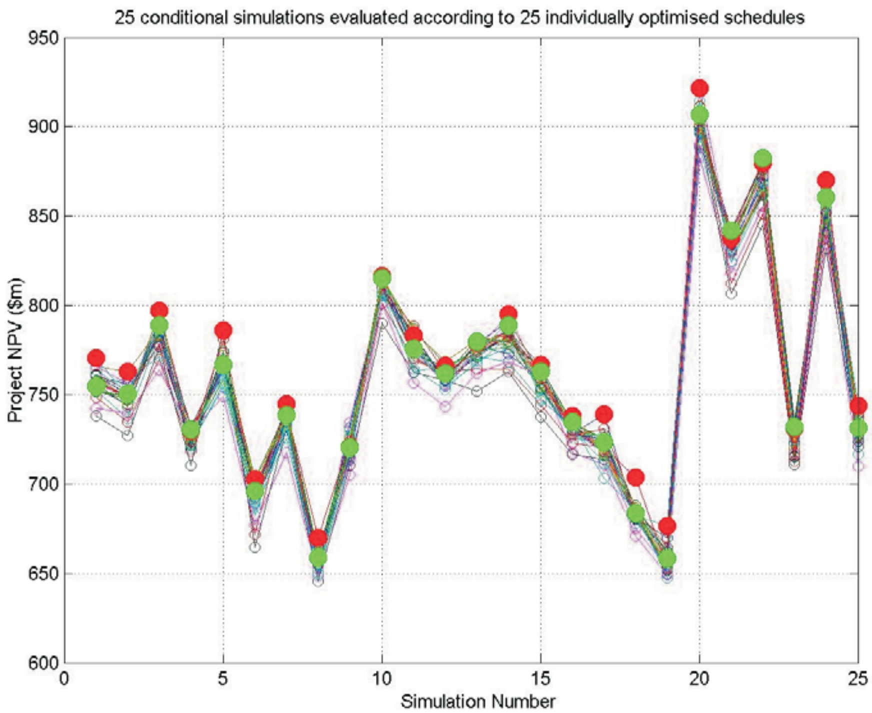


FIG 8 - Valuations of schedules: (i) individually optimised, (ii) optimising expected NPV.

The notion of VOIDI is an extremely useful quantification tool that formalises thinking on the matters of risk and uncertainty, and knowledge and information. Without such formal quantities, one's thinking can become very fuzzy. Of course, this analysis is only as good as the conditional simulations are at representing the true uncertainty in our current state of knowledge. If the conditional simulations do not capture the full uncertainty and provide an accurate sample of the full allowable variation of block values, then VOIDI will appear smaller than it really is.

Some final observations

1. An infill drilling program may delay the starting of mining. This will mean that NPVperfect knowledge may be lowered due to this delay. We have not taken this delay into account in our analysis, although any effect will be to reduce the value of $NPV_{perfect\ knowledge}$, and therefore lower the value of VOIDI.
2. One should bear in mind that VOIDI is a function of parameters such as:
   (i) mining rate;
   (ii) processing rate; and
   (iii) cut-off grade, and that under different conditions, the potential value of a drilling program may be more or less valuable.
   a. For example, a doubling of mining and/or processing rates will increase NPV through a more rapid mine exploitation. VOIDI will increase in proportion to the NPV increase; that is, both expected NPV and VOIDI will increase by a roughly equal percentage.
   b. The effect of changing cut-off grade may have a non-trivial impact on VOIDI.
3. We have assumed that the resource is contained within the boundary of outer drill holes. Clearly we cannot say anything about further value to be gained on extra drilling of resources which are not well contained within the existing drill hole boundary.
4. The conditional simulations we used were based on prescribed geological regions in the block model. Within each of these distinct geological regions a different variogram was used and the block grades were simulated independently of block grades in other regions. The regions arose from a single geological interpretation of the drill hole data. In order to capture the full variability, we require a rigorous method of computing multiple randomly generated volumes and boundaries for each geological region. Within each of these volumes we should conditionally simulate grade values as before. To our knowledge, the problem of properly performing conditional simulation of volumes has not been solved.
5. Our optimisation process produces a block schedule while in practice, blocks are removed as benches in phases or pushbacks. The block schedules that we have evaluated in this paper are valid in the sense that all slope precedence constraints are enforced; however, it is unlikely that our block schedules would be mineable in practice. A full analysis would require constructing phases or pushbacks from our K+1 optimised block sequences and then optimising a panel or bench schedule for each of the K+1 pushback designs.
6. In practice, one would not drill the entirety of the orebody to fully achieve the NPV increment promised by VOIDI. Rather, one wishes to target those blocks that if drilled, would lead to the greatest increment in NPV. Ideally, one would like to balance the drilling cost against the NPV increment and arrive at an optimal drilling program that is different to 'drill everywhere'. There are some rules of thumb about which blocks you might choose to selectively drill (eg those blocks with high grade variability and a mean grade around the cut-off grade⌘, or those blocks that are extracted in different periods when the different conditional simulations are individually optimised). To formulate the problem rigorously as an optimisation problem is difficult. One could for example:
   (i) select blocks to be drilled based on the above rules of thumb;
   (ii) turn to each of the K conditional simulations and fix the grades of those blocks;
   (iii) for each of the K conditional simulations, produce another K simulations using variograms constructed from the additional hypothetical drill holes, leading to K^2 simulations in all; and
   (iv) calculate VOIDI in an analogous way to that described earlier.

This procedure would value a putative additional drilling program. To identify rigorously optimal locations for future drill holes is a far more difficult problem. In this paper we have presented a rigorous valuation method that gives an idea of the 'size of the prize' if additional drilling were undertaken. Our method is a decision-making aid. On the basis of VOIDI, the decision of whether to drill further may become very simple.

⌘ Rendu (1970) used the block kriging variance to estimate the likelihood of a block being re-allocated to ore or waste in light of further drilling information. His work showed that there is little point in drilling a regular drill pattern for areas of 'known' waste or 'known' high-grade ore.

REFERENCES

Boucher, A and Dimitrakopoulos, R, 2007. A new efficient joint simulation framework and application in a multivariable deposit, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 345-354 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Caccetta, L and Hill, S P, 2003. An application of branch and cut to open pit mine scheduling, *Journal of Global Optimisation*, 27:349-365.

Cressie, N, 1991. *Statistics for Spatial Data* (Wiley: New York).

Deutsch, C V and Journel, A G, 1997. *GSLIB: Geostatistical Software Library and User's Guide* (Oxford University Press).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Godoy, M C, 2003. The efficient management of geological risk in long-term production scheduling of open pit mines, PhD thesis, 256 p, The University of Queensland, Brisbane.

Goovaerts, P, 1997. *Geostatistics for Natural Resources Evaluation* (Oxford University Press).

Johnson, T B, 1968. Optimum open pit mine production scheduling, PhD thesis, University of California, Berkeley, CA.

Lerchs, H and Grossmann, I, 1965. Optimum design of open pit mines, *Transactions CIM*, LXVIII:17-24.

Ramazan, S, 2007. Large-scale production scheduling with the fundamental tree algorithm — Model, case study and comparisons, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 121-127 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2004. Recent applications of operations research in open pit mining, *SME Transactions*, 316:73-78.

Remy, N, 2004. S-GeMS – Geostatistical Earth Modeling Software: User's Manual, Stanford University, 87 p. [online] Available from: <http://sgems.sourceforge.net>.

Rendu, J M, 1970. Some applications of geostatistics to decision making in exploration, in *Proceedings APCOM 1970*, pp 175-184.

Quantification of Geological Uncertainty and Risk Using Stochastic Simulation and Applications in the Coal Mining Industry

S Li[1], R Dimitrakopoulos[2], J Scott[3] and D Dunn[4]

ABSTRACT

Stochastic simulation is a recognised tool for quantifying the spatial distribution of geological uncertainty and risk in earth science and engineering. Metals mining is an area where simulation technologies are extensively used; however, applications in the coal mining industry have been limited. This is particularly due to the lack of a systematic demonstration illustrating the capabilities these techniques have in problem solving in coal mining.

This paper presents two broad and technically distinct areas of applications in coal mining. The first deals with the use of simulation in the quantification of uncertainty in coal seam attributes and risk assessment to assist coal resource classification, and drill hole spacing optimisation to meet pre-specified risk levels at a required confidence.

The second application presents the use of stochastic simulation in the quantification of fault risk, an area of particular interest to underground coal mining, and documents the performance of the approach. The examples presented demonstrate the advantages and positive contribution stochastic simulation approaches bring to the coal mining industry.

INTRODUCTION

Coal exploration, mine planning, economic valuation of coal assets, and coal production forecasting depend on the ability to effectively and reliably delineate, understand and assess coal resources and reserves. In turn, this ability supports investment decisions in exploration programs, development and production that are in the order of billions of dollars. Furthermore, Stock Exchange reporting of resources and reserves, aiming to benefit shareholders and attract the investment community, critically depends on the assessment of geological risk. Geological uncertainty is recognised as a critical factor in establishing accurate and reliable estimation, categorisation and economic assessment of coal resources and reserves, in terms of quality and quantity. Incomplete understanding of geological risk, including fault risk, is recognised as a major contributing factor to mining projects not meeting their financial expectations.

Stochastic simulation methods offer the technologies used to quantify geological risk. They are increasingly applied for this reason in metal mining and applications are widely reported (Dimitrakopoulos, in press; Dowd, 1997; Ravenscroft, 1992), including several papers in this volume. The practical application of simulation methods has been enhanced with the development of fast and efficient simulation algorithms better enabling the simulation of large, complex orebodies (Benndorf and Dimitrakopoulos, 2007, this volume; Boucher and Dimitrakopoulos, 2007, this volume) and their integration with mine planning, design and production scheduling (Godoy and Dimitrakopoulos, 2004; Ramazan and Dimitrakopoulos, 2007, this volume; Menabde *et al*, 2007, this volume).

When compared to metal mining, there have been limited applications of stochastic simulations in the coal mining industry. Stochastic simulation is now being adopted, recognising the inefficiencies of traditional approaches to:

1. model coal seams based on drill hole information,
2. assign and classify coal resources,
3. establish drill hole spacing requirements for resource classification, and
4. identify the location of faults.

Two new developments in modelling geological uncertainty and quantifying the related risk with applications to coal mining are presented herein. The first development, extensively reported in Dimitrakopoulos, Scott and Li (2005), refers to the use of stochastic simulation methods to quantify risk in coal seams estimated with conventional methods, to assist Competent Persons in classifying resources and report the level of error with a given confidence. In addition, the approach developed provides the means to test the performance of drilling patterns and optimise data collection based on the local characteristics of the seam considered and a pre-specified error and confidence level. The second development, detailed in Dimitrakopoulos *et al* (2001), examines the simulation of fault systems and quantification of fault uncertainty. The performance of the approach in a back analysis study at a mined out part of a longwall coal mine elucidates the method and documents the performance of stochastic modelling, its advantages and characteristics.

The methods and work presented in this paper were funded by the Australian Coal Association Research Program (ACARP Projects C7025 and C11042) as well as Anglo Coal Australia, BHP Billiton Mitsubishi Alliance, Coal and Allied (Rio Tinto Coal) and Xstrata (previously MIM).

QUANTIFICATION OF GEOLOGICAL UNCERTAINTY AND RISK IN COAL RESOURCE ESTIMATION AND CLASSIFICATION

The new JORC Code (2004) requires that resource reporting be related to the level of geological confidence, that is, quantified geological uncertainty, for mining companies listed on the ASX. These companies and their Competent Persons are required to ensure that the resource computations and classifications comply with the basic JORC requirements of transparency, materiality and competency. Traditional approaches to the classification of resource have tended to use subjective criteria to define the limits of measured, indicated and inferred resource polygons. Existing guidelines encourage resource classification based on the maximum distances between drill holes and the number of holes drilled, without sound, scientific justification. The stochastic simulation approach to quantifying errors at a specified confidence interval in coal resource estimation to assist Competent Persons is presented next.

1. CRCMining, The University of Queensland, 2436 Moggill Road, Pinjarra Hills Qld 4069, Australia. Email: s.li@crcmining.com.au

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

3. Roche Mining, PO Box 2569, Nerang MDC Qld 4211, Australia.

4. MAusIMM(CP), Manager Geological Services, BHP Billiton Mitsubishi Alliance, GPO Box 1389, Brisbane Qld 4001, Australia. Email: doug.l.dunn@bhpbilliton.com

A methodology for risk quantification

The method proposed for quantifying risk (Dimitrakopoulos, Scott and Li, 2005; Li, Dimitrakopoulos and Scott, 2004) involves the use of stochastic simulation to produce multiple coal resource models using all available drill hole data. With the simulated models representing the 'actual' deposit, a conventional orebody model can be assessed in terms of its ability to accurately predict reality. Figure 1 graphically illustrates the method. More specifically the method proceeds as follows:

1. Generate a high-resolution coal deposit model (the 'actual' deposit) using stochastic simulation based on all coal seam data and geological information.
2. Reblock the points in the simulated coal deposit model to blocks of the same size used in the estimated seam model below.
3. Use a conventional method to generate an estimated seam model based on coal seam exploration data at the desired block size.
4. Calculate the relative absolute error of each block in the estimated deposit developed in step three by comparing it to the reblocked simulated deposit in step two. The relative error of a unit block *j* is computed from:

$$\varepsilon_{ij} = \frac{|v_{sij} - v_{ej}|}{v_{sij}} \qquad (i = 1, \ldots, n;\ j = 1, \ldots, m) \tag{1}$$

where:

ε_{ij} is the relative absolute error of the unit block *j* with reference to the simulated deposit *i*

v_{sij} is the reblocked simulated value *i* of the unit block *j*

v_{ej} is the estimated value of the unit block *j*

n is the total number of simulated deposits

m is the number of unit blocks within the study area

5. Repeat for a large number of simulated deposits (eg 50 simulations).
6. Summarise results graphically to illustrate the expected difference between an estimate and possible seam attribute values and the relationship between drill hole spacing.

The outcome of the above process is the spatial distribution of relative errors associated with the estimated coal resource model given the available drilling patterns and the block size considered. The program 'GEOCOAL' implements the above process (Li, Dimitrakopoulos and Scott, 2004) and is based on the sequential Gaussian simulation method (Dimitrakopoulos and Luo, 2004; Journel, 1994).

A case study with coal seam thickness

The method described above is applied to a coal seam in central Queensland, Australia, to demonstrate how geological risk can be quantified in a practical situation. Figure 2a shows the coal seam thickness data in the study area, and Figure 2b shows one of the simulated models of coal seam thickness on a dense grid corresponding to step one of the method described above. Figure 3a shows the estimated coal seam thickness model for 50 by 50 m blocks from step three of the method above. The relative error associated with the conventional model is based on the estimated model and the reblocked simulated deposits using the formula given in step four above. Figure 3(b) shows the spatial distribution of the relative errors associated with the conventional coal seam thickness model. It is important to note that the confidence level for the relative errors shown is 95 per cent, and is derived numerically from the use of multiple simulated seam scenarios.

The quantified errors derived by the method used here reflect both the drill hole spacing as well as the *in situ* variability of the coal seam. For example in Figure 3b, the relative errors in the upper left section tend to be higher than those in the lower part of the seam (between ten per cent and 20 per cent) due mostly to the sparser drilling in that part of the study area. The lower part of the study area shows relative errors less than five per cent and, although denser drilled, these low errors mostly reflect the local low variability in coal seam thickness. It is clear that these two areas of the coal seam will require different drilling densities to generate the same level of errors at the same confidence level. Similarly to thickness, any other attribute of the coal seam can be modelled and errors assessed.

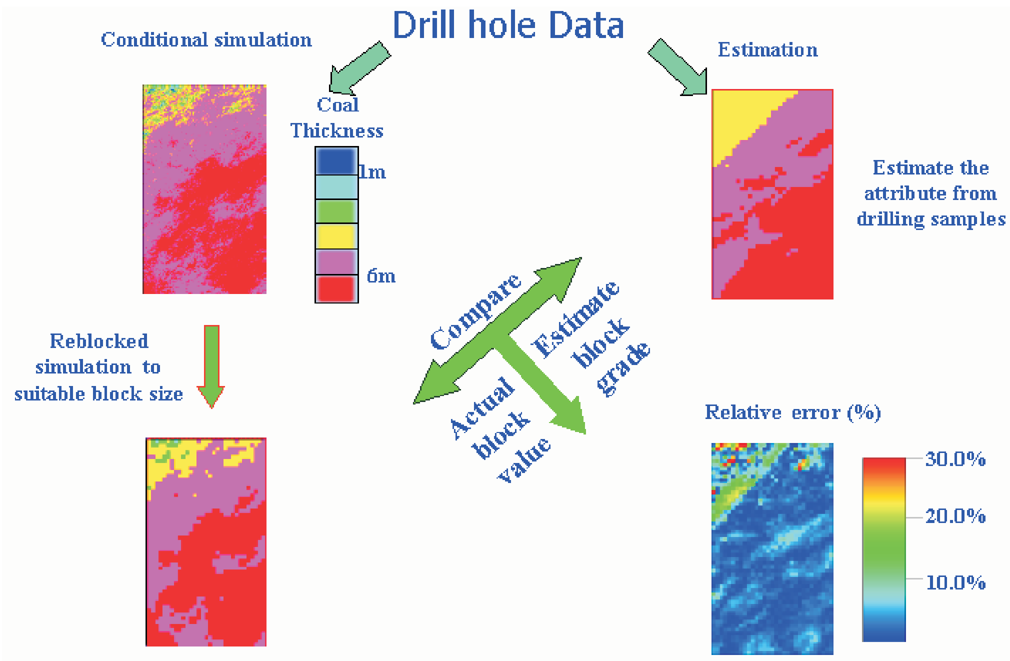


FIG 1 - Schematic representation of the method for the quantification of geological uncertainty in coal resource estimation.

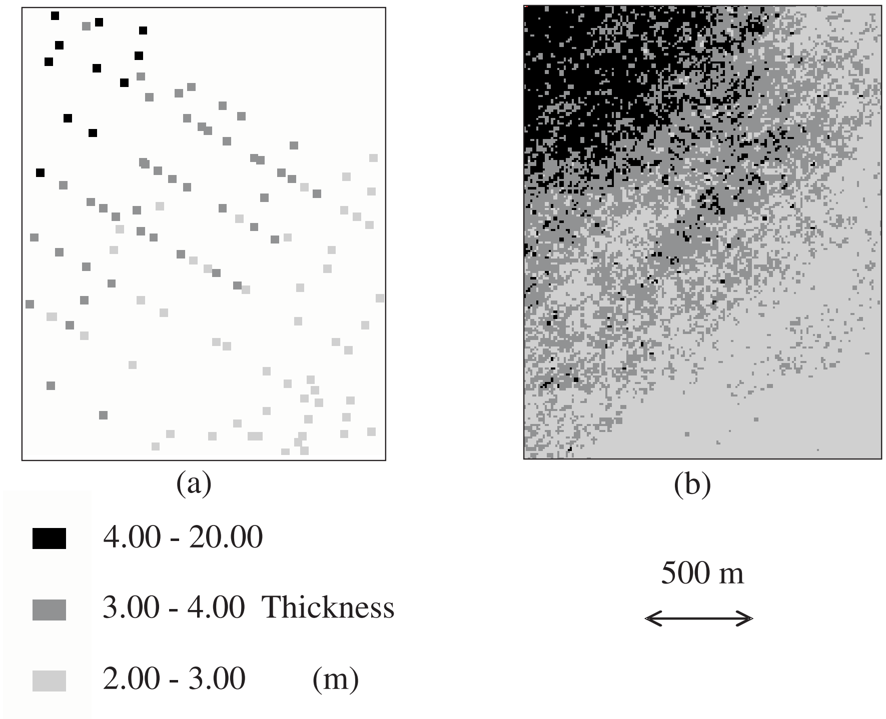


FIG 2 - (a) Coal seam thickness data; (b) one realisation of simulated coal seam thickness.

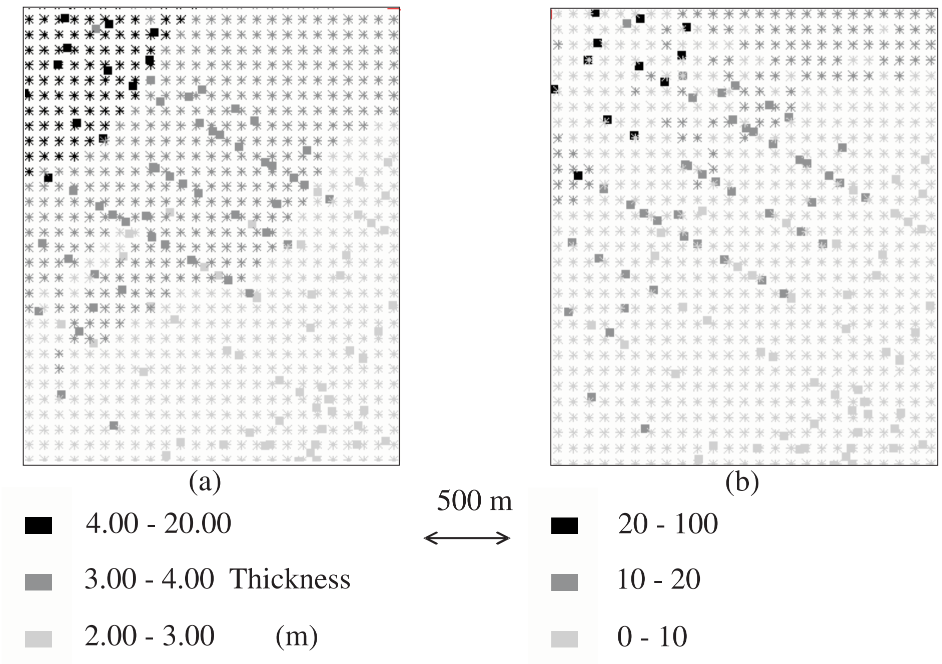


FIG 3 - (a) Conventionally estimated coal seam thickness in the study area; (b) errors associated with the conventionally estimated coal seam thickness model.

Extending the method to optimise drill hole spacing

The method presented above can be extended to assess the value of drilling campaigns before the drilling is conducted. The quantification of expected errors in estimates ahead of actual drilling would reduce over- and under-drilling. Desired criteria, such as the increase of expected confidence levels sought in resource estimates can be tested. For example, a drilling campaign can be designed to generate errors on estimates that are expected to be ±10 per cent at a 95 per cent confidence level.

In practice, alternative drilling patterns are designed, and all simulated deposits generated previously are sampled. The virtual samples are then used exactly as real data in the error quantification process previously described. Figure 4 plots the average relative errors of seam thickness associated with selected drilling densities (200 × 200 m^2, 300 × 300 m^2, 500 × 500 m^2, 800 × 800 m^2 and 1000 × 1000 m^2) for the same seam and study area shown earlier. The overall relative error of seam thickness associated with each drill hole spacing pattern up to 500 × 500 m^2 is less than five per cent at the 95 per cent confidence level, reflecting a general regularity in seam thickness. If an error

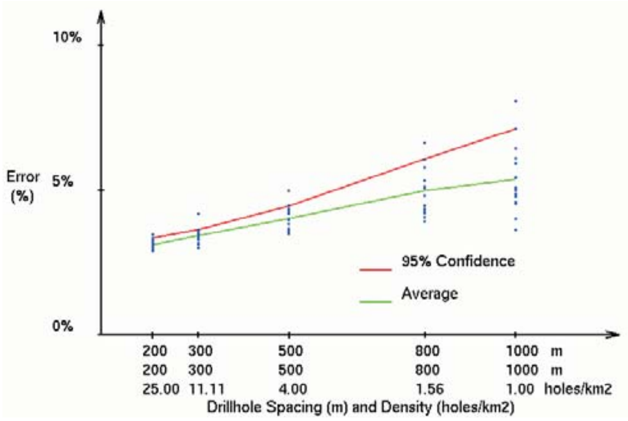


FIG 4 - Errors of seam thickness associated with various selected drilling densities.

of estimation less than ten per cent with a 95 per cent confidence is required, then the seam should be drilled at spacings over 1000×1000 m^2.

Figure 5 shows the spatial distributions of errors for two experimental drill hole spacing designs, 500×500 m^2 and 800×800 m^2, in the same study area with 95 per cent confidence levels. The estimation errors at the upper left in both Figure 5(a) and (b) are higher than those at the lower right, which is likely due to the higher seam variability in this area. This example graphically illustrates how the method proposed here can assist in identifying parts of a study area that may require a different spacing. More specifically, if for example an error less than ten per cent at 95 per cent confidence is needed, the drill hole spacing in the upper part of the area shown in Figure 5 should be, at most, 500×500 m^2, whilst the drill hole spacing in the lower-right part need not be less than 800×800 m^2. Alternative approaches to optimising drill holes are given in Dimitrakopoulos (in press) and Froyland *et al* (2007, this volume).

QUANTIFICATION OF FAULT UNCERTAINTY

A companion aspect to the uncertainty modelling of quantity and quality parameters of coal seams, as well as geological risk quantification for resource classification, is geological uncertainty and risk due to structural deformation. Faults are a major factor impacting particularly underground longwall mining. Unlike the so-called continuous parameters of coal seams that are stochastically simulated with a variety of methods for continuous variables (Dimitrakopoulos, in press), faults are 'discrete' objects and require the development of complex approaches, such as the one described in Scott *et al* (2007, this volume). The approach is based on fractal fault size distributions and length-throw statistical relations, combined with a probability field approach to 'thinning' a Poisson process so as to locate fault centres. The following sections visit this method in a 'back-analysis' case study that assesses the performance of the specific method and provides an insight to the stochastic simulation framework.

Stochastic simulation of faults and field testing

To assess the above-mentioned fault simulation method, a fully mined part of a longwall mine is used as detailed in Dimitrakopoulos and Li (2001), Dimitrakopoulos *et al* (2001) and Li *et al* (2001). Two data sets are formed:

1. the complete data set available, used as the ground truth to assess the fault simulation method; and
2. a subsample of this data set that resembles the level of fault mapping and information available at the time of the longwall design from 'exploration' sources (referred to here as the 'exploration' data set); this 'exploration' data set is used to generate statistics of fault population characteristics and simulate fault populations.

Figure 6(a) shows the complete fault data set in the mined out part of the corresponding longwall mine, and Figure 6(b) shows the 'exploration' data set.

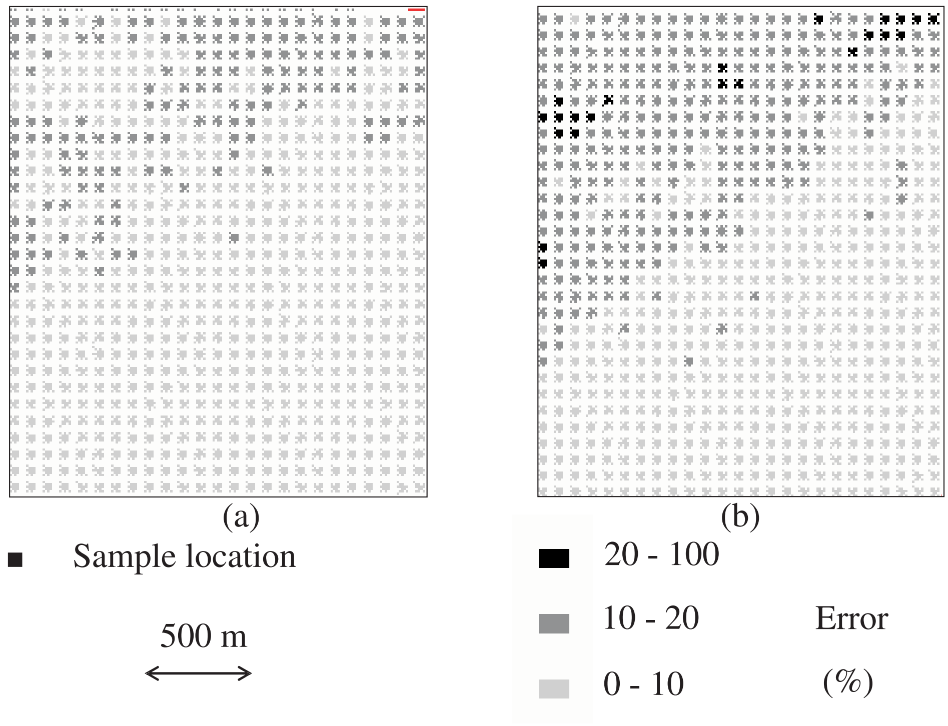


FIG 5 - Spatial distribution of expected estimation errors of seam thickness associated with drill hole spacings of (a) 500×500 m^2, and (b) 800×800 m^2.

FIG 6 - (a) Complete mapped fault dataset from the mined out part of a longwall mine; (b) 'exploration' fault dataset, a subsample of the complete data set (faults shown have a throw ≥1 m).

Two simulated fault populations using the 'exploration' data set are shown in Figure 7. The simulated fault populations reproduce the faults in the data set in Figure 6(b), and honour the fault characteristics derived from this 'exploration' fault data set; such as fractal characteristics, the power-law relationship between fault length and throw, and the fault strike distributions (Dimitrakopoulos *et al*, 2001). In comparing the simulated fault populations with the complete data set shown in Figure 6(a), the similarity between the simulated fault population and the complete fault data is evident, both in terms of the spatial distribution and density of faults.

A set of 50 simulated fault populations based on the 'exploration' data set is used to generate the fault probability map shown in Figure 8(a). Figure 8(b) shows the fault probability based only on the faults in the 'exploration' data set (70 faults with throw ≥1 m) and Figure 8(c) illustrates the fault probability using the complete data set (231 faults with throw ≥1 m). The conventional approach used for assessing or designing a longwall mine considers 'exploration' data sets only, resulting in the underestimation of actual fault risk. In contrast, the fault probability map based on 50 simulated fault populations corresponds to about 207 faults with throw ≥1 m and provides a realistic assessment of risk when compared to the true fault risk. Locations denoted by a '1' in Figure 8 indicate areas that have been accurately predicted to have a high fault risk. Locations denoted by a '2' are where the fault simulation method overestimates risk. Locations denoted by a '3' are where the fault simulation method has slightly shifted actual high-risk areas.

The example presented here provides a positive assessment in using simulation methods. Its ability to generate a more realistic assessment of fault risk than the spatially limited and incomplete exploration data set alone is apparent.

Integrating fault risk to resource classification

The ability of the above simulation approach to provide a realistic assessment of fault risk has ramifications to coal mining. One of these is the integration of quantified risk from different sources with respect to resource classification. It is relatively simple to combine assessments of resource risk as discussed earlier, such as coal resources estimation errors and fault probabilities. For example, Figure 9(a) shows the error map in coal tonnage in a lease and Figure 9(b) shows a map of the probability of faulting. Figure 9(a) indicates that estimation errors in coal tonnage are less than 20 per cent over the study area. If a threshold of 20 per cent were used for measured resources, the entire study area would be classified as a measured coal resource. However, Figure 9(b) shows that the fault probabilities in sections A, B, C, D and E are as high as 100 per cent and these sections should therefore be excluded from the measured resource classification. Conversely, in sections F and G the fault probabilities are between ten and 30 per cent implying that the coal resources in these sections could be measured rather than indicated, pending further drilling for fault detection. An alternative approach may be to consider assigning dollar values to different fault probabilities such that sections with a high probability of faults are assigned the highest cost of mining. This leads to the discounting of the value of a coal resource based on fault risk and allows coal resource classification to incorporate faulting information.

CONCLUSIONS

Stochastic simulation methods can assist in addressing the quantification of geological uncertainty adversely impacting various aspects of coal mining, including resource classification, drill hole spacing optimisation and quantitative fault risk assessment.

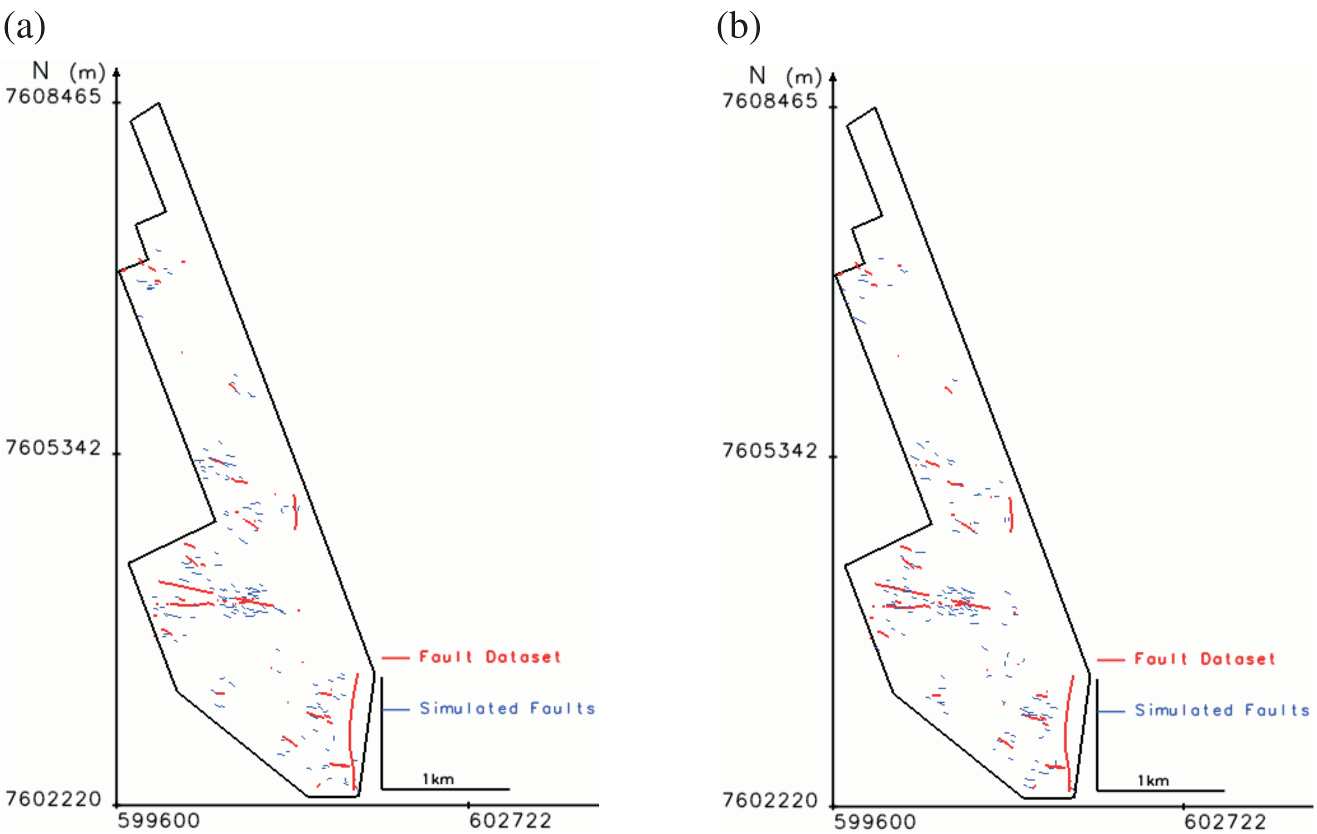


FIG 7 - Two fault realisations using the 'exploration' fault dataset (faults shown have a throw ≥1 m).

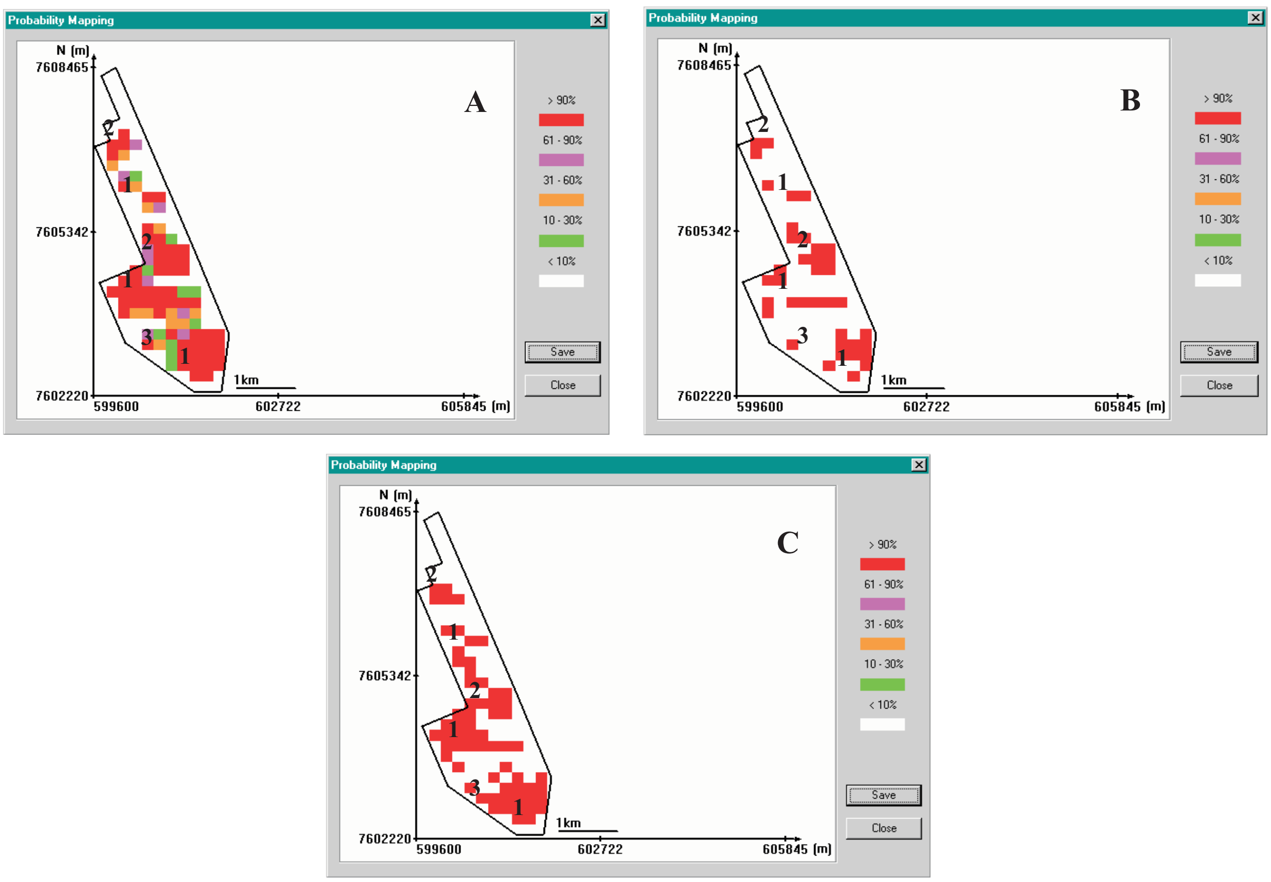


FIG 8 - (a) Fault probability map based on 50 fault realisations; (b) fault probability map based on the 'exploration' fault dataset; and (c) fault probability map based on the complete and mapped fault dataset; all faults shown have a throw ≥1 m.

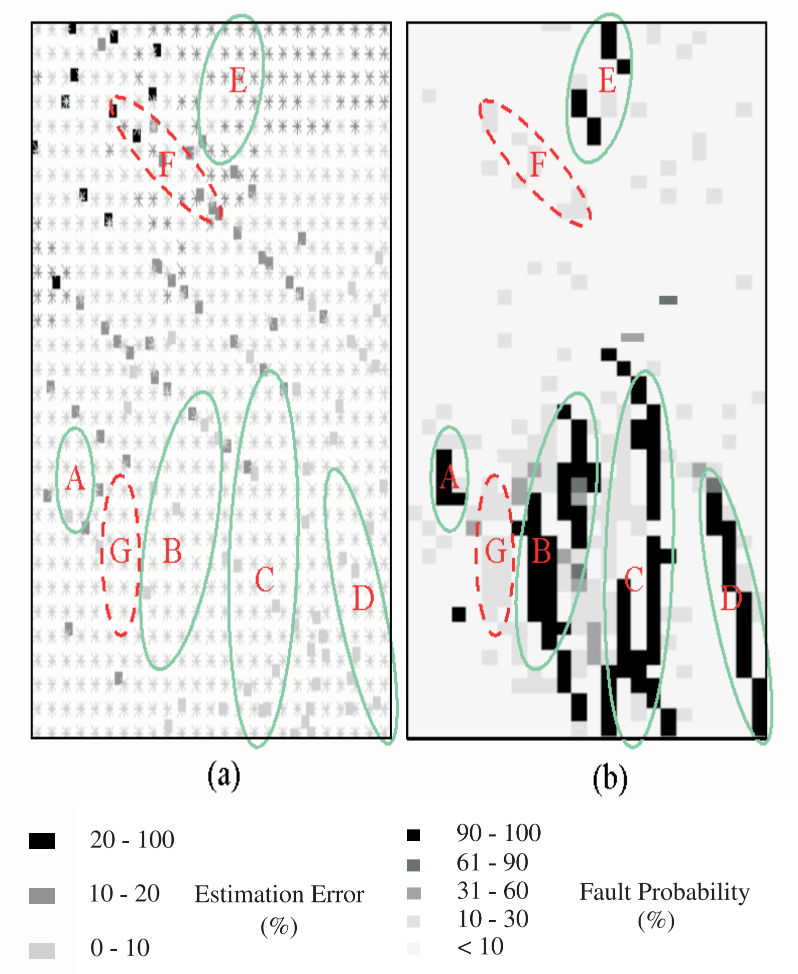


FIG 9 - (a) Errors in coal tonnage and (b) faulting probability map in a coal lease, and joint consideration to assist coal resource classification.

Two broad areas of applications in coal mining were presented. The first area refers to:

1. the quantification of uncertainty in coal seam attributes and risk assessment that can assist mining companies and their Competent Persons with resource classification, and
2. the application of quantified geological risk to the optimisation of drilling patterns to meet the desired risk level with the required confidence.

The simulation method presented provides a transparent and defendable approach to resource classification and provides a way to assess the drilling that may be required to generate models with a given error and confidence level.

The second application presented involved the stochastic simulation of fault systems and related quantification of fault risk. The work presented showed a back analysis study that demonstrated the ability of the fault simulation approach to quantify and assess fault risk. Quantification of fault risk can assist resource classification and be integrated with the simulation of other coal seam attributes.

ACKNOWLEDGEMENTS

The authors would like to thank Australian Coal Association Research Program, BHP Billiton Mitsubishi Alliance, Anglo Coal Australia, Coal and Allied (Rio Tinto Coal) and MIM (Xstrata) for funding this study and the contributions of key personnel: Andy Willson, Peter Forrestal, Darren Hope, Dianne Sommer, Sarum Peau and Andrew Paul.

REFERENCES

Benndorf, J and Dimitrakopoulos, D, 2007. New efficient methods for conditional simulation of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Boucher, A and Dimitrakopoulos, R, 2007. A new efficient joint simulation framework and application in a multivariable deposit, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 345-354 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R and Li, S, 2001. Quantification of fault uncertainty and risk management in underground longwall coal mining, in *Proceedings Geological Hazards* (eds: R Doule and J Moloney), pp 175-182.

Dimitrakopoulos, R, Li, S, Scott, J and Mackie, S, 2001. Quantification of fault uncertainty and risk management in underground longwall coal mining, ACARP Project C7025 Report, Volume I, W H Bryan Mining Geology Research Centre, The University of Queensland, 215 p.

Dimitrakopoulos, R and Luo, X, 2004. Generalized sequential Gaussian simulation on group size *v* and screen-effect approximations for large field simulations, *Mathematical Geology*, 36(5):567-591.

Dimitrakopoulos, R, Scott, J and Li, S, 2005. Quantification of geological uncertainty and risk assessment in coal resource/reserve classification, ACARP Project C11042 Report, Volume I, W H Bryan Mining Geology Research Centre, The University of Queensland, 250 p.

Dowd, P A, 1997. Risk in minerals projects: analysis, perception and management, *Trans Inst Min Metall*, Section A, Mining Technology, 107:A9-A20.

Froyland, G, Menabde, M, Stone, P and Hodson, D, 2007. The value of additional drilling to open pit mining projects, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 245-252 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions*, 316:43-50.

Joint Ore Reserves Committee of The Australasian Institute of Mining and Metallurgy, Australian Institute of Geoscientists and Minerals Council of Australia (JORC), 2004. Australasian Code for Reporting of Exploration Results, Mineral Resources and Ore Reserves (The Australasian Institute of Mining and Metallurgy, Melbourne) [online]. Available from: <http://www.ausimm.com.au/main/about/docs/jorc0105.pdf> [Accessed: 7 May 2007].

Journel, A G, 1994. Modelling uncertainty: some conceptual thoughts, in *Geostatistics for the Next Century* (ed: R Dimitrakopoulos) pp 30-43 (Kluwer: Dordrecht).

Li, S, Dimitrakopoulos, R and Scott, J, 2004. Quantification of geological uncertainty and risk assessment in coal resource/reserve classification, ACARP Project C11042 Report, Volume II, W H Bryan Mining Geology Research Centre, The University of Queensland, 77 p.

Li, S, Dimitrakopoulos, R, Scott, J and Mackie, S, 2001. Quantification of fault uncertainty and risk management in underground longwall coal mining, ACARP project C7025, Volume II, W H Bryan Mining Geology Research Centre, The University of Queensland, 88 p.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ravenscroft, P J, 1992. Risk analysis for mine scheduling by conditional simulation, *Trans Inst Min Metall*, Section A, Mining Technology, 101:A104-A108.

Scott, J, Dimitrakopoulos, R, Li, S and Bartlett, K, 2007. Fractal-based fault simulations using a geological analogue: quantification of fault risk at Wyong, NSW, Australia, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 87-93 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Risk Assessment in Strategic and Tactical Geomechanical Underground Mine Design

W F Bawden[1]

ABSTRACT

Mining is considered a high-risk industry for investment purposes, as the development of new mines is classed as a speculative venture. Time and effort are lavished on the study of a mine's feasibility, since the risks diminish substantially with the extent and quality of the analysis. A high degree of sophistication exists in techniques related to the analysis and management of risk, particularly in the areas of reserve estimation and finance. Once the commitment to mine development is made however, a more conventional design approach is generally followed. Although geomechanical considerations often dominate mine performance and profitability, the lack of analysis and appreciation of the potential impact of critical risk factors (particularly at mine operations where 'production rules') can result in inappropriate and very costly short-term decisions. The two necessary elements of risk are the hazard (ie what can go wrong, with what consequences) and the likelihood (ie probability).

In geomechanical mine design, hazards include factors such as ground falls, rockbursts, severe closure of development headings, dilution, slope failure, flooding, etc, with potential consequences including lost time injuries, fatalities, ore losses, increased rehabilitation, grade dilution, equipment damage, etc. The likelihood or probability of the various hazards can be assessed as due to one of three basic causes: random events, limited data (ignorance), and limited understanding of the processes at work (the other kind of ignorance).

The field of rock engineering (the scientific backbone of geomechanical mine design) falls in the 'data limited' class. As such, factors controlling the likelihood of a hazard occurring exist to varying degrees at every site. This paper discusses the need to develop a rational and robust risk assessment methodology (yet one that can be used easily and quickly) for tactical and strategic geomechanical mine design. The increasingly routine use of sophisticated geomechanical instrumentation (eg microseismic monitoring, instrumented support, extensometers, etc) provides a database that might act as the foundation for such a system. In the paper, selected mine case studies are used to illustrate how the lack of such routine quantitative geomechanical risk assessments can result in very costly short-term consequences.

INTRODUCTION

The four natural stages in the life of any economic mineral deposit move from prospecting through exploration to development and finally exploitation and closure. The decision to develop a new mineral deposit is preceded by prefeasibility and feasibility studies. Adoption of a feasibility report as a planning document, subject to modification as development progresses, represents the final step prior to what is generally a major capital expenditure commitment. At this stage general mining method(s) and mining plans are adopted and arrangement of financing, based on confirmed cost estimates from the feasibility report, is concluded. Time and effort are lavished on the study of a mine's feasibility, since at this stage the risks diminish substantially with the extent and quality of the analysis.

A high degree of sophistication exists in techniques related to the analysis and management of risk, particularly in the areas of reserve estimation and finance. At the feasibility stage the risk is 'financial' in nature. The nature of such projects nevertheless means that they remain in the 'data limited category'; ie actual grade measurements exist only at limited point source locations (drill intersections). Reserve estimation requires that some form of interpolation scheme be used to estimate grade distribution between measurement locations. This may be as simple as assuming that the influence from a measured grade diminishes with inverse distance squared or may invoke a more sophisticated geostatistics based approach. The nature of data limited problems is shown in Figure 1 (Holling, 1978). In this figure the vertical axis is a measure of the quality and/or quantity of available data while the horizontal axis measures the understanding of the problem to be solved. In region 1 there are good data but little understanding; this is where statistics is the appropriate modelling tool. In region 3 one has both the data and the understanding; this is where models can be built, validated and used with conviction. Regions 2 and 4 relate to problems that are data-limited in the sense that the relevant data are unavailable or cannot easily be obtained. Many mining geomechanics problems fall into the data-limited category; one seldom knows enough about the rockmass to model it unambiguously.

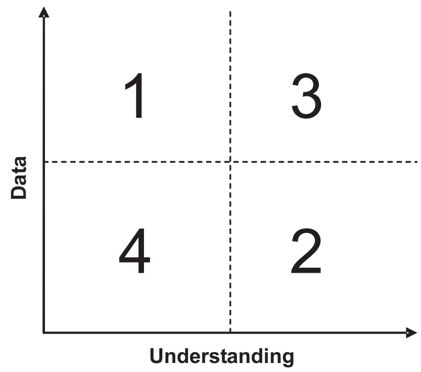


FIG 1 - Holling's classification of modelling problems.

With mine development and exploitation the nature of mining risk changes from being predominantly financial in nature to a combination of financial and safety. Financial risk may result from operational or non-operational (eg market condition) factors. Particularly in underground mining, geomechanics aspects strongly influence both financial and safety risk. With increasing extraction, grade prediction is expected to improve with the effectively ever increasing 'bulk sample' and improved geological modelling. Understanding of the mine geomechanics also increases with increasing extraction. Geomechanics however always remains in the 'data limited' category. This results largely from the fact that, with increasing extraction and depth, geomechanical conditions often change dramatically. These changes can have a significant impact on all aspects of mining related risk.

OPERATIONAL RISK

Operational risk arises from a variety of conditions related to the detailed nature of the orebody (eg depth, local geology, geological history, etc), the mining method employed (eg entry versus non-entry mining, cave mining, etc), the local stress regime, etc. Table 1 lists some of the more significant geomechanical risks that can impact a typical underground operation. As shown in Table 1, many of these factors involve elements of both safety and financial risk.

1. Pierre Lassonde Chair in Mining Engineering, University of Toronto, Room 118, Mining Building, 170 College Street, Toronto ON M5S 3E3, Canada. Email: bawdenw@ecf.utoronto.ca

TABLE 1

Typical underground geomechanical risk versus potential consequence (X – highly likely; U – unlikely; P – possible).

Risk	Potential consequence			
	Production interruption	Increased cost	Equipment damage	Lost time injury
Oversize dilution	X	X	X	U
Fall of ground	X	X	X	X
Strain burst	X	X	P	X
Pillar burst	X	X	P	P
Fault slip burst	X	X	X	X
Pillar (secondary stope) failure	X	X	P	U
Hole squeeze	X	X	U	U
Rehabilitation	X	X	P	P
Ore pass failure	X	X	P	U
Extreme closure of access	X	X	P	U
Stope caving	X	X	P	U

Over the life of a mine, extraction advances to greater depth and extraction ratios increase. Because of this, the risk associated with many of the items listed in Table 1 varies with time. In early mining (say the first 25 per cent of an underground deposit) the risk associated with many of the items listed in Table 1 is generally quite small. However these risk factors increase (often exponentially) with increasing extraction ratio and increasing depth.

The mine design itself exerts enormous influence over the control and management of risk associated factors. In underground mining the central platform of the mine design is the stoping sequence. The stope sequence exerts the dominant influence on mine induced stress redistribution, which in turn is the most important driver for most of the factors listed in Table 1. The practical impact of many of the operational problems listed in Table 1 is to force the mine to extract stopes out of sequence in order to meet short-term production objectives. This in turn is largely driven by pressures from the investment community, often leading to poor engineering decisions. This is an area where a more formal, quantitative risk analysis procedure could be helpful. Such analyses would help management and investors better understand long-term implications of short-term production decisions often taken to placate short-term investor demands.

In mines subject to difficult geomechanical conditions, either resulting from poor quality ground and/or from mining at great depth/high stress conditions, poor design decisions at the feasibility stage or in early mine life can have a serious impact on the ultimate net present value of the resource as shown in Figure 2. In the worst case, such decisions can result in complete loss of some or all of the resource.

PRACTICAL CASE STUDIES

Stepping out of sequence – sill pillar mining in highly stressed ground at the Williams Mine
(LeBlanc and Murdock, 2000)

The Williams Mine (the largest underground gold mining operation in North America) is the largest of the three gold mines located in the Hemlo region of north-western Ontario (Figure 3). Annual production is currently 2.1 million tonne from underground and 400 000 tonne from a surface pit operation, which generates approximately 400 000 ounces of gold.

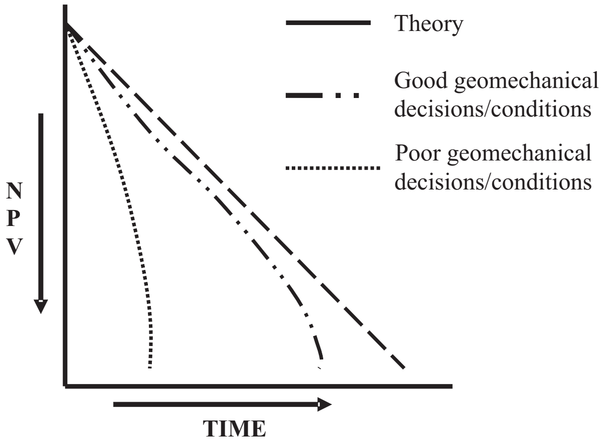


FIG 2 - Potential impact of geomechanical engineering decisions on mine net present value.

The uniformity of the orebody, with its steeply dipping orientation, lends itself well to a longhole open stoping mining method. The two main mining areas in the B Zone are Block 3 and Block 4, which are separated by a sill pillar. The mining configuration is a chevron shape. The chevron in Block 3 is open to the west but is bounded on the east by Newmont Canada's Golden Giant Mine. Block 4 is immediately below Block 3 and the mining configuration in this block also began as a full chevron. As mining progressed the east side of this chevron was advanced in an attempt to move to a half chevron retreating from east to west. Mining of Block 4 stopes up below the mined and filled Block 3 area resulted in a high stress island at the east boundary of the mine where the adjacent Golden Giant mine had also already mined and filled (Figure 4).

Initial indications of problems in the sill pillar began shortly after the removal of the first stope under backfill in 1994 (6-9415 stope). As mining progressed several sidewall failures occurred along with the first significant back failures. In November 1996, the first major ground failure occurred, which affected the mine's ability to produce from this area. As mining continued the frequency of ground falls increased. In all, from November 1996 to October 1997 there were four major ground occurrences in the Block 4 sill pillar area, which delayed the mining of approximately 1 000 000 tonne containing some 300 000 ounces and seriously hampered production from the mine.

Summary of major ground failures:

8-9-10 stope failures – caving area 1

In August 1996, following the cap blast in 10-9390 stope (52 000 tonne) a 14 000 tonne ground fall occurred from the back (Figure 5). As mucking progressed, stress arching and additional caving continued. Six weeks later the cap was blasted in the 9-9345 stope (72 000 tonne), which resulted in another massive failure from the back in this stope. Failures continued in both 9 and 10 stopes as mucking progressed. Prior to blasting 9 stope, a down cabling program was done in an attempt to hold the back, but the stope eventually failed up and into the 8 x/c on 9390. At this time mucking was halted in 9 stope in order to expedite the mucking and filling of 10 stope. Two weeks later, on 4 November 1996, a massive failure occurred throughout the 8-9-10 stopes in which the caving progressed through the sill pillar to the cemented sill of stopes at the bottom of Block 3. Five days later a second failure occurred in which the cave area broke into 11 x/c on the west side and halfway through the cemented fill. Some ore in this area has been recovered, but progress has been very slow as dilution from dry fill pulling through from Block 3 has become a major problem.

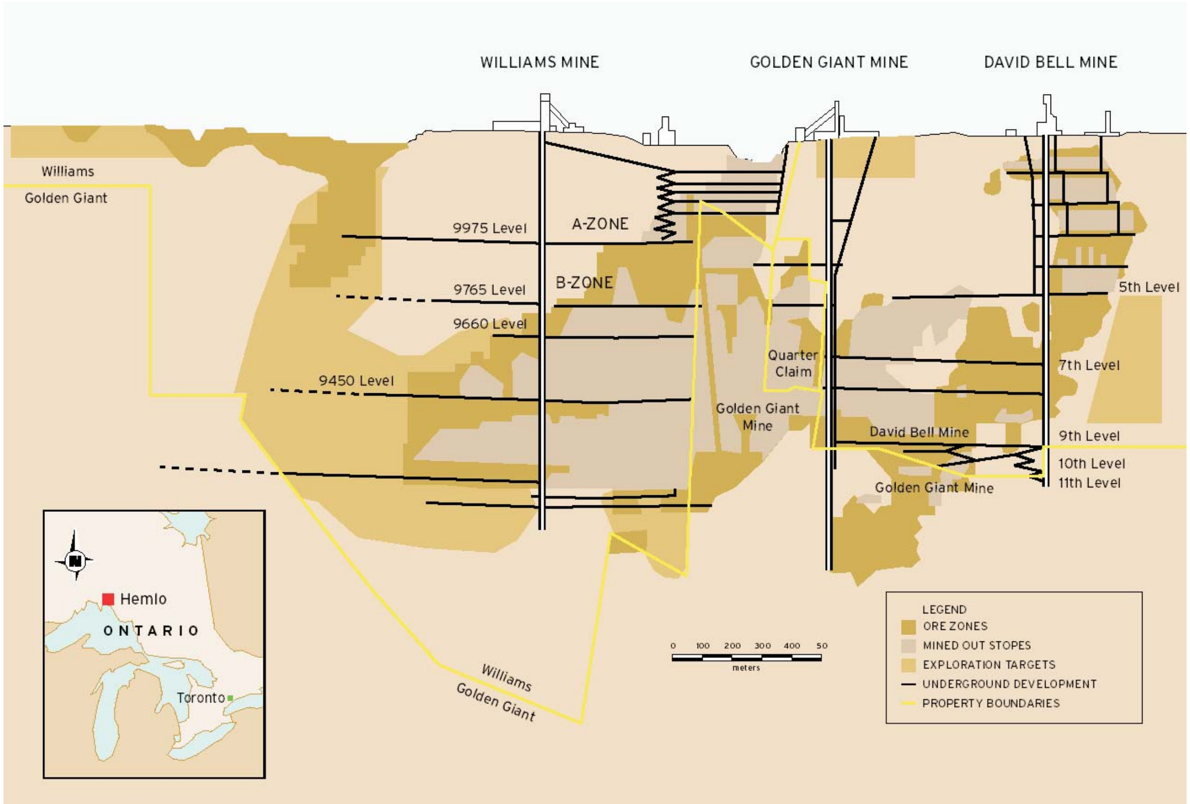


FIG 3 - Hemlo mining camp location map.

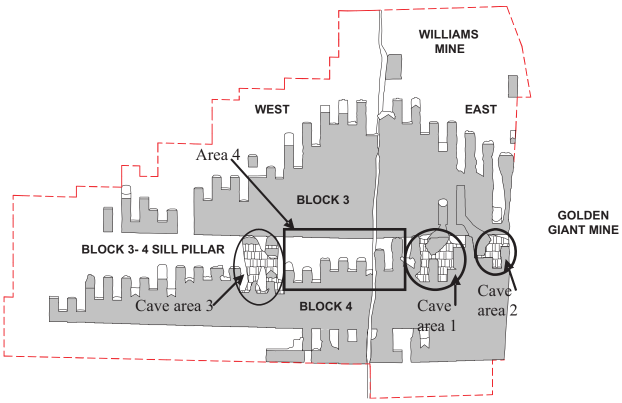


FIG 4 - Longitudinal of B-Zone looking north (LeBlanc and Murdock, 2000).

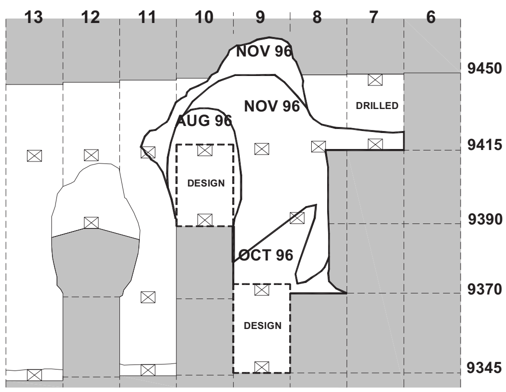


FIG 5 - 8-9-10 stope failures looking north (LeBlanc and Murdock, 2000).

2-3-4 stope failure – caving area 2

In March 1997 mining began in the 3-9370 stope (50 000 tonne) in the east end of the sill pillar (Figure 6). Following the cap blast, a movement of ~2 cm was noticed along the foot wall contact one level above. Mucking progressed and on 9 April 1997 a massive failure occurred in the 2, 3 and 4 stopes from 9390 to the cemented sill of Block 3. The ore block slid 4-10 m down a muscovite shear along the footwall contact. It remained relatively intact but recovery of this ore has not yet been possible. The resulting stress redistribution caused back failures in 2, 3, 4 and 5 stopes on the leading edge of mining in Block 3. This failure delayed the mining of some 275 000 tonne containing 95 000 ounces for a period of five to seven years. Recovery was expected to be difficult and costly with all mining and filling to be done in a top down fashion. This Block 3 ore has since been successfully mined.

28-30 stopes – caving area 3

Mining in this area was begun out of sequence to alleviate production constraints caused by the ground failures at the east side of Block 4 (caving area 1). In January 1997 the 28-9345 (30 000 tonne) stope was blasted without incident (Figure 7). The stope was 95 per cent mucked out before the initial ground failure occurred. Continuous slabbing and arching of the back occurred due to high mine induced stresses exacerbated by a series of small lamprophyre dykes that laced the back. The decision was made to suspend mucking and stabilise the stope with cemented rock fill. However, before the filling could be completed the failure occurred in the east end in the 3 stope area (cave area 2). To maintain production, a decision was made to blast the 30-9345 (32 000 tonne) stope on the same horizon (Figure 7). Initial failure in this stope followed the first lift blast, progressing to within 12 m of the over cut. Following the cap blast the stope continued to

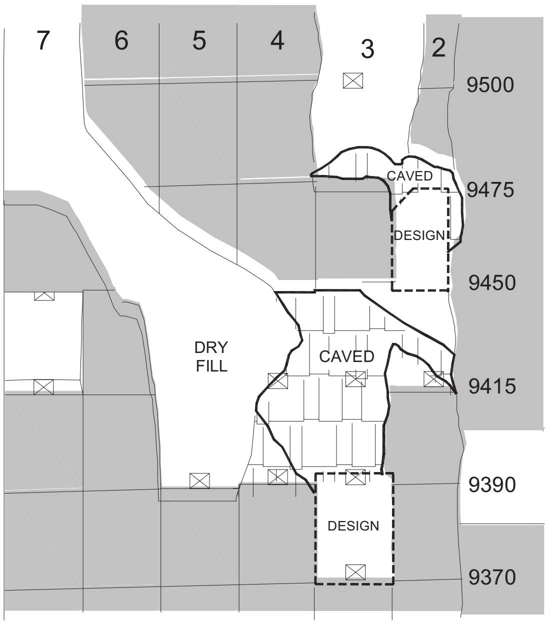


FIG 6 - 2-3-4 stope failures looking north (LeBlanc and Murdock, 2000).

work and spall, again due to the high stresses and the presence of a series of small lamprophyre dykes. The stope was allowed to cave and mucking was continued. The cave continued upward to the sill pillar and progressed up and into the hanging wall above the sill. It is estimated that approximately 190 000 tonne caved in this stope with some 110 000 tonne mucked out before the area was shut down. Only a small portion of the caved ore in this area has been recovered to date.

Impact on mining

By mid 1997 Williams Mine had experienced three major ground falls in the sill pillar in a nine-month span. In hindsight, two very highly stressed areas had been created, the long narrow sill pillar in Block 4 and the 'hanging pendant' in the east end of Block 3. The ground was controlling what and where mining could take place with the mine continually reacting to problems and adjusting mining plans accordingly. All efforts had to be focused on regaining control of the ground in order to resume mining in a controlled and orderly fashion.

Table 2 summarises reserves that were directly affected by the major ground falls in the sill pillar area. By the end of 1997 there were 37 reportable incidents, up considerably from ten in 1996. Four of these incidents in 1997 were considered major. Of prime importance were the tonnes of uncontrolled caving and the reserves whose recovery was affected by it. Table 3 summarises the tonnes of uncontrolled caving from 1995 to 1998 and the reserves whose recovery was delayed during this period. In 1998, with the efforts put into planning and ground control, there was a reduction in reportable incidents to 20; but, more importantly there was a major reduction in tonnes of uncontrolled caving. This trend continued from 1999 forward.

TABLE 2

Production delayed due to ground falls in the sill pillar area (LeBlanc and Murdock, 2000).

Area	Tonnes	Contained ounces
2-3-4 stope area	343 000	113 000
7 to 10 stope area	623 000	174 000
Central sill pillar: 19 to 26 stope	723 000	115 000
27 to 31 stope	499 000	68 000
Total	2 188 000	470 000

The ground falls in the sill pillar area during the fall of 1996 and the spring of 1997 had a major impact on the mine's ability to meet production targets. The areas affected contained 25 per cent of the planned tonnage and 40 per cent of the planned gold production for the year. In spite of all the difficulties the mine was able to adapt mining plans, change to panel mining and develop new mining areas in order to achieve budgeted production targets while maintaining cost effectiveness and one of the best safety records in the mining industry. The final impact on the mine reserve, however, is still not known.

Summary

The ground failures on the east side of Blocks 3 and 4 illustrate a case of 'limited understanding of the processes at work'. This shows how an increasing mine extraction ratio in an area of high mine induced stress can lead to an 'apparent sudden change in ground conditions and behaviour'. Indeed ground conditions in the affected area did change. This change, however, was not geological in nature, but rather was related to damage to the rockmass related to elevated mine induced stress in a local area

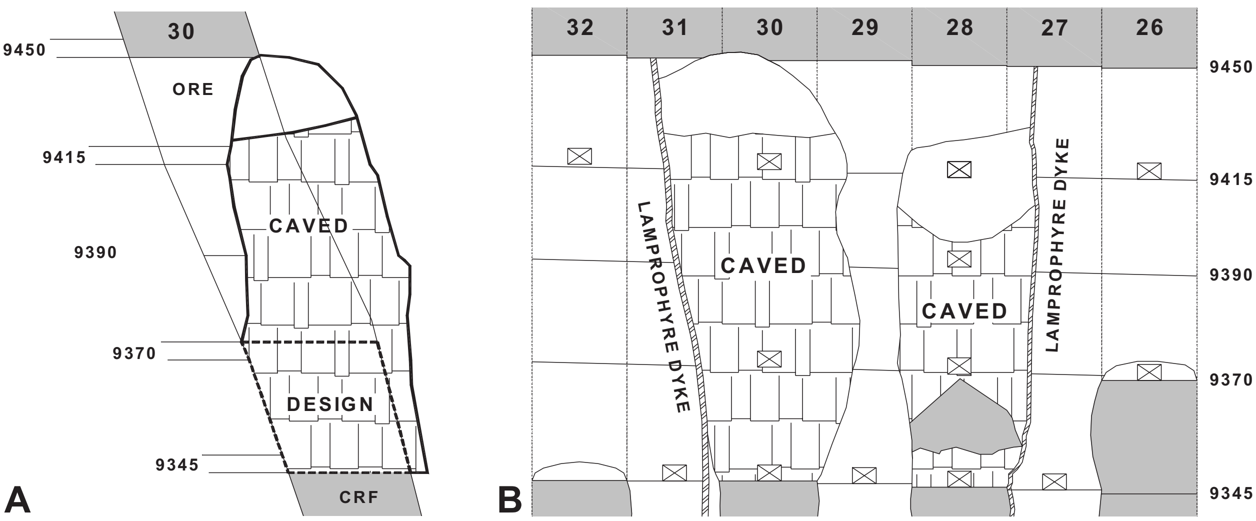


FIG 7 - (a) Cave in 30 stope looking west, (b) Cave in 28-30 stope area looking north (LeBlanc and Murdock, 2000).

TABLE 3

Uncontrolled caving and affected reserves (LeBlanc and Murdock, 2000).

	1995	1996	1997	1998
Uncontrolled caving: tonnes	120 000	235 000	553 000	127 000
Tonnes affected by ground problems (delayed recovery)	51 000	370 000	693 000	71 000
Ounces affected by ground problems (delayed recovery)	16 700	111 000	139 000	8 000

of the mine. While the mine was able to overcome these difficulties and maintain short-term production it was forced to 'step out of sequence' (initiate extraction in area 3, Figure 4). In this area relatively high mine induced stress conditions were compounded by a geological complication, local lamprophyre dykes, (random occurrence), and a short-term production decision (caused by failure in cave area 2) to blast a second primary stope before the adjacent primary was completely mined and filled, resulted in failure in area 3. This ground failure created another geomechanically based problem that is discussed in the next section of this paper.

While the risks in these stope failure cases was primarily financial (potential loss of reserve, dilution) there was also a finite safety risk. The non-entry mining method employed helped to minimise the safety risk in these cases. No formal risk assessment was undertaken for these areas.

Impact of rockbursting in a highly stressed sill pillar

The stope failures discussed in the previous section at the Williams mine resulted in the creation of another 'stress island' shown as area 4 on Figure 4 and referred to as the Block 4 sill pillar. On 29 March 1999 an unexpected event occurred in the Block 4 sill pillar, a rockburst of magnitude 3.0 Nuttli. The event was felt on surface and was picked up by the Geological Survey of Canada at several sites in Ontario. Previously, no event larger than an estimated 1.0 Nuttli had ever been experienced in the Hemlo camp. The location of the event was a major concern as previously all ground fall and seismic activity had taken place within the ore zone. In this case, the main damage zone was located in the footwall drift, centred between 18 x/c and 26 x/c on the 9415 level, one level below the cemented sill of Block 3 (Figure 8). Massive failures occurred in the back of the footwall drift at the cross-cut intersections, from 20 x/c to 26 x/c on 9415. Floor heave, buckling of the lower south wall and spalling of the upper north corner of the footwall drift occurred on the 9450 level throughout the same area. The centre of the damaged area was located in the shadow of the #3 ore pass system. Only minor damage occurred on levels above 9450 and below 9415. The only active mining ongoing in the area was 26-9370 stope, where the first lift had been blasted and removed one week earlier. Table 4 shows the direct cost of rehabilitation, new development, instrumentation, etc, resulting from this single event (after LeBlanc and Murdock, 2000).

TABLE 4

Costs associated with the 29 March rockburst (LeBlanc and Murdock, 2000).

Area	Cost
9450 Rehab	$500 000
9415 H/W access drift	$1 300 000
9390 Rehab	$600 000
9370 Rehab	$300 000
West end ramp	$1 100 000
Micro seismic systems	$560 000
Total	$4 360 000

The nature of this event (a fault slip rockburst) made the probability of additional events of a similar nature almost a certainty. Access had to be maintained through this general area and a support redesign to accommodate strong dynamic loading had to be developed. Indeed this part of the mine was subsequently subject to a number of additional large seismic events. Details on the support design and its response to later seismic events are given by Bawden and Jones (2002).

In this case, the risk from potential future seismic events included high financial risk (cost of rehabilitation, potential ore loss) and high safety risk since, in order to enable continued production from the west side of Block 4, by necessity there was relatively high exposure of personnel throughout the affected

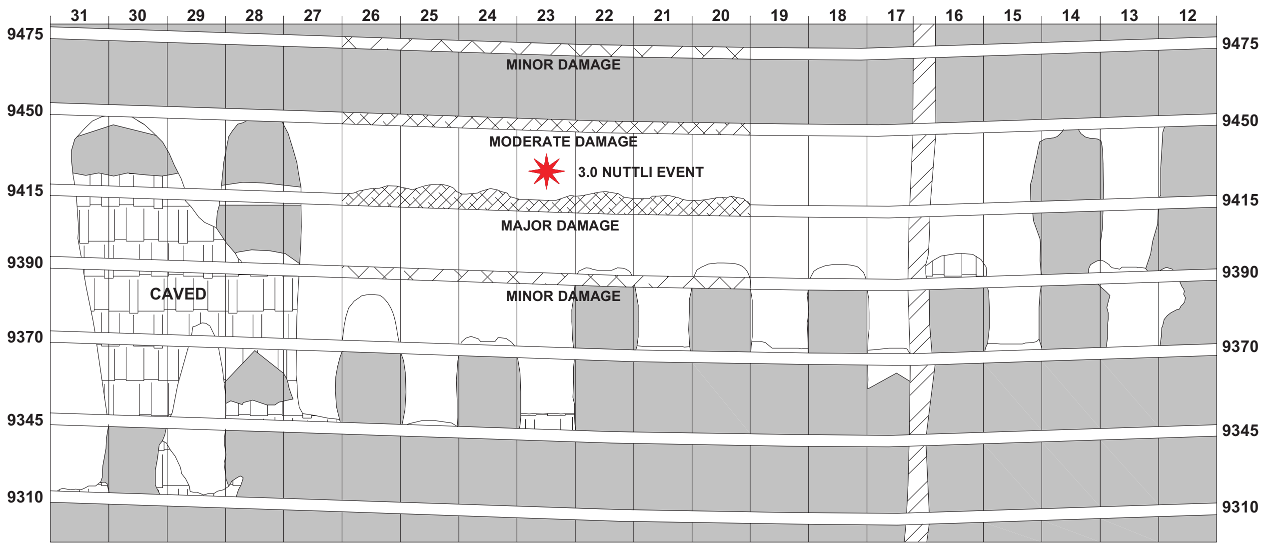


FIG 8 - Longitudinal section looking north of area affected by 29 March rockburst (LeBlanc and Murdock, 2000).

zone. A heavy, burst resistant ground support system combined with a very sophisticated instrumentation system was used to manage these risks. While confidence in the combined support-instrumentation system was gained early, this continued to be a high risk area until late 2003 when the entire sill pillar finally yielded (Bawden and Jones, 2005). No formal risk assessment procedure was used to quantify these risks. Rather, a conventional observational engineering approach was adopted and used to manage risk on a day to day basis. This approach was based on:

1. seismic decay rates: following nearby production blasting or any large seismic event, re-entry to the sill area was restricted until seismic event rates had decayed to close to background levels; and
2. instrumentation based rehabilitation: following any large seismic event in the sill, instrumented SMART cable bolt data was used to guide rehabilitation to ensure that safety factors remained within design guidelines throughout the high risk sill pillar area.

Summary

This is a case of 'random events' impacting production. None of the technical personnel involved foresaw the potential of seismic events of the magnitude that ultimately occurred. It is therefore unlikely that a risk analysis for this type of event would have been conducted 'a priori'. Nevertheless, it must be recognised that these rockburst occurrences were, at a minimum, exacerbated by the stope collapse problems discussed earlier, resulting in stepping out of sequence and inadvertently creating the Block 4 sill. Once the March 1999 event occurred and the nature of this event was understood, it was recognised that the probability (and hence the risk) of similar future events was nearly 100 per cent. In this case it is unclear what additional contribution risk analysis would have provided to the engineering decision-making process.

Extraction of shaft pillar ore while maintaining the shaft in full production – the Golden Giant shaft destress slot project
(MacMullan, Bawden and Mercer, 2004)

The Newmont Canada Golden Giant Mine is the central of the three mines shown on Figure 3. The Golden Giant shaft is a 6 m × 4 m timber shaft with the long axis oriented east-west. The shaft pillar mining area is a zone of ore that includes all stopes from 4500 - 4700 extending west to the Q8 stope and east to the S1 stope on the David Bell boundary (Figure 9). The zone is open above, but has been mined out below up to the 4600 elevation, except for the Q1-3 pillar east of the shaft which extends to 4500 elevation. In total, the shaft pillar mining area represents 660 000 tonne at 12.23 g/t (US$104 million at 400/ounce). Ore widths vary from approximately 5 - 12 m.

Potential future mine induced stress related problems with the shaft were first recognised following an early life-of-mine numerical analysis conducted by the Noranda Technology Center in the mid 1980s. This resulted in temporary sterilisation of a block of ground surrounding the area where the shaft penetrates the ore bearing formations (the shaft pillar – initial risk mitigation strategy). As Figure 9 shows, the shaft pillar ore zone is closely associated with the main production shaft. In easting terms the shaft is located roughly at the centre of the shaft pillar ore zone. At the 4600 level the shaft is located approximately 20 m south of the ore zone. As the zone dips to the north with depth, the plane of the ore zone, although not economic, cuts through the shaft at 4660 elevation. The closest mineable stope is located at a distance of 9 m from the main production shaft.

Over time this zone of the mine became a source of increasing concern due to the presence of high mining induced stresses and their proximity to the main production shaft. In 1998, preliminary modelling work indicated that the shaft and nearby infrastructure in the area of the shaft pillar ore zone was being subjected to increasing stresses due to stress shedding from

FIG 9 - Long-section of the shaft pillar ore zone and surrounding infrastructure (looking north) (MacMullan, Bawden and Mercer, 2004).

mining throughout the Hemlo camp. Review of these findings strongly suggested that this stress buildup would be detrimental to the shaft over the longer term and that a new extraction sequence would be required in order to safely extract the ore within the shaft pillar.

Shaft pillar extraction design

The original mine plan was to mine all ore at depth, abandon the shaft below the 4600 level, and extract the shaft pillar as the final mining block. Life of mine economic analyses indicated, however, that much of the high-grade shaft pillar ore would be lost if it could not be mined simultaneously with the deep ore. A new study was then commissioned to design an extraction rationale for the shaft pillar area that would not jeopardise the shaft's integrity. Based on this study (Curran *et al*, 2001), which incorporated detailed numerical modelling using the Examine3D software, it was concluded that mining of the shaft pillar ore while continuing to mine at depth would require the extraction of a destress slot to protect the shaft end walls from high mine induced stresses. The most desirable slot configuration required it to be created partly in ore and partly in waste between 4600 and 4690 m. The slot also had to be approximately 90 m high and 70 m wide to ensure that the shaft and the associated infrastructure were adequately protected. In the proposed extraction plan, the waste portion of the slot was to be excavated parallel to the main orebody, with a dip of 60° towards the shaft (Figure 10).

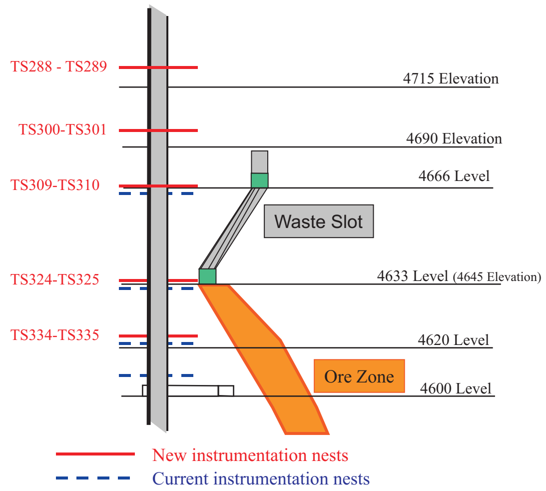


FIG 10 - Shaft instrumentation (conventional) sensor positions (MacMullan, Bawden and Mercer, 2004).

Design of the shaft pillar destress slot extraction and subsequent shaft pillar mining incorporated an extensive instrumentation package including microseismic monitoring and conventional instrumentation. The microseismic system was upgraded such that event locations with accuracies of <4 m were obtained at the 4633 level where slot excavation was to occur within 9 m of the shaft wall. Extensive instrumentation arrays including extensometers and stress monitoring instruments were installed at key locations in the walls of the shaft. Vibration monitoring and periodic shaft plumbing surveys were also conducted in the shaft. Cavity monitoring surveys (CMS) were conducted following each extraction stage of the destress slot and the most recent programmable electronic detonators were used for all blasts to control vibrations and ensure that the highest quality blasting results were achieved.

The purpose of this instrumentation was twofold:

1. to validate and calibrate the numerically based design, and
2. to monitor shaft behaviour through the end of mine life.

On this basis, extensometer, stress cell and microseismic data were all ported to surface in real time. Instrumentation data was monitored daily, with special attention following each blast of the destress slot. Shaft plumbing and CMS survey data was also critically reviewed as these were completed.

The design was subject to rigorous external review and a qualitative risk analysis. In this case the risk analysis was conducted as a back-check on the design, but was not incorporated as part of the design process itself. Following this, rigorous 'go – no go' decision points were incorporated into the process, with decisions being based on suitable conformity between instrumentation results and design predictions. To date, extraction of the destress slot has been successfully completed, with instrumentation results through this period showing exceptional correlation to numerical design predictions (MacMullan, Bawden and Mercer, 2004).

Summary

In this case the 'processes at work' were well understood and analysed. The problem nevertheless remained in the 'data limited' class and the risk of random events (ie large magnitude seismic events, etc) was one of the main issues of concern. A somewhat informal, qualitative risk analysis was conducted, albeit rather late, in the design process. This nevertheless helped the design team focus on critical 'go – no go' decision points during the shaft pillar destress slot extraction and to effectively communicate all of these issues to senior management.

A DISCUSSION ON GEOMECHANICAL MINE DESIGN RISK ANALYSIS

The present situation

Risk analysis in underground geomechanical mine design is currently an area of considerable interest. In some areas, such as open pit slope stability, formal risk analysis procedures are commonly applied as part of the slope design process. In underground mining however a relatively limited number of examples of practical risk analyses for underground excavations exist. Risk analysis for mine-induced seismicity probably represents the area where risk analysis in underground mining has received the most attention. Owen, Hudyma and Potvin (2002) provide an overview of practical risk analysis procedures for this area. Carter and Miller (1995) provide a detailed methodology for crown pillar risk assessment for underground mines. Pine and Thin (1993) discuss probabilistic risk assessment in mine pillar design for the South Crofty tin mine while Pine and Arnold (1996) discuss the application of risk assessment methods to underground excavations. The latter article incorporates examples of risk analysis for stope wall and pillar design for a hard rock application. Duzgun and Einstein (2004) provide a discussion of the assessment and management of roof fall risks in underground coal mines. All of these papers focus on rather narrow mining issues and none discuss the broader 'data limited' problem, particularly the 'stress probability distribution function' problem, although these issues impact, to varying degrees, all of the analyses discussed by these authors.

A possible interim solution

In a paper in 1997 in the *CIM Bulletin*, Davies (1997) states that:

> *Until recently, risk evaluations have tended toward a strongly mathematical basis. With this previous trend, it was essential to establish precise probability density functions to all assessed components or issues where little information was available. The combined effect of the seemingly*

complex analyses with corresponding insufficient data often resulted in either a poorly executed risk evaluation or no evaluation whatsoever; either result not acceptable for optimal decision-making.

Most deterministic design calculations inherently make the assumption that the component should not be allowed to fail. This is reasonable where the consequences of failure are intolerable. However, in cases where the possibility of some negative performance or 'failure' is not unacceptable and the cost of ensuring against it would be prohibitive, an alternative approach allowing a balance of cost against risk of failure is required.

For probabilistic risk analysis often the largest difficulties come from the manner in which estimates of probability, or likelihood, of event occurrence are developed. Ideally, statistically significant sample numbers based on an appropriate statistical model are required. In mining, however, spatial variability is very important, often meaning that statistically significant sampling may not be feasible on either a mass or volumetric basis. Davies then poses the question 'what kind of risk assessment, if any, is valid and practical for most mining applications?'

Pragmatic mining practice has followed the 'observational approach' and a more deterministic framework for risk evaluations. Davies states that the likely reasons for this are:

- statistically significant sampling is not feasible for most activities,
- most natural and man-induced mining associated processes do not tend to follow any readily available statistical distribution, and
- a formal framework whereby less rigorous but still realistic approaches to probabilistic risk assessments are possible has not been available to most mining decision-makers.

Davies then presents a simplified risk classification scheme for use in mining. He notes that the first requirement of a practical framework for risk assessment is a way to assign estimated probabilities. He recommends a judgement-based estimate. Although the most subjective, he claims that judgement is often the only feasible way, and in many cases, the best manner with which to estimate probabilities of events providing it is backed by logic, available and appropriate data and sound engineering principles.

A typical list of likelihood/probability descriptors suitable for mining projects is provided in Table 5. Davies notes that the consequence framework cannot and should not be universal in nature; rather, site specific and discipline specific consequences need to be established. Once the consequences and their severity are established the risk assessment can be carried out. Davies also assigns the severity/consequences of each potential problem a five step, N to E scale. To carry out the final step and allow some quantification of this judgement-based risk process, quantification of the estimated probabilities following the numerical equivalents for the five steps shown in Table 6 is recommended. Using this approach, judgement (subjective) values of risk and consequence can be determined and placed in the appropriate box in Figure 11. With this system, combinations

TABLE 5

Likelihoods or probabilities (Davies, 1997).

Negligible (N)	Essentially negligible occurrence potential, 'doubt it could ever happen'.
Low (L)	Not likely to occur, 'highly unlikely to happen'.
Moderate (M)	Moderate frequency of occurrence, 'it could happen'.
High (H)	Frequent occurrence, 'it has happened or it probably will happen'.
Extreme (E)	Very frequent occurrence, 'happens all the time'.

TABLE 6

Numerical equivalents for likelihoods or probabilities (Davies, 1997).

