

# Geostatistics in 12 Lessons

This course is designed to (1) provide a web-based training resource for testing, documentation, and development groups, (2) develop a glossary of geostatistical terms and key concepts, (3) develop workflow specifications for shared earth modeling. A table of contents including a preface and the twelve lessons is indicated below:

**Preface** A brief discussion about the course which indicates the lecture times, goals, and other important stuff.

**Lecture 1: Purpose / Motivation for Geostatistics** This first lecture discusses the need for geostatistics and its applications to reservoir management, decision making in the face of uncertainty throughout the life cycle of a reservoir, and portfolio management.

**Lecture 2: Basic Concepts** Lecture 2 deals with some of the basic concepts required for understanding geostatistics. Basic statistical tools, probability distributions, Monte Carlo simulation, stochastic modeling concepts are explained in this lesson.

**Lecture 3: Geological Principles for Reservoir Modeling** Lecture 3 discusses different geological structure types and associated modeling concepts along with illustrative examples.

**Lecture 4: Data Analysis** Reservoir data derived from wells and / or seismic is often unreliable and biased and therefore requires pre-processing. Lecture 4 discusses pre-processing issues such as declustering, trends, reconciliation of data, inference, and calibration of soft data.

**Lecture 5: Spatial Data Analysis** Geostatistics differs from regular statistics in deals with spatially correlated data. The most common tool used for describing spatial correlation (variability) is the variogram. Lecture 5 discusses variograms, their interpretation, and modeling.

**Lecture 6: Geostatistical Mapping Concepts** One application in geostatistics is making accurate maps. Lecture 6 will discuss using estimation and simulation for map making.

**Lecture 7: Structural Modeling** Lecture 7 discusses various aspects of structural modeling such as velocity uncertainty, thickness uncertainty, and how to handle faults.

**Lecture 8: Cell Based Facies Modeling** This lecture will discuss the methodology for cell based facies modeling.

**Lecture 9: Object Based Facies Modeling** This lecture discusses methods for cell based modeling with objects instead of cells. Specific geological environments of application include fluvial and deep water systems.

**Lecture 10: Petrophysical Properties** There are many methods to model continuous petrophysical properties such as porosity, permeability, and velocity. Lecture 10 discusses sequential Gaussian simulation, P-field / Cloud, transforms, and Indicator methods for modeling petrophysical properties

**Lecture 11: Seismic Data Integration** This lecture discusses calibration of seismic and well data, inference of cross correlation and various (multivariate) simulation techniques including cokriging collocated cokriging, kriging with external drift, and Bayesian upscaling. .

**Lecture 12: Post Processing** Lecture 12 will cover post-processing geostatistical models. Post-processing includes model validation, understanding uncertainty in models, ranking and selecting realizations, and scale-up.

# ***Preface to Geostatistics in 12 Lessons***

## **Introduction**

This web page is a set of companion notes to accompany the twelve lectures presented in the summer of 1999 For Landmark Graphics in Austin Texas. The lectures are intended to be an informal training seminar for those employees involved in the development, documentation, and testing of software that implement geostatistics.

## **Key Tasks**

There are some key tasks that will be accomplished by the end of the summer. These include:

- Run informal training seminars with members of the testing, documentation, and development groups.
- Develop a web-based training resource for testing, documentation, and development groups.
- Develop a glossary of geostatistical terms and key concepts
- Work with the testing manager to develop a test plan for the testing of geostatistical components.
- Develop workflow specifications for shared earth modeling.
- Document procedure and assumptions / underlying techniques in the Geostat SDK

## **Training Seminars**

The training seminars will be presented in 12 lectures. Lectures will be presented on Tuesday, and Thursday at 10:00 am until noon. The lectures will be presented in varying depth. In depth seminars will be held on Tuesday and Thursday, and light seminars will be held on Wednesday. The Wednesday seminars will be a high-level overview of the Tuesday and Thursday seminars. Each seminar will contain a lecture portion, an exercise / application, a quiz, and a review session. The training seminars will be held in the last six weeks of the summer.

## **Training Resource**

The material covered in the seminar will be available as an HTML document on the Landmark Graphics Intranet. The web site will be designed so that any employee can review the seminars, perform the exercises, and complete the quiz by themselves. Users of the web site will have the opportunity to submit their quizzes by email for marking or allow for self-marking.

## **Glossary of Terms**

A glossary of terms will be provided to assist the user. Words that appear in the training resource that also appear in the glossary will be hot linked.

## **Testing**

Familiarity with geostatistical concepts is essential in any testing strategy. Familiarity will come as a result of the seminars and the training resource. Common mistakes in the construction of a geostatistical model and items that indicate problems will be outlined. In depth pages will tackle theory and provide tools for proving algorithms by hand.

**Workflow Diagrams** The training resource will provide a diagrammatic illustration of a workflow for reservoir modeling. The training resource will be hot linked to the workflow model. Hot linking the workflow diagram will allow the user to go to specific lectures instead of having to browse through the entire training resource.

## Documentation of Assumptions / Procedures

The construction of a geostatistical model requires sometimes confusing procedures and assumptions. The training resource and the seminars will clearly state and explain all assumptions and procedures. In discussion with E. Gringarten and others, a 12 step geostatistical training / documentation plan was devised. This is summarized in the following table:

1. Purpose / Motivation for Geostatistics
2. Basic Concepts
3. Geological Principles for Reservoir Modeling
4. Data Analysis
5. Spatial Data Analysis
6. Geostatistical Mapping Concepts
7. Structural Modeling
8. Cell Based Facies Modeling
9. Object Based Modeling
10. Petrophysical Properties
11. Seismic Data Integration
12. Post Processing

Each of these lessons will be developed through the summer and will be delivered according to the following timetable:

Lecture Dates	
Lecture 1: July 6	Lecture 2: July 8
Lecture 3: July 13	Lecture 4: July 15
Lecture 5: July 20	Lecture 6: July 22
Lecture 7: Aug. 3	Lecture 8: Aug. 5
Lecture 9: Aug. 10	Lecture 10: Aug. 12
Lecture 11: Aug. 17	Lecture 12: Aug. 19

A "light" version will be presented on Wednesdays, that is lectures 1 and 2 will be presented in a seminar on Wednesday July 7.

A more detailed look at the 12 lectures is presented below:

### Lecture 1: Purpose / Motivation for Geostatistics

- o Qualitative reasoning
- o Reservoir planning
- o Elimination of surprises
- o The need for decision making

- Quantification of uncertainty
- Portfolio management
- The life cycle of a project

## **Lecture 2: Basic Concepts**

- Statistical tools
- Probability distributions
- Monte Carlo simulation
- Cell based modeling
- Object based modeling

## **Lecture 3: Geological Principles for Reservoir Modeling**

- Reservoir types and examples
- Modeling different structure types

## **Lecture 4: Data Analysis**

- Declustering
- Trends
- Reconciliation of data
- Inference
- Calibration of soft data

## **Lecture 5: Spatial Data Analysis**

- Variograms
- Variogram interpretation and modeling

## **Lecture 6: Geostatistical Mapping Concepts**

- Estimation
- Facies simulation
- Petrophysical property simulation

## **Lecture 7: Structural Modeling**

- Velocity uncertainty
- Surface flapping (thickness uncertainty)
- Fault handling

## **Lecture 8: Cell Based Facies Modeling**

- Methodology
- Sequential indicator methods
- Truncated Gaussian methods
- Cleaning cell-based realizations

## **Lecture 9: Object Based Modeling**

- Methodology
- Object based fluvial modeling
- Object based deep water systems

## **Lecture 10: Petrophysical Properties**

- Modeling petrophysical properties
- Sequential Suasion simulation
- P-field / Cloud
- Porosity permeability transforms
- Indicator methods

## **Lecture 11: Seismic Data Integration**

- Calibration of data
- Cross spatial variability
- Cokriging
- Simulation alternatives

## **Lecture 12: Post Processing**

- Model validation
- Understanding uncertainty
- Ranking and selecting realizations
- Scale-up and transfer to process simulation
- Decision making

## **Conflicts**

There are no foreseeable Landmark Graphics / civic events that conflict with the dates set for the lectures.

## **Motivation, Goals, and Benefits**

Landmark Graphics is beginning to develop a totally integrated numerical-geological modeling package. This package will have the ability to use geostatistics. A robust software package requires that all those involved in its creation have some knowledge about the purpose or the end use of the code being written, tested, and documented. The goal of this 12 lesson course in geostatistics is to provide a working knowledge of geostatistics, and fortify this newly acquired knowledge with exercises and quizzes. A global understanding of geostatistics will aid software testers in determining if an algorithm not only provides a result but the correct result. Developers will benefit by being more able to piece together the work flow. This website will assist documentation staff in preparing on-line help.

## **Conclusion**

The key tasks set forth in this proposal will be a worthwhile contribution to Landmark Graphics. The training resource will provide a useful resource for future and current reference, and the seminars will facilitate a good environment for questions, solutions and education in the science of geostatistics. The

skills gleaned from the seminars and the training resource will provide excellent tools for testing software. A glossary in HTML format will prove to be a valuable tool for understanding the terminology used in geostatistics. A hot linked workflow model will provide a "map" which will aid in the understanding of the process of events involved in the construction of geostatistically derived numerical geological model.

## **Disclaimer**

Parts of this website are patterned after the book *Geostatistical Reservoir Modeling*, a currently unpublished book authored by Dr. Clayton Deutsch. Dr. Deutsch is aware and has granted permission for the use of material.

## **Lecture 1: Purpose / Motivation for Geostatistics**

- [Qualitative Reasoning](#)
- [Reservoir Planning](#)
- [Elimination of Surprises](#)
- [The Need For Decision Making](#)
- [Quantification of Uncertainty](#)
- [Portfolio Management](#)
- [The Life Cycle of a Project](#)
- [Lecture 1 Quiz](#)

### **Introduction**

In view of Landmark's core business, i.e. reservoir characterization, geostatistics can be defined as a collection of tools for quantifying geological information leading to the construction of 3D numerical geological models to be used for assessment and prediction of reservoir performance.

Geostatistics deals with spatially distributed and spatially correlated phenomena. Geostatistics allows quantification of spatial correlation and uses this to infer geological quantities from reservoir data at locations where there are no well data (through interpolation and extrapolation). In addition, the main benefits from geostatistics are: (1) modeling of reservoir heterogeneity, (2) integrating different types of data of perhaps different support and different degrees of reliability, and (3) assessing and quantifying uncertainty in the reservoir model.

This course was not developed as a cookbook recipe for geostatistics. Each geostatistical study requires a certain degree of user-interaction, interpretation, customization, and iteration for a robust solution. This course offers a minimal guide to understanding the fundamental basics of geostatistics and the processes associated with the construction of numerical-geological models. The reader is expected to realize that the methods presented in this course are the foundation steps required for the development of reliable geostatistical models.

The reader will require an understanding of basic statistical concepts, low level calculus, and some understanding of linear algebra notation. This course is neither mathematically advanced nor difficult, but it may be challenging. You will be exposed to terminology specific to geostatistics, which is not used in everyday conversation. Due to its specific lingo, its use of statistical concepts and spatial correlation, geostatistics is still often perceived to be "black" magic or art. One of the many goals of this course is to remove the stigma and demystify geostatistics; "*Geostats for the masses*".

### **Qualitative / Quantitative Reasoning**

Geostatistics presents a probabilistic approach to the study of natural phenomena that vary in space. It was developed in the mining industry from a need to cope with earth science data in an intelligent and mathematically robust manner. It has, since its creation, been the preferred method for dealing with large data sets, integration of diverse data types, the need for mathematical rigor and reproducibility, and the need to make decisions in the face of uncertainty.

In the oil industry, there is, first of all, a need for reliable estimates of the original hydrocarbon volumes in a reservoir. These in situ volumes are important for: (1) determining the economic viability of the reservoir, (2) allocating equity among multiple owners, (3) comparing the relative economic merits of alternative ventures, (4) determining the appropriate size of production facilities [Deutsch, 1999]. A strength of geostatistics, as compared to more traditional interpolation techniques such as [inverse-squared distance](#), and [triangulation](#) is the quantitative use of [spatial correlation](#) / variability



models (e.g. variograms, see later lectures). Basically, the geostatistical interpolation routines will account for and reproduce geological interpretation information in estimating quantities at unsampled locations rather than blind interpolation between known data values.

For some there is significant confusion regarding the use (and abuse) of geostatistics. Geostatistics consists of a set of mathematical tools, comprising of data analysis components and interpolation / extrapolation routines. Like all problems requiring a solution, the solution will not arise by blindly throwing tools at it. The tools must be used intelligently to extract a solution. A geostatistical study consists of a series of (document) subjective (and interpretative) decisions. Many geoscientists are confused by the "geostatistical solution" because they do not understand how to cope with it. Geostatistics in recent years has come away from the idea of a single deterministic answer to earth science problems, rather, it focuses on the uncertainty associated with that answer. Geostatistics will not tell you to "drill a well two feet to your left", but rather "drill between 0 and 10 feet to your left with a best chance of hitting oil at 2 feet to your left". Geostatistics will not yield the precise volumes of oil in a reservoir. It will estimate this volume and the uncertainty associated with this estimate. Geoscientists and engineers must still make the educated decision of potential well locations, however, they are now armed with tools allowing them to quantify the uncertainty and risks associated with the decisions they have to make. The shift in "philosophy" is accepting the fact that there is tremendous uncertainty in most reservoir related quantities and that we will never know the true answer and any deterministic suggestion is very likely to be wrong (no matter how much "physics" is behind it). We have to learn to make decisions in the face of uncertainty and start thinking in terms of "probability of an outcome".

Traditionally, decisions have often been made by visual estimate, e.g. by "looking" at a contour map and deciding which is the best part of the reservoir, by eye-balling correlation across 2 wells. Difficulties arise when moving from 1 or 2D to 3D data sets. Qualitative decision making rapidly becomes impossible and unreliable. A quantification framework becomes necessary to deal with various (possibly correlated) 3D data types, sampled over different volumes, with different levels of precision and reliability, relating to different earth attributes. Geostatistics is a toolbox to do just this.

Like all toolboxes, there are appropriate uses and limitations of geostatistics. Geostatistics is very useful throughout the life of a reservoir, but it has the most impact early on when there exists tremendous uncertainty in geological quantities. As a reservoir matures and additional data / information become available (e.g. well logs, 3D seismic, production data), the uncertainty shifts focus and "hopefully" decreases. There is (usually) less uncertainty as more data comes about: additional data imposes more constraints on the model, yielding fewer degrees of freedom, thus less uncertainty. This is known as the information effect. As time progresses, more information becomes available further constraining the model and reducing uncertainty. Taken to the extreme, the reservoir will eventually be completely known and there will be no uncertainty; geostatistics will have reached the limit of its usefulness, as shown in Figure 1.1.

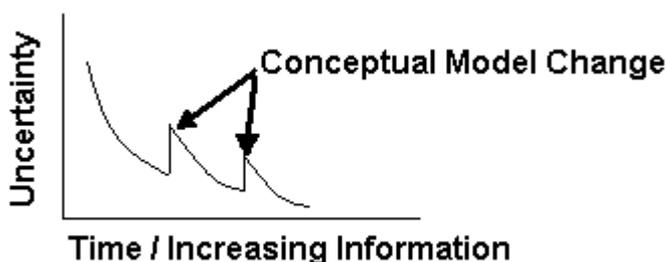


Figure 1.1: Effect of acquisition of new data on the uncertainty remaining on the geological model.

## Reservoir Planning

The motivation behind reservoir planning is maximizing the [net present value](#) of the reservoir by getting as much oil out as possible in the least amount of time. It consists of determining the best strategy for development of a reservoir or a field. In the early years of oil field exploration and reservoir production, a wide variety of different approaches were used to ascertain the best well locations and best exploitation strategies. Some were effective and others were not depending on the type of reservoir at hand and the complexity of the decisions to be taken, but mostly it was a hit or miss operation. Exploration and reservoir planning decisions were, at best, based on historical data from mature fields with similar properties and some geological information from outcrops. Around the same time frame as World War II, sonic tools and electric logs for geological exploration were developed. The data derived from these tools required different data management practices and paved the way for more rigorous workflows for reservoir exploitation leading to reservoir planning.

There are three distinct phases to reservoir exploitation: (1) the reservoir exploration phase, (2) the exploitation phase, and (3) the enhanced recovery / abandonment phase. At the beginning of the delineation phase, an oil reservoir has been discovered but its extent is not known. The aim of the exploration phase is to gain as much insight as possible about the size and boundaries of the reservoir. Past delineation practices were to drill step out wells in regular patterns until the well data showed that the limit of the reservoir had been reached. By enabling more accurate estimates of rock / petrophysical properties between wells, the use of geostatistics has helped reservoir delineation by allowing to increase the distance between step out wells. The exploitation phase is the strategic placement of additional wells to maximize the net present value. In the past, most wells were located using a single interpretation of the data. The use of geostatistics can help optimize well placement through sensitivity studies by considering the effects of infill wells on other existing wells, maximizing oil bearing lithofacies connectivity, determining optimal number of infill wells to drill, etc... By enabling multiple realizations of the reservoir and its heterogeneity, geostatistics provides tools to evaluate the probable impact of various drilling options and recovery schemes, thus quantifying the risks associated with different development scenarios. In the enhanced recovery / abandonment phase the reservoir is evaluated for the effectiveness of an enhanced recovery phase such as steam injection, chemical injection, and so on. If no enhanced recovery scheme proves fruitful then the reservoir is abandoned. The use of geostatistics spans the whole lifecycle of a reservoir, from the exploration phase to the decision of abandonment.

## Elimination of Surprises

Before we can discuss how geostatistics can be used to eliminate unwanted surprises, these must first be identified. Historically, reservoir modeling was performed in a [deterministic](#) mindset. In other words, a single model was created based on a unique interpretation of earth science data yielding a single deterministic response / solution to geological problems. In reality, due to the large uncertainty existing beyond the data and even within the data itself, one cannot express full confidence in a single deterministic response. Sources of uncertainty within the data arise not only from measurement or acquisition errors but also from interpretation errors. This applies to both direct (e.g. cores) and indirect measurements (e.g. logs, seismic). In addition, the resolution of some measurements (e.g. seismic) may be limited in that it either hides, blurs, or confuses important features. Also, well measurements represent a very limited sample of the reservoir. Wells are separated by very large distances as compared to the volume they sample, thus leading to tremendous uncertainty in interpolating geological quantities between wells. Above all, the biggest source of uncertainty may yet come from the high-level choice of the structural model and deposition environment. The uncertainty due to data and the subsequent surprise realized by putting too much faith in the data can be limited through rigorous data analysis exercises and proper calibration.

Some reservoir modeling packages allow for the addition of trends and other geologic information, but only geostatistical methods enable modeling of heterogeneity and the inclusion of uncertainty in both the data and the choice of model. Geostatistics provides stochastic techniques to generate alternative models each being a plausible solution ([equiprobable](#)). The value in modeling this way is the ability to quantify uncertainty in the model and to generate models that have a realistic level of heterogeneity. Heterogeneity is a component of uncertainty, defined as the magnitude of diversity in the reservoir. If the heterogeneity is poorly modeled the resulting reservoir model may be too smooth and provide inaccurate assessment of the uncertainty in the model and predictions of the reservoir behavior.

The surprises include but are not limited to the potential error of oil in place estimates, the flow characteristics, and water break through times. These surprises can catastrophic economic results. Using geostatistics prevents unwanted surprises creeping up in the modeling exercise through different means. Firstly, a geostatistical study forces practitioners to perform thorough data analysis and quality assurance steps before any modeling decision take place, furthermore, rigorous model validation is also recommended. But most importantly surprises in engineering / business decisions are reduced through the probabilistic approach taken by geostatistics: a full probability distribution of outcome is available rather than deterministic and subjective best, worst, most likely scenarios only.

## The Need For Decision Making

A geostatistical study, or any reservoir modeling exercise for that matter, should not happen unless it is dictated by a key business decisions aimed at increasing the net present value of a reservoir, an asset or even a whole portfolio. Major investment / divestment decisions must be made in the presence of significant uncertainty, geostatistics aims at providing a framework for working with uncertainty and managing it.

## Quantification of Uncertainty and Risk Qualified Decision Making

Uncertainty is the lack of assuredness about the truth of a statement or about the exact magnitude of a measurement. Uncertainty is the central concept in the decision making that follows geostatistical studies. Uncertainty is required in decision making because estimates do not agree with reality. For instance, an estimate with low uncertainty leads to easier decision making versus an estimate with high uncertainty. Figure 1.2 shows three histograms each of varying degrees of uncertainty. The left histogram shows a histogram with a mean of 0 with high uncertainty about the mean, the middle histogram shows decreased uncertainty about the mean, and the right histogram indicates little uncertainty about the mean.

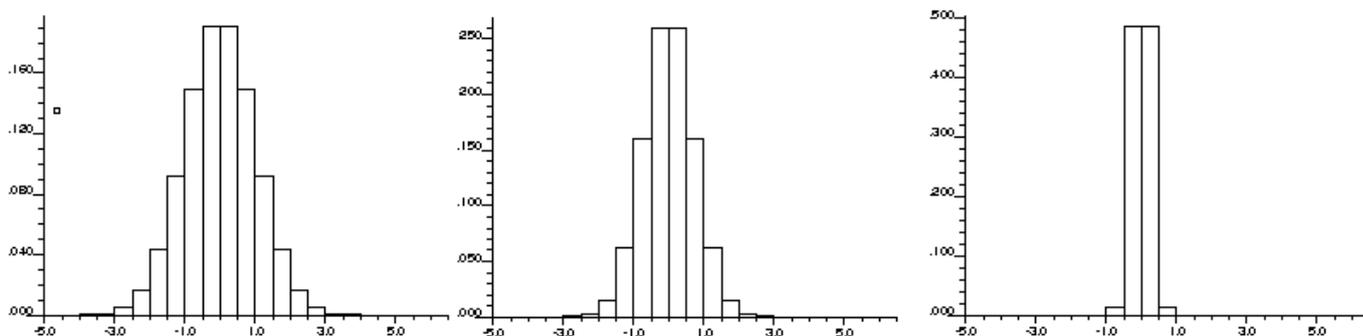


Figure 1.2 Three histograms of decreasing uncertainty, the greatest uncertainty being on the left, and the least uncertainty being on the right.

Risk-qualified decision making requires (1) a quantification of uncertainty, and (2) quantification of the loss associated with the decision. By way of example, Figure 1.3 illustrates the concept of risk

qualified decision making. A distribution of uncertainty is generated, and using a loss function the risk is assessed and an optimal estimate is determined, the estimate that incurs the least loss. Different loss functions can be used for pessimistic and optimistic estimates.

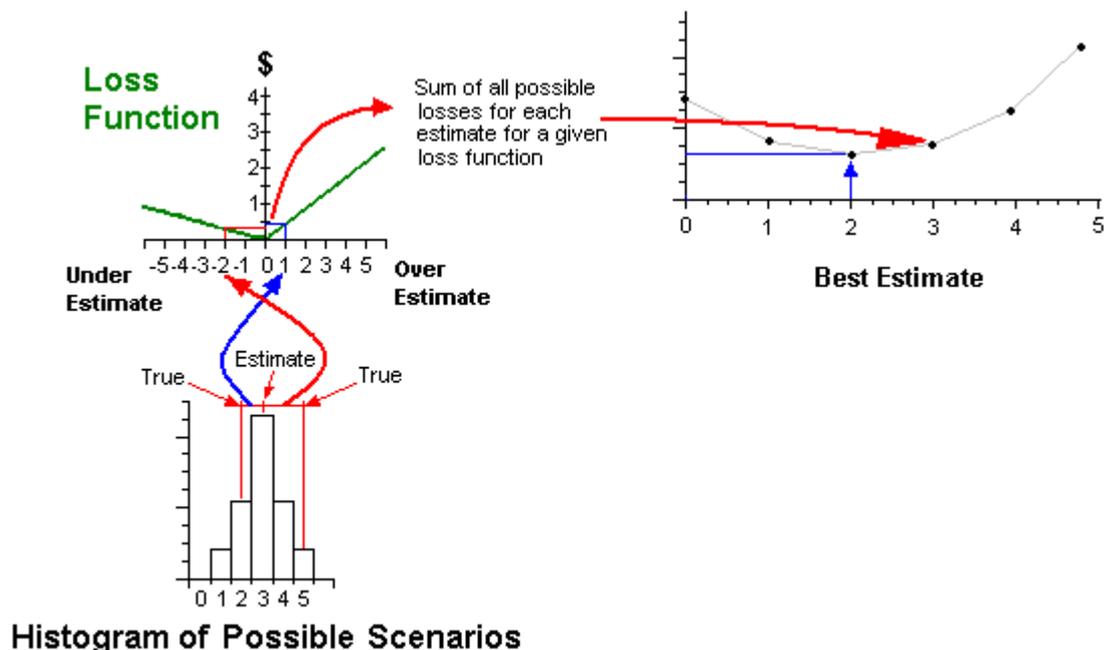


Figure 1.3, An illustration showing the concept of risk qualified decision making. Note that the loss function is scenario specific, and that the histogram of possible costs are in addition to those costs if the estimate were correct.

## Portfolio Management

Portfolio management requires that the best possible decisions be made in the face of uncertainty. Some of these decisions include:

- **Exploration License Bidding:** using limited seismic and well data, decide which zones to focus on and / or commit resources to.
- **Exploration Drilling:** given a few wells (1-3), decide whether or not the field warrants further investigation.
- **Drilling Campaign:** decide how many wells to drill, placement of wells, and timing of enhanced oil recovery tactics.
- **Development Planning:** decide how large support facilities must be, negotiate pipeline or sales agreements and contractor commitments.
- **Mature field:** decide on the value of infill drilling or the implementation of enhanced oil recovery schemes such as flooding and steam injection.
- **Abandonment / Sale:** timing of environmentally and economically sound closing of facilities.

These decisions are being made with less data and greater uncertainty for projects that are marginally profitable. Sound estimates backed by rigorous mathematical methods secures investors and fosters good economic relations.

## The Life Cycle of a Reservoir

Table 1.1 summarizes the various stages of the lifecycle of an oil reservoir, the types and quantity of data available, the technical / business decisions to be made, and the (geostatistical) numerical models that best support those decisions.

Lifecycle Stage	Data Available	Decisions to be Made	Types of study to consider
Early Exploration	1 or 2 wells  Maybe seismic (used primarily for structures/volumes)	Coarse volumetrics  Approximate flow rate  Number of platforms  Conceptual approach	<ul style="list-style-type: none"> <li>• Coarse 2D models, stochastic surface and fault models</li> <li>• Multiple realizations</li> <li>• Static display of volumes with uncertainty from Monte Carlo/bootstrap studies</li> <li>• Segment models for well behavior</li> </ul>
Early Development Planning	6 to 10 wells  (seismic attributes as well)  Some production data	Number and spacing of wells  Types of wells  Pressure support	<ul style="list-style-type: none"> <li>• Compartmentalization/surface modeling</li> <li>• Quick 3D full field models to understand interactions</li> <li>• Small scale high resolution models for water or gas coning</li> </ul>
Late Development Planning	More production data  Geologic understanding	Alternative recovery schemes	<ul style="list-style-type: none"> <li>• Realistic 3D models</li> <li>• Flow simulations</li> <li>• Segment models to look at EUR/costs</li> </ul>
Mature Field, Infill Well, IOR, Bypassed Oil		Well conversions  New processes  Abandonment	<ul style="list-style-type: none"> <li>• Small area/segment models</li> <li>• Static display of remaining volumes &amp; uncertainty</li> </ul>

## ***Lecture 1: Purpose / Motivation for Geostatistics, The Quiz***

### **Question 1**

List three major benefits that geostatistics offers, and describe what each mean and explain why they are important.

### **Question 2**

Decision making in presence of uncertainty is important. What are the two steps for risk-qualified decision making?

### **Question 3**

In general terms, explain the link between spatial variability (heterogeneity) and uncertainty.

### **Question 4**

In your own words describe the information effect and how it relates to uncertainty.

### **Question 5**

Geostatistics is useful at every point in the life cycle of a reservoir, but where is it most useful and why?

[solutions](#)

## ***Lecture 1: Purpose / Motivation for Geostatistics, The Quiz***

### **Question 1**

List three major benefits that geostatistics offers, and describe what each mean and explain why they are important.

Quantification of uncertainty: summarizes our lack of knowledge for better decision making

Rigorous mathematics: means that there are sound mathematical laws applied for repeatability

Data Integration: data of many types can be integrated using geostatistical tools

### **Question 2**

Decision making in presence of uncertainty is important. What are the two steps for risk-qualified decision making?

Quantification of uncertainty and then quantification of risk.

### **Question 3**

In general terms, explain the link between spatial variability (heterogeneity) and uncertainty.

As spatial variability increases heterogeneity increases and hence uncertainty increases

#### **Question 4**

In your own words describe the information effect and how it relates to uncertainty.

The information effect is the result of increased available information which leads to less uncertainty

#### **Question 5**

Geostatistics is useful at every point in the life cycle of a reservoir, but where is it most useful and why?

Geostatistics is most important in the early stages of the life cycle because it makes intelligent use of limited data and allows for decision making that is tempered with a knowledge and understanding of the uncertainty inherent in the numerical-geological model.

July 6, 1999

## Lecture 2: Basic Concepts

- [Statistical Tools](#)
- [Histograms](#)
- [Probability Distribution](#)
- [Categorical Variables](#)
- [Comparing Histograms](#)
- [Data Transformation](#)
- [Monte Carlo Simulation](#)
- [Bootstrap](#)
- [Geostatistical, and Other Key Concepts](#)
- [Numerical Facies Modeling](#)
- [Cell Based Modeling](#)
- [Object Based Modeling](#)
- [Lecture 2 Exercises](#)
- [Lecture 2 Quiz](#)

### Introduction

This lecture covers some of the fundamental geostatistical and statistical concepts that are required in the course. Essential statistical concepts and definitions that are the building blocks for Geostatistics, tools that will be used throughout the course, and concepts exclusive to geostatistics are covered here. The statistical content is modest and not difficult to master.

### Statistical Tools / Definitions

#### Statistics

Statistics is the science of collecting, processing, analyzing and interpreting numerical data. Statistics dilutes numerical information to provide (sometimes) clearer insights into a population.

#### Geostatistics

Geostatistics originally started as the study of phenomenon that vary in space but the science has evolved as a suite of mathematical tools for application to many other earth science problems. The strength of Geostatistics is its stochastic approach to numerical modeling. While not all of the tools in the geostatistical toolbox are stochastic in nature, most of them, or at least the workflows they describe are, and it is in this arena that Geostatistics has enjoyed the most success. In some ways geostatistics is the antithesis of traditional statistics; geostatistics takes sample data and infers a population rather than diluting sample information into a more digestible forms of information such as mean and variance. Geostatistics, unlike statistics, focuses on natural phenomena which are correlated in space a feature of all natural phenomena. Typical features of importance are spatial continuity (or variability), spatial anisotropy, and trends.

#### Variable

A variable is a symbol which can take any one of a prescribed set of values. A variable that can assume any real number value is called a [continuous variable](#) (often denoted  $z$  in geostatistical jargon); any variable that can only assume an integer value is called a [discrete or categorical variable](#). Porosity and permeability are continuous variables. Lithofacies classification are categorical and commonly denoted with the indicator variable  $i$ , where  $i$  is 1 if the category is present and 0 if not (more on this later). When a variable is distributed in space it is called a [regionalized variable](#). A regionalized



variable is often denoted  $Z$  in geostatistics. Permeability and porosity are two examples of regionalized variables, the value of the attribute is, as mentioned above, a particular realization of the regionalized variable and is denoted by  $z$ .

The regionalized variable is simply a function  $f(x)$  which takes on a possibly different value ( $z$ , or  $i$ ) at any location in space. One would think that it possible to examine a single mathematical function that characterizes a reservoir. More often than not, the variable varies so irregularly in space as to preclude any direct mathematical study of it [Journel, 1978]. Because earth science science phenomenon involve complex processes, and because the regionalized variable is so erratic the regionalized variable is considered a [random variable](#). The random variable is a variable which takes a certain number of numerical values according to a certain probability distribution. The set of permissible values that the random variable can take is called the [random function](#). Instead of attempting to model the regionalized variable analytically (mathematically) the regionalized variable is modeled as a random function.

For example, the result of casting an unbiased die can be considered as a random variable which can take one of six [equiprobable](#) values. The set of values that the die can take is called the random function. If one result is 5 then this value is called a particular [realization](#) of the random variable "result of casting the die". Similarly let us consider the permeability  $z(\mathbf{u}) = 4$  md, where  $\mathbf{u}$  denotes a location in space. This measure of permeability can be considered as a particular realization of a certain random variable. Thus the set of permeabilities  $z(\mathbf{u})$  for all points  $\mathbf{u}$  inside the reservoir (the regionalized variable  $z(\mathbf{u})$ ) can be considered as a particular realization of the set of random function  $Z(\mathbf{u})$ , for all locations  $\mathbf{u}$  in the reservoir.

## Minimum

The minimum is the smallest data value in the data set.

## Maximum

The maximum is the largest data value in the data set.

## Mean or Expected Value

The mean, or Expected Value, is the weighted average of a random variable (or sample data), where the weights represent the probability of occurrence of each sample. If the sampling technique sampled unbiasedly, that is sampling without preference, the data all have an equiprobable chance of being selected and all of the probabilities would be one, i.e. the mean is then obtained by adding all of the data and dividing by the number of observations. The Expected value is denoted by  $E[X]$ , or more simply  $m$ , and is defined by:

$$m = \frac{1}{n} \sum_{i=1}^n x_i = E[X] \quad (2.1)$$

where  $x$  is the value,  $n$  is the number of samples,  $E[X]$  is the expected value, and  $m$  is the mean.

Expected values have some important mathematical properties:

$$E[a * X] = a * E[X] \quad (2.2)$$

and

$$E[aX + b] = aE[X] + b \quad (2.3)$$

## Median

The midpoint of the ranked (i.e. sorted from smallest to largest) data. If there were 25 data, the median would be the 13th value. It also represents the 50th percentile in a cumulative histogram.

## Mode

The mode is the most commonly occurring data value in the data set

## Variance

The variance is a measure of spread. It can be thought of as the average squared-distance of the data from the mean. It can be found using the equation below:

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - m)^2 \quad (2.4)$$

## Standard Deviation

The standard deviation is the square root of the variance. It is sometimes the preferred measure of spread because it has the same units as the mean whereas the variance has squared units.

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - m)^2} \quad (2.5)$$

## Coefficient of Skewness

The coefficient of skewness is the average cubed difference between the data values and the mean. If a distribution has many small values and a long tail of high values then the skewness is positive, and the distribution is said to be positively skewed. Conversely, if the distribution has a long tail of small values and many large values then it is negatively skewed. If the skewness is zero then the distribution is symmetric. For most purposes we will only be concerned with the sign of the coefficient and not its value.

$$CS = \frac{\frac{1}{n} \sum_{i=1}^n (Z_i - m)^3}{s^3} \quad (2.6)$$

## Coefficient of Variation

The coefficient of variation is the ratio of the variance and the mean. While the standard deviation and the variance are measures of absolute variation from the mean the coefficient of variation is a relative measure of variation and gives the standard deviation as a percentage of the mean. It is much more frequently used than the coefficient of skewness. A coefficient of variation (CV) greater than 1 often indicates the presence of some high erratic values (outliers).

$$CV = \frac{s}{m}$$

## Quantiles (Quartiles, Deciles, Percentiles...)

Quartiles, Deciles, percentiles are used to break the data into quarters, tenths, and hundredths, respectively. Quantiles define any other user defined fraction. It is common to compare quantiles from two different distributions.

## Inter-Quartile Range

The difference between the 75th and 25th percentiles.

## Quantile-Quantile Plots

Quantile-quantile plots (Q-Q plot) are useful for comparing two distributions. A Q-Q plot takes the quantile value from one distribution and cross plots it against another. The result would be a straight line at a 45 line if the two distributions are the same shape. A change in slope indicates a difference in variance, and a parallel shift in any direction indicates a difference in the mean. Some uses of the Q-Q plot include core to log relations, comparing the results from different drilling campaigns, comparing the effects of declustering (to be discussed later), and comparing distributions by lithofacies.

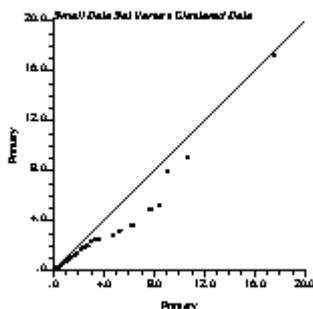


Fig. 2.1 A quantile-quantile plot.

## Covariance

The covariance is the expected value  $E[(X-m_x)(Y-m_y)]$  and is a measure of the linear relationship between the random variable  $X$  and  $Y$ . The thing to notice is that the covariance and the variance are the same if the variable  $X$  and  $Y$  are the same. Thus the two variables are called covariates. The covariance function can also be written as:

$$Cov(\mathbf{h}) = E[Z(x) * Z(x + \mathbf{h})] - m^2 \quad (2.7)$$

where the first variable is indicated by  $Z(x)$ , a data at one location, and the second variable is indicated by  $Z(x+h)$ , a data at another location. The variable  $m$  is the drift component (mean). The distinction to be noted here is that the covariates can be the same variable but in different places as indicated  $Z(x)$  (read the random variable at location  $x$ ), where  $x$  indicates the location specific nature of the covariates. Thus equation 2.7 can be read as the covariance between the two covariates of the same attribute but separated by a distance  $h$ .

## Scatterplot

The scatter plot is a bivariate display of two covariates, or the same variable separated by a distance. The values from one distribution are used as the X coordinate and the values from another are used as the Y coordinate to plot a point on the scatter plot.

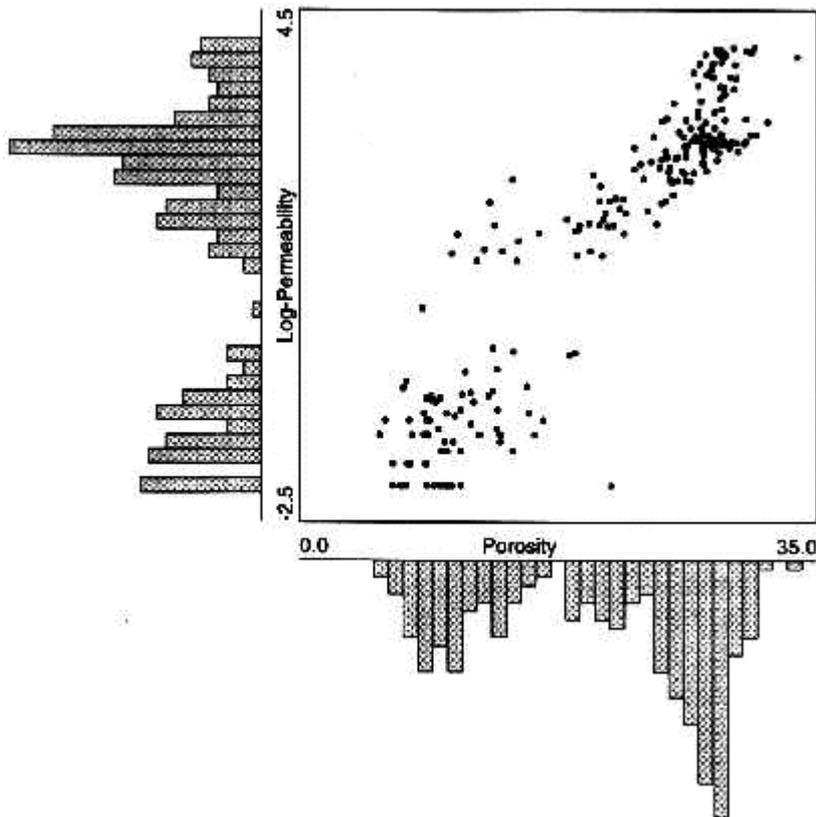


Fig. 2.2a, A scatterplot uses the a value from one distribution as the x coordinate and the value of another for the y coordinate to plot a point.