Negligible (N)	≤1%
Low (L)	10%
Moderate (M)	50%
High (H)	90%
Extreme (E)	≤99%

resulting in a blank square generally represent risks not worth much, if any, concern. On the other hand, a shaded result requires attention, either in the form of additional site information and/or an effective mitigation measure whereby risk is appropriately reduced. Although simplistic in nature, Figure 11 represents an excellent risk screening tool appropriate for many applications. In the same paper Davies goes on to describe a similar, although somewhat more sophisticated, practical risk analysis tool suitable for the mining industry called potential problem analysis (PPA).

Although the methodologies described above lack the numerical rigor of more classical probabilistic risk evaluation methods, they provide one way forward for pragmatic risk analysis for more general underground mining problems.

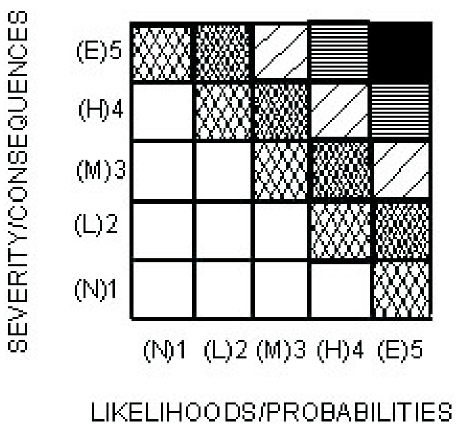


FIG 11 - Simplified risk classification scheme (Davies, 1997).

FUTURE DIRECTIONS

Through the examples discussed in this paper I have attempted to show how, even under the best conditions, underground geomechanical mine design issues generally remain within the 'data limited' category. This results largely from the inherently high degree of geological uncertainty and the resulting engineering ignorance (first kind of ignorance) factors involved in geomechanical mine design. When this is combined with 'limited understanding of the processes at work' (the other kind of ignorance) the resulting risk can escalate. Incorporation of 'random events' (eg seismic events) can exacerbate the risk dramatically.

In the case studies discussed, mine induced stress was a major factor driving geomechanical 'risk'. This results in additional problems from a quantitative risk analysis perspective. In quantitative risk analysis, one must deal with probabilities and probability distribution functions. Developing appropriate probability distributions in a data limited environment is a significant challenge. This is particularly problematic in the area of mine induced stress. In order to determine mine induced stress one must first measure far field stress. Far field stress measurement technology, however, is extremely costly, time consuming and unreliable. Developing a probability distribution function for far field stress data with existing stress measurement technology appears to be economically impractical.

The step from far field stress to mine induced stress involves numerical stress analysis of the mining step in question. Today such analysis involves 3D elastic or non-linear analysis (eg FEM, BEM, or DEM). Conducting multiple 3D analyses of multiple mining steps in order to derive mine induced stress distribution functions with today's technology is, from a mine operations perspective, totally impractical. With continuing advances in numerical computational power (both in hardware and software) however it is not outrageous to think that such capability could be available in the reasonably near future. The potential to calibrate such numerical data using seismic source parameter (ie stress drop) data offers an intriguing possibility that could strongly impact quantitative risk analysis in geomechanical mine design. To date, however, attempts at such correlations have not proven successful.

Running analyses such as discussed above on a 'one off' basis, while useful in a strategic planning sense (eg Golden Giant shaft destress slot design), is not what is needed for tactical mine design. Mining is a dynamic process and in order to be useful for day to day mine operations, risk analyses would have to be executable on a stope by stope basis, effectively in real time. In addition, for the technology to be reliable, suitable underground instrumentation data would also have to be available in real time for continuous validation and calibration of the mine design model.

Research into real-time 3D mine modelling and model validation forms the core of the 'digital mine' research project presently underway through the Lassonde Institute at the University of Toronto. This research, while promising, is several years from pragmatic day-to-day implementation at operating mine sites. From the perspective of geomechanical risk analysis, however, the far field stress data problem appears far more intractable. Serious research into this issue is needed if real time risk analysis in underground geomechanical mine design is to become a pragmatic tactical design application.

CONCLUSIONS

Standard risk evaluation techniques have tended toward a strongly mathematical basis. As such, it was essential to establish precise probability density functions to all assessed components or issues (Davies, 1997). Many areas of mining, however, reside in the 'data limited' regime. The combined effect of the seemingly complex analyses with corresponding insufficient data often results in either a poorly executed risk evaluation or no evaluation whatsoever; either result not acceptable for optimal decision-making.

While long-term research may alleviate these problems, more immediate decision-making requires an alternative approach. The judgement based methodology suggested by Davies (1997) offers one such approach. Modern computer based expert system techniques (eg neural networks) could potentially be used to make such a system tractable for use in tactical mine design planning.

REFERENCES

Bawden, W F and Jones, S, 2002. Ground support design and performance under strong rockburst conditions, in *Proceedings NARMS-TAC 2002, Mining and Tunnelling: Innovation and Opportunity*, Vol 1, pp 923-933.

Bawden, W F and Jones, S, 2005. The use of mine sequencing controlled through numerical modelling and instrumentation to destress a highly burst prone sill pillar, in *Proceedings Rockburst and Seismicity in Mines Conference*, Perth, March.

Carter, T G and Miller, R L 1995. Crown-pillar risk assessment – planning aid for cost effective mine closure remediation, in *Trans Inst Min Metall (Section A)*, Jan-April, 104:A41-A57.

Curran, J H, Yacoub, T E, Hammah, R E and Chew, J, 2001. Design of a destress slot at the Golden Giant Mine, in *Proceedings 38th US Rock Mechanics Symposium, DC Rocks 2001*, Washington DC.

Davies, M P, 1997. Potential problem analyses: a practical risk assessment technique for the mining industry, *CIM Bulletin,* 90(1009):49-52.

Duzgun, H S B and Einstein, H H, 2004. Assessment and management of roof fall risks in underground coal mines, *Safety Science*, 42(1):23-41.

Holling, C S (editor), 1978. *Adaptive Environmental Assessment and Management* (Wiley: Chichester).

LeBlanc, B C and Murdock, G M, 2000. Costs associated with sill pillar mining at Williams mine, in *Proceedings Canadian Institute of Mining and Metallurgy Annual General Meeting,* Toronto, Canada.

MacMullan, J, Bawden, W F and Mercer, R, 2004. Excavation of a shaft destress slot at the Newmont Canada Golden Giant mine, in *Proceedings Sixth North American Rock Mechanics Symposium – Gulf Rocks 2004,* Houston Texas.

Owen, M, Hudyma, M and Potvin, Y, 2002. Risk analysis for mining induced seismicity, in *Proceedings Sixth North American Rock Mechanics Symposium Mining and Tunnelling: Innovation and Opportunity,* Vol 2, pp 1079-1087.

Pine, R J and Arnold, P N, 1996. Application of risk assessment methods to underground excavations, in *Proceedings Eurock '96* (ed: G Barla) pp 1189-1196 (Balkema: Rotterdam).

Pine, R J and Thin, I G, 1993. Probabilistic risk assessment in mine pillar design, in *Proceedings Innovative Mine Design for the 21st Century,* (eds: W F Bawden and J F Archibald) pp 363-373 (Balkema: Rotterdam).

Geotechnical Risk Considerations in Mine Planning

P A Lilly[1]

ABSTRACT

At the planning and design stage of a mine, geotechnical engineers are faced with similar issues to those faced by resource estimation geoscientists. Both groups of professionals rely on a good geological model to underpin their work. They use the geological model to identify different domains within the rock mass. They are faced with having to interpolate between (and sometimes extrapolate from) widely-separated data points and are expected to develop relevant parameters from these sparse data sets. They make extremely important decisions in a highly uncertain environment and sometimes have difficulty in communicating the uncertainties and risks to non-specialist personnel. These high levels of uncertainty coupled with the magnitude of the decisions being made make it essential that risk-based approaches are adopted. This paper discusses the elements of geotechnical risk assessment, from sources of uncertainty through hazard identification and assessment to consequence assessment and tolerability.

INTRODUCTION

Traditional geotechnical engineering analysis focuses on the assessment of geomechanical stability through the estimation of a factor of safety. This deterministic approach compares the capacity of the geotechnical design with the anticipated demand to be placed on the design. If the ratio of capacity to demand is greater than unity, then the design is theoretically stable.

There is significant uncertainty, however, associated with the estimation of both capacity and demand in geotechnical engineering. Consequently, factors of safety significantly in excess of unity are traditionally applied. Depending upon the nature and sensitivity of the design and/or the consequences of failure, the selected factor of safety might typically range from anywhere between 1.1 and 2.0. However, the fact is that an estimate of factor of safety is only valid for the input values assumed in the calculation. We know that these values will change, not only from place to place within the same geotechnical domain but, in many cases, with time as well (due to creep and/or physio-chemical processes). If the values change then obviously the factor of safety changes and, in certain parts of the rock mass and/or at some point in time, its value may drop below unity. In other words, the factor of safety calculated using the mean values of the input parameters (or any other specific values, for that matter) is not likely to be correct either for most of the time or for most of the soil/rock mass because of this variability. As a consequence, a design having an apparently acceptable factor of safety could have a significant likelihood of failure.

Factor of safety, therefore, tells the mine designer and planner nothing quantitative about the chance of his or her design failing. In fact, if he or she were to double the factor of safety, for example, this would not necessarily mean that the likelihood of failure is halved. This realisation and understanding has led to the broad acceptance and use of probabilistic methods within mining geotechnical analysis. If one design has a probability of failure of ten per cent and the other five per cent then, assuming that the consequences of failure are similar, the risk of the former design is double that of the latter. Thus, we now have a direct relationship between stability analysis and risk that did not exist in the estimation of factor of safety. This implies that formal risk analysis is now embedded in the engineering analysis. However, there remain several issues associated with risk assessment and management in mining that need to be considered, and these are discussed below. The discussion focuses mainly on strategic and design risk issues rather than tactical and operational risk issues, although there is significant overlap between the two and this is manifested in the document.

1. FAusIMM(CP), Chief, CSIRO Exploration and Mining, PO Box 1130, Bentley WA 6102, Australia. Email: Peter.Lilly@csiro.au

UNCERTAINTY IN GEOTECHNIAL ENGINEERING

At the outset, it is worth giving some serious consideration to the various uncertainties associated with geotechnical engineering in the mine planning context. The reason for this is that if the geotechnical engineer or mine planner lacks an understanding of what he or she might not know, then the geotechnical risk can escalate rapidly. Conversely, if he or she takes a view or makes the assumption that everything is known with certainty, the design will be equally fraught. Surprisingly enough, the latter is not as uncommon as one might think.

McMahon (1985) identifies six types of geotechnical uncertainty. The first three are related to the fact that geotechnical engineers are working with geological materials, while the final three are related to human nature. To this list of six, the writer has added a seventh. The sources of uncertainty are as follows:

- Type 1 uncertainty refers to the risk of encountering an unknown geological condition,
- Type 2 uncertainty refers to the risk of difficulties arising due to the incorrect identification or selection of parameters for stability evaluations,
- Type 3 uncertainty relates to the risk that 'bias and/or variation in the estimated design parameters are greater than anticipated',
- Type 4 uncertainty is that due to human error,
- Type 5 uncertainty is that due to design changes,
- Type 6 uncertainty is that due to excessive conservatism, and
- Type 7 uncertainty is that associated with the fact that geotechnical engineering analysis is based on significant simplification of the actual situation.

In summary, one could classify these sources of uncertainty into those that relate to the geological model (Types 1 and 2), those that relate to the geotechnical model (Types 3 and 7), and those that relate to human factors (Types 4, 5 and 6). For the purposes of this section of the paper, the discussion will focus further on the uncertainties associated with the geological and geotechnical models associated with a design.

The main reasons for uncertainty in the geological model are usually a lack of data and/or a lack of geoscientific rigour prior to the engineering design process actually commencing. Mine geotechnical engineering at the design stage is almost without exception based on a severe paucity of data. Whilst it would be nice to have more data, it is usually not possible, which is why the input of experienced geoscientists is required to assist the engineer in understanding and articulating the geological environment that he or she is designing for and within. In many cases, however, even relatively basic geological information is lacking. Both scenarios lead to a rapid escalation in Type 1 uncertainties, where geological conditions (often deleterious structures or lithologies) are encountered that were not expected at all.

Similarly, uncertainty in the geotechnical model can be related to both a lack of data (on geotechnical parameters) and/or a lack of understanding or experience in formulating the design model based on what is known or has been experienced elsewhere in comparable environments. For example, Type 3 uncertainties often occur when the engineer lacks experience and/or data. Type 7 uncertainties, on the other hand, relate to the fact that, in most cases, geotechnical engineering deals with an extremely complex system of materials, structures, properties, forces and displacements. Consequently, geotechnical engineers often simplify rock mass properties because the reality is so complex as to be intractable. Or simplified numerical models may be used to simulate stability conditions because neither the analytical tools nor, for that matter, the data exist to model the actual conditions (even if we knew what those conditions where).

In summary, therefore, it is clear that geotechnical engineering occurs in a regime of significant uncertainty. The main reasons for this are as follows:

- the natural materials that geotechnical engineers work with are extremely complex when compared to man-made materials;
- there is a paucity of data (usually significantly less data exist for geotechnical design than do for orebody modelling, for example); and
- the analysis of stresses and displacements in complex mine excavation designs and sequences usually needs to be simplified for practical purposes.

Whilst properly educated geotechnical professionals usually have a good understanding of these issues the communication of these concepts of uncertainty becomes less clear when dealing with other professionals or the lay public. This is particularly manifested in situations where Resources and Reserves are quoted. For example, it is possible to envisage a situation where Measured Resources (in which the competent person has a high level of confidence), detailed unit mining and processing cost estimates and other ore reserve estimation parameters (in which the planning engineer may have a high level of confidence) are combined with, say, a slope design in which the geotechnical engineer may have a relatively low level of confidence. This combination of events could lead to a Proven Ore Reserve. However, is it reasonable to have a Proven Ore Reserve in an open pit design where the slope angles (which are key economic drivers) are known with a much lower level of confidence than the Resource or the mining parameters? Is there a better way for the geotechnical engineer to communicate uncertainty in much the same way as resource estimators do using the JORC Code?

The answer to this question is very much in the affirmative. Steffen (1997) and Haile (2004) provide excellent examples of frameworks for this type of discussion. In the latter case, for example, the geotechnical model is classified, with an increasing level of certainty, into Implied, Qualified, Justified and Verified, based on levels of knowledge of the geological and geomechanical conditions, and the opinion of the competent person. Then, depending on the type of mining method being mooted and its level of susceptibility to geotechnical risk, Haile (2004) identifies which level of geotechnical model is suited to which level of study (scoping, pre-feasibility, feasibility and operational). It is the writer's firm opinion that the mineral industry should pick up these concepts and use them.

GEOTECHNICAL HAZARD IDENTIFICATION AND ASSESSMENT

Assume that we now have geological and geotechnical models that are appropriate to the level of mine planning study that we are undertaking. An element of the next stage of the risk assessment process is to identify the geotechnical hazards that might beset the design. In mining environments at the strategic level these hazards are those leading to death or injury of personnel and/or loss of production, equipment, infrastructure and/or reserves. They also include geotechnical hazards that might lead to environmental or social impacts beyond those that the mining company has approval for, or those that would have a negative influence on the public perceptions of the company. Such hazards include, amongst others, mechanisms for:

- inter-ramp and/or overall pit slope failure,
- major stope or pillar failure,
- rockfalls in mine access excavations,
- backfill failure or liquefaction,
- rockbursts, and
- subsidence.

In other words, the geotechnical engineer must identify what could go wrong with the design (eg pit wall failure) and how it could go wrong (eg sliding on a fault plane). This includes considerations of individual mechanisms (such as those listed above) as well as multiple failure paths (eg more than one hazard being present or one hazard precipitating one or more other events). An excellent method of capturing these systemic inter-relationships is through Fault Tree Diagrams, which allows the engineer to analyse those elements that must take place before the top event occurs.

Once the hazards have been identified and placed into a framework such as a Fault Tree, it becomes necessary to estimate the probability of the lower level elements occurring so that the probabilities of the upper level elements can be calculated. In quantitative risk assessment of geotechnical hazards, this is often undertaken using stochastic, point estimation or first-order-second-moment methods. However, there are many situations (particularly in the initial design stages) where insufficient data exist to reliably develop the frequency distributions for the input parameters that are necessary for such simulations. There are two approaches that might be adopted here:

- frequency distributions of input parameters can be approximated based on the experience of expert personnel (eg the triangular distribution might be used initially to model a parameter, in which the minimum, maximum and expected value are estimated based on judgement and experience); and/or
- an expert or, more preferably, a group of people who have expert knowledge and experience makes a direct estimate of the probability of an event occurring.

The strength of these latter approaches should not be underestimated, as in most cases they are better than using small samples of data to try to identify appropriate frequency distributions.

Whilst it is possible (given sufficient data and experienced personnel) to identify the major hazards, what is less readily estimated is the timing of such hazards or events. If the probability of a geotechnical hazard occurring is thought to be, say, 25 per cent, then the question often becomes: 'When will this manifest itself?' How long, for example, will a large rock slope with a known likelihood failure stand before it, or more likely parts of it, actually start to 'fail'? This is an area of mining geomechanics where few reliable tools exist for engineers to use at the design stage. Even during operations, when engineers have the benefits of monitoring data, the prediction of time to failure is extremely complex and usually inaccurate.

CONSEQUENCE ASSESSMENT

Once the geotechnical hazards have been identified, placed in an appropriate framework and their probabilities of occurrence estimated, it then becomes necessary to assess the consequences that would ensue if the hazard(s) actually eventuated. In mining, these consequences relate mainly to impacts on the:

- workforce,
- public,
- environment,
- mine's production,
- mine's equipment and infrastructure, and
- ore reserves.

Semi-quantitative methods (such as the 5-by-5 Likelihood versus Consequence matrix) can be used to good effect, mainly to rank hazards and set management or design strategies for the higher risk hazards. However, in quantitative risk assessment, the Event Tree is a powerful means to identify what the key consequences are and where to focus management or design effort. In essence, Event Trees 'grow' out of Fault Trees and the combination of the two becomes a Cause-and-Effect Diagram.

The most common definition of risk is the product of probability and consequence, often within a given timeframe. That is, it is an expected value (a probability weighted average). Hazards with a high probability of occurrence and a low consequence (eg a loss of part of a berm crest in an open pit mine) can yield the same level of expected risk as hazards with a low probability of occurrence and a high consequence (eg the mass collapse of pillars in an underground mine leading to mine closure). This is clearly an unsatisfactory situation that must be assessed and managed carefully.

In this regard, Haimes (1999) notes that the risk of extreme events is misrepresented when it is solely measured by the expected value of risk. For example, a civil engineer wouldn't design an office building in Perth to withstand only expected (average) wind speeds because the first cold front of autumn would blow the building over. In the case of such potentially catastrophic risks, therefore, Haimes (1999) suggests that the question to ask is: 'What is the maximum risk?'

One element of geotechnical risk assessment is the need to ensure that we understand as far as possible the impacts that current decisions will have on future operations. In other words, are those geotechnical design and planning decisions being made today going to adversely affect the overall project in the future? Bawden (2004) notes that short-term demands often have the capacity to threaten long-term reserves. He gives the example of a stope sequence designed to maximise short-term cash flow that may be sub optimal in overall, project terms because the mining strategy might jeopardise parts of the Ore Reserve through increased levels of mining-induced seismicity. An example from the open pit environment is an aggressive, short-term open pit footwall slope design that ignores the fact that the decline portal to a future, longer-term underground operation will be located near the base of the same slope.

In summary, high consequence (catastrophic) geotechnical hazards must be identified, assessed and managed separately, and the short-term decisions made today must be considered in terms of their future (consequential) impact on overall project value.

TOLERABILITY OF RISK AND DESIGN OPTIMISATION

Having established the risk (probability and consequence) associated with a particular geotechnical hazard, the next question becomes: 'Is the risk tolerable?' In assessing tolerability, it is common to separate risks to people from those associated with equipment, infrastructure, reserves and the like.

When assessing risk to people (in particular, where the consequence is premature death), what is deemed to be tolerable by a particular social group is quite often known from other sources. For example, there is anecdotal evidence to suggest that most Australians would believe a risk to be intolerable when it is more dangerous than driving a car in Australia (a fatality rate of about 10^{-4} per year). Steffen (2004) notes that this rate is also equivalent to the lowest natural death rate for the least susceptible population group in North America (those in the age range ten to 14 years). On the other hand, the public generally feels comfortable with risks that have fatality rates similar to or lower than those associated with being killed by lightning strike (about 10^{-7} per year).

What this means is that as far as the general public is concerned, risk to the individual must be engineered out where it is greater than 10^{-4} per year, must be managed at be 'as low as reasonably practicable' or ALARP between 10^{-4} and 10^{-7} per year, and can be more-or-less ignored when it is less than 10^{-7} per year. From a geotechnical perspective, what the ALARP zone means is that, amongst other things, we must have ground control management plans in place to ensure that these risks are properly managed.

It is well known that the public becomes much more risk averse when multiple fatalities are possible. This is understandable since the consequence (that is, the likelihood of more people being killed) is greater. To assess societal risk, it is common to make use of diagrams (called FN diagrams) that plot annual fatality frequency rate (F) against the number (N) of fatalities.

In most other cases, the tolerability of risk is assessed via cost-benefit analysis. For example, Lilly and Villaescusa (2001) show how risk-based analysis can be used to optimise slope angles in open pit mines or stope sizes in underground mines. This involves comparing the benefits associated with steeper slopes or larger stopes with the costs (usually resulting from the management and control of geotechnical hazards) of these same slope angles and stope sizes. Based on quantitative risk assessment, it is possible to make better decisions that are more scientifically based.

Geotechnical hazards usually relate in one form or another to failure within the rock mass. However, what 'failure' actually means in terms of tolerability varies depending on the situation and the perception of the people involved. A 'failing' slope in one part of an open pit mine may be tolerable, but a similar situation in the wall in which the single access is located might not be. In this context, Harr (1987, p 162) has suggested that:

> *failure designates the inability of a system to perform its intended function. All systems fail eventually. However, from an engineering point of view it is the survival time before failure that determines whether the system was successful or not.' The reliability of a geotechnical design is, therefore, interpreted as the probability that it 'would perform adequately for at least a specified period of time and under specified operating conditions.*

CONCLUDING COMMENTS

At the planning and design stage, geotechnical engineers are faced with similar issues to those faced by resource estimation geoscientists. Both groups of professionals:

- rely on a good geological model to underpin their work;
- use the geological model to identify different domains within the rock mass;
- are faced with having to interpolate between (and sometimes extrapolate from) widely-separated data points;
- are expected to develop relevant parameters from these sparse data sets;
- need to use risk-based approaches to properly assess their output (orebody models and excavation designs, respectively);
- make extremely important decisions in a highly uncertain environment; and

- sometimes have difficulty in communicating the uncertainties and risks to non-specialist personnel.

The high levels of uncertainty coupled with the magnitude of the decisions being made make it essential that risk-based approaches are adopted.

> *To cope with hazards, whether natural or manmade, it is necessary to understand risk and try to quantify it. One has to devote attention to the various uncertainties associated with information used in engineering analysis and design. This can lead to an awareness of the nature of the engineering parameters and analytical models… Recognising our imperfect knowledge of the behaviour of engineering systems and the uncertainties associated with engineering parameters the limitations of deterministic approaches become obvious. These approaches do not permit the analysis of reliability and risk under conditions of uncertainty. New strategies and new approaches are, therefore, necessary to assess failure probabilities and risks associated with the environment and with human life* (Chowdhury, 1992, p 39).

ACKNOWLEDGEMENTS

During the International Conference on Orebody Modelling and Strategic Mine Planning held in Perth, Western Australia, in November 2004, a panel was convened to lead a discussion on the topic of geotechnical risk. This panel included Dr Oskar Steffen of SRK Consulting, South Africa, Professor Will Bawden of the University of Toronto, Canada, and the author. This discussion led to a number of points being made, some of which have been included in this paper. The author warmly thanks and acknowledges the valuable contributions of Dr Steffen and Professor Bawden to this discussion.

REFERENCES

Bawden, W, 2004. Personal communication, November.

Chowdhury, R N, 1992. Probabilistic risk analysis in geomechanics and water engineering, in *Geomechanics and Water Engineering in Environmental Management* (ed: R N Chowdhury), chapter 2 (Balkema: Rotterdam).

Haile, A, 2004. A reporting framework for geotechnical classification of mining projects, *The AusIMM Bulletin*, September/October, pp 30-37.

Haimes, Y Y, 1999. Risk management, in *Handbook of Systems Engineering and Management* (eds: Sage and Rouse) pp 137-173, (John Wiley and Sons).

Harr, M E, 1987. *Reliability-Based Design in Civil Engineering*, p 290 (McGraw-Hill).

Joint Ore Reserves Committee of The Australasian Institute of Mining and Metallurgy, Australian Institute of Geoscientists and Minerals Council of Australia (JORC), 2004. Australasian Code for Reporting of Exploration Results, Mineral Resources and Ore Reserves (The Australasian Institute of Mining and Metallurgy, Melbourne) [online]. Available from: <http://www.ausimm.com.au/main/about/docs/jorc0105.pdf> [Accessed: 7 May 2007].

Lilly, P A and Villaescusa, E, 2001. Optimising pit slope angles and underground stope spans using a risk-based approach, in *Proceedings Strategic Mine Planning Conference*, pp 71-74 (The Australasian Institute of Mining and Metallurgy: Melbourne).

McMahon, B K, 1985. Geotechnical design in the face of uncertainty, *Australian Geomechanics*, 10:7-19.

Steffen, O K H, 1997. Planning of open pit mines on a risk basis, *J SAIMM*, March/April.

Steffen, O, 2004. Personal communication, November.

Mine Design in Western Australia — A Regulator's Perspective

I Misich[1] and P Burke[2]

ABSTRACT

This paper relates only to the legislation relevant to mine planning and design in Western Australian mines that is administered by the Department of Consumer and Employment Protection (DoCEP) – previously the Department of Industry and Resources (DoIR). It does not address the legislative requirements of Federal agencies or other agencies in this state. The current relevant legislation is contained within the Mine Safety and Inspection Act, 1994 (MSIA) and supporting Mine Safety and Inspection Regulations, 1995 (MSIR), and the Mining Act, 1978 (MA) and corresponding Mining Regulations, 1981 (MR).

Although the current safety-based MSIA and environmentally-based MA models are generally performing better than earlier legislative models, concerns are held for significant incidents (resulting from inappropriate mine planning and design) that appear to be occurring at irregular intervals.

The major contributing factor in the mine design-related incidents is the lack of a systematic and flexible approach to mine planning and design. In some instances, a lack of understanding of the mechanisms that can potentially disrupt mine productivity and safety has led to inappropriate mine planning/mine designs being implemented – highlighting the need for continued research into some design issues.

Mine design processes need to be 'attuned' to the general mining conditions as they evolve through the full life of a mine. In conjunction, the process used to attune mine design is ideally pro-active and is best achieved by forward investigation of the factors that are likely to affect the mine performance.

Examples of inadequate mine planning and design that have led to potential safety issues or fatal injuries to mine personnel and sterilisation of ore are presented. Discussion is provided on possible actions open to the Western Australian State Government, through amendments to the existing mining legislation, to better protect mine personnel and Western Australia's valuable and finite resources[†].

INTRODUCTION – EXISTING RELEVANT LEGISLATION

The current regulatory requirements for mine planning and design in Western Australia are essentially limited to the generic safety requirements described in Section 9 of the MSIA, various regulations in the supporting MSIR, and environmental/land use requirements in various parts of the MA and corresponding MR.

From a mine safety/design perspective, MSIR 3.13 requires mining operations to present a formal document (a Project Management Plan [PMP]) to the State Mining Engineer (SME) for review prior to commencement or recommencement of mining at a particular site. The PMP is to include:

- a summary of the proposed mining operations, mineral processing and expected mine life;
- a broad assessment of the major risks associated with the mine and a summary of the strategies proposed to manage those risks;
- a summary of proposed ventilation, stoping and development systems for any underground mine; and
- details of proposed emergency management strategies for the mine.

To help facilitate the PMP process, the DoCEP provided guidance material, entitled *Project Management Plan Guideline* (DoCEP, 1997), available on the DoCEP website.

The MSIR provides definition of legislative requirements to safely manage various major hazards that may be present at a particular site. Examples of Regulations requiring proper planning and design include: Regulations 6.2 and 6.17 (maintenance, construction and general use of plant); Regulations 6.2 to 6.6 (designers of various plants); Regulations 7.27 and 7.28 (health of persons at the mine site); Regulation 9.14 and 9.28 (mine ventilation) and Regulations 10.28 and 13.8 (geotechnical issues). The MOSHAB-issued code of practice, entitled *Surface Rock Support for Underground Mines* (DoCEP, 1999a) also makes important note of the need for planning. Each regulation relevant to a particular site should be referred to when preparing a PMP.

The MA makes reference to the requirements of holders of various types of mining tenements to satisfy lease/licence conditions for each site, eg Section 84, relating to mining leases. A condition of grant on the majority of mine tenements is the requirement of the lessee/licensee to submit a plan (a Mining Proposal [MP]) of proposed operations and measures to safeguard the environment to the DoIR for assessment. Where relevant, the MP will also contain documentation on the safe design and management strategies for structures, such as pit walls near public infrastructure and tailings storage facilities. As with the PMP, the DoIR has provided guidance material entitled *Mining Environmental Management Guidelines – Mining Proposals in Western Australia*, which is available on the DoIR website (DoIR, 2006).

The DoCEP/DoIR has also published various guidelines (DoCEP, 1998, 1999b), such as *Guidelines on the Safe Design and Operating Standards for Tailings Storages*, to assist the industry with meeting the DoCEP standards with the intention of limiting potential for incidents, such as that illustrated in Figure 1.

The general intent of each part of the relevant legislation, and the requests for formal documents, is to encourage mining companies to logically think through the major issues related to the safety and environment at each mine site for the *full life* (and beyond) of a mining project. (The two-year incremental mine design model, generally favoured by the Western Australian mining industry in the past, does not meet with the general intent of current mining legislation.)

FIG 1 - A gateway to hell: the result of poor embankment design and operational planning for a tailings storage facility.

1. MAusIMM(CP), Senior Geotechnical Engineer, Mines Safety Branch, Resources Safety Division, Department of Consumer and Employment Protection, 100 Plain Street, East Perth WA 6004, Australia. Email: imisich@docep.wa.gov.au

2. Manager – Engineering Safety, Mines Safety Branch, Resources Safety Division, Department of Consumer and Employment Protection, 100 Plain Street, East Perth WA 6004, Australia. Email: pburke@docep.wa.gov.au

† See disclaimer at the end of this paper.

From a mine safety perspective, which is the main focus of this paper, the DoCEP/SME does not 'approve' or validate these precursory mine planning documents. However, if the DoCEP considers that certain details that could impact on the safety of mine personnel are lacking, additional information can be requested before the project commences. 'Validation' of the adequacy of mine plans, with respect to general mine safety, is largely achieved through site audits and general inspections.

From an environmental perspective, mining activities are not permitted to commence without the written approval from the DoIR; which is given on the understanding that the actions specified in the MP will be satisfactorily implemented. The MP is listed as a new condition of the mining tenement and all commitments in the MP become legally binding conditions of the lease(s).

The MSIA and MSIR do not require any modifications to the precursory planning strategies (given in the PMP) to be reported to the DoCEP (through the SME); whereas the MA, through the lease/licence conditions imposed, requires most variations to the MP to be formally presented to the DoIR.

The legislative approach in both Acts is largely non-prescriptive/self-enabling and rightfully places the responsibility for safe mining on mine management.

The current model is working better than its predecessors; however, it is still not 100 per cent effective. Current statistics show (eg Lang and Stubley, 2004) that incidents continue to occur, with and without injury, and fatalities have not been eliminated.

Most incidents seem to be associated with the actions of mine personnel. Some incidents, however, are directly related to mine planning shortcomings; particularly planning issues associated with the management of change. The DoCEP/WA Government advocates that mine design and planning incidents are avoidable and have consequently set out a legislative framework that aims at preventing such events. When mining incidents do occur and are seen to jeopardise the safety of any person at a mine, the legislation also has provision for penalising 'responsible persons'. Where it is clear to the Regulator that continued mining would further jeopardise the safety of mine personnel, the legislation provides the Regulator with 'powers' to call for cessation of mining (in specific areas of a mine, or the whole mine). In some instances, mining companies have voluntarily shut operations after a major hazardous event – and only recommenced operations once they (and the SME) are satisfied that similar events cannot reoccur.

CASE STUDIES

Examples of avoidable inappropriate outcomes resulting from mine design and planning incidents, in a changing mining environment, are discussed below. The selected examples represent a few of the major design issues that were the source of significant incidents. The selected design issues are water/slurry control, ground control and stope sequencing, ventilation and maintenance of plant. Comment is provided on the relevant legislative shortfall in each case.

Water/slurry control

Case Study 1

It is a curious fact that, whilst the climate in Western Australia is generally considered dry, water-related hazards have featured prominently in a number of serious incidents. In 1989 six people lost their lives when an exploration decline flooded during a severe rainfall event. In this case study, the hazard of water inrush had been recognised in the planning and layout of the mine and steps had been taken which, it was believed, would effectively eliminate any risk of flooding.

A disused open pit was being used for collection and storage of water for process purposes. This adjoined another pit, which was actively worked until a few weeks before the inrush took place. The direction of mining was toward the water storage pit. When mining ceased, a separating pillar remained between the pits; this was deemed to be adequate to provide a barrier of sufficient strength to resist an inrush. Additionally, it was anticipated that should the water storage pit overfill, excess water would flow into old underground workings, which were reputed to have a capacity of 300 000 m^3.

Some four months prior to the inrush an exploration decline was commenced from the operating open pit. The initial position for the portal was planned to be in excess of 20 m above the pit bottom. Some concerns regarding the stability of the host rock caused the intended location to be moved to within 3 m of the pit floor. Inexplicably, the actual decline entrance finished up at pit bottom level.

A prolonged and heavy rainfall event caused a large quantity of run-off water to enter the 'storage' pit. Eventually, the separating pillar was overtopped and water began to enter the active pit. A relatively small quantity of water diverted to the old underground workings. The overtopping rapidly eroded the separating pillar, allowing a progressively increasing flow to enter the active pit. In a very short time the decline filled up, drowning those below.

The sequence of events allowing the disaster to happen was:

- The potential volume of run-off water that could accumulate in the water 'storage' pit was greatly underestimated.
- The active pit was mined very close to the water storage pit.
- The assessment of the material that comprised the separating pillar would appear to have been superficial. Observations made after the incident indicate that the material forming the separating pillar may not have been '*in situ*' and if fact may have been old stope fill.
- An assumption was made that any excess water would find its way into the old workings.
- The decline entrance was located at the *bottom* of the open pit.

It is undeniable that deficiencies in planning, design and operational follow-through all contributed to this tragic event. At the time, the legislation did not contain any provisions to ensure that these issues would be addressed. The PMP regime established by the MSIA in 1994 is a response to this need.

Case Study 2

In 1992 a rain depression dropped large quantities of water on Western Australia in a very short time. One particular gold mine in the Kalgoorlie area, fortunate to have escaped the major downpour, was flooded by water runoff from upstream catchments some time after the rain depression had passed. It is understood that, as one of the pits looked likely to go 'under', it was encouraged to flood (by removing a small bund) so the pit could be used as fresh water storage for general mine use.

Towards the end of 1994, that mine began planning for underground workings in the pit that had previously flooded. After reviewing the PMP, the DoCEP was concerned that, given the design of the bunding intended for the proposed project and the history of water discharge through the area, that there was reasonable potential for another flood event during the life of the mine. After a series of discussions between the DoCEP and mine management, it was agreed that the bund height should be increased and that the construction methods used to build the bund be improved to meet with the intended scope of the project.

In February 1995, cyclone 'Bobby' performed a feat similar to the 1992 flood in the region, but on this occasion there was no inrush into the open pit or the new decline.

The diversion bund withstood the onslaught and, as Figure 2 illustrates, the additional precautions were vindicated. It is understood that, even with the additional earthworks, the water level peaked only 30 cm from the top of the diversion bund.

FIG 2 - Stranded haul trucks illustrate the effectiveness of well-designed flood bunding.

Case Study 3

A gold mine now closed, was not so fortunate. Bobby's floodwaters eventually overtopped a small safety bund around the pit and filled the open pit and underground workings near the base of the pit to within 20 m of the surface in a matter of hours. Fortunately, the notice given of the impending flood was sufficient to allow the withdrawal of all personnel from the mine. However, not all the mine equipment was recovered from the underground workings. The flood resulted in the closure of the mine, until it was sold off, pumped dry and re-worked by another mining company.

The power of rampaging floodwaters, illustrated in Figure 3 from this flood, must not be underestimated – even if a mine is located in a relative 'desert'.

It is clearly essential, during the mine design stage, that the issue of water drainage is looked at closely and accounted for adequately. Water diversion/control features must be designed to handle the volumes of water expected for an appropriate design rainfall event and must take into account the following aspects:

- the elevation, gradient and geometry of all flow paths and catchments;
- the likely flow rate;

FIG 3 - Rampaging floodwater at Case Study 3.

- the peak elevation of the flood waters at all locations along the drainage path;
- the nature of the materials that the flood waters will pass through or against;
- the construction methods and materials used to build diversion bunds; and
- the effect of any road works, earth mounds or water catchment and diversion features on the water-flow through the site.

Case Study 4

The most recent flooding disaster in the Western Australian mining industry was the release of approximately 22 000 m^3 of backfill into the workings of an underground mine, taking three lives. This was a modern operation in which planning and design were generally well resourced and implemented. A comprehensive MP document was submitted to the DoCEP and DoIR, which dealt with the preparation of backfill material and the arrangements for bringing it underground. Matters such as the separation and disposal of the slime fraction, together with cyanide destruction were fully addressed. As would be expected, the details provided in the environmentally-based MP ended at a distribution point 'somewhere' in the underground workings. The MP was the only formal detailed planning document that the DoIR needed for the backfill system, and at the time it met legislative requirements.

The original system – as defined by the MP – was based on a cemented rockfill model and was set up to work in that manner. The amount of design and planning work done on the underground features of the original system was significant.

Not long before the incident, operational changes were made to the system. At some point the backfill system became effectively a sand/clay fill model with no rock and no cement. This significant change seems to have been made 'on the run'; it is unlikely that the DoCEP would ever have been made aware of it. Consequently, even though the DoCEP had little formal knowledge of the original design, this paper in no way wishes to suggest that if the modified backfilling process had been 'vetted' by the DoCEP officers the disaster could have been avoided.

This example illustrates a regulatory dilemma – do we need to go back to more prescriptive legislation to prevent incidents such as these occurring? Prescriptive legislation, used for earlier regulatory models, was found wanting in Western Australia and was replaced by the current 'enabling' legislation in 1994. The origins of 'enabling' legislative environments, faithfully pursued by many agencies throughout the world, can be traced back to the 1976 Robens committee.

The move to enabling legislation (or self-regulation) has since generated considerable concern in some quarters, both within and external to Western Australia. In fact, the Royal Commission on the Longford disaster in Victoria considered that the self-regulatory regime in place at Longford actually contributed to the accident (Hopkins, 2000). Hopkins, in review of the Longford disaster, recommended the adoption of 'tighter' self-regulation.

In the inrush case study, three people, our colleagues, died. The mining company was prosecuted under Section 9 of the MSIA. Can our current legislation guarantee that this kind of event will not be repeated? Probably not; however, the current model is likely to be close to the mark. Mechanisms such as the PMP, which demonstrate to the regulatory authority that safety is being, or will be, effectively managed at a mining operation, could be described as the 'new prescription' mentioned by Hopkins (2000).

Examples of some potential legislative changes that could be implemented to alleviate some of these concerns are provided later in this paper.

Ground control

Case Study 5

The stoping method used at this case-study mine was cut-and-fill. The vertical height of each advancing cut was typically 3 to 4 m. Shortly after commencing mining of the fifth (and final) cut-and fill lift in one particular stope a rock burst occurred, killing two mine personnel. The rockburst event happened at approximately 600 m below surface.

The relevant section of the stope drive was much wider than the typical 4 m width (up to 8 m wide). In response to the wider stope drive span, mine management upgraded local rock reinforcement to include grouted cable dowels in conjunction with the standard reinforcement of 2.4 m long split sets and 1.8 m long universal bolts.

The rockburst induced a wedge-shaped fall approximately 3.5 m high (up-dip of the stope drive) and approximately 11 m long. It was evident that the wedge-shaped void was defined by three geological structures: the open footwall contact; an east-west joint that cut across the ore at roughly right angles to the stope; and a major fault that cut obliquely across the stope. A sketch of the resultant fall of ground is provided in Figure 4.

A seismic event measuring 1.4 on the Richter scale was recorded by AGSO at the time of this rockburst event.

The act of mining the fifth lift had three major adverse effects on the stability of the crown pillar:

1. Stresses in the crown pillar increased significantly. From the mechanical properties provided by the company, the DoCEP's own predicted 3D model stresses after Lift #5 (80 per cent greater than modelling stresses after 'mining' Lift #4) were more than twice the rock mass strength of the ore.
2. The effective strength of the crown pillar was reduced by reducing its slenderness ratio (pillar width to height) by 40 per cent. The effect of pillar strength in relation to its slenderness ratio is well documented (Salamon and Oravecz, 1967). The exact influence slenderness ratio has on pillar strength is site-specific. From personal experience and published literature, it would not be unreasonable to expect the strength of the crown pillar above Lift #5 to have been reduced by around 20 per cent.
3. A major, unfavourably oriented plane of weakness (a fault plane) was aligned diagonally across the length of the pillar. The effect of planes of weakness in rock on rock mass strength is also well documented. For example laboratory rock testing results provided in Hoek and Brown (1981) indicate that such features can lower rock mass strength by more than 50 per cent. Again, such effects are site-specific.

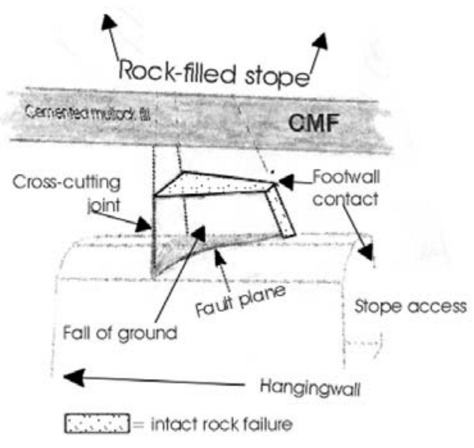


FIG 4 - Perspective view of rock failure from hanging wall side.

It is not hard to understand that the crown pillar would have been under far poorer stability conditions after mining Lift #5 (possibly up to 150 per cent worse). The mining company had knowledge of the relevant parameters and had developed modified working procedures – in accordance with Regulation 10.28 (3). However, it would appear that although the required information was available, through diligent geological mapping, etc mine management did not realise the clear potential for a quantum shift in the level of risk when deciding to mine the fifth and final lift. From the information provided, the mine did not systematically analyse and re-interpret the geotechnical data available to determine appropriate pillar dimensions. (As required by Regulation 10.28 (3), (a) and (b)).

As a result of this rockburst, the DoCEP instructed the mining company to cease mining in the stope and other stopes in similar stages of extraction until a detailed investigation was carried out on the suitability of the adopted mining method for the remainder of the mine life (which was in its latter stages). This area was never re-opened, and several other stopes with similar crown-pillar configuration were abandoned. A significant proportion of the mine's proven reserves (and Western Australia's valuable resource) was sterilised. The company was prosecuted under Section 9 of the MSIA.

Without commenting on the level of effort from mine management in this instance, it is clear that mine planning and design, formulated initially at the PMP stage from relatively limited geological and geotechnical information, needs to be regularly challenged and reassessed for the full life of the mine. Similarly, the continued use of empirically-based design criteria (eg the Q-system), that can be affected by site-specific characteristics of rock, must be verified at site level by rigorous investigation at all stages of mining.

Although it is to be expected that the amount of geological information available at the PMP stage will always be limited in comparison to the geological complexities of an orebody, mining companies are encouraged to place a greater emphasis on the use of diamond drilling to attain geotechnical and geological data. Crushed rock samples produced by other drilling methods provide little if any indication of rock conditions or structure. It could be argued that mine designs that have been based almost entirely on data attained from crushed rock samples contravene the MSIR.

Furthermore, the impact of geological structure on the mechanical strength of rock in operating mines appears to be either not well understood or underestimated – particularly in areas having elevated ground stresses – and strangely fails to receive the attention it deserves in Western Australian mines (Misich, 2002). It should be expected that, where the orientation and magnitude of principal stresses and geological structure are deleterious, a mine *will* have violent ejections of rock.

In such instances, the rock stress is redistributed around the rock defects (which can be viewed as a pre-existing failure plane) and mine void, creating tension or shear in the remaining intact rock. When the shear force of the redirected stresses exceed the strength of the cross-sectional area of remaining intact rock (that the relevant forces are being directed through), the rock can fail violently. Without such defects, that general rock mass would be expected to comfortably maintain a stable condition. It follows that larger defects (eg regional faults) and higher local stresses can result in shear through larger volumes of intact rock and thereby produce high-magnitude seismic events.

It is essential that the philosophical approach to the mine design process suitably take into consideration all factors that have any potential for deleterious effects on mine safety.

Clearly, there is a general need to adopt a more systematic approach to the research, investigation, analysis and documentation of rock failure events in order to better understand the underlying causes and their effects on mines (Misich and

Lang, 2001). This is a requirement of Regulation 10.28 and 13.8. Mines that do not undertake such an approach need a defensible reason for not doing so, or must adopt a very conservative mine design that eliminates the need to fully understand the characteristics of rock strength and behaviours, otherwise the operators risk having action taken against them under the MSIR.

There are many examples of mines successfully implementing a systematic approach to mine design in Western Australia and elsewhere (Falmagne *et al*, 2004). The 'never-ending' mine at Mount Charlotte is a good local example of the benefits of research and continued analysis of mine designs (Mikula and Lee, 2003). However, there needs to be a more wide-spread acceptance of this type of approach. Mine design and planning needs to be 'interactive'/convergent with the general mining conditions for the full life of the mine.

Interactive mine design and planning is ideally pro-active and not reactive. Although it may be feasible for mines to safely adopt a reactionary approach to some situations, the level of risk for significant loss is far greater with a reactionary approach.

Stope sequencing and design

Case Study 6

One example in Western Australia where reactionary change came too late was in a sublevel caving mine, where ore extraction took place radially from three access drives: a central access drive, and two drives located about one third of the distance of the strike length from the extremity of each level (see Figure 5).

This mine was experiencing major stress problems and associated ground stability concerns. Although the resultant stress regime was the product of a combination of factors, it is clear (from Figure 5) that the expected magnitude of stress would be unnecessarily raised by the method of mining (to retreat pillars) in conjunction with a square-shaped mine advancement with depth. Following a major rockburst that temporarily trapped a number of mine personnel (location shown in Figure 5) and ensuing discussions with the DoCEP, mine management agreed to change the mine design. The change was implemented slowly over a period of months, with no ore extraction being permitted from perceived areas of risk (most of the active sections of the mine). The changes subsequently improved the general ground conditions and reduced the frequency and magnitude of seismic events. However, the lateral extent of caving to the left of the earlier major collapse in the access drive (Figure 5) had been significantly diminished (retreat caving below this level would have resulted in untenable magnitudes of stress). As a result, the mine life was significantly reduced, an appreciable amount of ore was sterilised and the operations closed not too long afterwards.

Problems resulting from inappropriate stope sequencing and design are not uncommon to Western Australian mines. The Regulatory authority has directed several mines to cease stoping after experiencing a major seismic event and ground collapse. In one such mine, it took six months for mining to resume in the area of concern, and a major proportion of the upper orebody was effectively sterilised. The stope design that most commonly gives rise to serious problems is open stoping towards a central pillar – particularly when the mine is large and deep. Deep, large mine voids generate much greater stresses through solid ground such as pillars and stope abutments. The point at which stress-related problems occur will largely depend on the location of the mine (magnitudes of stress) and site specific characteristics of the rock. For example, research by Lee, Pascoe and Mikula (2001) show that, for a given depth below surface, stresses within the Yilgarn Craton are significantly greater than other parts of Western Australia, and in fact most parts of the mining world.

Major changes to mine design and planning, such as those required in Case Study 6, take considerable time to complete, and in the interim, can seriously reduce the profitability of a mine – as was the case here. Hence the need for a proactive approach to mine design. Proactive interactive mine design is best achieved by forward investigation of the factors that are likely to change the mine conditions (eg the stress field and changes in rock type, rock structure and rock temperature). It is unavoidable that many metres of diamond core drilling, core testing and a number of 3D stress measurements will be required in an extensive, long-term mine. It is essential to expect and systematically plan for change such that the required changes can be implemented prior to a significant incident actually happening (and the mine potentially being closed temporarily or permanently by the DoCEP).

VENTILATION

The most common ventilation regime used in the Western Australian mining industry is the so-called 'series' system (Figure 6). Series ventilation enjoys widespread use in all mining environments, particularly in primary development, where it is often the only practicable means available. The situation in Western Australia is somewhat unusual in that many mines are worked throughout their productive lives with series ventilation being the only system employed.

The principal weakness of this arrangement is the reliance on the re-use of potentially contaminated air through successive workplaces. This can be considered acceptable if the volume of air moved is sufficient to provide adequate dilution of any contaminants. Under normal operating conditions the system can perform satisfactorily. If, however, a major fire occurs in the principal intake airway, which virtually without exception in Western Australia, is the decline, catastrophic pollution of downstream workplaces is a very real possibility.

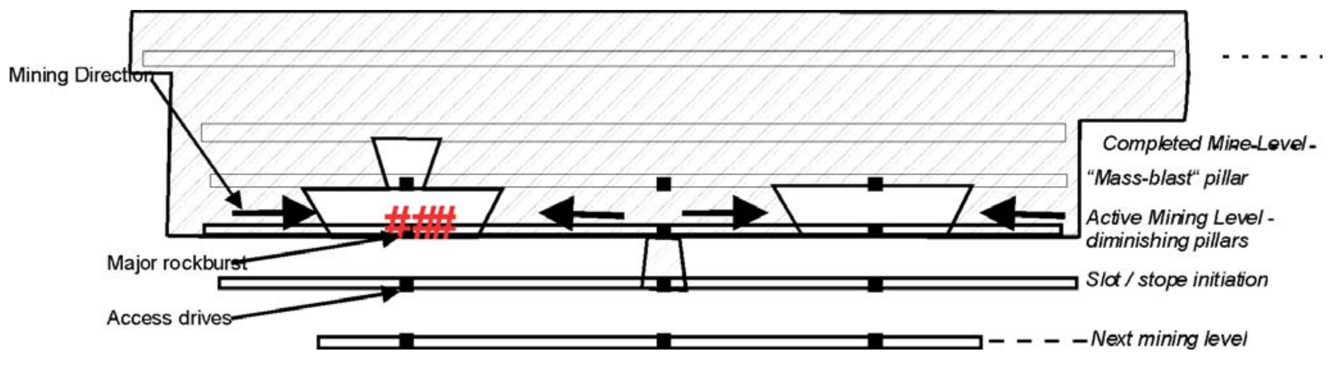


FIG 5 - Long-section view stoping sequence.

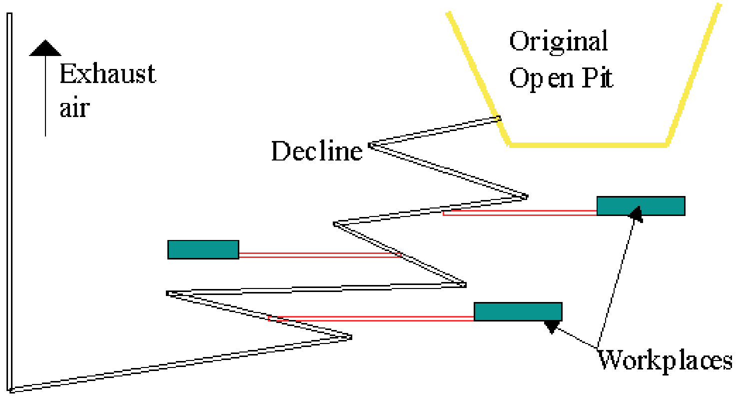


FIG 6 - Diagrammatic view of a series ventilation circuit.

For completeness, a diagram of a parallel ventilation system is included (Figure 7). It can be seen from this that all workplaces are provided with 'fresh' air and all contaminated air is quickly exhausted. It is equally obvious from this diagram that the implementation of this kind of system involves significant cost vis-à-vis the series model. Risks associated with fire underground are very much reduced by the parallel option. The smoke and fumes are exhausted directly to return by the shortest route, involving the least potential number of casualties. It is possible to extinguish the fire more rapidly because access is much easier.

The widespread use of series ventilation systems and the potential problems related to underground fires has provided Western Australian mining regulators with a major headache for many years. More that 30 per cent of all incidents reported in underground mines under MSIA Section 78 relate to fires (Lang and Stubley, 2004) – and predominantly fires on vehicles. In spite of this, no fatal injury to any person has been recorded in connection with any fire in a Western Australian underground mine. Taken at face value, it is difficult to argue for more stringent control. A view held by some in the industry – that risks relating to underground vehicle fires are effectively controlled by the measures in place and there is no cause for concern – illustrates this point.

For example, a Western Australian mine manager, required to present himself before a committee, which included the SME, to explain why his operation seemed to have so many vehicle fires, responded in a way that supports the above observation. His comments were: no serious safety consequence had ever arisen from these fires; the vehicles were fitted with AFFF (Aqueous Film Forming Foam) fire fighting systems, which were expected to deal successfully with the basic source of the fire; the mine had an active and well-equipped mine rescue team and had a mutual support agreement with associated mining operations; the mine also had effective warning systems – Stench Gas and PED; it had also purchased and installed state of the art underground refuge chambers. The manager did not see any problem. The problem was, of course, that despite these impressive but *reactionary* measures, no attempt was being made to address the fundamental issue of *fire prevention*.

It can be argued that the industry is a victim of the perception of its own success. Hopkins (2000), in referring to organisations that focus on their success in terms of safety, states:

> *Under the assumption that success demonstrates competence, people drift into complacency, inattention and habitual routines. They use their success to justify the elimination of what is seen as unnecessary effort and redundancy. The result for such organisations is that current success makes future success less probable.*

Mines are encouraged to research the causes of their underground fires, assess the risks associated with these fires and take the necessary precautions. One example of a mining company making successful inroads towards addressing this issue is given by Roberts (2003), where after a review of fire incidents in all of its Australian mines the company was able to reduce the incidence of fires by two thirds.

Risk is defined as:

Likelihood of event × *Severity of consequence*

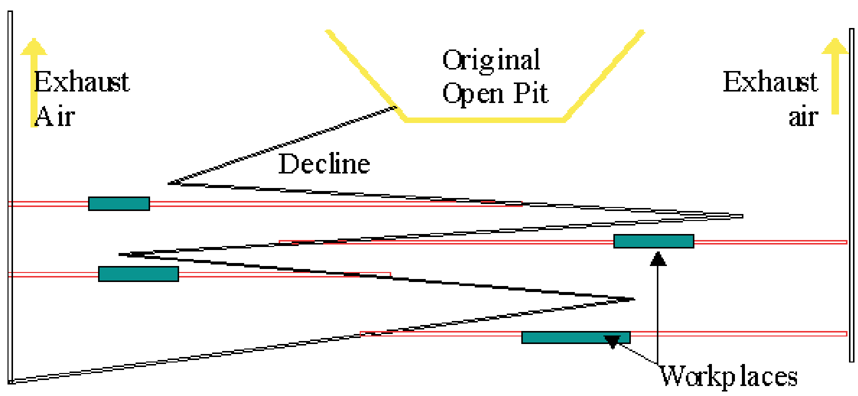


FIG 7 - Diagrammatic view of a parallel ventilation circuit.

The large number of underground fires reported each year suggests a high likelihood of that event occurring. Any fire underground is potentially a high consequence event – in a series ventilation environment the consequence of an underground fire is potentially catastrophic. The fact that no serious injury relating to a fire has so far been recorded should afford no comfort; we could be heading for a major disaster.

Plant design and maintenance

Another example where mine planning strategies require reassessment, or some form of change management approach is the maintenance of large equipment and high temperature and pressure plant.

Shifting loads, the wearing of parts and large fluctuations in the temperature of reaction chambers, etc can significantly change the pressure levels at specific focal points of the structure involved. There have been a number of incidents in recent times when, in the process of regular maintenance, mine personnel have paid dearly for the price of not being aware of the actual condition of the item they were dealing with. As mentioned earlier, certain Regulations require that both the principal employer and the designer of plant are obligated to assess the risk that personnel could be exposed to potential hazards and develop appropriate systems/plant design to manage that risk at all stages of the mine.

Working with new technologies makes it more difficult to assess and quantify the risks and control measures required to minimise the exposure of the workforce to unforseen hazards. Risk assessment processes work best when there is a large amount of information on the working life and performance of structures. Fixed items (eg pipes and valves, oil rigs and standard treatment plants) generally have sufficient performance data to allow management strategies to incorporate safety case or similar control measures.

When there is only limited information available on the expected performance of an item, it is not possible to determine an accurate likelihood of failure (see ventilation case study) and therefore not possible to determine the exact level of risk. In situations like these it is necessary to design the particular item more conservatively or be conservative in the manner in which personnel interact with that item. This should be taken into account when planning maintenance on certain items. The same need for a conservative approach applies to other areas of concern where there is not a full understanding of all the processes involved – eg the source of seismicity.

One example where the maintenance and operation of an engineering structure was found to be deficient was the collapse of a walkway at a shiploading facility (Figure 8, Anon, 1998) that caused six mine personnel to fall 8 m into the sea. Fortunately, no one died from the incident; however, some serious injuries were reported.

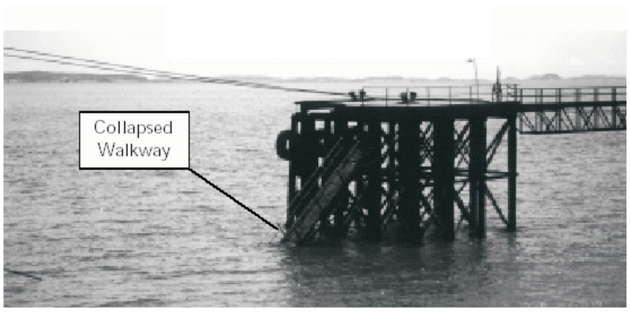


FIG 8 - View of the walkway structure that collapsed into the ocean.

Where the corrosion of supporting steelwork has diminished the original strength of the structure, a structural design engineer should be employed to assess the safety of the structure. If remedial work is recommended then the work should be carried out immediately. Structural design engineers must ensure design calculations for new and reconstituted plant conform to the relevant Australian Standards. Section 14 of the MSIA and Regulations 6.3 to 6.5 of the MSIR clearly define the duties of persons who design any plant for use at a mine.

POSSIBLE LEGISLATIVE MODIFICATION

Whilst there has been general acceptance of Western Australia's existing approach to regulating mines, there is room for improvement. A number of incidents are still occurring that potentially jeopardise the safety of mine personnel and sterilise the State's valuable finite resources. To make matters worse, some mine managers tend to 'keep their own counsel' on outcomes such as these. This makes it difficult for others to learn from their experience and increases the chances for similar occurrences at other mines – as indicated by the number of 'like' incidents being reported to the DoCEP. It is not acceptable for the industry and general community that mine management at each mine finds out the hard way (and by themselves) that certain aspects of mining may not work when certain conditions have changed – eg designing stope extraction from a central access at depth.

It is intuitively obvious that the best way to deal with these issues and to secure the long-term viability of underground mining in Western Australia is through systematic research, investigation and, where appropriate, open discussion by all involved in the mining industry.

Although most incidents arising from design faults have not resulted in injury to mine personnel, there was always the potential that they could. Furthermore, many design-fault incidents have unnecessarily sterilised large proportions of orebodies. Three examples of ore sterilisation have been given in this paper. In another instance, where the approach to mine design was to drive along the orebody to the extremity of ore, then strip the drives to ore width and 'chase off-shoots' (leaving very wide intersections), an ore drive collapsed and worked its way up to the stoping block above. The scale and location of the collapse made it uneconomic to safely extract any ore beyond the initial collapse, from both the level of the collapse and the upper stope level (which ended up having no floor).

As indicated by the case studies provided, many mining incidents can be attributed to the inability of mine management to identify potential serious problems and suitably manage changing mining environments. Under current legislation, the Department has limited ability to encourage mines to strive to improve their processes, look for change and implement appropriate change management strategies.

If mining incidents that have the potential to jeopardise the safety of the workforce and/or sterilise orebodies continue to occur, more stringent regulatory measures may need to be considered. For example:

1. Regularly review the original PMP (in a similar manner to the annual reviews of tailing storage facilities); whereby mines would be required to justify continuing with their adopted mine design/planning strategy.
2. Adopt the same approach as for the environmental MP for the PMP. The PMP could become a legally binding document; any changes would need to be reported to the State Mining Engineer as an addendum to the original PMP.
3. Penalise mining companies that have not mined to the average grade of the declared reserve reported in company documents to the stock exchange – as has been used in South Africa.

4. Financially penalise mining companies for ore lost within a given/defined stope.

Legislative provisions such as these could be more effective in coercing companies to plan forward, look and plan for change, and to abandon the 'incremental' two-year mine planning approach, which is seen to be a major contributor to incidents derived from poor mine planning and design. For example, a much greater amount of geomechanics information will be required before mine plans are formalised and shareholders are approached for funds.

By adopting a 'life of mine' design approach, prior to commitment to mining, other options such as shafts may become more attractive. The poor economic viability of sinking a shaft when more than half the orebody has been mined using decline access will, in almost all cases, result in mine management continuing with decline development to a stage where transport costs become prohibitive. The mine eventually closes prematurely – possibly leaving large proportions of ore in the ground that would otherwise have been mined if a shaft system was implemented early in the mine life.

To successfully implement changes such as these, additional regulators would be required and significantly more time and resources would have to be committed by mining companies. The impact of these ramifications to the mining industry and Government could be significant.

It is appreciated that attempts to control these kinds of issues by legislation are plagued by the danger that potential investors might be discouraged and take their business to where they perceive a more benign regulatory environment exists. The South African legislative example mentioned above has already been raised with some sections of the industry, causing some disquiet. That proposal has not been implemented but the underlying issue is very much alive.

CONCLUSION

Open pit and underground mines in Western Australia are getting larger and deeper; a fact that presents many challenges to the commercial viability of mines and workforce safety. To ensure the ongoing viability of the mining industry, these challenges will need to be met on a wide front through careful consideration, research and investigation.

Mine designers should plan for change. Mine design processes need to be 'attuned' to the general mining conditions as they evolve through the full life of a mine. In conjunction, the process used to attune mine design is ideally pro-active and is best achieved by forward investigation of the factors that are likely to affect the mine performance.

The State can no longer afford for each mine in Western Australia to arrive at the optimal mine plan through 'the school of hard knocks', the Regulator and the Industry must endeavour to learn more from the experience of others and start planning and designing for the future now.

DISCLAIMER

The views presented in this paper are strictly those of the authors and do not necessarily represent the views of the DoCEP, or the Western Australian Government.

REFERENCES

Anon, 1998. *MINEsafe Magazine*, September (Department of Minerals and Energy – now Department of Consumer and Employment Protection [DoCEP]: Perth).

Department of Consumer and Employment Protection (DoCEP), 1997. Project management plan guideline [online]. Available from: <http://www.docep.wa.gov.au/resourcessafety/Sections/Mining_Safety/pdf_/MS%20GMP/Guidelines/MSH_GMP_ProjectManag.pdf> [Accessed: 21 May 2007].

Department of Consumer and Employment Protection (DoCEP), 1998. Guidelines on the development of an operating manual for tailings storage [online]. Available from: <http://www.docep.wa.gov.au/resourcessafety/Sections/Mining_Safety/pdf_/MS%20GMP/Guidelines/MS_GMP_Guide_tailingsmanual.pdf> [Accessed: 21 May 2007].

Department of Consumer and Employment Protection (DoCEP), 1999a. Surface rock support for underground mines – Code of Practice [online]. Available from: <http://www.docep.wa.gov.au/resourcessafety/Sections/Mining_Safety/pdf_/MS%20LP/MS_LP_Code_SurfaceRockSupport.pdf> [Accessed: 21 May 2007].

Department of Consumer and Employment Protection (DoCEP), 1999b. Guidelines on the safe design and operating standards for tailings storage [online]. Available from: <http://www.docep.wa.gov.au/resourcessafety/Sections/Mining_Safety/pdf_/MS%20GMP/Guidelines/MS_GMP_Guide_tailstandard.pdf> [Accessed: 21 May 2007].

Department of Industry and Resources (DoIR), 2006. Mining environmental management guidelines – Mining proposals in WA [online]. Available from: <http://www.doir.wa.gov.au/documents/environment/ED_Min_GL_MiningProposalsInWA_Jan07.pdf>. [Accessed: 21 May 2007].

Falmagne, V, Gagnon, G, Oulett, D and Simser, B, 2004. Mining strategy for the retreat zone at Noranda's Bell-Allard Mine, *CIM Bulletin*, 97(1079).

Hoek, E and Brown, E T, 1980. *Underground Excavations in Rock*, pp 160-161 (The Institution of Mining and Metallurgy: London).

Hopkins, A, 2000. *Lessons from Longford*, pp 92-99, 141 (CCH Australia Ltd: Sydney).

Lang, A M and Stubley, C D, 2004. Rockfalls in Western Australian underground metalliferous mines, in *Proceedings Fifth International Symposium on Ground Support in Mining and Underground Construction*, Perth, September 2004.

Lee, M F, Pascoe, M J and Mikula, P A, 2001. Virgin rock stresses versus rock mass strength in Western Australia's Yilgarn greenstones, in *Ground Control in Mines Workshop*, June (The Chamber of Minerals and Energy of Western Australia Inc: Perth).

Mikula, P and Lee, M, 2003. Confirmation of Q Classification for use at Mt Charlotte Mine, in *Proceedings 1st Australasian Ground Control in Mining Conference*, November (University of New South Wales: Sydney). Note: Title of presented paper is different to the paper listed in the proceedings.

Misich, I, 2002. Deeper thought required, *Australia's Mining Monthly*, July.

Misich, I and Lang, A, 2001. Examples of rockburst damage in Western Australia, in *Proceedings Fifth International Symposium on Rockburst and Seismicity in Mines (RaSiM5)*, September (South African Institute of Mining and Metallurgy: Johannesburg).

Roberts, R, 2003. Newmont does its fire risk, *Australia's Mining Monthly*, August.

Salamon, M D G and Oravecz, K I, 1967. Rock mechanics in coal mining, Contract report to the Coal Mining Research Controlling Council, Chamber of Mines, South Africa.

A Practical Application of an Economic Optimisation Model in an Underground Mining Environment

I Ballington[1], E Bondi[1], J Hudson[2], G Lane[1] and J Symanowitz[1]

ABSTRACT

A prototype economic optimisation model has been developed to enhance the strategic planning process at Gold Fields Limited's deep level South African gold mines. This paper discusses briefly the building blocks of the economic optimisation model and then its application to the strategic planning process. A case study at one of Gold Fields Limited's shafts highlights how the model has been used within the scenario planning environment. A brief synopsis of mine optimisation theory is included.

INTRODUCTION

In an ever changing world, where the velocity of information flow can have an immediate effect on the economics of mining concerns, the lack of management decision tools may be disastrous for both the mining company and its stakeholders. In the open pit mining environment, there has been substantial development of optimisation tools. However, this has been lacking in the underground mining environment. Optimisation entails the allocation or configuration of resources, within the control of management, which will maximise or minimise a specifically desired objective. There is an optimal configuration of controllable variables (eg mining rates, capital expenditure, shift cycles and so on) for a given set of external or uncontrollable variables (eg geology, economic conditions, commodity markets, competitor activity). Optimisation requires a precise understanding of how changes in one or more of these variables or drivers will impact a single outcome or desired metric.

1. Cyest Corporation, PO Box 781090, Sandton, Gauteng 2146, South Africa.

2. Gold Fields Limited, 24 St Andrews Road, Parktown, Gauteng 2193, South Africa.

Input variables are 'influencers' that impact the performance of the enterprise, and can be classified into two distinct groups. The first group includes management levers, that is, factors that management can influence such as resource schedules. The second includes *environmental variables*, that is, external factors that also impact value, but that are outside of the direct influence of management; examples here include: competitor activity, demand, inflation, etc. Similarly, there are two output metric types. The first one includes *economic metrics*, such as profitability, costs, revenues, capital, etc. The second output metric type includes the *operational or diagnostic metrics*, the metrics that are not economic, but still indicate the performance of an operation; for example, volumes, market share, number of employees, etc. These metrics are also termed diagnostic metrics because they explain the economic metrics.

Generically, there are five ways that the profit of any enterprise function can be maximised (Figure 1).

Optimisation generally implies a trade-off – there are often two or more opposing effects or consequences of changing any one variable, thus mine planners are often faced with contradictory objectives. Typically, planning will attempt to balance maximum extraction of the resource, with maximum value from its exploitation. These two objectives often work counter to one another as full extraction is invariably achieved with a low margin and long-life, whilst value maximisation is generally focussed on large margins and somewhat shortened lives of the mining operation.

It has generally been observed that operators tend to focus more on costs than any other value drivers (Stoddart, 2002) when looking at maximising margins. However, an operation can acquire cost reductions through capital investments or by better utilisation of the mine infrastructure (eg shafts and metallurgical facilities). Further, full utilisation of infrastructure has the impact

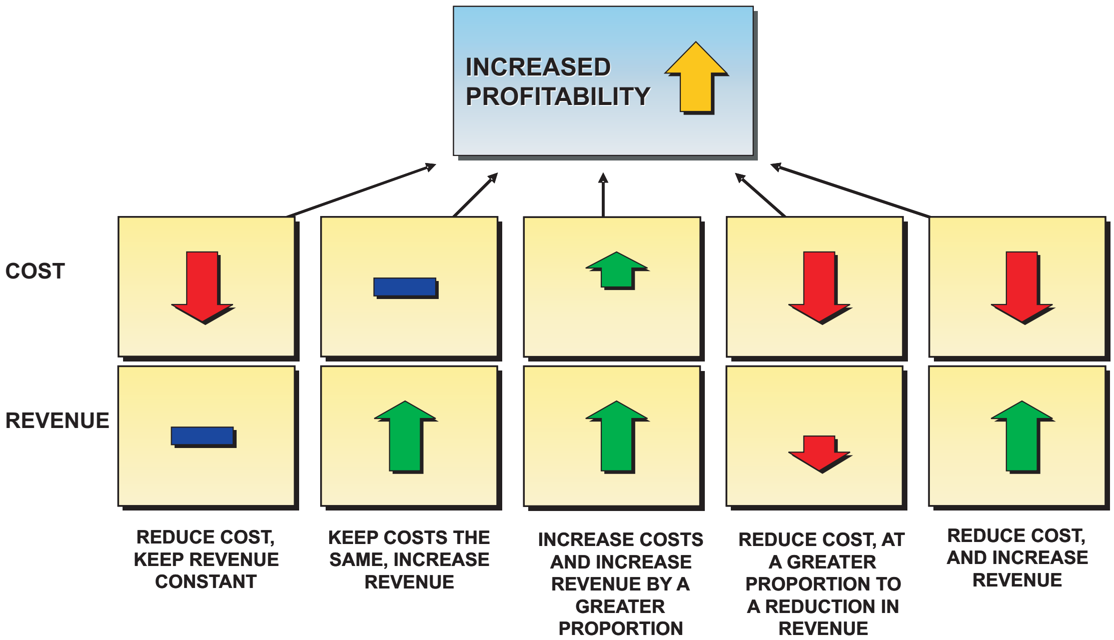


FIG 1 - The five ways to optimise an enterprise.

of diluting the fixed cost structure thus pushing down unit costs. This relationship, whilst understood, is often not quantified at the operational level and seldom considered at the planning level.

The planning process must quantify any such trade-off, so that consistent decisions can be made. The choice as to what the single objective function should be is a function of strategic judgment and appetite for risk, as well as prevailing uncontrollable variables. Therefore, optimisation is a constantly dynamic exercise. The Economic Optimisation Model (EOM) has been designed to allow for rapid, high-end comparison of scenarios and schedules at mines within Gold Fields.

AN ECONOMIC OPTIMISATION MODEL

Rationale for an EOM

The relationships between variables that are controllable, those that are not, and the physical and economic outcomes, are complex and often non-linear: these relationships comprise an economic system. In the mine optimisation context, although many of the relationships can be accurately quantified on a single-variable to single-variable basis, such relationships will not suffice in order to understand the dynamics of an entire mine. One has to specify and quantify the relationships between hundreds if not thousands of variables. Furthermore, in practice, these relationships change over time. Thus, to optimise a mine, it is necessary to develop an economic model simulating aspects of the mine to be optimised (Crawford, 2003). All relevant cause-effect relationships should be accurately reflected and return the 'potentially' desirable outcomes that will result from a change to any controllable or uncontrollable variable. Models used for optimisation must therefore capture the dynamics of the economic system in question, as well as anticipate changes to the relationships between variables.

For mine optimisation, the model must therefore incorporate the resource body, the infrastructure, the production resources – and the markets – as an integrated system modelled mathematically. It must evaluate any impact of production changes, commodity price fluctuations, efficiency changes, capital profiles, economy of scale benefits etc on costs, revenues, life of mine (LOM) and ultimately mine value.

Developing the EOM

The process of building an EOM is a collaborative exercise with representation from finance, production (both mining and metallurgical), mineral resource management, mine planning and engineering. A team encompassing all these disciplines was formed to establish rules for various activities on the mine. The EOM is thus not a 'black box', as all the rules and relationships are transparent and derived through a long consultative process with the mine personnel responsible for each rule. To facilitate the aforementioned, the total mineable area was modelled as comprising of discrete polygons (a maximum of 100 can be catered for in the model). Constraints, infrastructural configuration and production could then be modelled in terms of a common frame of reference. In this document these polygons are termed strategic planning polygons (SPPs). Microsoft Excel™ was selected as the platform for the prototype EOM, in order to reduce the overall project risk by reducing the capital investment requirement for overall development and to shorten the period between development, enhancement and usage. The platform offers a flexible environment in which to test rules and methodologies. The spreadsheet environment allows for relative ease of use and comprehension, and thus allows mining personnel to participate in the evaluation process yielding insights and value relatively quickly.

It was found that with a model this mathematically complex, audit ability was difficult, but not impossible. The eventual methodology adopted was to emulate each shaft's strategic plan production profile over LOM. The EOM output was then compared with the strategic plan's mining and financial outputs. A strong match was observed.

EOM components, relationships and modelling methodology

The developers implemented object based design within the spreadsheet environment: the model consisting of 'objects' with each having its own individual drivers and rules. Objects in the EOM include, amongst others, shafts, dumps, central, services, metallurgical facilities and SPPs. Each shaft is split up into SPPs, which are usually defined along geozone/facie boundaries. In some instances, a single geozone may comprise multiple SPPs. The LOM production schedule data per SPP is extracted from the group's spatial 3D planning systems. Using the geozones, the EOM models the grade/tonnage relationships – these are fundamental in the calculation of the shaft's revenue, and in the assessment of LOM. This relationship is also used to back-calculate the total resource required for the extraction of the SPP schedule (ie the scheduled reserve production data at a zero paylimit). Each individual shaft's costing data is derived from the mine's costing system. Costs are forecast in the model based on production, with the historical costs providing the basis. Cost drivers in the model are specified per line item and include square metres mined, tonnes broken, tonnes milled, metres developed and gold broken. Each line item is further modelled in terms of its unique fixed-variable behaviour, as well as gearing with distance (tramming distances). The user can also override the direct unit costs in any single year or multiple years. This allows for stress testing of the mining configuration under varying cost regimes.

Overhead costs, both central services and corporate are allocated back to production shafts based on the accounting practice on the mine. These costs are production driven and allocated, but the basis for allocation may differ from the cost driver if required. The interplay between production and reallocation of overhead costs makes for an interesting exercise. Decisions relating to shaft closure become far more complex when viewed in this context. The relationship is even more critical at the tail end of the mine's life where there is insufficient production to cover overhead costs and decisions made today may have a major impact on the future sustainability of the operation.

The EOM models costs in a real (un-inflated) financial environment. This may result in some underestimation of the mine's taxation liabilities (Ballington and Smith, 2002). Presently the nature of the South African gold mining taxation formula requires, for accuracy purposes, that economic evaluations be undertaken in nominal terms. This is particularly true for mines with relatively high unredeemed capital.

Services (hoisting, ventilation, refrigeration, water, pumping and compressed air) have been modelled as constraints. Required resources are determined from the production profile and the model flags the user when changes in volume, paylimit, configuration of infrastructure have exceeded capacity. For each service, constraints can be specified by shaft, level, SPP, or SPP group as appropriate. The decision-making steps that would be undertaken when constraints are breached are captured mathematically and when capacity is exceeded, the model invests the capital to enhance or upgrade the service. Large infrastructural upgrades, involving increases in shaft and metallurgical capacity, require manual intervention. This production-derived capital investment schedule is compared, by year, to the mine's own schedule and the greater of the two is returned as the shaft's capital investment schedule. This is then used in the determination of cash flow and valuation.

The EOM calculates the paylimit for the shaft, based on historical costs and the prevailing gold price. The paylimit is calculated as the grade mined where total revenue from mining equates to the total cost of mining (Storrar, 1977). The model thus captures the impact on the paylimit of changes in production, costs or commodity price. The EOM also models the relationship between pay and unpay as a function of paylimit, as well as consequent increased sterilisation of the orebody. The paylimit can either be hard-coded or calculated for each year by the model. The EOM models on and off reef development on a per SPP basis. When the mining rate is increased, an increase in development is required to sustain the new schedule. The model captures all further ramifications of the change in schedule. The increased reef tonnage being mined can result in a dilution of the fixed cost structure, depending on the required increase in development. If fixed costs become diluted then a decline in paylimit is to be expected. This in turn leads to a decrease in the unit revenue. However, the margin may yet increase if the decline in unit costs is greater than the decrease in unit revenue. Furthermore, increased profitability may occur if the absolute revenue increases in excess of the increase in absolute costs incurred by the increased mining. This relationship is a fundamental aspect of the OEM and is graphically depicted in Figure 2.

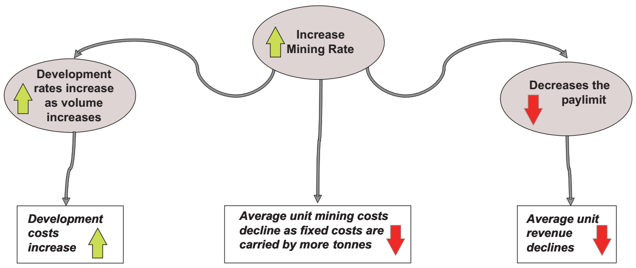


FIG 2 - Impact of a change in the mining rate.

The relationship between paylimit and development also needed to be accurately reflected. With increasing paylimit, a greater proportion of the orebody needs to be accessible so as to increase the mining flexibility. Thus an increase in the development rate should be expected, increasing the cost of development (Figure 3). Furthermore, increasing the paylimit may lead to a breach of the hoisting constraints, as more development tonnes must be hoisted, which may result in a reduction of ore mined.

The metallurgical facilities were modelled to accept underground material from each shaft, based on the rules for delivering material to plants. If the volume of underground material exceeds the plant's treatment capacity, then the EOM flags this constraint breach. The user can then either manually reallocate the material to other processing facilities or decrease

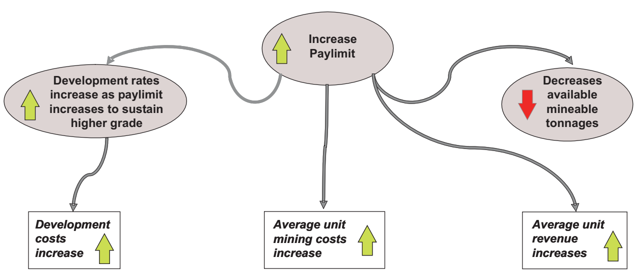


FIG 3 - Impact of a change in pay-limit.

the volume of material being mined at that shaft. Alternatively the user can use a 'Macro-driven' reschedule, which prioritises material based first on the delivery rules and then by grade. If spare capacity exists at a metallurgical plant then dump material is reclaimed, with the highest margin dump material being treated first.

USING THE EOM

Application of the EOM

The EOM allows for multiple operational and strategic scenarios to be quickly tested and compared. In addition, the robustness of the scenarios can be tested against changes in external market factors, and in so doing the risk associated with certain strategies can be quantified. Within each scenario, the model determines the value of each shaft, and the value of the mine, and when compared to a base case (current strategic plan) the impact on value is quantified. This methodology also allows for easy and explicit consideration of the trade off between maximisation of cash flows (short term) and increased sterilisation (long term). The EOM is also used to determine a shaft's sensitivity to the reallocation of SPPs to different shafts and the resultant re-assigning of services.

The model is also used to determine the optimal configuration, which derives the maximum net present value (NPV) over the LOM. The main methodology here was the development of value surfaces or 'Hills of Value'. Hall (2003) indicates 'Hills of Value' with relatively smooth value surfaces containing dips in value generally when capital investments are made. In this analysis it was found that these surfaces are sometimes anything but smooth for deep level mining activity, and in some instances were found to be highly erratic (Figure 4). The shading of the 'Hill of Value' reflects NPV zones. The absolute value reflected by these zones is of no consequence in this paper except that it serves to graphically depict the changes in the shaft's NPV. These erratic surfaces can be attributed to the capital investment requirements, change in mine life, non-mining of subeconomic polygons etc. Furthermore, a number of optimal configurations can be attained. High mining rates coupled with low paylimits might yield similar NPV values to a scenario with high paylimits and a low mining rate. This can make optimisation analysis problematic and this is where the human aspect cannot be underestimated due to the realities of deep level mining (rock engineering, spatial relationships, etc).

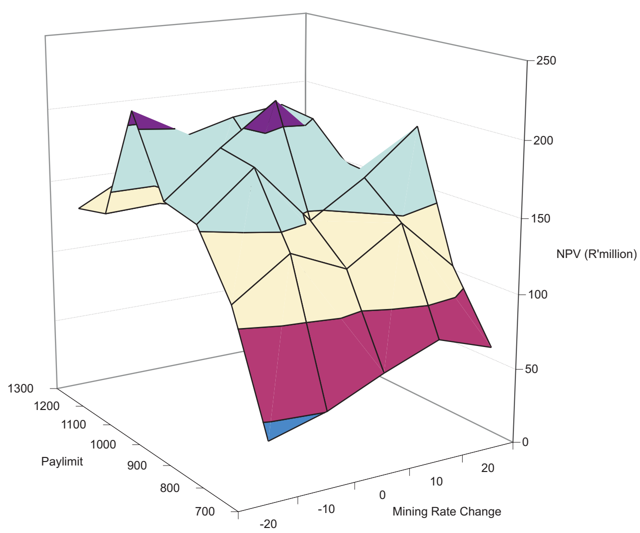


FIG 4 - Multiple optimal configurations may exist for a single orebody.

Impact on the planning process

The prototype EOM provided each operation with a tool to develop a more robust strategic and operational plan than has been proposed in the past. This has led to an adjustment in the company's strategic business planning process. The process now requires the establishment of multidisciplinary scenario workgroups at each operation to evaluate various scenarios and optimal configurations (Figure 5). A number of scenarios are then presented to the executive management (Exco) teams at each mine who then analyse and interrogate the results to determine which scenario fits best with the overall strategy of the group and mine. The optimal scenario can then be planned in detail to test its feasibility in terms of implementation. In the past, the time allocated for the traditional planning process generally resulted in the formulation of a single mine plan. This resulted in weaknesses and flaws in the viability of the plan over the long term.

The main function of the model is to provide direction and strategic intent at the beginning of the strategic planning cycle for each shaft. This then grants each planner strategic boundaries to work within for the shaft and mine, simplifying the design and scheduling effort going forward. This process includes the development of 'Hills of Value' for each shaft at various gold prices which are then presented to the mine's executive management, who then decide upon a specific path based on the group's strategic objectives.

A number of different scenarios, based on these strategic objectives, are then developed for each mine through a consultative process. These scenarios are then whittled down through a process of 'honing the best scenario'. This involves testing various production strategies and then filtering until the 'best' scenario is created that meets the company's overall strategic objectives. The main tool used during this 'honing' process is the EOM, but the process also involves testing the

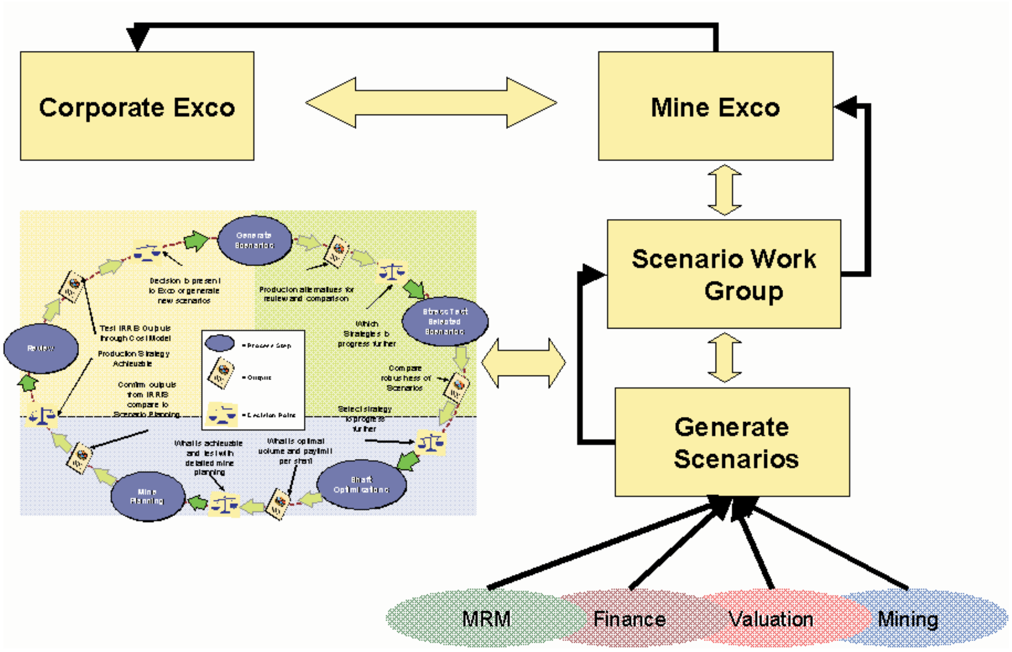


FIG 5a - Gold Fields Limited's new strategic planning process.