## Correlation Coefficient,

Correlation is the characteristic of having linear interdependence between RV's or any two data sets. In general two sets of data can be positively correlated, negatively correlated, or not correlated. A useful tool determining how two data sets are correlated is the scatter plot. The scatter plot is a plot of one variable versus another. The following diagram shows data sets that are positively correlated, not correlated, and negatively correlated:

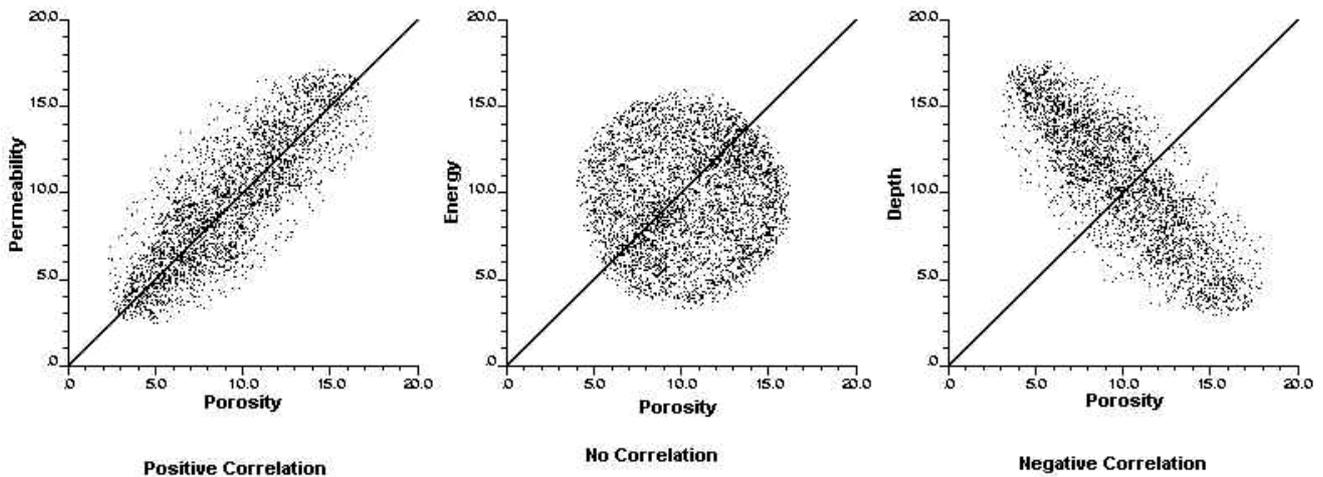


Fig. 2.2b The scatterplot on the left illustrates positive correlation, the middle one shows no correlation, and the right one shows negative correlation.

One measure of the extent of correlation is the correlation coefficient. The correlation coefficient can be calculated using the formula below:

$$\rho^2 = \frac{Cov(X,Y)}{s_x s_y} \quad (2.8)$$

where *Cov* is the covariance and *s* is the standard deviation. A correlation coefficient of 1 means that the two data sets are perfectly correlated. Negative correlation coefficients indicate negative correlation and positive coefficients indicate positive correlation. A correlation coefficient of 0 indicates no correlation.

## Rank Correlation Coefficient

The rank correlation coefficient or the Spearman rank correlation coefficient measures the correlation between two data sets of the same size by comparing the rank of each  $x_i$  to the rank of each  $y_i$ . The rank of the smallest value is 1 and the rank of the largest value is *n* for *n* data. The rank correlation coefficient is found by calculating the correlation coefficient for each of the values as they are compared by rank, and is independent of data values. In that sense, it is a more robust measure of correlation.

## Stationarity

Stationarity assumes that the data within the area of study are assumed to be statistically independent of location. This means that data at location  $\mathbf{u}$  is assumed to be derived from the same distribution or RV as data from location  $\mathbf{u}+\mathbf{h}$ , where  $\mathbf{h}$  is known as a lag vector separating two values by some distance in some direction. The assumption of stationarity is not always applied to the entire data set. It need only be applied to the instantaneous area of study or the search neighborhood for what is called *local stationarity*. All estimation methods assume local stationarity to some extent. The extent is dictated by the continuity of the data and the geology. Assuming stationarity over different lithofacies types can be unsafe and is identical to lumping the entire data set together.

There are at least two orders of stationarity (1) Strict stationarity; a RV is said to be under strict stationarity when its spatial correlation is invariant under translation, and (2) the expected value exists

and is not location dependent and for each pair of data the covariance exists and is dependent only on the separation distance  $h$ .

## Uncertainty

All numerical models would be found in error if we were to excavate that interwell volume and take exhaustive measurements; there is uncertainty. Uncertainty is defined as the lack assurance about the truth of a statement or the exact magnitude of a measurement or number. It is impossible to establish the unique true distribution of petrophysical properties between widely spaced wells. The uncertainty regarding the distributions we model is due to our lack of knowledge or ignorance about the reservoir. Geostatistical techniques allow alternative realizations (possible models) to be generated providing a method for quantification of uncertainty [Deutsch, 1999]. Uncertainty is the central concept behind the decision making that usually follows any geostatistical study [Olea, 1991]. It is important to note that uncertainty is not an inherent feature of the reservoir, it is a product of our ignorance.

## Histograms

A histogram is a bar chart comparing a variable to its frequency of occurrence. It is the most common way of graphically presenting a frequency distribution. The variable is usually organized into class intervals or bins. An example of a histogram is shown below.

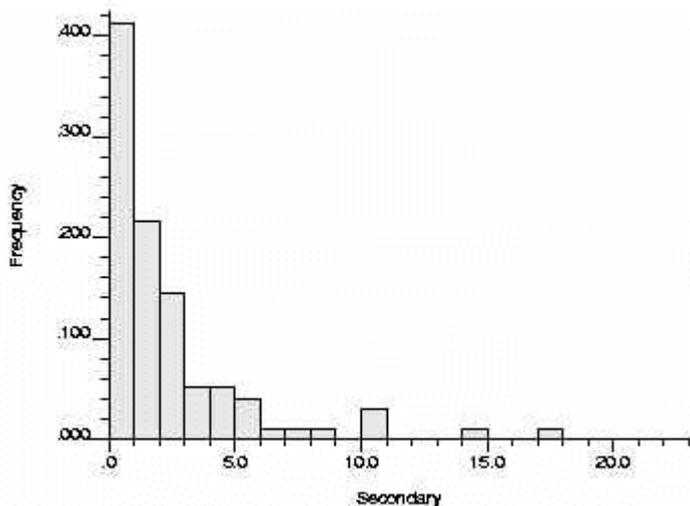


Fig. 2.3 This figure diagrammatically illustrates the essential components of a histogram.

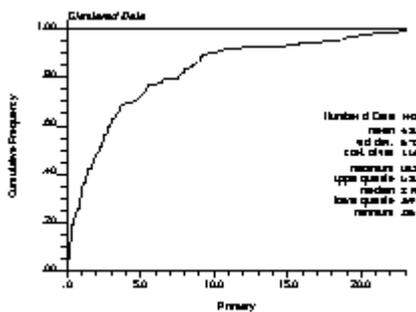


Fig. 2.4 A cumulative frequency plot

## Cumulative Distribution Function

A probability distribution summarizes the probabilities that the random variable will take a certain value. A probability distribution and a cumulative distribution function are the same. Probability can be defined as the relative frequency of an event in the long run. If we repeat the experiment many times the relative frequency of the outcomes should be the same as the random variables probability histogram. Cumulative distribution functions (cdf) are defined mathematically by:

$$F(z) = \text{Prob}\{Z \leq z\} \in [0,1] \quad (2.8a)$$

A cumulative frequency histogram is plotted using the cumulative distribution function (cdf) of the random variable  $Z$ . Some important features of the cdf include (1) its value is always between 0 and 1, (2) it is a non decreasing function, and (3) the values are not classified into bins. There are many different probability distributions each having different properties. We will work with the Gaussian distribution. The Gaussian distribution, or normal distribution, has qualities (integrates easily among others) that make it especially practical for use in geostatistics.

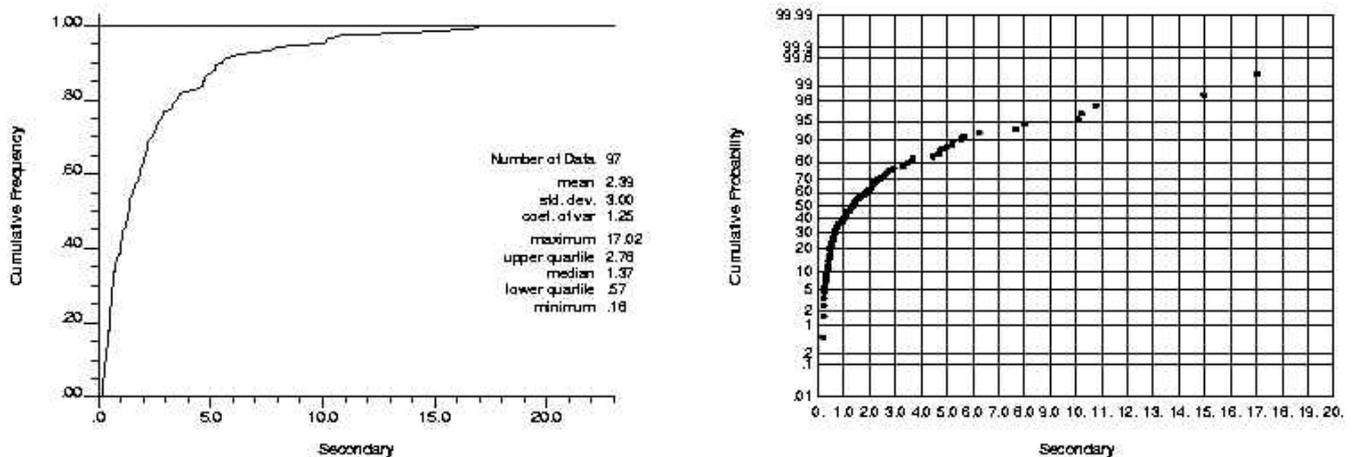


Fig. 2.5 A cumulative probability plot and a cumulative frequency plot are the same thing.

The probability distribution of a categorical variable is defined by the probability or proportion of each category.

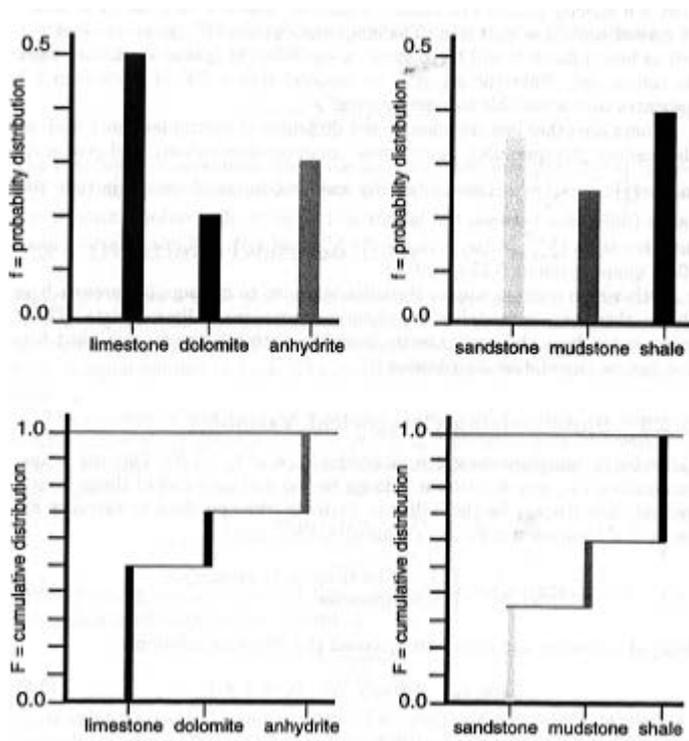


Fig. 2.6 Notice that it does not matter which way you order the data in a categorical histogram or a categorical cdf. It must be pointed out that the cdf is a non-decreasing function, and that it is cumulative.

## Probability Density Function

The probability density function (pdf) is the first derivative of the cdf. Extending the definition of probability to the continuous case the pdf is mathematically defined by:

$$F(z) = \int_{-\infty}^z f(x) dx \quad (2.8b)$$

Integrating between any two constants  $a$  and  $b$ , gives the probability that a random variable assumes a value between these limits.  $F$  however cannot be any old function, it is bound by

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (2.8c)$$

## Weighted Statistics

When a priori statistics are calculated, they are usually unweighted. In other words the data are assumed to be unbiased. We know that this is not always the case. It is less likely to roll two dice and have the sum equal two than for the sum to be 5. The statistics described above (mean, variance...) can again for a different weight for all data samples. In reservoir exploration, data are sample (i.e. wells are drilled) in preferential high pay zones. Resulting sample statistics are therefore biased, they overestimate the true average pay of the reservoir. Spatial declustering techniques are available to unbiased the sample statistics by assigning different weights to the sample data (see later).



## Categorical Variables

Consider  $k$  mutually exclusive categories (e.g. rock types)  $s_k$ , where  $k=1, \dots, K$ . The indicator transform is defined as:

$$i(\mathbf{u}, s_k) = \begin{cases} 1 & \text{if } (\mathbf{u}) \text{ is in category } s_k \\ 0 & \text{otherwise} \end{cases} \quad k = 1, \dots, K \quad (2.8)$$

Mutual exclusion properties and exhaustively state that  $i(\mathbf{u}; s_k) \cdot i(\mathbf{u}; s_{k'}) = 0$ . Which means if any data fall into category  $k$  and  $i(\mathbf{u}; s_k) = 1$  then  $i(\mathbf{u}; s_{k'}) = 0$  in all other categories and the sum of all  $i(\mathbf{u}; s_k) = 1$ . Which means that the sum of all proportions over all of the categories is equal to 1. In this way each of the categories are represented as proportions of the whole.

For continuous variables, we can define the indicator transform as:

$$i(\mathbf{u}, s_k) = \begin{cases} 1 & \text{if } z(\mathbf{u}) \leq s_k \\ 0 & \text{otherwise} \end{cases} \quad k = 1, \dots, K \quad (2.8a)$$

## Categorical Mean

The mean indicator for each category  $s_k$ ,  $k= 1, \dots, K$  is interpreted as the proportion of data in that category:

$$p_k = \overline{i(\mathbf{u}, s_k)} = \frac{\sum_{j=1}^n w_j * i(\mathbf{u}_j, s_k)}{\sum_{j=1}^n w_j} \quad (2.9)$$

where  $p_k$  is the proportion of the  $k$ th category,  $\mathbf{u}$  is the data location,  $s_k$  is category  $k$ ,  $w_j$  is a weighting factor, and  $i(\mathbf{u}; s_k)$  is the indicator transform. Simply decoded, the above formula says that the mean of a categorical variable is equal to the proportion of that category over all the data. Notice that the categorical mean can also be probability weighted just like the continuous variable.

## Categorical Variance

As with the mean, the categorical variance is found the same way in categorical variables as in continuous variables.

$$Var\{i(\mathbf{u}, s_k)\} = \frac{\sum_{j=1}^n w_j * [i(\mathbf{u}_j, s_k) - p_k]^2}{\sum_{j=1}^n w_j} \quad (2.10)$$

## Comparing Two Histograms

It is sometimes advantageous to compare two histograms so that one can see how the two distributions differ. This is done by cross plotting the quantile scores of the two distributions. The figure below diagrammatically illustrates one method for visually comparing two histograms:

**Histogram of 409 paired core and log derived porosity values**

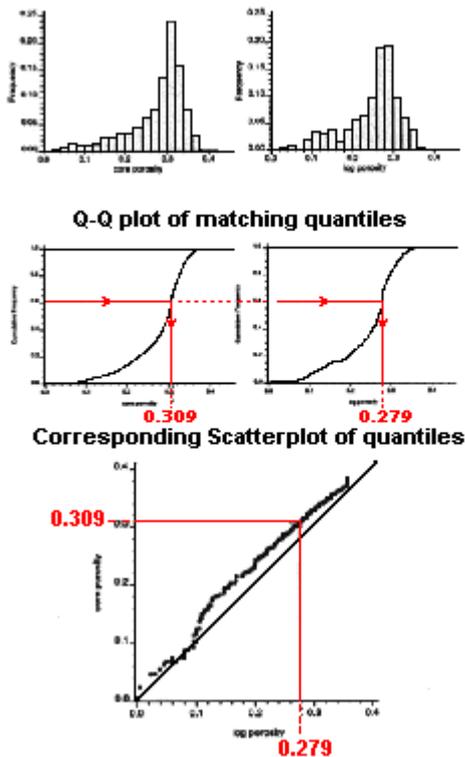


Fig. 2.7 To compare two histograms, one can compare them quantile by quantile, and plot the respective quantiles on a scatter plot.

## Data Transforms

Simulation methods such as sequential Gaussian simulation require that the cdf of the data be normally distributed, however not all data sets are normally distributed. This is not a problem though because in the same way as we compare two histograms, we can transform data from one distribution to a normal distribution or any another distribution. This is done using the following steps:

1. find the quantile value of the smallest data value from the cdf of the data
2. using the quantile from the first cdf find the respective normal scores value from the normally distributed cdf
3. repeat for all possible values in the data cdf

The Figure 2.8 diagrammatically illustrates data transformation into normal space:

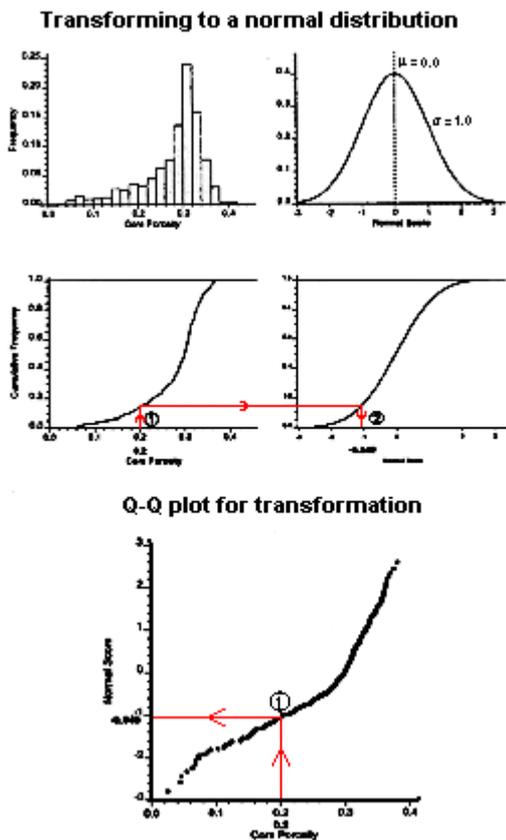


Fig. 2.8 Transforming data into Gaussian (normal) space is performed by converting the original quantile to the equivalent normal scores quantile.

## Monte Carlo Simulation

Monte Carlo simulation is any procedure that uses random numbers to obtain a probabilistic approximation to a solution. Monte Carlo Simulation proceeds in two steps:

1. A uniformly distributed random number between 0 and 1 is drawn
2. The random number is taken to represent a cumulative probability
3. the corresponding quantile is identified from the cdf

Figure 2.4 shows how Monte Carlo simulation is performed. For

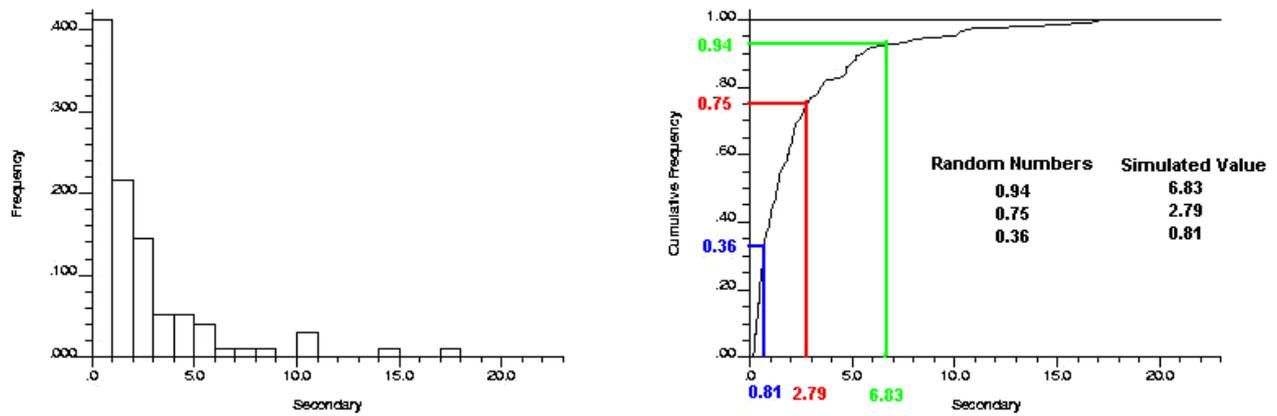


Fig. 2.9 Monte Carlo simulation consists of drawing a normally distributed number and recording the appropriate value from the cdf.

Monte-Carlo simulation is the foundation of all stochastic simulation techniques. Much care should be taken to ensure that the parent cdf is a representative distribution, as any biases will be translated into the results during the transformation.

## Bootstrap

The bootstrap is a method of statistical resampling that allows uncertainty in the data to be assessed from the the data themselves. The procedure is as follows:

1. draw  $n$  values from the original data set with replacement
2. calculate the required statistic. The required statistic could be any of the common summary statistics. For example we could calculate the uncertainty in the mean from the first set of  $n$  values
3. repeat  $L$  times to build up a distribution of uncertainty about the statistic of interest. For the example above we would find the mean of the  $n$  values  $L$  times yielding a distribution of uncertainty about the mean

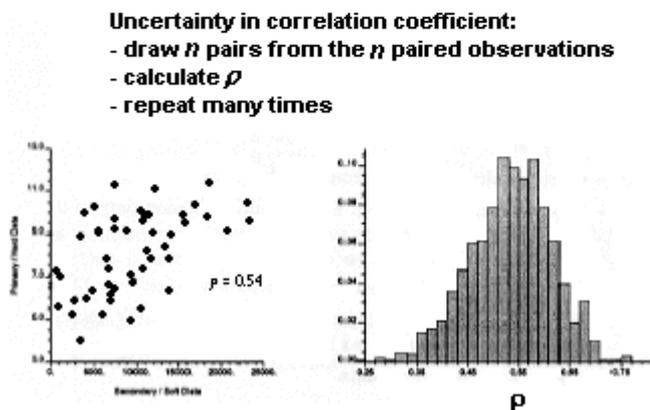
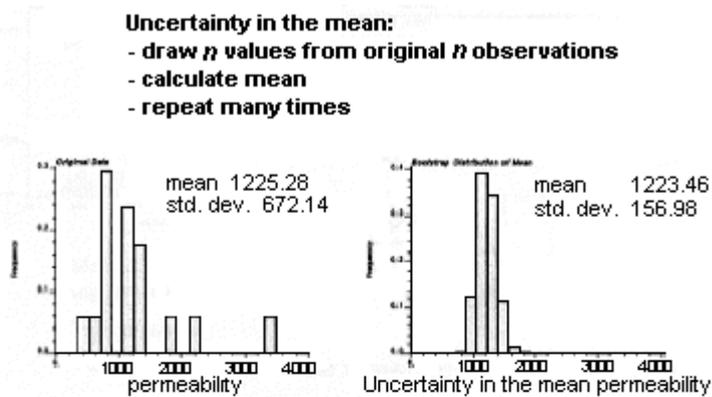


Fig. 2.10 The bootstrap is used to determine the uncertainty in the data itself. This diagram shows how the uncertainty in the mean is found. First randomly draw  $n$  values from the data set and calculate the mean. Repeat this many times, and the distribution of the mean quantifies the uncertainty about the mean.

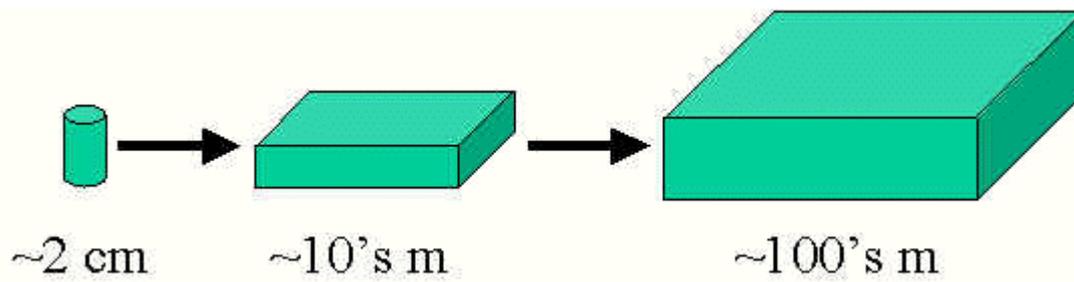
## Geostatistical, and Other Key Concepts

### Petrophysical Properties

There are three principle petrophysical properties discussed in this course: (1) lithofacies type, (2) porosity, and (3) permeability. [Hard data](#) measurements are the lithofacies assignments porosity and permeability measurements taken from core (perhaps log). All other data types including well logs and seismic data are called [soft data](#) and must be calibrated to the hard data. [Deutsch, 1998]

### Modeling Scale

It is not possible nor optimal to model the reservoir at the scale of the hard core data. The core data must be scaled to some intermediate resolution (typical geological modeling cell size: 100 ft \* 100 ft \* 3 ft). Models are built to the intermediate scale and then possibly further scaled to coarser resolutions for flow simulation.



## Uniqueness, Smoothing, and Heterogeneity

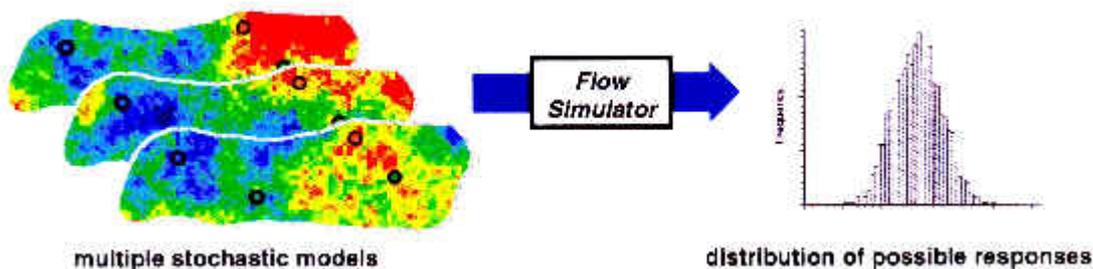
Conventional mapping algorithms were devised to create smooth maps that reveal large scale geologic trends; for fluid flow problems, however the extreme high and low values have been diluted and will often have a large impact on the flow response (e.g. time to breakthrough would be systematically under-estimated during a water flood). These algorithms remove the inherent variability of the reservoir; they remove the heterogeneity within the reservoir. Furthermore, they only provide one unique representation of the reservoir.

## Analogue Data

There are rarely enough data to provide reliable statistics, especially horizontal measures of continuity. For this reason analogue data from outcrops and similar more densely drilled reservoirs are used to help infer spatial statistics that are impossible to calculate from the available subsurface reservoir data.

## Dynamic Reservoir Changes

Geostatistical models provide static descriptions of the petrophysical properties. Time dependent processes such as changes in pressure and fluid saturation are best modeled with flow simulators because they take into account physical laws such as conservation of mass and so on.



## Data Types

The following list represents the most common types of data used in the modeling of a reservoir:

- Core data ( $\phi$  and  $K$  by lithofacies)
- Well log data (stratigraphic surfaces, faults, measurements of petrophysical properties)
- Seismic derived structural data (surface grids / faults, velocities)
- Well test and production data (interpreted  $K$ , thickness, channel widths, connected flow paths, barriers)
- Sequence stratigraphic interpretations / layering (definition the continuity and the trends within each layer of the reservoir)
- Spatial patterns from regional geological interpretation

- Analogue data from outcrops or densely drilled similar fields (size, distributions, and measures of lateral continuity)
- Knowledge of geological processes / principles established through widely accepted theories (forward geologic modeling)

[Deutsch, 1998]

## Numerical Modeling

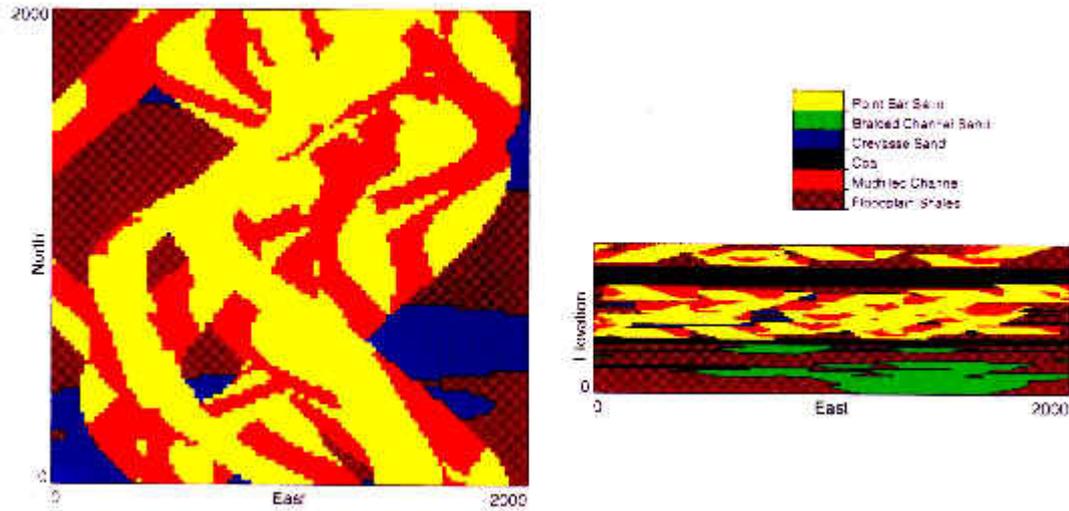
At any instance in geological time there is a single true distribution of petrophysical properties in a single reservoir. This true distribution is the result of a complex succession of physical, chemical, and biological processes. Although some of these depositional and diagenetic processes are understood quite well, we do not completely understand all of the processes and the associated boundary conditions in sufficient detail to provide the unique true distribution. Numerical geological modeling in a reservoir characterization context means arriving at a "gridded" picture of the reservoir, each grid or cell, containing a numerical value.

Reservoir characterization proceeds sequentially: modeling large-scale structures first (i.e. time-boundaries, major faults...), followed by the internal architecture due to lithofacies variations, finishing with petrophysical properties such as porosity and permeability. The first and second step are often the most important.

Lithofacies are considered extremely important in reservoir modeling because petrophysical properties are often highly correlated with lithofacies type. The term facies is perhaps the most frequently abused geologic term, thus making it difficult to define precisely [Hatloy, 1994]. Nevertheless, In this course we will define the term "facies" or "lithofacies" as any subdivision of lithology that is based upon an assessment of the grain size and / or mineralization. In this course we will consider modeling lithofacies in two ways (1) object-based lithofacies modeling, and (2) cell-based modeling:

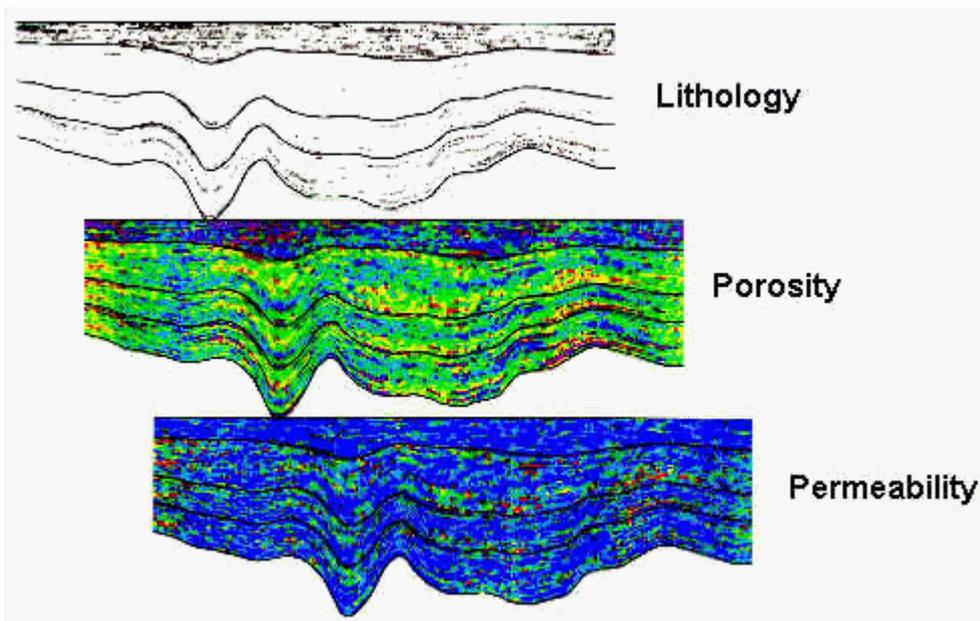
### Object Based Modeling

Object-based modeling considers that lithofacies can be represented by clearly defined geometrical shaped (e.g. channels created by rivers, lobes created by deltas,...). Facies are modeled as parametric objects each having a distribution of parameters for size, extend, etc... The stochastic simulation proceeds by "throwing" these parametric objects into the reservoir model and optimizing their location as to honor all available data. The methods are recommended for fluvial and deltaic environments when net-to-gross ratios are relatively low.



### Cell Based Modeling

On the other hand, cell-based modeling techniques do not consider that lithofacies have discernable shapes. They are applicable in high net-to-gross reservoir, shoreface environments, carbonates, etc. Algorithmically, they proceed in similar fashion to simulation techniques for continuous properties, using correlation functions.



July 6, 1999



## Lecture #2: Basic Concepts, The Exercises

### Question 1a

Using the following data:

0.00	1.20	2.19	3.46	7.38
0.00	1.50	2.55	3.96	13.14
0.00	1.70	2.63	4.21	28.48
0.00	2.13	3.26	6.25	50.64

calculate the population mean, variance (use  $n=20$  not  $n-1=19$ ), median, 25<sup>th</sup> percentile, 75<sup>th</sup> percentile, the interquartile range, and the COV.

### Question 1b

Repeat the above summary statistics first without the zero values (leaving 16 data), then without the outlier (leaving 19 values). Compare with the results from part b to those in part a and comment.

### Question 2

Using the data from question 1 draw the histogram using 5 bins (by hand) and the cdf (by hand). Also, indicate the interquartile range in both figures.

### Question 3

Assume the values in question 1 are really measures of permeability ( $K_i = z_i * 1000$ , ie.  $K_5 = 1.20 * 1000 = 1200$  md). A geologist has interpreted the core samples as follows: lithofacies 1 is shale having a permeability of 0.00 md, lithofacies 2 has permeabilities varying from 1000 md to 2999 md, lithofacies 3 has permeabilities varying from 3000 md to 3999 md, and lithofacies 4 has permeabilities varying from 4000 md and up. Using this information transform the data from question 1 into categorical form and show a categorical histogram including all four lithofacies.

### Question 4

Using the categorical information you derived in question 3 construct a pie chart showing the proportion of each lithofacies.

## **Lecture 2: Basic Concepts, The Quiz**

### **Question 1**

In your own words define random, uncertainty, and Monte Carlo simulation.

### **Question 2**

What does a distribution look like when the coefficient of skewness is; positive?, negative?, zero?. Can you provide sets of petrophysical properties that would present; a positive coefficient of skewness? a negative coefficient of skewness? a low coefficient of skewness?

### **Question 3**

Define the coefficient of variation. What does it mean when the coefficient of variation is greater than one?

### **Question 4**

Provide 2 geophysical properties that are positively correlated, negatively correlated, and not correlated.

### **Question 5**

How would interpret a quantile-quantile plot whose plotted points deviated from the reference line; in a parallel fashion? at an angle from the  $0^{th}$  quantile?

## **Lecture 1: Purpose / Motivation for Geostatistics, The Quiz**

### **Question 1**

List three major benefits that geostatistics offers, and describe what each mean and explain why they are important.

Quantification of uncertainty: summarizes our lack of knowledge for better decision making

Rigorous mathematics: means that there are sound mathematical laws applied for repeatability

Data Integration: data of many types can be integrated using geostatistical tools

### **Question 2**

Decision making in presence of uncertainty is important. What are the two steps for risk-qualified decision making?

Quantification of uncertainty and then quantification of risk.

### **Question 3**

In general terms, explain the link between spatial variability (heterogeneity) and uncertainty.

As spatial variability increases heterogeneity increases and hence uncertainty increases

### **Question 4**

In your own words describe the information effect and how it relates to uncertainty.

The information effect is the result of increased available information which leads to less uncertainty

### **Question 5**

Geostatistics is useful at every point in the life cycle of a reservoir, but where is it most useful and why?

Geostatistics is most important in the early stages of the life cycle because it makes intelligent use of limited data and allows for decision making that is tempered with a knowledge and understanding of the uncertainty inherent in the numerical-geological model.

July 6, 1999

## Lecture 3: Geological Principles for Reservoir Modeling

- [Reservoir Types and Examples](#)
- [Modeling Principles](#)
- [Petrophysical Properties](#)
- [Work Flow](#)
- [Lecture 3 Quiz](#)

Numerical geological models are built in an iterative refining fashion; we model the coarsest features first and revise the model by modeling progressively finer features. Modeling starts with a definition of the reservoir type then significant geological structures followed by petrophysical properties. This lecture will be discussed in the same fashion as a model would be constructed; coarse features followed by finer features. We will start with reservoir types and move onto the relevant structures followed by the modeling of petrophysical features.

### Reservoir Types and Examples

There are two types of reservoirs that we will concern ourselves with (1) siliciclastic reservoirs and (2) carbonate reservoirs. Siliciclastic reservoirs are those reservoirs with sandstone as host rock. Carbonate reservoirs are composed of either skeletal or non-skeletal debris from calcium carbonate secreting organisms. Siliciclastic reservoirs make up about 80% of the worlds known reservoirs and about 30% of the oil production world wide while carbonate reservoirs roughly make up the balance. We will discuss siliciclastic reservoirs first followed by carbonate reservoirs.