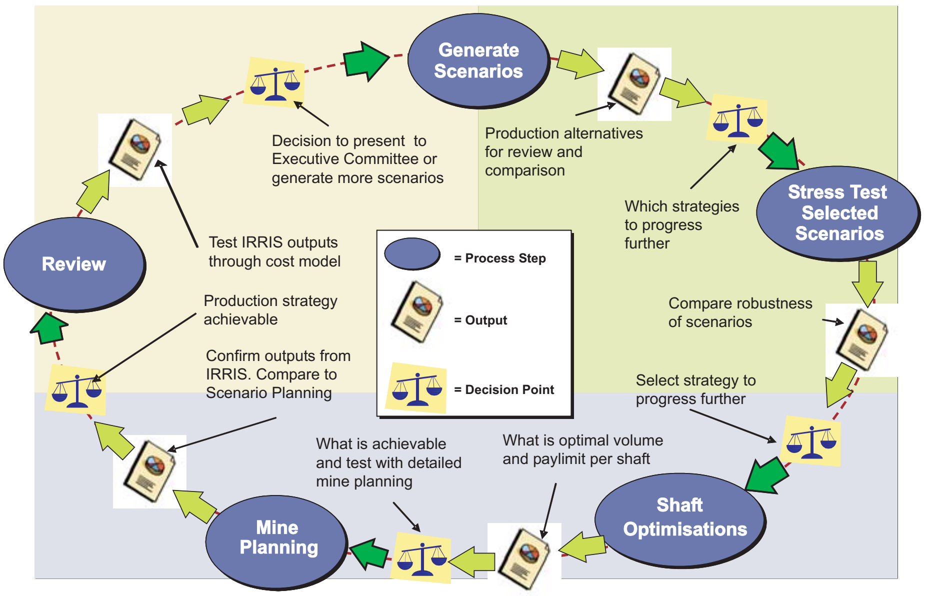


FIG 5b - The reiterative strategic planning process.

viability of selected scenarios, as determined by the EOM, in a new 3D mine planning environment that is currently being evaluated within Gold Fields.

Crawford (2003) indicates that true optimisation is rarely, if ever, achieved in the real-world. This is the case when the EOM is used for optimisation purposes, as the tool does not take into consideration full 3D spatial relationships. We find that the optimisation result, as depicted by the 'Hill of Value', is a road-map for the operations in pursuit of value accretion. Furthermore, optimisation is often only focussed on a single stakeholder, which may not be beneficial to all. The power of this filtering process is highlighted using results from an example 'Hill of Value' (Figure 6) where the operations were now able to challenge current production levels achieved by each shaft and decide on a more optimal configuration for the shaft in order to maximise NPV. The optimal points of the 'Hills of Value' are indicative of where the shaft should be heading and not prescriptive: there were instances where the mine teams found the optimal configurations were not practical and unrealistic as a result of the spatial dispersement of the orebody. However, in most cases, the insights gained through the use of the tool prompted a change in strategy.

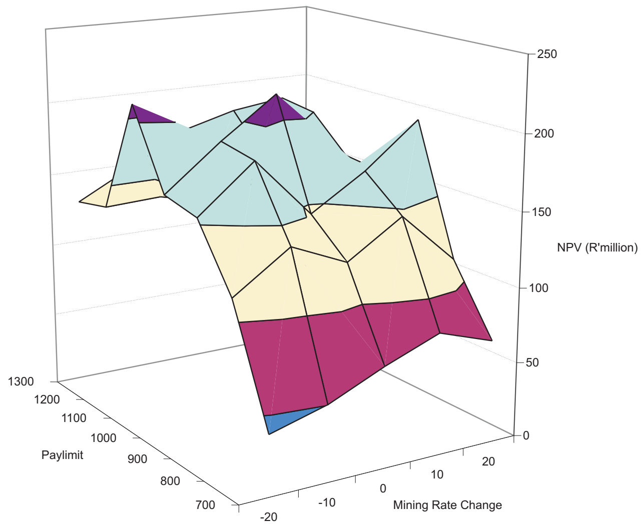


FIG 6 - A 'Hill of Value' indicating suboptimal configuration.

Another application of the EOM is for project valuation – where concept, prefeasibility and feasibility studies can be evaluated in the context of the overall operation and the reallocation of overhead costs. This reallocation of costs may result in the unlocking of value at existing marginal shafts within a ring fenced structure. In the concept and prefeasibility phase, multiple scenarios can be undertaken around new investment possibilities to determine optimal value. Stress testing the project against changes in economic parameters can be made immediately apparent to top executives and assist in understanding the project risk.

Optimising a shaft with the EOM

Shaft X (a shaft within one of Gold Fields' South African mines) was assessed using the current strategic plan square metres schedule. Only two variables can be altered during the development of the 'Hill of Value' (in this instance paylimit and mining rate). However, the EOM takes into account the impact of these changes and the interrelationships. In the strategic plan the shaft has a fixed paylimit of 1960 cm.g/t. For the development of the 'Hill of Value' it was decided to vary the shaft's paylimit at 100 cm.g/t intervals both above and below the current strategic paylimit. The mining rate was also increased from the current square metre schedule in increments of five per cent. The NPV of each scenario was then plotted (Figure 7). The NPV is thus a reflection of the shaft's sensitivity to changes in paylimit and tonnage, and the consequent impact that these two variables have on the shaft and mine.

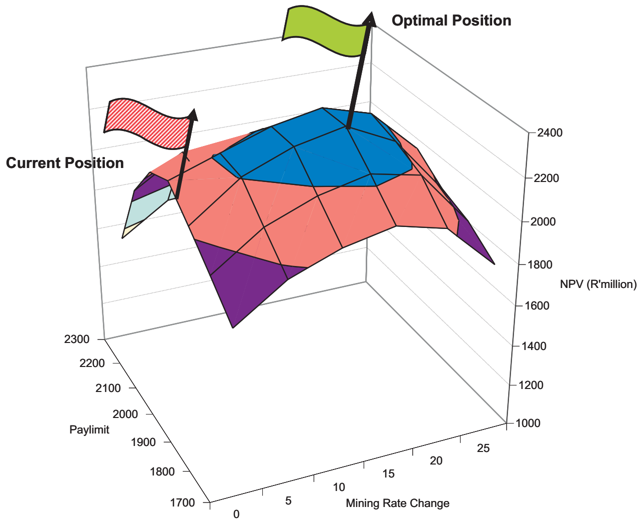


FIG 7 - The optimal configuration for the shaft requires a decline in pay-limit whilst the mining rate is increased.

On analysis, it was determined that the shaft should be reconfigured to mine at a higher rate: as can be seen, upside potential exists if the shaft is reconfigured to mine at a rate of 15 per cent above current schedule. This increased rate should result in a dilution of fixed costs. This is supported by the analysis which suggests that the paylimit can be lowered to 1900 cm.g/t. As a result of this new configuration, refrigeration had to be upgraded in excess of planned upgrades. Within the model, the shortfall was addressed by spending R20 M of capital on additional capacity. The Air constraint had a scheduled upgrade but as a result of the new mining schedule this upgrade was brought forward by one year, again based on the EOM constraint module. Hoisting incurred the greatest breach of capacity. Minor upgrades were undertaken within the model. Thereafter rock had to be cross trammed, at a cost, to other shafts for hoisting purposes; fortunately, sufficient spare hoisting capacity exists at the mine for this purpose.

The development schedule was altered under this new configuration. As seen (Figure 8) over the first few years, the development will need to be increased to cater for the increased mining rate. Thereafter, with the exception of a few instances, the development profile matches the Base Plan development profile. At lower paylimits, there is less sterilisation in some areas, leading to greater extraction with the same development. Furthermore, due to the lower paylimit, some SPPs that had already largely been developed could now be extracted. Overall the total development over LOM remained the same as in the base case, as the orebody is finite, though the timing changed.

Gold Fields is focused on maximisation of value and sustainable development. With the volatility in the domestic gold price, short-term value needs to be taken into consideration, but a mine can be severely hampered if the long-term strategic focus is not explicitly considered. In the above example, this trade off

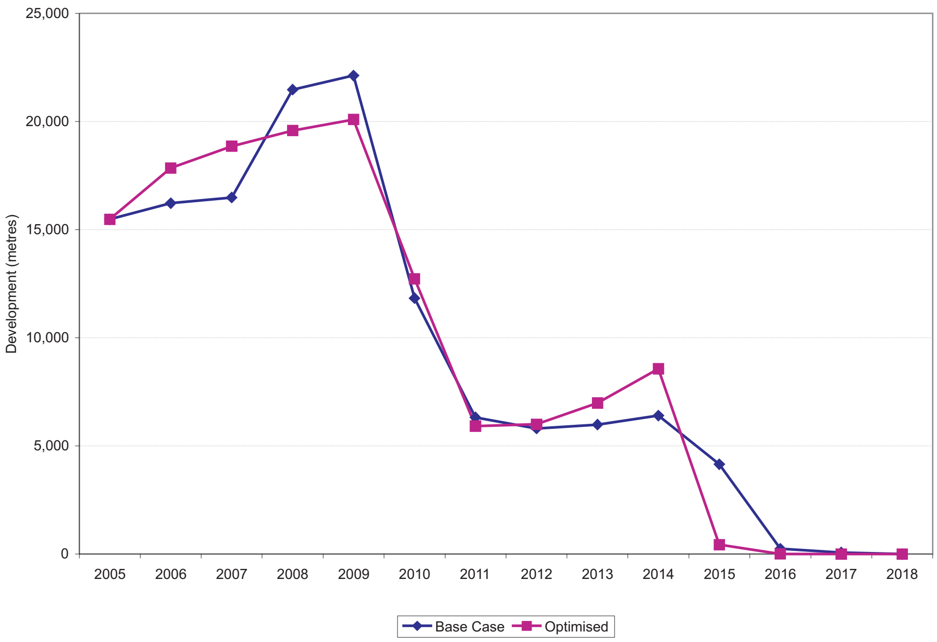


FIG 8 - The optimised scenario requires increased development during the first few years of mining. This allows the shaft to maintain the increased mining rate.

between long-term and short-term goals was clearly highlighted. The configuration that maximised NPV had a short-term margin that was ten per cent *lower* than the margin in the configuration that optimised short-term margins, ie cash flow. The scenario optimising short-term margins displayed a shortening of the shaft's life by three years. Furthermore, gold production was 35 per cent lower and the NPV showed a 38 per cent decline. At another mine within the group, the EOM was used for a shaft level and mine wide optimisation. The analysis suggested increasing paylimit and simultaneously increasing mining rates. What was interesting here was the near doubling of the shaft's LOM. The optimisation of the other shafts resulted in a reallocation of indirect costs – reversing the direction of negative cash flows. This together with shaft level optimisation results in an extension of the shaft's life by seven years.

The above analyses took several hours using the EOM. Without the use of such a model, the same analyses, particularly incorporating 'Hills of Value' may have taken several months.

What the future holds

The future application of this tool is to integrate the logic of the Microsoft Excel™ model into the 3D graphical mine planning software and costing system using a more robust platform leading to a seamless budgeting and reporting tool for short- and long-term planning. This would provide the operations with a tool to improve the quality of long-term and short-term management decision-making in line with the increasing volatility and uncertainty of the gold mining industry.

CONCLUSION

The importance of human interaction and stakeholder management during the development phase of a planning tool is critical to the acceptance and ability of embedding the tool into an organisation's business process. Once people understand the assumptions used to develop the model and trust the outputs, the black box syndrome is broken. The process of building this model also proved very valuable in bringing the multidisciplinary mine team together and collaboratively agreeing on rules and assumptions that had been taken for granted and often overlooked in the current planning process.

The original functionality requirements of the model were met. However, the size and complexity of the mathematical relationships within the model makes audit ability difficult but not impossible. The ability to run detailed multiple scenarios quickly is a major benefit to the strategic planning and project valuation processes, especially as market fluctuations become more frequent. The process of developing the EOM highlighted a common problem within the industry. It was found that strategic plans are not transformed into operational plans as a result of siloed planning where the requirements at corporate level do not dovetail with operational reality.

With the prototyping proving the viability of building an indepth optimiser for underground mines, the next step is to develop this tool into robust software that integrates with all existing mine budgeting and planning systems. The ultimate vision is to use the EOM to generate a budget instantaneously with the mine plan. A true rolling budget would now become a reality.

ACKNOWLEDGEMENTS

The authors would like to thank the management of Gold Fields Limited and Cyest Corporation for permission to prepare and present this paper. We would also like to thank all personnel, at both companies, who were involved in the developing of the Economic Optimisation Model.

REFERENCES

Ballington, I R and Smith, G L, 2002. Discounted cash flow analysis – methodology, inputs and sensitivity, in *Proceedings SAIMM Colloquium: The Valuation of Mineral Projects and Properties: An African Perspective*, Johannesburg.

Crawford, G D, 2003. Mine optimization and operations research, in *Pincock Perspectives*, 41(April).

Hall, B, 2003. How mining companies improve share price by destroying shareholder value, in *Proceedings CIM Mining Conference and Exhibition,* Montreal.

Stoddart, J, 2002. Benchmarking and optimal performance, paper presented to Third Annual Driving Down Costs Conference, Kalgoorlie.

Storrar, C D, 1977. *South African Mine Valuation*, p 145 (Chamber of Mines of South Africa: Johannesburg).

The Use of Extractive Blending Optimisation for Improved Profitability

C Wharton[1]

ABSTRACT

The last decade has seen major changes in mining operations. Metal price reductions and increased competition have driven operators to cut costs and improve efficiencies. There are many situations where improving the quality of the input into a process, through the use of blend optimisation or other linear programming techniques, can lead to better throughputs, reduced costs and increased recoveries. This paper will look at the ways in which extractive blending techniques can be used to improve process recoveries, generate better pitshells and even provide solutions for better utilisation of multiple plant capacities.

INTRODUCTION

The last decade has seen major changes in mining operations. Metal price reductions and increased competition have driven operators to cut costs and improve efficiencies. Imrie (2001) cites the need to look over the entire operation, not only individual components in the chain, and shows how changes in blasting can lead to improvements in recovery in the mill. King (1999) looked at scheduling rock types through the process based on their $/tonne contributions rather than just their economic grade. Indeed there are many situations where improving the quality of the input into a process, through the use of blending or other linear programming (LP) techniques, can lead to better throughputs, reduced costs and increased recoveries. Examples of these include:

- Mill throughput: can be sensitive to average rock hardness, work index or the ratio of materials such as clays. Control of these can lead to improved throughputs.
- Process improvement: control over sulfur content into autoclaves can result in reduced costs and faster reaction times. If the concentration is too low, there is not enough energy liberated for the process to continue and if the concentration is too high, there may be insufficient oxygen to continue the process.
- Process recovery: deleterious materials can reduce recoveries and increase processing costs.

The application of extra control may come at additional cost and complexity and may lead to reduced reserves but hopefully it will lead to increased net present value (NPV). Typically, the control factors can be included in the block model as additional elements and used in the extractive blend as constraints.

This paper will look at the way extractive blending techniques can be used to control input to the process, generate better pitshells and even provide solutions for better utilisation of multiple plant capacities. All the techniques described in this paper can be implemented in the Whittle Strategic Mine Planning package using the Blending module, which incorporates the necessary LP capabilities.

EXTRACTIVE BLENDING

Extractive blending is the term given to the use of blending, to meet constraints, prior to input into a mineral liberation and separation process. Figure 1 shows a typical extractive blend schematic. Material taken directly from the pit may be combined with material from one or more optional stockpiles, to create one or more desired blends, each of which is suitable as an input to a process. From a modelling point of view, the in-pit material is represented by a number of bench/phase panels, each of which has defined tonnage and grade characteristics. These panels have a defined availability depending on their position in the mine and the defined mining sequence. Material in stockpiles is likewise represented by tonnage and grades. The combination of panels and stockpiles forms the basis for a linear programming (LP) optimisation. The blend optimiser seeks to maximise cash flow from the processing of the available panels while taking user constraints into account. The constraints for the optimisation are:

1. the available mining, processing or product limits;
2. the blend minimum or maximum element grade limits;
3. stockpile quantity limits; and
4. block usage cannot exceed block tonnage.

The formulation can be expressed mathematically as:

$$
\begin{array}{c}
\begin{array}{cc} \text{Panel/bin sources} & \text{stockpiles} \end{array} \\
\begin{pmatrix}
v1 & v2 & . & . & . & vn & vm & . & . & vs \\
p1 & p2 & . & . & . & pn & pm & . & . & ps \\
s1 & s2 & . & . & . & sn & sm & . & . & ss \\
ma1 & ma2 & & . & & man & mam & . & . & mas \\
mi1 & mi2 & & . & & min & mim & . & . & mis \\
t1 & t2 & . & . & . & tn & tm & . & . & ts
\end{pmatrix}
\begin{pmatrix} x1 \\ . \\ . \\ . \\ . \\ xn \end{pmatrix}
\begin{matrix} \\ \le \\ \le \\ \le \\ \ge \\ = \end{matrix}
\begin{pmatrix} va1 \\ P1 \\ S1 \\ MA1 \\ MI1 \\ Tn \end{pmatrix}
\end{array}
$$

where:

v1..vs	are value contributions and va1 is derived objective function
p1..ps	are processing tonnages and P1 is process capacity constraint
s1..ss	are selling (production units) and S1 is selling capacity constraint
ma1..mas	maximum grade contributions and MA1 is maximum grade constraint
mi1..mis	minimum grade contributions and MI1 is minimum grade constraint
t1..ts	are usage constraints to ensure that each bin cannot exceed its own capacity
x1..xn	are the amounts of each item to take to maximise the objective function

There can be multiple P, S, MA, MI and T constraints.

The mining constraint is applied externally to the formulation and controls the amount of material supplied. The user can control whether a fixed amount of mining is used or the minimum of the available capacity and the amount required to meet production requirements. The blending LP solves the equations in a manner that maximises the objective function. The result defines, for each period, what proportion of each panel is blended, stockpiled or rejected and what proportion of each stockpile is used.

You must define one blend for each process and for each blend you must specify its allowable source rock types, stockpile material, constraints and blending costs. A key consideration is

1. Principal Consultant, Strategy Optimisation Systems Pty Ltd, 66 Rathmullen Quadrant, Doncaster Vic 3108, Australia. Email: Chris.Wharton@stops.com.au

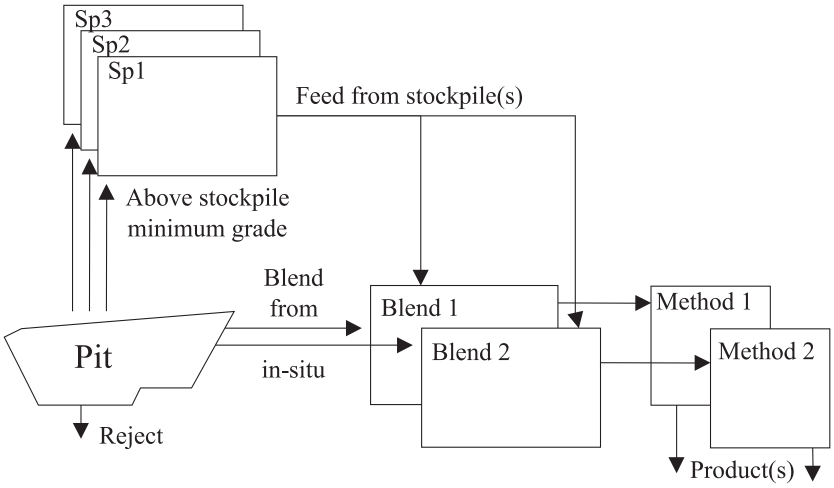


FIG 1 - Extractive blend schematic.

the definition of blend material. You can leave it up to the blend optimiser and only define element constraints or you can limit what is to be considered as blend material either by only providing parcels for material that you wish to consider for the blend in the block model, specifying a minimum or maximum grade in the process definitions, or by defining your own additional blend bin constraints.

BLEND BINS

The Whittle blend optimisation does not consider each individual block in the model, but accumulates material into a number of bench phase panels, then schedules from these accumulators. In the simplest case, the bench phase panel will have one average grade for each element for each rock type. In general, this does not lead to good blending; however, a user can create their own 'blend bins' and define up to 50 more accumulators on each panel. These blend bins can categorise the data into a series of grade ranges, which represent a degree of selectivity that is reflective of the actual mining conditions, to improve the chance of blending.

The 'bins' are defined in a similar fashion to stockpiles. They are defined for each rock type and have a minimum and maximum grade per element. If you add bins then the analysis program accumulates data for each bin in each panel and makes this extra information available to the LP optimisation. This results in better blending. You can control whether entry to a bin is based on grade minima and maxima or on equivalent metal grade. See example one and three for further discussion.

EXAMPLE ONE – MINIMISE CONTAMINANTS

The example detailed below uses a constraint of soluble copper; however, the same technique would apply to controlling rock hardness, work indexes, rock-type ratios or any other element limits. These techniques could work equally well with leach or flotation processes.

Lahtinen (2004) showed that a specific geological ore type can have a wide range of metallurgical recoveries depending on input copper concentrations. Tran *et al* (1997) state that each one per cent of soluble copper in an ore increases cyanide consumption by up to 23 kg/t so there is an obvious potential processing cost benefit to limiting the soluble copper. Furthermore, soluble copper inhibits the floatation of copper sulfide ores leading to potentially lower gold recoveries. If your metallurgist suggested that if the soluble copper content can be limited below 25 per cent then the extraction process could achieve better recoveries, then it would be worth investigating the feasibility of this action. In this example, we will examine the modelling of this behaviour and the application of blending strategies to deal with it.

This example is based on the Marvin porphyry copper mine model included in the Whittle demonstration data set. It contains gold (AU) and copper hosted in three ore types; oxide (OX), mixed (MX) and primary (PM). The model includes data for both copper sulfide (CuS) and soluble copper (CSol) concentrations.

Base case

Table 1 summarises the base case settings. The base case has non-linear recoveries for copper and has different recoveries for each of the rock types with maximum recoveries of 62 per cent, 82 per cent and 94 per cent for OX, MX and PM respectively. The base case life of mine generates an NPV of $273 M. Mine life is 12.7 years. As can be seen in Figure 2, the soluble copper grades are all elevated in the first five years (44 per cent, 42 per cent, 36 per cent, 32 per cent and 28 per cent respectively). This may possibly make blending a difficult proposition in the early years.

TABLE 1

Base case settings.

Item		Copper	Gold
Recoveries	OX	CROX	0.85
	MX	CRMX	0.5
	PM	CRPM	0.6
Income		$20.00/%M	$12.00/gram
Selling costs		$ 7.20/%M	$ 0.20/gram
Mining costs		$0.90/tonne	
Processing costs	OX	$4.00/tonne	
	MX	$4.00/tonne	
	PM	$3.85/tonne	
Throughput	Mine	60 M	
Limits/annum	Mill	20 M	
Investment capex		$250 M	
Discount rate		10%	

Copper recoveries vary with grade:

CROX max(0,(62-13.30/max(0.20,CuS.G)))/100

CRMX max(0,(82- 6.05/max(0.05,CuS.G)))/100

CRPM max(0,(94- 3.20/max(0.02,CuS.G)))/100

Maintain soluble copper grade under 25 per cent (with blend bins)

Let us assume, in this example, that when the soluble copper grade is kept below 25 per cent that the recoveries shown in Table 2 can be achieved by the process. Figure 3 shows the grade tonnage distribution for soluble copper in the Marvin data set. While the weighted average for the entire body might be below the target, the contribution from each rock type is quite different. The oxide, mixed and primary ore types have average copper soluble grades of 40 per cent, 35 per cent and 11 per cent respective and represent six per cent, 42 per cent and 52 per cent of the available ore. Blending the mixed ore is going to be the major issue. There will have to be some user-defined blend bins, for each rock type, to provide additional sources. The bin minimums for gold and copper (CuS) can be calculated from marginal cut-offs based on the revised recoveries. These are shown in Table 2. The bins will want to have a range of CSol grades to provide better blending capabilities. A possible spread of soluble copper grades might be: <15 per cent, 15 - 20 per cent, 20 - 25 per cent, 25 - 30 per cent, 30 - 35 per cent, 35 - 40 per cent and >40 per cent. These are shown on Figure 3 to illustrate the tonnages involved. There is only one blend required, which has a maximum constraint on soluble copper of 25 per cent and a blending cost of $0.05/tonne to allow for additional grade control costs.

The results of the optimisation are shown in Figure 4. As can be seen, the copper soluble grade is kept below 25 per cent in all periods. This, however, causes a major problem for supply to the mill in the first period. Despite this, NPV is increased to $385 M, which represents over $100 M improvement premium over the base case. Indicating that, despite the additional cost of blending ($6 M) and the stripping problem in the first period, blending to control soluble copper is (in this case) warranted. An aspect of this schedule, which is also of interest, compared to the base case, is that an additional 26 M tonnes of material is now rejected because of high soluble copper grade. By comparison, a blend without blend bins yields a solution of $334 M and clearly shows the advantages of using user-defined blend bins to assist with the blending.

This result warrants further investigation, and the next example examines the feasibility of prestripping and stockpiling material to provide a better feed to the plant in the first year of operation. It also demonstrates the potential of using stockpiles to smooth mining production and provide plant feed during periods of low ROM supply.

TABLE 2

Revised recoveries and blend bin settings.

Item		Copper	Gold
Recoveries	OX	0.62	0.85
	MX	0.82	0.7
	PM	0.92	0.7
Blend bin	OX	0.50	0.40
Minimum	MX	0.38	0.68
Cut-offs	PM	0.32	0.54

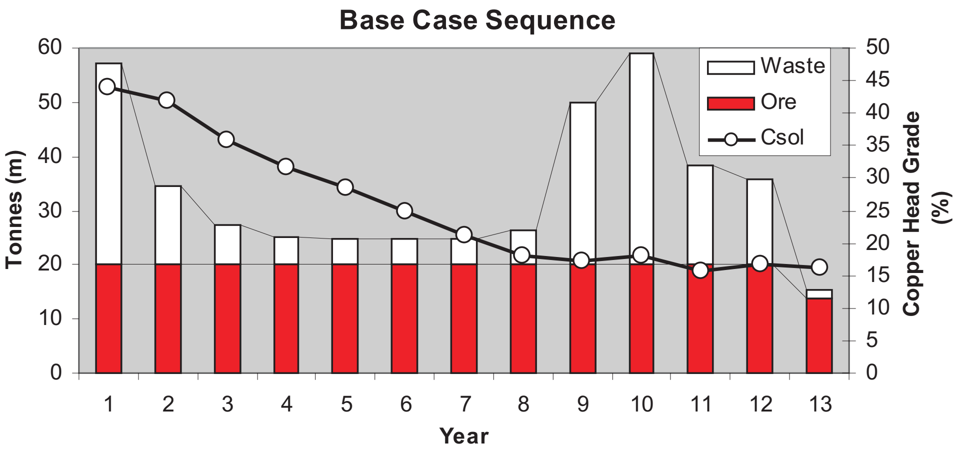


FIG 2 - Base case sequence.

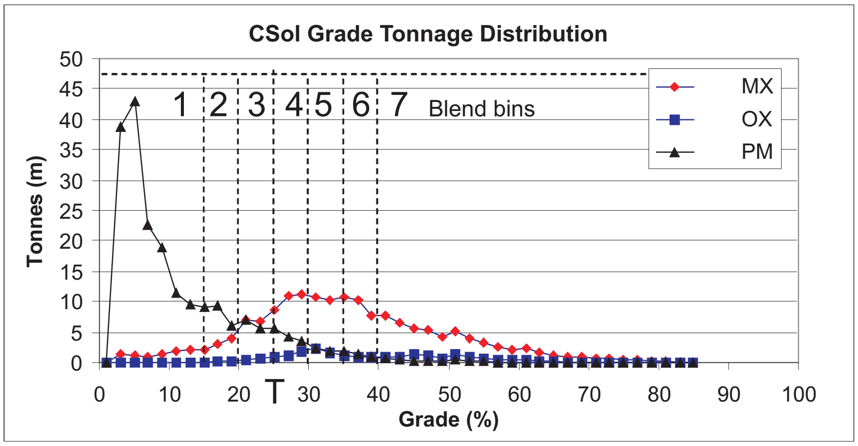


FIG 3 - CSol grade tonnage distribution.

Prestrip for one year and stockpile material

Gold and copper minimum cut-off grades need to be calculated for the stockpile by adding the stockpile rehandling costs ($0.20/tonne) to the processing costs (Table 3). Two stockpiles were set up for each rock type, with maximum soluble copper grades of 35 per cent and 40 per cent respectively. The mining rate was increased to 80 M tonnes to simulate a contract mining situation and then reduced to 35 M from year two onwards.

The prestrip life of mine sequence is shown in Figure 5. NPV, allowing for part delayed capital expenditure, is approximately the same as the previous case; however, the plant is 85 per cent full in the first year of operation. Furthermore, the reduced mining rate will keep the plant full throughout the rest of the mine life by feeding from stockpile during periods of low ROM supply. The stockpile reaches a maximum size of 20 M tonnes in year six and is predominantly used in periods seven, eight and nine. If stockpiling is not an issue then this might be attractive, especially with the better utilisation of the mining fleet.

TABLE 3
Stockpile settings.

Item		Copper	Gold
Stockpile	OX	0.53	0.42
Minimum	MX	0.40	0.71
Cut-offs	PM	0.34	0.57

EXAMPLE TWO – IMPROVED PITSHELL GENERATION

From the previous studies we have seen that some of the high CSol material is not required and is always hard to use in a blend. Could the pitshell generation be modified to try and avoid some of this material?

In this example, we will examine the use of two further tools: user-defined block value expressions and 'Pushback Chooser' a process whereby a large number of pushback alternatives are evaluated to find the best NPV. Traditional pit optimisation does not allow a user to do this easily. The latest release of Whittle (V3.2) allows a user to derive their own block values based on user expressions and these can therefore contain value conditions. A simple heuristic approach would be to exclude all material with CSol >35 per cent from the Lerchs-Grossmann pitshell optimisation calculation. Some blocks may be included in the pit because other blocks pay for their inclusion, but we want to avoid any pit expansion that is based on these blocks alone. Table 4 shows the user-defined expressions used to derive block value calculations. This is simply a sequence of calculations that calculates the recovered copper and gold for each parcel in a block and then determines parcel revenue and selling and processing costs. If net revenue is positive then the parcel processing values are used, otherwise the parcel is treated as waste and only has mining costs associated with it.

Running another pit optimisation based on these user-defined block values does indeed reduce the pit tonnage. A comparison of the pit volumes is shown in Figure 6. A and B mark the same

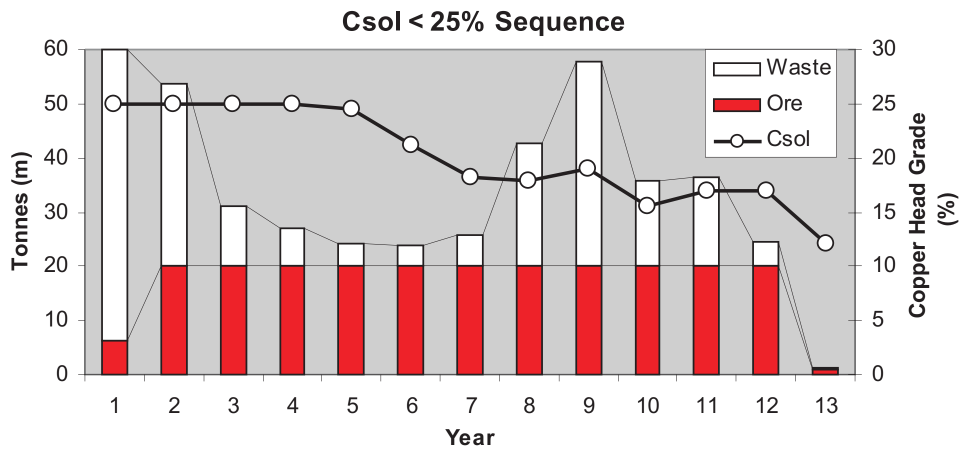


FIG 4 - CSol <25 per cent sequence.

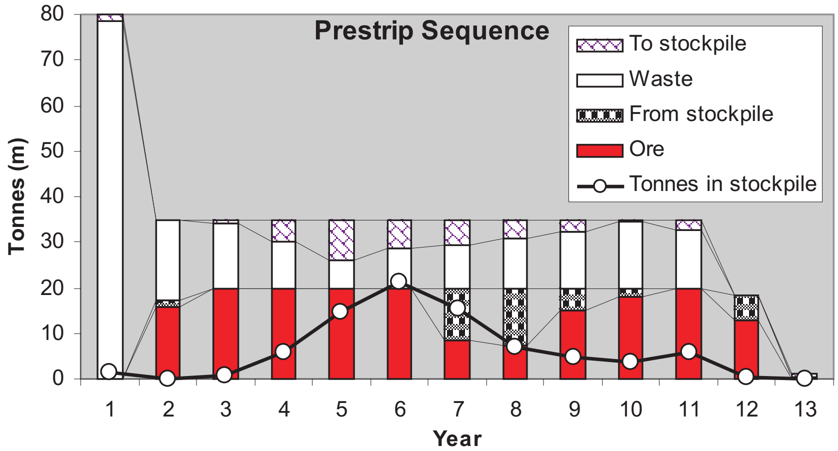


FIG 5 - Prestrip sequence.

TABLE 4

User-defined expressions for block calculation.

Code	Description	Expression
CUOR	OX copper recovery	if(CSol.g>35,0,OX.Q * MAX(0, CuS.g - 0.214516) * .62)
CUMR	MX copper recovery	if(CSol.g>35,0,MX.Q * MAX(0, CuS.g - 0.07378) * .82)
CUPR	PM copper recovery	if(CSol.g>35,0,PM.Q * MAX(0, CuS.g - 0.034043) * .94)
AUR	Gold recovery	if(CSol.g>35,0,OX.Q*au.g*.85 +mx.q*au.g*.5+PM.Q*AU.g * .60)
REVI	Element revenue	(CUOR + CUMR + CUPR)*20 * REVFAC + AUR*12 * REVFAC
SELL	Selling costs	(CUOR+CUMR+CUPR)* 7.20 + AUR * 0.20
PROC	Processing cost	(OX.Q*4+MX.Q*4.00+PM.Q*3.85)*BLOCKP
ROCA	Rock adjustment	-OX.Q*0.1-MX.Q*0.05
REVN	Parcel revenue	REVI -SELL -PROC
CSTM	Cost of mining	0.9
BLKV	Block revenue	SUMPARCEL(IF(REVN>0, REVN,0)) - (BLOCKT+SUMPARCEL(ROCA))*CSTM*BLOCKM

where:

.G	defines a grade
.Q	defines a quantity
BLOCKM, BLOCKP	are block mining and processing costs adjustment factors
BLOCKT	is block tonnage
REVFAC	is a revenue factor used to generate pitshells
SUMPARCEL	sums over all parcels in the block

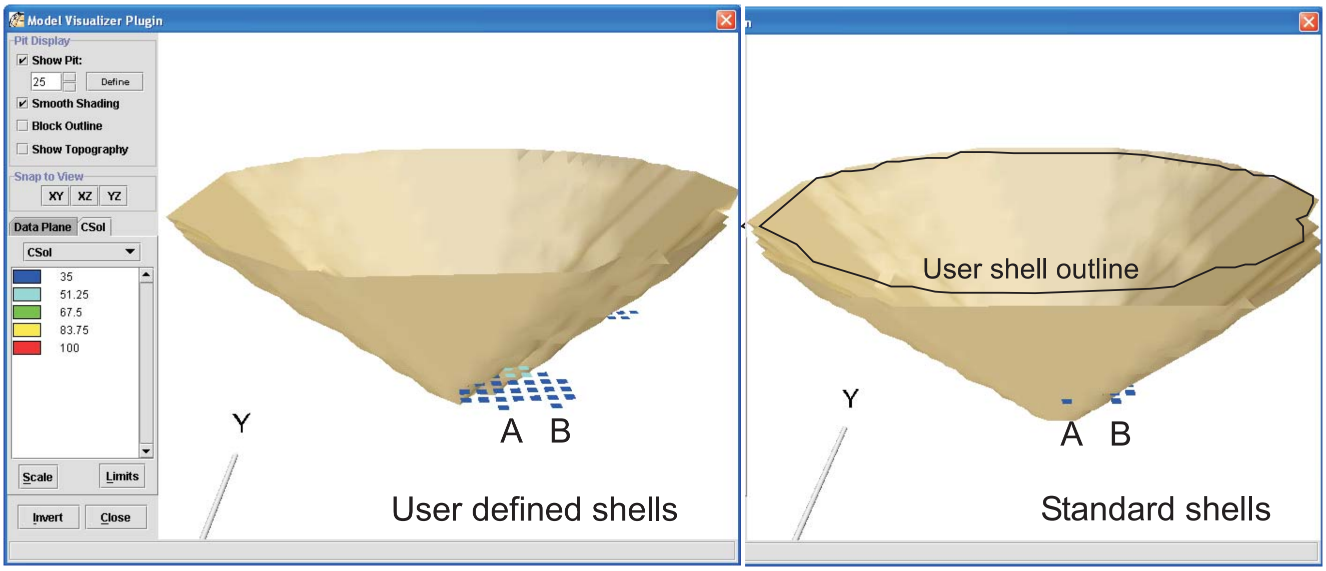


FIG 6 - Comparison of pitshells.

blocks in both the user-defined shells and the normal shells. The highlighted blocks have CSol grades above 35 per cent. As can be seen, some high CSol MX blocks are excluded from the bottom of the pit in the vicinity of A and this leads to a general reduction in pit size as shown by the overlay of the user-defined pitshell in the normal pit view. This is easier to see with a size versus value graph and Figure 7 shows NPV versus pit size for both normal pitshells and pitshells derived from the user-defined block values. What this shows is potentially a $30 M increase in NPV for the same pit size (A) or an 80 M tonne reduced pit size for the same NPV (B).

The above comparisons were done with the use of the benchmark scheduling method called 'Best Case', whereby each pushback is mined out sequentially. The basis for this is a set of nested pitshells, generated by a pit parameterisation technique. This scheduling approach leads to schedules with very high NPVs, but the very large number of pushbacks involved generally makes the schedule un-minable. The details of the approach and the importance of long-term scheduling on mine NPV, are described in various papers including Wharton (2000).

To generate a feasible schedule, it is necessary to further constrain the problem by limiting the number of pushbacks that can be mined during the life of the mine. Generally speaking, fewer pushbacks will lead to greater ease of mining, and lower NPV. It is an engineering/economic decision as to the level at which this compromise should be struck. Various heuristic methods for choosing pushbacks have been developed over the years, but none have been found which reliably lead to good pushback choices for a range of different orebody characteristics. The only certain way to determine the best selection of *n* pitshells, is to try every possible combination. The Pushback Chooser module in Whittle does exactly that – it tries every possible combination of *n* pushback choices and returns the selection that maximises NPV, subject to all other constraints in the simulation.

Figure 8 shows a life of mine schedule based on a Pushback Chooser sequence using the revised pitshells for the smaller pit option. While the mine life has been reduced, the NPV is marginally greater ($394 M). Further analysis of this pit might lead to a reduced fleet size and reduced capital expenditure.

Figure 9 shows a life of mine schedule based on keeping the pit size the same and choosing a new set of pushbacks for the larger mine. The mine life increases to 12.3 years and the NPV is $450 M. This really shows the power of using the revised pitshells, and also demonstrates the value of iterative analysis.

Table 6 summarises the results from all of these investigations and as can be seen, there has been a big impact on NPV and IRR through the use of blending and user-defined pitshell generation. There is more reject material but the quality going through the mill leads to improved profitability.

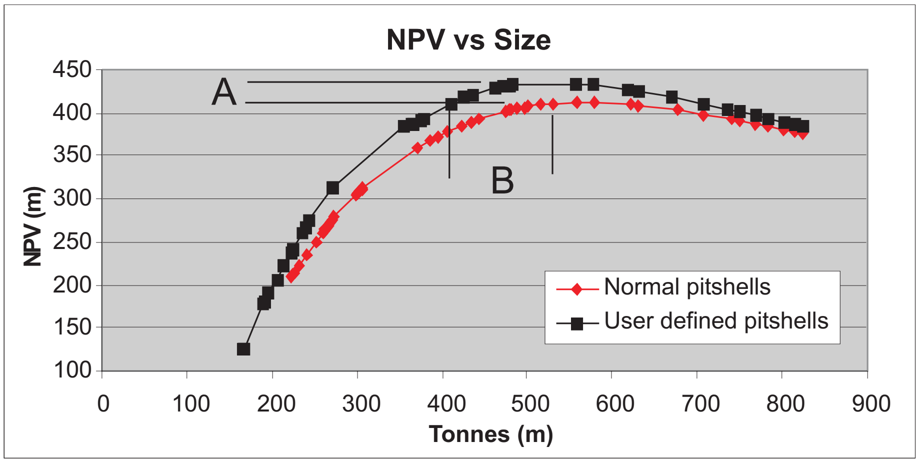


FIG 7 - NPV versus size.

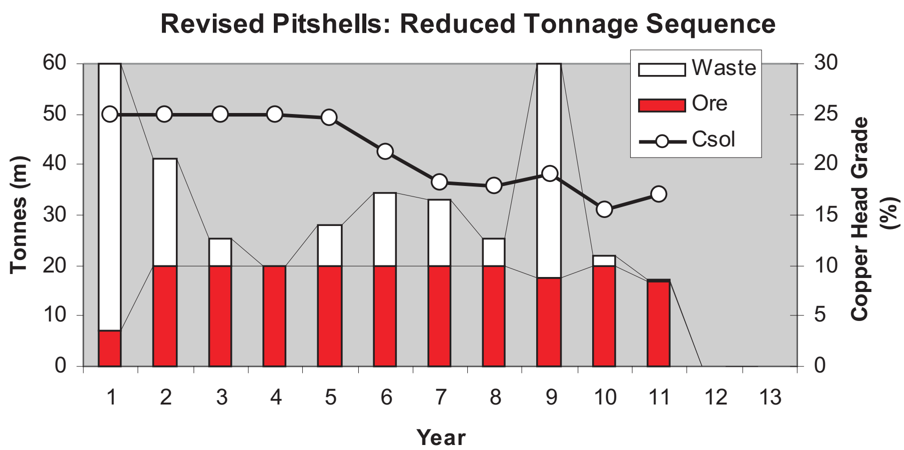


FIG 8 - Revised pitshells: reduced tonnage sequence.

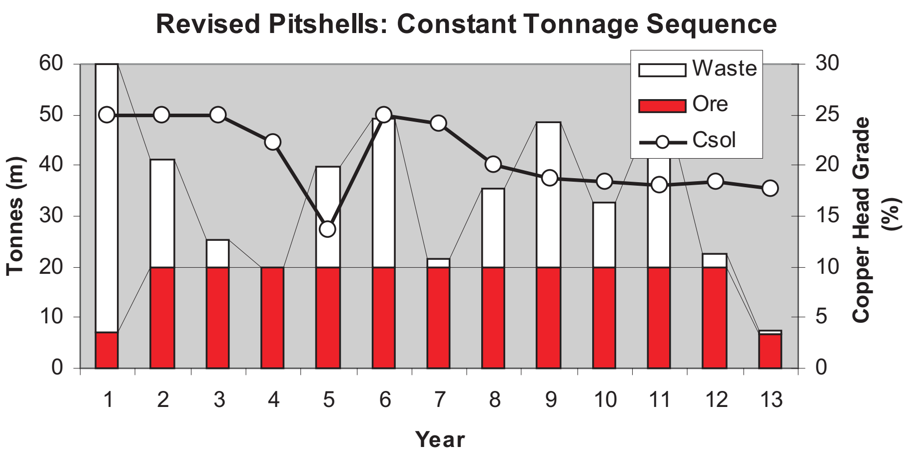


FIG 9 - Revised pitshells: constant tonnage sequence.

EXAMPLE THREE – PROCESS BALANCING

This example is included to show how a blending LP optimisation can be used to solve other problems on a mine site. For example, in many cases, companies have access to two or more process plants, which often have different operating costs and recoveries. If you just set up the two processes, each with their own capacity, costs and recoveries, you will find that they are not both used to full capacity, since material will be delivered preferentially to the lowest cost process. It is common when simulating these cases, to simplify the problem by averaging the costs and recoveries, on a weighted basis, to achieve a single plant model. However, when using this approach you will, in fact, be assigning some material that could be processed through the better plant, to waste, and putting some marginal material through the poorer process that should be assigned to waste.

A way around this dilemma is to set the problem up as an extractive blend. The key issue is how to control what material is sent to each of the processes as there are no overriding grade constraints to apply. This can be achieved by setting up the blend bins so that they only contain viable material. To illustrate the concept you can take the base case example and split the plant capacity up into 15 M through the new plant and 5 M through the old plant with the old plant having, say, five per cent increased costs. This will mean that the old plant cut-offs will be five per cent higher than the new plant.

The first step is to calculate the required cut-offs for each element, for each rock type, in the best process and apply these to the first blend bin(s). This exercise needs to be repeated for each process/rock-type combination. Table 5 shows the cut-offs required for this example. The next step is to create a blend for each process, but do not assign any other constraints or costs. The mining rate has been reduced to 50 M tonnes per annum to illustrate what happens when the processes are starved of material.

TABLE 5

Blend bin settings for old and new process.

Item		New mill		Old mill	
		CuS	Au	CuS	Au
Blend bin	OX	0.72	0.40	0.76	0.42
Minimum	MX	0.46	0.68	0.48	0.71
Cut-offs	PM	0.36	0.54	0.38	0.57

Figure 10 shows a typical schedule. Note that in period ten the mining rate is not sufficient to fill both mills. In this case the optimisation has reduced the input into the old mill, as it has higher costs and hence lower revenue per tonne.

CONCLUSION

The preceding examples are not intended to be final designs; they have been used to illustrate the various techniques that are available to current mining engineers. There are many practical situations on existing mine sites where improving the quality of the input into a process, through the use of extractive blending, can improve the bottom line for companies. This improvement can be based on better throughputs, reduced costs and/or increased recoveries.

The use of user-defined blend bins provides superior solutions and means that users can take existing models without having to re-categorise the data to provide required selectivity for blending. The examples have also attempted to show how user-defined block values can be used to produce more robust pitshells in cases where you wish to avoid deleterious material.

The techniques described in this paper can all be executed using the Whittle blending software and can lead to a better understanding of the mine dynamics and may lead to new ways of treating old problems.

TABLE 6

Life of mine comparisons.

Case description	Total tonnes (M)	Reject tonnes (M)	NPV ($M)	IRR (%)	Mine life
Base case	443.7	38.1	273	22.7	12.7
Constrain CSol < 25%	443.7	64.7	385	27.2	12.0
Prestrip and stockpile	443.7	76.5	385	30.6	11.9
Revised pitshells (A)	451.7	68.1	450	31.2	12.3
(B)	366.0	54.0	394	30.4	10.8

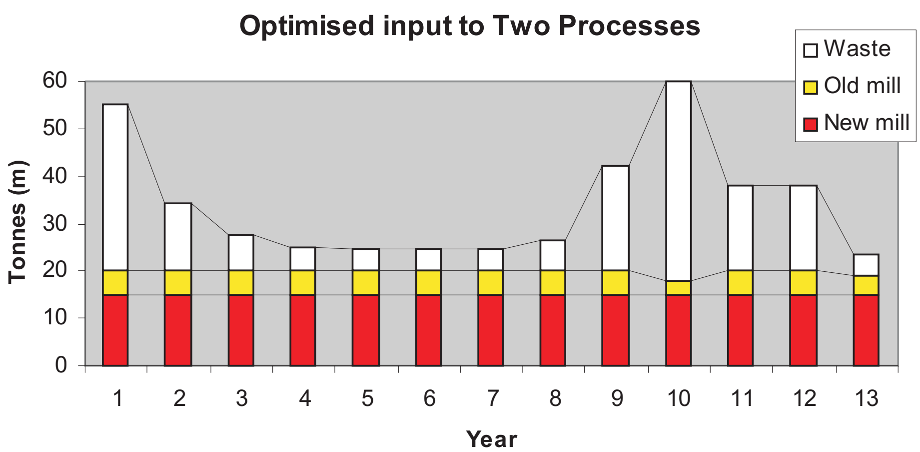


FIG 10 - Optimised input to two processes.

REFERENCES

Imrie, J O C, 2001. Ore flow optimization – Mine to mill [online]. Available from: <http://www.hatch.ca/Operations_Support/Ore%20Flow%20OptimizationMine%20to%20Mill.pdf> [Accessed: 7 May 2007].

King, B, 1999. Cash flow grades – scheduling rocks with different throughput characteristics, in *Proceedings Strategic Mine Planning Conference,* pp 103-110 (Whittle Programming Pty Ltd: Melbourne).

Lahtinen, M, 2004. Is your plant economically optimised to maximise profit? *The Australian Mining Club Journal,* pp 41-43.

Tran, T, Nguyen, H H, Hsu, Y J and Wong, P L M, 1997. Copper-gold interaction during the processing of copper-gold ores, in *Proceedings World Gold '97*, pp 95-98 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Wharton, C, 2000. Add value to your mine through improved long term scheduling, in *Proceedings Whittle North American Strategic Mine Planning Conference*, Colorado, August.

Optimising the Strategic Mine Plan — Methodologies, Findings, Successes and Failures

B Hall[1] and C Stewart[2]

ABSTRACT

Maximising shareholder value has become a common aim for mining companies. However, current cut-off policies at many mines ensure that shareholder value is not maximised, despite the stated corporate goals. Significant value gains are achievable, compared with the accepted strategy at the start of the study. Typically, the new optimal plan involves a significant increase in the cut-off grade, at least in the earlier years. An increase in the underground development rate or open cut stripping rate is often associated with this, at least in the short term, to establish the new strategy. Counter-intuitive results are often found. For example, the optimum cut-off is often relatively insensitive to changes in metal prices. Optimal cut-offs for different parts of an underground mine may be significantly different, even if mineralisation and cost structures are similar.

The paper describes a number of methodologies employed to identify optimum mine plans. In all studies undertaken, significant value improvement potential has been identified. In some cases, the company has adopted the recommended plan. However, in other cases, the status quo has been maintained. The paper identifies factors that the authors believe contribute to the likelihood of a company adopting or rejecting a new plan that has been demonstrated to add significant value, and therefore to the value of actually conducting such an analysis in the first place.

INTRODUCTION

Publicly stated corporate goals of mining companies typically include the concept of 'maximising shareholder value', often with associated goals of reducing costs and improving efficiency. Companies therefore commission studies (Table 1) to identify the optimum mine plan. This paper describes typical optimisation goals of a number of studies, methodologies used for optimisation, common findings, and factors contributing to successful and unsuccessful studies. Results from a number of case studies are used to illustrate these. Optimisation studies typically have maximisation of net present value (NPV) as a major goal. Other goals specified may include maximising the internal rate of return (IRR), accounting profits (eg EBIT, EBITDA, etc) and accounting returns, based either on total assets (or capital or funds employed) or on shareholders' equity (net assets) (eg ROFE, ROCE, RONA, etc). Minimising unit cost measures, such as cash costs per ounce or C1/C2/C3 unit costs, is a common goal, and achievement of output targets, such as maximising the metal produced, or exceeding specified minimum production targets, may be important.

Rarely expressed initially, but often coming into play when results of the study identify increasing the cut-off grade as a major strategy to increase value, is avoiding, or minimising, a reduction in publicly reported ore reserves. Also rarely expressed, but frequently implied and very important, are the risk management goals of maximising the ability to reap upside rewards, and minimising the danger of downside risks. Many companies have multiple, and often conflicting, corporate goals. To these may be added various external government and social goals, such as minimisation of greenhouse gas emissions, maximisation of taxation revenues, provision of local employment opportunities and infrastructure, and so on. To provide corporate decision makers with adequate information to select the optimum strategy, the optimisation process must be able to identify not only strategies that will deliver the various goals, but also the trade-offs required to best achieve a combination of various conflicting goals.

1. MAusIMM, Principal Mining Engineer, AMC Consultants Pty Ltd, 12/179 North Quay, Brisbane Qld 4000, Australia. Email: bhall@amcconsultants.com.au

2. MAusIMM, Principal Mining Consultant, AMC Consultants Pty Ltd, 19/114 William Street, Melbourne Vic 3000, Australia. Email: cstewart@amcconsultants.com.au

TABLE 1

Summary of optimisation studies conducted.

Level of detail of study	Locations	Minerals
• Scoping/conceptual • Prefeasibility • Feasibility	• Australia • Western Europe • East Africa • Central Asia • China • India	• Gold • Lead/zinc • Nickel • Mineral sands
Types of operations		
• Single underground mine and treatment plant • As above, plus other independent ore sources (pre-existing stocks and satellite mines) • Single open pit mine and plant with stockpiling • Single deposit and plant, with interacting underground and open pit mines • Multiple deposits with a single treatment plant • Multiple deposits with multiple treatment plants		

METHODOLOGIES USED

A number of modelling and optimisation techniques have been used for the optimisation studies described herein. In all cases, these have been implemented in Microsoft Excel™. Commercially available and relatively inexpensive 'add-ins' have also been used where appropriate, as described below. This is not necessarily the most efficient method, for both the model building and computational efficiency aspects of the study, but it has several advantages. These include:

- the unique conditions and concerns at each site can be built into the evaluation model as a matter of course;
- a provision does not have to be made in the model for matters that do not apply at the site;
- the modelling techniques that have been developed in previous studies can be quickly adapted for the study at hand;
- tabular and graphical outputs can be easily customised for the study at hand; and
- the model can be provided to the client, who can audit and use it with existing standard computer hardware and software.

Typically the client does not opt to implement the strategy that fully optimises one of the corporate goals only, but rather identifies the trade-offs between its various goals, and selects a strategy that best meets some or all of the conflicting goals. Using spreadsheet software makes it simple to evaluate the behaviour of the various 'goal' parameters as the values of a number of 'strategic decision' parameters are varied both separately and together.

Lane's methodology

The 'state of the art' cut-off grade theory was published by Lane 40 years ago (Lane, 1964) and made generally available in book form over 15 years ago (Lane, 1988). Despite the general knowledge of the existence of this work amongst relevant technical personnel, and further development of the theory and practice (eg King, 1998 and 1999), many mines have not applied the methodology. Rather, the use of simple operating cost breakeven grades as cut-offs is common. Even where Lane's methodology has been applied, it appears that the concepts have sometimes been misunderstood and therefore incorrectly applied. For example, comments to the effect of 'the mine can sell all it can produce, and therefore has no market constraint' miss the point that, for a 'Lane-style' analysis, 'market' deals with mineral or product. The 'market constraint' is anything that limits the production, handling and sale of product. Since most mine/mill operations can usually sell all that they can produce, the market constraint will typically be somewhere in the product side of the treatment plant circuit, such as the concentrate filters of a base metal plant, or the carbon stripping circuit of a gold plant.

Lane's methodology provides a rigorous analytical process, which, though requiring some iterative calculations, will converge to provide a cut-off policy (ie a planned sequence of cut-off grades over the life of the mine), which will maximise the NPV of the operation for a specified set of production rate and economic assumptions. The effects of, for example, different metal price forecasts, and various potential upgrades of mining and processing capacities, can be evaluated by repeating the process for each proposed scenario and comparing the costs and benefits as appropriate.

Lane's analytical methodology has been applied for some high-level studies, to give an indication of what may be achievable, but it has not formed the basis of a major strategy optimisation study, for one or more of three main reasons. First, it optimises only for NPV. The values of other 'goal' parameters can be determined for the strategy that maximises NPV, but there is no way within the methodology to identify how to optimise other goals, and how much NPV is lost by doing so. Second, the analytical process can handle a limited number of physical constraints. In most of the practical cases evaluated, there are more constraints than these, typically involving multiple products in polymetallic base metals operations, sulfur processing constraints in refractory gold operations, and product quality constraints. Lane (1988) identifies that in these cases his analytical process becomes unworkable, and use of a search technique to identify the optimum is necessary; and, third, stockpiling and grade-dependent recovery relationships introduce further complexities. Lane (1988) provides analytical procedures for the former, and for simple relationships for the latter, but again notes the complexities and the need to apply numerical techniques in more complex situations.

'Hill of Value' calculations

Because of the practical concerns noted above for application of Lane's analytical methodology, most optimisation studies conducted have used what has been called the 'Hill of Value' technique. This uses an Excel™ model, which has been constructed to be capable of handling all the combinations of various 'strategic decision' variables that can be independently specified. These typically include:

- cut-off grades, for either the whole mine, or for underground and open pit mines, or for various orebodies, lenses, areas or stages of the mine(s);
- production rate targets, for all or parts of the mining operation(s), and for the treatment plant(s);
- inclusion or not of various identified debottlenecking upgrades in the mines or treatment plants, and the timing of their implementation; and
- various mining method options, which may include different sizes of open pits (including no pit), and different methods or combinations of methods underground.

Other factors with the potential to impact on optimum strategy, and evaluated in some studies, include:

- alternative economic forecasts,
- varying degrees of exploration success,
- alternative haulage/hoisting systems, and
- various workforce productivity and equipment efficiency scenarios.

The evaluation models are constructed so that all of these parameters can be specified independently, and scheduling dependencies and constraints can be defined. The model logic ensures that realistic mining and production schedules are generated to honour all inputs. Capital and operating costs are then modelled by standard techniques appropriate to the mining and processing methods, using appropriate fixed and variable costs for the various physical quantities modelled. Revenues are estimated by calculations appropriate for the metals being produced. Sufficient information is then available within the model to calculate whatever measures of value may be required for the study.

The 'Hill of Value' technique and its application have been described in detail elsewhere (Hall, 2003; Hall and de Vries, 2003). It provides a clear picture of how a mine might change its strategy to optimise a particular goal parameter (Figure 1) or the trade-offs between various goals (Figure 2). Figure 1 is from an underground base metals study, and is typical of most of the studies conducted. The implications of this figure are described in the discussion of typical findings below. Figure 2 is from an underground gold study. It shows how different parameters are optimised by different cut-off strategies, and how there is a range of cut-offs that delivers close to the optimum for all five parameters of interest, with very little loss of one if another is optimised. Figures 3 and 4 illustrate how, with different price scenarios, deciding to target the upside potential may significantly increase the downside risk, particularly if suboptimal cut-off policies have been employed. These plots are schematic to emphasise the point being made, but case studies exhibiting the effect to a significant, though not as extreme, extent are discussed below.

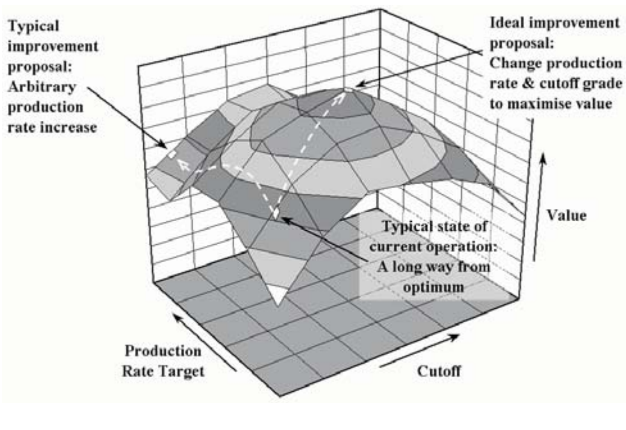


FIG 1 - Finding and climbing the 'Hill of Value'.

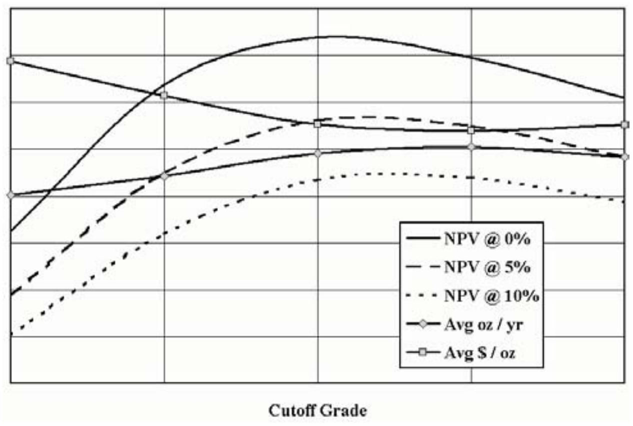


FIG 2 - Multiple parameters as functions of cut-off.

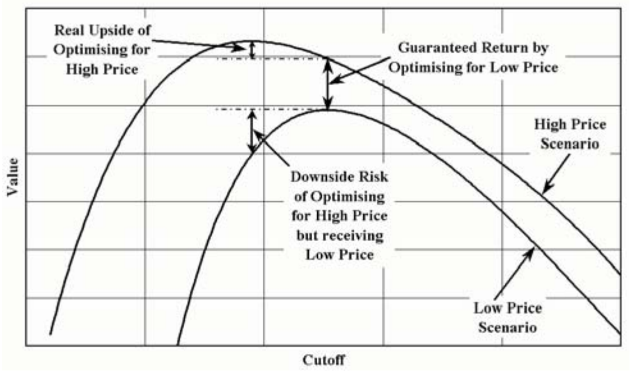


FIG 3 - Risks and rewards of optimum cut-offs.

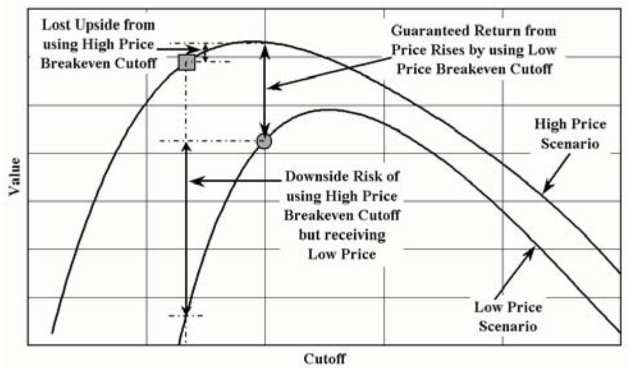


FIG 4 - Risks and rewards of incorrect price predictions and suboptimal cut-offs.

Working on the principle that, if it can be described, it can be modelled, robust spreadsheet modelling techniques enable realistic handling of the wide range of possible production strategies. These methods also enable reporting values of a number of 'goal' parameters for simultaneous changes in a number of 'strategy decision' variables, to generate Hills of Value for a number of parameters as in Figures 1 and 2. Despite the power of the 'Hill of Value' methodology to demonstrate how the values of a number of corporate goal parameters may vary with changes in operating strategies, its main drawback is the rapid increase in the number of cases to be evaluated as the number of 'strategy decision' parameters, and the number of options for each of these, increases. Other techniques then become necessary.

Genetic algorithms

Genetic algorithms (GA) have been described as one of the best techniques currently available for being reasonably sure of being reasonably close to an optimum solution, when there is no analytical method of finding the optimum, and when the number of cases is too great to permit evaluation of each to identify the best. If a GA analysis starts with the best results from a Hill of Value analysis, improvements of five per cent to 15 per cent in the value of the parameter being optimised are not uncommon, but there is no guarantee that a higher 'hill' will be found, even if it does exist. Paliside Corporation's Evolver™ and RiskOptimizer™, relatively inexpensive and easy to use add-ins for Excel™, have been used for GA optimisation.

Values of decision variables that have been flexed under the control of the GA have included in past studies:

- cut-offs applied to different mining stages, and to pits and underground mining areas;
- sizes of pits being mined;
- sequencing of mining various deposits;
- allocation of deposits to multiple treatment plants; and
- timing and size of plant upgrades.

Since the GA has to target one parameter to optimise, it cannot account for the trade-offs between various corporate goals. GA optimisations conducted typically focus on maximising NPV, though some analyses have also considered maximising return on assets, and minimising losses at 'bottom of cycle' prices. The alternative strategies identified in each case may be compared to identify similarities and differences, and decisions can be made to address any identified trade-offs. Results from GA analyses conducted have either confirmed the strategies already proposed on the basis of experience, previous studies, and engineering judgement, or identified new counter-intuitive strategies that add value. These are discussed in more detail in the section that follows.

Advantages of using a GA include:

- relative simplicity to implement;
- demonstration of how improvements can be made in existing strategies;
- identification of new strategies; and
- the use of logs of the analyses to identify not only the 'best' solution, but also factors common to many or all of both high value and low value cases, to guide further planning.

Disadvantages of using a GA include:

- the optimisation of a single parameter;
- there is no guarantee that the best possible solution has been found, but any identified gain is theoretically better than nothing; and
- conducting a reasonable number of calculations may take a long time.

Linear programming

Linear programming and other mathematical programming methods its derivatives are classical analytical techniques for maximising or minimising an 'objective function' subject to a number of constraints. It has been applied in a number of common mining industry problems, such as scheduling, and blending to account for quality constraints imposed on both the ore feed and the product. Increasing power of specialist software is making these methodologies more readily available. Relatively inexpensive add-ins for Excel™ have been demonstrated to deliver good results in suitable problems.

COMMON FINDINGS

A number of common outcomes are being found in all of the mine plan strategy optimisation studies conducted to date. Although the number of these studies is small relative to the number of mines in the industry, the consistency of these results and the commonality of planning processes employed by mining companies would suggest that these findings apply to a large number of mining operations.

An increase in cut-off leads to an increase in value

'Hill of Value' studies have consistently demonstrated potential value (NPV) increases of between ten per cent and 50 per cent over what is obtained by 'traditional' studies using breakeven cut-off analysis. Cut-offs are typically 30 per cent to 50 per cent higher. This has been found for a range of commodities and for both underground and open pit mines. Figure 1 is typical of many studies. Parameters other than NPV may also be used for decision-making. For gold producers, the cash cost per ounce is a commonly quoted metric. The cut-off that minimises unit cost is frequently higher than that which maximises NPV, as shown in Figure 2. This effect has also been seen in base metals studies. Figures 1 and 2 have been derived from underground studies. The ore production target from the mine is one of the independent variable axes in Figure 1, and is the same at all cut-offs in Figure 2. It is assumed that the development rate can and will be increased as necessary to account for the reduction in ore tonnes per metre of development as the cut-off increases. An injection of development 'working capital' in a short-term campaign may be required if an increase in cut-off at a working mine is proposed. Both of these increases in development costs are taken into account in the analyses shown in the figures.

An increase in mining rates leads to an increase in value

The cut-off is only one parameter in the mine plan strategy. It is usually not possible to change the effective cut-off alone and add value: changing a mining or treatment rate is also usually necessary. This is because most operations will adjust their mining plans, including the effective cut-off, to deliver the best result according to the constraints within which they are operating. It is those constraints that must be identified and changed if possible to permit the cut-off to be changed and value to be added.

The 'effective' cut-off is what is actually being applied. It is not uncommon to find that this is significantly different from the 'official' cut-off. An 'effective' cut-off as low as 50 per cent of the 'official' cut-off has been encountered, accompanied by comments to the effect of: *'the cut-off is 'x', but we can't fill the mill at 'x', so we have to use a cut-off of 'y' in practice to keep the mill full'*. The mining/treatment balance (as defined by Lane, 1988) is often the operating strategy used in practice, based on the common understanding that both the mill and mine usually need to operate at capacity to maximise profitability. This then becomes a self-fulfilling prophecy. If the cut-off has been specified by non-optimal means and the mill is being filled, then the waste stripping or development rate that maintains the equilibrium will be seen as acceptable.

Pressures to reduce costs frequently drive the waste mining and development rates as low as possible. However, a number of studies for both open pit and underground deposits have shown that improved value can be obtained by increasing the mining rate of total ore and waste. In open pits, the ability to treat higher-grade ore through increased mining rate, with stockpiling of lower grade material, brings forward the revenue stream and offsets the effects of increased mining expenditure, but only up to a point, as shown in Figure 5, which was derived for a large base metals open pit. In the case of an underground operation, an increased waste development rate, particularly in decline development, may allow access to more production areas. This again may allow higher grade material to be mined and value to be improved (Figure 6), even though lower grade material may be sterilised, rather than stockpiled for later treatment as in the open pit case. (Figure 6 was created in a high level study to make a case for a more detailed study for an underground gold mine. It was derived by a simplified analysis where the NPV took account of the cut-off grade, operating cost and revenue effects, but not the timing effect of the earlier mining of the decline at higher advance rates. A more detailed study would also demonstrate an optimum development rate at the peak of a rising then falling NPV versus decline advance rate curve). It is noted in passing that, when mines are operating in a low working capital mode with restricted waste development underground or waste stripping in the pit, a frequently suggested strategy is to increase the production rate so that the cut-off can be lowered and more ore treated profitably. This may involve some form of debottlenecking in the treatment plant to increase the ore treatment capacity.

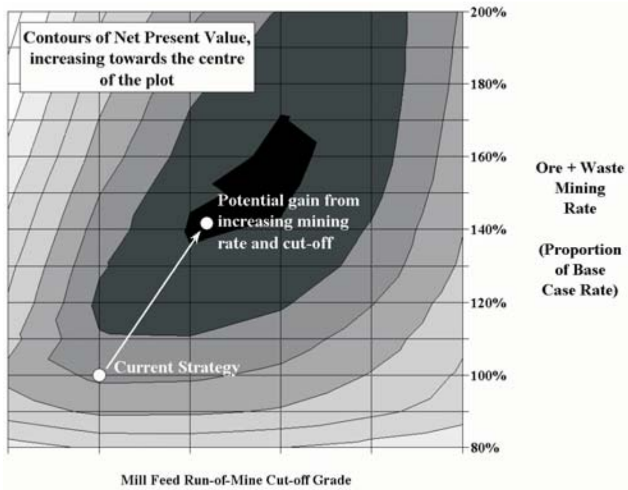


FIG 5 - Effects of changing mining rate and ROM cut-off in an open pit.

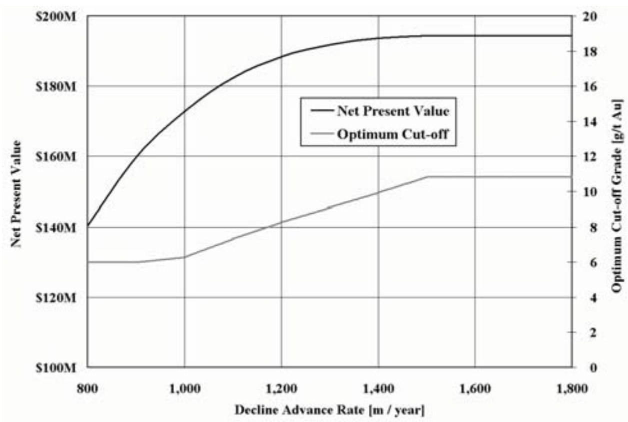


FIG 6 - Effects of changing decline advance rate.

Figure 5 clearly shows that significant gains in value may be achieved by simply increasing the rock mining rate and cut-off. A proposal to increase the ore treatment capacity should be evaluated by developing a second hill of value similar to Figure 5 for the new capacity, and including the capital cost of the upgrade. It would be expected that the peak of the new hill at any mining rate would be at a lower cut-off than for the original hill at the same mining rate, but the difference in optimum cut-off may not be as great as anticipated (see the discussion on the effect of changes in margin below for a potentially analogous situation). The optimum mining rate/treatment rate/cut-off strategy can only be determined by an examination of all the alternatives on the two Hills of Value thus derived.

Optimum cut-off is relatively insensitive to margin

Changes in both operating costs and metal prices will affect the margin obtained. It is common in sensitivity studies to find that the value of a project is much more sensitive to changes in revenue factors than cost factors. To identify how robust strategic decisions regarding cut-offs may be, Hills of Value are generated for variations in both metal price and cut-off. At the optimum cut-off grade, the strategy is typically relatively unaffected. Figure 7 shows results of a study for an underground gold mine, using gold prices of A$500 and A$600 per ounce, and illustrates a finding common to a number of studies. The volatility of the optimum cut-off is lower than the volatility of the breakeven grade when metal prices increase or costs fall. A 20 per cent change in price would result in a 17 per cent change in the breakeven grade. However, the change in the optimum cut-off in Figure 7 is only seven per cent, and the flatness of the curves near the optima is such that selecting the optimum for one price will generate only a small loss of potential value if the other price were to eventuate.

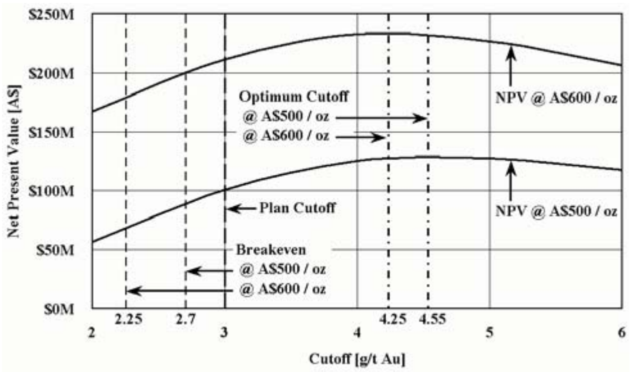


FIG 7 - Case study risk/reward trade-offs at different prices.

When determining strategic policy, there can be significant risks associated with the selection of the metal price to be used if lower than optimum cut-offs are selected. The operation studied had a 'planned' cut-off of 3 g/t Au. Breakevens at $500 and $600 were 2.7 and 2.25 g/t Au respectively. Using these three cut-offs, NPVs are respectively ten per cent, 15 per cent and 20 per cent less than those received by using the optimum cut-off of between 4 and 4.5 g/t Au, if the price received were A$600/oz. If the price received were A$500/oz, the proportional losses in NPV increase to 20 per cent, 30 per cent and 45 per cent at the three alternative cut-offs.

Similar mining areas may have different cut-offs

Conventional wisdom suggests that mining areas with similar characteristics (in terms of grade distribution, orebody characteristics, and cost structures, etc) would have the same cut-offs. Analyses have indicated that this is not necessarily so. Evaluation of the results of GA optimisations have indicated that, for underground mines with multiple mining areas, and open pit operations with a number of pits feeding a central plant, NPV may be maximised when different cut-offs are applied to different mining areas so that all are depleted at the same time. Figure 8 illustrates the principle. A common feature of scheduling mine production towards the end of the mine's life is a low production rate 'tail' of material that is not able to cover the fixed costs of the operation. Value is maximised in this case if the mine is closed when the production rate drops below the sustainable level. The tail in mining area A after the closure date contains high grade material, while the production from area A before the closure contains lower grade material which is nevertheless above the common cut-off used for both mining areas. Figure 9 shows the same operation with different cut-offs for both areas. In this figure, area B is assumed to be producing at the same production rate and cut-off as before. Area A is also producing at the same rate, but with a higher cut-off, so that at the time the operation closes, the best possible material has been mined.

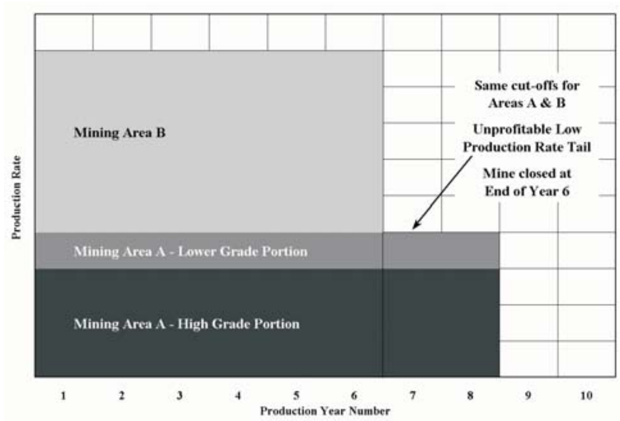


FIG 8 - Typical production profile with all mining areas using same cut-off.

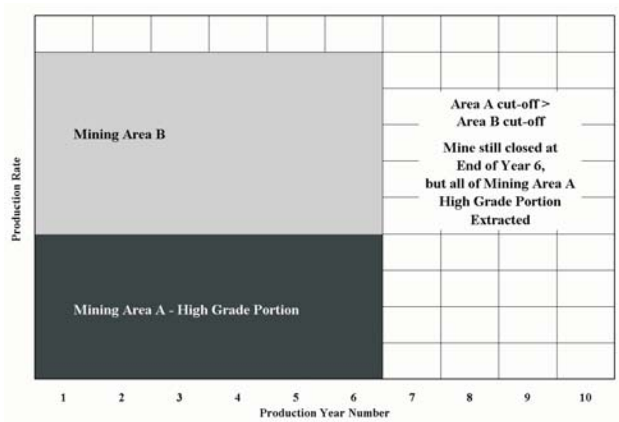


FIG 9 - Production profile delivering higher value, different mining area cut-offs.

Although this principle is easy to comprehend in theory, identifying optimum strategies in practice may not be trivial. Real case studies have exhibited more complex behaviour to take account of interdependencies between underground mining areas, and potential variations in sequencing of open pit deposits. GA optimisations have generated counter-intuitive results: some areas may have cut-offs significantly lower than the overall optimum, and deep parts of the mine may have lower cut-offs than shallower areas. Figure 10 shows schematically cut-offs applied to individual mining areas for an underground gold operation. Area cut-offs were integer values only, from 3 to 6 g/t Au, while the optimum mine-wide cut-off was 4 to 4.5 g/t Au. The NPV resulting from the use of area cut-offs was of the order of ten per cent greater than that at the optimum mine-wide cut-off found by a Hill of Value analysis.

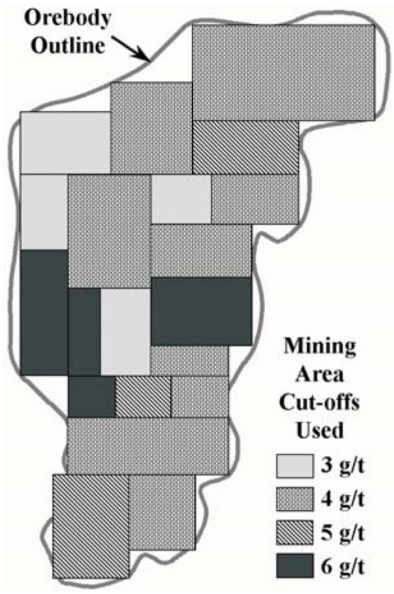


FIG 10 - Longitudinal projection of a case study deposit, showing cut-offs by mining area, for maximising NPV, as determined by genetic algorithm optimisation.

SUCCESSES AND FAILURES

All studies using these processes have demonstrated that significant gains in value can be realised by changing strategy. Three categories of the level of success of the studies have been identified and are as follows:

- 'successful' implies that the analysis has been completed and the client has implemented a new plan as a result;
- 'partly successful' means that the analysis has been completed and the client has accepted the results, however, has elected not to change strategy; and
- 'unsuccessful' indicates that the analysis has not been completed to the stage where firm recommendations can be made, and the study has been terminated.

An evaluation of the factors contributing to the success or otherwise of mining strategy optimisation studies indicates the following.

Characteristics of successful studies

For a study to be successful, it is critical that the company's senior management – those who will have to make the decision to change the operation's strategy, and implement the change – are fully involved in the study process from the beginning. They must understand the nature of the study, and its potential benefits. The study team must ensure that the management team is consulted on a regular basis, in particular to ensure that all factors of concern to decision-makers are addressed adequately in the study. The commitment of the company's senior management must be manifested practically by the commitment of technical staff to assist in the generation of the input data required for the study. The technical staff associated with the study must also understand the nature of the study, why the information they are preparing is important, and how it is to be used. Continuity of staff in these technical roles is highly desirable. The study team must then able to collate all the data, generate results, and present these to decision makers, in an understandable way, and in accordance with the project's timetable.

Characteristics of partly successful studies

There are two types of partly successful study. In one type, the potential benefits are clear, but the level of detail of the study is insufficient to be able to make a firm recommendation for change. In each case where this has occurred, a more detailed study has been commissioned to address those issues. The other type of partly successful study is typified by a major component of the proposed strategy being an increase in cut-off grade, together with a perception by the company's decision makers that the market would respond adversely to the consequent reduction in reported reserves. It is beyond the scope of this paper to discuss why the market should reward a strategy that reduces value and punish a strategy that adds value, or whether in fact this is even the case. However, this perception exists amongst decision makers in some mining companies. Cases have also been encountered where commitments made to financiers or governments may effectively preclude changes to cut-offs, at least in the short term. One could suggest that it would be wise where possible to identify optimum strategies before such commitments, and public announcements, are made.

Characteristics of unsuccessful studies

Unsuccessful studies typically exhibit one or more characteristics opposite to those of successful studies. Senior managers and decision makers may be ambivalent regarding the study. They may believe they already know the best strategy, and therefore do not need to conduct the study. Technical staff may not understand the potential benefits, may be too busy with other more urgent tasks, and/or may see the optimisation study team as a threat, if it is feared that the optimisation study may generate a 'better' plan than they have produced. If these are combined with lack of management commitment to ensure that the study proceeds at a satisfactory pace, the data required is not forthcoming, and the study grinds to a halt. Turnover of both management and technical staff may be a significant contributor to these problems.

Unsuccessful projects have ranged from a case where so little data was forthcoming that no useful modelling could be done, to another where the analysis was largely complete, but waiting on the provision of a few key data items. This latter case was almost in the category of the first of the two types of partly successful study noted above. Dummy data used to develop the model was reasonably accurate, and though real data would have changed the absolute values of reported numbers, conclusions and decisions were unlikely to be affected. Other delays and problems from other factors described above made it impossible to complete the study satisfactorily at the time.

CONCLUSIONS

Most mining companies have publicly stated corporate goals of 'maximising shareholder value', often with associated goals of reducing costs and improving efficiency. Many companies have multiple, and often conflicting, goals. Companies tend not to opt to implement the strategy that fully optimises one of the corporate goals only, but rather to select a strategy that best meets some or all of these conflicting goals. The strategy optimisation process must therefore be able to identify not only strategies that will deliver the various goals, but also the trade-offs required to obtain the optimum combination of various conflicting goals.

Despite the general knowledge of the existence of Lane's cut-off theory, many mines have not applied the methodology. Rather, the use of operating cost breakeven grades as cut-offs is common. Lane's analytical methodology has formed the basis of our detailed strategy optimisation studies, as it optimises only for NPV, and the analytical process can handle a limited number of physical constraints. Stockpiling and grade-dependent recovery relationships introduce further complexities.

The 'Hill of Value' technique is capable of handling all the combinations of various strategic decision variables that can be independently specified. These will typically include such things as cut-off grades, production rate targets, identified debottlenecking and upgrade stages in the mines or treatment plants, and various mining method options. Hill of Value studies have consistently demonstrated potential value (NPV) increases of between ten per cent and 50 per cent over what is obtained by 'traditional' studies using breakeven cut-off analysis. Cut-offs are typically 30 per cent to 50 per cent higher. Genetic algorithm optimisations have also been shown to add further value. It is usually not possible to change the cut-off alone and add value: changing a mining or treatment rate is also usually necessary. A number of studies have shown that improved value can be obtained by increasing the mining rate of total ore and waste in open pits, and development rates in underground mines. These are often reduced in practice to save costs, but optimisation studies indicate that the loss of value is often greater than the costs saved.

Changes in both operating costs and metal prices will affect the margin obtained. To identify how robust strategic decisions regarding cut-offs may be, Hills of Value are typically generated for variations in both metal price and cut-off. When determining strategic policy, there can be significant risks associated with the selection of the metal price to be used if lower than optimum cut-offs are selected. At the optimum cut-off grade, the strategy is relatively unaffected by cost or price changes.

Conventional wisdom suggests that mining areas with similar characteristics would have the same cut-offs. Analyses have indicated that this is not necessarily so. Evaluation of the results of GA optimisations have indicated that, for underground mines with multiple mining areas, and open pit operations with a number of pits feeding a central plant, NPV may be maximised when different cut-offs are applied to different mining areas so that all are depleted at the same time. For a study to be successful, it is critical that the company's senior management are fully involved in the study process from the beginning. The technical staff associated with the study must also understand the nature of the study and be committed to assist in the generation of the input data required for the study. There must also be a commitment to implement the strategies identified.

In some cases, the potential benefits have been clear, but the level of detail of the study is insufficient to be able to make a firm recommendation for change. Where this has occurred, a more detailed study has been commissioned to address those issues. Another type of outcome occurs when a major component of the proposed strategy is an increase in cut-off grade, but a perception by the company's decision makers that the market would respond adversely to the consequent reduction in reported reserves prevents the recommended strategy's implementation. Studies have been conducted for a variety of minerals, mining methods, and numbers of deposits and treatment plants, in a number of parts of the world. They indicate that significant improvements in value (however it is measured) and reduction in risk can be obtained by relatively simple but more comprehensive analysis, modelling a wider range of options, than is usually done. Decision makers can be provided with more, significantly better, information than they are used to receiving, to facilitate the optimisation of their strategic mine plans.