### Modeling Siliciclastic Reservoirs

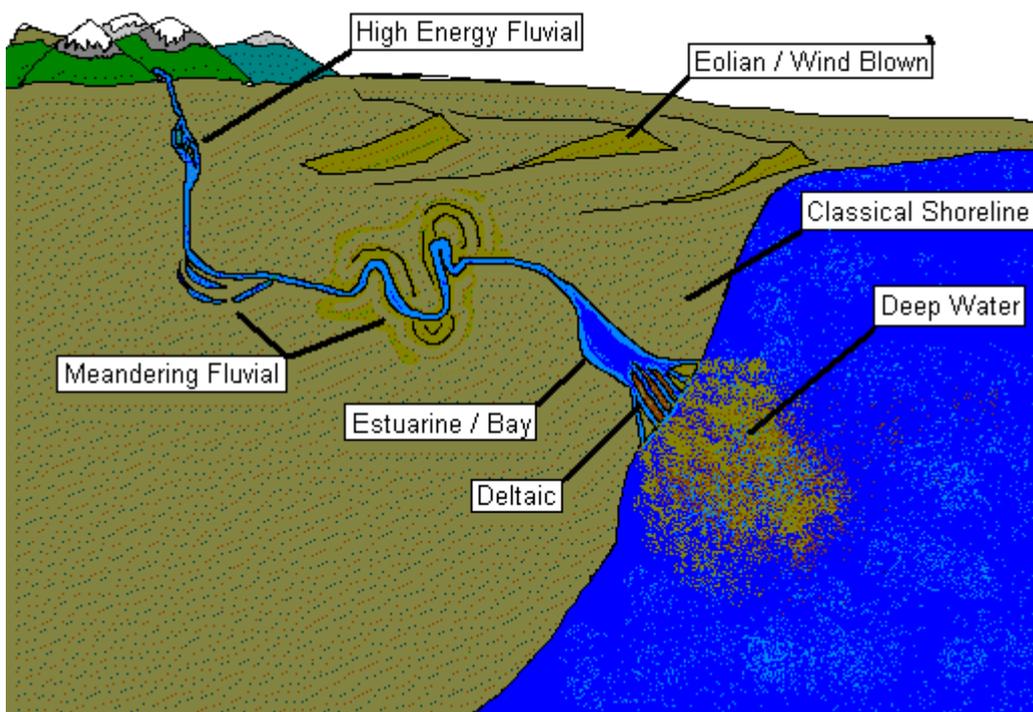


Figure 3.1, Seven different types of siliciclastic reservoirs

Other factors governing the modeling of siliciclastic reservoirs:

- Net to gross ratio. The higher the percentage of reservoir quality sand, the less important it is to model individual objects
- Diagenetic cements. The pore spaces may be filled in or cemented by later deposits.

Table 3.1 summarizes each of the seven different siliciclastic reservoirs that will be covered in this course

<b>Reservoir Type</b>	<b>Characteristic Shapes</b>	<b>Examples / Importance</b>	<b>Modeling Technique</b>
Braided, high energy fluvial	Braided stream channels	Majority of North Sea, excellent reservoirs <i>Important</i>	Object based, unless high N/G, then cell based. Model remnant shales
Eolian, windblown	Overlapping dune shapes, directional	Some in UK North Sea, US and Canada onshore. <i>Less important</i>	Sometimes classic dune shapes modeled as objects
Meandering fluvial	Meandering stream channels	Australia, Saudi onshore <i>Important</i>	Object-based, unless high N/G, then cell-based for remnant shales
Estuarine/ Bay	Tidal channels or bars, sometimes more deltaic	China. <i>May be very important in future</i>	Objects (tidal channels, sand bars), or cell-based
Deltaic	Less ordered, lower energy, fans	Gulf of Mexico Onshore <i>Less Important</i>	Cell-based indicator approach though some clearly defined objects
Classical shoreface	Facies change horizontally: foreshore (beach), upper shoreface, lower shoreface, offshore	East Texas Field, Gulf of Mexico Onshore & Shallow Water <i>Important</i>	Cell-based techniques that can handle ordering: truncated Gaussian or transition probability
Deep water	Turbidites, fans, storm caused slumps—look like braided	Gulf of Mexico, Offshore California	Customized object based or

	channels that evolve into sheets.	<i>Currently very important</i>	hybrid approaches.
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## Carbonate Reservoirs

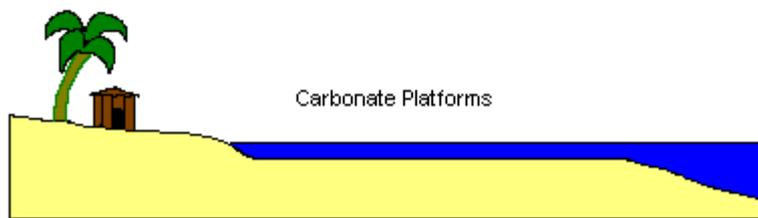
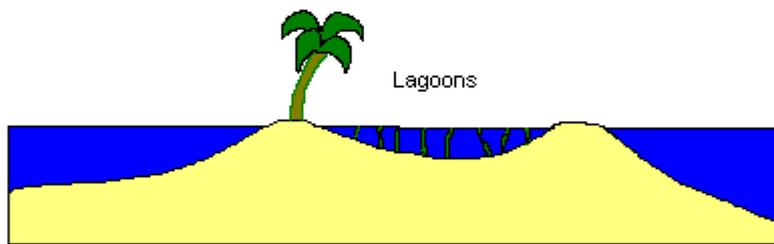
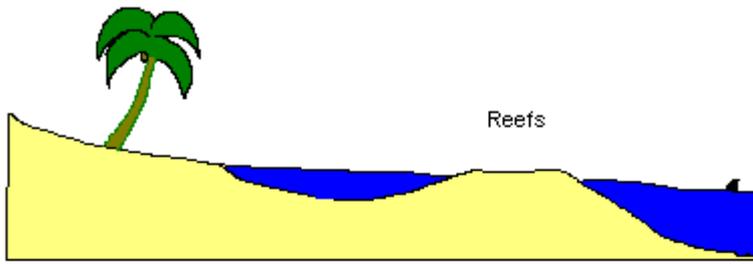
By definition carbonate (limestone) rocks are those have greater than 50% carbonate material. The carbonate material is either derived from organisms that secrete carbonate as skeletal material or as fecal matter, or precipitated out of solution. Limestone is chemically unstable and is easily converted to dolomite when hydrothermal fluids rich in magnesium pass through it. Limestone is also easily metamorphised into other rock types such as marble. Most (90%) of carbonate reservoirs can be modeled using cell based indicator simulation to model limestone / dolomite conversion.

Dolomitization often has a directional trend. Fluid flow in rock is almost always directional and the magnesium required for dolomitization is carried by hydrothermal fluids. The fluid flows through the rock and magnesium replaces calcium creating dolomite. The trends can be seen with seismic (dolomitized limestone has different acoustic properties than limestone). Because there are at least two rock types (limestone and dolostone) we must use estimation methods that make use of multiple variables. Trends such as the conversion of limestone to dolostone may also show up in the geologic contour maps from the wells.

In other cases, reefs may sometimes be modeled as objects, and there may be areal trends associated with these as well. (A change in sea level may cause a reef to die out and another to form further in or out.)

Table 3.2 Table for Carbonate reservoirs

<b>Reservoir Type</b>	<b>Characteristic Shapes</b>	<b>Examples / Importance</b>	<b>Modeling Technique</b>
Carbonate	Elongated Reefs, lagoons, and platforms.	Most of Russia's reservoirs, North Danish Sea  <i>Important now and in the future</i>	Sometimes modeled as objects with areal trends (death and regeneration of reefs), but usually modeled with cells



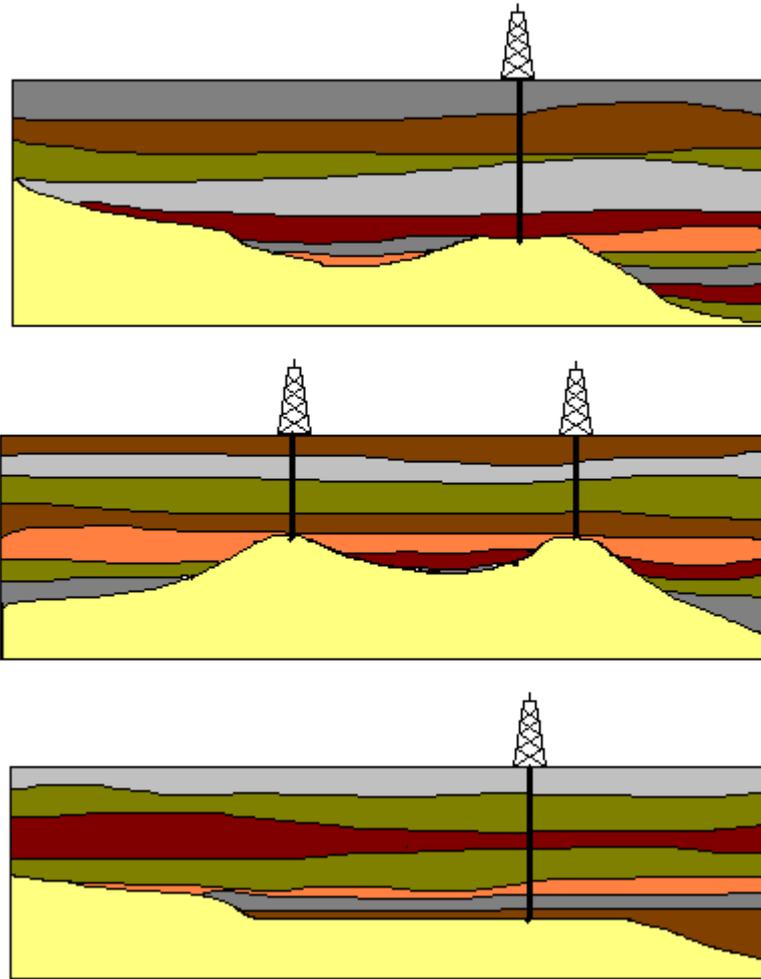


Fig. 3.2 , Different carbonate structures considered in this course

### ***Modeling Principles***

The first step modeling an oil bearing geological structure is to define the topology. The topology defines the the coordinate system, grid dimensions, the orientation of the axis, and the cell dimensions. Standard practice is to use the Cartesian coordinate system. The Cartesian coordinate system defines a location in space by an  $x$ ,  $y$ , and  $z$  coordinate. In terms of notation, geostatistics uses  $\mathbf{u}$  to denote location. The grid dimensions are the maximum and minimum coordinates that the grid must cover to post all of the data. A good first approach is to plot the data points on a location map. In order to do this the minimum and maximum data locations are required so that the extents of the plot can be set. One way to determine these parameters is to plot a histogram and calculate the summary statistics for each of the coordinate axis. The advantage to this approach is that the minimum, maximum, and mean location for each axis is posted allowing you to determine the required parameters for the location map and get a feel for the sampling scheme. The distribution of locations can reveal a biased sampling scheme. Consider the following 2-D example:



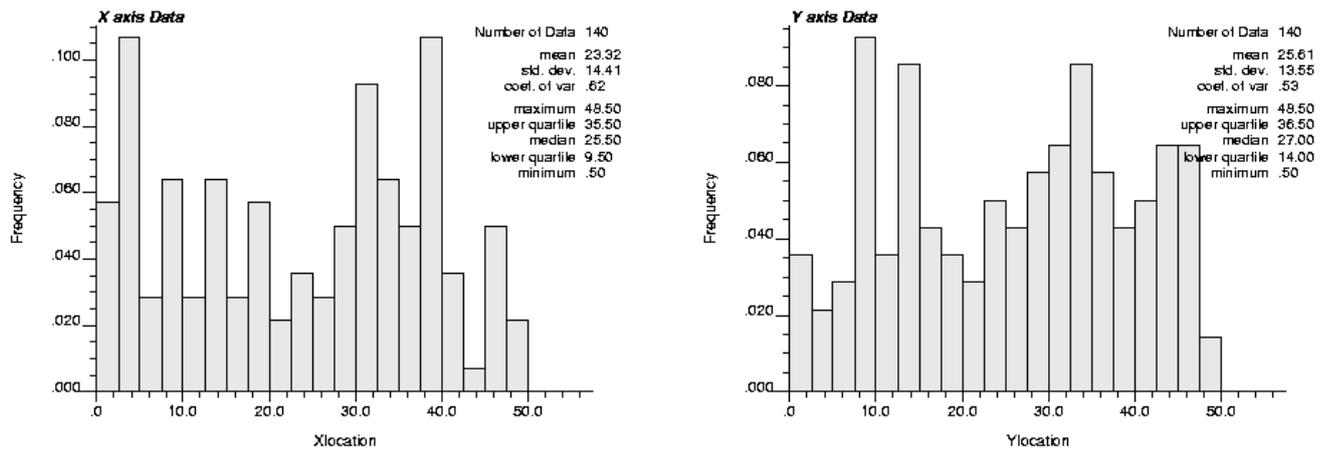


Figure 3.3, A histogram of the X and Y data.

Notice that the x axis data seems well distributed while the y axis data seems a little skewed. This implies that the sampling scheme was a little biased toward the northerly end of the map. The corresponding location map is:

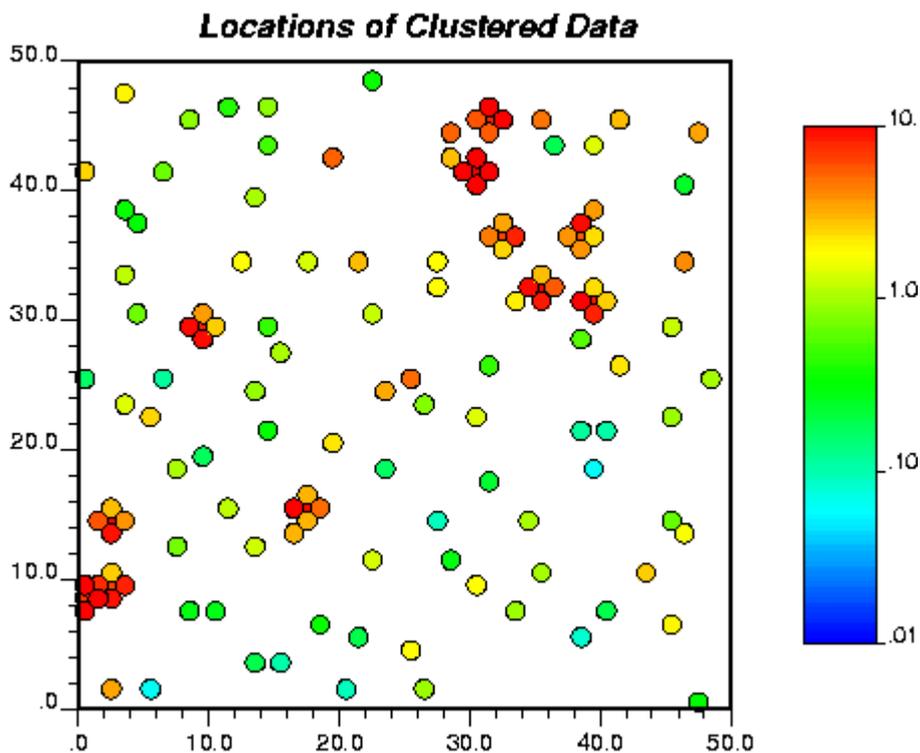


Figure 3.4, A location map of a sample data set.

The assessment of the sampling scheme was correct, there is a northerly bias in the sampling scheme. It is useful to draw a contour map of the data. A contour map helps gain some insight to the nature of the data, and can sometimes reveals important trends. The map below shows that most of the sampling occurred in areas of high potential. The map in Figure 3.5 does not reveal any trends but illustrates the value of a contour map.

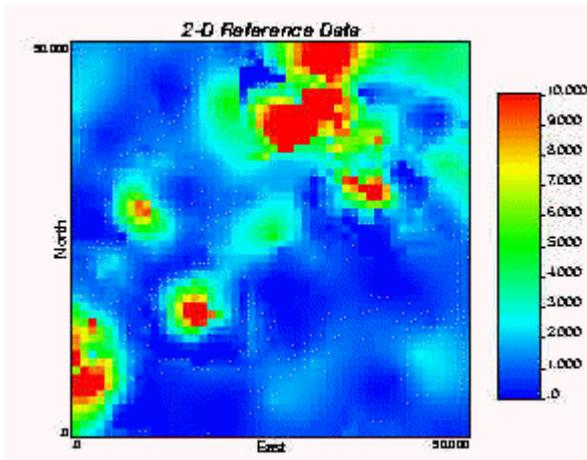


Figure 3.5, A contour map using the sample data set. The accuracy of the map is not critical. Its purpose is to simply illustrate trends.

The contour map illustrates that any areas of high potential (red areas) are heavily sampled; a biased sampling procedure. The contour map also illustrates that we may want to extend the map in the east direction.

It is common practice to use the Cartesian coordinate system and corner-point grids for geological modeling. The corner-point grid system is illustrated in Figure 3.6.

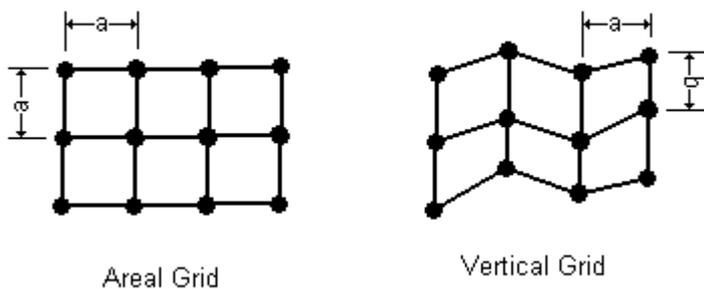


Fig. 3.6 The standard grid system used for geological modeling

Notice that the  $Z$  dimension  $b$  in Figure 3.6 is not the same as the dimension  $a$  in the areal grid, but the  $XY$  dimension for both the areal and vertical grids are the same. For the sake of computational efficiency the stacked areal grids are aligned with  $Z$  axis, but for flexibility the  $Z$  axis need not be of the same dimensions as the areal grid. This technique proves valuable for:

1. modeling the hydrocarbon bearing formation as a stack of stratigraphic layers: It is intuitively obvious that a model should be built layer by layer with each layer derived from a homogenous depositional environment. Although each depositional environment occurred over a large span of time in our context the depositional environment actually occurred for only a brief period of geological time and for our purposes can be classified as a homogenous depositional environment.
2. volume calculations: The model must conform to the stratigraphic thickness as closely as possible. Modeling the formation as a "sugar cube" model leads to poor estimates.
3. flow calculations: Flow nets must have equipotential across facies. A "sugar cube" model would yield erroneous results.

This permits modeling the geology in stratigraphic layers. The stratigraphic layers are modeled as 2-D surface maps with a thickness and are then stacked for the final model. Thus having a non regular grid in the  $Z$  direction allows for conformity to thickness permitting accurate volume calculations, also allows for flow nets (must be equipotential across any face).

Geological events are rarely oriented with longitude and latitude. There is usually some azimuth, dip, or plunge to the formation. If the angle between the formation and the coordinate axis is large there will be error  $a'$  in the cell dimensions as indicated by Figure 3.7. Also, it is confusing to have to deal with the angles associated with azimuth, dip, and plunge, so we remove them and model in some more easily understood coordinate system.