ACKNOWLEDGEMENTS

The authors wish to thank the management of AMC Consultants Pty Ltd for permission to prepare and present this paper, and the secretarial staff for assistance with its preparation. Past and present members of the AMC Mine Optimisation Group are thanked for their input into this paper, both directly and through their contributions to the development of the overall concepts.

REFERENCES

Hall, B E, 2003. How mining companies improve share price by destroying shareholder value, in *Proceedings CIM Mining Conference*, Montreal, paper 1194 (CIM: Montreal).

Hall, B E and de Vries, J C, 2003. Quantifying the economic risk of suboptimal mine plans and strategies, in *Proceedings Mining Risk Management Conference*, pp 59-69 (The Australasian Institute of Mining and Metallurgy: Melbourne).

King, B M, 1998. Impact of rehabilitation and closure costs on production rate and cut-off grade strategy, in *Proceedings APCOM '98, 27th International Symposium of Computers in the Minerals Industry*, pp 617-629 (The Institution of Mining and Metallurgy: London).

King, B M, 1999. Cashflow grades – scheduling rocks with different throughput characteristics, in *Proceedings Strategic Mine Planning Conference*, Perth (Whittle Programming Ltd: Melbourne).

Lane, K F, 1964. Choosing the optimum cut-off grade, *Q Col Sch Mines*, 59(4):811-829.

Lane, K F, 1988. *The Economic Definition of Ore* (Mining Journal Books Ltd: London).

Optimising Multiple Operating Policies for Exploiting Complex Resources — An Overview of the COMET Scheduler

R Wooller[1]

ABSTRACT

The COMET cut-off grade and schedule optimising software is the culmination of decades of research and development in this area. It has been applied at several of the world's major mining operations, in minerals as diverse as coal and diamonds. The paper provides an introduction to COMET for engineers interested in applying it or similar techniques at their operations. COMET's role in the development of optimal mine plans is discussed, placing it in context with respect to other mine optimising software. Its development can be traced from the early work on cut-off grade theory within the Rio Tinto group, through various systems leading finally to COMET. A case study is used as the basis for the description of COMET. Starting with phase designs and the data to be extracted from them, the paper describes its inputs, the optimisation process and the outputs. A valuable aspect of the software is its macro programming feature in combination with the power of Excel. Once a base model has been developed it is easy to create and run a set of scenarios to test, for example: the impact of changes to the mining fleet, process capacity or costs on the project's value and policies employed. The paper then discusses the assumptions made in COMET's optimisation algorithm, and their likely impact on the optimisation process. Further sections cover the types of deposits and operations where COMET can be applied to obtain the maximum benefit.

INTRODUCTION

The objective of this paper is to provide a general introduction to the application of Commercial Optimal Mine Exploitation Technology or COMET, the mine planning software that simultaneously optimises mine schedules and operating policies such as cut-off grade and mill throughput and recovery. The paper aims to assist engineers who are involved in mine optimisation and need to know if COMET is a suitable tool for them. This is done by posing and answering a series of questions these engineers would ask, starting with where does comment fit into the mine planning process.

Currently, there is no tool or technique for providing a totally optimum mine plan for any deposit. Available tools concentrate on optimising a single aspect of a mining operation. Where packages offer more than one tool, these are invariably applied sequentially. Otherwise, engineers apply several tools sequentially, often iteratively, in order to seek an optimal solution.

A typical mine optimisation study would commence with pit optimisation, using algorithms such as Lerchs-Grossmann (eg Muir, 2007, this volume) to delimit the ultimate pit shell. Following this, phase optimisation breaks the ultimate pit into working pushbacks using the same algorithms as pit optimisation (Wharton and Whittle, 1997). Access ramps are designed, possibly with the aid of a haul road optimiser (Gill, 1999). Sequence optimisation then delivers the best sequence of benches for value and blending. Finally, cut-off grade optimisation determines the optimum operating strategy within the capacity constraints of the operation. COMET is applied for the last two of these steps to optimise the mining sequence, cut-off grade and processing options.

WHAT IS COMET'S BACKGROUND?

COMET traces its history back to research into optimum cut-off grades in the 1960s and 1970s within the Rio Tinto group. This research lead to the development of Optimum Grades for Resource Exploitation (OGRE) (Lane, 1988). OGRE was used on major projects within the Rio Tinto group throughout the 1970s and 1980s. However, OGRE's one limitation was that it optimised cut-off grades within a fixed sequence of material, represented as grade/tonnage data. A scheduler was required to generate that sequence.

In 1988, Rio Tinto developed OGREPlus for the Lihir project. The 'Plus' part was a scheduler that took a set of pit phases (pushbacks) and generated an annualised sequence for OGRE to optimise, the resulting cut-off grade policy being 70 per cent fed back to the scheduler. The engineer performed successive iterations between the scheduler and OGRE until the process converged on an optimised schedule (Wooller, 1995). OGREPlus was further developed during the 1990s with the addition of a graphical user interface (GUI) to enhance its utility.

In 1996, while working for Rio Tinto, King (1999a) developed the theory that lead to the development of Cosmos (COnStraint MOdelling Scheduler), a combined scheduler and cut-off grade optimiser. Cosmos became widely used within the Rio Tinto group both at mine sites and for project evaluation by Rio Tinto Technical Services. After leaving Rio Tinto, King created a next generation schedule optimisation software tool. COMET was first demonstrated in November 2001 when a comparison with Cosmos was made on the Escondida project in Chile. Since this time, COMET has been used by several large Rio Tinto and BHP Billiton operations and their technical services groups.

HOW DOES COMET WORK IN PRACTICE?

Probably the best way of describing how COMET works is through an example case study. This exercise involves an open pit copper mine – the Symposium Mine. Optimisation studies have determined the ultimate pit which the engineer has split into three pushbacks. The block model has been exported to an ASCII file. Table 1 gives a summary of the deposit and its operating parameters.

Running COMET

There are two primary components in COMET: the engine and the GUI. The engine is a program written in C++ which performs the optimisation. It reads its data from two sources: text parameter files written by the GUI and phase data files. Its output is also written to text files. The GUI is an Excel workbook, which manages all the input and output for COMET. By working in Excel the user is free to utilise formulae, text formatting, comments and links to other spreadsheets. Comments written into the GUI provide an explanation of each parameter.

Once the model parameters have been entered, the Optimise button writes them to COMET's input file and starts the engine. The engine reads the parameters together with the phase data and performs the optimisation. Meanwhile the GUI waits for a signal from the engine that the optimisation is complete at which stage it reads the output generated by the optimisation and loads the results into the report sheets. These are accessed via the menu buttons at the bottom. Figure 1 shows a summary of the results of the last optimisation loaded.

1. 32 Bader Close, Yate, Bristol BS37 5UA, UK.
Email: Richard.Wooller@blueyonder.co.uk

The Wizard

A quick way to get started with COMET is to use the Wizard. The Wizard comprises a set of dialogues that asks the user about the project and enters the parameters into the input sheets.

The Wizard requires a block model of the mine that has been subject to phase or pushback designs. These phases must be flagged in the block model as a phase number. The block model is exported from a general mining package (GMP) as a text file.

TABLE 1

Symposium Mine parameters.

Item	Value
Total tonnage	295 Mt
Ore	98 Mt
Average grade	1.12% @ 0.5% cut-off
Mining	
Mine capacity	20 Mt/y
Mining waste and dumping	$1.00/t
Mining LG and stockpiling	$0.81/t
Mining ore	$0.80/t
Stockpile	
Cut-off grade	0.3%
Reclaim from stockpile	$0.22/t
Processing	
Plant capacity	10 Mt/y
Recovery	92%
Processing cost	$5.00/t
Overheads	
G&A	$20 M
Closure	$200 M
Revenue	
Copper price	$2000/t
Discount rate	10%

The Wizard then analyses the range of grades, generating a set of cut-off grade bins to fit the range. Finally a set of dialogues prompts for the project details as set out in Table 1. On completion the Wizard generates the phase data files.

Input parameters

At this stage COMET is ready to run an optimisation. Before doing so, a review of the input parameter sheets that the Wizard has populated, starting with the Project information and working round to the Settings sheet, will prove instructive.

Project

The Project sheet contains the parameters such as the title, case, discount rate and terminal value. The convention in COMET is to enter costs as negative values and revenues as positive values. Closure costs are therefore entered as a negative terminal value.

Periods

The Periods sheet defines the start time of each period in the schedule, the end time and therefore duration of a period by the start time of the subsequent period. COMET uses the last period defined as the template for all subsequent periods. Fixed Value holds the annual fixed costs in addition to any capital that might be expended in those years.

Attributes

The Attributes sheet describes the material's properties that are be required for calculating costs and revenues, applying constraints and for reporting.

Attribute data are often in metal grades such as gold (g/t) and copper (per cent). Mass is used by default to weight these grades as, internally, COMET stores quantities (eg gold metal, copper metal), which it accumulates during scheduling. Either quantities or grades may be reported.

Attributes are not confined to metal grades. A recovery attribute would be weighted by the attribute being recovered to derive the recovered quantity, eg copper recovery being weighted by copper grade to give recovered copper. Truck hours per tonne

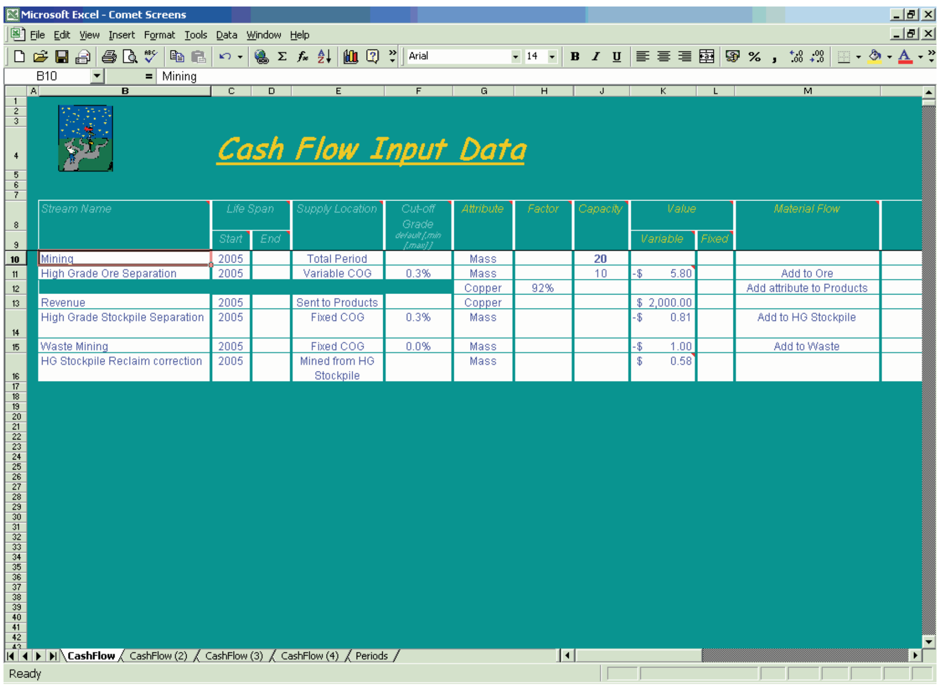


FIG 1 - COMET cash flow sheet.

can be used to calculate the size of the truck fleet required for a schedule and for applying a truck fleet capacity constraint. Mill hours per tonne can be used to model different ore hardness which can affect mill productivity (Wooller, 1997 and 1999).

Phases

In COMET, phases represent sources of material, destinations (targets) and stockpiles. The Phases sheet contains information on each phase. For sources (open pit pushbacks and/or underground sequences of mining) this will include the name of the phase data file together with dependence, precedence, undercutting and lag constraints. Source phase can have a development time and capital to bring them into production.

Target phases define destinations where material can be accumulated for subsequent processing or reporting. Stockpile phases are both a target and a source, and can be initialised via a phase file.

Source phases on input are divided into increments. Reblocking parameters can specify the increment size globally and individually for phases. Increments are discussed later in the paper.

Cash flow

Of all the inputs, the *Cash Flow* sheet (Figure 1) is the most crucial. It is used to model the flows of material (streams) within the operation; in essence, it is a description of the operation being modelled. It contains costs and revenue parameters for calculating cash flow and capacity constraints for the various processes. It is worthwhile examining this sheet in detail with the aid of the Symposium Mine example.

Each stream is given a set of attributes:

- Name – used to identify the stream for reporting.
- Life Span – limits the periods during which the stream's parameters are applied.
- Supply Location – the source of material being processed by the stream. Valid locations are selected from a drop down list generated from the phases defined in the Phases sheet. The Variable COG supply location forces a search for the cut-off grade which maximises value, the material above the cut-off being processed in the stream.
- Cut-Off Grade – where a cut-off grade is applied to the source, this defines the default value.

Each stream is divided into one or more products. Each product may have one or more of the following:

- Attribute – the name of the attribute to which the following are applied.
- Factor – a scaling factor applied to the product quantity.
- Capacity – an annual capacity constraint which limits the amount of the product that can be scheduled.
- Value – the monetary value to be applied to the product quantity. Costs are input as negative values, revenues as positive values.
- Material Flow – an optional destination (target or stockpile) phase for the product for further processing in subsequent streams and/or reporting. Valid destinations, ie those defined in the Phases sheet, are selected from a drop down list.
- Custom Function – the above define a set of standard processes to be applied to each product. Custom functions, some general and some site specific, add a range of exceptional processing to be applied.

Streams are processed from top to bottom, so care should be taken with the position of the information. It is advantageous to put constraints close to the top of the stream list so that unnecessary calculations are avoided.

Figure 1 shows the streams generated by the Wizard for the Symposium Mine. To understand how the mine is modelled in COMET these are examined starting from the first line, labelled Mining:

- Mining – applies a 20 Mt/y capacity constraint to total movement from the mine. As in all streams, this applies from 2005, the start of the schedule, until the end of the mine life.
- High-Grade Ore Separation – takes as its source all material mined above a variable cut-off grade. A capacity limit of 10 Mt/y is applied to the tonnage (mass) and a value of -$5.80 applied, representing the sum of the ore mining and processing costs. Add to Ore places this material in a target phase called Ore for reporting. The next line applies a 92 per cent plant recovery to the Copper in the ore stream and Add attribute to Products places the recovered copper in a target phase called Products for subsequent processing and reporting.
- Revenue – takes as its source the material Sent to Products and applies a $2000/t copper price.
- High-Grade Stockpile Separation – takes as its source all material between a 0.3 per cent cut-off grade and the cut-off applied at High-Grade Ore Separation, and adds it to the HG Stockpile. In this case the cut-offs are the same, hence no material is sent to the stockpile. However, should a higher cut-off grade been applied to the high-grade ore, material sent to the stockpile will incur a $0.81 cost.
- Waste Mining – takes as its source all remaining material (ie down to a cut-off grade of 0.0 per cent). The waste mining cost of $1.00/t is applied and the material added to the Waste target for reporting.
- HG Stockpile Reclaim correction – takes as its source the material reclaimed from the HG Stockpile and applies a credit of $0.58/t. This may seem odd. However, as this material will be processed in the High-Grade Ore Separation stream it will incur a direct mining cost of $0.80/t. To negate this, a credit of $0.80/t must be applied and set against the stockpile reclaim cost of $0.22/t to give the $0.58/t credit.

Sequence

At times it may be necessary to fix the timing of some or all of the sequence of the increments being scheduled. For example, in order to compare a COMET schedule with another methodology (eg manual generated schedules) or to ensure those early periods of a schedule follow a current mine plan. The Sequence sheet is used to manually specify a schedule or part of a schedule.

Formats

The Formats sheet is used to specify what information is reported in the Summary and Detail reports.

Settings

The Settings sheet is used to specify general parameters not directly associated with a project.

Running the optimisation

Now that all of the input parameters have been entered, the next stage is to return to the Control sheet and run the optimisation by clicking the Optimise button.

Before the optimisation engine is started, all previous result files in the working directory are deleted to ensure information is current. The parameters contained in the GUI are then written to a text (ASCII) file. While writing the parameters, checks are performed on the input parameters and, if an error is found, the sheet with the error is displayed and the cell in error selected.

The COMET engine executes in a command window. As the optimisation progresses COMET reports the progress of its iterations and the NPV calculated at each stage, saving the highest NPV schedule to files in the working directory.

Meanwhile, the COMET GUI monitors the progress of the optimisation and, once it detects that it has completed, automatically imports the results into a set of sheets for reviewing.

COMET reports

COMET provides a series of reports to view the results of the optimisation.

Summary (period schedule)

Reports cash flow, cut-off grades, quantities and grades of material mined by period. Periods reported are defined in the Periods sheet and the items reported in the Formats sheet. The period schedule also provides the source data for the COMET charts workbook.

Detail (increment schedule)

Reports details of all increments scheduled, including the period in which they were mined. Although this report can be viewed to check the details of individual increments, its primary function is the source data for two Pivot Table reports – Phases and Timing.

Constraints

This report shows the usage of capacity constraints and dependencies and is particularly useful in understanding why capacity targets were not met.

Reserves

A check on the tonnage input via the phase files and the tonnage scheduled.

Phases (phase summary)

Implemented as a pivot table, this report gives a more detailed breakdown of scheduled quantities by phase and by period.

Timing (phase timing)

Designed to give a quick pictorial view of the schedule via a bar chart, it also shows the relative value of what is scheduled.

COMET charts

COMET schedules may be presented as charts. The COMET Charts Excel workbook is designed to link to the Summary report in a GUI to generate a chart. Figure 2 is an example from the Symposium Mine optimisation.

The Symposium Mine case study

Figure 2 shows the results of the fixed cut-off grade schedule at a 0.3 per cent copper cut-off. Displayed are the total movement, ore tonnage, copper grade and cut-off grade. The present value (no start up capital has been included) is $469 M. Note that at 20 Mt/y total movement the mine has difficulty filling the mill in 2012.

The next stage is to generate a schedule with an optimised cut-off grade policy. Going to the Cash Flow sheet, the Supply Location for the High-Grade Ore Separation stream is changed from Fixed COG (Figure 3) to Variable COG (Figure 4). Rerunning the optimisation generates the schedule illustrated in Figure 5. The present value has increased to $486 M. In spite of this the mine still fails to fill the mill in 2011, 2012 and 2016.

To overcome this problem it is decided to increase the mining fleet by adding 10 Mt/y additional capacity in 2009 at a cost of $10 M. In the Cash Flow sheet (Figure 6) a copy of the Mining stream row is inserted below it. In the first line the End time is set to 2009 (start of 2009). In the second line the Start time is set to 2009 and the End time left blank (to the end of the schedule). The Capacity is increased to 30 (Mt/y). Capital for the expansion is accounted for in the period fixed value in 2009.

The resulting schedule (Figure 7) manages to keep the mill filled throughout the life of the mine. This has increased the present value to $521 M. Apart from 2009, when it takes a sharp drop in order to fill the mill, the cut-off grade exhibits a typical declining policy over the life of the mine. Cut-off grades are generally higher throughout the mine life, leading to more material being placed on the stockpile to be reclaimed at the end.

The next stage could be to test if expanding the mining fleet in 2008 would add even more value or even to investigate the size of the expansion. If plant throughput/recovery data were available these could be varied to find the optimum plant configuration.

The model could be refined with the addition of more attributes. Truck hours/tonne would enable the number of trucks to be calculated and the schedules constrained within the truck fleet capacity. This exercise gives a hint of how a project starts with a simple COMET model which, using the basic building blocks outlined above, is expanded and refined to model all significant aspects of a mining operation.

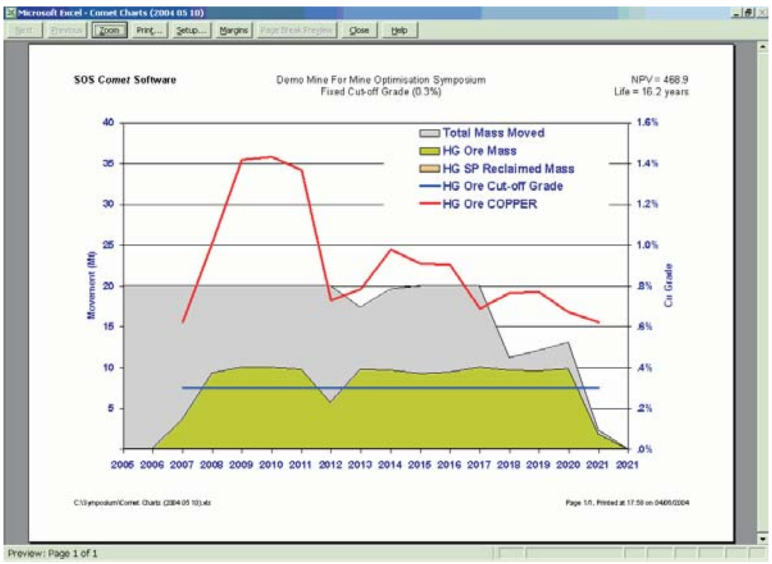


FIG 2 - Fixed cut-off grade schedule.

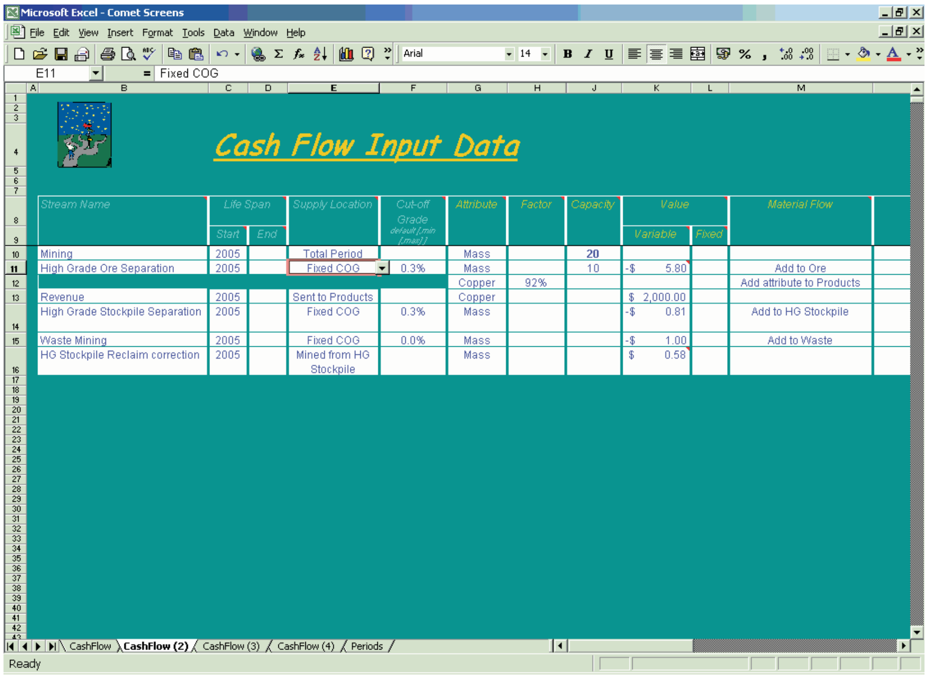


FIG 3 - Cash flow sheet for fixed cut-off grade.

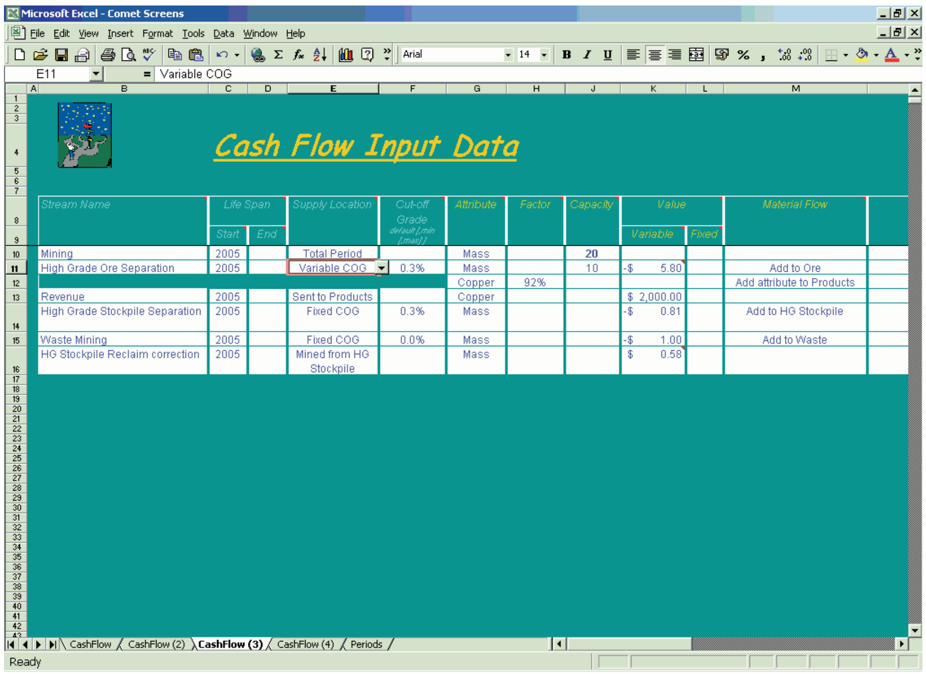


FIG 4 - Cash flow sheet for variable cut-off grade.

CAN COMET BE USED FOR SENSITIVITY STUDIES?

Once a base model has been developed in COMET it is easy to create and run a set of scenarios to test, for example, the impact of changes to the mining fleet, process capacity or costs on the project's value and policies employed.

The Symposium Mine is a simple example in which the optimisation can be completed in under a minute for ten iterations. The size and complexity of some of the projects where COMET has been applied can lead to optimisations that run for hours. In the Symposium Mine example only one mine expansion scenario was considered. What if other options such as different capacity increments or start years needed to be studied?

COMET has a macro programming facility built in whereby several cases can be run in succession, changing parameters between each run. The resulting schedules can be saved and recalled at will. The Macro sheet records a summary of each case run, including the NPV and life, so that the best cases can be selected.

HOW IS DATA PREPARED FOR COMET?

COMET takes as its input a set of phase data files, each file in the set representing a single phase or pushback in an open pit. Files are in ASCII format. This means that they can be viewed and, in simple cases, created in a text editor. Fields can be in fixed or free format, separated by white space characters (Figure 4).

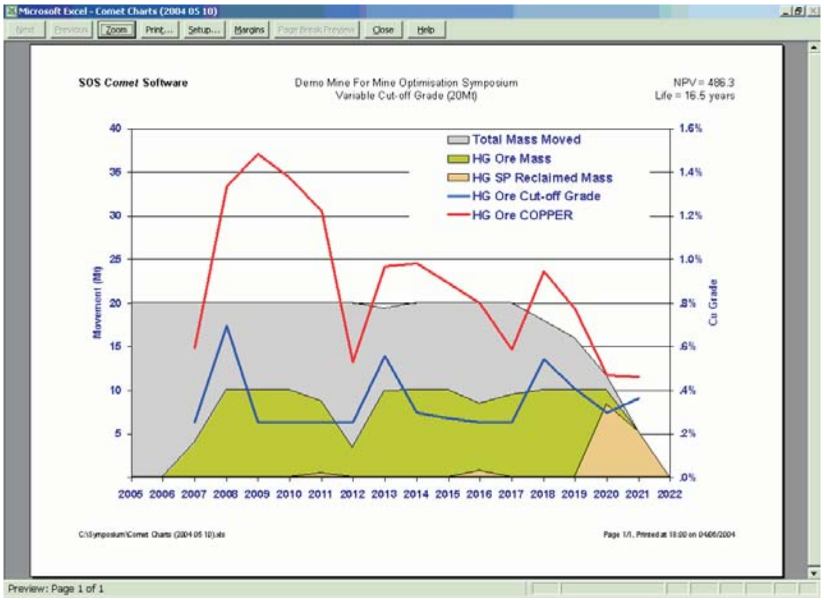


FIG 5 - Variable cut-off grade schedule.

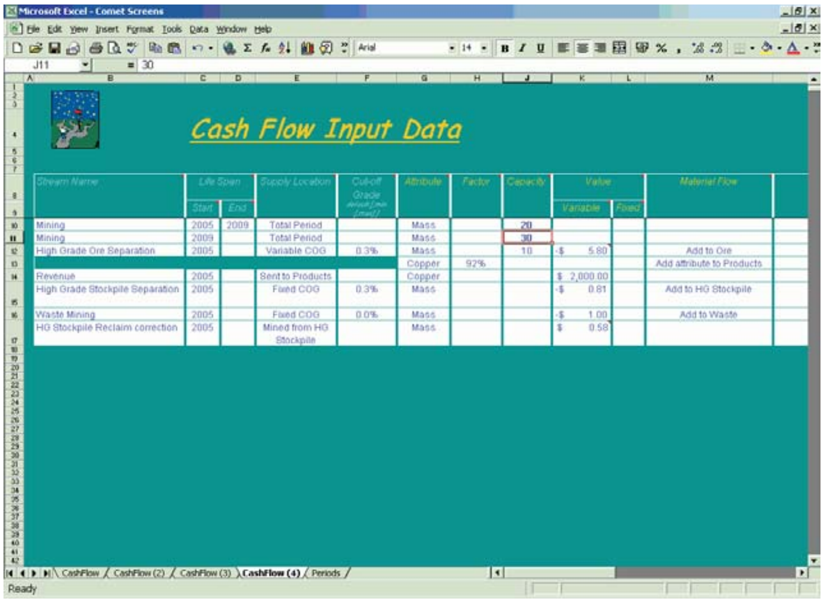


FIG 6 - Cash flow sheet for expanded mine capacity.

The choice of attribute with which to rank material requires careful consideration. Ore should be ranked on increasing value/tonne. For a single element, metalliferous deposit grade serves as a proxy for value. However, in multi metal mines it may be necessary to substitute a dollar value and use that for the cut-off ranking. King (1999b) has shown that additional value can be obtained by considering the rate at which revenue is generated when ranking ore for deposits where different rock hardness impacts the mill throughput.

The description above is based on an open pit mine. However, the same logic can be applied to extraction sequences in an underground mine providing the records are ordered in the sequence of extraction. This technique has also been successfully applied to multiple pit scheduling where each pit has been treated as a phase.

No commercially available general mining package produces output in COMET format though, for historical reasons, MEDSystem comes close. It is up to the engineer to provide files in the required format. As was seen above, COMET provides a Wizard that creates phase files from block model data provided in ASCII format.

Not all attributes modelled in COMET are to be found in block models. For example, haul cycle times could be provided in tables keyed by pushback and bench. Over the years, COMET practitioners have built up a library of code that combines data from several sources to build phase files. As these data are invariable found in tables, most of this code is written in Visual Basic in either Microsoft Excel or Microsoft Access. It can be readily adapted for new projects.

OPTIMISATION ALGORITHM ASSUMPTIONS

A mine plan for the life of the operation has a substantial number of independent variables to consider (eg based on five phases with 20 benches, 20 periods and 50 cut-off grades, ${}^{5000}C_{100} = 10^{211}$ alternatives). Evaluating every one of these alternatives to find the highest value schedule would be impractical with current technology.

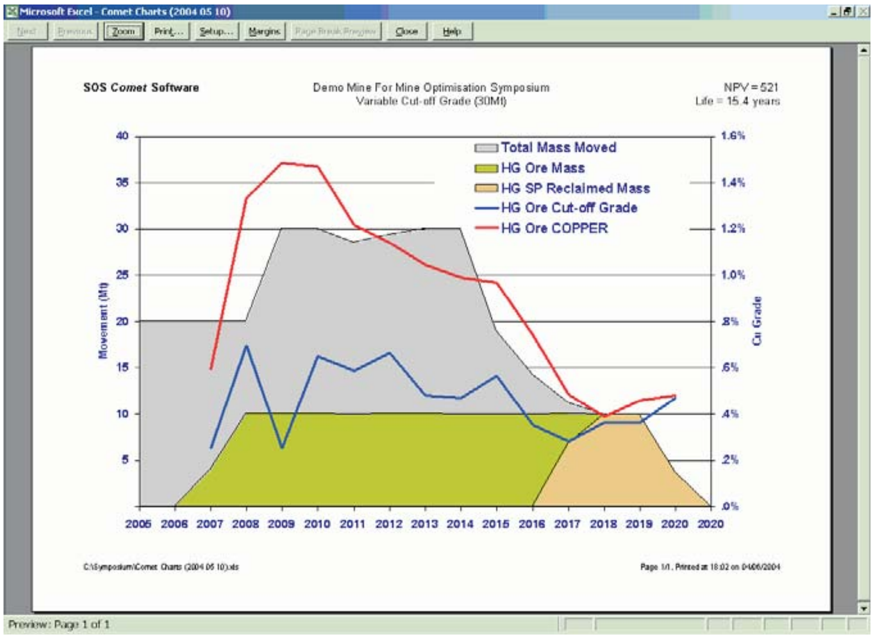


FIG 7 - Expanded mine capacity schedule.

King (2000) identified that the mine scheduling problem was amenable to optimisation using traditional dynamic programming, thus considerably reducing solution times. This traditional approach working from the last increment had two limitations: time dependent properties meant that it was impossible to calculate the value of the final increment; multiple sources, such as alternative pushbacks and stockpiles, meant that it was impossible to know the final increment mined. To overcome these, King applied an iterative algorithm in COMET, mathematically known as a 'successive approximation dynamic programming' solution. This technique optimises successive schedules but does not guarantee always converging on the highest value schedule.

Through successive iterations the algorithm searches for an operating policy and sequence that maximises the value of a resource as given by the general expression:

$$V(R) = \max_{\substack{0 \le r \le R \\ \text{for all } \omega}} \{c(r,\omega) + V(R-r)\}$$

where:

V = maximum discounted value

R = entire resource

r = resource increment

c = discounted cash flow

ω = operating policy

In each iteration, COMET generates period operating policies until the depletion of the resource R. While $c(r,\omega)$ can be determined, the value $V(R-r)$ is an estimate based on information about the remaining increments from the previous schedule iteration.

The operating policy optimised by Lane (1988) and Whittle and Wharton (1995) is cut-off grade, the selection criterion for processing material as ore. However, operating policies can also include the mill throughput/recovery policy, itself a form of cut-off grade (Wooller, 1999) or the choice of process routes such as heap leach versus concentration. King (2000) showed the generic nature of this algorithm by adding dilution/recovery and blast design into the optimisation. This work also recognised the importance of terminal values such as closure costs in generating optimum operating policies.

WHAT ARE THE COMET ASSUMPTIONS?

By understanding the assumptions of optimisation tools engineers are better able to apply them to obtain the maximum benefit. The following list highlights those used in COMET:

- The discount rate is constant for the life of the mine, so is not considered a time dependent variable.
- The design of the phases has the highest value blocks grouped in the same phase. If this is not done then low value material may disguise higher value material and the scheduler may not be able to compensate for poorly sequenced phase designs.
- There must be a seed schedule, ie one based on a set of default operating policies such as a fixed cut-off grade and mining sequence, which generates a positive value.
- Mining, processing and market policies, such as cut-off grade and grind, are fixed within each period of mining.
- Changes in NPV between cut-off grades and processes must be continuous. The presence of step changes in NPV between different cut-off grades and processes should be small compared to changes in value attributable to each increment.

WHAT TYPES OF DEPOSIT ARE BEST SUITED TO COMET?

The algorithms used in COMET are best suited to base metal operations such as copper and gold deposits. In poly-metallic deposits a dollar value, such as net smelter return, can be used in place of a metal grade for ranking material. COMET is most beneficial when applied at mines with more than five years life remaining and several mine plan options that would prove too onerous to evaluate manually.

Normally, deposits where blending is required to produce the final product would not be expected to be amenable to optimisation using COMET. In practice, COMET has been successfully applied at coal, iron ore and industrial mineral operations. The choice of COMET may be due in part to its sequence optimisation and also to the general lack of suitable life-of-mine scheduling software.

WHAT MINING METHODS ARE WELL SUITED TO COMET?

COMET is ideally suited to multiple-pushback, surface operations where cut-off grade optimisation algorithms have historically been applied. Often these have several processing routes and stockpile options. Some operations do not use the cut-off grade optimisation capability but instead use COMET for its capability to sequence the pushbacks within realistic mining and treatment constraints.

Underground operations have been included with open pit phases when it has been necessary to integrate underground and surface operations, eg when ore from each is treated by the same processing plant. Typically, underground operations are scheduled as fixed sequences of material to be processed alongside ore from the surface operation. A number of underground sequences may be evaluated to determine the policies that maximise the value of the combined project, determining the optimum transition from surface to underground mining.

IS COMET BEST SUITED TO SHORT-, MEDIUM- OR LONG-TERM PLANS?

Although the algorithms used by COMET will, in theory, work in any time frame, the reserve, cost, revenue and constraint assumptions are normally only appropriate for long-term plans. This means schedules in annual periods covering the entire life of the operation.

CAN COMET BE USED FOR BLEND OPTIMISATION?

For blend optimisation problems, such as those with both minimum and maximum constraints, the answer is probably no. The algorithms COMET uses are well suited to value based objective functions with maximum constraints but not minimum constraints. In blending problems as defined above, some grades must be kept within a certain range (eg sulfur or phosphorous between an upper and lower limit). The maximum boundary can be modelled in COMET but the minimum boundary cannot. However, by applying a value based objective function to reflect the minimum boundary or reducing the maximum limit, the minimum grades in the schedule may be increased to the desired level.

While not claiming to produce an optimum blended schedule, COMET's optimisation algorithm can achieve reasonable results. Used in conjunction with its tools for generating multiple schedules, COMET has been applied with some success in operations where different types of material are blended to produce a single or multiple products.

CAN COMET BE USED FOR UNDERGROUND OPERATIONS?

The principles for optimising cut-off grades are the same for underground operations as they are for open pit mines. However, the formulation of the problem for open pit mines often disguises the fundamentals of the theory, obscuring the steps needed to build a suitable model for underground operations.

Even without cut-off optimisation, an underground operation can be viewed as a set of fixed sequences of material for COMET to schedule.

CONCLUSION

COMET is an integrated mine sequence, mill throughput/recovery and cut-off grade optimiser. It can be used to optimise schedules for a wide range of mining methods, minerals and processes. Its user interface, written in Excel, enables complex models to be easily constructed from just a few basic building blocks. Its utility has been demonstrated in some of the major mining operations in the world.

One of its major strengths is its macro programming facility which enables optimal schedules to be quickly generated for a wide range of scenarios, invaluable for strategy studies.

ACKNOWLEDGEMENT

I wish to thank Dr Brett King for his assistance in preparing this paper.

REFERENCES

Gill, T, 1999. Express road routing: the application of an optimal haul road generator to real world data, in *Proceedings 1999 Optimising with Whittle: Strategic Mine Planning*, pp 71-79 (Whittle Programming Pty Ltd: Melbourne).

King, B M, 1999a. Schedule optimisation of large complex mining operations, in *Proceedings APCOM*, Denver, Colorado.

King, B M, 1999b. Cash flow grades – scheduling rocks with different throughput characteristics, in *Proceedings 1999 Optimising with Whittle: Strategic Mine Planning,* pp 103-109 (Whittle Programming Pty Ltd: Melbourne).

King, B M, 2000. Optimal mine scheduling policies, doctoral thesis (unpublished), pp 62-87, 110-155 (University of London).

Lane, K F, 1988. *The Economic Definition of Ore* (Mining Journal Books Ltd: London).

Muir, D C W, 2007. Pseudoflow, new life for Lerchs-Grossmann pit optimisation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 113-120 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Wharton, C and Whittle, J, 1997. The effect of minimum mining width on NPV, in *Proceedings 1997 Optimising with Whittle: Strategic Mine Planning* (Whittle Programming Pty Ltd: Melbourne).

Whittle, J and Wharton, C, 1995. Optimising cut-off grades, in *Proceedings 1995 Optimising with Whittle: Strategic Mine Planning* (Whittle Programming Pty Ltd: Melbourne).

Wooller, R, 1995. Where Four-D ends…, in *Proceedings 1995 Optimising with Whittle: Strategic Mine Planning* (Whittle Programming Pty Ltd: Melbourne).

Wooller, R, 1997. Who needs mine and mill constraints?, in *Proceedings 1997 Optimising with Whittle: Strategic Mine Planning* (Whittle Programming Pty Ltd: Melbourne).

Wooller, R, 1999. Cut-off grades beyond the mine – optimising mill throughput, in *Proceedings 1999 Optimising with Whittle: Strategic Mine Planning*, pp 217-229 (Whittle Programming Pty Ltd: Melbourne).

Integrating Multiple Simulations and Mining Dilution in Open Pit Optimisation Algorithms

A Richmond[1]

ABSTRACT

Mineral grade uncertainty and mining dilution have important implications when considering the financial risk of an open pit limit. A method for preserving the financial influence of short-scale grade information provided by multiple conditional simulations in large-scale block models is proposed. Furthermore, this novel re-blocking technique can also account for ore loss and mining dilution by pre-processing the simulations with an automated dig-line algorithm. The methodology is demonstrated for open pit optimisation with, firstly, the floating cone algorithm in which multiple simulations are dealt with by averaging pay-off matrices, and secondly, a local search algorithm that works with the distribution of pay-off matrices to estimate the financially efficient set (frontier) of open pit limits.

INTRODUCTION

The vast majority of open pit optimisation techniques, such as floating cone (Lemieux, 1979), Korobov (1974), Lerchs-Grossmann (1965) and network flow algorithms (Johnson, 1968), involve a 3D grid of blocks that is converted *a priori* into a pay-off matrix by considering a 3D block model of mineral grades, economic parameters and some form of cut-off grade(s). Much has been written about the choice of block size used in block models for optimal pit design (eg Whittle, 1989; Cai, 1992; Dowd, 1994a). However, the discussions seem to revolve around the ability of different block sizes to describe the geometry of the open pit limit and the orebody, as decreased computing time will result from larger block sizes. The disadvantage of larger blocks is the loss of definition of grade variations within the orebody (as described by the support effect or volume-variance relationship), hence the incorporation implicitly of ore loss and mining dilution into the orebody model. For example, an orebody model for selective mining units (SMUs) of say 6 × 6 × 3 m will more accurately reflect mining recoverability than an orebody model with 30 × 30 × 15 m blocks can. In addition, the former will yield a more accurate revenue block model (pay-off matrix), which in turn will yield an open pit limit that will be closer to the true optimum.

From an estimation viewpoint, the overwhelming restriction on block size is the relative spacing of data available to estimate the grade of blocks. The numerical grade, hence the expected revenue values assigned to blocks are estimated from the available sample data and they invariably have an error associated with them. In general, for a given amount of data, the smaller the block size, the greater the error of estimation of its grade value (Armstrong and Champigny, 1989), which in turn introduces greater unreliability into the pay-off matrix. Farrelly and Dimitrakopoulos (2001) attempted to retain the smaller-scale grade information by building indicator kriging models with large block dimensions that incorporated grade distributions corrected for SMU dimensions with the indirect log-normal support correction method. Then, from a theoretical viewpoint, individual SMUs represent a portion of the corrected grade distribution. However, there is no means of confirming if such a portion of the corrected grade distribution actually exists, or that such a contiguous unit is in fact shaped like the SMU.

1. MAusIMM, Golder Associates Pty Ltd, 611 Coronation Drive, Brisbane Qld 4066, Australia. Email: arichmond@golder.com.au

To overcome the inadequacy of smoothed estimates in open pit optimisation, Dowd (1994b) and Farrelly and Dimitrakopoulos (2001), among others, proposed re-blocking simulated grade values. For example, the distribution of blocks for simulated and E-type copper values at SMU support (6 × 6 × 3 m) and re-blocked to 30 × 30 × 15 m are shown in Figure 1. These histograms are from a case study which will be discussed in detail in following sections. Note that the distribution variance decreases as the block size increases (volume-variance relationship). Furthermore, the E-type distribution variances are smaller than corresponding simulated distribution variances, illustrating the smoothing associated with the E-type estimates. The mining and financial implications related to the volume-variance relationship and smoothing may vary significantly with the cut-off grade. For example, when applying a cut-off grade of say 0.35 per cent copper to the histograms in Figure 1, significantly different proportions of the deposit would be considered as ore. In this example, 61 per cent of the simulated SMU copper values are greater than 0.35 per cent. However, 90 per cent of the E-type SMU copper values and 82 per cent of the 30 × 30 × 15 m re-blocked simulated copper values are greater than 0.35 per cent, ie both smoothed estimates and straightforward re-blocking significantly over-estimate the ore tonnage. The first problem that is addressed in this paper is how to re-block simulations such that the mining and financial implications related to small-scale grade variations is maintained in large blocks.

An explicit assumption when generating the pay-off matrix is that blocks are selected *a priori* as ore or waste using the block grade values and cut-off grades. Thus, Figure 1 also demonstrates an additional problem in open pit optimisation. If block estimates (E-types) are used to identify the pay-off matrix, the corresponding set of block selections overestimates the ore tonnage. On the other hand, when considering multiple simulations, alternative sets of block selections may be derived by the application of cut-off grades to each simulated block grade value. The second problem that is addressed in this paper is how to incorporate multiple simulations into the pay-off matrix, which facilitates objection functions other than the traditional maximum pay-off to be incorporated into open pit optimisation algorithms.

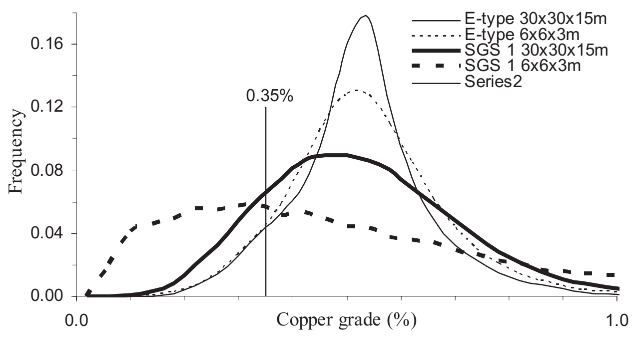


FIG 1 - Histograms of copper grade values.

ACCOUNTING FOR ORE LOSS AND MINING DILUTION

To overcome the deficiencies associated with the straightforward re-blocking of simulations, many authors (Whittle, 1989; Farrelly and Dimitrakopoulos, 2001) suggest calculating large block revenues as the linear average of the spatially equivalent SMU revenues. This approach to preserving the small-scale grade information assumes free selection. However, ore extraction involves dig-lines of irregular geometry that are unlikely to be represented by contiguous groups of SMUs on a Cartesian grid. Consequently, several approaches to automating dig-line generation have been proposed. For example, 2D adaptations of the floating cone algorithm (Srivastava, Hartzell and Davis, 1992; Richmond and Beasley, 2004) and constrained optimisation of a polygon (Norrena and Deutsch, 2001).

This study adopted the floating circle algorithm of Richmond and Beasley (2004). Their idea was to float a circle representing the dig-line constraint, for example, the minimum turning radius of the mining equipment employed, around the problem area, and if the average grade of blocks that fell within the current circle perimeter were greater than the optimal cut-off grade, then the ore dig-line was extended outwards to the circle perimeter. Thus, dig-lines generated by this floating circle approach incorporate explicitly ore loss and mining dilution. Richmond and Beasley (2004) also described algorithms for generating dig-lines for multiple ore types, multiple simulations, alternate mining strategies and risk-based objective functions.

To reflect mining recoverability more accurately, it is proposed to incorporate ore loss and mining dilution into re-blocked orebody models (pay-off matrices) by assimilating the geometrically irregular dig-line solutions, based on small-scale simulations, into large-scale geometrically regular blocks as follows:

1. for each bench level, corresponding to the vertical dimensions of a large-scale block model, generate a dig-line using the simulated grade values; and
2. for each large-scale block, and considering only the spatial equivalent of the dig-line solution from Step 1, calculate the block revenue as the linear sum of the simulated grade revenues.

To illustrate graphically the proposed re-blocking method, consider the 324 values simulated on a 1 × 1 m support for an 18 × 18 m 'parent' block shown in Figure 2. In Figure 2a the simulated values are plotted as ore or waste, based on the cut-off grade value. Now consider re-blocking the simulated values to 6 × 6 m SMUs, ie the parent block contains nine SMUs on a regular grid, shown in Figure 2b. If the average grade of the simulated values contained within an SMU is greater than the cut-off grade, then that SMU is classified as ore (south-east and northernmost three SMUs in Figure 2b), otherwise it is classified as waste (eg south-west SMU in Figure 2b). For this example, re-blocking to SMUs introduces significant ore loss (dark grey blocks) and mining dilution (light grey blocks) into the SMU model. Ore loss involves an opportunity cost of lost revenue for rejected ore blocks, and mining dilution realises the loss of some operating costs. Furthermore, the financial losses associated with these miss-classified blocks are propagated through to the financial pay-off of the parent block, calculated as the linear sum of the nine SMU revenues (each of which in turn are the linear sum of 36 simulated block revenues). Now consider the dig-line shown in Figure 2c. Any simulated block within the dig-line is viewed as ore, otherwise it is regarded as waste. The financial pay-off of the parent block is then calculated as the linear sum of the 324 simulated block revenues. Note that, the ore loss and mining dilution is significantly less than in Figure 2b and corresponds to operational practice. Hence, Steps 1 and 2 above will yield more accurate large-scale revenue block models.

Note that the financial difference between re-blocking to SMUs and the proposed re-blocking methodology may be significant throughout the orebody when the mineralisation is erratic. For mineral deposits with strongly gradational spatial variations in the mineral content, the financial differences are

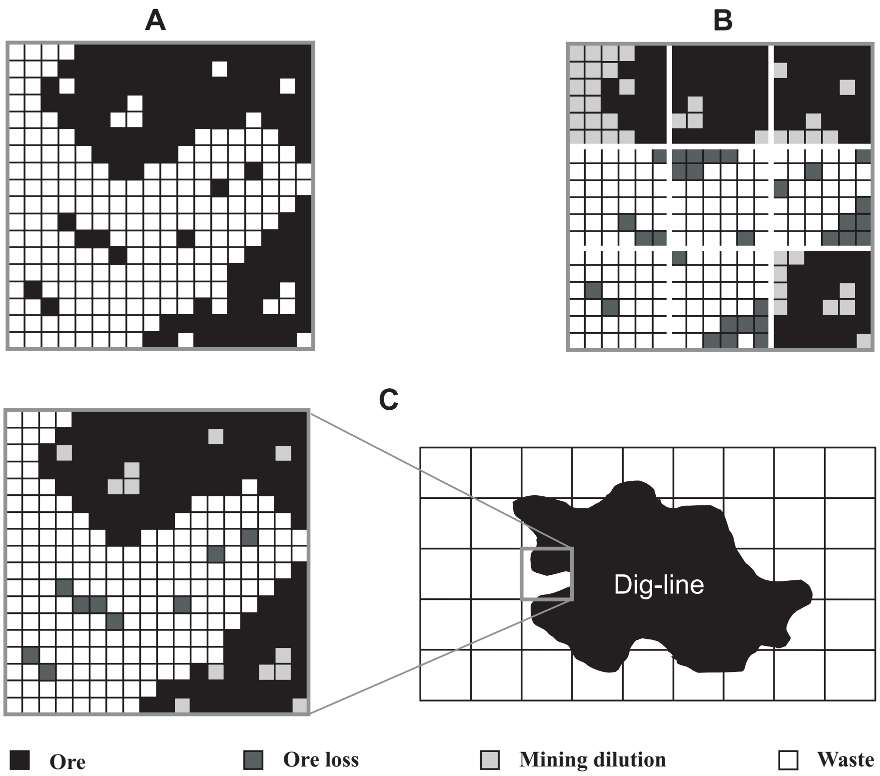


FIG 2 - Graphical illustration of re-blocking: (a) simulated values; (b) re-blocking to SMUs; (c) proposed methodology.

only of significance in the vicinity of the grade contour that corresponds to the cut-off grade value. One drawback of the proposed methodology is that the *a priori* dig-line considered during re-blocking may not honour mining equipment constraints for blocks at the extremities of the open pit limit. In the computational results reported below this feature was rarely observed, and it was limited to the lowest benches in the open pit limit.

ASSIMILATING MODELS OF GRADE UNCERTAINTY

A key difference between ore selection and open pit optimisation is that, in practice, ore selections using dig-lines will actually be mined. For the open pit limit adopted, optimal for ore selections assumed explicitly by traditional open pit optimisation algorithms, additional information (blasthole assays) will become available at some future point in time and mining will actually be based on subsequent ore selections. There is no guarantee that these new ore selections will correspond to the ore selections assumed for open pit optimisation. To account for this uncertainty, it is proposed to assume possibly different sets of local processing decisions, each corresponding to an individual simulation.

Let:

- K be the number of mutually exclusive and exhaustive processing options, where without loss of generality we include treating a location as waste (k=1) as a processing option
- L be the total number of simulations
- X be the total number of simulated locations (also conveniently thought of as discrete blocks)
- B be the total number of large-scale blocks
- P be the total number of feasible open pit limits
- i_{xk} =1 if it is decided to process location x using option k, and zero otherwise
- S represent the set of processing options chosen for the entire set of locations, with $s(x)$ being the option chosen for location x
- B_p be the set of blocks that identify an open pit limit p
- F_p be the distribution of financial pay-offs for p

In most pit optimisation studies found in the literature L=1, and elements of the decision set $s(x)$ are conditional on outcomes of a binary decision variable i_{xk} based on estimated grade values z^*. The simple decision rule is i_{xk}=1 if $g_k \le z^*(x) < g_{k+1}$ and zero otherwise, where g_k is the cut-off grade for processing option k (with by convention g_1=0). It is proposed to vary the decision variable i_{xk}, and consequently $s(x)$, using either cut-off grades applied to the simulated values or dig-lines, ie:

$$i_{xk}^l = \begin{cases} 1, & \text{if } g_k \le z^l(x) < g_{k+1} \\ 0, & \text{otherwise} \end{cases}, \; k = 1,...,K; \; l = 1,...,L \tag{1}$$

or

$$i_{xk}^l = \begin{cases} 1, & \text{if } s^l(x) = k \\ 0, & \text{otherwise} \end{cases}, \; k = 1,...,K; \; l = 1,...,L \tag{2}$$

where:

- $z^l(x)$ are the simulated values
- $s^l(x)$ are elements (local processing decisions) of a dig-line involving only simulation l

Equation 1 assumes that the locations can be selected independently (free selection), whereas Equation 2 more accurately accounts for ore loss and mining dilution. Although Equation 2 is advantageous in permitting explicitly ore loss and mining dilution based on mining equipment constraints in open pit optimisation algorithms, Equations 1 and 2 are both appropriate as they consider future flexibility (practical reality) in the decision process when the blasthole information becomes available and mining actually takes place.

From Equation 1 or 2 and the re-blocking method discussed previously, pay-off matrices $\{w^l(b|(S^l)),\ b=1,...,B,\ l=1,...,L\}$ can be calculated as linear sums:

$$w^l(b|(S^l)) = \sum_{x \in b} \sum_{k=1}^{K} i_{xk}^l q_x [m r_k z^l(x) - c_k], \; l = 1,...,L \tag{3}$$

where:

- q_x is the quantity (weight) of material associated with location
- m is the mineral value per concentration unit
- r_k is the proportion of the mineral recovered using processing option k
- c_k is the mining and processing cost per unit weight for processing option k

Assimilating multiple simulations into open pit optimisation algorithms that employ a pay-off matrix can then be achieved by estimating a single pay-off matrix $\{w^*(b|(L)),\ b=1,...,B\}$ corresponding to the linear average of the pay-off matrices associated with the individual simulations, ie $w^*(b)=\Sigma w^l(b)/L$. Alternatively, a distribution of financial pay-offs for a given open pit limit $F_p=\text{Prob}\{w(B_p|(L))\le w\}$ can be derived from the L results:

$$w(B_p|(l)) = \sum_{b \in B_p} w^l(b|(S^l)), \; l = 1,...,L \tag{4}$$

RISK-BASED OBJECTIVE FUNCTIONS IN OPEN PIT OPTIMISATION

In practical terms, modern pit optimisers are employed solely to find the maximum net present value (NPV) pit at a given moment in time for a defined set of economic, physical and mining constraints, eg capital cost, discount rate, commodity price, slope angles and mining capacity. A distribution of potential financial pay-offs (Equation 4) permits the search for an efficient frontier of open pit limits that maximise the objective function:

$$O(F_p) = \lambda E\{F_p\} - (1-\lambda) r(F_p) \tag{5}$$

where:

- E is the usual expectation operator
- $r(\ )$ is a real-valued risk function
- λ ($0 \le \lambda \le 1$) is the weight to be attached to expected pay-off

In other words, a set of open pit limits that represent the best possible trade-off between financial risk and expected pay-off can be found by solving Equation 5, varying λ from $0 \rightarrow \infty$. The ratio $(1-\lambda)/\lambda$ represents the degree of risk aversion. In other words, in mean-risk space, with risk on the x-axis, an indifference curve will plot as a line with slope $(1-\lambda)/\lambda$. The intercept of this indifference curve with the y-axis is interpreted as the utility.

There are several different approaches to measuring risk, such as variance, semi-variance, expected loss and probability of loss. In mining, measuring financial risk by the probability weighted dispersion of potential pay-offs below a target is appealing, since it recognises the finite non-renewable nature of the resource in question and the desire to achieve a minimum acceptable pay-off from the resource. In this paper, the downside risk function proposed by Fishburn (1977) is adopted as a suitable measure of risk. This downside risk function is:

$$r(F_p) = \int_{-\infty}^{t} (t-w)^{\alpha}\, dF_p(w) \tag{6}$$

where:

t is the target pay-off

α (≥0) is a measure of the impact of failing to reach the target pay-off t

Exact solutions to Equation 5 are computationally inefficient; however, estimates of the efficient frontier can be found heuristically. A Local Search Heuristic (LSH) that can estimate the efficient frontier of open pit limits involves a stochastic (conic) backfill and excavate perturbation mechanism as follows (Richmond, 2003):

1. generate an initial solution B_p, for example, the maximum pay-off open pit limit identified by the floating cone algorithm;
2. calculate the initial objective function $O(F_p)$ associated with B_p using Equation 5;
3. modify B_p by randomly backfilling part of the pit and/or excavating the pit deeper to create a new open pit limit B_{p*};
4. accept the modification if $O(F_{p*})$- $O(F_p)$>0, ie set $B_p = B_{p*}$; and
5. repeat Steps 3 and 4 until additional modifications do not significantly increase the objective function further.

Alternate estimates of open pit limits on the efficient frontier can be made by varying the initial solution in Step 1 above, the weight λ, or the random number seed used to backfill/excavate the pit in Step 3 above.

COMPUTATIONAL RESULTS

This section demonstrates the proposed concepts for a large copper deposit. For simplicity, the number of ore processing options in this study was restricted to two (ore and waste), ie K=2. Sequential Gaussian simulation (SGS) was used to generate twenty-five realisations on a 6 × 6 × 3 m (X, Y, Z) grid. The E-type estimates and SGS simulations were re-blocked into 30 × 30 × 15 m blocks, the latter dimension being the assumed bench height. These large-block models included the proportion of 6 × 6 × 15 m blocks above the cut-off grade and within the ore dig-line, which are referred to as the undiluted and diluted models respectively. Figure 3 shows the 6 × 6 × 15 m local selections for the E-type model and the first simulation (SGS 1). Ore selections based on cut-off grades (Figures 3a and 3b) show numerous horizontal transitions between ore and waste, especially for SGS 1. From an operational perspective, vertical transitions between ore and waste are not significant as the benches will be mined independently. Differences between the dig-lines for the E-type estimates (Figure 3c) and SGS 1 (Figure 3d) are less pronounced than for the cut-off grade solutions.

Floating cone algorithm

Thirty-three open pit limits were identified for the diluted E-type model with the floating cone (FC) algorithm by varying the copper price from $1300/tonne to $2100/tonne in $25 increments. The financial evaluation of the open pit limits was based on a copper price of $1700/tonne. The pay-offs for the 33 nested open pits using this single estimated grade (E-type) model approach are plotted in Figure 4a as filled triangles. The pay-offs for the same open pit limits were calculated using the multiple grade model approach with the diluted SGS model, plotted in Figure 4a as filled circles. Note that as the open pit limit increases in size, the multiple grade model approach values the open pit at increasingly larger expected pay-offs relative to the single estimated grade model approach. The maximum pay-offs for the diluted E-type and SGS models are $457 M and $491 M respectively.

Pay-offs associated with open pit limits identified by FC using the undiluted models of grade uncertainty are shown in Figure 4b. The difference between the expected pay-offs for the undiluted E-type and SGS models represents both the flexibility in local selection decisions and free selection that is implicit to cut-off grade solutions. The difference between the expected pay-offs for the undiluted E-type and SGS models is considerably greater than for the diluted case (Figure 4a), suggesting that free selection accounts for most of the difference with the undiluted models.

Figure 4 also shows the pay-offs for the individual simulations. Note that these pay-offs are significantly greater for the undiluted simulations and may have important implications when considering the financial risk of an open pit limit. For example, consider a minimum acceptable threshold value of $425 M, shown as a dashed line in the plots. For the undiluted simulations (Figure 4b), once the open pit size exceeds 12 000 blocks the probability that the pay-off is less than this threshold value is zero. However, for the diluted simulations (Figure 4a), the probability that the pay-off is less than $425 M for all the identified open pit limits is always greater than zero. These differences justify the use of the re-blocking method discussed earlier.

LSH algorithm

For this study, LSH was implemented with the downside risk function parameters α=2 and t=$425 M. To identify the efficient frontier, $(1-\lambda)/\lambda$ was varied from 1.0×10^{-10} to 1.0×10^{-2} with fifty λ values. Figure 5 presents the heuristic efficient frontier for LSH as filled diamonds. Note that the optimal FC solution (shown as a cross in Figure 5) is dominated, ie LSH found the maximum expected pay-off open pit limit. Figure 5 shows that the LSH minimum risk solution returns 99.2 per cent of the maximum expected pay-off at 81.7 per cent of the corresponding downside risk.

CONCLUSIONS

This paper proposed novel methods for incorporating ore loss and mining dilution and multiple simulations into traditional open pit optimisation algorithms. Small-scale and irregular dig-line solutions that accounted for ore loss and mining dilution associated with equipment constraints were upscaled into the large regular blocks, which are required for computationally effective open pit optimisation. Multiple simulations accounted for the multivariate uncertainty in grade estimates, permitted flexibility in local ore selections and quantified the financial uncertainty for a given open pit limit. The computational results presented showed that, for a given open pit limit and set of economic parameters:

1. the pay-off associated with a grade model composed of 'smoothed' E-type values is not the linear average of the pay-offs associated with the individual simulations used to calculate the E-type values; and

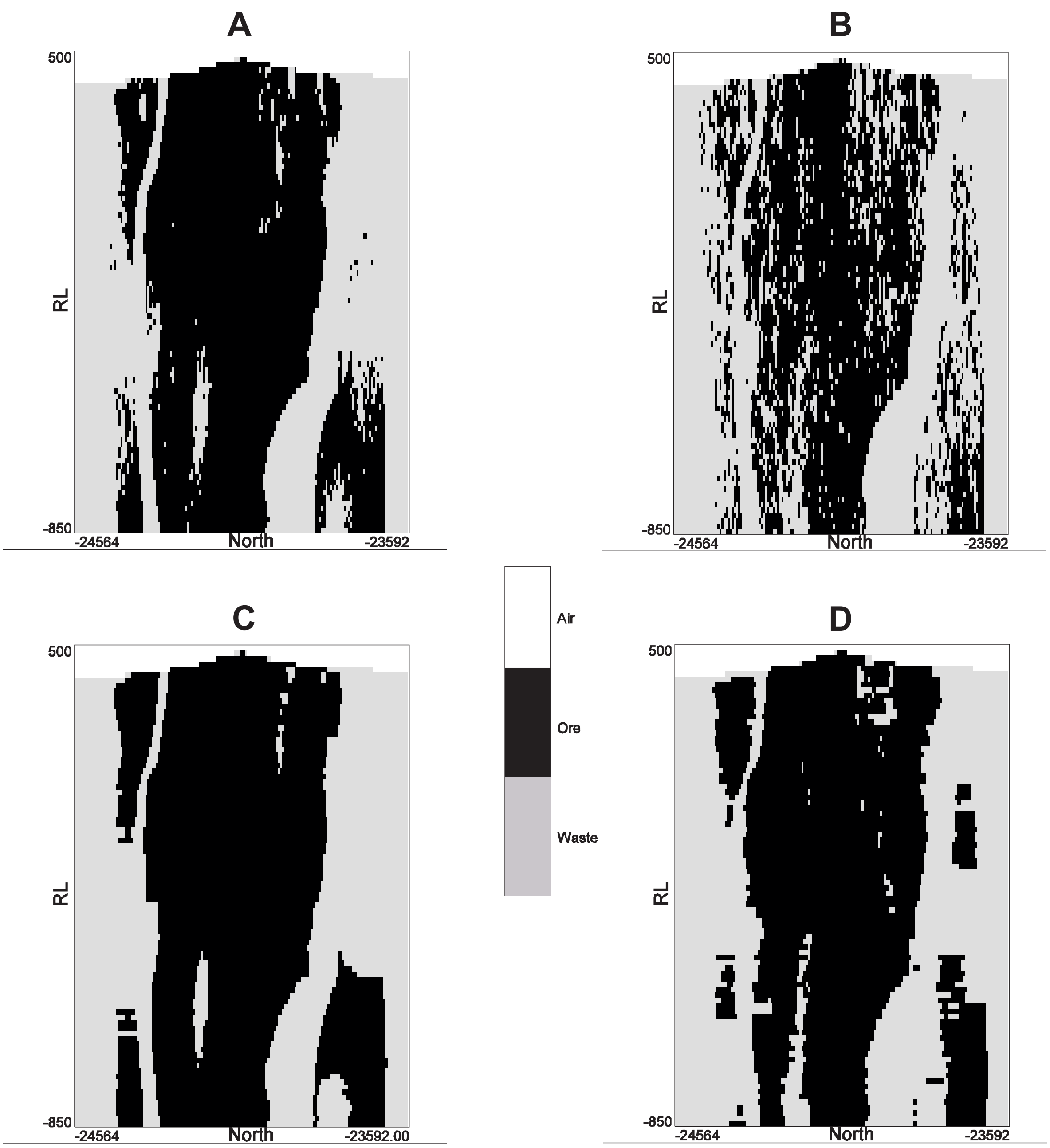


FIG 3 - Local selections used for re-blocking: (a) undiluted E-type; (b) undiluted SGS 1; (c) diluted E-type; (d) diluted SGS 1.

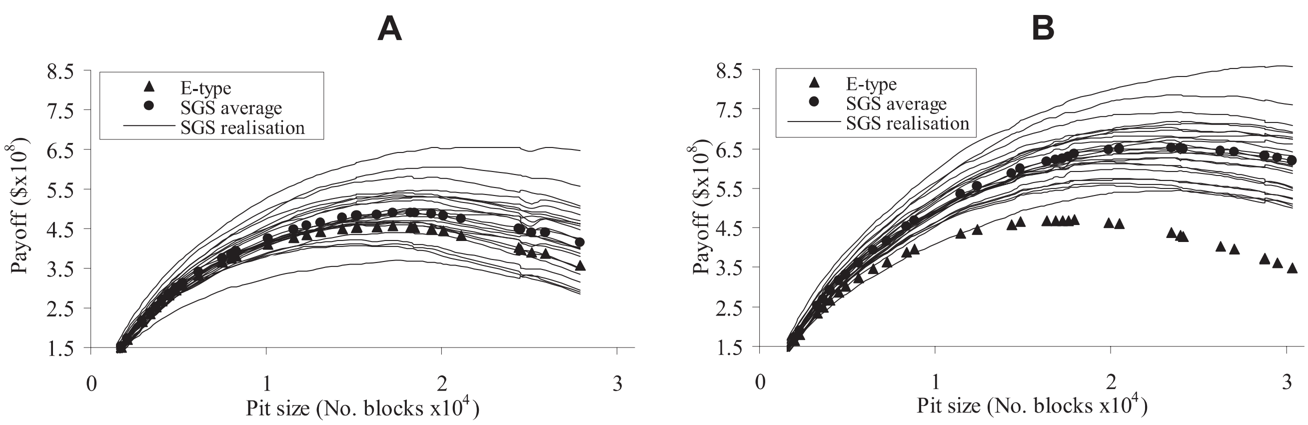


FIG 4 - Pay-off versus pit size using floating cone method: (a) diluted models; (b) undiluted models.

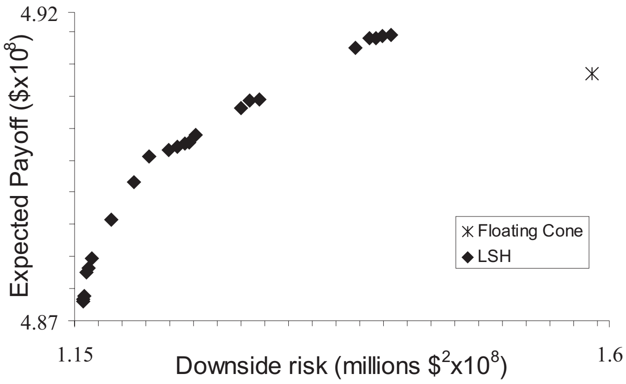


FIG 5 - LSH efficiency plot (note: dominated solutions are not plotted).

2. incorporating small-scale grade variability into the open pit optimisation process with models of grade uncertainty, without accounting for mining dilution, significantly increases the expected pay-off.

Finally, a new generation of open pit optimisation algorithms (eg LSH) that are able to work with pay-off distributions was demonstrated. LSH provided a means of incorporating financial risk measures directly into the optimisation process. The resulting efficient set of open pit limits provides the decision-maker with the best possible trade-off between expected pay-off and financial risk.

ACKNOWLEDGEMENTS

The author would like to thank Rio Tinto Technology Group (Bristol) for providing financial support, mining data and constructive criticism.

REFERENCES

Armstrong, M and Champigny, N, 1989. A study on kriging small blocks, *Bull CIM*, 82:128-133.

Cai, W L, 1992. Sensitivity analysis of 3-D model block dimensions in the economic open pit limit design, in *Proceedings 23rd APCOM*, pp 475-486.

Dowd, P A, 1994a. Risk assessment in reserve estimation and open-pit planning, *Trans Inst Min Metall*, Section A, Mining Technology, 103:A148-A154.

Dowd, P A, 1994b. Optimal open pit design: sensitivity to estimated block values, in *Mineral Resource Evaluation II: Methods and Case Histories* (eds: M K Whately and P K Harvey), pp 87-94, Special Publication No 97 (Geological Society).

Farrelly, C T and Dimitrakopoulos, R, 2001. Recoverable reserves and support effects in optimising open pit mine designs, *Int J Surface Min, Recl and Envir*, 16(3):217-229.

Fishburn, P C, 1977. Mean-risk analysis with risk associated with below-target returns, *Am Econ Rev*, 67(2):116-126.

Johnson, T B, 1968. Optimum open pit mine production scheduling, PhD thesis (unpublished), University of California, Berkeley.

Korobov, S, 1974. *Method for Determining Ultimate Open Pit Limits* (Department of Mineral Engineering, Ecole Polytechnique: Montreal).

Lemieux, M, 1979. Moving cone optimizing algorithm, *Computer Methods for the 80s*, pp 329-345 (SME of AIMMPE: New York).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open pit mines, *Bull CIM*, 58:47-54.

Norrena, K P and Deutsch, C V, 2001. Automatic determination of dig limits subject to geostatistical, economic and equipment constraints, in *Proceedings 2001 SME Annual Meeting.*

Richmond, A J, 2003. Financially efficient mining decisions incorporating grade uncertainty, PhD thesis (unpublished), Imperial College, London.

Richmond, A J and Beasley, J E, 2004. Financially efficient dig-line delineation incorporating equipment constraints and grade uncertainty, *Int J Surface Min, Recl and Envir*, 18(2):99-121.

Srivastava, R M, Hartzell, D R and Davis, B M, 1992. Enhanced metal recovery through improved grade control, in *Proceedings 23rd APCOM*, pp 243-249.

Whittle, J, 1989. *The Facts and Fallacies of Open Pit Optimisation*, (Whittle Programming Pty Ltd: Melbourne).

Hybrid Pits — Linking Conditional Simulation and Lerchs-Grossmann Through Set Theory

D Whittle[1] and A Bozorgebrahimi[2]

ABSTRACT

A technique that leverages some of the statistical properties of conditionally simulated models and the technical characteristics of Lerchs-Grossmann pits has been developed and tested. The result is not one pit but a nested set of pits, termed 'hybrid pits' in this paper, each of which has a definable statistical characteristic that ultimately reflects risk. In the application of the hybrid pit designs one further key element is introduced, and it is the propensity for knowledge to increase over time. At the time of planning, a certain amount of orebody data is available leading to an estimable degree of uncertainty in the model. As time passes and mining progresses, the amount of information increases (eg due to additional drilling), thus the degree of model uncertainty should decrease. This paper includes an explanation of how the hybrid set of pits can be used as design guides to allow a degree of risk avoidance, associated with the higher uncertainty in early times. The benefit of applying this methodology is a managed reduction in risk, contributing to higher project values.

INTRODUCTION

Applying the Lerchs-Grossmann (L-G) pit optimisation algorithm (Lerchs and Grossmann, 1965) to an orebody model means applying a process that will guarantee to find the optimal pit outline. This is the pit outline that maximises the dollar value for a given input orebody model and a given set of economic and geotechnical conditions. The L-G algorithm can only be applied to a single orebody model and cannot directly take account of uncertainty associated with that model.

Conditional simulation has emerged as a methodology to provide more meaningful models of orebodies, taking into account the uncertainty inherent in the sampling and interpolation process, and providing multiple representative models for any given set of data. The question that arose early in the development of conditional simulation is: how can the additional information that the process provides be used to better design mines? Van Brunt and Rossi (1997, 1999) describe the general nature of conditionally simulated models and their application in mine design and describe a construct and an analysis method that are both relied upon in this paper. Dimitrakopoulos, Farrelly and Godoy (2001) extend the application of the Van Brunt and Rossi (1999) efforts, including analysis techniques described in the context of a case involving 50 simulations of a gold deposit typically found in disseminated low-grade epithermal quartz breccia. Dimitrakopoulos (2003) also outlined a technique similar to one described by Van Brunt and Rossi (1999), but extended it to include the evaluation of each simulated model against each of the multiple L-G pits. The objective of this approach was to find an L-G pit that delivered the highest average dollar value when compared with the full set of simulations. Existing methods involve the generation of optimal pits, each for a separate model. The value of these methods revolves around the way in which the multiple optimal pits are generated and evaluated. In each case, the pit that is finally chosen will be an L-G pit that has been generated for a single model. That is, a pit that is optimal for a representative model such as a kriged model or E-type model described by Van Brunt and Rossi (1999), or a best performing pit as described by Dimitrakopoulos (2003).

Building on previous work, Dimitrakopoulos, Martinez, and Ramazan (2007, this volume) attempted to use the information gained about grade and reserve uncertainty in the model using conditional simulation to enhance pit optimisation with Whittle Four-X. They introduced a technique to measure the maximum upside and minimum downside of optimum pits for different simulated models, each an equally probable representation of the orebody. The technique quantifies the grade risk for the selected key project performance indicator, such as net present value (NPV). Then, it calculates the upside potential and downside risk for selected project indicators. Menabde *et al* (2007, this volume) developed a new method for simultaneous optimisation of the extraction sequence and cut-off grade policy for a set of conditionally simulated orebody realisations. The method uses a combination of integer mathematical programming and commercial software packages to produce the best possible expected NPV. A similar method is presented in Ramazan and Dimitrakopoulos (2007, this volume).

In early 2000, David Whittle worked on a methodology that led to the creation of pits that were directly influenced by a set of conditionally simulated models, and produced preliminary software specifications to enable the technique to be developed. The expectation at the time was that this methodology, called 'hybrid pits' in this paper, would provide a useful mechanism for relating the variance information inherent in conditionally simulated resource models into a reserve context. In the following sections, the theory of hybrid pits is presented leading into an examination of the issues associated with implementation of hybrid pits to conditionally simulated orebodies. A trial case study is presented next and conclusions follow.

HYBRID PITS

Conditional simulation produces multiple orebody models, each being an equally probable estimate of the real resource. It is possible to generate optimal pit outlines for each of these orebody models using the Lerchs-Grossmann algorithm (Lerchs and Grossmann, 1965). This means that the pit obeys the pit slope constraints as modelled by the structure arcs in the graph model in the L-G algorithm and that the dollar value of the pit is maximised for the inputs provided. The value of the pit can then be estimated by applying the whole family of simulated orebody models to the pit and recalculating its monetary value (Dimitrakopoulos, 2003). However, the optimality of the Lerchs-Grossmann pit relates to the individual orebody model used for the generation of the pit, rather than to the family of pits. In the creation of the shape of the pit, no account was taken of the family of possible orebody models representing the deposit.

The authors propose the use of hybrid pits that are derived from the family of pits, which are generated from the family of simulated orebody models. The hybrid pits derived from L-G pits are technically feasible and have specific probabilistic characteristics. In order to describe the derivation of the hybrid pits, it is necessary to establish certain principles, and set theory provides a useful framework for doing this.

1. MAusIMM, Mine Engineering Manager, Business Excellence, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: david.whittle@bhpbilliton.com

2. President, AnoMineTech Limited, Mining Engineering Services, 1602 - 1675 West Hastings Street, Vancouver BC V6B 1N2, Canada. Email: anoush@anominetech.com

Set theory model for hybrid pits

Let the Universal Set be the set of blocks in a block model framework.

Universal Set U – {x|x is a block in the model framework}

A {x|x is a block being a member of pit A, which is optimal for simulation *a*}

B {x|x is a block being a member of pit B, which is optimal for simulation *b*}

C {x|x is a block being a member of pit C, which is optimal for simulation *c*}

D {x|x is a block being a member of pit D, which is optimal for simulation *d*}, etc.

The sets defined above will be referred to as the 'o-sets' (original sets representing L-G pits for single simulations), to make the distinction between this type of set and other types of sets that will be discussed. In o-set A, for any block x, it can be said that the set includes all the blocks that must be mined if x is to be mined. This is true because the application of the Lerchs-Grossmann algorithm, through which the set was defined, requires it to be true. The same can be said for o-sets B, C, D etc. Let us call the set of all blocks that must be mined if x is mined X. It is true to say that for any given x, X is unique. There cannot be two sets of blocks which satisfy the condition of needing to be mined if x is mined.

Principle 1 – Intersections of o-sets represent feasible pits

An illustration of an intersection of three pits is shown in Figure 1. In the Set Theory model, this is the intersection of three o-sets. If block x is an element of o-sets A and B, then both A and B must include X (all the blocks which must be mined if x is mined). So, if x is an element of o-sets A and B, then X must be a subset of the intersection of A and B. This is true for all X sets, for all incidents of x that are members of both A and B. Accordingly $A \cap B$ (the intersection of o-sets A and B) will constitute a pit that can be mined, as it obeys the precedence rules of mining. The same can be said of $B \cap C$, $C \cap D$ etc. With the application of the associative law, the intersection of any combination of o-sets will lead to a set that represents a feasible pit.

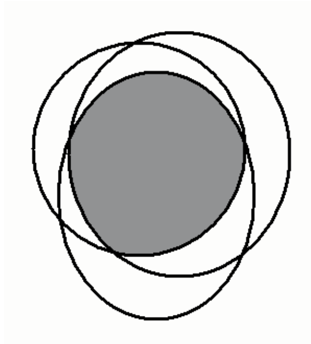


FIG 1 - An intersection of technically feasible pits will produce a technically feasible (hybrid) pit.

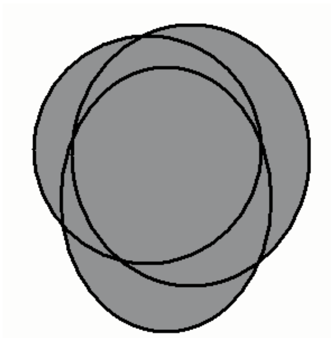


FIG 2 - A union of technically feasible pits will produce a technically feasible (hybrid) pit.

TABLE 1

Explanation of Principle 3 as it relates to four sets (A, B, C and D).

Symbols

$\cap$ 'Intersection'. The intersection of two sets, A and B ($A \cap B$), is the set of all elements that are common to both A and B.

$\cup$ 'Union'. The union of sets A and B ($A \cup B$), is the set of all elements that are members of set A and/or set B.

The set of blocks that are members of one or more o-sets

All elements of $A \cup B \cup C \cup D$ (the union of sets A, B, C and D) must be a member of one or more o-sets.

The set of blocks that are members of one or more o-sets is equal to $A \cup B \cup C \cup D$ which, in accordance with Principle 2 is a feasible pit.

The set of blocks that are members of two or more o-sets

If x is a member of two or more o-sets, then it must be a member of one or more of the following:

$$A \cap B \quad A \cap C \quad A \cap D \quad B \cap C \quad B \cap D \quad C \cap D$$

All members of one or more of the above sets are members of two or more o-sets. Accordingly, the set of all blocks that are members of two or more o-sets is equal to:

$$(A \cap B) \cup (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D) \cup (C \cap D)$$

With the application of both Principle 1 and 2, it follows that the set of all blocks that are members of two or more o-sets represents a feasible pit.

The set of blocks that are members of three or more o-sets

If x is a member of three or more o-sets, then it must be a member of one or more of the following:

$$A \cap B \cap C \quad A \cap B \cap D \quad A \cap C \cap D \quad B \cap C \cap D$$

All members of one or more of the above sets are members of three or more o-sets. Accordingly, the set of all blocks that are members of three or more sets is equal to:

$$(A \cap B \cap C) \cup (A \cap B \cap D) \cup (A \cap C \cap D) \cup (B \cap C \cap D)$$

With the application of both Principle 1 and 2, it follows that the set of all blocks that are members of three or more o-sets represents a feasible pit.

The set of blocks that are members of four sets (all sets)

If x is a member of four o-sets, then it must be a member of $A \cap B \cap C \cap D$. With the application of Principle 1, this represents a feasible pit.

Principle 2 – Unions of o-sets represent feasible pits

An illustration of a union of three pits is shown in Figure 2. In the Set Theory model, this is the union of three o-sets. A union of o-sets will constitute a feasible pit. For any block x, the o-set or o-sets to which it is a member must include X (all the blocks that must be mined if x is to be mined). The pit can only become not feasible if blocks are removed from an o-set, and determining the unions of sets will not lead to the removal of any blocks.

Principle 3 – The set of all blocks which are members of more than or equal to m o-sets (A, B, C, D etc) represents a feasible pit.

Principle 3 is explained fully for the case in which there are four (A, B, C and D) o-sets, but the logic is extendable to any number of o-sets. The explanation is included in Table 1.

Principle 4 – The set of blocks which are members of m or more o-sets, is a subset of the set of blocks which are members of m-1 or more o-sets.

Principle 4 is explained fully for the case in which there are four o-sets (A, B, C and D), but the logic is extendable to any number of o-sets. The explanation is included in Table 2. An illustration of a set of four o-sets is provided in Figure 3.

Hypothesis 1 – The set of blocks which are members of exactly *m* o-sets may be made up of spatially distinct subsets

Hypothesis 1 is explained fully for the case in which there are four o-sets (A, B, C and D) (refer to Figure 4). The numbers represent the number of o-sets to which blocks belong. The lowercase letters represent different spatial subsets delineated by the overlapping boundaries of the o-pits. The labels A, B, C and D have been removed for clarity. Regions *d*, *f*, *w* and *z* each include blocks which are members of only one o-pit. These regions do not overlap. There is no block in any subset (*d*, *f*, *w* or *z*) which is also a member of another subset. The subsets *e*, *j*, *y* and *t* each represent the intersection of two o-pits and subsets *g*, *v*, *u* and *h* each represent the intersection of three o-pits. They are all spatially distinct as are *d*, *f*, *w* or *z*. Any of the subsets described above may be empty sets.

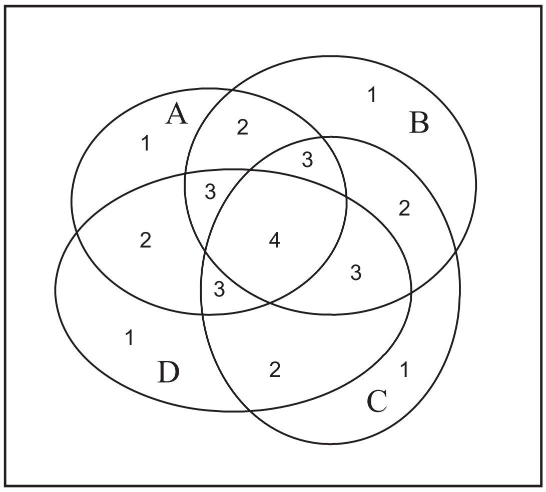


FIG 3 - An illustration of four overlapping o-sets (A, B, C and D). The numbers in the diagram indicate the number of o-sets to which blocks belong. The block outlines are not drawn.

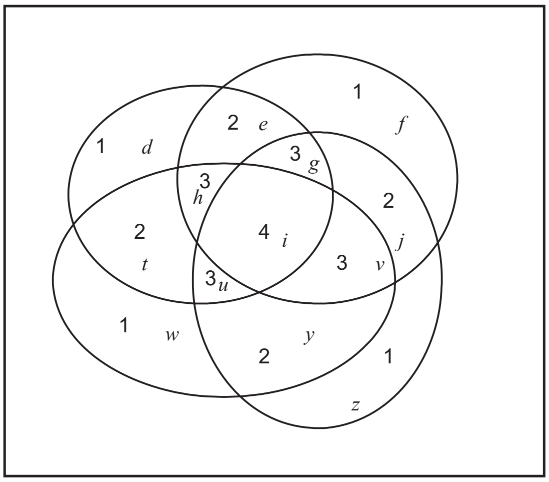


FIG 4 - Four o-sets. The numbers represent the number of o-sets to which blocks belong. The letters indicate different spatial regions delineated by the overlapping boundaries of the o-pits.