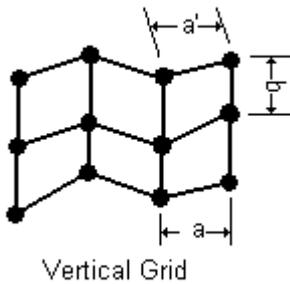


Figure 3.7, Notice that with large deviations in dip that there will be some cell dimension error.

It is common practice to rotate the coordinate axis so that it aligns with the direction of maximal continuity. The direction of maximal continuity can be derived from the contour map. A note about continuity; it is assumed that the direction of maximal continuity is that direction which the formation has the greatest continuity, and the direction of minimal continuity is perpendicular to the direction of maximal continuity. The rotations are performed in two steps. The first step removes the azimuth the second removes the dip. In the event that there is plunge to the formation the procedure for removing dip is repeated. The procedure is illustrated in Figure 3.8. The transform for removing the azimuth is indicated in Matrix 3.1, where  $x_i, y_i$  are the original coordinates,  $x_i^0, y_i^0$  are the translated coordinates for the origin,  $\alpha$  is the azimuth angle,  $x_2, y_2$  are the new coordinates.

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x_1 - x_1^0 \\ y_1 - y_1^0 \end{bmatrix} \quad (3.1)$$

The dip is removed using the transform indicated in Matrix 3.2, where where  $x_2, z_1$  are the coordinates from the first transform (note that the first transform rotates about the  $Z$  axis so there is no change in the original ( $z_i$ )  $z$  coordinate), and  $z_1^0$  is the translated coordinate for the origin,  $\beta$  is the dip angle,  $x_3, z_2$  are the new coordinates. Note that we are now rotating about the  $Y$  axis and thus there is no change to the  $y$  coordinate. Removing the plunge is the same process as removing the dip except that instead of using the  $x$  coordinate the  $y$  coordinate will be used because the axis is rotated about the  $x$  axis.

$$\begin{bmatrix} x_3 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x_2 \\ z_1 - z_1^0 \end{bmatrix} \quad (3.2)$$

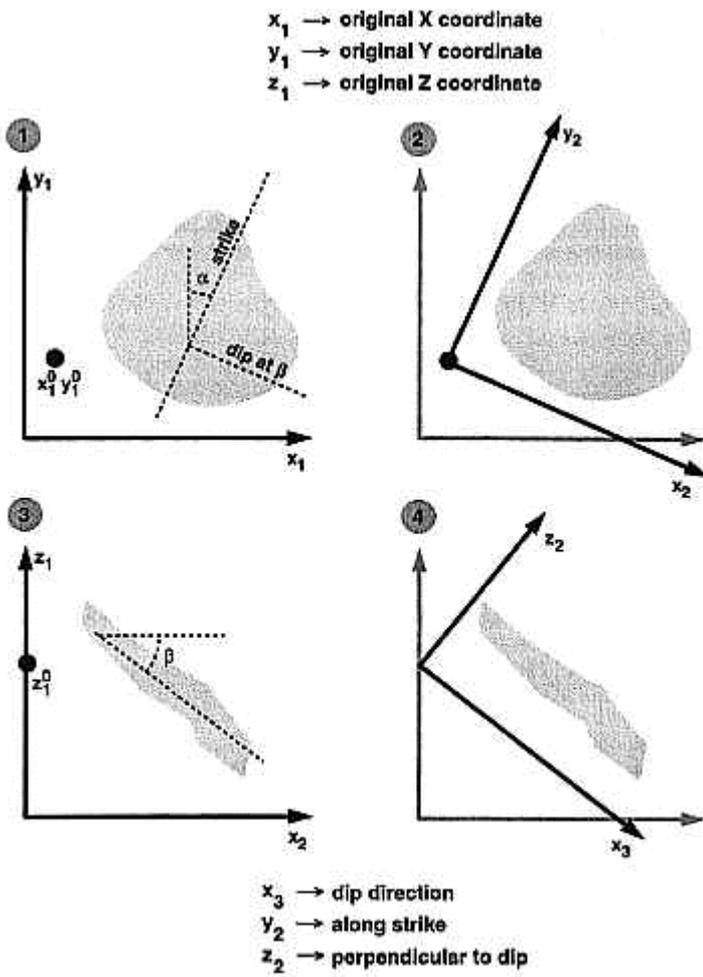


Figure 3.8 Illustrates the process of rotating the coordinate axis to be aligned with the major axis of the reservoir. First the axis are rotated about the z axis to accommodate the the azimuth of there reservoir, second axis are rotated about the y axis to accommodate dip in the reservoir.

The rotations can be removed using the the transforms indicated in Matrix 3.3 and 3.4.

$$\begin{bmatrix} x_2 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} x_1 \\ z_1^0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_1^0 \end{bmatrix} \quad (3.3)$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} x_1^0 \\ y_1^0 \end{bmatrix} \quad (3.4)$$

Sometimes fluvial channels can be difficult to model because they deviate significantly. In this case it is possible to straighten the channel using the transform in Equation 3.5. Figure 3.8 diagrammatically illustrates the straightening transform.

$$x_1 = x - f^c(y) \quad (3.5)$$

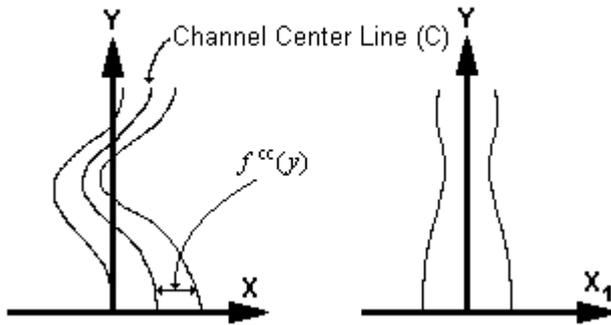


Figure 3.8, Transforming a twisty channel into a straight channel.

### Stratigraphic Coordinates

Reservoirs often consist of stratigraphic layers separated by surfaces that correspond to some sequence of geologic time events, much like growth rings in a tree. The bounding surfaces that differentiate the strata are the result of periods of deposition or periods of deposition followed by erosion. The surfaces are named according to these geologic events:

- **Proportional:** The strata conform to the existing top and base. The strata may vary in thickness due to differential compaction, lateral earth pressures, different sedimentation rates, but there is no significant onlap or erosion (Deutsch, 1999).
- **Truncation:** The strata conform to an existing base but have been eroded on top. The stratigraphic elevation in this case is the distance up from the base of the layer.
- **Onlap:** The strata conform the existing top (no erosion) but have "filled the existing topography so that a base correlation grid is required.
- **Combination:** The strata neither conform to either the existing top or bottom surfaces. Two additional grids are required.

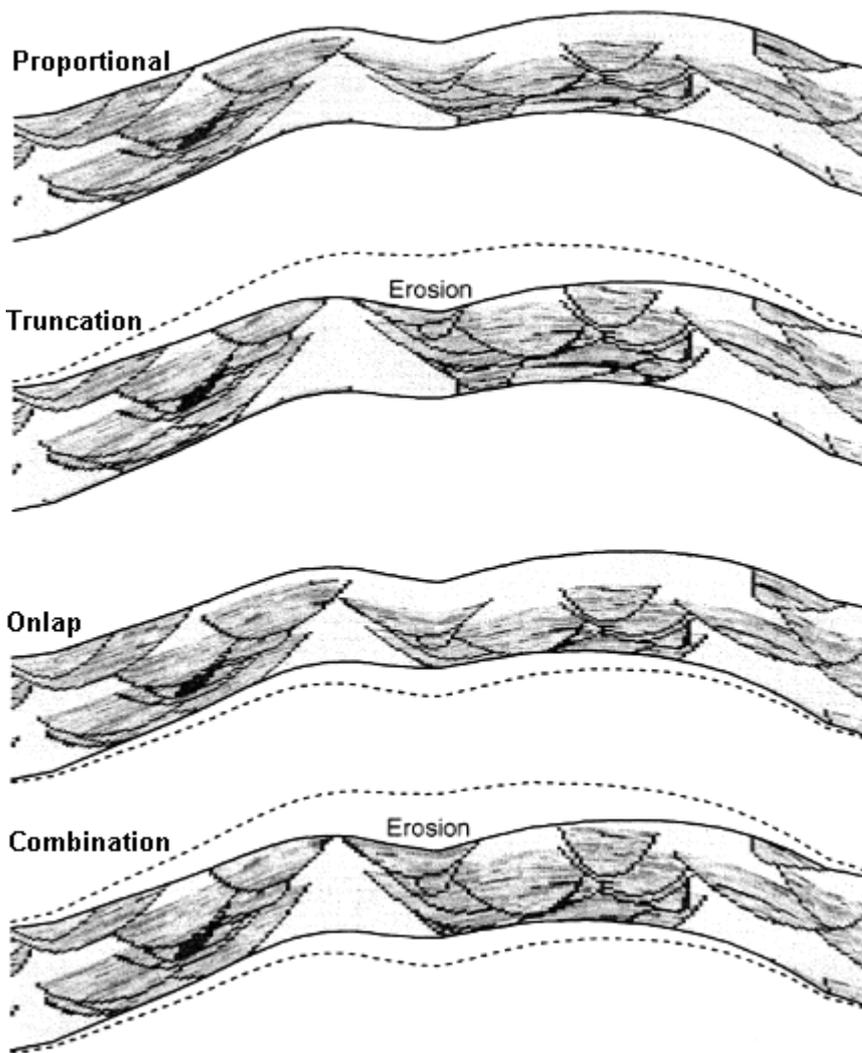


Figure 3.9 Illustrates proportional, truncation, onlap, and combination type correlation surfaces.

The stratigraphic layers must be moved so that they conform to a regular grid. This is done by transforming the z coordinate to a relative elevation using:

$$z_{rel} = \frac{z - z_{rb}}{z_{rt} - z_{rb}} \cdot T \quad (3.5)$$

Figure 3.10 shows how the strata is moved to a regular grid. Note that features remain intact, just the elevation has been altered to a relative elevation.

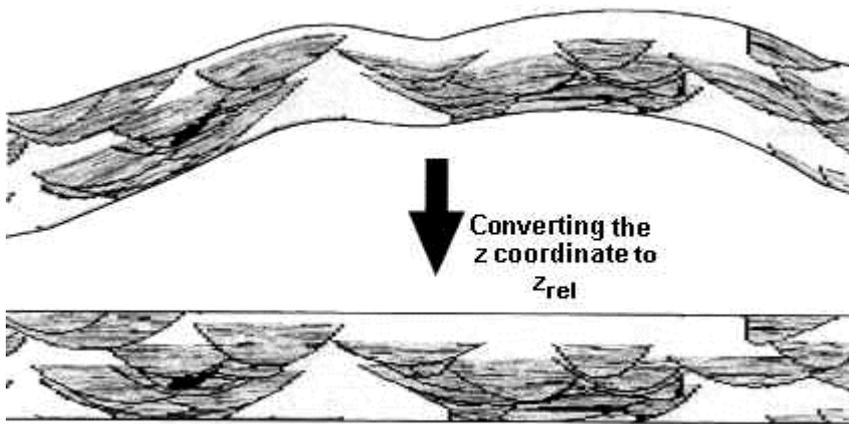


Figure 3.10 Illustrates the result of transferring the z coordinate to a regular grid using formula 3.9.

It is important to note that these are coordinate transforms; the data is transformed to a modeling space and transformed back to reality. There are no property or distance changes here, just the movement from reality to some virtual space then back to reality.

## Cell Size

The cell size used in the model is a serious issue. If the cell size is too small an enormous number of cells will be required to populate the model. Too many cells holds the consequence of having a model that is too difficult to manipulate and very taxing on the CPU. If the cells are too large then important geological features will be removed from the model. As processing power increases model size is of lesser and lesser importance, but with today's computers models that range from 1 million cells to 5 million cells are appropriate.

## Work Flow

The specific process employed for 3-D model building will depend on the data available, the time available, the type of reservoir, and the skills of the people available. In general, the following major steps are required:

1. Determine the areal and vertical extent of the model and the geological modeling cell size
2. Establish a conceptual geological model and define zones for modeling
3. For each zone:
  - a) Define stratigraphic correlation
  - b) Define the number of rock types, the data, and the spatial correlation
  - c) Generate 3-D rock type model
  - d) Establish porosity and permeability values and the spatial correlation
  - e) Generate 3-D porosity models

- f) Generate 3-D permeability models
- g) Merge and translate back to real coordinates
- 4. Verify the model
- 5. Combine zones into a single model

Figure 3.11 illustrates the modeling concepts discussed in this lecture:

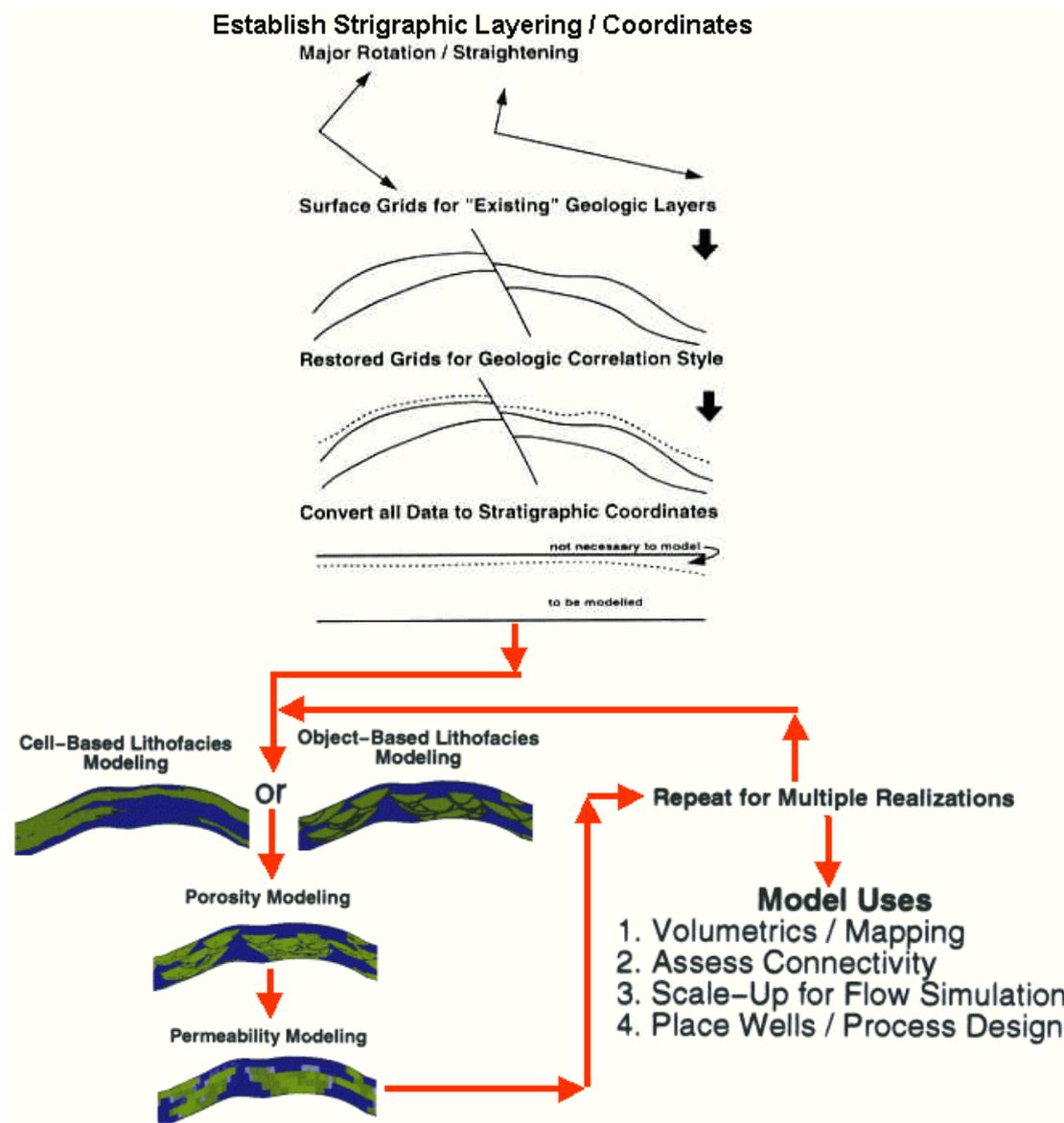


Fig. 3.11 A flow chart showing the geostatistical work flow for Lecture 3.

July 9, 1999



## ***Lecture 3: Basic Geology, The Quiz Solutions***

### **Question 1**

How important are geological constraints in oil reservoir characterization? Should all known geological constraints (on the spatial distribution of petrophysical properties) be incorporated deterministically or left to geostatistics?

### **Question 2**

What are the challenges in object-based modeling of geological facies?

### **Question 3**

Reservoirs are modeled in a hierarchical fashion, summarize the steps.

### **Question 4**

What are the primary factors when considering cell based modeling or object based modeling?

### **Question 5**

Why are most carbonate reservoirs modeled using cell based modeling instead of object based modeling?

[solutions](#)

July 12, 1999

## ***Lecture 2: Basic Concepts, The Quiz***

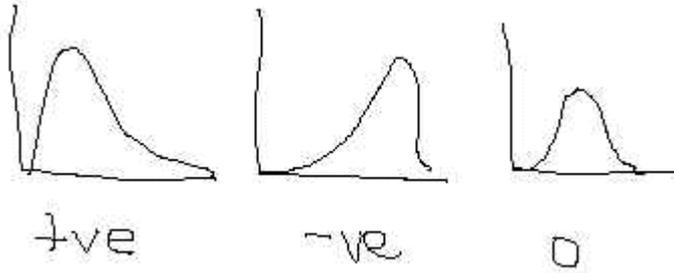
### **Question 1**

In your own words define random, uncertainty, and Monte Carlo simulation.

uncorrelated numbers, lack of assuredness about the truth of a statement or the exact magnitude of a measurement, any procedure that draws a random numbers to arrive at a probabilistic solution.

## Question 2

What does a distribution look like when the coefficient of skewness is; positive?, negative?, zero?. Can you provide sets petrophysical properties that would present: a positive coefficient of skewness? a negative coefficient of skewness? a low coefficient of skewness?



, a) permeability, b) water

saturation, c) porosity

## Question 3

Define the coefficient of variation. What does it mean when the coefficient of variation is greater than one?

$$CV = \frac{s}{m}$$

there are a lot of outliers

## Question 4

Provide 2 geophysical properties that are positively correlated, negatively correlated, and not correlated.

+ve: porosity and permeability

-ve: impedance and porosity

not: permeability and the weather

## Question 5

How would you interpret a quantile-quantile plot whose plotted points deviated from the reference line; in a parallel fashion? at an angle from the 0<sup>th</sup> quantile?

a parallel deviation indicates a difference in the mean, and a change in slope indicates a difference in variance

July 6, 1999

## **Lecture 4: Data Analysis**

- [Data Analysis](#)
- [Inference](#)
- [Declustering](#)
- [Trends](#)
- [Reconciliation of Data](#)
- [Soft Data Declustering](#)
- [Work Flow](#)
- [Lecture 4 Quiz](#)

### **Data Analysis**

Data analysis is the gathering, display, and summary of the data. Data analysis is an important step for building reliable numerical models. Important features of the data are realized, erroneous data, and outliers are revealed. The issues addressed by a Data Analysis are:

- Understanding and cleaning the data
- Global resources assessment: first order / back-of-the-envelope calculations of volumetrics, facies/zone summary statistics, used to estimate and confirm magnitude of expected results
- Identification of geological populations
- Identification of geological trends
- Reconciliation of different data types: e.g. transform log-derived porosity measurements to match the core-derived data
- Inference of representative statistics / distributions
- Calibration of soft data

### **Inference**

In recent years the growth of Geostatistics has made itself felt more in the petroleum industry than any other, and an important feature of this growth is the shift in philosophy from deterministic response to stochastic inference. Stochastic inference concerns generalizations based on sample data, and beyond sample data. The inference process aims at estimating the parameters of the random function model from sample information available over the study area. The use of sample statistics as estimates of the population parameters requires that the samples be volume / areally representative of the underlying population. Sampling schemes can be devised to ensure statistical representativity, but they are rarely used in reality. It is up to the geoscientist to repair the effect of biased sampling, integrate data of different types, cope with trends, etc., and in general ensure that the data truly is representative of the population.