TABLE 2

Explanation of Principle 4 as it relates to four sets (A, B, C and D).

Additional symbols (Also refer also to Table 1 for the explanation of symbols) $\subset$ 'Is a subset to'. Indicates that the set on the right of the symbol contains (at least) all the elements that are members of the set on the left of the symbol. For example $G \subset H$ means that all elements in set G can also be found in set H. U 'The Universal Set'. This is the set of all elements that could be members of any of the sets under consideration. In the context of this discussion, the Universal Set is the set of all blocks in the block model.
The set of blocks that is a member of one or more o-sets, is a subset of the set of blocks that are members of zero or more o-sets. $(A \cup B \cup C \cup D) \subset U$
The set of blocks that is a member of two or more o-sets, is a subset of the set of blocks that are members of one or more o-sets. $(A \cap B) \cup (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D) \cup (C \cap D) \subset (A \cup B \cup C \cup D)$
The set of blocks that is a member of three or more o-sets, is a subset of the set of blocks that are members of two or more o-sets. $(A \cap B \cap C) \cup (A \cap B \cap D) \cup (A \cap C \cap D) \cup (B \cap C \cap D) \subset (A \cap B) \cup (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D) \cup (C \cap D)$
The set of blocks that is a member of four (all) o-sets, is a subset of the set of blocks that are members of three or more o-sets. $(A \cap B \cap C \cap D) \subset (A \cap B \cap C) \cup (A \cap B \cap D) \cup (A \cap C \cap D) \cup (B \cap C \cap D)$

Hypothesis 2 – Each of the spatially distinct subsets of order *m* can only be mined if adjacent subsets of order <*m* are also mined

By example: with reference to Figure 4, *i* must be mined before any of the other subsets are mined. Subset *u* may be mined if *j* is mined. The mining of *j* is not dependent on the mining of any of the other subsets of order three, as these are spatially distinct.

GENERAL BEHAVIOUR OF HYBRID PITS AS APPLIED TO CONDITIONALLY SIMULATED MODELS

The *real resource* is the actual mineralisation that exists in the ground, but which cannot be absolutely known by the modeller, as the modeller only has samples of it rather than absolute knowledge of it. Conditional simulation seeks to generate *n* equally probable models of the real resource, where *n* is sufficiently large that the full set of simulations is representative of the whole population of possible models. The models are equally probable representations of the real resource, which at the time of modelling is not absolutely known; it is only known through the samples of it. One of the conditionally simulated models will be the *most representative* of the real resource, but it is not known which model this is. If *n* is sufficiently large then it is highly probable that at least one of the models will be sufficiently representative of the real resource, such that in the process of mining the real resource should behave, for all economic and operational purposes, exactly like that model. The chance of any of the models being the most representative of the real resource is 1/*n*.

Pit optimisation generates sets of blocks which represent the reserve and necessary stripping, such that the dollar value of the pit is maximised. If an optimal pit is generated for each of *n* models, and if *n* is sufficiently large then any one of these pit optimisations has a 1/*n* chance of being optimal for the *real resource.* If a pit is suboptimal for the real resource then it must include material that should not be mined, or it must not include material that should be mined, or both. In other words, if the real resource was absolutely known, and the pit optimisation proceeded on the basis of a precise model of it, the pit would be different from, and have a higher value than any of the suboptimal pits.

With the application of Principle 2 (established above), and if *n* is sufficiently large, the union of all o-sets is a feasible pit which is a superset that will contain the optimal pit for the real resource.

With the application of Principle 4 (established above) the set of blocks for $m \geq 1, m \geq 2, m \geq 3, ..., m \geq n$, are progressively more likely to be subsets of the optimal outline of the real resource. Principle 4 also establishes that the pits represented by the set of blocks for $m \geq 1, m \geq 2, m \geq 3, ..., m \geq n$ progressively nest (that is, they are each supersets containing the next set).

PROJECTED APPLICATION OF HYBRID PITS

With reference to the above discussion, a hybrid pit will now be defined. **H-Pit(*m*)** is the set of all blocks which are members of *m* or more o-sets.

Outer bound pit – H-pit(1)

It is possible to produce a pit outline that is feasible, by finding all blocks that are members of any of the original pits. This is referred to as H-Pit(1), meaning that it includes all blocks which are members of one or more o-pits. If *n* is sufficiently large this pit will *almost certainly* include the pit outline that would be optimal for the real resource. Such a pit provides an outer bound for the optimal outline.

Inner bound pit – H-pit(n)

It is possible to produce a pit outline which is feasible by finding all blocks that are members of all of the original pits. This is referred to as H-Pit(*n*), meaning that it includes all blocks which are members of all o-pits. If *n* is sufficiently large, the resulting outline will almost certainly be a subset of the optimal outline for the real resource, and be associated with high confidence that H-Pit(*n*) will not exceed the boundaries of the optimal outline for the real resource. Such a pit provides an inner bound for the optimal outline for the real resource.

High-confidence reserve pit

The inner bound pit H-Pit(*n*) may be used as a type of high-confidence reserve pit; that is, this pit is unlikely to over-mine the real resource. By the time the high-confidence reserve pit is mined, better geological data will be available for the remaining resource, providing a much better position for determining the direction in which to expand the pit. This is an example of the way in which the hybrid pits technique can be used as a design guide to allow a degree of avoidance of the risk associated with the higher uncertainty in early times.

An estimator of the impact of geological uncertainty

One of the impacts of geological uncertainty is that it leads to uncertainty as to the shape and size of the final pit. Application of the hybrid pit approach provides high confidence that the optimal pit for the real resource will be both a superset containing H-Pit(*1*) and a subset of H-Pit(*n*). The area bounded by H-Pit(*1*) and H-Pit(*n*) represents the area in which the optimal outline for the real resource can exist. If the area is large, it indicates that the orebody model variance (as expressed in the simulations) leads to a high degree of uncertainty as to the position of the optimal pit boundary. If the area is small, it indicates that there is a low degree of uncertainty as to the position of the optimal pit boundary.

The set of blocks for, $m \geq 1$, $m \geq 2$, $m \geq 2$, $m \geq 3$, …, $m \geq n$, are progressively more likely to be subsets of the optimal outline of the real resource. It is envisaged that this model will be applied in the future to determine pit shapes that exhibit a known and acceptable compromise between certainty (risk reduction) and size (reserve maximisation).

INITIAL TRIAL OF HYBRID PITS ON A CONDITIONALLY SIMULATED MODEL

Methodology of the trial

In order to assess the theory of hybrid pits, a block model related to a gold deposit was selected. Based on this original model, five separate block models were created using a simulation program. Although five simulations would not be adequate for normal modelling purposes, this number is considered adequate for the purposes of experimenting and testing the mathematical propositions put forward in this paper. Each of the five simulated models has a chance of representing a possible real resource. In the next step, optimisation analysis was performed on the simulated models using Whittle software. In order to be consistent in optimisation, the same parameters were applied in this analysis. The optimised pit shells were then exported to GEMS software, where they were used to modify new attributes within the block models. The values of these attributes were then exported into spreadsheets with their block numbers. Using the theory explained above, a pit-list file for hybrid pits was created. This pit list was then imported into the Whittle program, where a cash flow analysis was performed for each simulated model and the hybrid-pits. Figure 5 shows the procedures of this experiment.

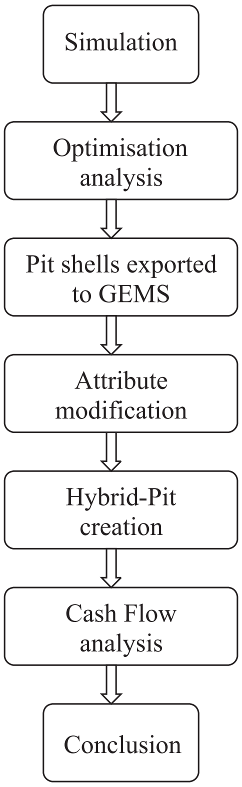


FIG 5 - Procedure of the experiment.

Simulation

Due to the lack of a real variance distribution model within the block model selected, the following methodology was applied to create a simulated block model. Using a random generator, a normal distribution was applied to the original block model with the following parameters:

- $\mu = Original\_Grade$
- $\sigma = 0.3 \times Original\_Grade$

Using this method, five different simulated block models were created. The grades were controlled in order to avoid entering into the negative territory. All simulated models have the same origin and block size.

Optimisation analysis

Optimisation analysis was performed on the models using Whittle 3.2. The same parameters were applied in each analysis. Table 3 shows the parameters that were used for this analysis.

TABLE 3

Parameters used in optimisation analysis.

Parameter	Value
Block size	10*10*8
Number of blocks in each model	63 296
Gold price	380 $/oz
Mining cost	1 $/tonne
Processing cost	22 $/tonne
Mining recovery	95%
Processing recovery	95%

Based on this analysis, five optimal pits were extracted into GEMS. Using GEMS, some new attributes were defined and then modified by these optimal pits. These attributes actually contained a value of 1 or 0 that defined whether the related cell is part of the pit or not.

Hybrid pit creation and technical feasibility

Calculations were done on the attribute explained above to create hybrid pits. The mathematics indicated that the hybrid pits would be technically feasible; that is they would nest and they would not violate pit slope constraints. The trial supported this. The test for nesting was performed by a visual inspection of the pits bench by bench, and section by section. It was possible to verify that there was no violation of the nesting rule by this method.

The test to determine whether any pit slope constraints were violated by the hybrid pits was performed with a modified version of the Whittle Mining Width module. The module reapplies the pit slope constraint to each shell and changes block allocation in the event that a pit slope constraint is violated. By running the hybrid pits through this module it was possible to verify that no pit slope constraints were violated.

Results of the trial

The five original optimal pits and the five hybrid pits were all evaluated against the five simulated models, giving a total of fifty evaluations. The results are summarised in Table 4. In the table, Pit 1 is the pit which is optimal for Simulation 1; Pit 2 is the pit which is optimal for Simulation 2; and so on. As expected, for each of the simulations, the pit which performs best is the corresponding original optimal pit. H-Pit(5) is the inner bound pit. This can be used as a high-confidence reserve pit, that is, it is highly likely that the optimal pit for the real resource will be a superset containing H-Pit(5). H-Pit(1) is the outer bound pit. It is highly likely that the optimal pit for the real reserve does not extend beyond the perimeter of H-Pit(1). The section above titled 'Projected application of hybrid pits' includes a discussion of the

TABLE 4

Summary results of the Hybrid Pit trial.

	Pit 1	Pit 2	Pit 3	Pit 4	Pit 5	H-Pit(5)	H-Pit(4)	H-Pit(3)	H-Pit(2)	H-Pit(1)
mT	19.5	20.5	19.3	20.3	20.3	18.2	19.6	20.2	20.6	21.3
$'000s	Pit 1	Pit 2	Pit 3	Pit 4	Pit 5	H-Pit(5)	H-Pit(4)	H-Pit(3)	H-Pit(2)	H-Pit(1)
Sim 1	**27 098**	25 943	25 770	25 911	25 928	25 157	25 951	26 479	**26 578**	26 486
Sim 2	25 294	**26 589**	25 091	25 387	25 291	24 106	25 213	25 955	26 164	**26 214**
Sim 3	24 702	24 972	**26 218**	24 851	25 088	24 219	24 872	25 565	**25 689**	25 487
Sim 4	25 756	26 054	25 973	**27 365**	25 669	24 707	25 895	26 602	26 756	**26 857**
Sim 5	23 509	23 871	23 666	23 884	**25 240**	22 691	23 570	24 386	24 745	**24 766**
Average	25 272	25 486	25 344	25 480	25 443	24 176	25 100	25 797	**25 986**	25 962

intended interpretation and application of these results. Figure 6 shows the spatial relationship between H-Pit(1) and H-Pit(5) in an elevation view. As can be seen, the effect of the variance in the model (as expressed in the five simulations) leads to uncertainty as to the position of the pit wall on the right hand side of the diagram, as well as some minor uncertainty as to the pit wall position on the left hand side.

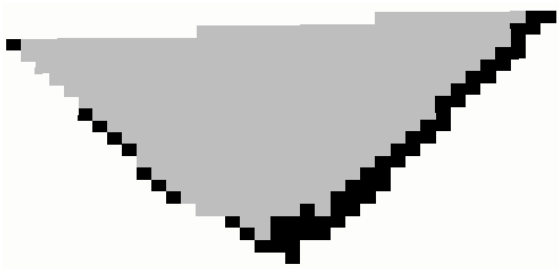


FIG 6 - Section showing the spatial relationship between H-Pit(5) and H-Pit(1).

Table 4 provides the average value of each o-pit (Pits 1 through 5) as evaluated against the five simulated models. With the application of the Dimitrakopoulos (2003) technique, Pit 2 could be chosen as a design pit, as it provides the highest mean value (meaning that it is the most likely to return the highest value) of all the o-pits. Table 4 also provides the average value of each of the hybrid pits as evaluated against the five simulated models. In this trial H-Pit(2) produced the highest average value. It is interesting that, evaluated in this way, an H-pit outperforms all of the o-pits in this trial. However, it is not a central aspect of this paper, and no attempt has been made to determine theoretically whether this may always be the case.

CONCLUSIONS

Considering uncertainties presented in any geological model, this paper has shown that the hybrid pits technique leads to the creation of pit outlines with quantifiable probability characteristics, with respect to their spatial relationship to the optimal pit for the real resource. The hybrid pits can be used as design guides to allow a quantifiable degree of risk avoidance, associated with the higher uncertainty in early stages of a mine development.

The validity of the hybrid pits technique is supported by the set theory model explained in this paper, and by the trial of the technique on a small data set. The pits were found to obey the pit slope constraints, thus are technically feasible. The pits were also found to nest, which is an inherent quality of the pits if they are to conform to the theory presented in this paper.

The trial produced hybrid pits that were quite similar in terms of overall size. It is not known, on the basis of this sample of one, whether this is common, or whether it is more common for the gap between H-Pit(n) and H-Pit(1) to be great. The size of the gap will certainly be a function of the variance of the models, but it will also depend on a great many other economic, geotechnical and geological factors.

The operations required to complete the trial were found to be relatively straightforward, though large in number. For this very small trial, five pit optimisations were performed, and fifty life of mine schedules, with a good deal of associated data manipulation in Whittle, GEMS and Excel. To repeat the same exercise for a family of 25 simulated models would require only 25 pit optimisations, but 1250 economic evaluations.

Generally, the trial produced results that were in line with expectations. One pleasant surprise was finding that one of the hybrid pits outperformed all the original optimal pits when evaluated against the family of simulated models. Prior to the trial, it had not been possible to form a hypothesis as to whether hybrid pits would outperform original optimal pits in this manner. However, in this one case, it was found to be so. Only experience will tell whether it is a common or an uncommon outcome.

ACKNOWLEDGEMENTS

Thanks are in order to Dr Ing Leszek Jurdziak of the Instytut Górnictwa, Politechnika Wroclawska, Poland, for his insights on an earlier version of this paper. His comments led to the improvement of this publication.

REFERENCES

Dimitrakopoulos, R, 2003. Personal communication, November.

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2001. I'd rather be approximately right than precisely wrong: grade uncertainty, risk effects and decision making in open pit design, *in Proceedings Strategic Mine Planning Conference*, pp 35-42 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopoulos, R, Martinez, L and Ramazan, S, 2007. Optimising open pit design with simulated orebodies and Whittle Four-X — A maximum upside/minimum downside approach, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 201-206 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Lerchs, H and Grossmann, I F, 1965. Optimum design of open pit mines, *The Canadian Mining and Metallurgical Bulletin,* 58(January):47-54.

Menabde, M, Froyland, G, Stone, P and Yeates, G A, 2007. Mining schedule optimisation for conditionally simulated orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 379-383 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Van Brunt, B H and Rossi, M E, 1997. Optimizing conditionally simulated orebodies with Whittle 4D, in *Proceedings Optimising with Whittle Conference,* pp 119-128 (Whittle Programming: Melbourne).

Van Brunt, B H and Rossi, M E, 1999. Mine planning under uncertainty constraints, in *Proceedings Optimising with Whittle: Strategic Mine Planning Conference*, pp 181-196 (Whittle Programming: Melbourne).

Global Asset Optimisation

G Whittle[1]

ABSTRACT

Although there are now many tools and techniques available for optimising various parts of the mining and processing stream in isolation, so far an integrated approach that simultaneously addresses the various components has not been available.

In the last four years Jeff Whittle has focused on expanding the boundaries of integrated optimisation for the resource industry. The result is an approach that applies business and operational modelling techniques to construct integrated geological, mining, processing, transport and market models, which are then optimised by allowing powerful optimisation algorithms to control the values of those variables that are considered negotiable.

Confidently referred to as 'global optimisation', due to the number of variables that are simultaneously controlled, the result is a powerful business tool that can be used as a platform to support strategic decision-making at many levels.

In this paper, the author outlines a variety of modelling techniques applied during recent projects, the optimisation mathematics employed and the typical characteristics of a 'globally optimised' business plan.

INTRODUCTION

There are now many tools (distributed by the mining software vendors or resulting from mining company's internal developments) and techniques for optimising various parts of the mining and processing stream in isolation. However, the last frontier is to make it all happen simultaneously.

In the last four years, Jeff Whittle (Whittle, 1999) has focused on expanding the boundaries of integrated optimisation, concentrating on the issues faced by large and complex mining and processing operations. By using advanced business modelling and analytical techniques, an integrated geological, mining, processing, transport and market model can be constructed, which is then manipulated mathematically to optimise the values of those variables that are considered negotiable. Utilising this procedure, it is possible to develop long-term plans that maximise the value of large geological and technical plant asset portfolios. As such the approach is a powerful business tool, which can be used as a platform to support strategic decision-making at many levels.

Not every part of a mining/processing operation can yet be simultaneously optimised, but the following work is confidently referred to as Global Asset Optimisation due to the increasing range of variables and the scope of assets that are considered together.

In this paper, the author outlines a variety of modelling techniques applied during recent projects, the optimisation mathematics employed and the typical characteristics of a globally optimised business plan.

THE NATURE OF THE PROBLEM

Global Asset Optimisation addresses the issues raised in mining and processing operations with multiple pits/mining faces/ underground mines, multiple elements, stockpiling opportunities, blending issues and alternative processing and/or product options. The combination of these dimensions creates significant long-term planning and analytical challenges that often exceed the capabilities of readily available mining optimisation tools.

1. Managing Director, Whittle Consulting Pty Ltd, Suite 13, 333 Canterbury Road, Canterbury Vic 3126, Australia. Email: gerald@whittleconsulting.com.au

The factors that make mine planning more complex than other business planning challenges are:

1. *The link between time periods*. An orebody being mined is a depleting resource. When we decide what to mine and process in one period, we determine the starting surface for the next period, and therefore we limit the options of how to operate it.

   This inescapable link between time periods creates the need to determine an integrated chain of events, which results in a chosen path through the orebody with all the associated capital and operating decisions involved. Two different plans might ultimately mine and process the same tonnes and grades of ore and result in the same overall production, total revenues and costs. However, the order and timing of these activities and cash flows can make one plan far superior to all others in terms of financial viability and performance.
2. *Blending*. In many circumstances, individual parcels of material cannot be evaluated in isolation. Their value will depend on what other parcels are available in the orebody, and the timing of such availability. The blending possibility creates extensive mathematical permutations and interdependencies between the variables, significantly complicating the optimisation mathematics.
3. *Stockpiling*. Flexibility is created (at a cost) when it is possible to separate the time at which an ore parcel is mined, which might be driven by the parcels that surround it, and when it is used. Stockpiling creates more mathematical permutations to consider and complicates the links between time periods.
4. *Alternatives*. If material can be used or not, or used in different ways, more options and flexibilities are created, and once again more mathematical permutations to consider.
5. *Variation and uncertainty.* Nature dictates that grades and physical characteristics are distributed with little consistency within an orebody. This often defies our attempts to categorise, describe simply and predict. With less than complete information we are forced to make approximations as to what material there is and how it will perform when mined, handled and processed. The inaccuracies and risks that arise from this must be understood and the resulting consequences carefully managed.

The aim of the modelling phase is to capture the details of the geological, mining process, mineral processing and market alternatives, using particular modelling techniques. The result is effectively an integrated business model that embodies the existing knowledge on geological, engineering, metallurgical and financial issues. This model is then controlled by a powerful mathematical optimiser that can handle the nature and scale of the system defined.

MODELLING METHODOLOGY

The focus is on strategic scheduling. Every situation is different and, although the modelling techniques outlined below have all been applied in more than one situation, the procedure cannot yet be described as 'generalised'.

Let us envisage a Global Asset Optimisation exercise for a situation involving several deposits, several processing options and alternative products.

Pit shell optimisation

When preparing for a Global Asset Optimisation, conventional techniques are used to determine pit shells (eg using Whittle Four-X software) for each deposit. In this process it is necessary to take a single and initially isolated view on the definition of ore, and how it will be treated. It is necessary to make assumptions about what material will qualify for the blend, via what processing method it will travel, and which product the ore will ultimately report to. Pit shell optimisation is a static piece of analysis in that no attempt is made to determine *when* a block of material will be mined, so it is not possible to consider the fact that prices, costs, capacities and recoveries may change over time. These factors can only be considered during schedule optimisation.

It is not actually necessary to determine the ultimate pit shell with any degree of certainty at this stage. This statement may seem ironic as determining the ultimate pit has been the hallmark of Whittle methodology. In a Group Asset Optimisation, however, the ultimate pit for a particular deposit will be influenced by factors outside the deposit itself, and can therefore only be confirmed during detailed schedule optimisation of the total system.

The approach is, therefore, to develop nested pit shells that are efficient in terms of:

1. stripping ratio; and
2. prioritising ore based on its value, given its expected most likely outcome.

The schedule optimisation of the total system will at some point run out, or reach a break-even point, or a point of inadequate cash flow or returns. What has been mined at this point is therefore deemed to be the ultimate pit, and this will change as assumptions in the overall scenario are modified. The ability of a deposit to participate in a group schedule will determine its timing, rate and ultimate size.

Pit shell design is by no means a perfect procedure in the context of the Global Asset Optimisation, but we need to start somewhere. Once a round of schedule optimisation has been performed, a different view of what the most likely outcome for different geological materials may develop, in which case another iteration of the pit shell optimisations may be warranted.

It is implicit in a pit that phases can be mined consecutively or concurrently, subject to the rule that an outer phase cannot overtake an inner phase in descending at any point of time. Details of any required minimum/maximum lead/lags, earliest start dates, start-after rules, alternative mining methods, tonnage rates limits, vertical rate limits, costs, dilutions, etc must also be considered.

Underground mine design

The Global Asset Optimisation does not attempt to get inside and control the specific mining activities within an underground mining area or 'block'. In a Global Asset Optimisation an underground block will be one of the components of the overall system involving many other underground blocks and/or pits.

An existing local schedule is taken for an underground mining block, which typically involves upfront capital development and time, maintenance of access and ventilation during ore mining and periods of interspersed backfilling. This schedule is summarised as a quarterly (ie three-monthly) schedule, of costs and tonnes/grades of ore produced. A quarter of underground mining activity is, in a mathematical sense, no different than a bench in a pit when it comes to scheduling, in that it represents an inventory of ore that can be obtained in a certain sequence, at a certain cost and rate.

The Global Asset Optimisation schedule will determine when and how a particular underground mining block will feature in the master plan, by considering how it relates to all other sources of material in the total system. As with pit shells, once a round of scheduling has been performed, this may present some feedback with which to re-do the specific design and local internal schedule of a particular underground block, to enable it to fit better into the big picture.

Pit geology

In the case of pits, geological blocks are consolidated into 'grade bands' within a bench of a phase/pushback (see Figure 1). Grade banding techniques are designed to summarise ore data, but maintain a relevant segregation of mineralogy, and either a

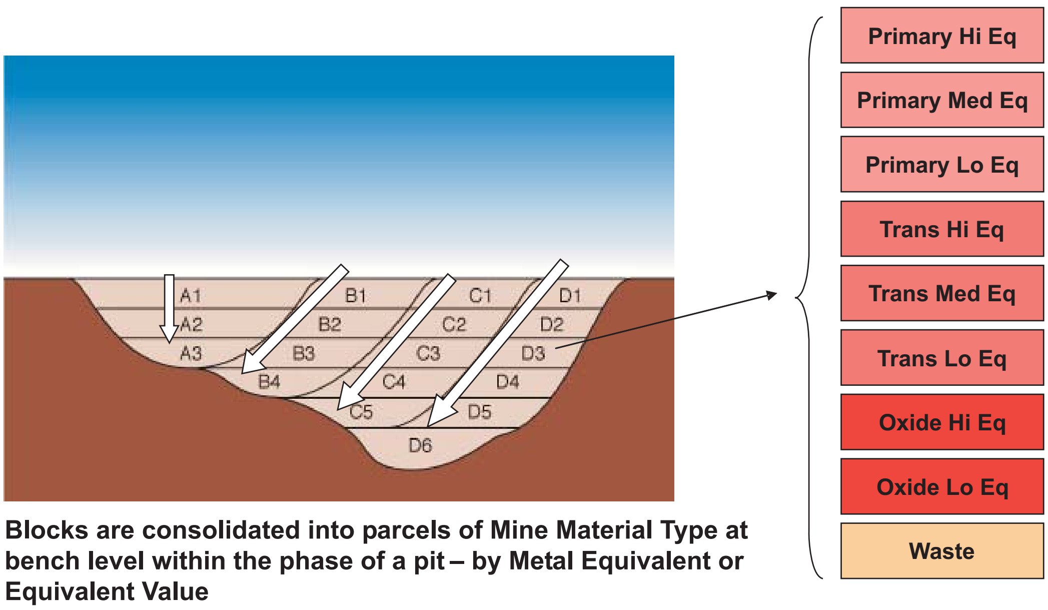


FIG 1 - Mining by phase and bench.

matrix of relevant ore grades/attributes or a ranking of ore based on an equivalent metal or net-value calculation can be used. It is typical to work with between eight and 20 grade bands, depending on what is relevant for cut-off grade, stockpiling or blending, and the actual operational grade control capability that exists. A geological model with many millions of block records will therefore reduce to a grade-banded database of several thousand records. There may be several deposits in the system being modelled and the grade banding approach is likely to differ for each.

The level of consolidation chosen for grade banding influences the resolution at which ore/waste can be defined, stockpiling versus immediate processing, and processing path selection will be made (see Figure 2).

It is likely that a particular band of Mined Material will report to different destinations in different periods, as the decision will be influenced by what else is going on in the global optimisation at the time a parcel is mined.

Grade banding is a subject in its own right and is the key to significant value in the schedule optimisation, by facilitating appropriate decisions on cut-off grade, stockpiling, processing path selection, blending and product mix.

Grade banding is important for the scheduling of pits in a system and could theoretically be applied to the ore generated by an underground mine. Our experience is, however, that underground operations by nature focus on premium high-grade ore of one type and do not generate the wide spectrum of grades and ore types that pits tend to, so banding is less relevant.

Ore processing

A Processing Summary model is developed, which captures the cost, throughput and recovery relationships for each type of ore and each of its potential processing paths. This summary will cover between say three and 50 channels, and allows us to capture in great detail the metallurgical sensitivities. There will be separate channels for each plant and for different groups of ore types if they have different costs, throughput or recovery in that plant. Different channels can be created for the same plant operating in different modes.

Non-linear expressions, multi-stage paths, recycle loops, etc can all be accommodated. Processing models have been developed to cover mills, concentrators, acid leach, smelters, refineries and to include consideration of mineralogy, grades, blending limits, synergy from blending, hardness, sizing, SG, density, viscosity, rejects, by-products, intermediate stockpiles, additives, consumables, maintenance, sustaining capital, shutdowns, purchase/sales of intermediates, etc with changing capacities, availability and performance over time.

In a group asset situation, it is typical for some ore types to be eligible for more than one processing method. These methods may change in availability, capacity, cost and performance over time – all of which will be captured. Rules, which are applied as filters based on one or more characteristics of the ore, will be formulated to define what categories of Mined Material can go through each processing path, and what will happen when it does. At this stage we are just capturing all the alternatives, not attempting to determine what makes sense or what is best under what circumstances – the optimiser will do that. We are not even presuming that material will be processed by one of these paths; the optimiser may choose to discard it.

Processing turns Mined Material into one or more ‘Blend Feeds’ (see Figure 3), which may simply be rock, lump and fines, slurries, concentrates, rejects, by-products, or even fully extracted metal – depending on the operation and how we have chosen to model it. Different processing paths may produce the same Blend Feeds (perhaps with different qualities, quantities, or cost) or totally different ones. Blend Feeds are not necessarily the finished product, but they are available for further use in the system.

Blend Feeds can be allowed to be stockpiled, allowed to be discarded, or forced to be used.

Blending to products and market

Blending may simply be the adding together of the available Blend Feeds, with or without set criteria on the characteristics of the combined product. Alternatively, it may involve more complex stages of extraction such as leaching, smelting, refining, or combinations of all of these.

‘Blending’ is the concept of being able to determine the required criteria of the resulting product, which may involve the combination of Blend Feeds (ore, concentrates, etc) with attributes or characteristics that are complementary. In building a blend, it is likely that many of the components that participate would not qualify alone. Blending is a very powerful mechanism, which represents a significant opportunity but also a significant challenge to plan and optimise. Just as Mined Materials can have alternative Processing Paths, so too can Blend Feeds have alternative Blending Paths and/or more than one product destination (see Figure 4).

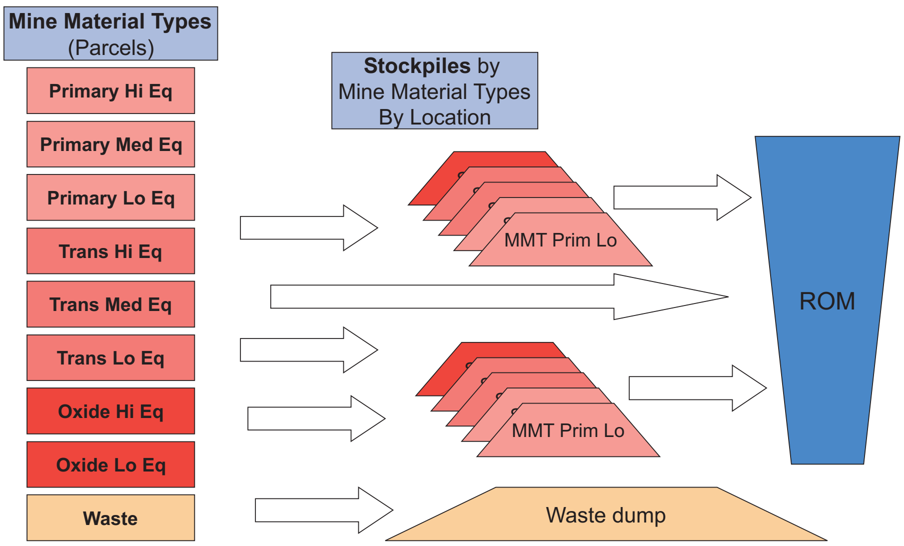


FIG 2 - Waste/stockpile/process.

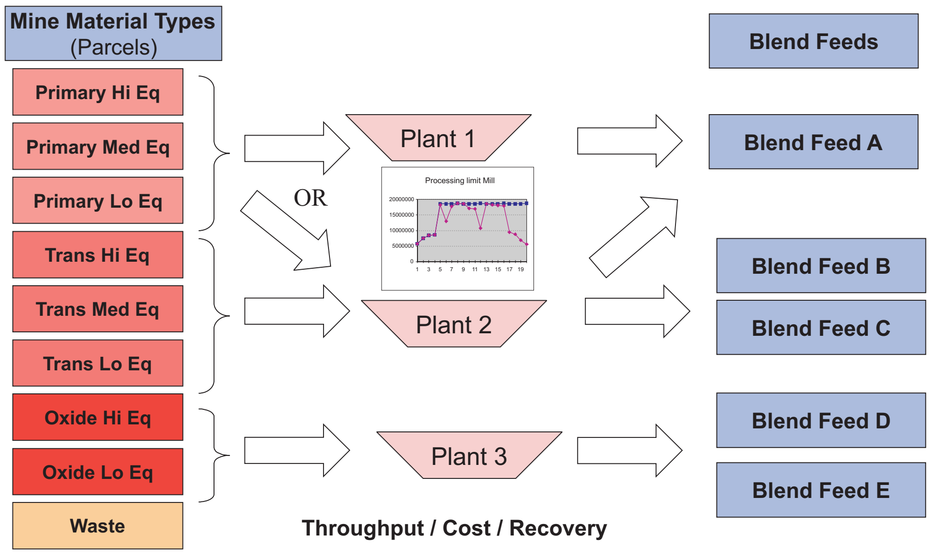


FIG 3 - Alternative processing paths.

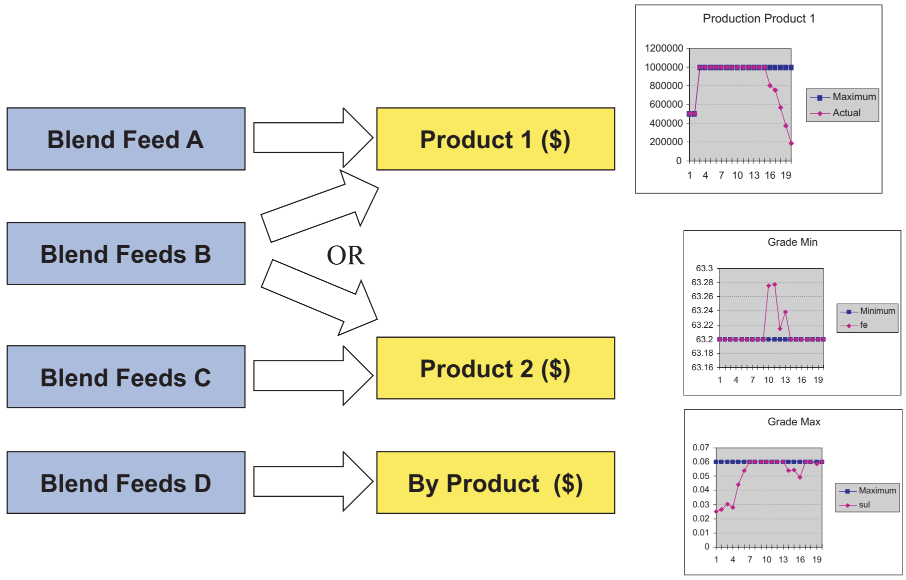


FIG 4 - Multiple products.

Product attributes determine the blending criteria and may be defined as strict limits (upper, lower or both) on particular grades or other attributes of the final blend, or with flexibilities, with or without penalties and rewards for variations. The valuation of the final product may involve constant or changing prices, exchange rates, royalties, transport and further treatment/refining allowances.

The model therefore contains all the material under consideration, the rules by which it can be accessed, and the details of all the options by which it could be treated and blended. No attempt is made to draw conclusions about the solution, only to comprehensively lay out all the possibilities.

CONSTRUCTING THE MODEL

No model is perfect – but some models are useful

Although it sounds complicated, the methodology is well developed, and mining companies usually have sufficient existing information and knowledge within the organisation to consolidate into a comprehensive model of this kind. Information is never perfect, so it is a matter of making the best use of the information that is available and understanding its risks and weaknesses.

Increasing numbers of templates are being developed that help deal with a range of situations/challenges without having to revert to research or problem-solving mode. Upfront discussion on appropriate scope and level of detail is important to ensure the focus is kept on the material and relevant issues. The capacity of the optimiser, and indeed our/your mental ability to deal with complexity, is large, but not unlimited – nor are budgets for this type of work. Choices have to be made that involve judgement on where to focus and to what level of detail.

OPTIMISATION

Modelling is not rocket science, the optimisation of such a model is

A model of any scale with these mechanisms can exceed the capacity of conventional mathematical optimisation tools, including linear programming and the various mining and scheduling optimisation software packages available.

At this point, specialised proprietary mathematical procedures are required to control the variables in the model that are considered negotiable, in order to maximise the objective function of net present value (NPV). Development of such an optimisation capability and the limits and flexibilities these provide on various modelling techniques, has been the subject of Jeff Whittle's research and development program for the last four years. The result has been two optimisers referred to internally as 'Z3' and 'Prober'.

From the outset we have used adaptations of the 'Z3' optimiser. Z3 is the combination of a mathematical search algorithm that works in conjunction with a linear programming evaluation routine. The search algorithm samples the feasible domain of alternative life-of-mine mining plans; the evaluation routine determines the optimal cut-off grade, stockpiling, processing selection, and blending and production plan that the specified mining plan can support, and determines the NPV. Based on the NPV fed back by the evaluation routine, the search algorithm applies complex decision rules to focus on combinations of mining variables giving good results, and discarding combinations showing poor results. The Z3 optimisation procedure is well developed and is used to routinely optimise models with several thousand mining variables, and many more processing variables.

The recently announced development of the 'Prober' optimiser accelerates and increases the capacity of this type of optimisation. It uses a random sampling and local optimisation approach, which is faster than but not yet as comprehensive as Z3, (but rapidly developing). As the processing of a sample is relatively fast, hundreds of samples can be optimised. When there are many results within a small tolerance in terms of NPV, we are confident that the overall optimal result has been located. This gives the capacity to handle larger and more complex models, with faster and more consistent results.

Optimisation of a comprehensive model can take between half a day and several days to process using the latest PCs. An advantage of Prober over Z3 is that each sample is independent, so the program can be run across many PCs in parallel to get a result within a shorter elapsed time.

The models being optimised are large and complex.

> *The breakthrough in optimisation has been to set out to 'find' the overall optimal answer using a search algorithm, rather than to try to formulate the problem and 'calculate' the answer.*

This philosophy is common to both Z3 and Prober methodologies.

WHAT A GLOBAL ASSET OPTIMISATION DOES

This approach involves the construction of a detailed business model. This would in itself be a useful exercise, because the model could be used to perform consolidations of different strategies and test the merits of different scenarios on a trial and error basis. Combined with an optimisation capability, however, such a model finds its own best configuration with an apparent intelligence that cannot be achieved by humans. This makes it a very powerful analytical and business-planning tool.

By using a Global Asset Optimisation model to assist the planning process, an integrated business plan can be developed, which combines and links the geological, operational and economic dimensions. The mining schedule will respond to the detailed options, opportunities to earn value-in-use and sensitivities within the various streams mined material could take to get the metal in the orebody to market. It can be considered that the various ore parcels have to compete for space in the (limited) processing streams that they are eligible for, and in the interests of increasing the value of the total system not all of them will get their first preference.

Within a run, the optimiser will make precise trade-offs and determine simultaneously the following:

- mining schedule: where and at what rate to mine;
- cut-off grade: what to discard, stockpile or process;
- stockpiling recovery;
- processing path selection;
- blending and product destination; and
- production quantity, mix and timing,

whilst considering the consequences on all periods (which are inextricably linked), using discounted cash flow as the measurement.

By iteration, questions of capital scale and timing, operational configuration and the impact of market scenarios can be addressed.

SOME EXAMPLES OF GLOBAL ASSET OPTIMISATION MODELS

The decision to proceed with the creation of a Global Asset Optimisation model has been prompted by a range of situations:

1. a desire to look for the next level of value in an asset portfolio, having already optimised all the components individually;
2. a new project that has a range of options in terms of scale and configuration, with too many permutations to consider using manual techniques; and
3. an existing operation that is contemplating expansions or changes to its geological or technical asset base, or is experiencing changes in technical performance or market factors of which it wants to fully understand the implications and opportunities.

Although the initial construction of a complex model typically involves several man weeks of work, once completed it is generally the quickest and surest way of evaluating a range of scenarios, sensitivities and business issues.

In some cases the Global Asset Optimisation model is the only consolidated technical expression of the group's activities, and can serve as a medium for communication between the different functions and across divisions in a large organisation.

Some Global Asset Optimisation models constructed have had the following dimensions:

1. An iron ore operation with a large central pit with a dozen phases and several surrounding satellite pits. Selective beneficiation helped achieve a range of products with strict blending criteria. Transport capacity expansions were foreseen.
2. A multi-seam truck-shovel coal operation, wishing to expand its product range and output, concerned about the timing and rate of commencing operations in adjacent orebodies.
3. A base metal producer, with 30-plus existing/foreseen underground and open pit operations in separate divisions, contemplating a major pit development that would yield a combination of metals affecting the currently independent production streams and involving both the closing down and development of new technical infrastructure.
4. A nickel/cobalt producer with over 100 potential pit sites, wishing to optimise the mining/cut-off/stockpile strategy to maximise the value throughput of their extensive ore-processing/metal-extraction plant investment.

5. A metal business unit with several pit and underground mines, faced with timing and capacity decisions for existing mine and plant expansions and introduction of new processing technologies to suit the changing mix or ore types.

In each case the model-building project and the model itself has been tailored to meet the business needs of the subject organisation. Model construction, validation and interpretation of results have therefore involved a cross-section of participants within the organisation (Whittle, 2001).

SOME TYPICAL RESULTS

Every situation is different, but some examples of the characteristics of a globally optimised group of assets include:

1. Pit phases and underground blocks (ie mining sequences) tend to have negative cash flows (waste stripping or capital development) in front of positive cash flows (from rich ore). The optimiser is NPV-driven, so it will wait until it is justified to incur the negative cash flow to start a phase, but, having done so, it then will mine the sequence at the maximum rate in order to get the best value. This compresses the negative and positive cash flows in time, to maximise the NPV. This behaviour is both logical and convenient, as it minimises the number of active locations and means that local operations are performed at their foreseen rates.
2. The above may not occur for a sequence that is contributing a key characteristic to a blend, in which case it may trickle in over a long period of time to compensate as required for characteristics of ore from other sources.
3. Although there may be dozens of constraints built in to the model (mining, processing, blending), the system is likely to be limited by two or three of these at any one time. The active set of constraints will overlap and change dramatically over time. For example, a system may be mining limited in early years due to waste stripping; refinery limited once the high-grade ore is accessed; mill throughput limited once the highest grade ore is depleted; and grade blending limited when only poor ore with excessive contaminants are left. The introduction of a new orebody or plant expansion during the time frame can shift the bottlenecks dramatically.
4. Through ore source prioritisation, grade control and stockpiling activity, the head grade processed tends to look like a typical 'Ken Lane' descending curve (Lane, 1988). Changes in capacities, costs, prices, recoveries, and orebody access can make significant bumps and irregularities in this curve.
5. In some cases the last bit of capacity is not used, even at the bottleneck in the system. Point one above works in reverse as well, in that many sequences are not economic if they cannot be mined at a sufficient rate. If mined slowly, the delay between the upfront negative cash flows and the following positive cash flows is so great that the discounting effect reduces or negates the NPV. In this case the optimiser will choose to under-produce, rather than add components at uneconomic rates.
6. In a blending situation, it is not uncommon to have ore taken from a pit shell that is regarded as outside the expected ultimate pit, and for seemingly economic ore to be left in the bottom of pit shells within the ultimate pit. Blending is a time-dependent activity and ore needs to be accessible at the right time to contribute to the blend.
7. Large pushbacks with high pre-strip and deep ore create large waves in the NPV terrain being searched. Sometimes relatively minor changes in parameters can cause large pushbacks to flip in or out of the schedule with dramatic effects on total tonnage and mine life, but with minor impact on the NPV.
8. There is generally more than one plan that will give a result very close to the maximum NPV. The search algorithms give a variety of results and it is worth looking at the best few to understand the similarities and differences. There may be five or ten slightly, or very different, schedules produced that have an overall NPV within a fraction of a per cent of each other. They will tend to have some similar characteristics – those that are fundamental to a high-value schedule. They will also have some differences – which indicate some flexibility that will have little impact on the overall value, but may have other implications. The choice between these schedules should be made on criteria other than NPV, as they all qualify almost equally on that basis, so it is important to consider practicality, risk, consistency, political, social, environmental, etc issues, not all of which will have been fully incorporated in the model.

CONCLUSION

The work described in this paper has advanced the ability to achieve integrated or global optimisation by several degrees, providing new insights into the operations to which it has been applied. Much has been achieved, but there is still a lot to be learnt about the management of large groups of mineral and technical assets in a dynamic market. Although the mathematical objective of each optimisation run is NPV, the real benefit of this type of study is the understanding gained on the drivers and sensitivities of value within the system. It is just as important to eliminate less important projects/ideas from the management agenda as it is to prioritise the good ones or develop new ones.

The Global Asset Optimisation approach is helping to develop new insights into complex problems and is increasing knowledge and understanding of the opportunities and options.

REFERENCES

Lane, K F, 1988. *The Economic Definition of Ore* (Mining Journal Books: London).

Whittle, G, 2001. Strategic mine planning – the cross functional approach, in *Proceedings Fourth Biennial Conference: Strategic Mine Planning,* pp 19-22 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Whittle, J, 1999. A decade of mining optimization – the craft of turning algorithms into packages, in *Proceedings APCOM '99.*

A Multi-Stage Approach to Profitable Risk Management for Strategic Planning in Open Pit Mines

M Godoy[1] and R Dimitrakopoulos[2]

ABSTRACT

Design and production scheduling of open pit mines deals with the quest for the most profitable mining sequence over the mine's life. The dynamics of mining ore and waste and of interactions with spatial grade uncertainty make the prediction of the optimum mining sequence a challenging task.

This paper examines an optimisation approach to open pit production scheduling based on the effective management of waste mining to maximise net present value (NPV) and in relation to the presence of grade uncertainty and related risk management. The approach considers an economic model, the specific mine set-up, mining and processing specifics, including production equipment, as well as the development of a combinatorial optimisation formulation that integrates multiple grade realisations of the deposit. The efficient use of grade uncertainty and mining rates leads to schedules that are risk-resilient, as well as substantial improvements in project NPV compared with conventional methods. A case study with data from the Fimiston open pit gold mine demonstrates the approach and illustrates the potential economic benefit from risk quantification and management.

INTRODUCTION

In surface mining, the need to assess and manage risk for project valuation and decision-making translates to the need to assess and manage risk in any pertinent parameter of the design and production scheduling of a pit. This can only be achieved if uncertainty is quantified and taken into account in the mine optimisation process. Geological uncertainty is seen as a major contributor to not meeting project expectations. The problem of quantifying geological uncertainty in open pit design and production scheduling can be addressed in the context of stochastic simulation (Dimitrakopoulos, in press). Optimisation in mine planning has been accepted as a set of techniques that introduce analytical mathematical methods into mine planning (Lane, 1999). In the presence of risk, effective optimisation calls for the use of advanced mine optimisation techniques that are able to take into account the probabilistic nature of several influencing variables and constraints. Although there has been limited practical success to date in developing such techniques, the efforts are continuing (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2007, this volume; Jewbali, 2006).

This paper presents an optimisation approach that integrates grade uncertainty into the optimisation of long-term production scheduling. A general framework for long-term production scheduling is reviewed and extended through combinatorial optimisation to enable the risk of not achieving production targets due to geological uncertainty to be effectively minimised. The approach has the ability to minimise deviations from production target variables to acceptable ranges. An application developed in a large open pit gold mine is presented to show the potential economic benefits of the proposed approach.

1. MAusIMM, Senior Ore Reserves Analyst, Golder Associates SA, Avenida 11 de Septiembre 2353, Piso 2, Providencia, Santiago, Chile. Email: mgodoy@golder.cl

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

Some of the production scheduling concepts considered in the approach proposed herein originate from Russian mining (Rzhenevisky, 1968) and are considered in Tan and Ramani (1992) in formulating optimisation models. More recently, Godoy (2003) and Godoy and Dimitrakopoulos (2004) revisit the concepts in the context of modern open pit scheduling optimisation and, in particular, scheduling optimisation based on nested pits (Whittle and Rozman, 1991). The framework considers the open pit production scheduling optimisation process as the determination of a sequence of depletion schedules involving the removal of at least two types of material: ore and waste. Two major technical constraints involved in the determination of such schedules are:

1. the feasible combinations of ore and waste production, and
2. the ore production rate that meets the mill feed requirements.

In theory, the determination of an optimal production scheduling in an open pit mine must be done within a so-called 'feasible' domain. The current framework establishes this domain based on the two extreme cases of deferment of waste mining. The worst case corresponds to mining out each successive bench before starting the next. This schedule is assumed to provide the maximum quantity of waste that can be removed from the pit in order to recover a certain amount of ore (highest stripping ratio). The best case corresponds to the sequential mining that just extracts the minimum necessary quantity of waste to uncover a certain amount of ore (lowest stripping ratio), while still providing the necessary working room and the safety of the operations. The solution domain of ore production and waste removal can be represented in the form of a cumulative graph, bounded by the curves of the best and worst mining cases. The solution domain accounts for all physically possible combinations of stripping ratios. An example of such a graph, developed for a gold deposit, is shown in Figure 1.

An optimal schedule, in terms of net present value (NPV), will follow the curve representing the least possible quantity of waste. A key variable involved in the determination of an optimal waste removal curve is the mining capacity. Tan and Ramani (1992) proposed a linear programming (LP) model to solve such an optimisation problem. This LP model is used herein, extended to include periodic stabilisation of mining rates, so as to avoid solutions with impractical fluctuations in mining capacity and metal optimisation. The optimisation model delivers a life-of-mine schedule of waste removal and a prescription for the formation of mining capacity, given a predefined ore demand function and a set of possible models of mining equipment. This schedule maximises the project's NPV for a set of economic and technological parameters. However, the formulation, similarly to that in Tan and Ramani (1992), does not provide the physical mining sequence and therefore does not provide a complete solution to the long-term scheduling problem.

To overcome this limitation, a procedure is proposed herein that consists of splitting the long-term scheduling problem into two subproblems:

1. the determination of optimum best mining rates for the life of the mine, and
2. the generation of a detailed mining sequence constrained by the previously determined mining rates.

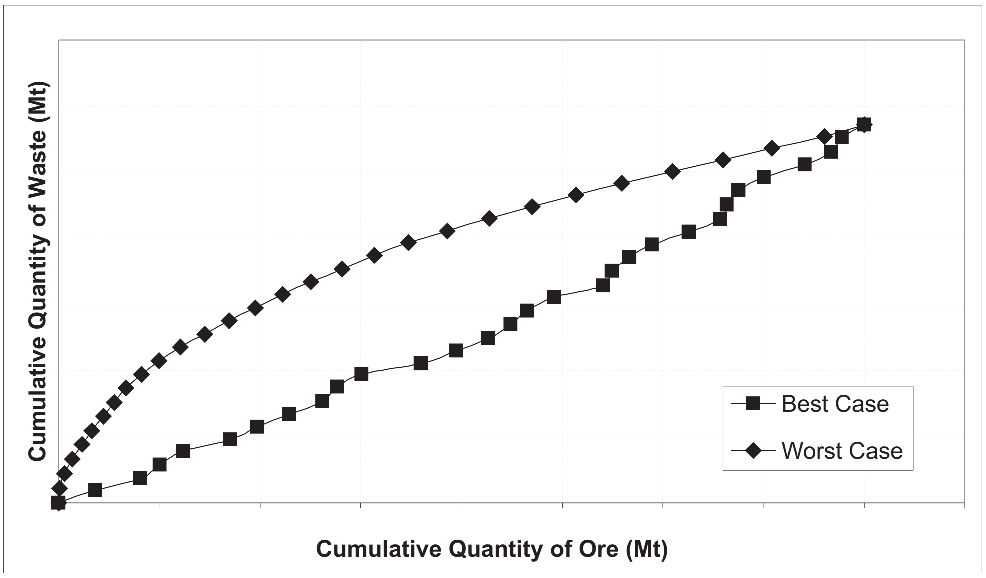


FIG 1 - Feasible domain of ore production and waste removal.

The approach is general and independent of the scheduling formulation used to produce the detailed mining sequence. The first subproblem deals with the objectives of ore production, stripping ratios, major investment in equipment purchase and average operational costs. The second subproblem focuses on the spatial evolution of the mining sequence and the equipment use and provides a more precise assessment of operational costs.

The following sections first present the proposed multi-stage approach with emphasis on the combinatorial optimisation part that generates the risk-based life of mine production schedule. Then an application at an open pit gold mine elucidates the practical aspects of the approach and provides a comparison with the traditional optimisation approach.

A NEW RISK-BASED APPROACH TO PRODUCTION SCHEDULING

The probabilistic approach to production scheduling optimisation proposed here is conceptually very different from the traditional deterministic one. In all traditional approaches, an optimisation formulation processes a single estimated orebody model to produce a mining schedule. Since this type of estimated orebody model is based on incomplete and imperfect geological knowledge, estimation errors are propagated to the various mining processes involved in the optimisation. The final result is a single, and often biased, forecast for the economic outcome of the life-of-mine production schedule (Dimitrakopoulos, Farrelly and Godoy, 2002). In probabilistic approaches, geological uncertainties are characterised by a series of equally probable models of the orebody, as produced by conditional simulation techniques (Dimitrakopoulos, 1998). The multi-stage optimisation algorithm presented herein takes all these simulated orebody models into account to produce an optimal and risk-robust mining schedule. Instead of providing a biased forecast for the economic outcome linked to the schedule, it yields a range of possible outcomes along with a single risk-managing schedule. One of the most important features of the method is its ability to minimise the ranges of variation of these outcomes, allowing for the minimisation of geological risk associated with the generated schedule.

The proposed algorithm first generates a series of mining schedules, each corresponding to a simulated realisation of the spatial distribution of grades. These mining sequences are optimised within a common feasible domain and post processed to provide a single mining sequence, which minimises the chance of deviating from the target production figures. The algorithm proceeds as follows:

1. derive a solution domain of ore production and waste removal common to all simulated models of the distribution of grades;
2. determine the optimum production rates for the life of the mine within the solution domain, derived in the first stage using the LP formulation;
3. for each one of the simulated models, generate a physical mining sequence constrained to the mining rates derived in the second stage (suboptimal mining schedules); and
4. combine, using simulated annealing, the mining sequences generated in the third stage to produce a single optimal mining sequence that minimises the chances of deviating from production targets.

First stage: derivation of the stable solution domain

The first stage is based on the consideration of N equi-probable simulated orebody models $S_1,\ldots,S_N$ mapping the space of geological (grade) uncertainty; an ultimate pit limit; and a sequence of cutbacks. A series of N cumulative graphs of ore production and waste removal, one for each simulated orebody model, is then generated. The 'stable solution domain' (SSD) is defined as the intersection of all feasible domains (Godoy and Dimitrakopoulos, 2004). This solution represents a domain that provides 100 per cent confidence on the contained reserves. The procedure is general and independent of the objectives driving the production scheduling optimisation. Figure 2 illustrates the SSD in an open pit gold mine.

Second stage: schedule optimisation

The second stage corresponds to the optimisation of the production schedule in terms of ore production and waste removal, under uncertainty. This stage incorporates the LP optimisation model discussed in the previous section. Note that a main difference here from Tan and Ramani (1992) is that the solution domain is now based on a series of simulated orebody models. The economic parameters of unit purchase, and ownership costs of each type and model of mine equipment available are also included in this stage. Stabilisation of the mining rate over time periods is therefore determined as a search

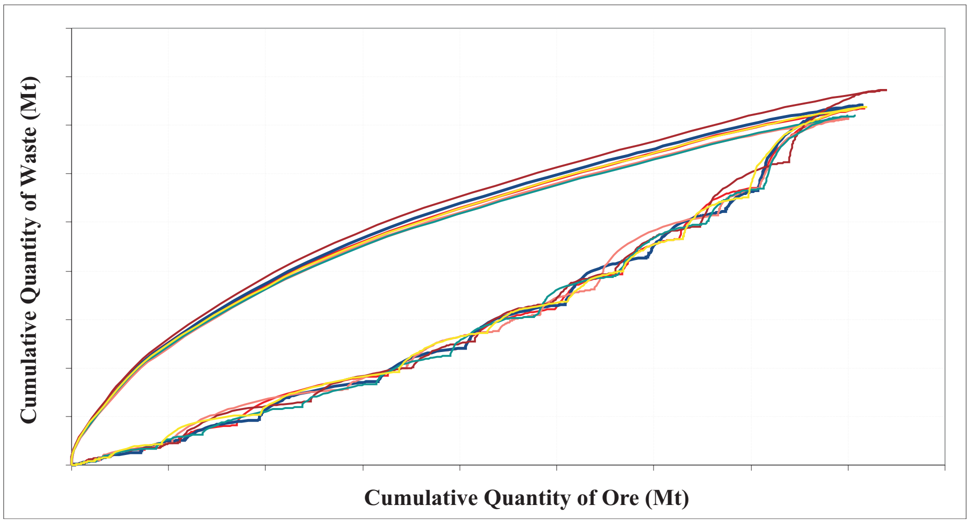


FIG 2 - Example of the stable solution domain (SSD) derived from six simulated orebody models.

for the balance between the purchase and ownership costs of the production capacity. This represents a direct incorporation of the capital investments in the production scheduling optimisation. Further details on the optimisation model in this second stage are given in Godoy and Dimitrakopoulos (2004).

Third stage: mining sequencing

This third stage aims to produce a series of physical schedules describing the detailed spatial evolution of the working zones over the life of the mine. Any formulation able to perform mining sequencing can be used for this task. The requirement is the ability to constrain the sequencing to obey slope constraints, maximise the equipment utilisation and meet mill requirements while matching the mining rates previously derived by the optimisation. The procedure consists of producing multiple mining sequences, one for each simulated orebody model. These multiple alternative mining sequences present two specific properties that will allow the derivation of a single sequence:

1. they are all technically feasible solutions that maximise the project's NPV within a common solution domain, and
2. they are based on distinct but equi-probable models of the spatial distribution of grades within the mineral deposit.

Fourth stage: combinatorial optimisation

The fourth stage requires a combinatorial optimisation algorithm to be developed to generate a single mining sequence from the alternative sequences produced in the third stage. The algorithm that has been developed is based on simulated annealing, a technique for solving combinatorial optimisation problems such as the minimisation of functions of many variables (Kirkpatrick, Gelatt and Vecchi, 1983). The key idea is to continuously perturb a suboptimal configuration until it matches some predefined characteristics expressed in an objective function at an acceptable level.

The optimisation starts by selecting an initial mining sequence where blocks with maximum probability of belonging to a given period are frozen for that period. The maximum probability threshold is user defined. Subsequently, the selected initial sequence is perturbed by randomly swapping selected blocks between candidate periods. All favourable perturbations (ie where the objective function is lowered) are accepted, whilst all unfavourable perturbations are accepted with an exponential probability distribution. The optimisation is considered complete when additional perturbations do not lower the value of the objective function or when a specified minimum objective function value is reached.

The objective function is defined as a measure of the difference between the desired characteristics and those of a candidate mining sequence. In this case, the measure is the average deviation from the production targets for a given mining sequence over a series of S simulated grade models. It is defined as the sum of N components:

$$O = \sum_{n=1}^{N} \left[\sum_{s=1}^{S} \left| \theta_n^*(s) - \theta_n(s) \right| + \sum_{s=1}^{S} \left| \omega_n^*(s) - \omega_n(s) \right| \right] \tag{1}$$

where:

n=1,…,N are the component objective functions, each corresponding to one of the production schedule periods

For each n component, the objective function measures the average deviation of ore and waste production $\theta_n^*(s)$ and $\omega_n^*(s)$ of the perturbed mining sequence from the target productions θ_n(s) and ω_n(s) over all S simulated grade models with s=1,…,S. The decision of whether to accept or reject a perturbation is based on the change to the objective function in Equation (1).

Recalculations of the global objective function can be replaced by a selective update of the component objective functions involved in the perturbation. The resulting sequence meets the production target for each period with minimum chance of deviation, ie this mining sequence will achieve the production targets, within the prescribed mining rates, given any of the simulated orebody models. The swapping mechanism is an important aspect of the annealing procedure above. To guarantee the feasible final solution, the perturbation mechanism must be able to recognise the spatial evolution of the mining sequence. To accomplish this, the swapping mechanism is set to limit the candidate periods for any given block to only those that will have physical access to the block without violating slope constraints.

In addition to the objective function and the perturbation mechanism, a critical aspect of simulated annealing-based algorithms is a prescription for when to accept or reject a given perturbation. The acceptance probability distribution is given by the Metropolis criterion (Metropolis *et al*, 1953):

$$P\{accept\} = \begin{cases} 1, & if\ O_{new} \leq O_{old} \\ e^{\frac{O_{old} - O_{new}}{T}}, & otherwise \end{cases} \tag{2}$$

All favourable perturbations updated from O_{old} to O_{new} ($O_{new} \leq O_{old}$) are accepted and unfavourable permutations are accepted with an exponential probability distribution. The algorithm is stopped when a low objective function value (O_{min}), indicating convergence, is reached. A so-called 'cooling' function controls the rate of decrease in time of the parameter T, also known as 'temperature', of the exponential distribution. The higher T is, the greater the probability that an unfavourable perturbation will be accepted.

The parameter T must not be lowered too fast because the mining sequence may get trapped in a suboptimal situation and never converge. However, if T is lowered too slowly, the convergence may be unnecessarily slow. The specification of how to lower T is known as the annealing schedule. The idea is to start with an initially high t_o and lower it by a multiplication factor λ whenever enough perturbations have been accepted (K_{accept}) or too many have been tried (K_{max}). The algorithm is stopped if K_{max} is reached S_{tp} times, where S_{tp} is the parameter known as 'stopping number'. The algorithm is also stopped if a maximum number of swaps are reached or after reaching a maximum number of swaps with no change in the objective function. These parameters are named *MaxSwap* and *MaxNoChange* respectively (see Table 1).

The method presented in this section provides a framework for the derivation of a single schedule that minimises the chances of deviating from production targets, given the uncertainty assessment from the available information. Precedence constraints built into the perturbation mechanism are designed to recognise the spatial evolution of the mining sequence, which is restricted by pit slope constraints. These mining sequences are produced by an external mining sequence algorithm and must reflect mining practices and technological constraints.

TABLE 1

Parameters used for the annealing schedule.

Parameter	Value
t_o	0.000 000 001
λ	0.1
K_{max}	85 000
K_{accept}	50 000
S_{tp}	20
O_{min}	100 000
MaxSwap	100 000 000
MaxNoChange	100 000

APPLICATION IN AN OPEN PIT GOLD MINE

An application of the proposed method is carried out for the Fimiston open pit mine in Western Australia; Australia's premier gold mine. This application starts with the development of a 'base case' schedule for the life of the mine. The aim is to produce a base case that is a benchmark, against which the potential economic benefits of the new risk-based optimisation approach can be evaluated.

The base case schedule was developed using a traditional estimated model for the distribution of grades, as used in the conventional approach. The mining capacity was formed with a combination of Komatsu PC8000 face shovels, CAT994 loaders and CAT793C trucks. A constant mining capacity of 85 Mt per year was adopted. The schedule of ore production was identified with the mill demand. Both the schedule of mining capacity and ore production for the base case are presented in Figure 3. Note that Figure 3 also presents the mining capacity and ore production from the second stage of the proposed multi-stage approach and is discussed in a subsequent paragraph. It is important to note that the fluctuations in ore production do not indicate a variable mill production rate. The mill production rate is constant over the life of the mine. Rather, periods characterised by a reduction in ore demand indicate there is input of ore from external sources such as, for example, underground operations and stockpiles. A risk analysis on the base case schedule, using a set of simulated models, was also carried out. This risk analysis was developed by taking the base case mining sequence, which indicates which blocks are to be mined in each period, and evaluating the schedule outcome for each one of the simulated orebody models, representing the potential deposit in the ground. The procedure generates a distribution of responses or a range of alternative outcomes for key project indicators and is similar to that employed in Dimitrakopoulos, Farrelly and Godoy (2002).

Figure 4 presents the base case predictions and the risk profile obtained for the annual and cumulative discounted cash flows, respectively. The expected NPV is approximately 11 per cent less than that indicated by the initial predictions of the base case schedule. Figure 5 presents the results obtained from the risk analysis on the ore production and the initial predictions of the base case schedule. The average expected deviation from the base case prediction shows a deficit of approximately 1.3 Mt per year. This result shows that the base case schedule is unable to meet the predicted mill feed tonnage. Note that the use of optimal mining rates, using the approach in the second stage without grade risk and in combination with the conventional methods used in the base case, provides a relatively small

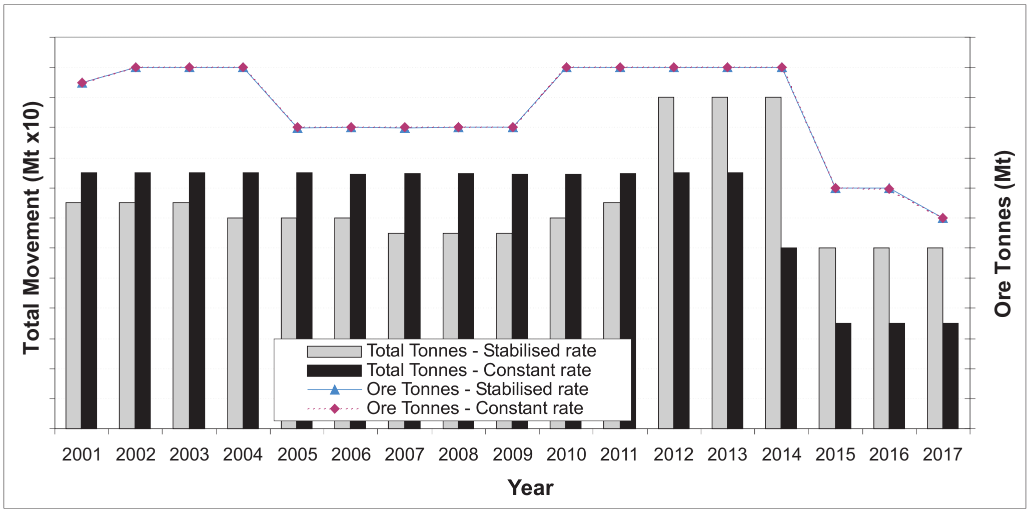


FIG 3 - Schedules of mining capacity and ore production for constant mining rates from the base case (black), and periodically stabilised mining rates from the second stage (grey).

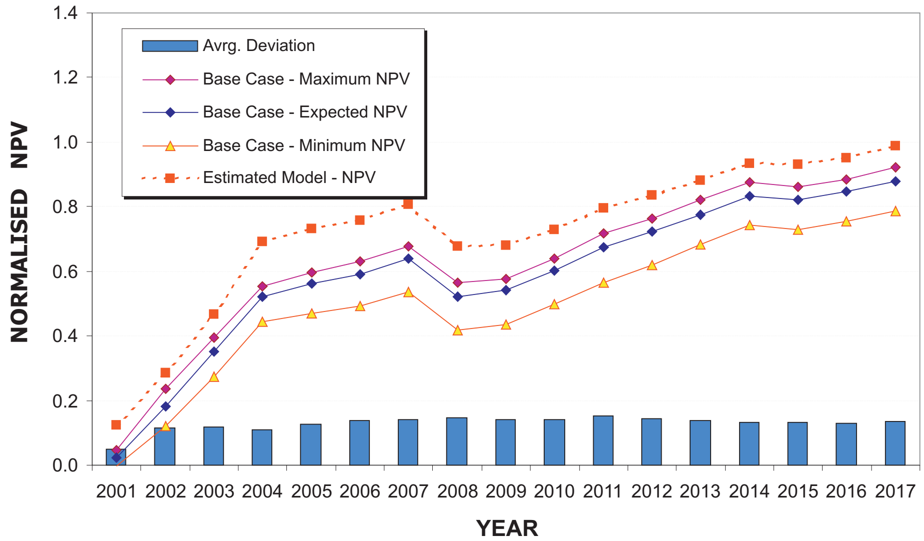


FIG 4 - Cumulative cash flow and average deviations from the forecast for the base case schedule.

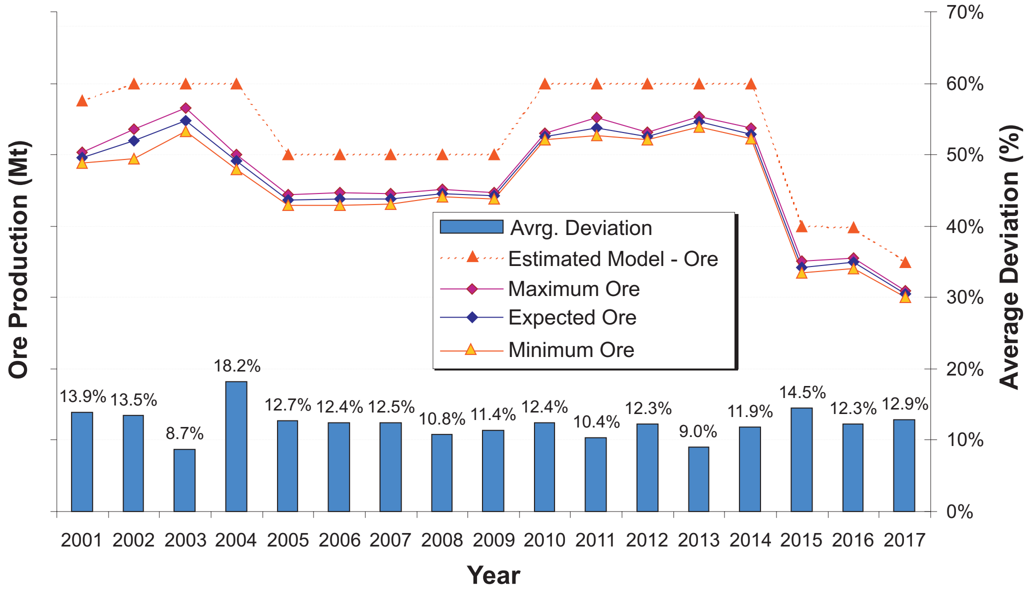


FIG 5 - Base case schedule: uncertainty in ore production and expected deviation from target.

improvement and leads to some improvement of total NPV. However, similarly to the base case schedule, it does not meet production targets, as detailed in Godoy (2003).

The application of the risk-based approach starts, as outlined previously, with the derivation of the SSD in the first stage. The optimisation in the second stage is carried out within the SSD. The resulting prescription of ore and waste production and the selection of mining equipment forming the required mining capacity (Figure 3) are used to generate the mining sequences, one sequence for each simulated model. Last, the simulated annealing procedure is used to combine these multiple mining sequences. Table 1 shows the parameters used for the annealing process. Figure 6 shows the component objective functions versus the number of attempted perturbations in the present application. The optimisation stopped after 202 669 perturbations, with 8716 being accepted, as it reached the maximum number of attempts with no change in the objective function.

The exceptional performance and effectiveness of the proposed method and the effects of managing risk are further demonstrated in the results shown in Figure 7. The figure shows the final schedule and the risk profile obtained from the risk analysis in ore production. The bars indicate the absolute average deviation from the target. The largest deviations belonging to years 2002, 2005 and 2008 are respectively 357 000 t, 347 000 t and 265 000 t. The magnitude of these deviations is considered very small and could be easily managed by rehandling ore from alternative sources, especially for those years presenting a shortfall.

In terms of NPV, shown in Figure 8, the expected outcome corresponds to an increase of 28.3 per cent in relation to predicted NPV for the base case schedule. This difference reflects the deferment of waste mining, the reduction in the life of the mine and the ‘blending’ of grade risk. One of the major contributions to the increased NPV comes from the recovered metal. The risk-based schedule recovers the same metal quantity first predicted by the base case, but it does so sooner.

An important aspect of the case study is that it demonstrates how risk-inclusion leads to a counter-intuitive risk reduction and simultaneous increase of NPV, which are both substantial. In addition, the case study makes a distinct case for risk-based optimisation against what is seen as the inherent limits of conventional technologies.

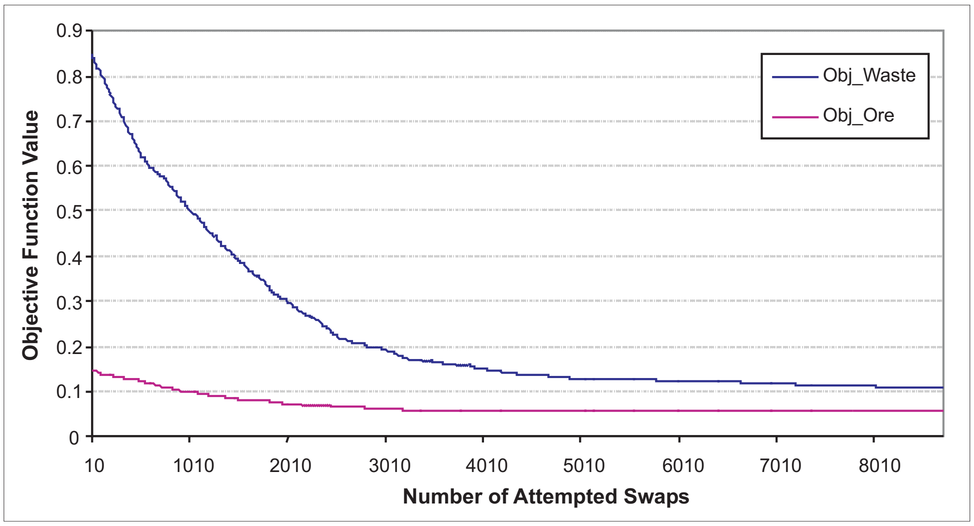


FIG 6 - Simulated annealing: component objective function values versus attempted number of swaps.

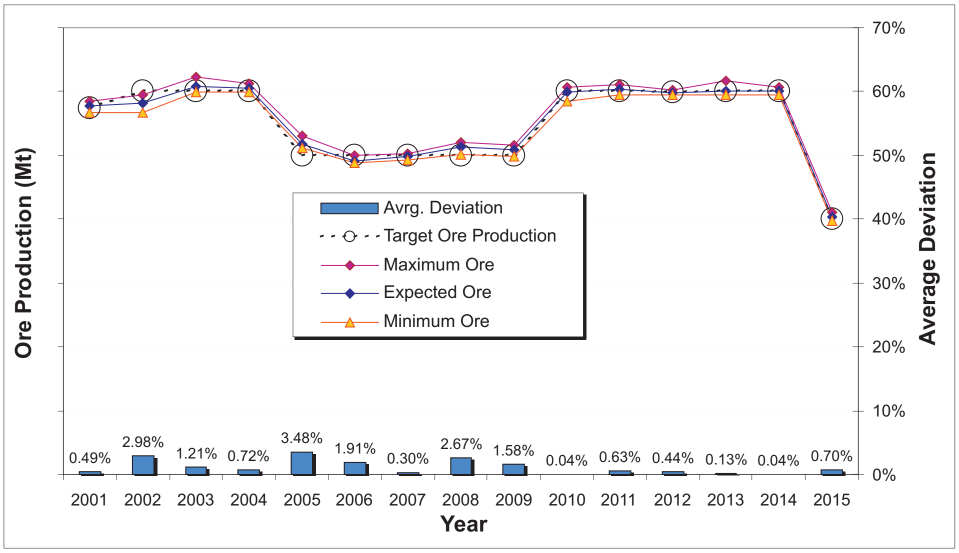


FIG 7 - Assessment of the risk profile for ore production in the final risk-based schedule.

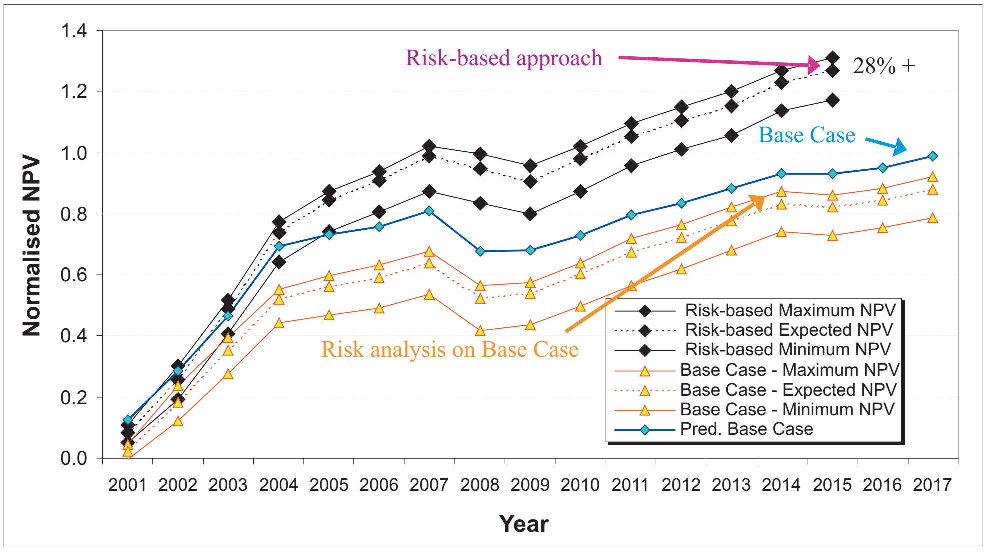


FIG 8 - Risk profile on cumulative NPV for the schedule produced by the risk-based approach (top three lines), cumulative NPV as forecast by the base case schedule (single middle line) and risk profile on cumulative NPV from risk analysis on the base case schedule (bottom three lines).

CONCLUSIONS

This paper has presented a new risk-based optimisation formulation for long-term production scheduling in open pit mines. The new multi-stage formulation was used to produce a minimum-risk life-of-mine schedule for Fimiston, the largest Australian open pit gold mine. This optimisation approach was found to have the potential to considerably improve the economic outcome of the mine and forecasts for the life of the mine, when compared with conventional (non risk-based) scheduling practices. That is, the results not only show a potential increase of 28.3 per cent in the value of the mine, but also provide a schedule that minimises the chance of deviating from mill-feed requirements.

The approach has been shown to be capable of capitalising on both the deferment of waste mining and the assessment (as well as inclusion) of grade uncertainty, leading to maximised economic returns and driving the mining sequence through zones where the risk of not achieving the target ore production is minimised. The approach provides not only a risk-resilient solution to the production scheduling problem but also an increase in asset value by considering an inherent source of uncertainty and risk. This ability represents a major advance in the risk management of open pit mining and makes a convincing case for the need to implement and further develop risk-based optimisation approaches as an alternative framework to conventional optimisation.

ACKNOWLEDGEMENTS

The work in this paper was funded from the Australian Research Council grant #C89804477 to R Dimitrakopoulos, 'General optimisation and uncertainty assessment of open pit design and production scheduling'. Support from Kalgoorlie Consolidated Gold Mines Pty Ltd (KCGM), WMC Resources Ltd and Whittle Programming Pty Ltd is gratefully acknowledged. Thanks to K Karunaratna, P de Vries, W Li and C Reardon of KCGM, for facilitating research, providing data and comments, and last but not least, to Jeff Whittle for technical comments and encouragement.

REFERENCES

Dimitrakopoulos, R, 1998. Conditional simulation algorithms for modelling orebody uncertainty in open pit optimization, *Int J Surface Mining, Reclamation and Environment*, 12:173-179.

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R, Farrelly, C and Godoy, M C, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open pit mine design, *Trans Inst Min Metall*, Section A, Mining Technology, 66:A82-A89.

Dimitrakopoulos, R and Ramazan S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions*, 316:106-112.

Godoy, M C, 2003. The effective management of geological risk in long-term production scheduling of open pit mines, PhD thesis, 256 p, The University of Queensland, Brisbane.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling in open pit mines, *SME Transactions*, 316:43-50.

Jewbali, A, 2006. Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines, PhD thesis, The University of Queensland, Australia, 280 p.

Kirkpatrick, S, Gelatt, C D and Vecchi, M P, 1983. Optimization by simulated annealing, *Science*, 220:671-680.

Lane, K, 1999. Optimisation: is it the best? in *Proceedings Third Biennial Conference: Strategic Mine Planning,* pp 1-7 (Whittle Programming Pty Ltd: Melbourne).

Metropolis, N, Rosenbluth, A, Rosenbluth, M and Teller, E, 1953. Equation of state calculations by fast computing machines, *J Chem Phys*, 21(6):1087-1092.

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Rzhenevisky, 1968. *Open Pit Mining*, 312 p (Nedra Publications: Leningrad) (in Russian).

Tan, S and Ramani, R V, 1992. Optimization models for scheduling ore and waste production in open pit mines, in *Proceedings 23rd APCOM Symposium*, pp 781-791 (SME-AIME: Littleton).

Whittle, J and Rozman, L I, 1991. Open pit design in the 90s, in *Proceedings Mining Industry Optimisation Conference,* pp 13-19 (The Australasian Institute of Mining and Metallurgy: Melbourne).

A New Efficient Joint Simulation Framework and Application in a Multivariable Deposit

A Boucher[1] and R Dimitrakopoulos[2]

ABSTRACT

Ore mineralisations frequently contain more than one mineral or element of interest that are spatially related. As a result, they require the use of joint geostatistical simulation techniques that generate models conserving this correlation. Although joint simulation methods have long been available, they are impractical when it comes to more than two variables and mid to large size deposits. This paper presents a new framework for joint conditional simulations of non-Gaussian vectors of variables and stresses, in particular, the joint simulation directly at the block-support scale. The proposed framework is based on minimum/maximum autocorrelation factors (MAF) that decorrelate variables at all lags, thus allowing the simulation of independent variables. The MAF approach is combined with the direct block simulation framework presenting a new algorithm termed 'DBMAFSIM'. It permits computationally efficient joint simulation of large, multivariable deposits. The proposed method is then applied at the Yandi iron ore deposit in Western Australia, with five major correlated attributes being successfully simulated and validated on block support. The application shows the efficiency and excellent performance of the method.

INTRODUCTION

Geostatistical simulation methods are used to quantify geological uncertainty in mineral deposits and assess risk in various aspects of mining project development and operation. Although methods for simulating individual attributes are generally efficient (see Benndorf and Dimitrakopoulos, 2007, this volume), existing methods for jointly modelling multivariable deposits are in practice limited, particularly when dealing with medium to large deposits having more than two attributes of interest. For example, a realistic model of an iron deposit must account for silica, alumina and phosphorus in addition to iron, and reproduce the joint local variability of the attributes of interest. Thus, there is a need to consider new efficient and practical joint simulation methods.

Available approaches for joint simulation include early developments based on the model of linear coregionalisation and conditioning of simulated correlated fields (Chilès and Delfiner, 1999); extension of the conditional univariate LU decomposition method of Davis (1987) to joint simulation (Myers, 1988); and combination of the LU vector simulation and sequential simulation for large joint simulation of two variables (Verly, 1993), a method which becomes cumbersome for more than two variables. The major drawback with the above methods and their variations is that they require considerable computer processing capacity to solve the large systems of equations per simulated node, in addition to the inference of cross-variograms, and issues arising from data management. An alternative to the impractical common joint simulation methods is to factorise the variables involved to uncorrelated (orthogonal) factors that can be simulated independently of each other. Subsequent back-transformation of simulated factors to simulated realisations of variables aims to indirectly restore the histograms, variograms and cross-variograms of the data in the respective realisations. This type of a factorisation approach is introduced by David *et al* (1984) as a principal component analysis (PCA) data transformation (David, 1988; Suro-Perez and Journel, 1991). However, this transformation decorrelates variables only at lag zero, and is limited in practice (Wackernagel, Petitgas and Touffait, 1989; Goovaerts, 1993).

Desbarats and Dimitrakopoulos (2000) and Dimitrakopoulos (in press) present a major improvement of the PCA approach to joint simulation of multiple variables by replacing PCA with the minimum/maximum autocorrelation factors (MAF), a factorisation method originally developed for remote-sensing applications (Switzer and Green, 1984). The advantage of MAF is that it produces uncorrelated factors at all lags, when the variogram model of the related variables follows the linear model of coregionalisation with two structures. The method gives access to a substantially wider range of variables that can be jointly simulated than is possible with PCA factorisation. Joint simulations of mineral deposits based on MAF are shown to be effective, relatively efficient, flexible and practical (eg Boucher, 2003; Dimitrakopoulos and Fonseca, 2003). The efficiency of joint simulation with MAF could be further enhanced if it were possible to simulate directly on a block-support scale.

The block support on which an orebody is being numerically represented and modelled differs from the support size of the available data, thus requiring modelling and change of support. The current approach to change of support is safe but cumbersome. It consists of simulating points and then averaging them to the blocks needed, which has two computational drawbacks. First, the algorithm needs to process, store and manage large sets of data and files (several gigabytes). Second, the algorithm involves an additional operation (averaging) that can be time-consuming for large orebodies. An alternative simulation method is proposed by Godoy (2003), and it is termed 'direct block simulation'. The method minimises the information stored in memory by retaining in memory only block values, a procedure that significantly speeds up the simulation process and also reduces the size of the output files, facilitating efficiency in data storage and management. Advantages of the method are that there is no assumption for change of support, and it is substantially more efficient than other existing methods (Benndorf and Dimitrakopoulos, 2007, this volume). The direct block simulation can be extended to the joint direct block simulation of multiple variables using MAF, an approach shown to be very efficient and effective (Boucher, 2003).

This paper focuses on a new and efficient joint simulation framework. First, it outlines the MAF approach to joint geostatistical simulations at the conventional point-support scale and, subsequently, shows the extension of the approach to the direct joint simulation at the block-support scale. An application at the Yandi Central 1 iron ore deposit, Western Australia, follows and shows the joint simulation of iron content, silica, alumina, phosphorus and loss on ignition, directly at the block-support scale. Comments on the performance of the approach and conclusions follow.

1. Department of Geological and Environmental Sciences, Stanford University, 450 Serra Mall, Braun Hall, Building 320, Stanford CA 94305-2115, USA. Email: aboucher@pangea.stanford.edu

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

JOINT SIMULATION ON POINT AND DIRECT BLOCK SUPPORT-SCALE WITH MIN/MAX AUTOCORRELATION FACTORS

Point support

The joint simulation of multiple variables based on minimum/ maximum autocorrelation factors (MAF) presented herein proceeds as follows. First, a stationary vector random function (RF) **Z** (u) is transformed into its Gaussian equivalent **Y** (u); this transformation is $\mathbf{Y}(u) = \phi(\mathbf{Z}(u))$ or:

$$\mathbf{Y}(u) = \{Y^1(u), ..., Y^P(u)\} = \{\phi_1\left(Z^1(u)\right), ..., \phi_P(Z^P(u))\} \tag{1}$$

The resulting vector RF $Y(u)$ is composed of p Gaussian RFs that are assumed to be multi-Gaussian. Then, the MAF are derived as a new vector RF, $\mathbf{M}(u) = \{M^1(u), ..., M^P(u)\}$, where the p RFs are independent and obtained from the multi-Gaussian vector RF **Y** (u) using the set of vectors **A** of coefficients derived from:

$$\mathbf{M}(u) = \mathbf{A}^T\mathbf{Y}(u) \tag{2}$$

with the MAF approach described below. Note that the MAF, **M** (u), is a linear function of **Y** (u), which is a non-linear transformation of the original data **Z** (u) such that:

$$\mathbf{M}(u) = \mathbf{A}^T\phi(\mathbf{Z}(u)) \tag{3}$$

The orthogonalisation coefficients matrix **A** are generated from:

$$2\Gamma_Y(h)\mathbf{B}^{-1} = \mathbf{A}^T\Lambda\mathbf{A} \tag{4}$$

with:

$$\begin{aligned} \mathbf{B} &= \mathrm{cov}[\mathbf{Y}(u), \mathbf{Y}(u)] \\ 2\Gamma_Y(h) &= \mathrm{cov}[\mathbf{Y}(u)-\mathbf{Y}(u+h), \mathbf{Y}(u)-\mathbf{Y}(u+h)] \end{aligned} \tag{5}$$

where:

B is the variance/covariance matrix of $\mathbf{Y}(u)$, a multi-Gaussian RF

$\Gamma_Y(h)$ is the variogram matrix at lag h

The above derivation of **A** is equivalent to performing two successive principal component (PCA) decompositions (Desbarats and Dimitrakopoulos, 2000).

The use of MAF at the point-support scale is a straightforward application of Equation 3, which transforms the data to MAF; independent conditional simulation of each MAF, with any point simulation algorithm; and subsequent back-transformation of the generated MAF realisations using the coefficients **A,** which generates realisations of the variables being simulated in the original data space. Two points related to the application of MAF are worth noting. First, the application of MAF in mineral deposits has been shown (eg Dimitrakopoulos and Fonseca, 2003) to work particularly well and reproduce the statistical, spatial continuity and cross-continuity, as well as scatter-plots, in spite of being based on simulating independent factors. Second, despite the major efficiency improvements, reasonable size deposits with several correlated variables require further efficiency improvement. That improvement is achievable if the joint simulation with MAF is extended to simulating directly at the block-support scale.

Block support

The direct simulation with MAF at the block-support scale is based on the RF that is the scaled-up vector $\mathbf{Z}_V(x)$ obtained from:

$$\mathbf{Z}_V(x) = \frac{1}{N}\sum_N \phi^{-1}\left(\mathbf{A}^{-T}\mathbf{M}(u)\right) \tag{6}$$

A simulated block is obtained by simulating all N points $\mathbf{m}^*(u_\alpha), \alpha = 1, ..., \mathrm{N}$ discretising a block independently for each MAF. The simultaneous simulation of the points is performed with an LU algorithm. The block values are computed with:

$$\mathbf{z}_V^*(x) = \frac{1}{N}\sum_N \phi^{-1}\left(\mathbf{A}^{-T}\mathbf{m}^*(u)\right) \tag{7}$$

The above relations allow the extension of the direct block simulation of Godoy (2003) to the joint direct block simulation algorithm outlined next.

The DBMAFSIM algorithm

The joint conditional simulation of correlated attributes of deposits directly at the block-support scale using MAF is termed 'DBAMFSIM' and is graphically illustrated in Figures 1 and 2. The algorithm is as follows:

1. transform the data **Z** (u) to the normal-score data **Y** (u),
2. transform **Y** (u) with the MAF transformation to **M** (u), and
3. simulate sequentially the N groups of points **m** (u_l) representing the deposit with an LU decomposition:

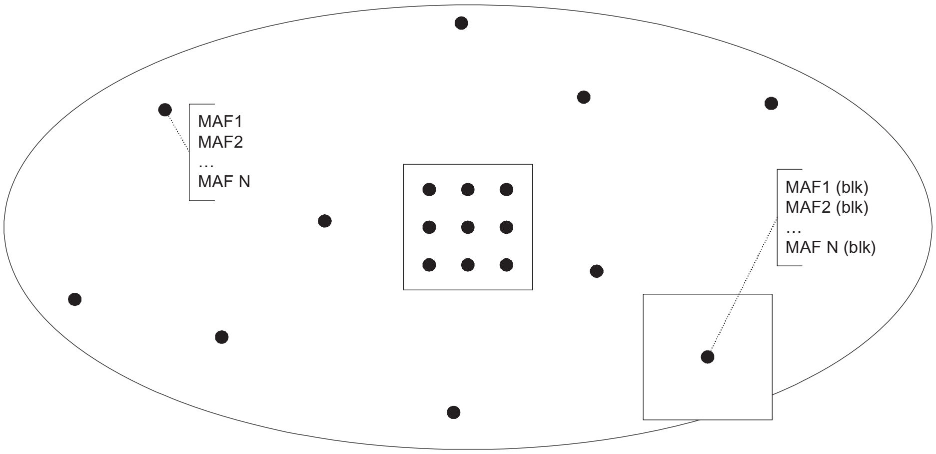


FIG 1 - Search neighbourhood for the multivariate direct block simulation.

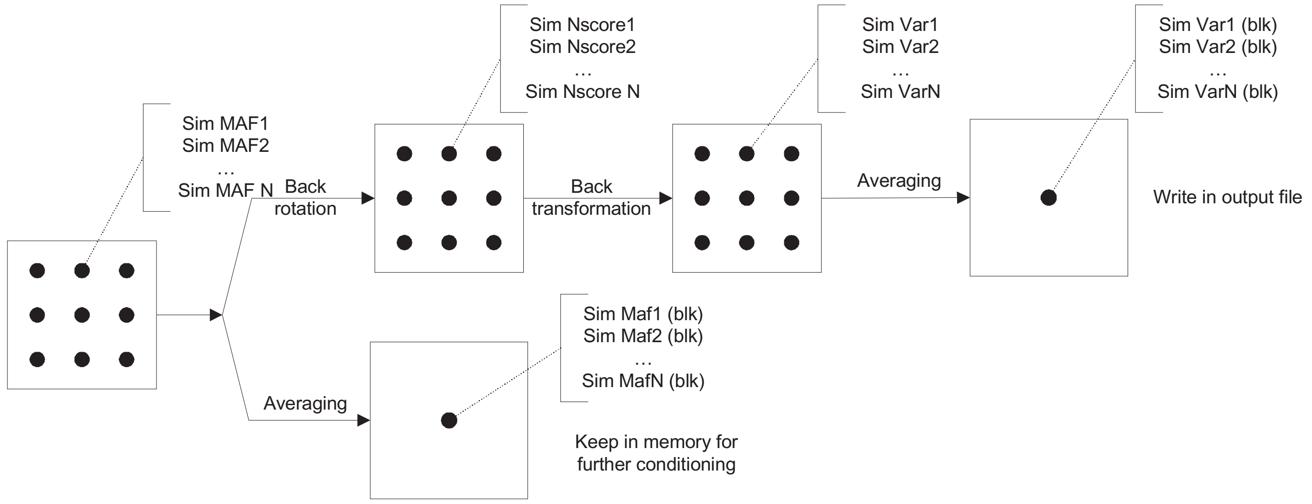


FIG 2 - Process for simulation of multivariate RF on block-support scale.

- For each factor $M^i(u)$ in $\mathbf{M}(u)$ simulate realisations of points $m^i(u_l)$.
- Average the points $\mathbf{m}(u_l)$ over the group to obtain $\mathbf{m}_V(u_{lv})$ at the block support. Introduce $\mathbf{m}_V(u_{lv})$ to the data set feeding the sequential simulation process.
- Back transform $\mathbf{m}(u_l)$ to $\mathbf{z}(u_l)$.
- Average the points $\mathbf{z}(u_l)$ into $\mathbf{z}_v(u_{lv})$ and output.

Figure 1 shows the discretised block to be simulated, the search neighbourhood, and the two types of conditioning data: the samples and the previously simulated blocks. When the conditioning data contains a previously simulated block, the point-to-block and block-to-block regularised variogram values are calculated from the input point variogram value (Journel and Huijbregts, 1978). Figure 2 shows the workflow of the proposed algorithm. First, the N internal points of a block are simulated for every factor. Then, the process is split into two parts:

1. the block values for the MAF are obtained by averaging the simulated factors at point support; and
2. the internal points are first back-transformed to normal-score space, subsequently back-transformed into data space and, finally, averaged to the desired block value.

The block value in part one is kept in memory for further conditioning, while the value from part two is the final output for the block and is written to a file.

The procedure described above contributes to an increase in speed and efficiency by reducing the number of neighbourhood searches. Only a single search per block to be simulated is needed, instead of the N searches that the point-wise approach would require. In addition, when the number of discretising nodes used in blocks is correctly chosen, the simulation with the LU algorithm is substantially faster than the solution of the system of N cokriging equations (Dimitrakopoulos and Luo, 2004).

APPLICATION AT YANDI CENTRAL 1, IRON ORE DEPOSIT, WA

The Yandi Central 1 iron ore deposit is a part of the larger Yandicoogina-Marillana detritic channel deposits in Western Australia. The mine is located around 120 km north-west of Newman, in the Hamersley province, an area providing 97 per cent of Australian iron. BHP Billiton commenced operations at Yandi in 1992 and has developed the deposit into one of the world's top ten iron ore mines, with estimated resources of about 1500 million tonnes of high-grade iron ore. Iron is derived from the erosion of the banded iron formation of the Hamersley Province and trapped in paleochannels incisions within the Weeli Wolli Formation that formed the surface approximately 40 to 50 million years ago (Stone *et al*, 2002). The deposit is composed of cemented masses of concretionary iron oxides, largely goethite (Hall and Kneeshaw, 1990; Stone *et al*, 2002). The main ore zone of Yandi Central contains a consistently high level of iron, together with high and low levels of silica and generally low levels of alumina.

Study area and data

The section of the channel at Yandi Central 1 considered in this study is 4.1 km long by 500 m wide. This study is concerned with a section of the main ore zone located above the water table, a section 30 m thick on average. The part of the deposit modelled is discretised into 40 698 blocks, each 25 m × 25 m × 2 m, filling a volume of about 50.87 million cubic metres. This is the most economically important zone within the deposit because of its volume, high iron content and low silica, alumina and phosphorus content. The study area is covered by 961 drill holes (Figure 3), representing 7126 two-metre long composites of iron content (Fe), silica (SiO_2), alumina (Al_2O_3), phosphorus (P) and loss on ignition (LOI), all present in the available composites. Drilling density is approximately on a regular 50 m grid, with the exception of a densely drilled section in the middle of the orebody. In this study, the main ore zone is divided into the external and internal zones shown in Figure 3. The statistics of the data in the two zones are given in Table 1. Note that the internal zone differs from the larger external zone by having higher Fe and Al_2O_3 content and slightly lower SiO_2 content. The SiO_2 in the external zone also displays higher variability.

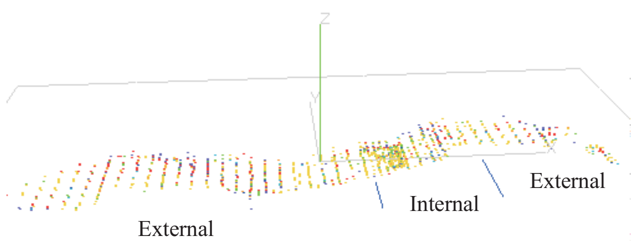


FIG 3 - Drilling patterns of Yandi Central 1 and division into the external and internal zones.

TABLE 1

Summary statistics for the data in the external and internal zones.

External zone (3873 samples)								
Field	**Mean**	**Std dev**	**Min**	**Max**	**0.25Q**	**0.5Q**	**0.75Q**	**0.975Q**
Fe	58.27	2.37	40.00	61.80	57.49	58.70	59.68	60.93
P	0.03	0.006	0.007	0.08	0.03	0.03	0.03	0.04
SiO_2	5.04	2.41	1.71	20.00	3.29	4.53	6.24	10.46
Al_2O_3	0.97	0.84	0.17	5.00	0.52	0.70	1.05	4.06
LOI	10.32	0.94	6.80	17.00	9.70	10.36	10.97	12.10
Internal zone (2886 samples)								
Field	**Mean**	**Std dev**	**Min**	**Max**	**0.25Q**	**0.5Q**	**0.75Q**	**0.975Q**
Fe	58.85	2.29	40.00	61.77	58.35	59.40	60.10	61.10
P	0.03	0.006	0.007	0.06	0.02	0.03	0.03	0.04
SiO_2	4.20	3.60	2.14	1.700	20.00	2.79	4.92	9.64
Al_2O_3	1.10	0.94	0.18	5.00	0.560	0.77	1.20	4.67
LOI	10.27	0.74	6.60	14.26	9.80	10.27	10.79	11.64

The correlations and rank correlations between the elements in the data set are shown in Table 2. Three strongly correlated elements are present: Fe, SiO_2 and Al_2O_3. The rank correlation is important because the normal-score transformation is a rank transformation, and thus tends to only preserve the rank correlation. This has an impact when the two correlations differ significantly, as is the case with the Fe-Al_2O_3 and SiO_2-Al_2O_3 correlations.

Minimum/maximum autocorrelation factors

To generate uncorrelated MAF, Equation 5 is used. The variance/covariance matrix $\Gamma(h)$ is computed using pairs located at a minimum of 110 m and a maximum of 155 m apart in the horizontal plane, but with no more than 2 m of vertical separation. This distance is, on average, just larger than the range of the first spherical structure and much shorter than the second one for the five variables. The vertical constraint of 2 m is necessary to avoid the various local vertical trends and an artificial increase in variance.

The factor coefficients **A** are presented in Table 3. In both zones, Fe has the highest coefficient and it is always important in the determination of all MAF, followed by SiO_2. P is the least important element, but it is also the one having the least correlation with others elements. The two most important factors, MAF1 and MAF2 (representing almost 50 per cent of the variance), are essentially a combination of Fe and SiO_2 with the addition of LOI for MAF2.

Examples of experimental and model variograms of the MAF for the external zone are shown in Figure 4. It is interesting to note that the range of a factor seems to be inversely proportional to its contribution to the variance. With slightly less than 15 per cent of the global variance, MAF5 has the longest range at 680 m and 770 m for the external and the internal zones respectively. On the other hand, MAF1 has a range of 160 m and 200 m for the two zones, but represents around 25 per cent of the global variance. Representative examples of cross-variograms are shown in Figure 5 and, as the method suggests, the decorrelation between the factors is excellent.

Joint conditional simulation at the external and internal zones

The external and internal zones are separately simulated 20 times using the DBMAFSIM method and subsequently merged. Each realisation contains 40 698 blocks, each 25 m × 25 m × 2 m, with simulated Fe, P, SiO2, Al_2O_3 and LOI content. For comparison, at the point-support scale this corresponds to 2 122 752 points for the external zone and 481 920 points in the internal zone per realisation (a sum exceeding 2.6 million points). In addition to visual inspection, the simulations are subsequently validated by:

1. quantile-quantile plots between data and simulated point-support values,
2. block variogram validation with their respective scaled-up (regularised) data variogram models, and
3. assessment of the vertical profiles of the simulated realisations at the block support used.

Figure 6 shows a horizontal section for each element from one of the jointly generated realisations. The correlation between Fe, SiO_2 and Al_2O_3 is apparent from the realisations, for example the low levels of SiO_2 and Al_2O_3 are present where there is a high level of Fe. Examples of quantile-quantile plots for the five elements highlighting the relationship between nodes of a given simulation and the sample data are shown in Figure 7. In general, the reproduction of data histograms is excellent.

TABLE 2

Pearson's correlation (left) and rank correlation (right) between variables.

External zone									
	Fe		**P**		**SiO_2**		**Al_2O_3**		**LOI**
Fe	1								
P	-0.03	-0.21	1						
SiO_2	-0.9	-0.86	-0.02	-0.08	1				
Al_2O_3	-0.81	-0.34	0.02	0.25	0.56	0.16	1		
LOI	-0.12	-0.18	0.15	0.15	-0.19	-0.22	0.08	-0.02	1

TABLE 3

*Coefficients **A** of MAF for the external zone.*

External zone						
	Fe	**P**	**SiO_2**	**Al_2O_3**	**LOI**	**% Var**
MAF1	-1.328	-0.168	-1.096	0.676	-0.491	25.42
MAF2	3.049	-0.145	2.654	0.950	1.712	22.24
MAF3	1.031	0.871	0.614	0.312	-0.119	20.53
MAF4	0.859	0.123	1.734	-0.022	0.273	16.93
MAF5	-0.904	0.524	-0.575	-0.419	0.376	14.86

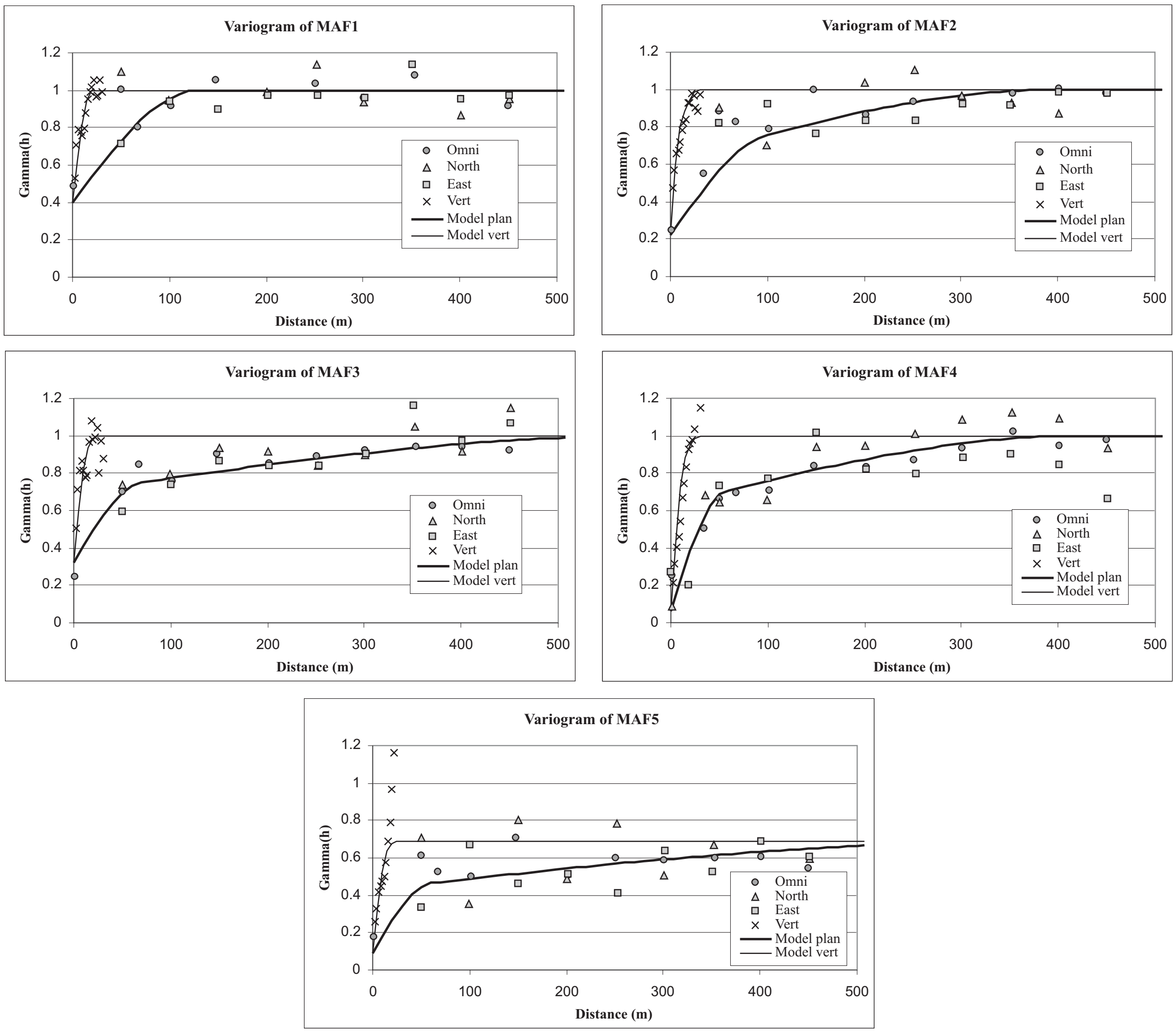


FIG 4 - Variography of MAF for the external zone.

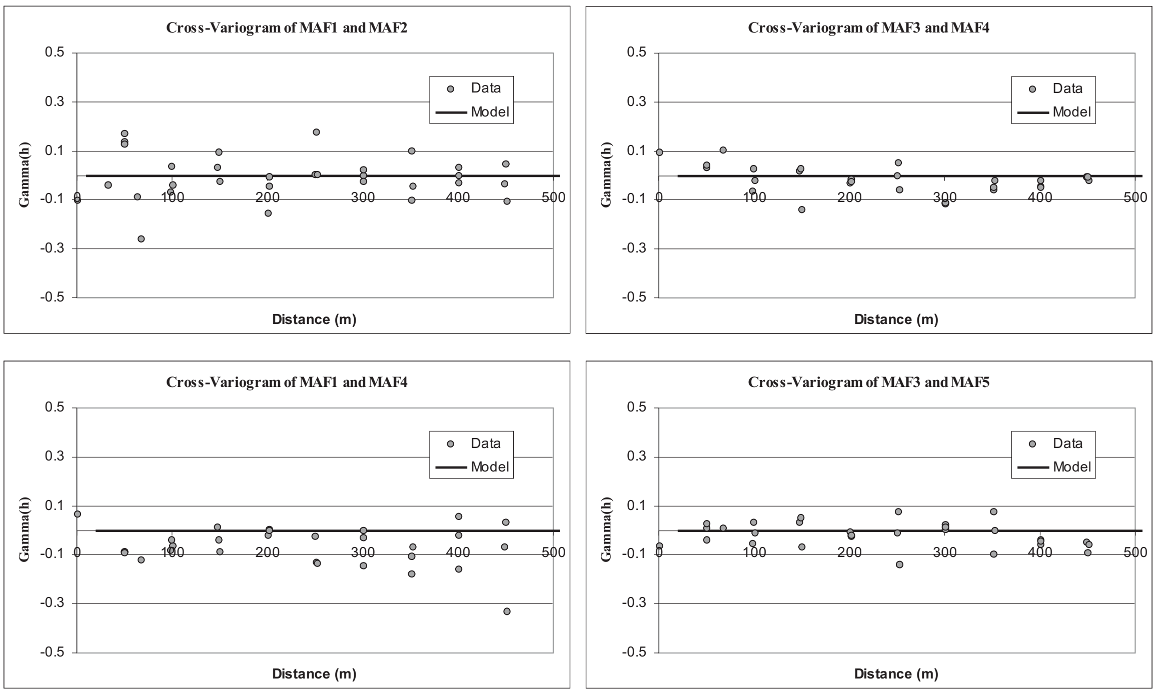


FIG 5 - Examples of cross-variograms of MAF.

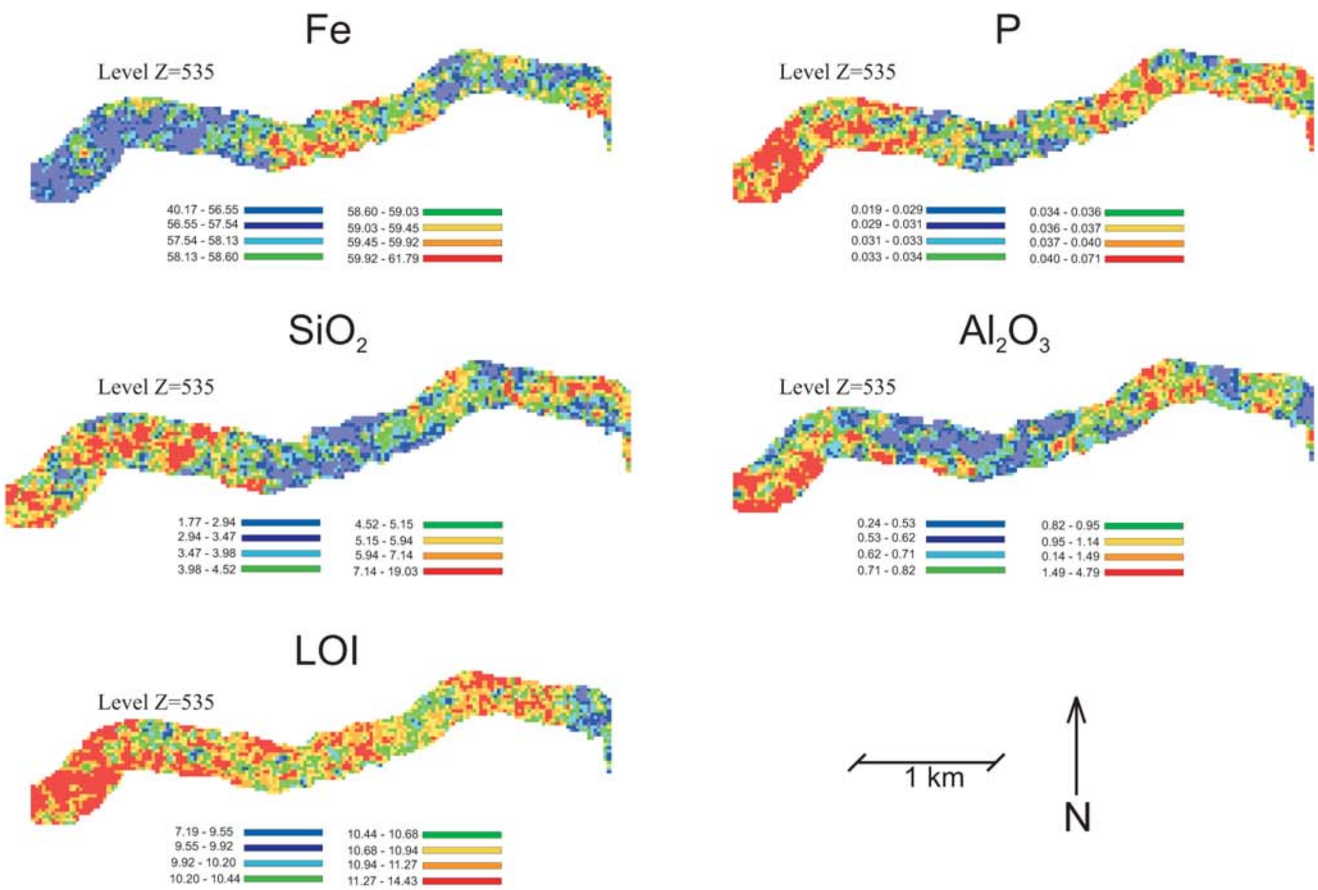


FIG 6 - Horizontal section of a joint simulation at level 535.

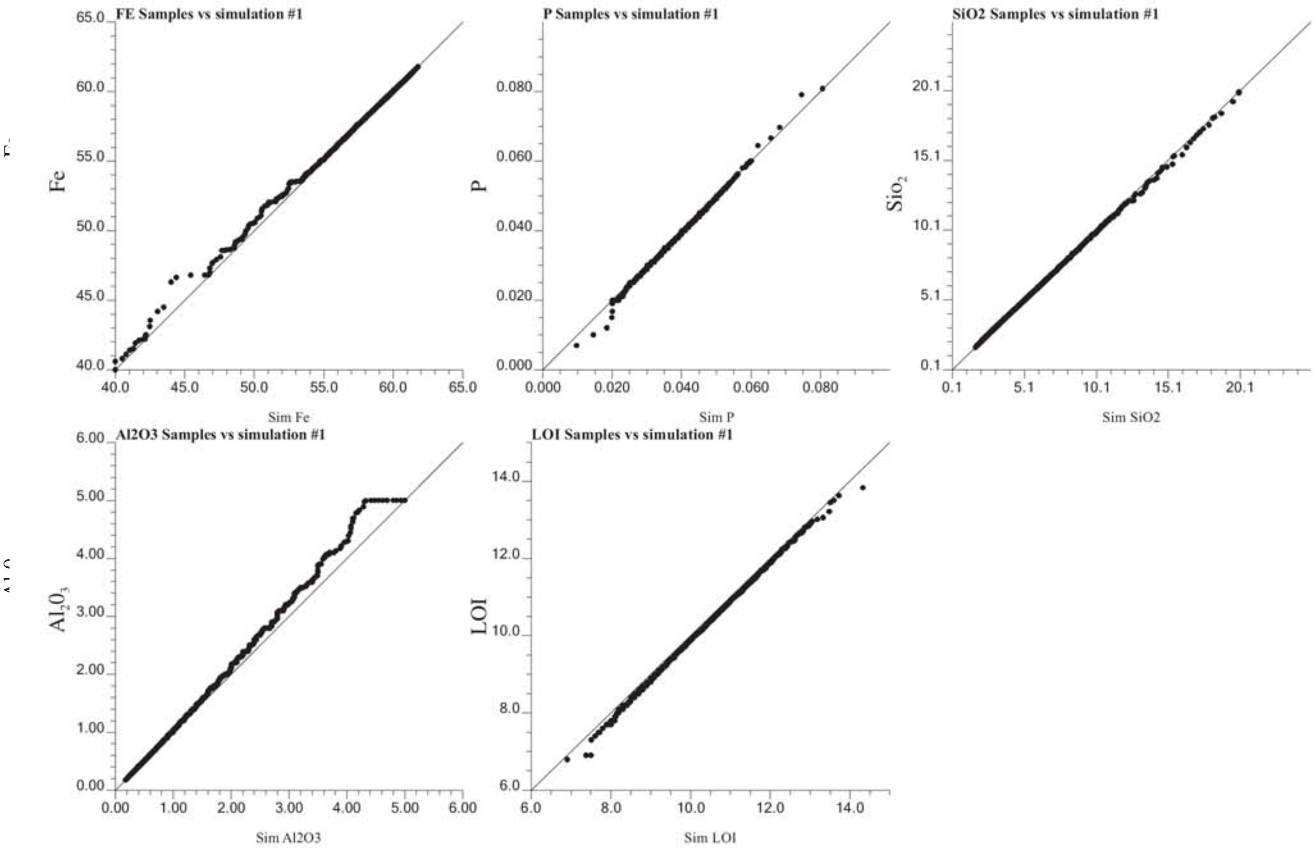


FIG 7 - Quantile-quantile plots for the external zone.

Validation of variogram and cross-variogram reproduction is performed in the data space at the block-support scale. For each element the regularised data variogram and cross-variogram models at block support (25 m × 25 m × 2 m) are compared with their corresponding variograms of the 20 simulations. Figure 8 shows representative examples from the simulations. As with all the preceding validation checks, the figures show a very good level of reproduction of the desired spatial statistics with the exception of Al_2O_3, where a problem arises from the normal-score transformation (further examined in a subsequent section). Of particular importance is the correlation between Fe and SiO_2. This correlation is very well preserved because the standard and the rank correlations are very close. Details of the simulation at Yandi presented elsewhere (Boucher, 2003) show that the method does not introduce spatial correlation between variables where it does not exist. The simulations reproduce very well, despite the various transformations; spatial features of the original data have not been explicitly modelled. This observation has also been made in point simulations using MAF (Dimitrakopoulos and Fonseca, 2003).

Further validation in the normal-score space is considered for Al_2O_3 and the related variogram and cross-variogram reproduction by the simulation process. As noted earlier, normal-score transformations are rank transformations and preserve only the rank correlations when the data are normalised, hence the poor reproduction of the cross-variograms for a few elements. The variograms of the simulated normal-score blocks (Figure 9) fit very well with the regularised model of the normal-score transformation. The cross-variogram between SiO_2-LOI is presented as an example in Figure 9, as the block cross-variogram has a slightly better fit with the regularised data variogram in the normal-score space than in the data space.

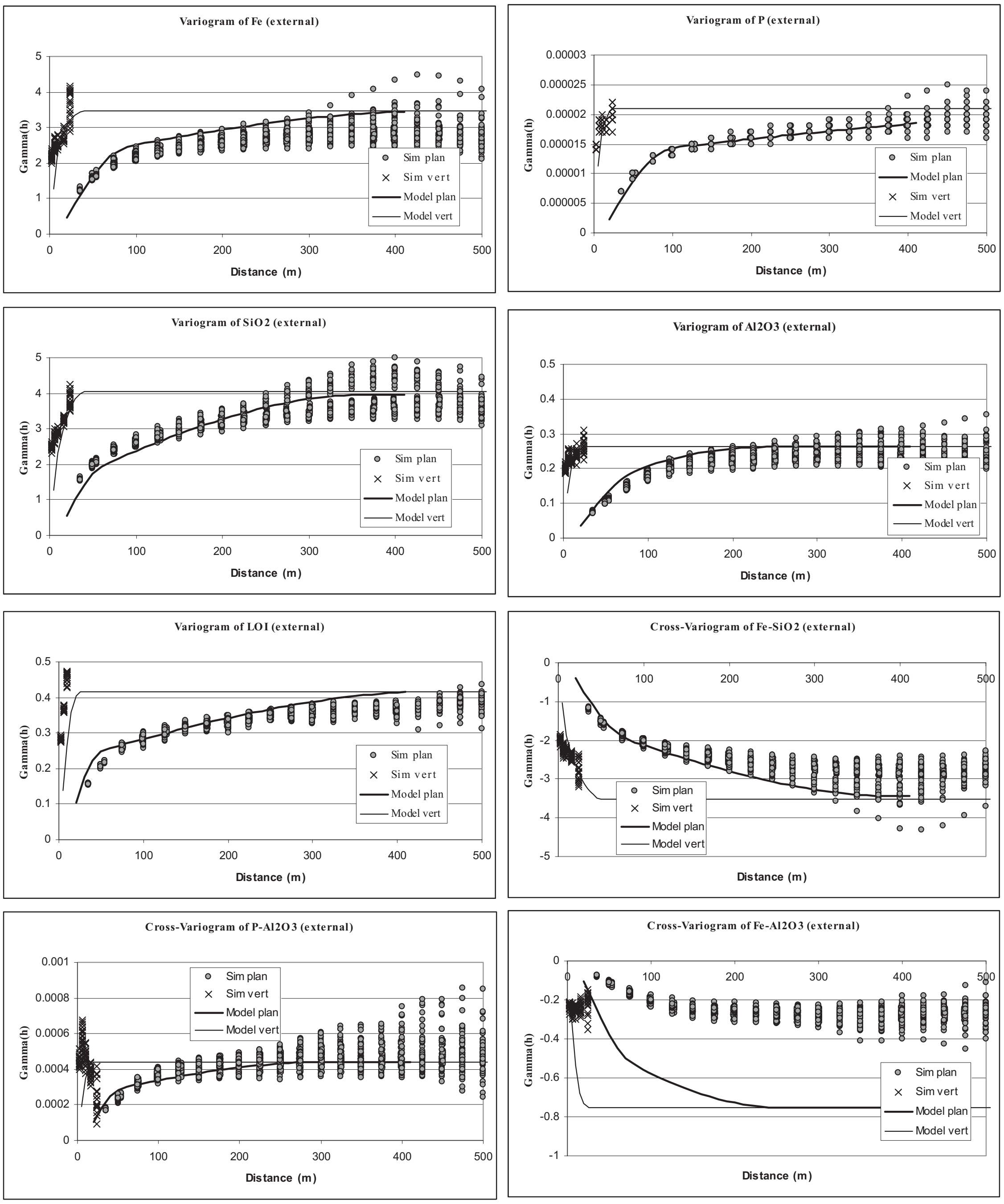


FIG 8 - Selected variograms and cross-variograms from a joint simulation and regularised variograms for blocks of the external zone.

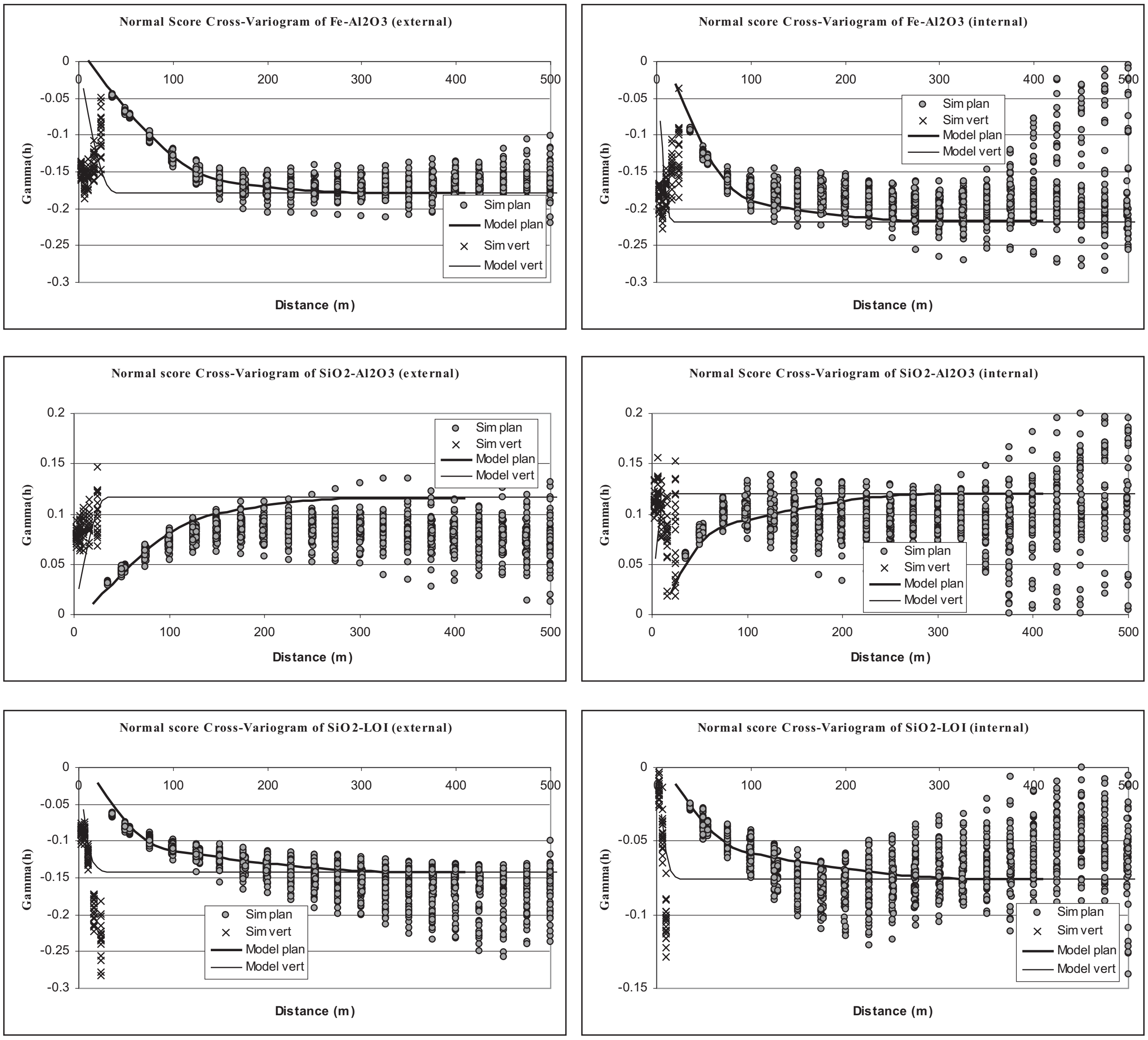


FIG 9 - Normal-score and regularised cross-variograms for block support of Fe-Al_2O_3, SiO_2-Al_2O_3 and SiO_2-LOI for the external and internal zones.

Of additional interest is the reproduction of vertical trends, common in the attributes of Fe ore deposits. Figure 10 represents examples from the external zone that show the reproduction of vertical trends to be excellent. Of further interest, Figure 11 shows examples of the reproduction of cross-plots of data in simulated realisations. Details may be found in Boucher (2003).

Discussion of the results

In general, the simulations show excellent results, especially when considering the geological complexity of this type of deposit. It is possible that in all cases used, the normal-score transformation may limit the reproduction of cross-variogram sills in the data space, specifically when the rank correlation does not equal the standard correlation. The effect may occur because the back-transformation from normal-score space to the data space tends to reproduce the rank correlation irrespective of the original standard correlation. This effect, nevertheless, may be desirable and affects any method that requires Gaussian data and normal-score transformation. An alternative could be the use of a direct sequential simulation algorithm, where the normal-score transformation is not used (Soares, 2001).

COMPUTATIONAL EFFICIENCY

A key reason for considering joint simulations based on MAF transformations, as well as developing the joint simulation method on a block-support scale, is the expected computational efficiency. This case study has documented this in detail (Boucher, 2003). For instance, consider the reduction in size of an output file. The 20 simulations of the external zone, containing a total of 663 360 blocks, produced a 60 megabyte file. However, if simulated on a point-support scale and subsequently post-processed to the same size blocks, a ten gigabyte file containing over 43 million points for each of the five elements is required. It is extremely difficult to work with and transfer such large files. On a relatively smaller scale, the size of the block output of the much smaller internal zone is only 13 megabytes, while its point equivalent is larger than 2.3 gigabytes. With respect to time required on a desktop computer with an Intel 2 GHz processor and 530 megabytes of RAM,

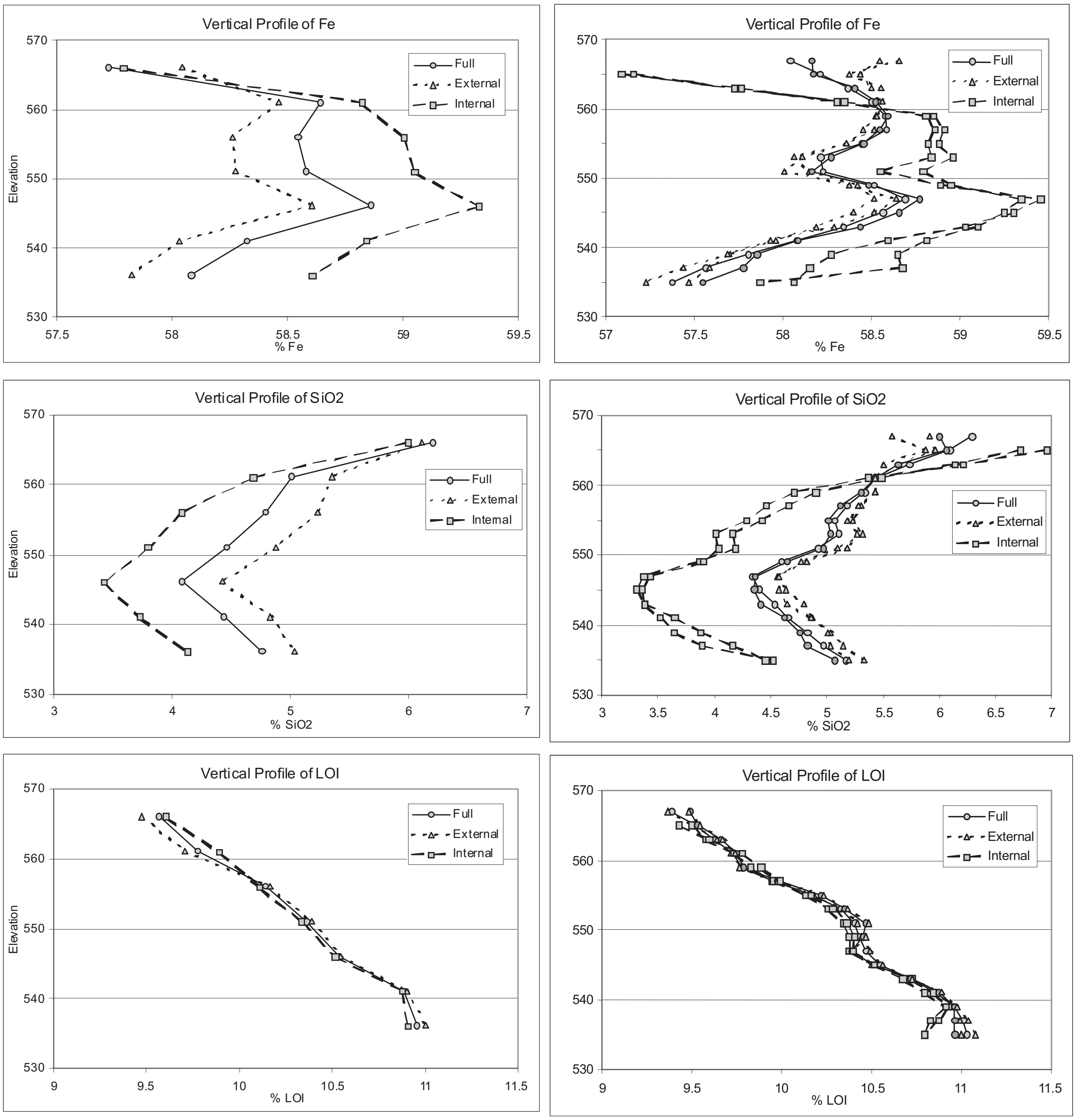


FIG 10 - Example of vertical profiles (trends) in the original data (left column) and their reproduction in simulations (right column).

it took approximately 25 hours to produce the 20 simulations of all five elements for both zones. This is a fraction of the time that conventional joint simulation methods would require. Clearly, the method presented in this paper removes major constraints in the handling of industrial-size joint simulations of multivariable deposits.

CONCLUSIONS

This paper has outlined a new and efficient framework for the joint simulation of correlated variables based on minimum/maximum autocorrelation factors. These factors are used to transform a set of deposit attributes of interest to independent factors that are then simulated independently. The simulated realisations of these factors are subsequently back-transformed to jointly correlated simulations. The joint simulation directly at the block-support scale using MAF was particularly stressed here. The DBMAFSIM algorithm presented is very efficient and offers the possibility of a completely new range of applications. With realistic simulated representation of multivariable deposits, new and more realistic complex transfer functions for mine planning and optimisation can now be used or developed. The application at the Yandi Central 1 iron ore deposit showed the excellent performance of the modified MAF approach as well as the exceptional efficiency of the method in terms of computation and storage costs.

ACKNOWLEDGEMENT

The work in this paper is part of ARC Grant # LP0211446 to R Dimitrakopoulos and is also funded by Anglo Gold Ashanti, BHP Billiton, Rio Tinto and Xstrata. Thanks are in order to Andrew A Bailey (BHP Billiton).

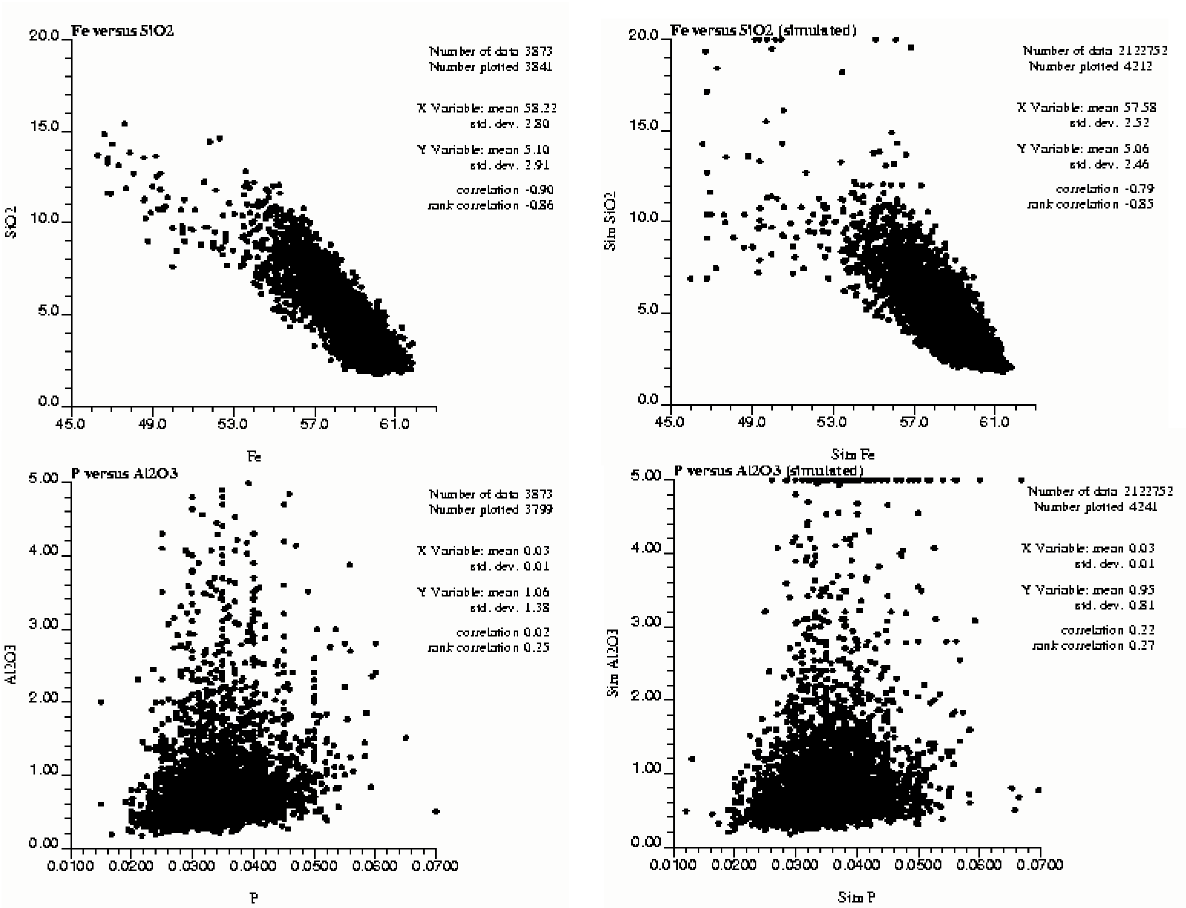


FIG 11 - Example cross-plots of data (left column) and joint simulated nodes (right column).

REFERENCES

Benndorf, J and Dimitrakopoulos, R, 2007. New efficient methods for conditional simulation of large orebodies, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 61-67 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Boucher, A, 2003. Conditional joint simulation of random fields on block support, MPhil thesis, University of Queensland, Brisbane, 161 p.

Chilès, J P and Delfiner, P, 1999. *Geostatistics, Modelling Spatial Uncertainty,* 695 p (John Wiley and Sons: New York).

David, M, 1988. *Handbook of Applied Advanced Geostatistical Ore Reserve Estimation*, 216 p (Elsevier: Amsterdam).

David, M, Dagbert, M, Sergerie, G and Cupcic, F, 1984. Complete estimation of the tonnage, shape and grade of a Saskatchewan uranium deposit, in *Twenty-seventh International Geology Congress*, pp 154-486 (VNU Science Press).

Davis, M D, 1987. Production of conditional simulations via the LU triangular decomposition of the covariance matrix, *Mathematical Geology*, 19(2):91-98.

Desbarats, A J and Dimitrakopoulos, R, 2000. Geostatistical simulation of regionalized pore-size distributions using min/max autocorrelation factors, *Mathematical Geology*, 32:919-942.

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R and Fonseca, M B, 2003. Assessing risk in grade-tonnage curves in a complex copper deposit, northern Brazil, based on an efficient joint simulation of multiple correlated variables, in *Proceedings APCOM/SAIMM*, pp 373-382.

Dimitrakopoulos, R and Luo, X, 2004. Generalized sequential Gaussian simulation on group size *v* and screen-effect approximation for large field simulations, *Mathematical Geology*, 36:567-591.

Godoy, M, 2003. The effective management of geological risk in long-term scheduling of open pit mines, PhD thesis, University of Queensland, Brisbane.

Goovaerts, P, 1993. Spatial orthogonality of the principal components computed from coregionalized variables, *Mathematical Geology*, 25:281-302.

Hall, G C and Kneeshaw, M, 1990. Yandicoogina-Marillana pisolitic iron deposits, in *Geology of the Mineral Deposits of Australia and Papua New Guinea* (ed: F E Hughes), Vol 2, pp 1581-1586 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Journel, A G and Huijbregts, C J, 1978. *Mining Geostatistics*, 600 p (Academic Press: London).

Myers, D E, 1988. Vector conditional simulation, in *Geostatistics*, pp 283-292 (Kluwer Academic Publishers: Avignon).

Soares, A, 2001. Direct sequential simulation and cosimulation, *Mathematical Geology*, 33(8):911-926.

Stone, M S, George, A D, Kneeshaw, M and Barley, M E, 2002. Stratigraphy and sedimentary features of the tertiary Yandi Channel iron deposits, Hamersley Province, Western Australia, in *Proceedings Iron Ore 2002*, pp 137-144 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Suro-Perez, V and Journel, A G, 1991. Indicator principal component kriging, *Mathematical Geology*, 23:759-788.

Switzer, P and Green, A A, 1984. Min/max autocorrelation factors for multivariate spatial imagery, Technical Report No 6, Department of Statistics, Stanford University.

Verly, G W, 1993. Sequential Gaussian cosimulation; a simulation method integrating several types of information, in *Geostatistics* (ed: A Soares) Vol 5, pp 543-554 (Kluwer Academic Publishers: Dordrecht).

Wackernagel, H, Petitgas, Y and Touffait, Y, 1989. Overview of methods for coregionalization analysis, in *Geostatistics*, pp 409-420 (Kluwer Academic Publishers: Dordrecht).

Modelling the Geometry of Geological Units and its Uncertainty in 3D from Structural Data — The Potential-Field Method

J-P Chilès[1], C Aug[1], A Guillen[2] and T Lees[3]

ABSTRACT

Most 3D geological modelling tools were designed for the needs of the oil industry and are not suited to the variety of situations encountered in other application domains. Moreover, the usual modelling tools are not able to quantify the uncertainty of the geometric models generated. The potential-field method was designed to build 3D geological models from data available in geology and mineral exploration, namely the geological map and a DTM, structural data, borehole data and interpretations of the geologist. This method considers a geological interface as a particular isosurface of a scalar field defined in the 3D space, called a potential field. The interpolation of that field, based on universal cokriging, provides surfaces that honour all the data.

Due to the difficulty of inferring the covariance of the potential field, the first implementation of the method used an *a priori* covariance given by the user. New developments allow this covariance to be identified from the structural data. This makes it possible to associate sensible cokriging standard deviations to the potential-field estimates and to express the uncertainty of the geometric model.

Practical implementation issues for producing 3D geological models are presented: how to handle faults, how to honour borehole ends, how to take relationships between several interfaces into account, how to integrate gravimetric and magnetic data.

An application to the geological modelling of the Broken Hill district, Australia, is briefly presented.

INTRODUCTION

The resource evaluation of a mining deposit is often decomposed in two steps:

1. delimitation of the boundaries of the units corresponding to the various geological formations or ore types, and
2. estimation of grades within each unit.

In simple cases (eg a series of subhorizontal layers), the geometric model can be built using 2D geostatistical techniques (kriging or cokriging of the elevations or thicknesses of the various horizons), which also quantify the uncertainty of the model. A lot of effort has been undertaken to develop 3D modelling tools capable of handling more complex situations (eg Mallet, 2003). Most of them were designed to fulfil the needs of the oil industry, namely for situations where a draft of the underground model can be defined from seismic data. Deterministic methods are also available to interpolate between subparallel interpreted cross-sections.

When assessing resources, the knowledge of the degree of uncertainty of the estimation is as important as the estimate itself. The uncertainty on the boundaries and volumes of the various units is often a major part of the global uncertainty. When 2D geostatistical techniques can be used, the quantification of that uncertainty by an estimation variance is a valuable by-product of the estimation process. By contrast usual 3D modelling tools are not able to quantify the uncertainty attached to the interpolated model, whereas that uncertainty can be quite large.

1. Centre de Géostatistique, École des Mines de Paris, 35 rue Saint-Honoré, 77305 Fontainebleau Cedex, France.

2. Bureau de Recherches Géologiques et Minières (BRGM), BP 6009, 45060 Orléans Cedex 2, France.

3. School of Geosciences, Monash University, Victoria, Australia.

The potential-field method was designed to build 3D geological models from data available in geology and mining exploration, namely:

1. a geological map and a digital terrain model (DTM),
2. structural data related to the geological interfaces,
3. borehole data, and
4. interpretations from the geologist.

It is not limited to sedimentary deposits and does not require seismic data (such data would be useful but is seldom available in geological, mining and civil engineering applications). It can be linked to inverse methods to take gravimetric and/or magnetic data into account.

The potential-field method defines a geological interface as an implicit surface, namely a particular isosurface of a scalar field defined in the 3D space – the potential field. The 3D interpolation of that potential field, based on universal cokriging, provides isosurfaces that honour all the data. Since no data measures the potential field itself, its covariance cannot be inferred directly, so that the method was used with a covariance chosen by the user, thus making the method a conventional one, among others. Recent developments allow that covariance to be determined from the structural data, which makes it possible to associate sensible cokriging standard deviations to potential-field estimates and to translate them into uncertainties on the 3D model.

We will first recall the basic principle of the method, present the inference of the potential-field covariance from the structural data and explain how the uncertainty of the 3D model can be quantified. We will then examine several practical issues: how to handle faults, how to honour borehole ends, how to take relationships between several interfaces into account, how to link 3D geometrical modelling and inverse modelling of gravimetric and magnetic data. We will end with a brief presentation of an application to the geological modelling of the Broken Hill district, Australia, and a short discussion.

BASIC PRINCIPLE OF THE POTENTIAL-FIELD METHOD

The basic method – which will be generalised in the sequel – is designed to model a geological interface or a series of subparallel interfaces I_k, $k = 1, 2, \ldots$ (Lajaunie, Courrioux and Manuel, 1997). Its principle is to summarise the geology by a potential field, namely a scalar function $T(\mathbf{x})$ of any point $\mathbf{x} = (x, y, z)$ in 3D space, designed so that the interface I_k corresponds to an isopotential surface, ie the set of points $\mathbf{x}$ that satisfies $T(\mathbf{x}) = t_k$ for some unknown value t_k of the potential field. Equivalently, the geological formation encompassed between two successive interfaces I_k and $I_{k'}$ is defined by all the points $\mathbf{x}$ whose potential-field value lies in the interval defined by t_k and $t_{k'}$. In figurative terms, in the case of sedimentary deposits, T could be seen as the time of deposition of the grain located at $\mathbf{x}$, or at least as a monotonous function of that geological time, and an interface as an isochron surface. This figurative interpretation can be adequate in some applications but is not necessary for the development of the method.

Data types

$T(\mathbf{x})$ is modelled with two kinds of data, as shown in Figure 1:

1. points known to belong to the interfaces $I_1, I_2, \ldots$, typically 3D points discretising geological contours on the geological map and intersections of boreholes with these interfaces; and
2. structural data: in the case of sedimentary rocks whose stratification is parallel to the geological horizons, this data is polarised unit vectors normal to the stratification; similarly they can be unit vectors orthogonal to foliation planes for metamorphic rocks; this data is measured on outcrops or in boreholes, either on the interfaces or anywhere within a formation.

For the interpolation of the potential field, this data is coded as follows:

1. Since the potential value at $m + 1$ points $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n$ sampled on the same interface is not known, this data is taken as m increments $T(\mathbf{x}_\alpha) - T(\mathbf{x}'_\alpha)$, $\alpha = 1, \ldots, m$, all valued to 0. Two classical choices for $\mathbf{x}'_\alpha$ consist in taking either the point $\mathbf{x}_0$ whatever α, or the point $\mathbf{x}_{\alpha-1}$ (the choice has no impact on the result; other choices are possible provided that the increments are linearly independent). Since the sampled data can be located on several interfaces, let M represent the total number of increments (it is equal to the total number of data points on the interfaces, minus the number of interfaces).
2. The polarised unit vector normal to each structural plane is considered as the gradient of the potential field, or equivalently as a set of three partial derivatives $\partial T(\mathbf{x}) / \partial u$, $\partial T(\mathbf{x}) / \partial v$, $\partial T(\mathbf{x}) / \partial w$ at some point $\mathbf{x}_\beta$. The coordinates u, v, w are defined in an orthonormal system; this system can be the same for all the points or a specific system can be attached to each point (the result does not depend on the choice provided that the three partial derivatives are taken in consideration). In the sequel let $\partial T(\mathbf{x}_\beta) / \partial u_\beta$ denote any partial derivative at $\mathbf{x}_\beta$ and N denote the total number of such data (in practice N is a multiple of three and the $\mathbf{x}_\beta$ form triplets of common points). Let us recall that the $\mathbf{x}_\beta$ do not necessarily coincide with the $\mathbf{x}_\alpha$ (the latter are located on the interfaces whereas the former can be located anywhere).

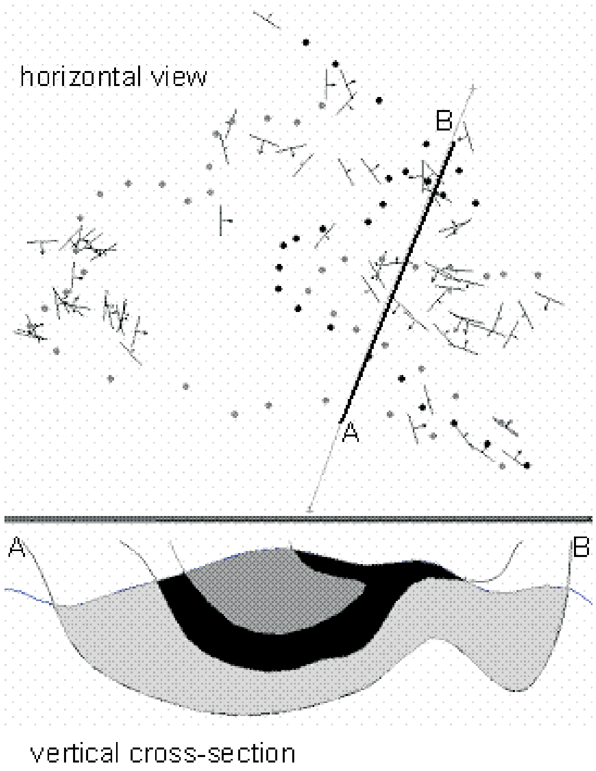


FIG 1 - Principle of the potential-field method. Top: surface data – points at interfaces and structural data; bottom: vertical cross-section through the 3D model (Courrioux *et al,* 1998).

Interpolation of the potential field

The potential field is then only known by discrete or infinitesimal increments. It is thus defined up to an arbitrary constant. So an arbitrary origin $\mathbf{x}_0$ is fixed and at any point $\mathbf{x}$ the potential increment $T(\mathbf{x}) - T(\mathbf{x}_0)$ is kriged. The estimator is in fact a cokriging of the form:

$$T^*(\mathbf{x}) - T^*(\mathbf{x}_0) = \sum_{\alpha=1}^{M} \mu_\alpha \big(T(\mathbf{x}_\alpha) - T(\mathbf{x}'_\alpha)\big) + \sum_{\beta=1}^{N} \nu_\beta \frac{\partial T}{\partial u_\beta}(\mathbf{x}_\beta)$$

where the weights μ_α and ν_β, solution of the cokriging system, are in fact functions of $\mathbf{x}$ (and $\mathbf{x}_0$). One may wonder why the potential increments are introduced in that estimator since their contribution is nil. Because, and this is key, the weights ν_β are different from weights based on the gradient data alone. Conversely, the gradient data also play a key role, because in their absence the estimator would be zero whatever $\mathbf{x}$ may be.

Cokriging is performed in the framework of a random function model. $T(.)$ is assumed to be a random function with a polynomial drift:

$$m(\mathbf{x}) = \sum_{\ell=0}^{L} b_\ell f^\ell(\mathbf{x})$$

and a stationary covariance $K(\mathbf{h})$. Since the vertical usually plays a special role, the degree of the polynomial drift can be higher vertically than horizontally and the covariance can be anisotropic. For example, if we model several subparallel and subhorizontal interfaces, it makes sense to assume a vertical linear drift of the form $m(\mathbf{x}) = b_0 + b_1 z$, ie with two basic drift functions $f^0(\mathbf{x}) \equiv 1$ and $f^1(\mathbf{x}) = z$. A geological body with the shape of an ellipsoid would correspond to a quadratic drift, ie to the ten basic monomial drift functions with degree less than or equal to two. Note, however, that the drift function $f^0(\mathbf{x}) \equiv 1$ shall be forgotten in any case since the potential increments as well as the partial derivatives filter b_0. In theory, sinusoidal terms could be added too (Dimitrakopoulos and Luo, 1997), but in usual applications geology is not regular enough for that.

Once the basic functions $f^\ell(\mathbf{x})$ of the drift and the covariance $K(\mathbf{h})$ of $T(.)$ are known, we have all the ingredients to perform a cokriging in the presence of gradient data, as shown in Chilès and Delfiner (1999, section 5.5.2). Indeed, the drift of $\partial T(\mathbf{x}) / \partial u$ is simply $\partial m(\mathbf{x}) / \partial u$, ie a linear combination of the partial derivatives $\partial f^\ell(\mathbf{x}) / \partial u$ with the same unknown coefficients b_ℓ as for $m(\mathbf{x})$, the covariances of partial derivatives are second-order partial derivatives of $K(.)$, and the cross-covariances of the potential field and partial derivatives are partial derivatives of $K(.)$.

Implementation of the cokriging algorithm

Since the potential increment data in fact do not contribute to the final cokriging estimate, the estimator can be seen as an integration of the gradient data. To preserve the spatial continuity of the cokriging estimates it is wise to work in unique neighbourhood, namely to effectively include all the data in the cokriging of $T(\mathbf{x})$, whatever $\mathbf{x}$ may be. If we are not interested in the cokriging variance, cokriging can be implemented in its dual form, which has two advantages:

1. the cokriging system is solved once for all, which saves computing time; and

2. that form is especially suited when cokriging is considered as an interpolator, because it allows an easy estimation of $T(\mathbf{x}) - T(\mathbf{x}_0)$ at any new point $\mathbf{x}$.

The latter property is very useful to display 3D views of the geological model with an algorithm such as the marching cube, which starts from the estimation of $T(\mathbf{x}) - T(\mathbf{x}_0)$ at the nodes of a coarse regular grid and then requires intermediate points to be predicted to track the desired isopotential surface.

INFERENCE OF THE COVARIANCE OF THE POTENTIAL FIELD

In usual geostatistical applications, the covariance or variogram of the variable under study is modelled from the sample variogram of the data. In the present case, we have no measurement of the potential $T(\mathbf{x})$, and the potential increments used for the interpolation cannot be used for the inference of K since they all have a zero value. In its first implementation, the algorithm was used heuristically with a covariance model arbitrarily chosen by the user. That choice had been more or less rationalised according to the following considerations:

1. At the scale considered, geological interfaces are smooth rather than fractal surfaces, which implies that the covariance is twice differentiable. A cubic model was considered as a good compromise among the various possible models, because it just has the necessary regularity at the origin and has a scale parameter that can accommodate various situations.
2. The scale parameter a and sill C of the covariance $K(\mathbf{h})$ determine the sill of the variogram of the partial derivatives: it is equal to $14\ C / a^2$ in the case of an isotropic cubic covariance considered here. When there is no drift and the geological body is isotropic (eg a granitic intrusion), the unit gradient vector can have any direction so that its variance is equal to one. The variance of each partial derivative is then equal to one third. A consistent choice for C once the scale parameter a has been chosen is thus $C = a^2 / 42$. That value shall be considered as an upper bound for C when the potential field has a drift, because in that case the mean of the potential gradient is not equal to zero so that its variance is shorter than one (its quadratic mean is zero by definition).
3. Sensible measurement variances can also be defined (nugget effects).

The use of a heuristic model, however, implies two limitations:

1. The choice is usually not the best one.
2. More importantly, this precludes any evaluation of the magnitude of the interpolation error. A means to infer the covariance is thus a core issue of that approach.

Since K cannot be inferred from the potential increments, its inference shall be done with the gradient data. This is possible because the covariances of the partial derivatives derive from that of the potential field. In the case of an isotropic covariance $K(\mathbf{h})$, which for simplicity will be denoted $K(r)$ as a function of $r = \|\mathbf{h}\|$, the covariance of, say, $\partial T(\mathbf{x}) / \partial u$ and $\partial T(\mathbf{x}+\mathbf{h})/\partial u$ is $-K''(\|\mathbf{h}\|)$ when $\mathbf{h}$ is parallel to the u axis, $-K'(\|\mathbf{h}\|) / \|\mathbf{h}\|$ when $\mathbf{h}$ is orthogonal to the u axis.

The assumption of an isotropic covariance model is of course too restrictive and shall be relaxed. In practice the covariance $K(\mathbf{h})$ is supposed to be the sum of several cubic components $K_p(\mathbf{h})$, each one possibly displaying a zonal or geometric anisotropy. To avoid a too great complexity, the main anisotropy axes u, v, w, are supposed to be common to all the components. More general formulae than the above ones are available for that model.

Thanks to these formulae the covariance parameters of K (nugget effect, scale parameter of each covariance component in the three main directions, sill of each component) are chosen so as to lead to a satisfactory global fit of the directional sample variograms of the three components of the gradient. An automatic fitting procedure based on the Levenberg-Marquardt method has been developed to facilitate that task (Aug, 2004).

Figure 2 shows an example of such a fitting. 1485 structural data was sampled in an area of about 70 × 70 km^2 in the Limousin (Massif Central, France). The main (u, v, w) coordinates here coincide with the geographical (x, y, z) coordinates. Since the structural data is all located on the topographic surface, the variograms have been computed in the horizontal plane only. Note that the sill of the variogram of the vertical component is much lower than that of the horizontal components. This is due to the fact that the layers are subhorizontal so that the vertical component of the gradient displays limited variations around its non-zero mean. The model K includes three components, the second of which only depends on the horizontal component of $\mathbf{h}$ and the third one on the N-S component (zonal anisotropies).

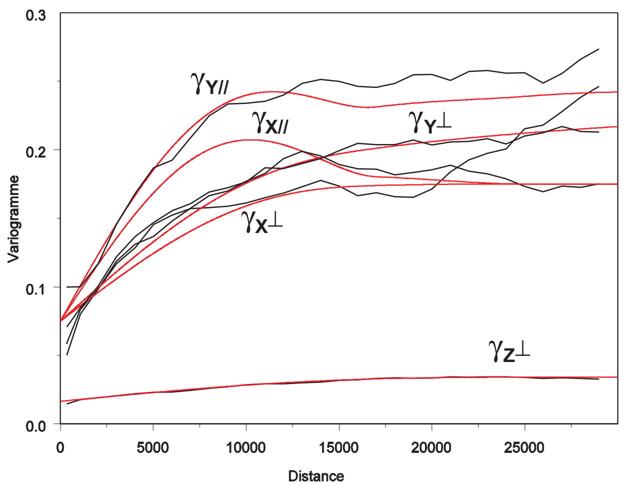


FIG 2 - Example of fitting of the covariance of the potential field from the sample variograms of the partial derivatives of the potential field. Limousin dataset, Massif Central, France. $\gamma X_{//}$ and $\gamma X^{\perp}$ denote the variogram of the partial derivative $\partial T/\partial x$ respectively along and orthogonal to direction x (Aug, 2004).

UNCERTAINTY OF THE 3D MODEL

Case studies have shown that the use of a sound covariance model improves the model in comparison with the use of a conventional model. An additional interest of using a covariance fitted from the data is the possibility to obtain sensible cokriging standard deviations. Indeed, when the covariance model is *a priori* chosen by the user, cokriging is a conventional interpolator, among others, which cannot claim for optimality, and the cokriging variance is a mere configuration index.

When the 'true' covariance of the potential field is known, a meaningful cokriging standard deviation $\sigma_{CK}(\mathbf{x})$ can be associated with the cokriging of $T(\mathbf{x}) - T(\mathbf{x}_0)$. The calculation of that standard deviation requires the use of the standard form of the cokriging system, which calls for more computing time than its dual form (this is the price to pay for knowing the uncertainty attached to the geological model). Let us suppose that some geological formation is defined by the set of points $\mathbf{x}$ such that $T(\mathbf{x}) - T(\mathbf{x}_0)$ is comprised between two values t and t'. Under the assumption that the potential field is a Gaussian random function

– an assumption that seems reasonable in the present context – the probability that a given point **x** belongs to that formation is:

$$\Pr\{t \le T(\mathbf{x}) - T(\mathbf{x}_0) < t'\} = G\left(\frac{t' - (T^*(\mathbf{x}) - T^*(\mathbf{x}_0))}{\sigma_{CK}(\mathbf{x})}\right) - G\left(\frac{t - (T^*(\mathbf{x}) - T^*(\mathbf{x}_0))}{\sigma_{CK}(\mathbf{x})}\right)$$

where:

G is the standard normal cumulative distribution function

Similarly, if we are interested in the interface passing by the point $\mathbf{x}_0$, namely in the set of points **x** such that $T(\mathbf{x}) - T(\mathbf{x}_0) = 0$, the variable $R(\mathbf{x}) = [T^*(\mathbf{x}) - T^*(\mathbf{x}_0)]/\ \sigma_{CK}$ measures the likelihood that **x** belongs to the interface. Indeed, writing the obvious relation:

$$T(\mathbf{x}) - T(\mathbf{x}_0) = T^*(\mathbf{x}) - T^*(\mathbf{x}_0) + \text{cokriging error}$$

we see that **x** belongs to the interface if and only if $T^*(\mathbf{x}) - T^*(\mathbf{x}_0)$ is equal to minus the cokriging error, or equivalently if $R(\mathbf{x})$ is equal to minus the standardised cokriging error (the ratio of the error by $\sigma_{CK}(\mathbf{x})$). The value of that error is not known but it is a variable with zero mean and unit variance.

For example, assuming again that the potential field is Gaussian, the area defined by $|R(\mathbf{x})| < 2$ includes about 95 per cent of the actual interface. Figure 3 displays $R(\mathbf{x})$ for the top of the lower gneiss unit in the Limousin. The black line corresponds to $R(\mathbf{x}) = 0$, ie to the isovalue surface of the cokriged potential field passing by the data points sampled on that interface. The true interface is likely to be found in the light-coloured area, whereas the darkest area can be considered as a forbidden area.

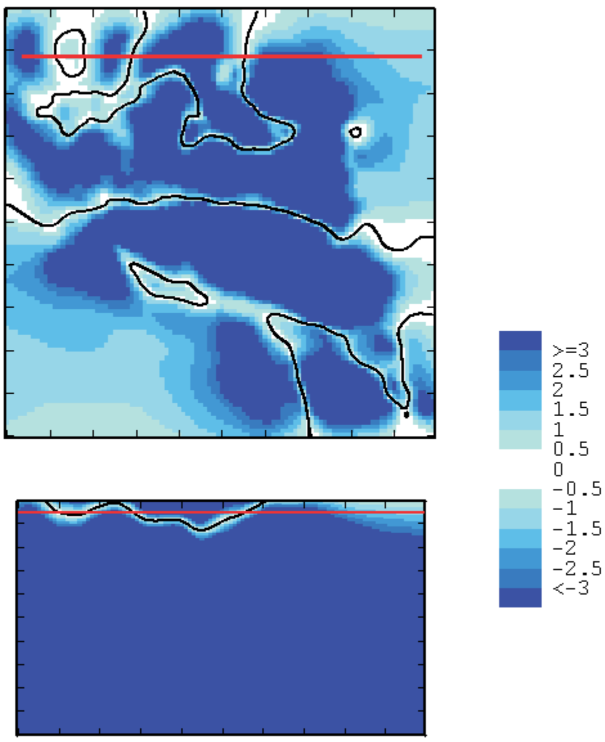


FIG 3 - Representation of the uncertainty of the top of a geological unit by the variable $R(\mathbf{x})$ (upper gneiss unit, Limousin). The data (geological map and structural data) are all located on the topography. Top: map of a zone of 65 km × 65 km in the horizontal plane with elevation 500 m; bottom: vertical E-W cross-section with 62 km extension and 34 km depth. The black curve represents the kriged interface. The true interface is in fact in the shaded zones, with a smaller probability as the zone is darker. The darkest zones can be considered as exclusion zones (Aug, 2004).

PRACTICAL IMPLEMENTATION ISSUES

The potential-field method has been implemented in 3D Geological Editor, software developed by BRGM (the French geological survey). In order to model real-world situations a number of practical implementation issues had to be solved.

Modelling several interfaces

In practical applications several interfaces shall be modelled, and all of them are not subparallel. Several potential fields are then used. A stratigraphic column is defined by the geologist to determine how to combine the various potential fields. That column defines the chronological order of the interfaces as well as their nature, coded as either 'erode' or 'onlap'. An 'erode' potential field is used for example to mask the eroded part of the previous formations or to model an intrusive body.

Faults

Several methods can be envisaged to handle faults. If they delimit blocks and the potential field is not correlated from one block to the other, it obviously suffices to process each block separately. Another conventional technique is to consider faults as screens. This technique cannot be used in unique neighbourhood. The method used in 3D Geological Editor is thus different. It is a transposition to 3D potential fields of the method proposed by Maréchal (1984) to handle faults in the 2D interpolation of the elevation of interfaces, where faults are entered as external drift functions. This method requires the knowledge of the fault planes and also of the zones of influence of the faults.

Let us start with a very simple example, a normal fault intersecting the whole study zone and dividing it in two subzones D and D'. That fault induces a discontinuity of the potential field, whose amplitude is not known. Cokriging can accommodate that discontinuity whatever its amplitude by introducing a drift function complementing the L polynomial drift functions, for example:

$$f^{L+1}(\mathbf{x}) = 1_D(\mathbf{x}),$$

or equivalently, in a symmetrised form:

$$f^{L+1}(\mathbf{x}) = 1_D(\mathbf{x}) - 1_{D'}(\mathbf{x}).$$

If the polynomial drift functions include the monomial $f^1(\mathbf{x}) = x$ (first coordinate) due to the presence of a linear trend of the potential field, and we have good reasons to suspect not only a discontinuity but also a change of slope of the drift when crossing the fault, it is advisable to also introduce an additional drift function such as:

$$f^{L+2}(\mathbf{x}) = x\ 1_D(\mathbf{x}).$$

A finite fault can be modelled with a drift function with a bounded support, and whose value vanishes on the support boundaries; inside that support, the function takes on positive values on one side of the fault plane, with a maximum at the centre of the fault, and negative values on the other side. Figure 4 illustrates in 2D how that method takes faults into account.

In real-world applications a fault plane is not exactly a planar surface. It is often only known by some points on its surface and unit vectors orthogonal to it. Its geometry can thus be modelled by a potential field too.

Borehole ends

When processing borehole data, only the intersections of the boreholes with the interfaces are usually entered as data, whereas the borehole also carries the information that all the points between two successive interfaces belong to the same horizon.

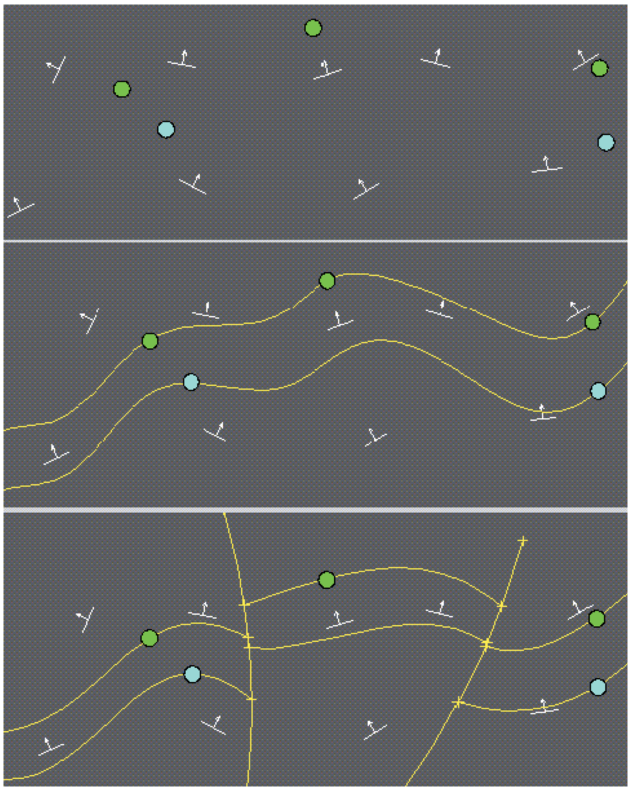


FIG 4 - Handling faults. Top: data points located on two interfaces and structural data; middle: model built without introducing any fault; bottom: model taking faults into account.

That additional information is usually redundant, so it can be legitimately disregarded. A noticeable exception is the end of a borehole: it does not coincide with an interface and gives the information that the next interface is deeper than the borehole. It is important to take that information into account, because otherwise the model can place the interface at a lower depth than the borehole. If that interface is modelled by a 2D interpolation of its elevation, such information is simply an inequality about the elevation value at the (x, y) location of the borehole. In the case of a 3D modelling of a potential field, it can also be expressed as an inequality about a potential-field increment.

Such inequalities can be taken into account by first replacing the inequality data with hard data and then applying the standard cokriging method. The critical step is of course the first one. The hard value replacing an inequality datum must be consistent with the inequality and all the other data (the hard data and the other inequality data) and with the spatial variability of the potential field. The method is rigorous when the inequality is replaced by the mathematical expectation of the potential increment conditional on all the hard and inequality data (Freulon and de Fouquet, 1993; Chilès and Delfiner, 1999). This is done with an iterative method, which is a direct application of the Gibbs sampler. Note that contrary to the usual potential field data, this new increment data is not equal to zero.

The practical implementation of the iterative process is based on a simple kriging algorithm. It is rigorous if the potential field is a Gaussian random function with known mean, because in that case kriging coincides with the conditional expectation. A Gaussian assumption does not look unnatural in our applications, but the potential fields considered usually include an unknown global drift. Aug (2004) has shown that the algorithm remains robust in the applications we consider when simple kriging is replaced by an ordinary or universal cokriging of our data. Figure 5 illustrates the consequences of using or not using that algorithm.

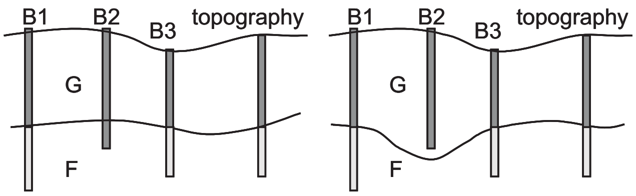


FIG 5 - Handling borehole ends. Left: end of borehole B2 not taken into account; right: end of borehole B2 taken into account.

Coupling with an inverse modelling of geophysical data

In geological and mining exploration applications, seismic profiles as well as gravity and magnetic data are often available. Interpreted seismic cross-sections directly provide data that can be processed by 3D Geological Editor. This is not the case for gravity and magnetic data. Presently, the geological model provided by the use of the potential-field method is considered as the initial state of a constrained inverse modelling of this data.

That inversion is based on an iterative method presented by Guillen *et al* (2004), which is applied to a discrete version of the domain under study. The domain is subdivided in cubic cells, with a geological formation and a physical property (density or magnetic susceptibility) attached to each cell. At the initial state the formation derives from the potential-field model, and the value of the physical property, eg density, is randomly chosen in an *a priori* distribution for that formation. The gravity response of the model at the location of the gravity data is computed. A cell is then randomly chosen and a tentative new state is proposed by changing the formation and/or density of that cell (a formation change is proposed only if it does not alter the topology of the model); that tentative state is accepted as a new state according to whether or not it improves the response of the model in comparison with the gravity data. That procedure is iterated millions of times. In fact the decision of accepting or not accepting a proposed state is taken according to a Metropolis-Hastings dynamic, which accepts some deterioration of the gravity response of the model, especially in the early iterations, to avoid a convergence of the algorithm to a local optimum.

APPLICATION TO THE BROKEN HILL DISTRICT

3D Geological Editor has been mainly used for geological modelling at a regional scale, especially in the Alps and the Massif Central. We present here the results of an application to geological data from the Broken Hill district, Australia.

Geological context

The project area is a 20 km × 20 km area (Figure 6) extending to a depth of 5 km. The rock units and their relationships, listed in Table 1, are based on the GSNSW synthesis (Willis, 1989); it is noted, however, that there is the possibility of major structures within the stratigraphy (Noble, 2000; Gibson and Nutman, 2004).

Two geological questions concerning the geometry of these units were posed at the beginning of this study:

1. Do the major units flatten at depth?
2. What is the relative importance of the different units and are they regionally extensive?

The objective was therefore to use the geological modelling tool to evaluate various geological hypotheses.

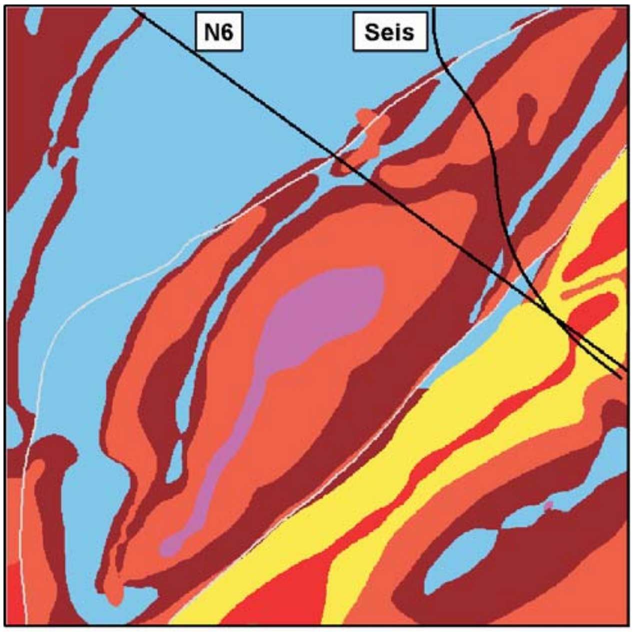


FIG 6 - Plan view of the Broken Hill geological model. The shades correspond to the geological units shown in Table 1. The presence of a fault is indicated by a white line. The location of Section N6 (Figure 7) and the seismic line are shown as black lines. The project covers an area 20 km x 20 km, with the coordinates for the top-left corner being 535 000E 6 470 000N (GDA94, MGA54).