### **Outliers and Erroneous Data**

Erroneous data and outliers can affect the summary statistics and subsequently the geostatistical model. Therefore it is sometimes necessary to remove or modify outliers. This section discusses how one finds outliers what constitutes an outlier. The following steps indicate one strategy for looking for outliers.

- Plot a histogram of the data and zoom in on extreme values.
- Perform summary statistics on the global statistics with and without the extreme values.
- Plot the cdf and examine extreme values
- Look at the probability plot for extreme values

- Search the location map for the outliers. Do they appear to be all in the same location? Do they appear to be inappropriate?
- Show a cross plot of the local averages versus the data (every point is mapped versus the average of the surrounding data).



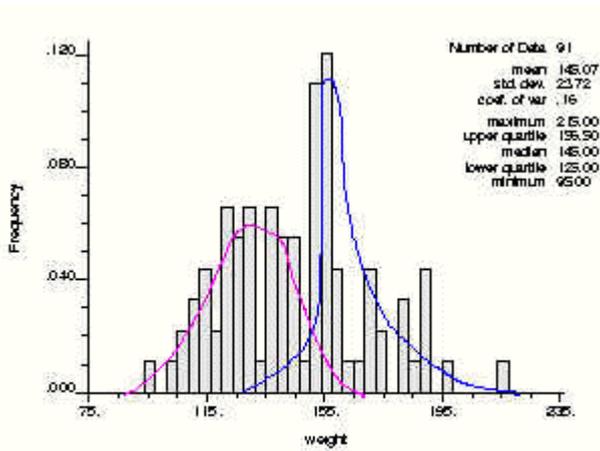
Figure 4.0a, a mean that differs from the mode significantly, a maximum that is significantly higher than the mean, or even a posting that sticks out indicates the presence of outliers.

Figure 4.10 shows some of the things that indicate an outlier. A mean that deviates significantly from the median, a maximum that deviates significantly from the mean or the median, or a single posting way out in the middle of nowhere. There are three possible solutions for coping with outliers (1) we can decide to leave them as they are, (2) we can remove them from the data set, (3) or we alter the value to something more appropriate to the surrounding data. Each solution is left to professional judgement.

## Lumping or Splitting Populations

Sometimes it is strategically wise / unwise to lump all of the data together as a single data set. When performing a "back of the envelope" reservoir characterization it is wise (in some cases) to lump all of the data together. In the case of a full blown geostatistical study with multiple distinguishable lithofacies it is not wise. The question is how do we decide when to lump and when not to lump? The following points illustrate one strategy for determining when and when not to lump:

- Plot a histogram and look for multiple modes (peaks)
- Examine a probability plot and look for kinks or breaks in the distribution
- Plot condition histograms for each of the suspect distributions and overlay for comparison
- Look at the spatial location of the suspect distributions. Is it feasible that there is more than one distribution?



**Two Different Distributions within a single data set.**

Figure 4.0b, Should two distributions be separated? It depends on the study.

Decisions made in the geostatistical study must be backed up by sound practice and good judgement. The strategies indicated only serve as an aid in justifying your decisions you must also document your decisions; you know when you may need to justify them.

**Declustering**

Earth science data is rarely collected with statistical representativity in mind. Data are often taken preferentially from areas that are high or low in value. There is nothing wrong with collecting data in this fashion, one would like prefer to have areas of high or low value delineated. Unfortunately this sampling practice leads to location biased sampling. Figure 4.1, the location map of the sample data illustrates location biased sampling. The areas of low potential are not as well represented as the areas of high potential.

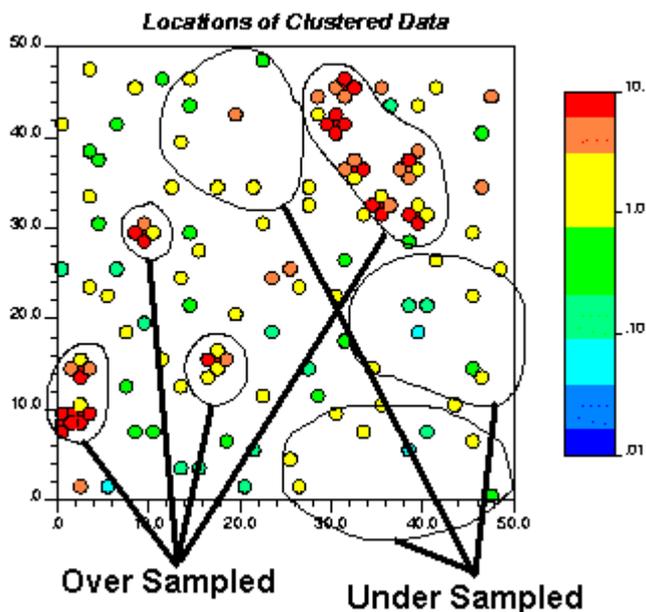


Figure 4.1, Note that all areas are not sampled unbiasedly. Some are areas are heavily sampled and others are poorly sampled.

Declustering corrects the distribution for the effect of location-biased sampling. Declustering assigns a weight to each data and calculates the summary statistics using each weighted data.

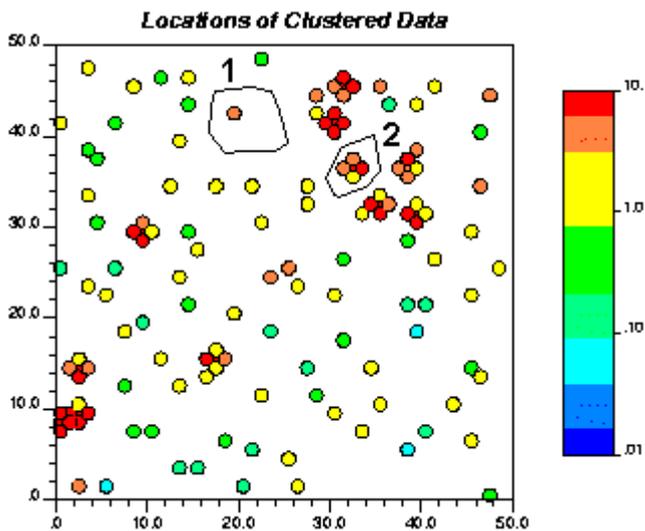


Figure 4.2, The single data in Area 1 informs a much larger area than the 5 data in Area 2.

Figure 4.2 illustrates location-biased sampling. The single data in Area 1 informs a larger area than the 5 data of Area 2. Intuitively one would weight each of the data in Area 2 by one fifth and the data in Area 1 by one. Calculating the weights this way is effective but a more efficient way is to overlay a grid and weight each data relative to the number of data in the cell area defined by the grid using Function 4.1 below:

$$w_i^{(c)} = \frac{1}{n_i \cdot L_0} \quad (4.1)$$

where  $w_i^{(c)}$  is the weight,  $n_i$  is the number of data appearing in the cell, and  $L_0$  is total number of cells with data. Figure 4.3 shows how geometric declustering, or cell declustering works.

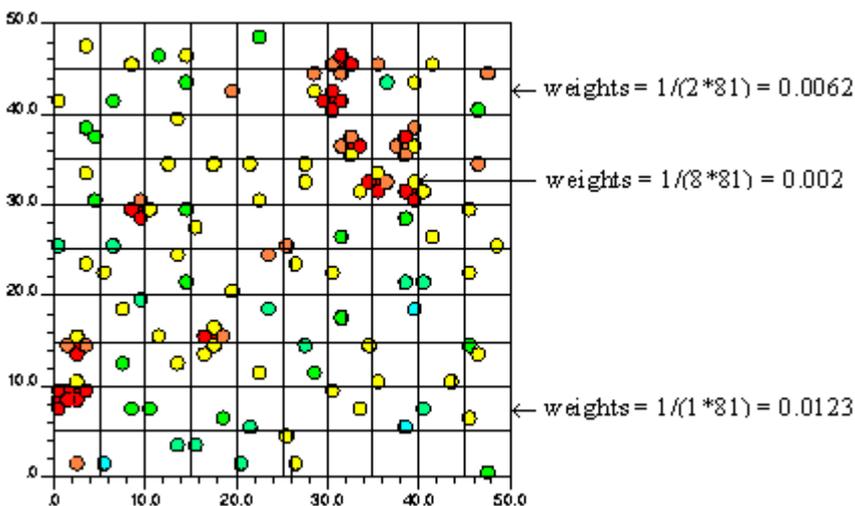
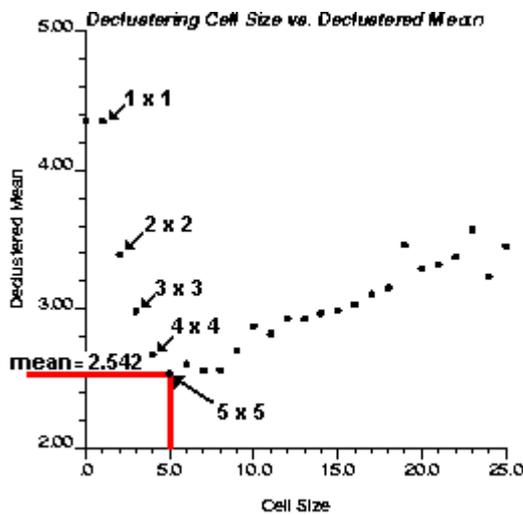


Figure 4.3 The declustering weights for three different cells has been calculated, all other declustering weights are found in the same way.

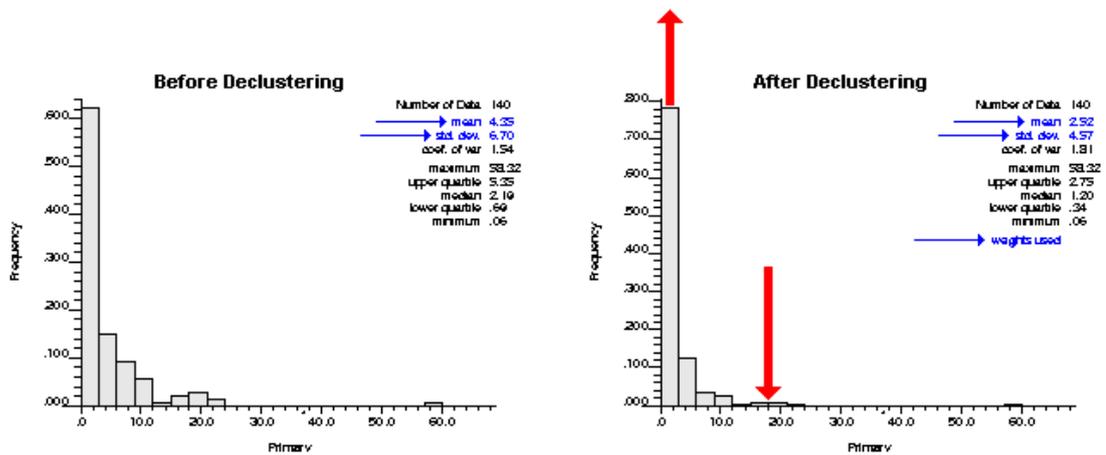
The declustering algorithm can be summarized by:

1. choose an initial declustering cell size that includes about one datum per cell and calculate the mean. The goal to finding the optimal declustering cell size is to find the lowest mean for data sets where the high potential areas have been over sampled or the largest mean for data sets where the low potential areas have been over sampled, and there are two parameters that can effect the declustered mean (1) the declustering cell size and the location of the grid origin. To ensure that the optimal declustering cell size is chosen several cell sizes and grid origin locations should be used to calculate the mean. The cell size that yields the lowest / largest mean is chosen as the optimal declustering cell size. Figure 4.4 shows the declustered mean and declustering cell size using 24 different cell sizes between 1 and 25 units and 5 grid offsets. The posted declustered mean is the average of the mean calculated from 5 grid offsets.

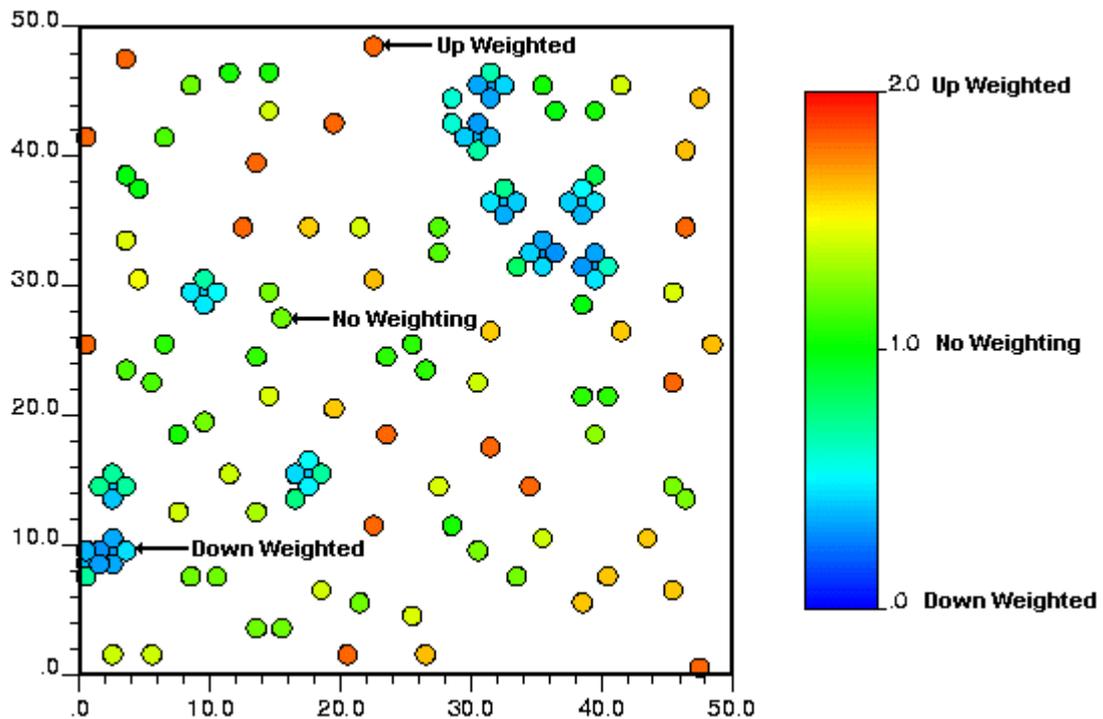


2. Figure 4.4, The scatter plot shows the declustered mean for a variety of grid sizes. The minimum declustered mean is the lowest mean. The declustering cell size is a 5 x 5 cell yielding a declustered mean of 2.542
3. Using the optimal declustering cell size decluster the data. Recall that step 1 only determines the optimal declustering cell size, it does not decluster the data. Figure 4.5 shows histograms before and after declustering. Notice that the low values (the poorly represented values) are now more represented and the high value (the overly represented values) are less represented in the declustered histogram. Also notice that the declustered mean and variance are lower than the clustered statistics. Figure 4.6 is a map showing the magnitude of weight applied to each data. Note the clustered data are under weighted, the sparse data are over weighted, and the well spaced data are not weighted.





4. Figure 4.5, Histograms of the data before and after declustering. Points to note are the clustered mean is larger than the declustered mean, which is the expected result, and the clustered variance is larger than the declustered variance, also an expected result.



5. Figure 4.6, A map of the magnitude of weight. Notice that sparsely sampled areas are up weighted, clustered areas are down weighted, and well spaced are not weighted.

## Soft Data Declustering

Geometric declustering is the preferred method of declustering, but what should be done when there are too few data to decluster, or when there is systematic preferential drilling? One method is to use a soft data set to develop a distribution for the hard data in locations where there is no hard data, only soft data. Data is weighted by the value of the secondary variable at its location. Soft data declustering requires:

- Gridded secondary variable, that is defined over the area of study
- Calibration cross-plot
- Mathematical manipulation

Consider a reservoir that has a porosity known to be negatively correlated with depth, such as that indicated in Figure 4.7. The wells were preferentially drilled along the crest of the structure, and a model of the reservoir is to be built. Clearly the well data will not represent the reservoir unbiasedly, but seismic data has been collected over the entire area of interest.

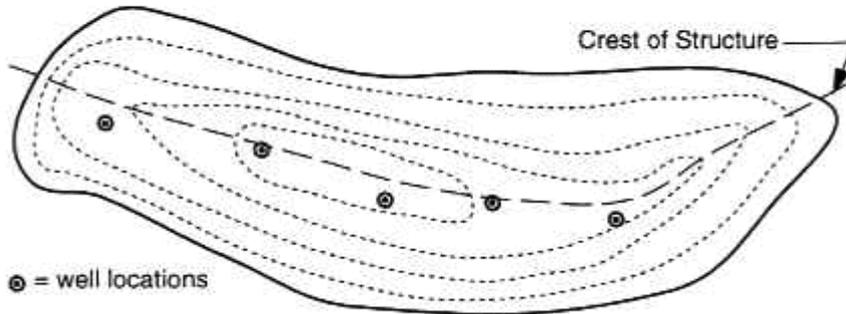


Figure 4.7, A location map showing the locations of well and seismic data. The well data is not representative of the entire reservoir and there is very little data to decluster, thus soft data declustering is used.

The protocol for soft data declustering is:

- Map the secondary variable over the area of interest
- Develop a bivariate relation between the primary and secondary data, such as that in Figure 4.8a. The calibration is critical to this operation and great care must be exercised in developing it.
- construct a representative distribution by accumulating all of the conditional distributions. Figure 4.8b illustrates the concept of adding distributions to create a single distribution.

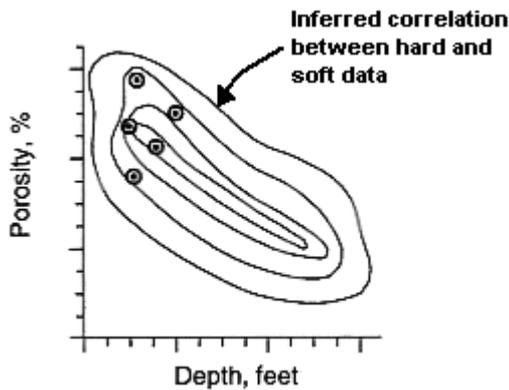
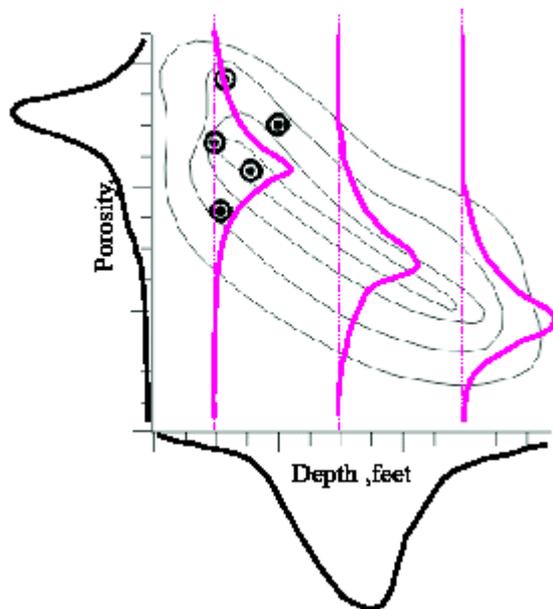


Figure 4.8a, A scatter plot of the hard porosity data versus the seismic depth data. The inferred correlation between the hard and soft data is shown. The relation is inferred from analogue data and the data themselves.