TABLE 1

Geological units, relationships.

Map symbol	Geological unit	Relationship
	Alma Gneiss	Intrusive (erosional)
	Paragon Group	Unconformable (onlap)
	Sundown Group	Unconformable (onlap)
	Broken Hill Group	Unconformable (onlap)
	Thackaringa Group	Unconformable (onlap)
	Thorndale Gneiss	Unconformable (onlap)
	Clevedale Migmatite	Unconformable (onlap)
	Rift Series	Unconformable (onlap)

3D Geological Editor allows the rapid construction and editing of 3D geological models that are based on input observations, supplemented by various hypothetical observations. The 3D volumetric model proposed by Pasminco (Archibald *et al*, 2000) was used as a starting point.

Lithology

The published geological map (Willis, 1989) was a primary input along with five regional-scale geological cross-sections. A seismic section was also used as a backdrop for digitising 'observations' for the Rift unit (Gibson *et al*, 1998). The final 3D model is shown as a geological map in Figure 6, and on Section N6 in Figure 7 and Section N7 in Figure 8.

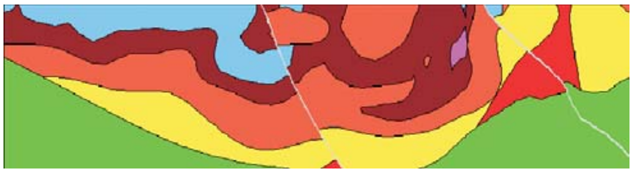


FIG 7 - Section N6 through the 3D model (length: 20 km; depth: 5 km; V/H=1). The shades of the units are as shown in Table 1.

The Alma Gneiss is an intrusive body. Note that it is presented to 3D Geological Editor as being at the top of the sequence, with an erosional relationship, to be properly represented.

Structure

Management of faults is a key issue in constructing a realistic 3D geological model. The number of faults introduced into the model was minimised. It was found that a satisfactory geological model at this broad scale could be constructed with just two faults. Other more extensive and detailed models with up to ten faults are in preparation. Complex structural effects of the Broken Hill terrain that are represented in the model include 'retrograde' shears, high temperature shears, boudinage and transposition.

The 3D geological model encompasses a parallelepiped 20 km long, 20 km wide and 5 km deep (Figure 9). More details are given by Guillen *et al* (2004), along with a constrained gravity inversion where that model is considered as the *a priori* geological model.

DISCUSSION

The potential-field method has now reached a development level that enables it to model real-world situations, even in complex cases. For example, Maxelon (2004), and Maxelon and Mancktelow (2005), used it to model foliation fields and a juxtaposition of nappes with a strong folding in the Lepontine Alps. Some issues deserve a mention, as follows.

Rather than a cokriging we can be interested in conditional simulations. The method can be straightforwardly generalised to conditional simulations if we assume that the potential fields are Gaussian, which is not a strong assumption in this kind of application.

The covariance fitting has some part of uncertainty. To take it into account, a Bayesian approach has been developed by Aug (2004). It consists in defining an *a priori* distribution for the joint distribution of the covariance parameters and defining the corresponding distribution from the data. That posterior distribution can be incorporated in the cokriging or conditional simulation process.

A better integration of the geometric modelling and the geophysical inversion would be welcome. This could be done by starting from a conditional simulation of the geometric model and defining the state changes with regard to the uncertainty of that geometric model, to the spatial structure of the potential field, and to the spatial structure of the physical variables.

Last but not least, the gradient of a random function is only by chance a unit vector. Considering the vectors defined by the structural data as unit vectors is thus somewhat abusive. The ideal would be to sample both a structural direction and a structural intensity, but this is possible only in very specific cases. Aug (2004) has shown on simulations of actual situations that replacing actual gradients by unit vectors usually has a minor impact on the determination of the covariance and the cokriging. It could be useful, however, to improve the inference method, which could be done at least with the use of simulations.

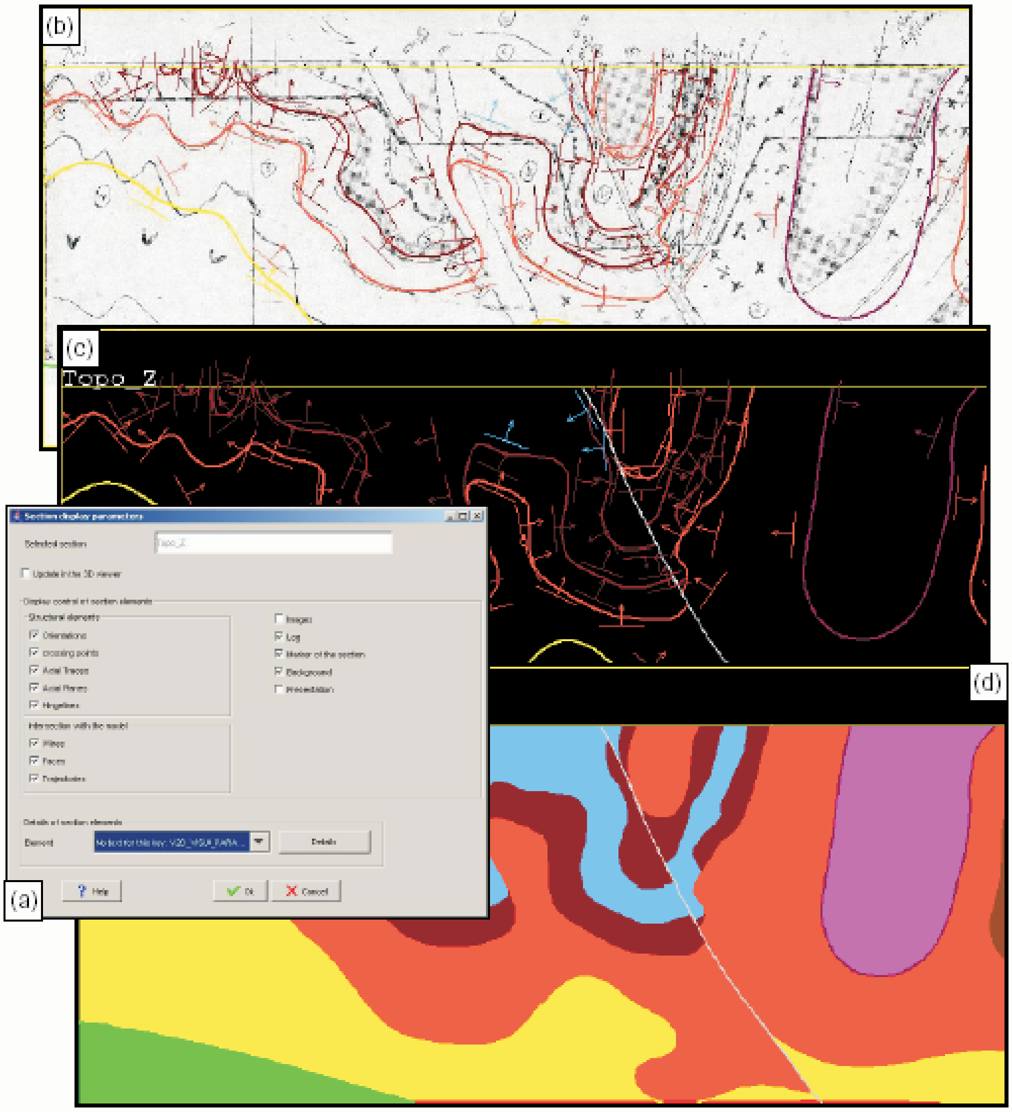


FIG 8 - Part of the geological cross-section N7. Various 'layers' can be presented in 3D Geological Editor's map and section presentations, and each of these can be turned 'on', or 'off' (a). Image (b) shows the model geology rendered as lines onto the geologist's original working section. Image (c) shows the model geology as lines, together with the orientation data. Image (d) shows the model geology as solid-geology. The user can control the plotting resolution, to achieve either fast plots, or high-resolution images, such as this one. Section length: 12.7 km, V/H=1.

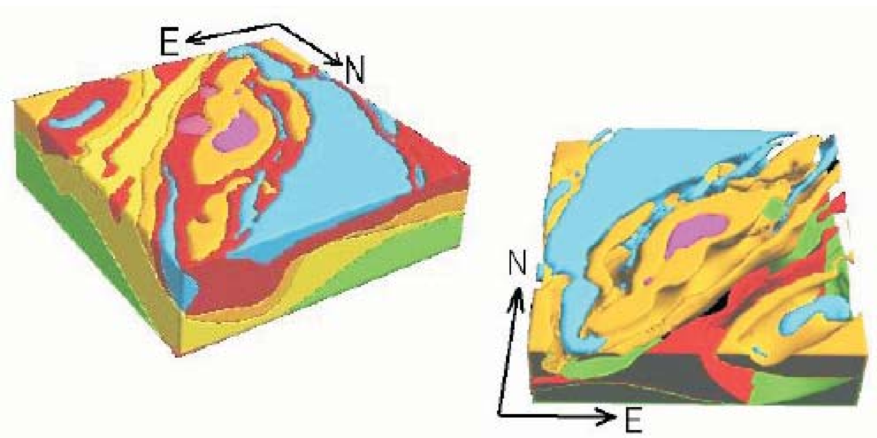


FIG 9 - Perspective view of the 3D geological model, viewed from the northeast and south west. The shades of the units are as shown in Table 1.

ACKNOWLEDGEMENTS

The research work carried out at the École des Mines de Paris was funded by BRGM. The Broken Hill geological modelling work was undertaken by Intrepid Geophysics and was funded by an *Innovation Access Programme* grant under the Australian Government's innovation statement.

REFERENCES

Archibald, N J, Holden, D, Mason, R and Green, T, 2000. A 3D geological model of the Broken Hill 'line of lode' and regional area, Unpublished report, Pasminco Exploration.

Aug, C, 2004. Modélisation géologique 3D et caractérisation des incentitudes par la méthode du champ de potentiel, PhD thesis to be defended in December 2004, École des Mines de Paris.

Chilès, J-P and Delfiner, P, 1999. *Geostatistics: Modeling Spatial Uncertainty*, 695 p (Wiley: New York).

Courrioux, G, Lajaunie, C, Chilès, J-P and Lazarre, J, 1998. Foliation fields and 3D geological modelling, in *Proceedings 3D Modeling of Natural Objects, A Challenge for 2000's*, ENS de Géologie, Nancy, Vol 1.

Dimitrakopoulos, R and Luo, X, 1997. Joint space-time modelling in the presence of trends, in *Geostatistics Wollongong '96* (eds: E Y Baafi and N A Schofield) Vol 1, pp 138-149 (Kluwer: Dordrecht).

Freulon, X and de Fouquet, C, 1993. Conditioning a Gaussian model with inequalities, in *Proceedings Geostatistics Tróia '92* (ed: A Soares), Vol 1, pp 201-212 (Kluwer: Dordrecht).

Gibson, G, Drummond, B, Fomin, T, Owen, A, Maidment, D, Gibson, D, Peljo, M and Wake-Dyster, K, 1998. Re-evaluation of crustal structure of the Broken Hill inlier through structural mapping and seismic profiling, AGSO Record 1998/11.

Gibson, G M and Nutman, A P, 2004. Detachment faulting and bimodal magmatism in the Palaeoproterozoic Willyama Supergroup, south-central Australia: keys to recognition of a multiply deformed Precambrian metamorphic core complex, *J Geol Soc (London)*, 161:55-66.

Guillen, A, Courrioux, G, Calcagno, P, Lane, R, Lees, T and McInerney, P, 2004. Constrained gravity inversion applied to Broken Hill, in extended abstracts, *ASEG 17th Geophysical Conference and Exhibition*, *Integrated Exploration in a Changing World*, Sydney, August.

Lajaunie, C, Courrioux, G and Manuel, L, 1997. Foliation fields and 3D cartography in geology: principles of a method based on potential interpolation, *Mathematical Geology*, 29(4):571-584.

Mallet, J L, 2003. *Geomodeling*, 599 p (Oxford: Oxford).

Maréchal, A, 1984. Kriging seismic data in presence of faults, in *Geostatistics for Natural Resources Characterization* (eds: G Verly, M David, A G Journel and A Maréchal), Part 1, pp 271-294 (Reidel: Dordrecht).

Maxelon, M, 2004. Developing a three-dimensional structural model of the lower Lepontine Nappes – Central Alps, Switzerland and Northern Italy, PhD thesis (unpublished), ETH Zurich.

Maxelon, M and Mancktelow, N S, 2005. Three-dimensional geometry and tectonostratigraphy of the Pennine zone, Central Alps, Switzerland and Northern Italy, *Earth Science Reviews*, 71(3-4):171-227.

Noble, M P, 2000. The geology of the Broken Hill synform, NSW, Australia, MSc thesis (unpublished), Monash University, Victoria.

Willis, I L, 1989. *Broken Hill Stratigraphic Map* (New South Wales Geological Survey: Sydney).

Planning, Designing and Optimising Production Using Geostatistical Simulation

P A Dowd[1] and P C Dare-Bryan[2]

ABSTRACT

The full potential of geostatistical simulation as a tool for planning, designing and optimising production is only realised when it is integrated within the entire design and production cycle. In the planning and design stages this involves the simulation of components of the production cycle that depend on (simulated) grades and geology. In the production stage it involves integration with the mining method and the type and use of equipment.

This paper explores the general concepts of integrated geostatistical simulation and illustrates them with particular reference to blast design, equipment selection and the associated quantification of ore loss, ore dilution and the ability to select ore on various scales. The critical component of most metalliferous open pit mining operations is ore selection, ie the minimisation of ore loss and ore dilution during extraction. In general, extraction comprises drilling, blasting and loading, all of which are planned and designed on the basis of uncertain models of geology and grade.

The application describes the integration of geostatistically simulated grade, geological and geomechanical models with blast modelling to provide a link between the estimated *in situ* characteristics of the orebody and the locations of the same (displaced) characteristics following the blast. This approach provides a means of evaluating different types of selection and thereby enables planners to optimise the selection process in terms of blast design, type and size of loading equipment, maximisation of ore recovery and minimisation of ore loss and dilution. This conversion of the *in situ*/block model resource to a realistically recoverable reserve may, in many instances, be the most significant source of uncertainty in reserve estimation.

SIMULATION

Geostatistical simulation is rarely an exercise in its own right and is usually undertaken to provide a model for further studies. In the simplest applications the purpose may be to estimate ore reserves; or to assess the uncertainty associated with mine planning based on specified drilling densities; or to assess the effect on recoverability of various sizes of selective mining units. In more complex applications a simulated orebody model may be used to assess the effects of sequences of downstream activities. All of these applications, in one way or another, are assessing uncertainty and its operational consequence – risk.

An effective evaluation of risk must include adequate quantifications of all sources of uncertainty. Too often in these applications the quantification of uncertainty is limited to *in situ* grade and geological variables, with little attention to the uncertainties that arise from the technical processes that are applied to extract ore from the *in situ* material. The usual assessment of recoverable reserves, for example, is limited to a simple volumetric exercise in which ore recovery is assessed as a function of applying a range of selection volumes to a simulated orebody. This simplistic approach ignores the practicalities of the actual mining, selection and loading processes – blast design, behaviour and performance; equipment type, size and operation; ore displacement during blasting and loading; and ability to identify ore zones within a blast muck pile. In many applications, the uncertainties introduced by these technical processes are at least as significant as those that derive from the *in situ* spatial characteristics of grades and geology.

In mining applications, the full effectiveness of geostatistical simulation can only be realised by integrating it with adequate and realistic simulations of the technical processes. The authors demonstrate this argument with an application to selection and recovery of ore in open pit mining. The *in situ* simulation of geology and grades can be achieved by any of the standard algorithms. Ore, however, is not selected and recovered from this *in situ* mass, but from the broken and displaced components of the mass that results from the blasting process. The integration of the simulation of blasting, selecting and loading with the simulation of *in situ* grade, geology and geomechanical characteristics provides a realistic means of evaluating selection and recoverability, as well as an effective basis for mine planning and equipment selection.

THE METHOD

The method comprises:

- generation of an *in situ* model of the orebody comprising the grade, geology, geomechanical properties and grade control variables within sufficiently small volumes determined by the smallest selectable volume within a blast muck pile;
- definition of a blast volume comprising a large number of the *in situ* model volumes, and subjecting it to a blast simulator, which effectively moves each of the component model volumes to its final resting place in the blast muck pile; and
- application of selective loading processes to the simulated muck pile to determine the degree of selectivity that can be achieved by various sizes of loader and types of loading and to quantify ore dilution and ore loss.

The *in situ* model, representing perfect knowledge at all relevant scales, is obtained by geostatistical simulation. An *in situ* model that represents the reality of knowing only the data and information that are available from specific grade control drilling and sampling grids can be obtained by sampling the geostatistically simulated model on a specified grid. The volumes comprising the *in situ* model are then populated by estimates based only on the data corresponding to the specified grade control drilling and sampling grids. Different drilling and sampling grids can be used to generate different models, each reflecting the levels of data and information available. Selectivity can then be assessed as a function of the drilling and sampling grids as well as the size and type of loader. Performance is assessed against the ideal selectivity that can be achieved on the perfect knowledge model, comprising the simulated values of each component volume. Applying costs, prices and financial criteria enables an optimal selection of the grade control drilling grid, size of loader, type of loading and even blast design.

Blast simulation

A discrete block modelling approach was used in the work reported here. The discrete block model is based on the SCRAMBLE code (Sophisticated CRA Model of Blasting with Explosives) developed by CRA (now Rio Tinto PLC) Advanced Technical Development from the ICI SABREX code (Scientific

1. FAusIMM(CP), Executive Dean, Faculty of Engineering, Computer and Mathematical Sciences, The University of Adelaide, Adelaide SA 5005, Australia. Email: peter.dowd@adelaide.edu.au

2. Orica Australia Pty Ltd, 1 Nicholson Street, Melbourne Vic 3000, Australia.

Approach to Blasting Rock with Explosives) (Harries and Hengst, 1977; Jorgenson and Chung, 1987; Kirby, Harries and Tidman, 1987; Chung and Tidman, 1988; Mohanty, Tidman and Jorgenson, 1988). The code has been revised to include, *inter alia*, a fragmentation model based on the Bond Work Index. Details of the basis of the blast simulation are given in Appendix A.

A standard regular block model is input to the blast simulator, which then moves each block to its final position within the muck pile. Although the block effectively remains intact in the muck pile, it is assigned an estimated degree of fragmentation. Movement and final position are determined from models of the behaviour of explosive gases, energy release, heave mechanics, fragmentation, throw and velocity of movement as functions of, *inter alia*, bench height, burden, hole spacing, hole diameter, rock density and rock fracture density.

This approach becomes more realistic as the block size becomes smaller and approaches the average size of particles in the muck pile. In principle, the block size can be made as small as desired but in practice the size is limited by computing constraints.

Simulating the loading process

The Floating Stope Optimiser (FSO) routine in the Datamine mine planning software was used to simulate an optimised selective loading process on the muck pile block model generated from a blast design. The FSO procedure is similar to the 'floating cone' method of open pit optimisation and provides a flexible means of locating optimal envelopes of block model grades (Randall and Wheeler, 1998a, 1998b).

To apply the FSO to a selective mining operation, the envelope size is defined as the selective mining unit for the excavation of the muck pile. The subcell size, which defines the grid spacing at which the envelope is successively positioned throughout the block model, is determined by the minimum digging width of the excavator used.

As an excavator works through a muck pile the broken rock continually recovers the natural angle of repose. Thus, to recover a pocket of ore near the bottom of a muck pile a 'cone' of material, projected up from the ore pocket, must be removed with it. To incorporate this in the selection process a slope of 45° is applied to the four vertical sides of the cube envelope from its base in the XY plane, generating the envelope shape shown in Figure 1.

The output from the FSO flags all blocks as ore or waste. These are then processed to generate total tonnes mined, tonnes excavated as ore and waste, head grade of ore and tonnes of metal in ore. Multiple runs are taken for each muck pile over a range of cut-off grades to find the optimum.

Optimisation procedure

A blast design is applied to the complete geostatistically simulated blast volume (the 'reality') and to the estimated block models for the blast design. Once the block models have been heaved to generate the corresponding muck piles, the muck pile block models, with associated block grade values, are entered into the FSO to evaluate the ore/waste excavation boundaries to give the optimum head grade based on a selected cut-off grade and selective mining unit size. The region of the bench that is to be excavated as ore is evaluated on the basis of the total tonnes of metal/mineral within that region minus the portion of metal/mineral expected to be lost in the processing operation.

The 80 per cent passing size of the resulting muck pile (cf Appendix A) is then used to adjust the standard cost per tonne values for the downstream processes of loading, hauling and primary crushing. The total mining cost for the bench comprises the drilling and blasting costs derived from the blast design, the revised loading and hauling costs and the mining services costs, all as a cost per tonne blasted (cf Appendix B). The total processing cost comprises the adjusted primary crushing costs and the remaining processing operations costs, which are expressed as a cost per tonne processed.

The value of the bench is thus the value of the concentrate output from the processing plant less the mining and processing costs.

CASE STUDY

The case study is based on the Minas de Rio Tinto SAL (MRT) open pit copper mine at Rio Tinto, southern Spain, which is typical of a low-grade operation in the later stages of its life. The application described here is to the low-grade Cerro Colorado mineralisation. Ore/waste delineation for selective mining is particularly difficult because the head grades are near the economic cut-off grade and there are no clear geological controls on the mineralisation.

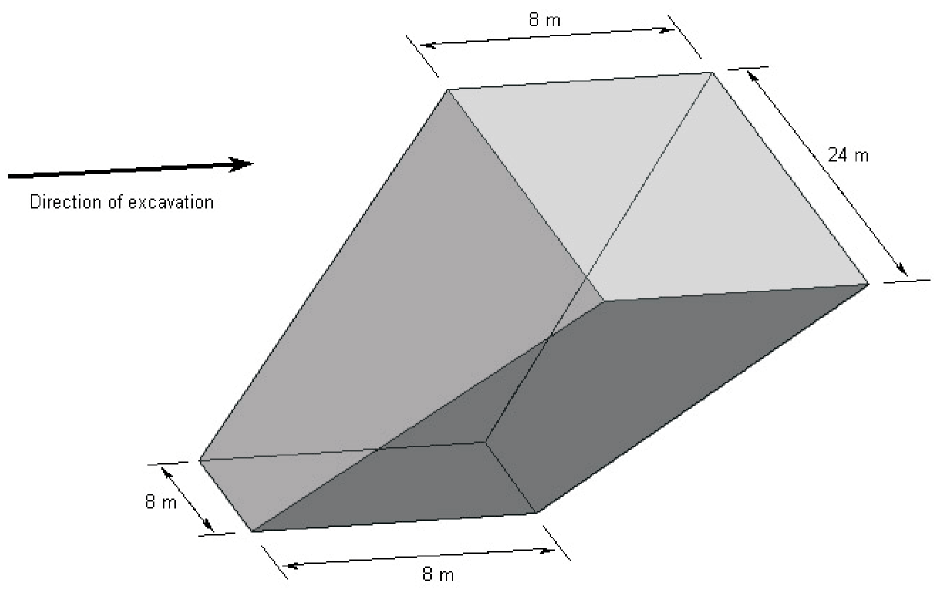


FIG 1 - Envelope shape for Floating Stope Optimiser.

The mining operation has been temporarily closed pending an increase in the copper price. During operation the mine produced concentrate with an average grade of 24 per cent copper.

Geological setting

The Rio Tinto deposit lies in the eastern Iberian Pyrite Belt. Submarine volcanic activity created an anticline structure, the edges of which formed pyroclastic rocks, where the massive sulfide mineralisations are located. The volcanic mass is buried under carboniferous slates, but subsequent folding has exposed the volcanic sequence locally in the eastern half of the anticline to form the Cerro Colorado deposit (Pryor, Rhoden and Villalon, 1972).

The Cerro Colorado mineralisation is a stockwork of sulfide accumulations, fed by several near-vertical brecciated feeder pipes. The predominant sulfides are pyrite and chalcopyrite, with galena, sphalerite, tetrahedrite, arsenopyrite and cassiterite present in much smaller quantities.

Mining method

The operation at MRT used traditional drilling and blasting on 10 m and 12 m benches that were drilled with two Buycrus Eyre 45R rigs and one 60R rig drilling 250 mm holes to a depth of 11.2 m or 13.7 m depending on the bench height. A square blast pattern was employed with burden and spacing dimensions ranging from 5.5 m × 6.5 m to 6.6 m × 8.0 m. The holes were charged with heavy ANFO because of water problems in the lower benches. P&H 2100 BL electric face shovels and Caterpillar 994 wheel loaders were used for loading and Caterpillar 789 dump trucks used for hauling. Two blasts, B4053 and B4056, were selected for this study.

Generating block models

Experimental semi-variograms were calculated from the blasthole data using a conical search. As no significant directional anisotropies were detected within the two blast volumes, all directional semi-variograms for each blast were combined into a single omni-directional semi-variogram for modelling purposes. For both blast volumes a two-structure, spherical semi-variogram model was fitted to the experimental semi-variograms as shown in Figure 2.

Sequential Gaussian simulation (Journel and Alabert, 1989, 1990), with the blasthole grades as conditioning data, was used to generate a realisation of each entire bench on a block grid of 0.5 m × 0.5 m × 0.5 m, the grid determined on the basis of blast and selection criteria. The histograms of simulated values and conditioning data are shown in Figures 3a and 3b; corresponding statistics are given in Table 1. There were no significant differences between the input variogram models shown in Figure 2 and those fitted to the simulation outputs for the two blasts.

The simulations for both benches used ordinary kriging and an octant search strategy with an isotropic search radius of 60 m. A minimum of two and a maximum of ten conditioning values (original data plus previously simulated values) were specified for each octant with a minimum of three informed octants. The maximum proportion of previously simulated values in each set of conditioning values was set to 70 per cent and the coordinates of the original data were retained, ie data was not assigned to simulation grid nodes. Linear extrapolation was used in the upper and lower tails for back transformation of the Gaussian simulated values.

The simulation provides a realisation of the grade distribution throughout the bench on the scale required for the blast simulation. For each specified blast design, new 'sample hole data' is taken from the simulation block model of the bench. This sample data is then used to generate ordinary kriging estimates of the block grades to produce an estimated block grade model of the bench. The semi-variogram used for kriging is the model fitted to the experimental semi-variogram of the sample 'data' taken from the simulation block model.

Blast modelling parameters

The simulated heaving action and muck pile generation were adapted to replicate the muck piles generated by the actual blasts, based on the data available for throw and the overall shape of the muck pile profile.

The blast pattern specifications for the two blasts used in this study are shown in Table 2 and the geomechanical data used is summarised in Table 3. The modelling was calibrated against the original blast designs for B4053 and B4056 using the input data in Tables 2 and 3 and the muck pile profiles from field data.

TABLE 1

Statistics of conditioning data and simulated values.

	Blast B4053		Blast B4056	
	Conditioning data	Simulated values	Conditioning data	Simulated values
Mean	0.403%	0.401%	0.494%	0.489%
Variance	$0.075\%^2$	$0.072\%^2$	$0.113\%^2$	$0.105\%^2$
No of values	1440	288 000	1200	240 000

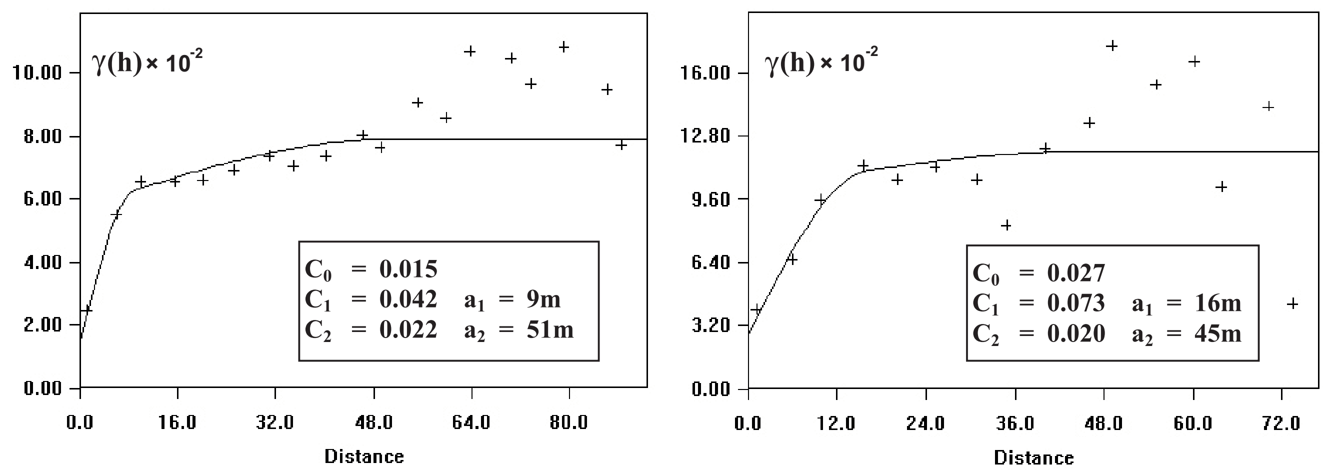


FIG 2 - Experimental semi-variograms and two-structure spherical models for B4053 (left) and B4056 (right) blasthole data.

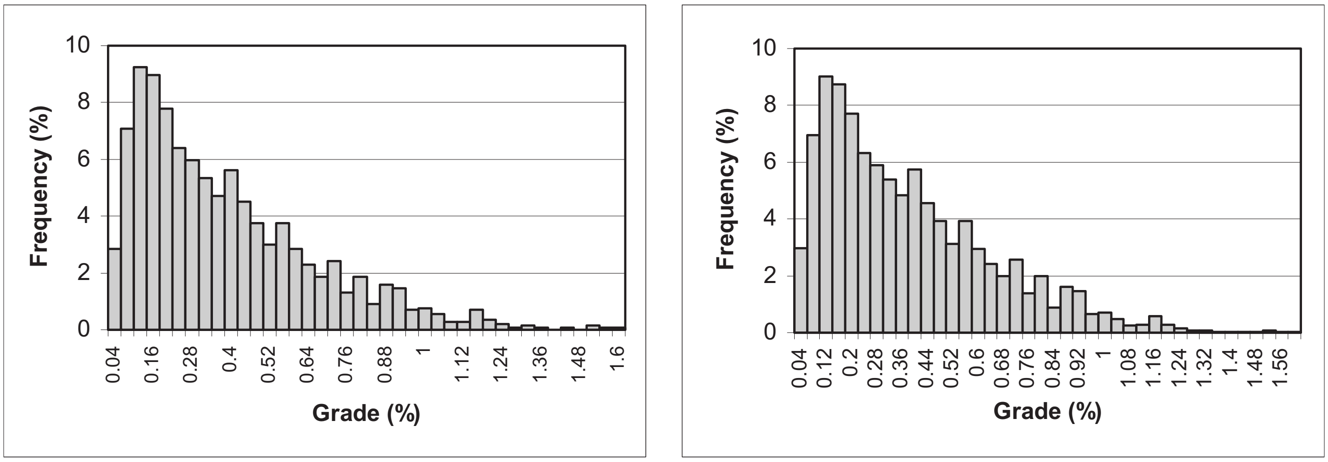


FIG 3a - Histograms of blasthole grades for blast B4053; data (left) and simulated values (right).

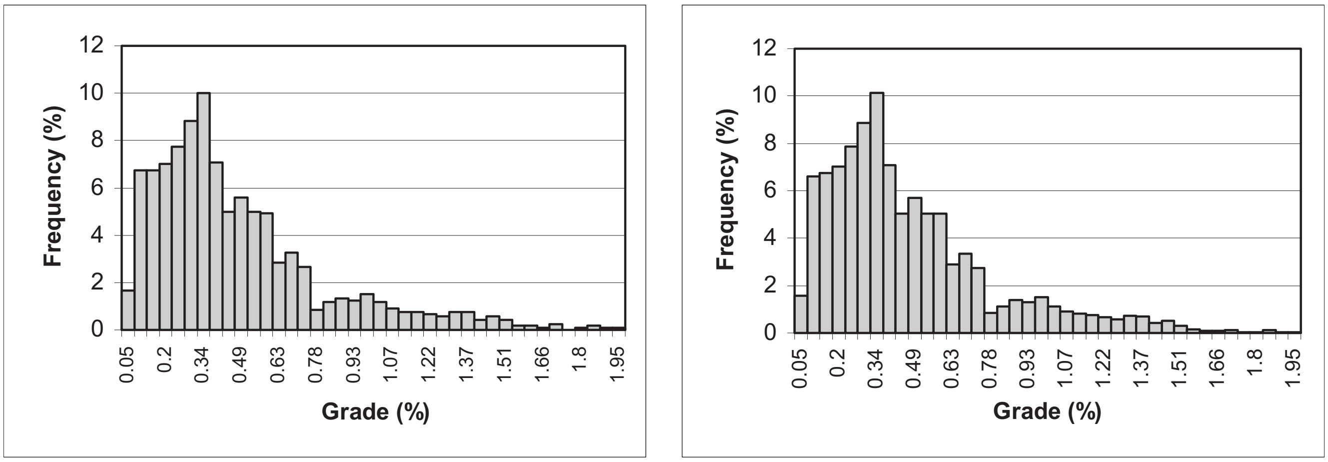


FIG 3b - Histograms of blasthole grades for blast B4056; data (left) and simulated values (right).

TABLE 2

Blast pattern specifications used in case study.

Burden	6.5 m	Main explosive charge	540 kg ANFO
Spacing	8.0 m	Initiation sequence	S1
Bench height	12 m	Inter-hole delay	50 ms
Vertical blasthole length	13.7 m	Inter-row delay	100 ms
Hole diameter	250 mm		

TABLE 3

Geomechanical data used in case study.

Young's modulus	750 kbars
Poisson's ratio	0.25
Uniaxial compressive strength	1.2 kbars
Rock density	2.75 g.cm^{-3}

Selection of ore/waste boundaries in muck piles

For the excavators used at MRT, with a bucket size of 13 m^3, an FSO envelope of 8 m × 8 m × 8 m was selected with a subcell size of 2.7 m. More selective loading was also assessed using a 6 m × 6 m × 6 m envelope.

Costs of the blasting and selection processes

For a given blast design it is relatively straightforward to calculate the costs associated with drilling and blasting, by summing the constituent costs. However, the composition of the muck pile produced by the blast directly affects the downstream processes of loading, hauling and primary crushing, and the overall cost evaluation of a blast must include the costs of these processes. It is not possible to quantify directly the effect of different quality blasts on the downstream processes, and the best common variable for comparisons is the degree of fragmentation achieved by the blast. It is generally recognised that the costs of the downstream processes, including operation and maintenance, decrease as fragmentation improves (MacKenzie, 1966). A common practice is to use a functional relationship, formulated through in-pit operational assessment, to adjust the cost per unit weight worked as a function of the degree of fragmentation. A summary of the cost functions and their derivation is given in Appendix B.

Optimisation procedure

The flow diagram in Figure 4 shows the procedure applied to each bench. The chosen blast design is applied to the standard geostatistical simulation and estimated block models for that blast design.

Once the block models are heaved to generate the corresponding muck piles, the muck pile block models, with their associated block grade values, are entered into Datamine. Within Datamine, the FSO is applied to the block models to

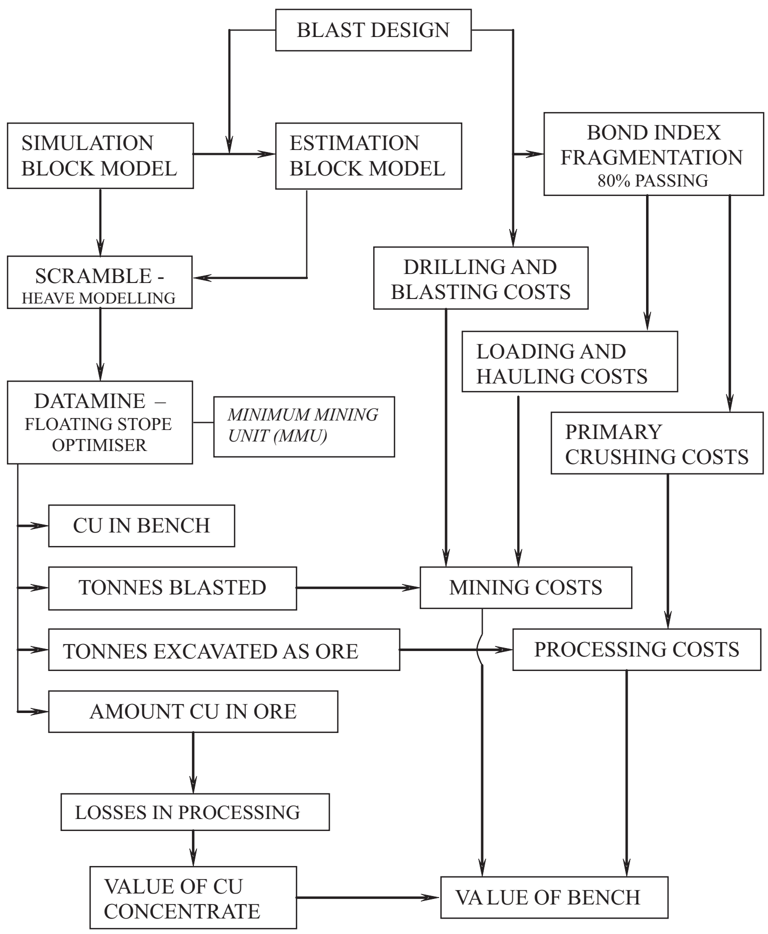


FIG 4 - Flow chart for optimisation procedure.

evaluate the ore/waste excavation boundaries to give the optimum head grade based on a selected cut-off grade and selective mining unit. The region of the bench that is to be excavated as ore is evaluated on the basis of the total tonnes of copper within that region, minus the portion of copper expected to be lost during processing.

The flow chart in Figure 4 is an example of what might be termed a transfer function that transforms the idealised/*in situ*/simulated, and/or estimated, block grades into realistically recoverable grades and tonnages. These transfer functions are not generally linear and in most cases their effects cannot be approximated by simple dilution factors.

Ore reserve statements, or resource statements expressed in terms of production units, that are derived by selecting blocks directly from *in situ*/block models ignore some of the most significant sources of uncertainty. There may be other highly non-linear transfer functions (eg some types of mineral processing operations) that have significant effects on recoverability, but generally the extraction and loading processes are the most significant.

Results

By way of example, Figure 5 shows colour-coded simulated grades of sections of the 0.5 m × 0.5 m × 0.5 m blocks that comprise bench B4056 and Figure 6a shows the muck pile generated by applying the blast modelling process to this bench.

Figure 6b shows the muck pile that results from applying the blast modelling to the same bench but with the component block grades kriged from the simulated grades on the 6.5 m × 8 m drilling grid. The smoothing effect of kriging is clearly evident when comparing Figure 6a and 6b. Figure 6a represents the muck pile given complete information, whereas Figure 6b is the interpretation of the composition of the muck pile on the basis of the data. Selection is planned and implemented on the basis of Figure 6b but the volume selected will have the grade and tonnage of the equivalent volume in Figure 6a.

Figure 7 shows the corresponding muck piles generated from simulated and estimated block grade models for B4053. Figures 6 and 7 clearly show the significantly different spatial distribution of grades in the two muck piles with consequent implications for selection.

By way of example, when selection is applied via the FSO to the two muck piles shown in Figure 7, the volumes selected are those shown in Figures 8 and 9.

For each bench there are nine block models: the simulated block grades, taken as ‘reality’, and eight models of estimated block grades kriged from simulated values on various drilling grids, together with variations in other blast design parameters as summarised in Table 4.

The grades of the blocks that comprise the two benches are similar in terms of histograms (cf Figure 3) but they differ significantly in their spatial distributions within the respective benches. It is the latter that has the major effect on the spatial distribution of the grades in the muck pile and consequently on the ability to load selectively.

Bench B4053 is subeconomic for some blast designs but must still be blasted to allow continuing mine development. Having blasted this bench, any losses are minimised by processing the ore in the muck pile. Bench B4056 is economic for all blast designs and is mined and processed in the normal manner.

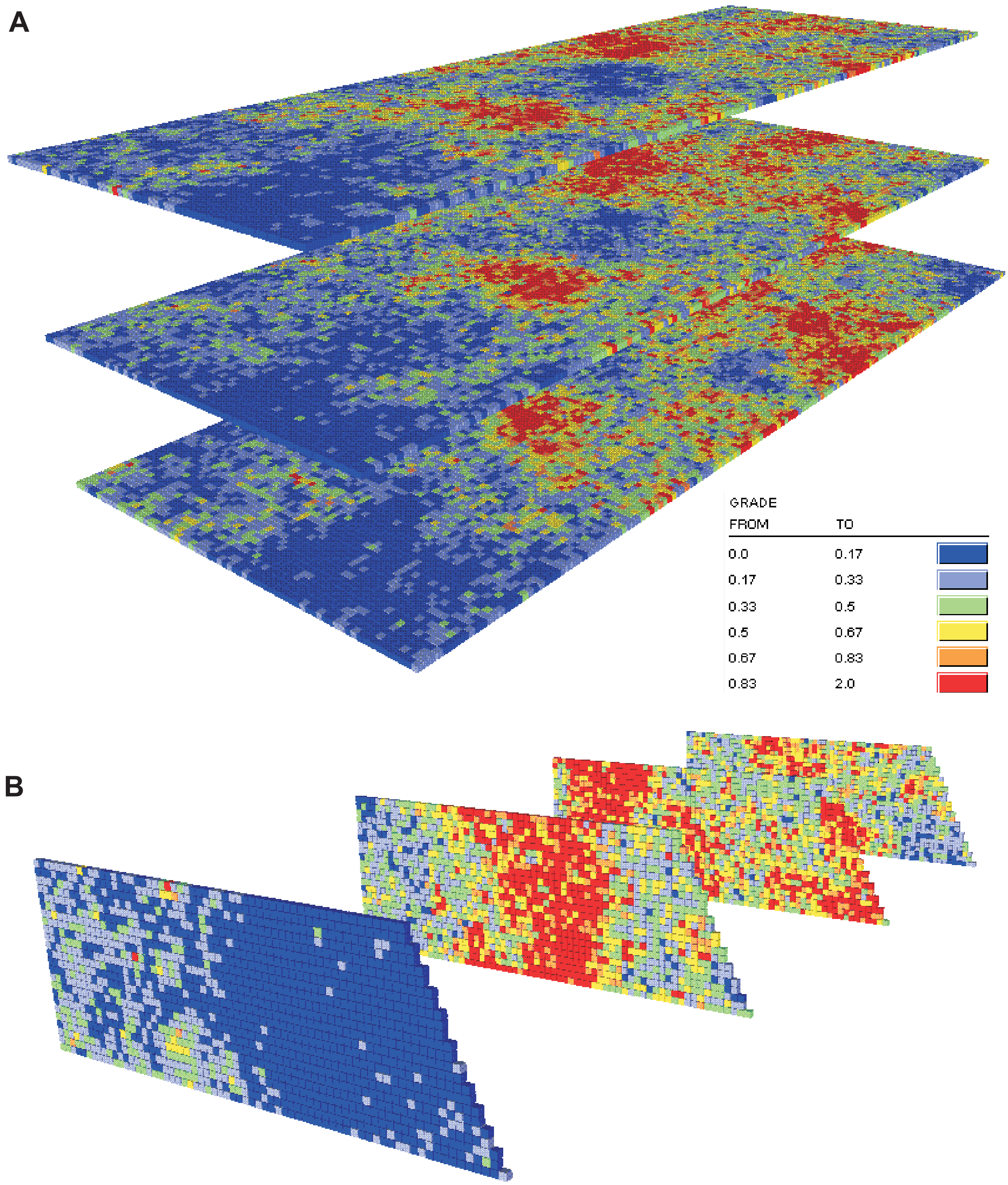


FIG 5 - Representations of the simulated *in situ* bench grades for B4056 showing colour-coded grade ranges on (a) horizontal planes and (b) cross-sectional planes. Horizontal planes are top and bottom of 12 m bench and 6 m mid-plane. Vertical planes are extremities (0 m and 80 m) and intermediate planes at 26 m intervals.

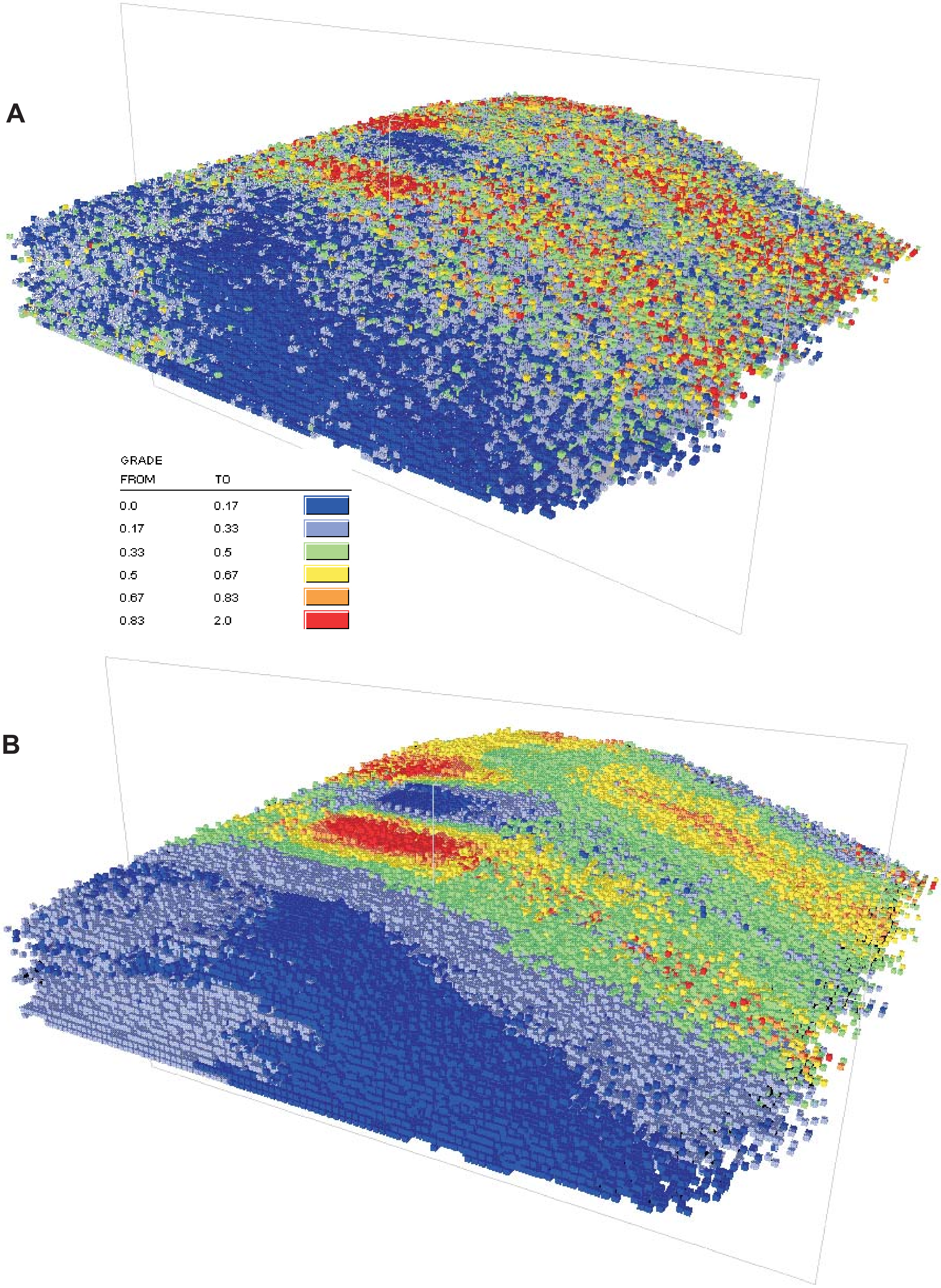


FIG 6 - B4056: Muck piles generated by blast design number one from (a) simulated bench grades and (b) from kriged bench grades.

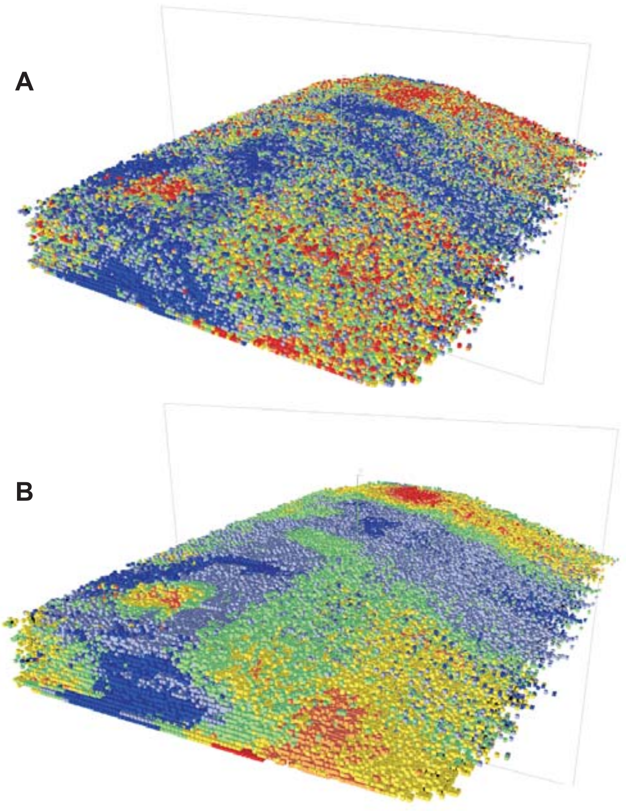


FIG 7 - B4053: Muck piles generated by blast design number one from (a) simulated bench grades and (b) from kriged bench grades.

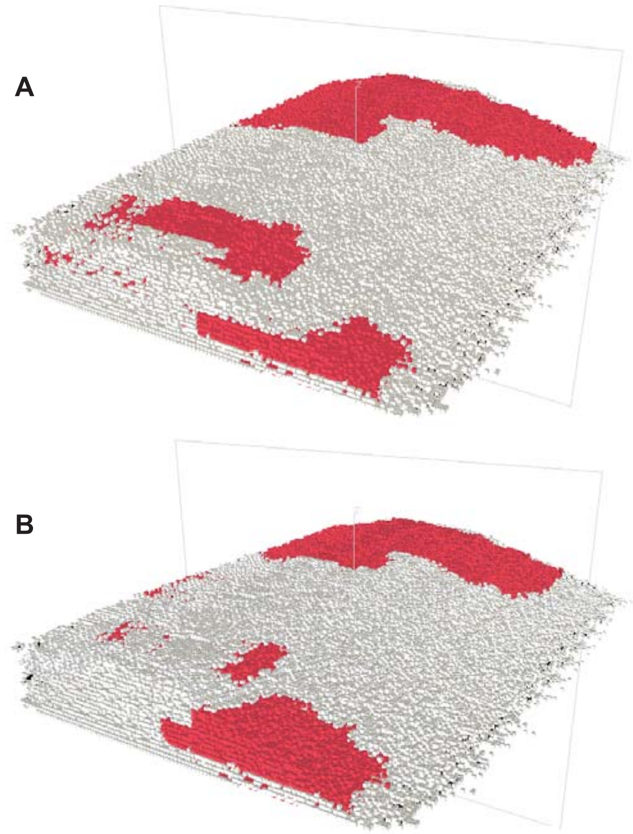


FIG 8 - Muck piles for B4053. (a) Muck pile generated from simulated block grades (reality). (b) Muck pile generated from estimated block grades using blast design one. The darker shade indicates exposed selected ore and the lighter shade is non-selected broken rock.

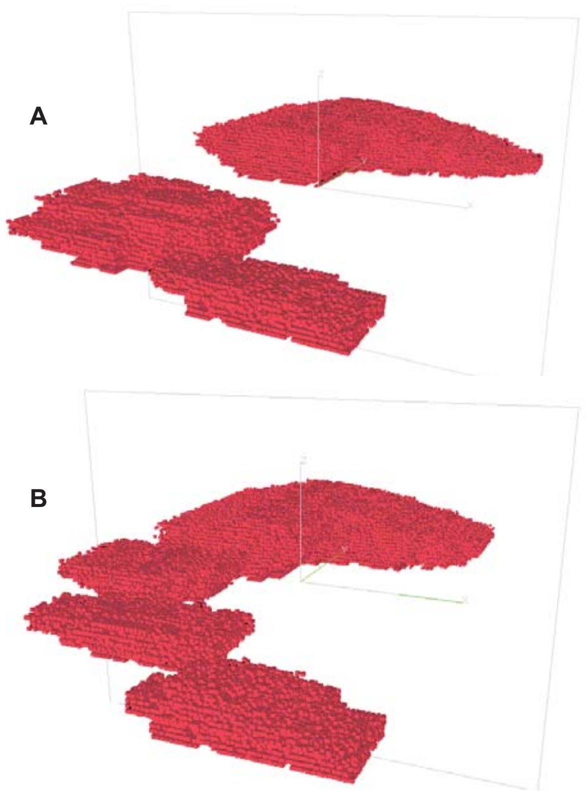


FIG 9 - Volumes of ore selected from muck pile for blast design 1, generated from (a) simulated block grades and (b) from estimated block grades.

The financial performances of each blast design against the 'reality' of the simulated block model are summarised in Figure 10 for B4053 and in Figure 11 for B4056. These figures show the ideal or maximum bench values corresponding to the simulated block grades, together with the actual bench values achieved by selecting from the muck piles generated from the estimated block grades for the various blast designs.

Figures 12 and 13 show the tonnages of copper within the ore selected from the muck pile generated from the simulated block grades, together with the actual tonnages recovered from the muck piles generated from the estimated block grades for the various blast designs.

TABLE 4

Blast designs used in study for estimated block grades.

Blast design	Design changes	Burden (m)	Spacing (m)	Powder factor (kg.tonne^{-1})	Hole diameter (m)
1	Control	6.5	8	0.31	0.25
2	Changing hole diameter	6	7.5	0.31	0.23
3		7	9	0.31	0.27
4		8	9.5	0.31	0.30
5	Increasing powder factor	6	7.5	0.37	0.25
6		5.5	7	0.43	0.25
7	Decreasing powder factor	7.5	9	0.25	0.25
8		8.5	10.5	0.19	0.25

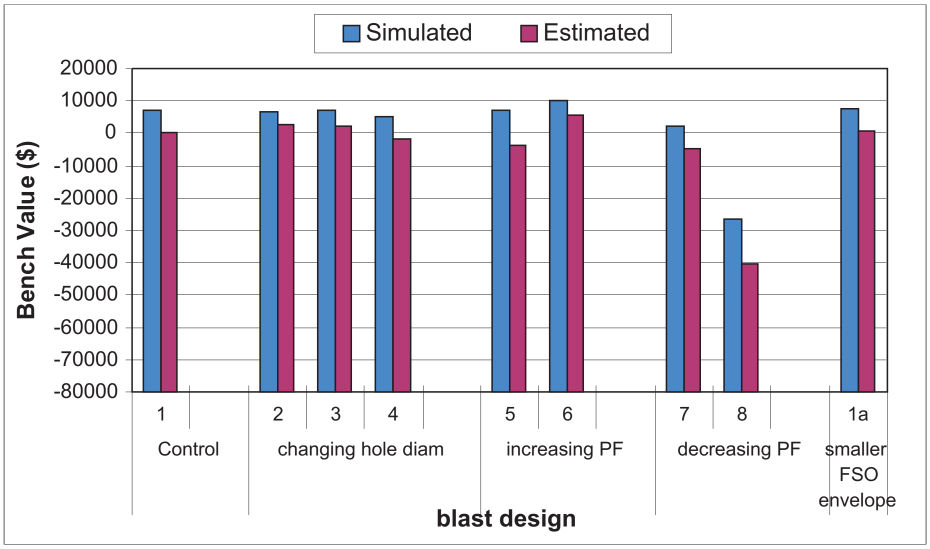


FIG 10 - B4053 optimised bench values for blast designs.

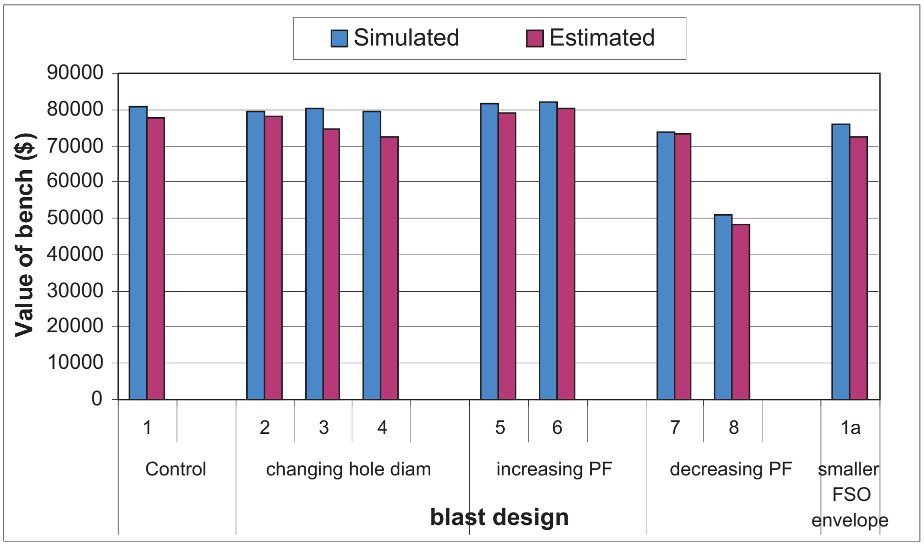


FIG 11 - B4056 optimised bench values for blast designs.

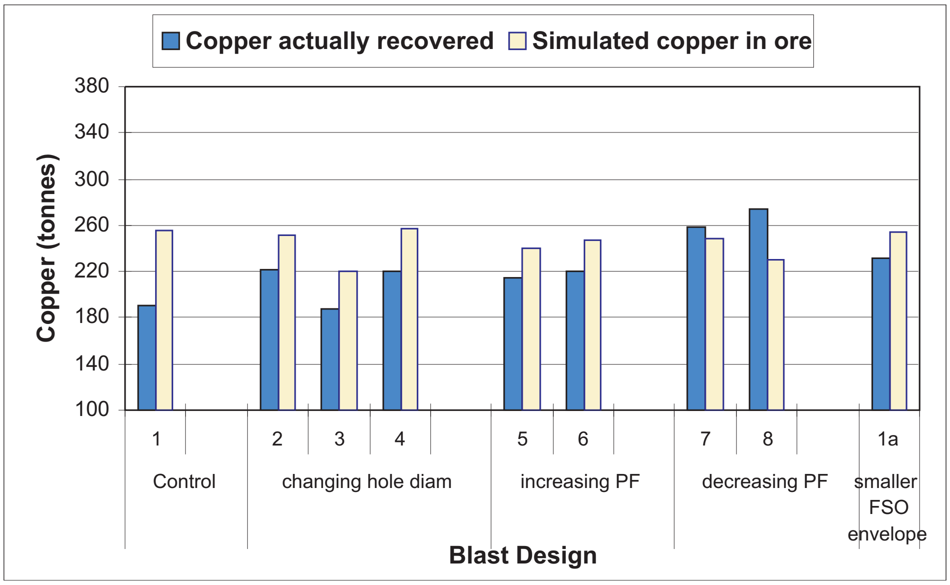


FIG 12 - B4053: simulated ('actual') copper in ore selected from muck pile and amounts recovered on the basis of estimations from various blast designs.

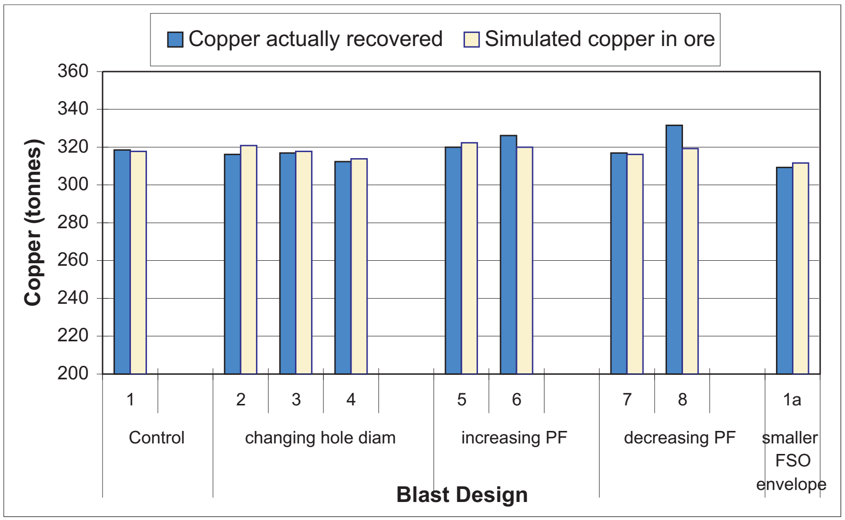


FIG 13 - B4056: simulated ('actual') copper in ore selected from muck pile and amounts recovered on the basis of estimations from various blast designs.

Numbers on the horizontal axes of Figures 10 - 13 denote the blast designs given in Table 1. Blast 1a (smaller FSO envelope) is a smaller selection envelope applied to blast one, in which the envelope corresponds to smaller-scale selection (6 m × 6 m × 6 m) using a wheel loader.

Note that in some cases, more copper is recovered from the muck pile generated from the estimated block grades than from the muck pile generated from the simulated block grades (eg blast designs seven and eight in Figure 12). This is, however, at the expense of diluting the ore with additional waste, which reduces profit (eg as indicated by the bench values for blasts seven and eight in Figure 10).

The differences between ideal selection and selection based on estimated block grades are more significant for B4053 because the economic grades are more widely dispersed through the bench and the muck piles than they are for B4056. The differences are large and critical for B4053, as planning on the basis of the estimated block grades leads, more often than not, to financial loss.

The real effects on the operation can be quantified by comparing the expected performance against the actual performance. Figures 14 and 15 show, for each blast design and for selection based on the estimated block grade models, the difference between the estimated copper content and the actual copper content of the selected ore regions, together with the difference between the estimated and actual financial values of the selected ore regions. It is these differences between planned and actual performances that have the greatest impact on the viability of the operation.

The results summarised in Figures 14 and 15 are functions of the complex relationships among block grade values, heave mechanics of the blasting process, the spatial distribution of ore and waste blocks in the muck pile and the method of selecting from the muck pile. The absolute values of the bars shown in Figures 14 and 15 are the deviations from planned outcomes and are measures of the ability to plan the operation to acceptable levels of accuracy and of the consequences of not being able to do so. The larger differences for B4056 (Figure 15) are a function of the more distinct ore/waste boundaries in the resulting muck pile, which in turn provide a greater propensity for ore loss and ore dilution with small changes in the selection volumes. By contrast, the greater dispersion of the ore throughout the muck pile generated from B4053 offers less scope for selectivity and less adverse consequences arising from changes in the selection volumes.

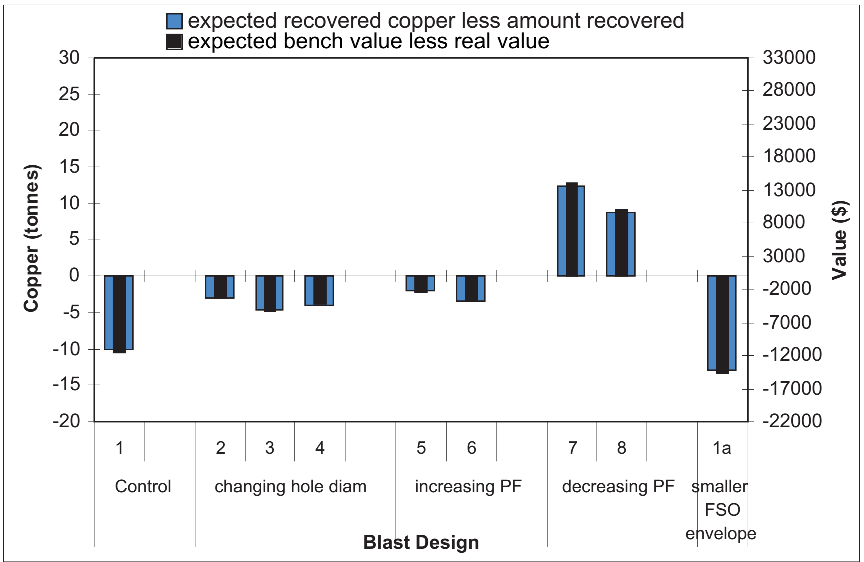


FIG 14 - Differences between planned and actual performance for B4053.

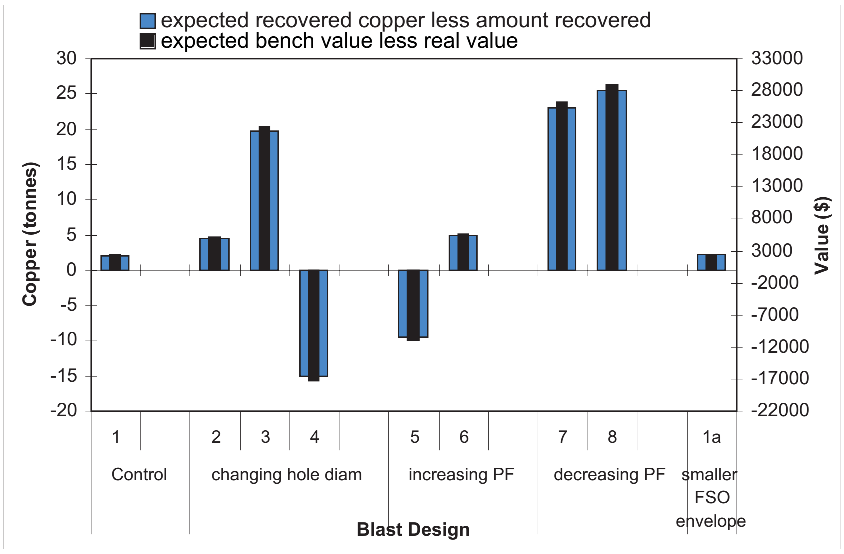


FIG 15 - Differences between planned and actual performance for B4056.

SUMMARY AND CONCLUSIONS

This study demonstrates the potential of geostatistical simulation in the optimisation of blasting and loading in selective mining processes. In particular, it provides a means of quantifying the effects of grade distribution smoothing on blast design and the selection of ore regions within the resulting muck pile. It also provides a means of assessing the financial consequences of ore loss and dilution arising from planning and implementing specific blasting and loading practices on the basis of various drilling grids.

Although a very specific blast modelling process has been used in this study, it could readily be replaced by any other type of modelling, either to provide a more realistic simulation of heave mechanics and fragmentation or to simulate other types of blasting and selection. Similarly, other types of geostatistical simulation could be used and multiple variables, including qualitative geological variables, could be simulated and incorporated into the selection procedure, for example by selecting gold-bearing ore on the basis of observable quartz veins and fracture networks in the muck pile (Dowd, 1995). The methods and approach used in this study do not limit the generality and practical potential of the application.

A real-time, virtual reality version of the approach described here could also be used to guide loader operators in making optimal selections from muck piles. Real-time applications would require very rapid capture of accurate survey and locational data, which could readily be provided by GPS.

More generally, the application described here demonstrates that the full effectiveness of geostatistical simulation can only be realised in mining applications by integrating it with adequate simulations of the technical processes that turn the simulated *in situ* characteristics into mined products. This is an important issue in determining and reporting reserves.

REFERENCES

Chung, S H and Tidman, J P, 1988. Effective modelling for cast blasting, in *Proceedings International Symposium for Mine Planning and Equipment Selection* (ed: R K Singhal) pp 357-360 (A A Balkema: Rotterdam).

Dowd, P A, 1995. Björkdal gold mining project, northern Sweden, *Trans Inst Min Metall*, Section A, Mining Technology, 104:A149-A163.

Harries, G and Hengst, B, 1977. Use of a computer to describe blasting, in *Proceedings 15th APCOM Symposium*, pp 317-324 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hustrulid, W, 1999. *Blasting Principles for Open Pit Mining,* Vol 1, General Design Concepts (A A Balkema: Rotterdam).

Jorgenson, G K and Chung, S H, 1987. Blast simulation surface and underground with the SABREX model, *CIM Bulletin*, 80:37-41.

Journel, A G and Alabert, F, 1989. Non-gaussian data expansion in the earth sciences, *Terra Nova*, 1:123-134.

Journel, A G and Alabert, F, 1990. New method for reservoir mapping, *Journal of Petroleum Technology,* 42(2):212-218.

Kirby, I J, Harries, G and Tidman, J P, 1987. ICI's computer blasting model SABREX – the basic principles and capabilities, in *Proceedings 13th Conference on Explosives and Blasting Technique*, (ed: R D Boddorff), pp 184-198 (Society of Explosives Engineers).

Leiper, G A and Plessis, M P, 1987. Describing explosives in blasting models, in *Proceedings Second International Symposium on Rock Fragmentation by Blasting*, (eds: W L Fourney and R D Dick) pp 462-474 (Society for Experimental Mathematics).

MacKenzie, A S, 1966. Cost of explosives – do you evaluate it properly?, *Mining Congress Journal*, pp 32-41.

Mohanty, B, Tidman, J P and Jorgenson, G K, 1988. Advanced computer simulations – the key to effective blast designs in open pit and underground mines, in *Computer Applications in the Mineral Industry* (eds: K Fytas, J L Collins and R K Singhal) pp 41-48, Rotterdam.

Nielsen, K, 1983. Optimisation of open pit bench blasting, in *Proceedings First International Symposium on Rock Fragmentation by Blasting*, Vol 2, pp 653-664 (Society for Experimental Mechanics).

Pryor, R N, Rhoden, H N and Villalon, M, 1972. Sampling of Cerro Colorado, Rio Tinto, Spain, *Trans Inst Min Metall*, Section A, Mining Technology, 81:A143-159.

Randall, M and Wheeler, A, 1998a. Balancing the books, *Mining Magazine*, pp 337-342.

Randall, M and Wheeler, A, 1998b. Where did it go? *Mining Magazine*, pp 245-249.

Van Zeggeren, F and Chung, S H, 1975. A model for the prediction of fragmentation, patterns and costs in rock blasting, in *Proceedings 15th Symposium on Rock Mechanics* (ed: E R Hoskins) pp 557-569 (The American Society of Civil Engineers: Reston).

APPENDIX A: BLAST MODELLING

The adapted version of the SCRAMBLE/SABREX blast modelling code used in this study is an energy-based approach comprising two separate models: heave mechanics and fragmentation. The heave mechanics are based on the energy released from the adiabatic expansion of the explosive gases following detonation. Fragmentation is based on the powder factor (ratio of charge weight in kilograms to mass in tonnes of rock broken by the charge) converted to an energy equivalent via the Bond Index.

The velocity of detonation for each blasthole is taken as infinite and the wall is allowed to expand until it reaches a state of equilibrium determined by the isotropic expansion characteristics of the quasi-static gas pressure and the elastic resistance of the rock. The expanded blasthole sets up hoop stresses in the surrounding rock, creating a system of radial cracks that, because of tensile failure, spread away from the hole. The radial fractures, together with any pre-existing geological discontinuities, define the damage created in the rock mass by the blast.

The gaseous detonation products flow into the fractured rock mass at the local speed of sound until the gas vents through a free face; at this stage a rarefaction wave travels back toward each blasthole decompressing the cracks. As the rarefaction wave travels through the rock, the pressurised crack system imparts an impulse, which heaves the broken rock mass out from the bench.

In generating the muck pile, empirical routines are used to limit the angle of repose whilst producing a smooth surface and adding swell factors.

Equation of state for explosive gases

The equation of state for the gaseous products of detonation is:

$$p = \frac{\alpha E \rho (1 + \beta\rho)^3}{100(1 + 2\beta\rho)} \tag{A1}$$

where:

p is the gas pressure in kbars

ρ is the gas density in g.cm^{-3}

E is the available energy in J.g^{-1}

α and β are dimensionless constants

The available energy E is the work done by the explosive gases in expanding adiabatically from the density ρ to ambient conditions, and is obtained from:

$$1\text{n}\left(\frac{E}{E_o}\right) = \alpha\left(\frac{(\beta\rho)^2 + 5\beta\rho}{4} - \frac{(\beta\rho_o)^2 + 5\beta\rho_o}{4} + \frac{1}{8}1\text{n}\left[\frac{\rho(1 + 2\beta\rho_o)}{\rho_o(1 + 2\beta\rho)}\right]\right) \tag{A2}$$

where:

ρ_o is the initial gas density after detonation (equal to the explosive density)

E_o is the initial available energy

The values for E_o, α and β can be obtained from an ideal or non-ideal detonation model. An ideal detonation model is adequate for the large diameter holes used in this study; more accurate data could be obtained from non-ideal models such as CpeX (Leiper and Plessis, 1987).

Equation A1 reduces to the ideal gas law for small gas densities and, together with Equation A2, allows available energy and pressure to be generated as a function of their density during the expansion process.

Heave mechanics

All regions within the gas envelope have a common gas density and pressure. The leading edge of the envelope is regarded as the gas front, which is assumed to move at the local speed of sound (m.s^{-1}) given by:

$$c = \left(\frac{100\,000\gamma p}{\rho}\right)^{\frac{1}{2}} \tag{A3}$$

where γ is the adiabatic exponent for the gases at pressure *p (kbar) and the density* ρ(g.cm^{-3}), γ is given by:

$$\gamma = \left(\frac{1 + \alpha(1 + \beta\rho)^3}{1 + 2\beta\rho}\right) + \left(\frac{3\beta\rho}{1 + \beta\rho}\right) \tag{A4}$$

and is derived from the equation of state given in Equation A1.

To calculate the necessary density and pressure of the gas within the envelope the volume of rock within the envelope is assumed to be in a state of hydrostatic compression at pressure p. The resultant reduction in the volume of rock is given by:

$$\Delta V + \frac{Vp}{G} \tag{A5}$$

where:

V is the initial volume (m^3)

G is the bulk modulus

ΔV is the volume increase in the envelope contributing to the reduction in gas density and pressure

Another small increase in volume is associated with the gas pressure compressing the rock below and behind the blasthole.

As the gas expands with the moving gas front, the local speed of sound in Equation A3 falls and a time-stepping loop is used to track the expansion of the gas. The time steps used are defined by:

$$\Delta t = \frac{b + \Delta b}{c} \tag{A6}$$

where:

$b + \Delta b$ is the equilibrium blasthole radius

Equation A6 shows that, although the time steps can vary, the corresponding spatial steps are constant and equal to the equilibrium borehole radius.

The time-stepping procedure is:

1. calculate the initial local speed of sound from Equations A3 and A4 prior to the expansion of gas into the rock mass;
2. calculate the appropriate time step from Equation A6 and generate the appropriate gas front profile;
3. calculate the increase in volume from Equation A5 and then calculate the new gas pressure and density using Equations A1 and A2;
4. recalculate the local speed of sound using Equations A3 and A4; and
5. repeat the steps while keeping track of the total elapsed time.

Venting of the explosive gas begins when the gas front meets a free face. At that time the gas fronts retrace their original paths and, during this period of contraction, the gas density, pressure and speed of sound are assumed to be constant within the volume of the gas envelope. The respective constant values are those that were calculated at the time of venting, while the pressure beyond the gas fronts is assumed to be insignificant.

At the time of venting, the rock mass is assumed free to move, reacting to a momentum impulse that is imparted on the rock mass. The calculated impulse is based on the assumption that the rock mass does not start to move until the gas fronts have contracted. The total impulse imparted on the rock mass (kg.ms^{-1}) is given by:

$$10^8 \int_{t_v}^{t_0} p(t_v)A(t)dt = M.v \quad \text{(A7)}$$

where:

t_V is the time (s) at which gas venting takes place

t_O is the time (s) at which the contracting gas fronts reach their blastholes

$p(t_V)$ is the gas pressure (kbar) in the gas envelope

$A(t)$ is the area (m^2) over which the pressure is applied

M is the mass (kg) associated with each blasthole

v is the velocity (ms^{-1}) with which the rock mass is heaved

t is the time (s)

To derive heave velocities from Equation A7 an expression for M can be applied for a vertical free face to calculate the mass of rock associated with each blasthole using:

$$M = B.S.H\rho_R.1000 \quad \text{(A8)}$$

where:

B is the burden (m)

S is the hole spacing (m)

H is the bench height (m)

ρ_R is the rock density ($g.cm^{-3}$)

In practical situations the highwall of a bench is not vertical and the program has an input variable for face angle to calculate the true mass of rock associated with the first row of holes.

The momentum impulse for each blasthole is resolved into the vertical and horizontal directions on the basis of the areas defined by the gas envelope. For the vertical impulse the area at the base of the envelope is used in Equation A7. However, due to the angled highwall, the front row has an inconsistent burden and the area is taken as an average of the areas at the top and bottom of the explosive column length.

Two impulses are computed in the horizontal direction. The first is the section of rock between the toe of the bench and the top of the explosive column, and the second impulse is the region at the top of the bench where the blasthole is filled with stemming material.

A similar averaging process is used to account for the effect of the front row of holes in the calculation of the horizontal impulse, which results in three horizontal heave velocities defining the heave velocity profile. On subsequent rows the effective free face is assumed to be vertical.

For the heave action, the blocks comprising the block model are treated sequentially within a time-stepping loop using a raster pattern starting at the toe of the bench with priority given in order to z, x and then y. For each run through the time-stepping loop all block positions and velocities are recalculated from ballistic trajectory equations and the revised values are stored in three-dimensional arrays; in-flight interactions with other blocks are not modelled. Each block remains in the time-stepping loop until it travels to a point in space at which, ahead or below it, another three-dimensional array describing the mine floor has a positive value, defining that volume of space as containing a block.

When a block drops out of the time-stepping loop to form part of the muck pile it immediately comes to rest on the ground and becomes part of the array that defines the floor and the developing muck pile. The input value for maximum angle of repose ensures that if the defined angle is exceeded in the generation of the muck pile then the block is moved down the surface of the muck pile until it reaches a point of stability.

When all blocks have come to rest, swell is applied to the muck pile by raising each block by a pre-defined factor proportional to the change in vertical height the block underwent in moving from the bench to the muck pile.

Fragmentation

The Bond Index equation from comminution theory is used to assess the effect of different blasting practices on the degree of fragmentation resulting from a blast (Van Zeggeren and Chung, 1975; Nielsen, 1983). The equation relating energy input to degree of comminution is:

$$W = K_B\left[\frac{1}{P^{1/2}} - \frac{1}{F^{1/2}}\right] \quad \text{(A9)}$$

where:

W is the energy input to a machine reducing particle size ($kWh.t^{-1}$)

F is the feed size, measured in microns (10^{-6} m), and defined as the mesh size of a screen that allows 80 per cent of the material to pass

P is the product size in microns also at 80 per cent passing

K_B is a constant determined for a specific feed material

The constant K_B is determined by rearranging Equation A10 to give:

$$K_B = W\left[\frac{P^{1/2}F^{1/2}}{F^{1/2} - P^{1/2}}\right] \quad \text{(A10)}$$

and the amount of energy required to reduce a known feed size to a given product size is measured. For MRT the amount of energy needed to reduce the secondary crushed product from -19 mm to a final product size of -210 microns was, on average over a two-month period, 16.10 $kWh.tonne^{-1}$. As the Bond Index works on 80 per cent passing size, the feed and product sizes are taken as 16 300 microns (16.3 mm) and 180 microns respectively. Substituting these values into Equation A10 gives:

$$K_B = 16.10 \times \left[\frac{180^{1/2} \times 16\,300^{1/2}}{16\,300^{1/2} - 180^{1/2}}\right] = 241.4 \text{ kWh - micron}^{1/2}.\text{tonne}^{-1}$$

Equation A9 can also be rearranged to calculate the energy required to reduce an infinite feed size ($F = \infty$) down to any product size P. This is referred to as the total energy (W_t) and is given by:

$$W_t = K_B\left[\frac{1}{P^{1/2}} - \frac{1}{\infty^{1/2}}\right] = \frac{K_B}{P^{1/2}} \quad \text{(A11)}$$

Based on Equation A11, the Bond Work Index (W_i) is the amount of energy required to reduce an infinite feed size down to an 80 per cent passing size of 100 microns. This is used as a common basis of comparison across different materials and processes and is given by:

$$W_i = K_B\left[\frac{1}{100^{1/2}} - \frac{1}{\infty^{1/2}}\right] = \frac{K_B}{100^{1/2}} \quad \text{(A12)}$$

Substituting the calculated K_B in Equation A12 gives:

$$W_i = \frac{241.4}{100^{1/2}} = 24.1 \text{ kWh.tonne}^{-1}$$

From Equations A11 and A12 it is possible to calculate the energy required to reduce material from an infinite size down to the desired 80 per cent passing size as:

$$W_t = W_i\left[\frac{100}{P}\right]^{1/2} \tag{A13}$$

If it is assumed that the only factor that influences the degree of fragmentation in blasting is the amount of energy imparted to the rock mass, and that the energy distribution and initiation variables can be ignored, then Equation A13 should give a good representation of the energy input from the explosive in a blast, based on the resulting fragmentation.

For the 6.5 m × 8 m MRT blast designs, the material in the resulting muck piles had an 80 per cent passing size of approximately 0.5 m. From Equation A13 the energy imparted by the explosive is:

$$W_t = \left[\frac{100}{5\times10^5}\right]^{1/2} = 0.34 \;\; \text{kWh.tonne}^{-1} = 1.23 \text{ MJ.tonne}^{-1}$$

The energy supplied by the explosive acting on the rock mass can be derived from the known powder factor (PF) at 0.31 kg.tonne^{-1} for the blasts and the energy contained in the explosive used. The energy for the heavy ANFO used, with specific density 1.2 g.cm^{-3}, is 4.5 MJ.kg^{-1}. The explosive energy per tonne is therefore:

$$\text{PF} \times \text{Explosive Energy} = 0.31 \times 4.5 = 1.40 \text{ MJ.tonne}^{-1}$$

This value of 1.40 MJ.tonne^{-1} compares favourably with the value of 1.23 MJ.tonne^{-1} derived using the Bond Index for comminution (Hustrulid, 1999). If it is assumed that the difference in values is due to slight differences in the efficiencies of the two processes then it is reasonable to reconcile the two values by applying a factor (α) that is appropriate over a range of energies.

By rearranging Equation A13 and applying the correction factor α (Van Zeggeren and Chung, 1975) the equation for product size from the powder factor used in the blast design is:

$$P = \left(\left[\frac{W_i}{W_t}\right]\times\alpha\right)^2 \tag{A14}$$

where:

W_t is the energy equivalent of the powder factor

APPENDIX B: COSTING THE BLASTING AND SELECTION PROCESSES

To simplify calculations, all process costs are calculated as cost per tonne worked.

Drilling costs

Drilling costs are expressed as a cost per metre drilled (DC_m) for the 250 mm hole diameter used in this study. The tonnage of rock associated with each blasthole, taken as a standard for a specific hole pattern, is given by Equation A8, divided by 1000 to give tonnes. Cost per tonne (DC_t) is then:

$$DC_t = \frac{DC_m \times HL}{M} \tag{B1}$$

where:

HL is the hole length (m), including subdrill

Blasting costs

The initial costs are calculated for a single hole and are divided into fixed costs per hole – booster, detonator, surface connection and manpower costs – and the variable cost of the main charge placed in the hole. The main charge costs (EX_m) in dollars are calculated using:

$$EX_m = EX \times (A_h \times EC_l)\times\rho_e \tag{B2}$$

where:

EX is the cost of the explosive (\$.kg^{-1})

A_h is the cross-sectional area of the hole (m^2)

EC_l is the charge length of the explosive in the hole (m)

ρ_e is the density of the explosive used (g.cm^{-3})

Loading costs

The loading costs for the original blast design are taken as \$0.14/tonne for a muck pile with 80 per cent passing size of 0.5 m. Within reasonable limits, as passing size decreases loading costs decrease, due mainly to an increase in the ease of digging, which leads to faster loading rates and reduced maintenance costs. MacKenzie (1966) reports a linear relationship between cost per unit loading and degree of fragmentation for Quebec Cartier's 16-D iron ore mine. Van Zeggeren and Chung (1975) found that their data followed a square root relationship and Nielsen (1983) used a variable exponent selected by the user. For this application, with too few operational data to derive an appropriate relationship, the linear equation is:

$$C_l = (D80\times\alpha) + (SC_l - \beta) \tag{B3}$$

where:

C_l is the adjusted loading cost (\$.tonne^{-1})

$D80$ is the calculated 80 per cent passing size (m) using Equation A14

SC_l is the standard loading cost (0.14 \$.tonne^{-1})

α, β are constants

The incorporation of the standard loading cost (SC_l) in Equation B3 allows the loading cost relationship to be adjusted for different loaders with different attributes.

Haulage costs

Haulage costs also decrease with muck pile particle size because the truck is more completely filled, providing the ore density allows it. The relationship used for haulage costs, (C_h), in \$/tonne is:

$$C_h = \chi . e^{D80} \tag{B4}$$

where:

χ is a constant

Primary crushing costs

Because variations in feed size to the primary crusher affect power costs much more than general maintenance and plate replacement costs, the Bond Index Equation A9 was used to calculate crushing costs:

$$C_{cr} = \delta \times 241.4 \times \left(\frac{1}{16\,300^{1/2}} - \frac{1}{D80^{1/2}}\right) \tag{B5}$$

where:

C_{cr} is the adjusted crushing cost ($\$.\text{tonne}^{-1}$)

δ is a constant

Costs unaffected by blasting practices

Costs incurred in producing a saleable product that are not affected by blasting practices include mining services and the entire mineral processing operation downstream of the primary crushing. These values, also expressed as $/tonne, are assumed to remain constant.

Mining Schedule Optimisation for Conditionally Simulated Orebodies

M Menabde[1], G Froyland[2], P Stone[1] and G A Yeates[3]

ABSTRACT

Traditionally the process of mine development, pit design and long-term scheduling is based on a single deterministic orebody model built by the interpolation of drill hole data using some form of spatial interpolation procedure, eg kriging. Typical steps in mine design would include the determination of the ultimate pit, the development of a number of mining phases (pushbacks) and then the development of a life-of-mine schedule. All of these steps would have the aim of maximising the mine's net present value (NPV), along with meeting numerous other business and physical constraints.

There are a number of software packages commercially available and widely used in the mining industry that deal with some or all of these issues. The methods employed by all of these packages treat the process described above in a strictly deterministic way. In reality, given the sparse drill hole data, there is usually significant and variable uncertainty associated with a single or unique deterministic block model. This uncertainty is not captured or used in the planning process.

This paper describes work undertaken by the Exploration and Mining Technology Group within BHP Billiton to develop a new mathematical algorithm for mine optimisation under orebody uncertainty. This uncertainty is expressed as a number of conditionally simulated orebody models. This optimisation algorithm is implemented in a new software package. The software uses a number of proprietary algorithms along with the commercially available mixed integer-programming package ILOG CPLEX. The development targets all phases of mine optimisation, including the NPV optimal block extraction sequence, pushback design, and simultaneous cut-off grade and mining schedule optimisation.

INTRODUCTION

This paper describes the development and implementation of a new software package for open pit mine development and scheduling optimisation under conditions of orebody uncertainty and is based on the mixed integer programming method. The approach uses multiple conditionally simulated realisations of the orebody as input to characterise the orebody along with the uncertainty in the estimate.

Traditionally open pit mine planning, pit design and long-term scheduling is based on a block model of the orebody built by interpolation techniques such as kriging from the drill hole sample data. This single model is assumed to be a fair representation of reality and is used for mine design and optimisation. The design process consists of four main steps:

1. Determining the ultimate pit shell to define the scheduling universe.
2. Finding the block extraction sequence which produces the best net present value (NPV) whilst satisfying the geotechnical slope constraints.
3. Designing the practically minable mine phases (pushbacks) which are roughly based on the optimal block sequence.
4. Optimising the mining schedule and cut-off grades (COG) within a set of business and operational constraints. The NPV of this 'optimal' schedule is considered as a main criterion of the economical viability of the project.

In reality, there are many uncertainties in the models and parameters used in optimisation. Thus, the adoption of a single economic criterion for a project can be very questionable. One of the most important sources of uncertainty is the block model itself. The drill hole data for a mining project is typically sparse, particularly at the scale of the selective mining unit and could support a range of possible outcomes for the orebody. A unique deterministic block model will often be a good representation of the global resource, but will not be representative of the potential local variability or the uncertainty in the estimate. An approach that quantifies both the local variability and the potential uncertainty is to use multiple conditional simulation realisation to represent the orebody (see Dimitrakopoulos, 1997). This approach allows the generation of a number of equally probable realisations of the block model, at the selective mining unit (SMU) scale, with all of them honouring the drill hole data along with the first and second order statistics of the orebody represented, respectively, by the probability distribution and variogram (eg Isaaks and Srivastava, 1989).

The simplest and most straightforward use of this set of orebody realisations is to estimate the variability in the project NPV associated with the orebody uncertainty by valuing the 'optimal' schedule obtained from the kriged deterministic model through each of the conditionally simulated realisations.

The more interesting question is whether it is possible to use the set of conditionally simulated realisations to produce a better mine design and production schedule. By 'better' we mean here a higher expected NPV (which becomes a random variable in case of multiple realisations of the orebody model) and/or less variability from one realisation to other (ie lower variance of NPV). A new promising approach to this problem is presented in Ramazan and Dimitrakopoulos (2007, this volume); Jewbali (2006).

In this paper we address one particular aspect of the optimisation under uncertainty, namely the simultaneous optimisation of the extraction sequence and COG. The use and importance of optimal (variable) COG to mining projects has been known for a long time (eg Lane, 1988). It will be demonstrated here that the use of variable COG optimised under uncertainty, using the set of equi-probable realisations of the orebody can provide a substantial improvement in terms of expected NPV. The approach based on mixed integer programming techniques can provide a truly optimal schedule, as opposed to various heuristic methods used in most of the commercially available mining optimisation software packages.

MINING SCHEDULE OPTIMISATION AS A MIXED INTEGER PROGRAMMING MODEL

Typically, the orebody block model contains between 50 000 to 5 000 000 blocks, which must be scheduled over a period of say five to 25 years. The objective of any scheduling procedure is to find the block extraction sequence, which produces the maximum possible net present value (NPV) and obeys a number of constraints. The latter include:

1. BHP Billiton, GPO Box 86A, Melbourne Vic 3001, Australia.
2. School of Mathematics, The University of New South Wales, Sydney NSW 2052, Australia. Email: froyland@maths.unsw.edu.au
3. FAusIMM(CP), Global Manager – Mineral Resource Development, Business Excellence, BHP Billiton Limited, PO Box 86A, Melbourne Vic 3001, Australia. Email: gavin.yeates@bhpbilliton.com

1. geotechnical slope constraints, which are modelled by a set of precedence arcs between individual blocks;
2. mining constraints, ie total maximum amount of rock which can be mined in one time period (usually one year);
3. processing constraints, ie maximum amount of ore which can be processed through a given processing plant in one time period; and
4. the market constraints, ie the maximum amount of metal that can be sold in one time period.

The mathematical formulation of the scheduling procedure in terms of binary decision variables describing in which period the particular block is extracted and what its destination is (either processing plant, stockpile or waste dump), is quite straightforward. The size of the problem is, however, prohibitively large. Apart from the computational difficulties, the hypothetical optimal block extraction sequence may be completely impractical due to the requirements for the mining equipment access and relocation.

Because of these problems the mine scheduling is done using much bigger elementary units that are typically aggregations of hundreds or even thousands of blocks. The aggregation of blocks is a nontrivial problem. For example, simply combining rectangular blocks into a larger rectangular block with dimensions multiples of that of individual blocks can effectively reduce the size of the problem but will provide a very poor approximation for the geotechnical slopes. An interesting approach to block aggregation based on the concept of 'fundamental trees' has been recently developed by Ramazan (2007, this volume). In this method the aggregations of blocks – fundamental trees – obey the slope constraints and can substantially reduce the number of integer variables required for the scheduling model. However, the number of these aggregations is not user controllable and in many cases the problem can be still too big to be solved by a direct application of the mixed integer programming techniques.

We have recently developed a new algorithm for block aggregation, which preserves the slope constraints, and is very flexible allowing the user to fully control the size and shape of these aggregations. The details of this algorithm will not be discussed here. The optimisation procedure, however, can be applied to any aggregation of blocks with a set of precedence arcs, prescribing which blocks should be extracted before the given one. As an example we consider here the scheduling of mining phases.

In practice, the open pit mine is divided into a number of mining phases, which are mined bench by bench, each bench represented by a horizontal layer of blocks within the given mining phase and having the same elevation. A bench within a mining phase is sometimes referred to as a 'panel'. The mining phases can be mined one by one from top to bottom, however this kind of schedule is usually suboptimal. Mining several phases simultaneously and applying variable COG can produce much better results in terms of NPV. There are several commercially available packages, which use proprietary (and undisclosed) heuristics to optimise the schedule and COG. It is difficult to estimate their effectiveness, as the upper theoretical limit on NPV remains unknown. Moreover, these methods can only be used on a single orebody representation and cannot be directly used on a set of conditionally simulated orebody realisations.

The standard optimisation technique widely used in many industrial applications is the linear and integer programming (eg Padberg, 1995). The main difficulty in its application to mining scheduling is that the optimisation with variable COG in its direct formulation leads to a non-linear problem, which is much harder to solve. Our approach provides an effective linearisation of this problem, making it possible to use a mixed integer programming (MIP) formulation for a simultaneous optimisation of the extraction sequence and COG for a number of conditionally simulated orebody models. The MIP formulation we use here is similar to the one used by Caccetta and Hill (2003) but is generalised to include the multiple realisations of conditional simulations and variable cut-off grades. This approach also allows one to estimate the gap between the obtained solution and the upper theoretical limit.

We consider the simplest case when we have one rock type containing one metal type, which can be processed through one processing plant. Generalisation to the case of multiple rock types, metals and processing streams is cumbersome but straightforward. For simplicity we consider here only the case of a discrete set of COGs, though it is possible to generalise the results to the continuous COG case. We use the following notations:

T is the number of scheduling periods

N is the number of simulations

P is the total number of panels

G is the number of all possible cut-off grades

R_i^n is the total rock in the panel i in simulations n

Q_{ij}^n is the total ore in the panel i, simulation n, when mined with the COG j

V_{ij}^n is the value of the panel i, simulation n, when mined and processed with the COG j

R_t^0 is the maximum mining capacity in period t

Q_t^0 is the maximum processing rate in period t

Si is the set of panels that must be removed before starting the panel i

d^t is the time discount factor

$xijt$ is the fraction of the panel i is extracted with the COG j in period t

y_{it} is a binary variable equal to 1 if the extraction of the panel i has started in periods 1 to t, and equal to 0 otherwise;

δ_{jt} is a binary variable controlling the selection of the COG applied in period t

The MIP formulation is:

$$Maximise\left(\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j=1}^{G}\sum_{t=1}^{T}V_{ij}^n x_{ijt} d^t\right) \tag{1}$$

subject to the following constraints:

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j-1}^{G}R_i^n x_{ijt} \le R_t^0, \quad for\ all\ t \tag{2}$$

$$\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{P}\sum_{j=1}^{G}Q_{ij}^n x_{ijt} \le Q_t^0, \quad for\ all\ t \tag{3}$$

$$y_{i,t-1} \le y_{it}, \quad for\ all\ i\ and\ t \tag{4}$$

$$\sum_{\tau=1}^{t}\sum_{j=1}^{G}x_{ij\tau} \le y_{it}, \quad for\ all\ i \tag{5}$$

$$y_{it} \le \sum_{j=1}^{G}\sum_{\tau=1}^{T}x_{kj\tau}, \quad for\ all\ i,\ t\ and\ k \subset S_i \tag{6}$$

$$\sum_{j=1}^{G}\delta_{jt} = 1, \quad for\ all\ t \tag{7}$$

$$x_{ijt} \le \delta_{jt}, \quad for\ all\ i,\ j\ and\ t \tag{8}$$

The objective function (1) represents the discounted cash flow. Constraints (2) and (3) enforce the mining and processing limits on average. Constraints (4) – (6) enforce the panel extraction precedence constraints, and constraints (7) and (8) ensure that the same COG is applied to all panels extracted in any given time period.

This MIP formulation is solved by the commercially available software package CPLEX version 9.0, by ILOG Inc.

CASE STUDY

To test the algorithm we have chosen ten conditional simulations of a block model containing one type of metal and using one processing plant. Because of confidentiality requirements, all the economic parameters were rescaled and do not represent reality. All of the relative characteristics which demonstrate the potential of this new method are not affected by this rescaling. The ultimate pit for the design is chosen by applying the Lersch-Grossmann algorithm (Lersch and Grossmann, 1965) in a procedure similar to that used in Whittle Four-X software. The ultimate pit contains 191 million tonnes of rock and 62.9 ± 2.7 million tonne of ore (above the marginal COG = 0.6 per cent). The undiscounted value in the ultimate pit (if processed with the marginal COG) is $(1316 ± 99) million. It was divided into six mining phases and scheduled over 12 years. The mining rate was set to 30 Mtpa and the processing rate to 5 Mtpa. The initial capital investment was assumed to be $300 million, and the discount rate ten per cent. The base case optimisation was done using the marginal COG applied individually to all conditional simulation. The NPV for this case was $(404 ± 31) million. The mining schedule is shown in Figure 1. The second optimisation was done using the variable COG, but was based on the mean grade block model, ie it was similar to an optimisation generated by using a single deterministic model. This schedule was evaluated against all ten realisations of orebody model and produced the NPV = $(485 ± 40) million, an increase of 20 per cent over the base case. This mining schedule is shown in Figure 2. The third optimisation was done using the algorithm described in earlier, using all orebody realisations as input to the optimisation and produced the NPV = $(505 ± 43) million, a further increase of 4.1 per cent over the case of mean grade based optimisation. This mining schedule is shown in Figure 3. The relative variability of NPV in all cases was roughly the same, about eight per cent. The cumulative NPV graphs for the three different schedules are shown in Figure 4, and the comparison between expected NPVs and their variability is shown in Figure 5. Another important result of the variable COG policy is that the pay-back period (defined here as the time when the cumulative NPV becomes equal to zero) is decreased from five to three years (see Figure 4).

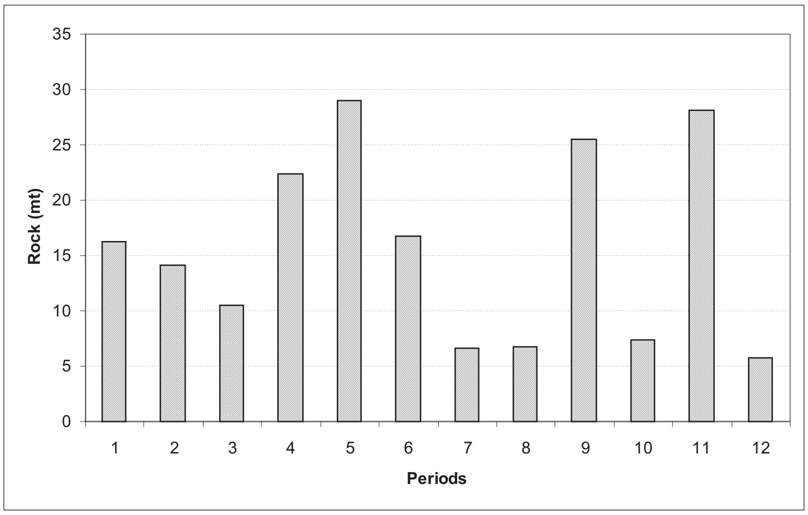


FIG 1 - Mining schedule optimised with the marginal COG.

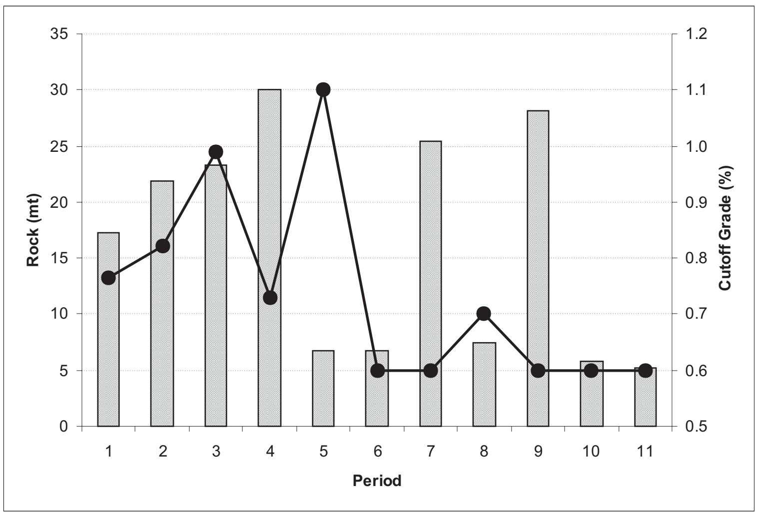


FIG 2 - Mining schedule optimised with the mean grade model.

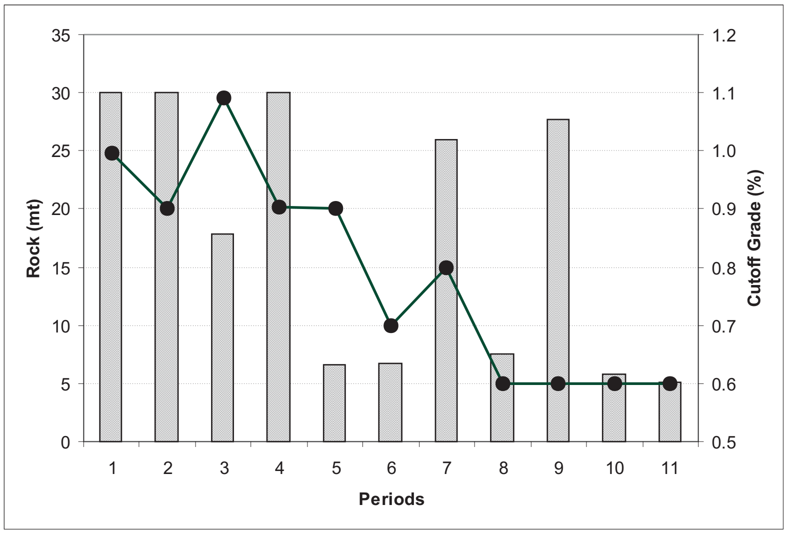


FIG 3 - Mining schedule optimised with the set of conditional simulations.

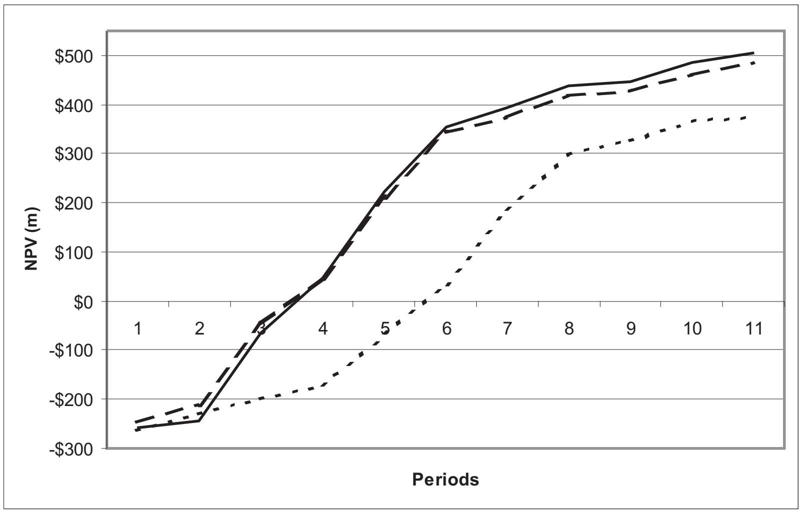


FIG 4 - Cumulative NPV for different mining schedules (solid line – variable COG on conditional simulations; dashed line – variable COG on the mean grade model; dotted line – marginal COG).

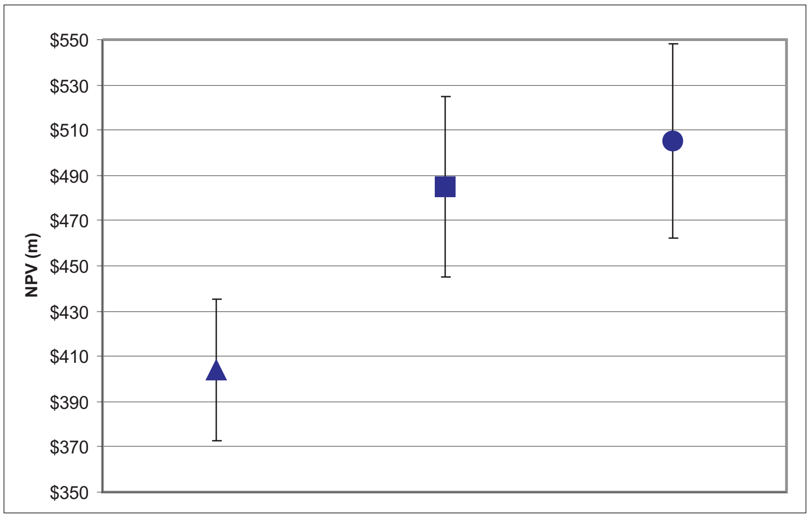


FIG 5 - Comparison of expected NPVs and their variability for different mining schedules (circle – variable COG on conditional simulations; square – variable COG on the mean grade model; triangle – marginal COG).

The increase of 4.1 per cent in NPV may be not seen as a very substantial, but it should be mentioned that the block model considered does not have a high variability. The relative variance in the undiscounted value of the ultimate pit is only 7.6 per cent. There are many deposits that have variability of the order of 20 - 30 per cent. For these kind of deposits the potential improvement in the expected NPV may be substantially higher.

CONCLUSIONS

A new method for simultaneous optimisation of the extraction sequence and cut-off grade policy for a set of conditionally simulated orebody realisations has been developed and demonstrated. This method is based on the mixed integer programming model and uses the commercially available software package CPLEX by ILOG Inc. The goal of the optimisation is to find the extraction sequence and cut-off grade policy, which, when evaluated through the whole set of conditionally simulated orebodies (representing the range of possible outcomes), will produce the best possible expected NPV. The degree of accuracy of this optimised schedule can be estimated precisely, in contrast to a number of heuristic routines used in current commercially available mining optimisation software packages. A fully functional software prototype that uses the new optimisation method has been developed.

In this study, we were using the expected NPV as the objective function and the mining and processing constraints were applied to the mean rock and ore tonnages. Some of the possible extensions of this method may include some kind of penalty functions in the objective function in order to find a schedule with a reduced variability in NPV, defining hard constraints bounding the NPV from below, or defining a lower bound on the annual cash flows. Another very interesting generalisation may include a stochastic price model for metals and adjustable cut-off grade policy.

REFERENCES

Caccetta, L and Hill, S P, 2003. An application of branch and cut to open pit mine scheduling, *Journal of Global Optimization*, 27(2-3):349-365.

Dimitrakopoulos, R, 1997. Conditional simulations: tools for modelling uncertainty in open pit optimisation, in *Proceedings 1997 Optimizing with Whittle: Strategic Mine Planning Conference,* pp 31-42 (Whittle Programming Pty Ltd: Melbourne).

Isaaks, E H and Srivastava, R M, 1989. *Applied Geostatistics*, 561 p (Oxford University Press: New York).

Jewbali, A, 2006. Modelling geological uncertainty for stochastic short-term production scheduling in open pit metal mines, PhD thesis, The University of Queensland, Australia, 280 p.

Lane, K, F, 1988. *The Economic Definition of Ore*, 147 p (Mining Journal Books Ltd: London).

Lerchs, H and Grossmann, L, 1965. Optimum design of open-pit mines, *Trans CIM,* LXVII:17-24.

Padberg, M W, 1995. *Linear Optimization and Extensions*, 449 p (Springer: New York).

Ramazan, S, 2007. Large-scale production scheduling with the fundamental tree algorithm — Model, case study and comparisons, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 121-127 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Ramazan, S and Dimitrakopoulos, R, 2007. Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 385-391 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Stochastic Optimisation of Long-Term Production Scheduling for Open Pit Mines with a New Integer Programming Formulation

S Ramazan[1] and R Dimitrakopoulos[2]

ABSTRACT

Conventional approaches to optimising open pit mine design and production scheduling are based on a single estimated orebody model, which does not account for geological variability. Conditional simulation can be employed to quantitatively address the resulting grade uncertainty.

Multiple simulated orebody models provide a suitable input for stochastic integer programming (SIP), a type of mathematical programming that generates the optimal result for a defined set of objectives under uncertainty. In the case of production scheduling, the objectives are to maximise the total net present value (NPV) and to minimise unsatisfied demand for processed ore. Using a set of multiple simulated orebody models as input into an SIP model allows for the integration of *in situ* deposit variability and uncertainty directly into the production scheduling optimisation process.

INTRODUCTION

Stochastic integer programming (SIP) is a type of mathematical programming and modelling that considers multiple equally probable scenarios and generates the optimal result for a set of defined objectives within the feasible solution space bounded by a set of constraints. SIP is defined as an extension of mixed integer programming (MIP) with uncertainty in one or more of the related coefficients (Escudero, 1993). This tends to increase problem size and complexity when compared with scheduling formulations based on MIP (Ramazan, 2001). Different approaches in SIP formulations are discussed in Birge and Louveaux (1997); however, the existing developments in the technical literature are not directly applicable to mining problems.

The effects of orebody uncertainty and *in situ* geological variability on approaches to optimising open pit mine design have been shown in recent studies. Dimitrakopoulos, Farrelly and Godoy (2002) show the substantial conceptual and economic differences of risk-based frameworks. Dowd (1997) proposes a framework for risk integration in surface mining projects. Ravenscroft (1992) discusses risk analysis in mine production scheduling, where the use of stochastically simulated orebodies shows the impact of grade uncertainty on production scheduling, and states that conventional mathematical programming models cannot accommodate quantified risk. The need for optimisation methods that can integrate uncertainty raises the need for efficient simulation methods, as discussed in Boucher and Dimitrakopoulos (2007, this volume), Godoy (2003) and Dimitrakopoulos and Luo (2004). Pursuing this line of thought, Ramazan and Dimitrakopolous (2004a) developed efficient MIP formulations to generate feasible mining patterns of optimised probabilistic production schedules.

Although all these studies represent substantial developments in the field, they do not directly integrate uncertainty in the optimisation process. Dimitrakopoulos and Ramazan (2004) propose a probabilistic long-term scheduling optimisation method based on linear programming to deal with uncertainty. The proposed method accounts for risk through probabilities of being above or below a cut-off; however, it still does not directly and explicitly account for orebody uncertainty. Godoy and Dimitrakopoulos (2004) developed a new risk-inclusive long-term production scheduling approach based on simulated annealing and achieved significant improvement in the total NPV of a large gold mine project. Their model does not consider the issues of grade blending and controlling the risk distributions for production targets; although it does minimise the risk of not meeting periodical ore production targets.

This paper presents an efficient new SIP mathematical model that generates optimum long-term production schedules for open pit mines for a defined objective function, considering the operational requirements at the mine. The SIP model takes multiple simulated orebody models, without averaging the grades, and maximises the total NPV when considering geological uncertainty caused by grade variability. The geologic risk discounting concept (Dimitrakopoulos and Ramazan, 2004; Dimitrakopoulos, in press) is incorporated within the SIP model to control the risk distribution between production periods. The penalty parameters for deviations from targets are implemented to control the geological risk distribution in terms of magnitude and variability. This SIP model has been developed as part of an ARC-Linkage project, initially reported by Ramazan and Dimitrakopoulos (2003).

STOCHASTIC INTEGER PROGRAMMING MODEL

The SIP model developed herein accounts for uncertain inputs by considering simulated grade realisations in the optimisation process. It can thereby minimise the risk of a mine not meeting production targets as a result of geological uncertainty. The model contains an objective function and a set of constraints representing the operational requirements of the mine. Within these constraints, the model performs the necessary calculations to reach the objective. The objective function is defined as the maximisation of the total NPV of the project minus the cost of deviations between the planned amount of ore tonnage, grade and quality and the amount of those produced from the actual operation. The NPV values of individual blocks in the objective function are calculated from the average of the undiscounted economic values in the simulated orebody models (Godoy and Dimitrakopoulos, 2004), not from the average of the grade. The parameters that are included in the objective function to account for deviations are assigned for each simulated orebody model and for each time period for each type of production target, such as maximum periodical grade of ore, minimum grade of ore, maximum ore processing capacity and minimum ore tonnage that has to be processed. These deviation factors are calculated in the related constraint formulations that consider individual simulated orebody models for each of the production periods.

Definition of symbols and terms

Two basic concepts for the set-up of the SIP program and model are:

- *'Variable'* is a factor whose value will be determined by solving the mathematical model. The solver CPLEX is used to solve SIP/MIP/LP type mathematical models in this study.
- *'Constant'* is a factor whose value has to be provided to the mathematical model by the user.

1. MAusIMM, Rio Tinto, GPO Box A42, Perth WA 6000, Australia. Email: salih.ramazan@riotinto.com

2. MAusIMM, COSMO Laboratory, Department of Mining, Metals and Materials Engineering, McGill University, Frank Dawson Adams Building, Room 107, 3450 University Street, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

The variable and constant factors used in the SIP model are defined below:

P is the total number of production periods, or mine life; *constant*

N is the number of blocks considered in modelling; *constant*

b_i^t is a variable representing the percentage of block i mined in period t; if a b_i^t variable is defined as binary (0 or 1), it is assigned 1 if block i is mined in period t and assigned 0 otherwise; *variable*

M is the total number of simulated orebody models; *constant*

d_{su}^{to} is the excess amount of ore tonnage produced above a desired tonnage, or upper limit, in period t if the deposit has the same characteristics defined in the orebody model s; *variable*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

c_u^{to} is unit cost of d_{su}^{to} for the objective function; *constant*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

d_{sl}^{to} is the deficient amount of ore tonnage produced below a desired tonnage, or lower limit, in period t if the deposit has the same characteristics defined by the orebody model s; *variable*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

c_l^{to} is unit cost of d_{sl}^{to} for the objective function; *constant*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

f_l is the orebody risk discounting rate used to calculate c_l^{to} and c_l^{tg} values; *constant*

f_u is the orebody risk discounting rate used to calculate c_u^{to} and c_u^{tg} values; *constant*

f is used in this project as the orebody risk discounting rate: $f=f_l=f_u$; *constant*

R is the periodical economic discount rate, which is set to ten per cent in this case; *constant*

$E\{(EV)_i^0\}$ is the expected economic value to be generated in the future time t if block i is mined in period t; *constant*. The expected value of block i is calculated as follows:

$$E\{(EV)_i^0\} = ((EV_i)_1^0 + (EV_i)_2^0 + ... + (EV_i)_M^0) / M$$

$E\{(NPV)_i^t\}$ is the expected discounted value to be generated if block i is mined in period t; *constant.* It is calculated as follows:

$$E\{(NPV)_i^t\} = E\{(EV)_i^0\} / (1 + R)^t.$$

V_i^t is a representation of $E\{(NPV)_i^t\}$; *constant*

G_{si} is the grade of block i in orebody model s; *constant*

O_{si} is the ore tonnage inside block i in orebody model s; *constant*

G_{min} and G_{max} are the targeted minimum and maximum average grade of the ore material to be processed in a period; *constant*

m_{sl}^{to} is the dummy variable used to balance the equality constraints when the ore tonnage produced is more than the minimum required amount for the orebody model s; *variable*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

m_{su}^{to} is the dummy variable used to balance the equality constraints when the ore tonnage produced is less than the maximum amount for the orebody model s; *variable*. Note that *g* instead of *o* in this term refers to grade and *q* to metal quantity

Y_i number of blocks overlying ore block i considered for setting the slope constraints; *constant*

The objective function

The objective function of the SIP model is constructed as the 'maximisation of a profit function'. The profit function is defined as the total expected NPV minus the cost of deviations from planned production targets. It is expressed as follows:

$$\text{Max} \sum_{t=1}^{P} [\underbrace{\sum_{i=1}^{N} V_i^t b_i^t}_{\text{Part 1}} - \underbrace{\sum_{s=1}^{M} (c_u^{to} d_{su}^{to} + c_1^{to} d_{sl}^{to} + c_u^{tg} d_{su}^{tg} + c_l^{tg} d_{sl}^{tg} + c_u^{tq} d_{su}^{tq} + c_l^{tq} d_{sl}^{tq})]}_{\text{Part 2}} \quad (1)$$

Part 1 of the objective function is used for maximising the total discounted economic value while Part 2 is used for managing the risk of not meeting production targets using conditionally simulated orebody models. Traditionally, one orebody model, a smooth image of the deposit, is used for maximising NPV. However, when the expected deviations from the planned amount of ore tonnage having a planned grade and quality in a schedule are high in actual mining operations, the traditional model is unlikely to achieve the resultant NPV of the planned schedule. So, the NPV to be generated from actual mining can be far from optimal even if the schedule is optimised using a traditional true optimiser, MIP model (Ramazan, 2001; Ramazan and Dimitrakopoulos, 2004a). Therefore, the SIP model is developed to consider the minimisation of the deviations together with the maximisation of NPV to generate achievable NPV.

For constructing the objective function, initially, a constant value is assigned for each of the cost parameters representing the cost at time 0 (*base cost*). Then, the risk discounting parameter (*f*) is introduced to determine the cost at different time periods by discounting the *base cost* using *f* (Dimitrakopoulos and Ramazan, 2004). The risk-discounting concept is then incorporated into the SIP model (Ramazan and Dimitrakopoulos, 2003; Dimitrakopoulos, in press).

If *f* is set to 0, the deviations in production targets can be expected to result in more or less the same level between different production periods because the cost of a unit deviation will be the same in all periods. However, the distribution of deviations will also depend on how the variability in grade and ore tonnage is distributed over the deposit and on how the relative magnitude of the costs for the deviations used in the SIP model compare with the economic values of the blocks.

The model constraints

The deviation parameters are calculated within the SIP model by using the related constraints that consider each of the simulated orebody models. In this paper, equality-type constraints that use simulated multiple orebody realisations are called 'stochastic constraints' or, more specifically, 'soft stochastic constraints' because they are feasible for any value of the decision variables (b_i^t).

Stochastic constraints related to grade blending are used to satisfy not only the grade requirement at the mill but also the requirements for quality parameters, for example, the combination of elements like aluminium and magnesium in nickel mines, or silica in iron ore mines. This type of constraint can be expressed by:

$$\sum_{i=1}^{N}(G_{si}-G_{min})O_{si}b_i^t + d_{sl}^{tg} - m_{sl}^{tg} = 0 \qquad \text{Lower Bound} \quad (2a)$$

$$\sum_{i=1}^{N}(G_{si}-G_{max})O_{si}b_i^t + d_{su}^{tg} - m_{su}^{tg} = 0 \qquad \text{Upper Bound} \quad (2b)$$

These constraints are written for each of the M – equally probable orebody models (s=1, 2,…,M) and P – time periods (t=1, 2,…,P). The stochastic constraints for ore tonnage and metal can also be written in a similar way as grade constraints (Ramazan and Dimitrakopoulos, 2003). Other operational constraints (Ramazan and Dimitrakopoulos, 2004a) are also included in the model although not discussed in this paper.

TESTS ON A HYPOTHETICAL TWO-DIMENSIONAL DATA SET

This section presents applications of the SIP scheduling model using different cost parameters for the deviation factors on a hypothetical two-dimensional single-element gold deposit. The deposit considered herein is a subvertical orebody model that requires mining with a 45° slope angle. The model contains 200 square blocks, 20 and ten along the horizontal and vertical axes, respectively. The gold deposit is simulated by generating 50 orebody models that represent the deposit with equal probabilities (Ramazan *et al*, 2004). The grades in these simulated orebody models are then averaged, generating an orebody model referred to herein as the 'e-type' orebody model. This orebody model is the equivalent of an estimated model that is a smoothed image of the deposit, which is often used as input in traditional optimisation methods. The grade distribution of three simulated orebody models and the e-type model are shown in Figure 1. The figure shows that, although there are some similarities in the general characteristics of the grade distribution, there are local differences between the simulated equi-probable orebody models. All the simulated models have the same histogram and spatial continuity.

Implementation of the SIP model

The artificial deposit is scheduled to be mined for three years of production using the SIP model. During the tests performed, two different cost parameters are used: one aims to penalise the deviations in ore production (c^{to}) in period t and the other aims to penalise the deviations in the average grade of the ore produced (c^{tg}) in period t. In this study, the excess ore production (d_{su}^{to}) and grade (d_{su}^{tg}), and shortage in ore production (d_{sl}^{to}) and grade (d_{sl}^{tg}) are penalised equally ($c^{to}= c_l^{to}= c_u^{to}$ and $c^{tg}= c_l^{tg}= c_u^{tg}$). The orebody risk discount rate (*f*) of eight per cent is used to distinguish the cost of the deviations over the production periods. All the blocks in the deposit model are considered for the scheduling. Even the blocks at the edges are assumed to be mineable for the purpose of illustrating the new SIP concept, although they would not be feasibly mined in actual operation.

The average grade of the ore tonnage mined in each period is constrained to be between 4.7 and 5.2, and the minimum and maximum periodical ore tonnage production is limited to be between 260 tonnes and 290 tonnes.

Generating multiple schedules with different risk distributions

Table 1 shows the values calculated from the e-type orebody model corresponding to the summary information of the schedules obtained assigning different values for the c^{0o} and c^{0g} parameters. The first column, S, shows the schedule number, which corresponds to a schedule generated by using the cost parameters given in the table. The values for cost parameters are selected by trial and error. Initially, zero is assigned as the cost of deviations from both ore production and grade targets. Then, the values are randomly increased to generate different risk profiles. In some cases, such as models 1, 3, 5 and 6, the same scheduling result is generated by using different cost parameters for ore and grade deviations in the objective function. Although it is possible to calculate the actual cost of not producing a certain amount of metal in this case, it is not the best way of using the SIP model proposed. The purpose of the SIP model is to generate schedules

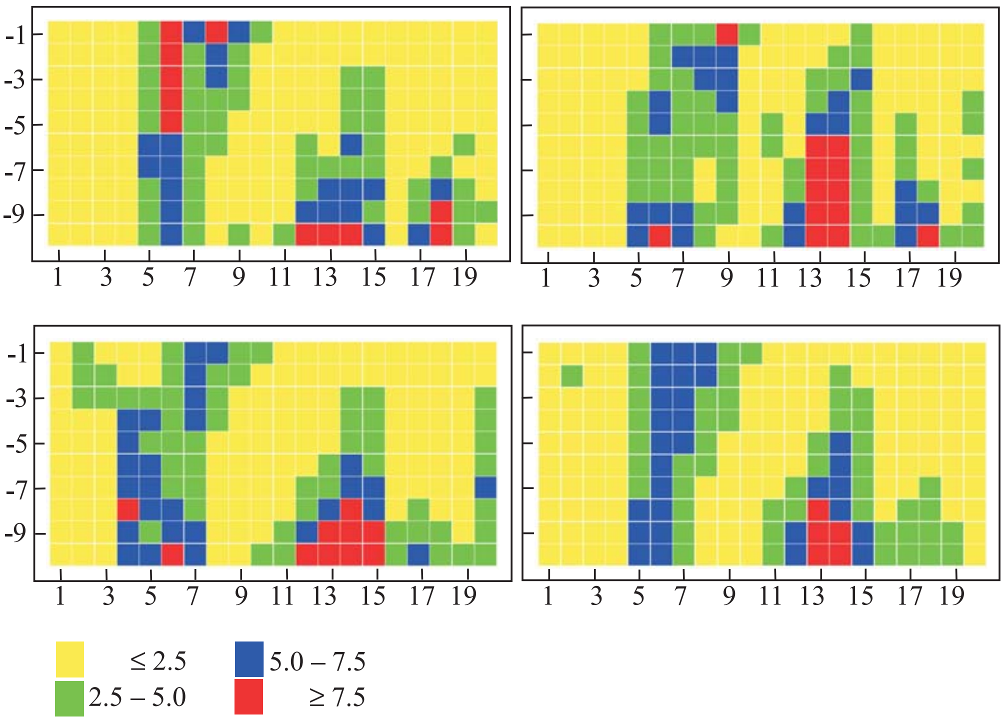


FIG 1 - Grade distributions of the hypothetical 2D deposit in three simulated realisations of the orebody and the e-type orebody model generated from the averaging of 50 simulated realisations (at bottom right).

TABLE 1

Summary of the six different SIP schedules generated with different cost parameters. Schedule S7 is the traditional schedule.

S	c^{0o}	c^{0g}	Period	Value	NPV	Grade	Ore	Waste	Sum
1	0.0	0.0	1	354.6	328	4.592	340	270	610
	20.0	0.0	2	193.2	166	5.269	310	460	770
			3	-14	-11	4.946	180	370	550
			Sum/Mean	**533.8**	**483**	**4.921**	**830**	**1100**	**1930**
2	0.0	0.5	1	-550	-509	0.000	0	550	550
			2	333.8	286	4.949	310	280	590
			3	750	595	4.905	520	270	790
			Sum/Mean	**533.8**	**372**	**4.921**	**830**	**1100**	**1930**
3	20.0	0.1	1	63.9	59	4.688	290	480	770
	20.0	0.2	2	187.1	160	5.066	270	360	630
	20.0	0.3	3	272.8	217	5.028	270	270	540
	20.0	0.4	**Sum/Mean**	**523.8**	**436**	**4.921**	**830**	**1110**	**1940**
4	20.0	0.5	1	53.8	50	4.773	290	500	790
			2	291.4	250	5.192	280	290	570
			3	188.6	150	4.795	260	310	570
			Sum/Mean	**533.8**	**449**	**4.921**	**830**	**1100**	**1930**
5	25.0	0.0	1	87.7	81	4.647	300	470	770
	30.0	0.0	2	234.6	201	5.237	260	310	570
			3	211.5	168	4.923	270	320	590
			Sum/Mean	**533.8**	**450**	**4.921**	**830**	**1100**	**1930**
6	10.0	0.5	1	77.4	72	4.804	290	480	770
	2.0	0.1	2	267.8	230	5.160	280	310	590
			3	188.6	150	4.795	260	310	570
			Sum/Mean	**533.8**	**451**	**4.921**	**830**	**1100**	**1930**
7	-	-	1	388.4	360	4.718	280	140	420
			2	52.0	44	4.930	280	500	780
			3	173.4	138	5.125	270	380	650
			Sum/Mean	**613.8**	**542**	**4.921**	**830**	**1020**	**1850**

with optimal NPV and control the risk distribution. This is because of the fact that different mines may have different preferences of risk distribution, and management should be able to decide the most suitable risk distribution for the specific mine. For example, if there is budget for more exploration drilling after a few years, it may be preferable to mine the risky part of the deposit later; if the mine's overall profit is not very high, it may be best to keep the risk as low as possible, but if the mine's profit looks reasonably high, it may be better to tolerate some risk if it has a significant potential in generating higher NPV. Therefore, there is no method available to determine optimal values for the cost of deviations for any mine. The important issue is to generate a schedule that will produce the optimal NPV for a desired risk distribution rather than the optimality of the costs for deviations.

In this paper, Schedule S7 is generated by applying a general form of MIP formulations with the NPV maximisation objective in a single estimated orebody model that is considered as traditional scheduling.

The first schedule S1 is generated by assigning 0 to both of the cost parameters (c^{0o}=0 and c^{0g}=0), which makes the ore tonnage and grade constraints inactive. Schedule S1 violates ore processing capacity constraints in all the periods by large amounts, and grade constraints significantly. The resultant NPV from the mathematical model cannot be achieved through the actual operation due to the high deviations. This scheduling model is considered to be infeasible and unrealistic due to the resulting high deviations.

The schedule S2 is also not realistic due to the fact that it produces no ore in the first period for two main reasons. The first reason is that the cost of grade deviations, c^{0g}, is too high, dominating and destroying the effect of the NPV parameter in the objective function. The second reason is that assigning zero *base cost* for deviations in ore tonnage disables the processing capacity constraints. Schedules S1 and S2 show that cost parameters are crucial, and assigning wrong values to them may generate infeasible schedules.

The scheduling periods of the schedules S3, S4, S5, S6 and S7 are depicted in Figure 2. The figure shows that the traditional schedule (S7) mines fewer blocks than the other schedules. This occurs because, in the SIP models, a block is classified as ore if it is considered to be ore in more than 40 per cent of the simulated orebody models, and this has resulted in the classification of more blocks as ore than are so classified in the e-type model in this case.

Quantification of uncertainty within schedules

In a schedule, average deviations, average of non-zero deviations and probability to deviate from the production targets according to the simulated orebody models are considered as the uncertainty measurement parameters in this study. Table 2 shows the percentage of deviations for each of the schedules. The third column 'Deviations (per cent) e-type' is the per cent deviations with respect to the e-type orebody model.

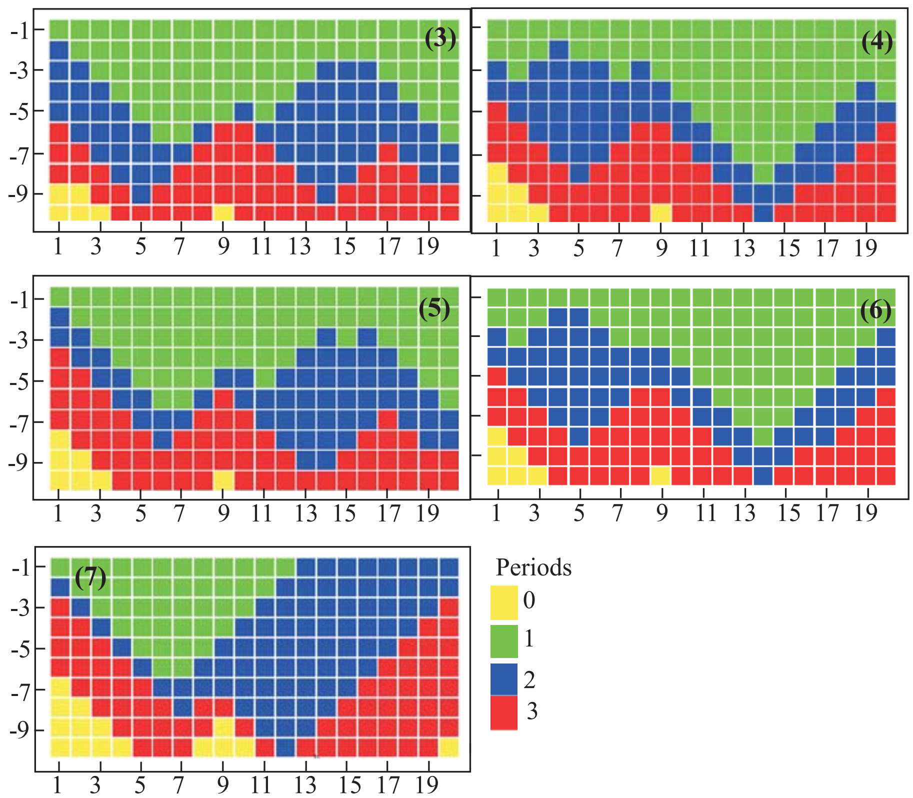


FIG 2 - Cross-sectional view of the schedules at 45° slope constraint.

The values reported in the fourth column are determined with respect to simulated models as follows:

1. Assuming the values in the actual deposit are exactly the same as the simulated orebody model 1, calculate the resultant ore tonnage (R_o) and the resultant average grade of that ore tonnage (R_g) within each production period.
2. Calculate the percent deviations D_o for ore tonnage and D_g for grade per period:

$$D_o(\%) = \begin{cases} (R_o - 290)\ \ 100/290, & \text{if } R_o > 290 \\ (R_o - 260)\ \ 100/260, & \text{if } R_o < 260 \\ 0, & \text{otherwise} \end{cases}$$

$$D_g(\%) = \begin{cases} (R_g - 5.168)\ \ 100/5.168, & \text{if } R_g > 5.168 \\ (R_g - 4.675)\ \ 100/4.675, & \text{if } R_g < 4.675 \\ 0, & \text{otherwise.} \end{cases}$$

3. Repeat Step 2 for all the remaining simulated orebody models.
4. Find the average of the calculated D_o and D_g values for each period and report under ore and grade columns in the table, respectively.

The third column 'deviations (per cent) e-type' in Table 2 is calculated by determining the R_o and the R_g values for e-type orebody model and using the equations in Step 2.

The fifth column 'Average of non-zero deviations (per cent)' is generated as follows:

1. Perform the above Steps 1 through 3.
2. Count the number of simulated orebody models that deviations are greater than 0, for ore (N_o) and grade (N_g) for each period.
3. Sum up the deviations, D_o and D_g values, and report sum(D_o)/N_o and sum(D_g)/N_g under ore and grade columns.

This fifth column 'average of non-zero deviations (per cent)' provides a quantity in terms of actual magnitudes of the deviations by not including the orebody models with 0 deviations in the averaging process.

The last column 'probability to deviate' shows the probability of each schedule to deviate in each production period. Since there are 50 simulated orebody models used here, the values of ore (P_o) and grade (P_g) in the table are calculated as follows:

$$P_o = 100\ \ N_o / 50, \quad P_g = 100\ \ N_g / 50$$

The SIP scheduling model is designed in such a way that it does not take ore production and average grade constraints in the last period into consideration, because this doesn't affect the optimality of the schedule (Ramazan and Dimitrakopoulos, 2004b). Therefore, the schedules are compared and analysed on the basis of the first and the second periods only, which leads to the infeasible schedules S1 and S2 being excluded from further discussion.

Analysis of the results

Table 2 shows that traditional schedule S7 has the highest total deviations in ore production, 22 per cent, for the first and the second periods among the schedules considered. The total average deviations in ore production in SIP schedules S3, S5 and S6 are about 15 per cent, and 17 per cent in schedule S4. Total average of the non-zero deviations in ore production are 45 per cent in schedules S5 and S6, and 46 per cent in S4, which are slightly less than the 50 per cent in the stochastic schedule S3 and traditional schedule S7. Traditional schedule S7 also has the highest total non-zero grade deviations, 34 per cent. The average probability of having the deviations in ore tonnages and grades is highest in the traditional schedule, at 73 per cent and 69 per cent respectively on average for the first two periods. Stochastic schedules S5 and S3 have lower average probability to deviate in ore production at 55 per cent and 58 per cent respectively, while

TABLE 2

Deviations in ore production in scheduled periods and average grade.

S	Period	Deviations (%) e-type		Average deviations (%)		Average of non-zero deviations (%)		Probability to deviate	
		Ore	Grade	Ore	Grade	Ore	Grade	Ore	Grade
1	1	18.2	-1.7	13	8	26	16	86	82
	2	7.3	2.1	20	3	35	11	86	58
	3	29.1	0.0	16	6	37	17	86	64
2	1	94.5	-95.0	79	26	79	46	100	88
	2	7.3	0.0	13	3	25	12	76	48
	3	83.6	0.0	80	4	80	13	100	54
3	1	0.0	0.0	10	8	28	17	66	80
	2	0.0	0.0	5	4	22	15	50	48
	3	0.0	0.0	9	2	23	10	76	52
4	1	0.0	0.0	10	5	25	16	78	64
	2	0.0	0.5	7	4	21	10	60	68
	3	0.0	0.0	8	4	24	11	66	58
5	1	3.6	-0.6	11	8	28	17	64	82
	2	0.0	1.4	4	4	17	14	46	22
	3	0.0	0.0	9	3	23	11	76	60
6	1	0.0	0.0	8	5	23	17	76	62
	2	0.0	0.0	7	3	22	11	62	68
	3	0.0	0.0	8	4	24	11	66	58
7	1	0.0	0.0	8	8	23	20	60	80
	2	0.0	0.0	14	4	27	14	86	58
	3	0.0	0.0	9	3	27	13	64	46

schedules S4 and S6 have 69 per cent average probability during the two periods. These results illustrate that the traditional schedule, which uses a single estimated input orebody model, performs poorly compared with the stochastic scheduling models. The poor performance of the traditional model is the result of its lack of ability to incorporate grade uncertainty in the optimisation process and of a single input orebody model not being able to represent the grade variability.

Schedule 5 seems to perform better than other schedules in terms of meeting the grade and ore production targets as shown in Table 2. Table 1 shows that schedule S5 is assigned higher cost for deviating from the ore production targets than the other SIP schedules. Although zero cost is assigned for the deviations in average grade, the grade deviations are similar to the deviations in the other schedules, and the probability to deviate in grade as an average of the first two periods is lower than in the other methods. This may be due to the fact that this schedule produced more balanced ore tonnage between periods with the blocks that have higher probability for being ore. In this specific case study, producing ore tonnage with the risk-robust ore blocks may have resulted in low-grade variations.

Figure 3 presents the production of ore tonnes from simulated orebody models, the average of these ore tonne, and the minimum (LO) and maximum (UO) limits of ore production constraining each of the schedules being considered for the first and the second periods. It is shown that the average of the expected ore tonnes using the traditional scheduling method is less than the lower limit, indicating a higher risk in falling short in the first period. Since stockpiling is not considered in these schedules, producing more ore tonnage than the maximum processing capacity, as is the case in schedules S3 and S5 in the first period, should also be considered as undesirable and costly. Schedules S4 and S6 can be considered better than the others from the analysis of ore production in the first period. The variations in the possible ore production of schedules S4, S6 and S7 are slightly less than those of schedules S3 and S5 in the first period. However, schedule S7 should be considered undesirable due to its higher possibility of not producing sufficient ore to feed the mill.

The variations in the ore tonnage production during the second period indicate that schedules S3 and S5 have relatively less risk of producing less than the lower limit, or more than the upper limits of ore. The variability is not particularly large among the stochastic schedules, but it is very large in the traditional schedule. The traditional schedule also has a high probability of exceeding the maximum mill capacity during the second period. Figure 3 indicates that the traditional schedule S7 has the highest risk of deviating from the planned ore production.

There is not a significant difference in the total NPV of the project among the simulated orebody models. The difference is that since the traditional model is not likely to produce the planned amount of metal, the NPV may not be realised, but the proposed method is likely to achieve the planned NPV.

CONCLUSIONS AND FURTHER WORK

This paper has presented a new and efficient SIP model formulation that can consider multiple simulated orebody models to optimise long-term production scheduling. The objective function in this model is constructed as maximising NPV of the mining operation, with a managed risk of not meeting production targets in terms of ore tonnes, grade and quality. The scheduling method developed here allows the decision-maker to define a risk profile based on the existing uncertainty quantified by simulated orebody models. The decision-maker has the option of minimising the risk in each of the production periods, or tolerating some risk in certain periods, or all periods. In the traditional scheduling model, geological risk is randomly

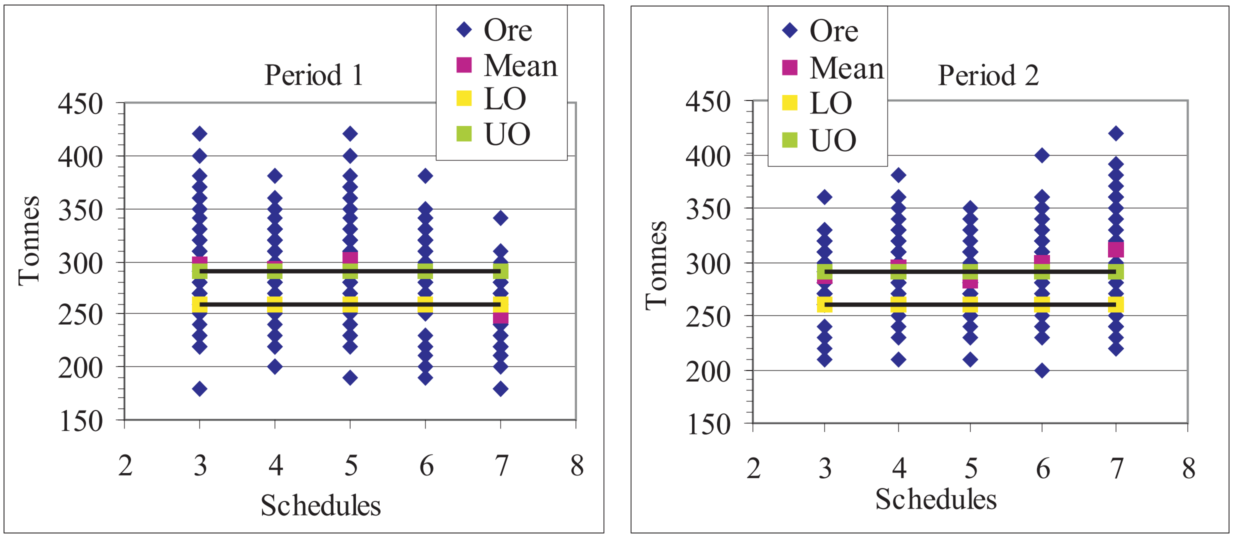


FIG 3 - Possible outcomes of ore tonnages generated by the schedules S3, S4, S5, S6 and traditional schedule S7. LO and UO shown in the two horizontal lines represent lower and upper bounds of ore production per period and are 260 tonnes and 290 tonnes, respectively.

distributed over the periods and can be significantly large. The new SIP model allows the selection of the best mine design based on the resultant NPV and the risk profile defined. The SIP method contains substantially less binary variables than traditional MIP mine scheduling models and the SIP model is efficient in terms of solution time. Although a hypothetical data set has been used to illustrate the strength of the new SIP model in this paper, the model is applicable to large open pit mines.

SIP models are proven to have significant economic benefits compared with traditional models that use deterministic inputs (Birge and Louveaux, 1997; Ramazan and Dimitrakopoulos, 2003; Ramazan *et al*, 2004; Dimitrakopoulos, in press). A recent example from applications with substantial monetary benefits from the use of the stochastic models presented in this paper are available in Jewbali (2006). The stochastic programming and modelling concept is useful not only for optimising the production scheduling process, but also for investigating various stages of the whole mining process, such as finding the value of an additional drilling campaign as discussed in Froyland *et al* (2007, this volume).

ACKNOWLEDGEMENTS

The work in this paper is part of ARC Grant # LP0211446 to R Dimitrakopoulos and was also funded by AngloGold Ashanti, BHP Billiton, Rio Tinto and Xstrata.

REFERENCES

Birge, J R and Louveaux, F, 1997. *Introduction to Stochastic Programming*, 421 p (Springer: New York).

Boucher, A and Dimitrakopoulos, R, 2007. A new efficient joint simulation framework and application in a multivariable deposit, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 345-354 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Dimitrakopoulos, R, in press. Applied risk analysis for ore reserves and strategic mine planning: Stochastic simulation and optimisation, 350 p (Springer – SME: Dordrecht).

Dimitrakopoulos, R, Farrelly, C T and Godoy, M, 2002. Moving forward from traditional optimisation: grade uncertainty and risk effects in open-pit design, *Trans Inst Min Metall*, Section A, Mining Technology, 111:A82-A88.

Dimitrakopoulos, R and Luo, X, 2004. Generalized sequential Gaussian simulation on group size *v* and screen-effect approximations for large field simulations, *Mathematical Geology,* 36(5):567-591.

Dimitrakopoulos, R and Ramazan, S, 2004. Uncertainty based production scheduling in open pit mining, *SME Transactions,* 316:106-112.

Dowd, P A, 1997. Risk in minerals projects: analysis, perception and management, *Trans Inst Min Metall*, Section A, Mining Technology, 106:A9-A18.

Escudero, L F, 1993. Production planning via scenario modelling, *Annals of Operations Research*, 43:311-335.

Froyland, G, Menabde, M, Stone, P and Hodson, D, 2007. The value of additional drilling to open pit mining projects, in *Orebody Modelling and Strategic Mine Planning*, second edition (ed: R Dimitrakopoulos), pp 245-252 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Godoy, M C, 2003. The efficient management of geological risk in long-term production scheduling of open pit mines, PhD thesis, University of Queensland, Brisbane, 256 p.

Godoy, M C and Dimitrakopoulos, R, 2004. Managing risk and waste mining in long-term production scheduling, *SME Transactions,* 316:43-50.

Jewbali, A, 2006. Modelling geological uncertainty for short-term production scheduling in open pit mines, PhD thesis, University of Queensland, 280 p.

Ramazan, S, 2001. Open pit mine scheduling based on fundamental tree algorithm, PhD thesis, Colorado School of Mines, Golden.

Ramazan, S and Dimitrakopoulos, R, 2003. Stochastic integer programming based modelling for long-term production scheduling of open pit mines, ARC-Linkage Report N-6002-1, University of Queensland, Brisbane.

Ramazan, S and Dimitrakopoulos, R, 2004a. Traditional and new MIP models for production scheduling with in-situ grade variability, *International Journal of Surface Mining, Reclamation and Environment,* 18(2):85-98.

Ramazan, S and Dimitrakopoulos, R, 2004b. Recent applications of operations research in open pit mining, *SME Transactions,* 316:73-78.

Ramazan, S, Dimitrakopoulos, R, Benndorf, J and Archambault, L, 2004. Extension of SIP modelling for long term production scheduling with stochastically designed stockpiles and multiple ore processors, ARC-Linkage Report N-6003-1, University of Queensland, Brisbane.

Ravenscroft, P J, 1992. Risk analysis for mine scheduling by conditional simulation, *Trans Inst Min Metall*, Section A, Mining Technology, 101:A104-A108.

Uncertainty and Risk Management in Research and Development

A Cockle[1]

ABSTRACT

Despite the major general improvements in the mining sector productivity and conditions, the same cannot be said for the funding of mining research and development. In a business that is increasingly reliant on technology, rather than a technical work force, there will be fewer people conducting, producing, being trained and reporting on research. In Australia, and internationally, there is a general downturn in business spending on research and development as a proportion of gross domestic product and this is significantly more evident in the mining industry. A number of issues that underpin this downturn include the segmented nature of the various mining operations, dwindling funds for university research and curricula, reduced opportunities for employee involvement in research and the failure of the industry to compensate for deficits in public funding.

Without investment in people and appropriate supportive funding for research and development there is a major risk that innovation in the mining industry will be limited. A number of strategies are suggested in this paper to address some of this uncertainty and to manage the ongoing risk associated with these issues.

INTRODUCTION

The underlying theme of this volume and the recent Symposium on Orebody Modelling and Strategic Mine Planning (held in Perth, WA, on 22 - 24 November 2004) has been uncertainty and risk management. It is useful to review the definitions of these terms before discussing related concepts further. Uncertainty refers to a state of doubt, hesitance, an unpredictability; while risk means an exposure to the chance of injury or loss, the probability of loss. While many perceive of these in a mathematical sense, this paper will discuss the concepts and related aspects from a slightly different angle. More specifically, the paper addresses a state of doubt, hesitancy, and unpredictability in the mining business regarding the health of research and development. The next sections draw on the author's long career in the mining business as a basis for observations that may be useful as the industry tries to sustain this aspect of business.

TECHNOLOGY DEVELOPMENT

Every day the industry faces the challenge of developing, maintaining and utilising research and pursues answers because of the desire to:

- have longer-lived assets,
- convert lower grade resources from an existing reserve,
- face the reality of recovering metals from complex ores,
- keep the net present value on the table,
- achieve project targets, and
- avoid becoming victims of the metal markets.

These outcomes are valued because the industry strives to keep business successful. Research and development are used to achieve this value through incremental improvements or, hopefully, significant step changes, such as some of the technology changes suggested in this volume. The process used usually involves hiring or funding the 'right' people, giving them continuous funding for work to be undertaken and looking for innovation from them through the sustained efforts they give to the task. Given these conditions, it is useful to consider where any chance of injury or loss exists – in the broad sense.

The mining industry faces a world where it must be successful with a workforce that has less technical skills and, because of labour shortages or less interest in the business, the same work will be done with less manpower. There is, therefore, a real unpredictability as to how research and development needs will be met. The cold reality is that there will be fewer and fewer people in the engineering field reporting on technical work. Without the right people, there is a risk of injury or loss to business and the industry. Some facts can be used to elaborate further. On 7 September 2004, the *Australian Financial Review* published a lead article entitled 'Business research and development: Spending grim' (*The Australian Financial Review*, 2004). The article presented data released by the Australian Bureau of Statistics (ABS) that showed a decline in business sector research and development (BERD) as a proportion of the gross domestic product. The article noted that 'the downturn was slight but continued a ten-year downward trend in business spending'.

Upon researching the more complete ABS report further (Australian Bureau of Statistics, 2004) mining lagged most other businesses. Some of the key statistics that reflect performance are presented in Tables 1 to 3. Further, the different aspects of funding are shown in Figures 1 and 2.

To be specific, Table 1 notes the sum of all research funding in Australia for the period 2002 - 2003 ($5.98 billion). Further, it lists the subset of the mining industry portion of that total – $536 million or nine per cent. In addition, the major portion of spending is documented as occurring in Western Australia and Queensland. Complementary information, shown in Figure 1, indicates the inconsistent funding and current downtrend in spending for the mining sector for the last three reporting periods.

TABLE 1

Overall spending (for the period 2002 - 2003).

	A$	Percentage of all business spending	Percentage of all mining spending
All business	$5.98 billion		
Mining industry	$536 million	9%	
WA Queensland	$154 million $134 million		29% 25%

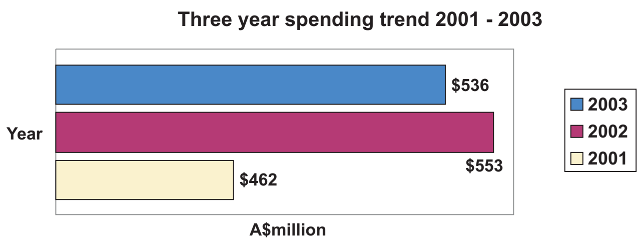


FIG 1 - Three-year spending trend (for the years 2001, 2002, 2003).

1. MAusIMM, Vice President Technical Services, Newmont Mining Corporation, 10101 East Dry Creek Road, Englewood CO 80101, USA. Email: allen.cockle@newmont.com

In addition to the general trend in spending, some of the more important figures in the BERD report relate to human resources committed to research (Table 2), spending by activity (Figure 2) and source of funds (Table 3).

Table 2 data indicates that there has been a dramatic decline in the number of people committed to research efforts since 2001. Figure 2 confirms this in part by its tabulation of the amount of dollars committed to experimental development (63 per cent) versus applied research (29 per cent) versus basic research (eight per cent). And finally, Table 3 identifies the paltry effort by the mining industry to use government funds to gain leverage for its research initiatives.

TABLE 2

Human resources devoted to R&D (2001 - 2003).

	2001	2002	2003
Mining	1194	836	608 in person years

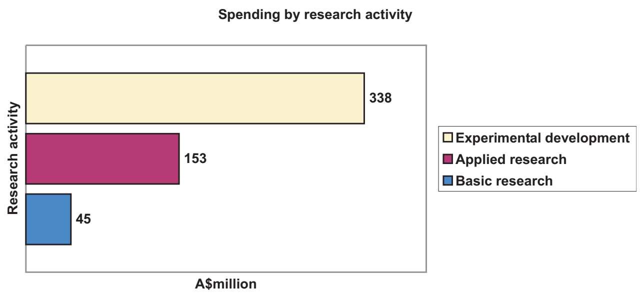


FIG 2 - Spending by activity.

TABLE 3

Source of funds (collaboration).

Mining industry's own funds	$ 528 million
Commonwealth	$ 8 million
State and local government	$ 0

There is more data in the ABS Report 8104.0, but for the purpose of understanding business spending in research and development, the data and trends in the tables and figures above are a strong indication that the industry effort is not healthy. The data specifically reveal that:

- As a per cent of GDP the industry commitment to research and development is declining.
- Most funds are going into experimental development (92 per cent). Only eight per cent is applied to basic research.
- Industry commitment of human resources has been halved.
- Industry members do not collaborate.

Some may suggest that these figures reflect the cyclical history of the mining industry and the business climate, where structural changes have made commitment to research and development unpredictable. This may be true if we consider five fundamental traits I have seen in the industry over a long career.

First, it has been a much segmented industry in that mining companies are too centric. Coal mining is driven by different objectives to alumina, gold or base metals mining. This situation is reflected in the microcosm of this volume, as it was in the Perth Symposium, where one presenter indicated that ore body modelling and mine planning methods common in gold and base metals have not yet had wide circulation in the coal industry. While this may be a technical point, on a management scale this verifies a fractured, short-term focus.

Second, consolidation of the industry has changed the base from which universities derive research funds in Australia and internationally. This denies them a base for support as much as for the industry to access the right people. Dwindling funds have limited the ability of institutions to create an innovative environment. At their present levels, universities can neither sustain facilities, retain professors, support post-graduate work, nor can they maintain mining, metallurgical and geology curricula. This grim reality is confirmed by the figures shown in Figure 2 where basic research forms only eight per cent of the industry funding.

Third, leaner organisations have hurt the ability of current and future employees to participate and/or lead research and development efforts. The organisational trend has been to place less emphasis on mentoring in general and 'engineer-in-training' programs in particular. Additionally, funding internships for university students has decreased. This has led to a 'dumbing down' of the practice of engineering in mining operations and is not a conducive environment in which research and development can be nurtured.

Fourth, these same leaner organisations have limited their attraction of young high school students to the industry. In recent times, scholarships for high school students, and focus on engineering as a basis for a scholarship, have both been minimised.

Finally, industry has not compensated for the abandonment of research originally supported by government. This is perhaps particularly evident in the US, where public funding was used to support a great deal of mining/metallurgical research. When that was abandoned in the mid-1990s no industry-equivalent program evolved. The parallel in Australia might be the 125 per cent versus 150 per cent tax deduction that some argue determines whether there is enough incentive for industry to focus and sustain research and development.

Strategies needed to reverse the trend

While some may conclude from the data I have used there is an inevitable, irreversible loss and threat to the mining industry, the situation is reversible and certain steps can be taken that can counter the trends.

In the first instance, our professional society continues to acknowledge this risk. Recently and in the past and just this year, The AusIMM held two forums on 'The Future of the Minerals Industry'. Themes discussed included:

- Generation Y study – attraction and retention of this demographic group,
- graduate school supply and demand,
- employment practices and turnover, and
- attracting high calibre personnel to the mining industry.

These investigations are a continuing effort by our professional leaders to answer the questions asked at the time '*Back from the Brink*' was written in 1998. As industry professionals we should adapt our research models and management practices to the results identified from these studies.

Further, working as employee, managers or executives, there are four basic areas where a positive impact on research and development can be made:

1. Businesses should fund research on a continuous basis with a concerted effort made to develop five, seven and ten year collaborations that commit the industry to longer range research. This form of funding can assure that work will continue despite the business cycle. Further, fundamental questions not addressed by short-term, solitary work can be developed and answered.

2. Concurrently, local, state and federal governments should be brought into the collaboration. These organisations can bring great value by extending the research dollar and being a partner, rather than an adversary in business.
3. In the workplace, continuous improvement methods need to be used to drive the pace of development as this process often defines research topics and drives funding and a commitment for results. As part of this workplace culture, technical committees need to be developed that drive the direction and amount of work because research and development requires vision and attention. A committee can serve as a liaison to executives and upper management so that understanding is maintained and the commitment is sustained.
4. A research and development ethic must be created for employees, and this should start from the undergraduate university years. This ethic will encourage employees to remain abreast of industry/educational research. It will also encourage and support presentation of ideas in forums and symposia, as well as bring educators and people with ideas into the workplace. A research and development ethic can provide funding for industry forums as well as fund continuing education for current employees. One additional aspect of the research ethic is to invigorate, support and maintain a knowledge management system. In many organisations the collapse of the knowledge base and the loss of corporate history has been due to poor record-keeping or the failure to retain/transfer knowledge. By maximising the use of external knowledge networks the research ethic can be enhanced as many new electronic databases are available. However, some may suggest the need to revert back to basics and retain research librarians to manage this activity.

Last but not least, the industry must plan for its posterity. The researchers of the future will only come to the mining sector if it develops and funds outreach programs to high school and undergraduate programs and engages future employees in the excitement of technical work.

CONCLUSION

In summary, this Spectrum Series volume, and the Symposium on Orebody Modelling and Strategic Mine Planning it originates from, has been a truly outstanding forum and spans some 40 years of orebody modelling and strategic mine planning experience.

While there will be fewer in the field, a sense of doubt and unpredictability to sustaining research and development is evident. Statistical data on business spending indicate that the mining industry is inconsistent at supporting research and development. This brings a chance of injury or loss that may be profound. One solution may be that companies and industry that remain cease being centric in their thinking. There are many industry-wide problems that will require sustained research and aggregate groups must be built and collaboration with government is needed to achieve solutions.

Each member of the mining industry must be an advocate for research and development. Working as individuals, professionals, managers or executives there is a need to nurture a research and development culture that includes continuous training of professionals. More importantly, the youth in our community must be sought out and shown the excitement and reward of working in the industry.

The fruits of research and development are the result of investment in people and a commitment of funds to them so that innovation will follow. The challenge must be taken up so that in the future the mining industry as a whole will look fondly upon a sustained research and development effort so that future symposia and special volumes will describe another 40 years of progress as has been demonstrated through this volume and related symposium.

REFERENCES

Australian Bureau of Statistics, 2004. 2002-2003 research and experimental development businesses, Report #8104.0, 6 September [online]. Available from: <http://www.abs.gov.au/AUSSTATS/abs@.nsf/ProductsbyReleaseDate/87EA9096F4389558CA25708900757484?OpenDocument> [Accessed: 14 May 2007].

The Australian Financial Review, 2004. Business R&D spending grim, 7 September, p 3.

Reviewers

The papers in this volume were peer reviewed. Thanks are in order to the following reviewers:

Editor: Roussos Dimitrakopoulos

Michael Andrews

Jorg Benndorf

Alexandre Boucher

Gary Froyland

Marcelo Godoy

Nikki Grieco

Arja Jewbali

Brett King

Mark Knoppe

Mustafa Kumral

Andre Leite

Shuxing Li

Peter Lilly

Conor Meagher

Merab Menabde

Ute Mueller

Richard Peattie

Salih Ramazan

Jean-Michel Rendu

Sabry Abdel Sabour

Michael Samis

Peter Stone

Olivier Tavchandjian

Jeff Whittle

Rodney Wolff

Mark Zuckerberg

An initiative of

AusIMM
AUSTRALASIAN INSTITUTE OF MINING & METALLURGY
http://www.ausimm.com

McGill COSMO
Stochastic Mine Planning Laboratory
http://cosmo.mcgill.ca

The South African Institute of Mining and Metallurgy
Founded in 1894

CIM ICM

GAA

SME

Second Edition Sponsors

Companhia Vale do Rio Doce

BARRICK

First Edition Sponsors

AngloGold Ashanti

bhpbilliton

De Beers
A DIAMOND IS FOREVER

HAMERSLEY IRON

Newmont
The Gold Company

Rio Tinto

Whittle
STRATEGIC MINE PLANNING

xstrata

Orebody Modelling and Strategic Mine Planning: Uncertainty and Risk Management

International Symposium, Perth, WA, 22 - 24 November 2004

AN OUTSTANDING SUCCESS!

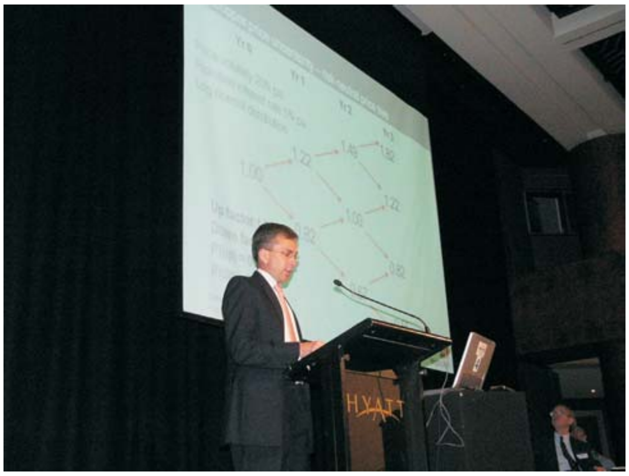
By Damon Frith and Tina Thornton

The International Symposium on *Orebody Modelling and Strategic Mine Planning: Uncertainty and Risk Management* was held in Perth in November 2004 and attracted 260 delegates from Australia, the Americas, Europe, Africa and Asia. Symposium attendees included senior representation from the major mining companies, including BHP Billiton, Rio Tinto, AngloGold-Ashanti, De Beers, Newmont, Hamersley Iron, Xstrata, and Anglo American, as well as the leading academics in the field. Whittle Programming had a strong presence as co-sponsor of the symposium. The symposium was supported by leading mining professional organisations, namely The AusIMM, SME, CIM, SAIMM and GAA.

Mining is arguably a high risk/high reward business, and the aim of the symposium was to add value to the industry by demonstrating how quantification of uncertainty and risk management can be used to capture maximum upside potential while minimising downside risk. New trends in pit optimisation modelling and the international shortage of skilled labour in the workforce were among the many issues debated. In his introduction of the event, Symposium Chairperson, Professor Roussos Dimitrakopoulos, described it as ‘a remarkable assembly’ of participants. The symposium sessions started with themes such as ‘Why strategic risk management?’ continued with ‘Integrated large-scale applications’ and concluded with ‘New concepts, technologies and directions’. Parallel sessions focused on specific issues and techniques ranging from conditional simulation to mining operations research and optimisation, and global asset optimisation. Geotechnical risk in mine planning and optimisation was addressed in a special forum and the final forum session on new challenges addressed issues stemming from the Symposium.

Symposium presenters used traditional thinking to improve the currently available tools within the mining community to optimise mine plans and designs. They combined this with attempts to challenge delegates with new concepts for radically changing how mine planners develop and optimise their mine plans in an uncertain, changing, and increasingly complex, global economic environment combined with the equally complex orebodies being mined.

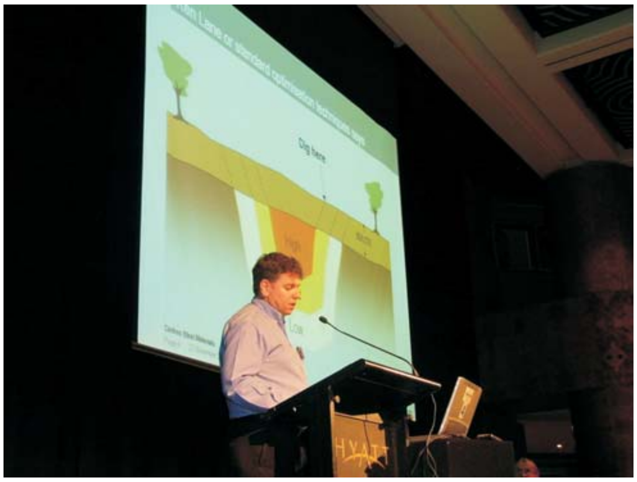
In his keynote opening address, Peter Monkhouse, BHP Billiton Vice President Strategy for Carbon Steel Materials, highlighted that, when planning and developing an orebody, there was a need to look beyond the *Economic Definition of Ore* by Ken Lane, which has been used as the mining industry’s ‘bible’ for the past 17 years. Monkhouse said the standard practice of taking a single set of assumptions – like a single orebody, mining costs, exchange rates, and metal prices and applying them to optimise mine designs and production schedules – is inherently wrong. The assumptions would undoubtedly be erroneous over the 20 or more years of mine-life typical for a large operation. He said that BHP Billiton was adapting its mine developments to incorporate ‘robust mine plans’. These he described as flexible in absorbing changes in a mine plan or the underpinning assumptions with minimal impact, while optimising over a wide range of assumptions. The concept is to recognise the changing world economic environment and to manage risk, sensitivities and mine development using tools such as ‘real options’, sensitivities and mine development. Real options was described as a valuation method used to calculate project net present value while accounting for uncertainty, risk and the time value of money.



Peter Monkhouse – Opening Address *Beyond Naive Optimisation* (Vice President Strategy for Carbon Steel Materials – BHP Billiton).

L-R: O Tavchandjian (Inco Ltd), D Campbell (Anglo American plc) and V Chamberlain (AngloGold Ashanti).

BHP Billiton Resource Evaluation Manager and co-speaker, Gavin Yeates, stressed the need to encapsulate a plan that covered all contingencies in the long-term development of a large-scale mine. While high-grading a deposit in the initial years of operation may lead to early profits, a lack of pre-stripping and other long-term goals could lead to ‘value destruction’ for both the



Gavin Yeates (Resource Evaluation Manager, BHP Billiton).

company and the host country. This is particularly evident if changing world conditions make the mine uneconomic to mine mid-grade or low-grade zones. According to Yeates, questions to be resolved in the pre-feasibility stage include what is waste and what is ore? How much excess capacity should be built into the plant? Should the operation be selectively mined or use bulk mining? How much exposed ore should be carried? What level of stockpiles should be maintained? Mines had to work in the real world, which means shutting down production near the bottom of the metals cycle if required; and achieving mine flexibility through solutions such as early pre-stripping of the orebody, and not just the area that makes up the initial pit. A failure to recognise 'real world' impacts on a mining operation was an invitation to create 'value destruction'.

The perspective of the developer and consultant operating in the 'real world' came from Jeff Whittle, founder of Whittle Programming. He asserted that the value of a developer's and consultant's contribution was in providing a mining company with a competitive edge by using the mine planning and evaluation modelling and optimisation tools developed to meet each client's individual needs of their operations or systems, thus maximising their return on funds invested.

Symposium Chairperson Roussos Dimitrakopoulos continued this theme of optimisation of resources and summarised public statistics suggesting that about 70 per cent of plants operate at less than 70 per cent of planned capacity in their first year of operation, while 60 per cent of mining operations operate at less than 70 per cent of their planned rate. Dimitrakopoulos commented that given the risky nature of the mining business – the uncertain demand for raw materials and equally uncertain supply from partially known orebodies – we are better off accepting uncertainty and managing risk to our benefit. 'Uncertainty is good for us', he stated, 'uncertainty creates opportunities for strategic planning decisions'. This leaves us with the goal of developing new frameworks and technologies for mine planning and design, technologies that accurately quantify risk in all key sources of information used.

Peter Ravenscroft, General Manager Resource Planning, Hamersley Iron, stressed some of these difficulties in orebody modelling and strategic mine planning. He calculated that the level of information available from drilling for the average mine plan and mine production forecasting is equivalent to generating a daily forecast for a month from looking out of the window once, and for a couple of seconds!

In his closing address, Allen Cockle, Corporate Director, Mining, Newmont Mining Corporation stressed issues on research and development from the angle of uncertainty and risk management. Cockle suggested that, despite the general improvements in the mining sector's productivity and conditions, the same cannot be said for the funding of mining research and development. In a business that is increasingly reliant on technology, rather than a technical work force, there will be fewer people conducting, producing, and being trained and reporting on research. In Australia, and internationally, there is a general downturn in business spending on research and development as a proportion of Gross Domestic Product and this is significantly more evident in the mining industry. Business expenditure on research and development (in current price terms) has decreased from 0.81 per cent of GDP in 2001-02 to 0.79 per cent of GDP in 2002-03; within the mining industry research and development expenditure decreased by three per cent in current price terms (ABS, 2004). A number of issues that underpin this downturn are evident and include the segmented nature of the various mining operations, dwindling funds for university research and curricula, reduced opportunities for employee involvement in research and the failure of the industry to compensate for deficits in public funding.

Cockle emphasised that the fruits of research and development are the result of investment in people and a commitment of funds to them so that innovation will follow. Without investment in people and appropriate supportive funding for research and development there is a major risk that innovation in the mining industry will be limited. A recent statement on monetary policy by the Reserve Bank of Australia (February 2005) reports that overall mining investment in Australia has substantially increased since the downturn in the 1990s. A resurgence in global demand and world commodity prices from 2003 has also provided impetus for resource-led investment. The challenge for the mining industry will be to ensure that research and development attracts some of this investment.

Cockle's address was followed by a forum discussion led by Roussos Dimitrakopoulos, Allen Cockle (Newmont), Peter Ravenscroft (Hamersley Iron), Jeff Whittle, Gavin Yeates (BHP Billiton), Wynand Kleingeld (De Beers) and Martin Whitham (Rio Tinto) – see photo.

L-R - R Dimitrakopoulos, A Cockle (Newmont), P Ravenscroft (Hamersley Iron), J Whittle, G Yeates (BHP Billiton) and W Kleingeld (DeBeers) – Forum on New Challenges.

The open forum discussion tackled the growing problems of skills shortage, research and development, as well as the transfer of technologies to practice in an environment where a lack of skilled labour was a real threat to the growth of the industry, much as Newmont's Allen Cockle described how over 1200 person-hours were spent in research and development in Australia in 2001, but that in 2003 the figure had plummeted to

103 person-hours. He said governments were looking for short-term fixes but failed to contribute long-term funding to research. The industry was also fragmented and did not collaborate to help generate interest in the industry for young people or research programs. De Beer's Wynand Kleingeld added his voice to a call for collaboration and said the mining industry had a propensity to re-invent the wheel every time it brought in a new manager to an operation. Kleingeld also stressed the need for undergraduate and postgraduate training, as well as the need for internationally located Centres of Excellence in the field that are linked and collaborate. Industry, he said, must and will support these types of initiatives.

Dimitrakopoulos stated that there is a need for undergraduate studies in mining that reflect and are relevant to today's environment; the structure of engineering courses had changed little in the last decades. Allen Cockle also lamented the lack of a mentoring age group within the industry and noted its limited attraction to women, with a representation of only two per cent of the workforce. Peter Ravenscroft and Martin Whitham stressed the need for transfer to, and application of, new technologies such as the developments presented at the conference. Whitham stressed that, to capitalise from minerals, new techniques need to be sufficiently demonstrated and proven to convince mine management of the benefits to their business; whilst the software currently available is often not adequately designed for ease of use by the user. He also stressed that we need to reduce the gap between development of new technology/techniques and their application – an excellent 'engagement model' needs to be developed to addresses technical robustness, ease of use, implementation training, and maintenance options.

Continuing the debate on new challenges, Yeates said that BHP Billiton is already putting a number of leading edge ideas raised at the conference into practice, including 'stochastic' methods for mine planning and optimisation. The world's leading mining company is seeking to introduce robust mining plans at all of its major operations with responses made to 'real world' changes in market conditions, uncertain orebodies, and other variables. Its operations are becoming a litmus test of how some of the new principles raised operate in a dynamic mining plan. Yeates stressed the need for more and focused research funding, as well as the effort and funding of development of tools after research. Whittle commented that development requires a different flair to research as much as different skills. The person that developed a new method is not the same person that will develop a commercial product, he stressed.

A Journel and A Boucher (Stanford University).

So where to from here? As the Chairperson, Dimitrakopoulos summarised that in time a collective group working in orebody modelling and strategic mine planning will form and collaborate to move the current state of discussions forward. While that may take months to achieve, a follow up conference is already being planned for 2007/08. This concluded a very successful conference, acknowledged by all attendees.

In response to the success of this symposium and wide demand and requests from the mining industry, The AusIMM is publishing a Spectrum Series Volume on *Uncertainty and Risk Management Models in Orebody Modelling and Strategic Mine Planning.*†

† This article was first published in *The AusIMM Bulletin* – May/June 2005, No 3, pp 69-72.