## Trends

Virtually all natural phenomena exhibit trends. Gravity works; vertical profiles of permeability and porosity fine upward within each successive strata (Deutsch, 1999). Since almost all natural phenomena exhibit a trend it is not always appropriate to model using a stationary RV. Reducing the size of the study area to a size where the assumption of stationarity is appropriate or reducing the assumption of stationarity to the search radius are two methods for coping with trends. Universal kriging, an adaptation of ordinary kriging system produces good local estimates in the presence of trend. Universal kriging can also be used to calculate a trend automatically, but its use should be tempered with good judgement and sound reasoning instead of just accepting the result. The best method for coping with a trend is to determine the trend (as a deterministic process) subtract it from the observed local values and estimate the residuals and add the trend back in for the final estimate. (Mohan, 1989). Often it is possible to infer areal or vertical trends in the distribution of rock types and/or petrophysical properties, and inject this deterministic information into the model.

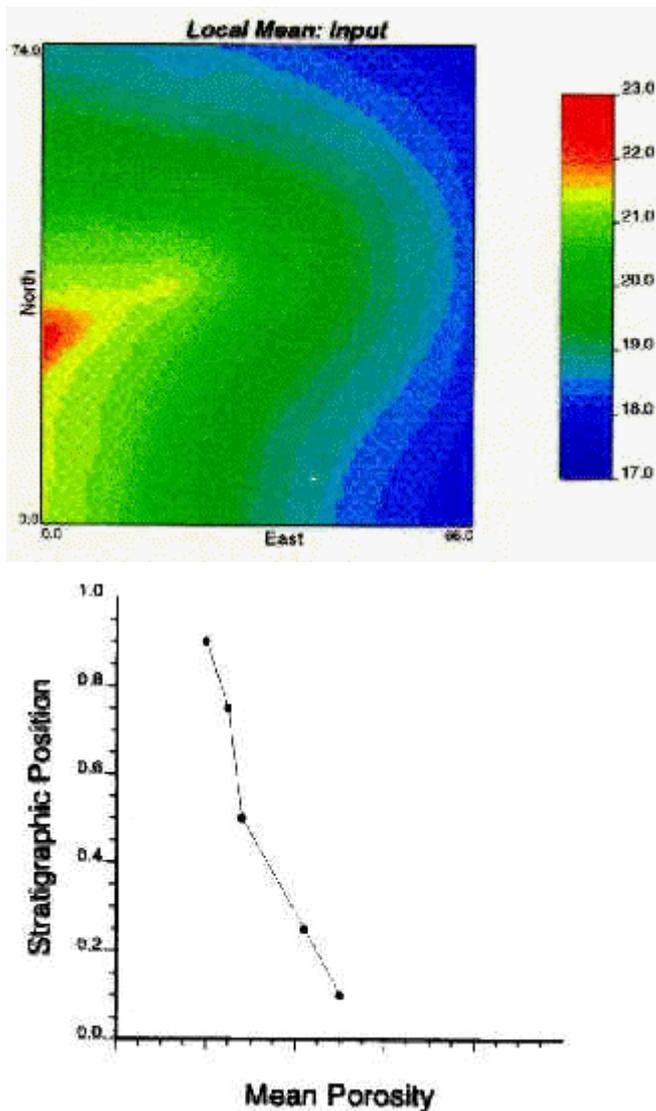


Figure 4.8a, Trend mapping is important for at least two reasons: (1) it is wise to inject all deterministic features and a trend is a deterministic feature, and estimation requires stationarity and a trend implies that there is no stationary mean. Thus, remove the trend and estimate the residual.

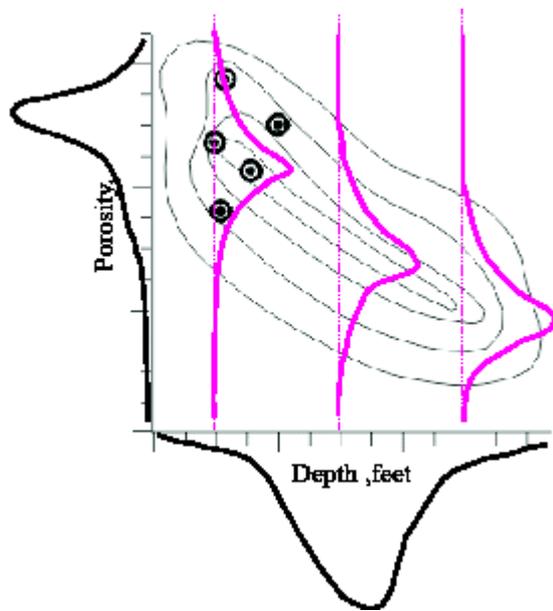
## Calibration of soft Data

Soft data calibration is performed in much the same way as declustering using soft data, but the goal is somewhat different. The goal here is to derive meaningful information from soft data (seismic or log data) that may not directly provide information about a petrophysical attribute. For example core porosity measurements can be calibrated to seismic data. The result is a fortified distribution of porosity for all map locations. The algorithm is as follows:

- Map the secondary variable  $X$  at all locations
- Develop a bivariate relationship between  $X$  and the  $Y$  variable of interest
- generate a distribution of  $Y$  by combining the conditional distributions:

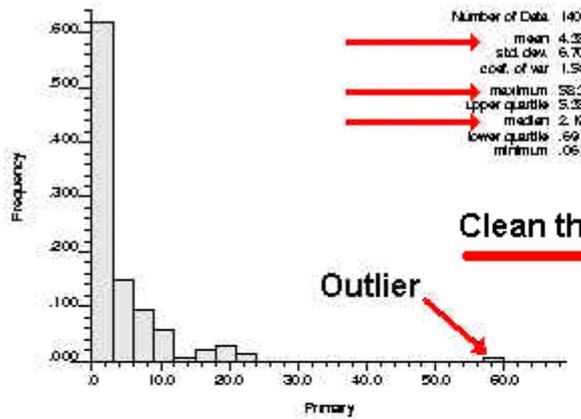
$$f_Y^*(y) = \sum_{\mathbf{u} \in A} \frac{1}{C} f_{Y|X}(y | X = x(\mathbf{u}))$$

, where  $c$  is a normalizing constant.



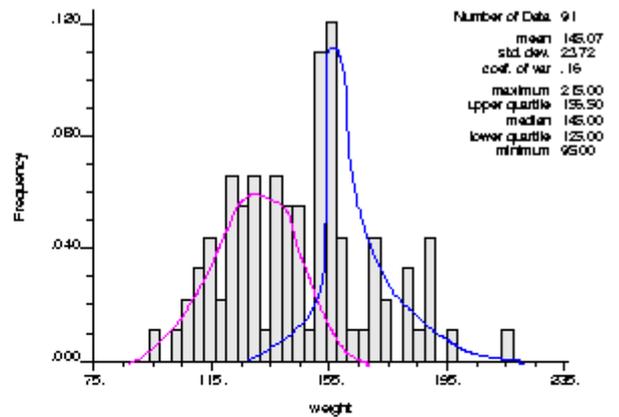
As with using soft data for declustering, the calibration is difficult to infer. This methodology, however, is a useful empirical calibration approach to a difficult problem and it is better than ignoring the problem.

# Work Flow

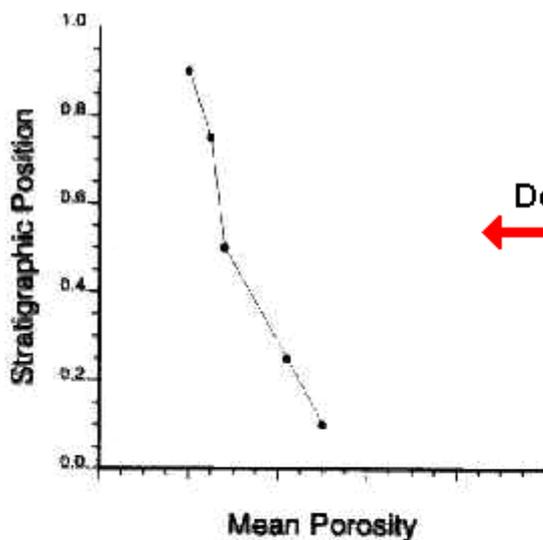


Clean the data

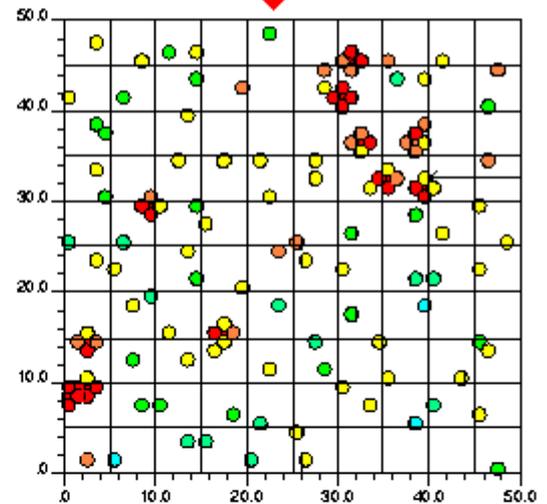
Outlier



Separate the data (if necessary)



Declustering



Trend Analysis

Soft Data Calibration → Variograms

July 10, 1999

## ***Lecture 4: Data Analysis, The Quiz***

### **Question 1**

Briefly explain the principle of declustering? Why is declustering important in geostatistical oil reservoir characterization?

### **Question 2**

Should all outliers be automatically be removed? Why or why not?

### **Question 3**

What tools you use to split data sets into different facies? How would you proceed?

### **Question 4**

Why bother using soft data to decluster hard data?

### **Question 5**

When we speak of inference in the context of data analysis what are we striving for? Is it important? Why?

## **Lecture 4: Data Analysis, The Quiz Solutions**

### **Question 1**

Briefly explain the principle of declustering? Why is declustering important in geostatistical oil reservoir characterization?

Weighting the data so that each is volume representative of the volume of interest. It is important because we require that the data used is as representative of the reservoir as possible. Garbage in garbage out.

### **Question 2**

Should all outliers be automatically be removed? Why or why not?

No, not all outliers are bad. You must refer to the location of the data. Is it surrounded by other high ranking data? Is it sitting out in the middle of nowhere? Is it surrounded by data that would imply that there should be a low / high value instead of what is there?

### **Question 3**

What tools you use to split data sets into different facies? How would you proceed?

Histograms, scatterplots, quantile-quantile plots, location (base maps), contour maps. Start with a location map and a contour map. Separate those data that appear to show trends or directional continuity. look at the histogram, do the trends and the peaks (indicating different distributions) correspond to those locations on the maps? once separated, look at the scatterplots. Are the data sets well correlated?

### **Question 4**

Why bother using soft data to decluster hard data?

We know that the data is not representative and it must be declustered. If we ignore the problem then we are building models that are not accurate, and there is no way to tell how inaccurate. Soft data declustering may seem crude but it is much better than ignoring the problem.

### **Question 5**

When we speak of inference in the context of data analysis what are we striving for? Is it important? Why?

We are striving for summary statistics that are as representative as possible. It is very important. The entire estimation / simulation process relies on an accurate representation of the reservoir.



## Lecture 5: Spatial Data Analysis

1. [Variograms](#)
2. [Variogram Interpretation and Modeling](#)
3. [Lecture 5 Quiz](#)
4. [Emmanuel Gringarten's Paper on Variogram Analysis](#)

### Introduction

Geostatistics focuses on natural phenomena which are correlated in space, a feature of all natural phenomena. Geostatistical modeling requires quantitative measures of spatial correlation for estimation and simulation, and the most commonly used tool for measuring spatial correlation is the semivariogram. For variogram calculation it is essential to work with data that are free of outliers, trends, and are oriented in an appropriate coordinate system. The lecture will begin with a qualitative look at the variogram, followed by the parameters of the variogram, then a quantitative look at variograms, and conclude with variogram interpretation and modeling.

### Variograms

We will start with a qualitative understanding of variograms. What is important here is to gain an understanding of the concept of spatial correlation. Recall that a variogram is chart that converts distance to [correlation](#). Figure 5.1 shows an [experimental variogram](#). Scatterplot 5.1a in Figure 5.1 shows that at short distances correlation is high. Scatterplot 5.1b shows that as distance increases the correlation decreases, and scatterplot 5.1c shows at some distance there is no correlation among the data.

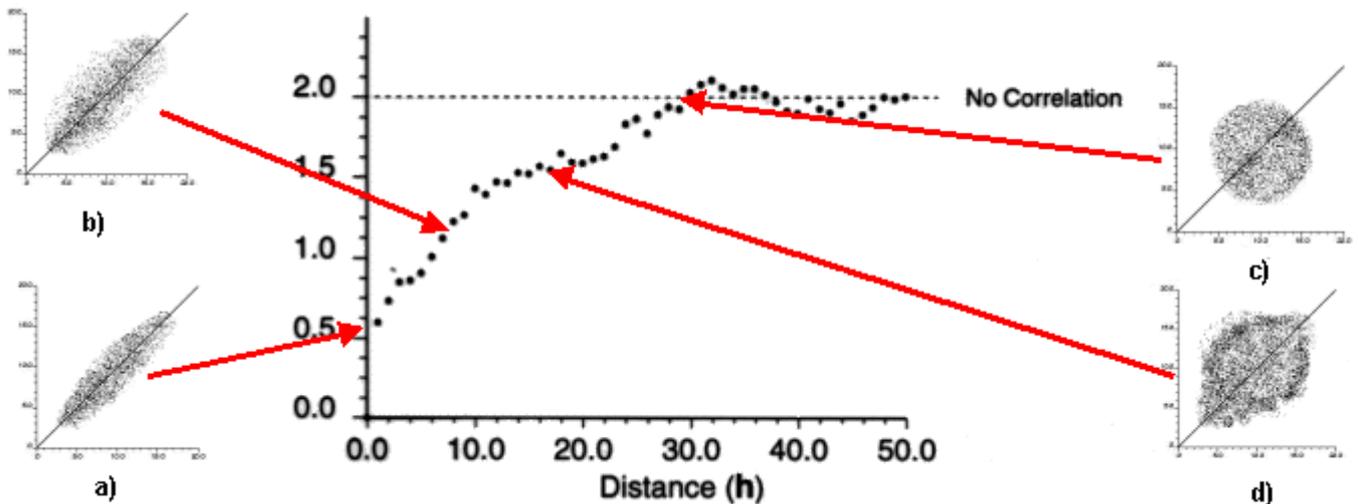


Figure 5.1 Each point in the experimental variogram relates to the scatterplot of two data separated by a distance  $h$

Consider the variogram and the resulting maps in Figure 5.2 below:

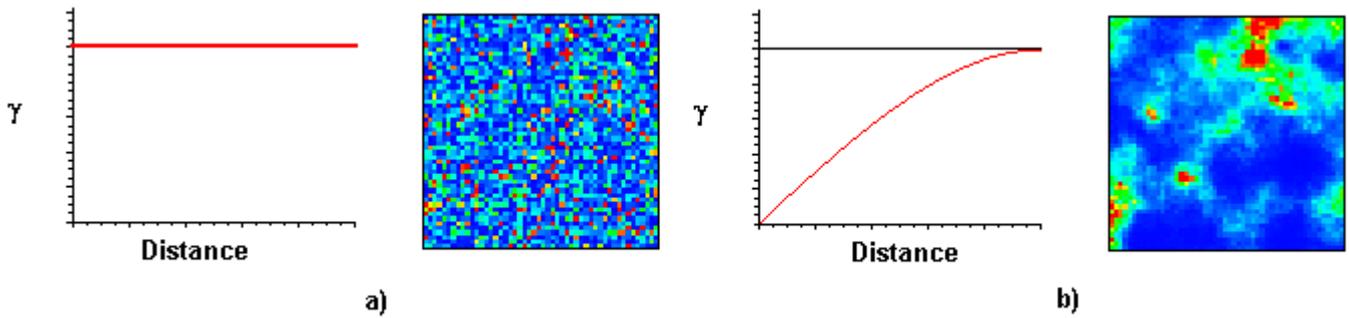


Figure 5.2 Two variograms and the corresponding maps. The variogram on the right shows no spatial correlation and the resulting map is random. The variogram on the right is very continuous showing extensive spatial correlation and the relevant map shows good spatial correlation.

Figure 5.2 shows a map that was made using the variogram 5.2a. The variogram indicates that the data have no correlation at any distance, and hence image *a* is a random map. Image 5.2b was made using variogram 5.2b. Variogram 5.2b indicates that the data are well correlated at long distances and image 5.2b shows some correlation at long distance. Figure 5.3 shows a close up the two images in Figure 5.2. Notice that in figure 5.3b the colors gradually change from blue to green to orange then red, and that this is not the case in figure 5.3a. In Figure 5.3a the colors change randomly with no correlation from one pixel to the next. A pixel in image 5.3a is not well correlated to a neighboring pixel, whereas in image 5.3b neighboring pixels are well correlated.

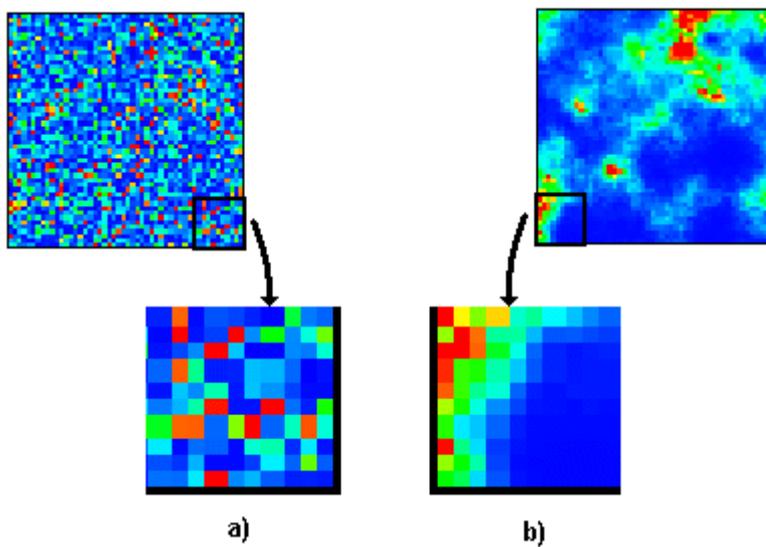


Figure 5.3 A close up of an area on each map shows that map a using the variogram having no spatial correlation is random whereas the map on the right which used a variogram that is continuous and thus the map shows spatial correlation.

Correlation is the characteristic of having linear interdependence between random variables or between sets of numbers. Between what variables is the variogram measuring correlation? In the variograms presented so far, correlation is being measured between the same variable, but separated by a distance approximately equal to  $h$ . Figure 5.4 shows conceptually how an experimental variogram is calculated. The lag distance or distance  $h$  is decided upon by the practitioner. The two variables are (1) the data at the head of the vector, and (2) the data at the tail of the vector. The data tail of the vector (the circled end in figure 5.4) is called  $z(\mathbf{u})$  (the random variable at location  $\mathbf{u}$ ) and the data at the head of the vector is called  $z(\mathbf{u}+\mathbf{h})$  (the random variable at location  $\mathbf{u}+\mathbf{h}$ ). Starting with the smallest lag distance the algorithm visits each data and determines if there are any data approximately one lag away. If there are, the algorithm computes variogram value for one lag. After each data has been visited, the

algorithm doubles the lag distance and repeats the calculation. In this way the experimental variogram quantifies the spatial correlation of the data.

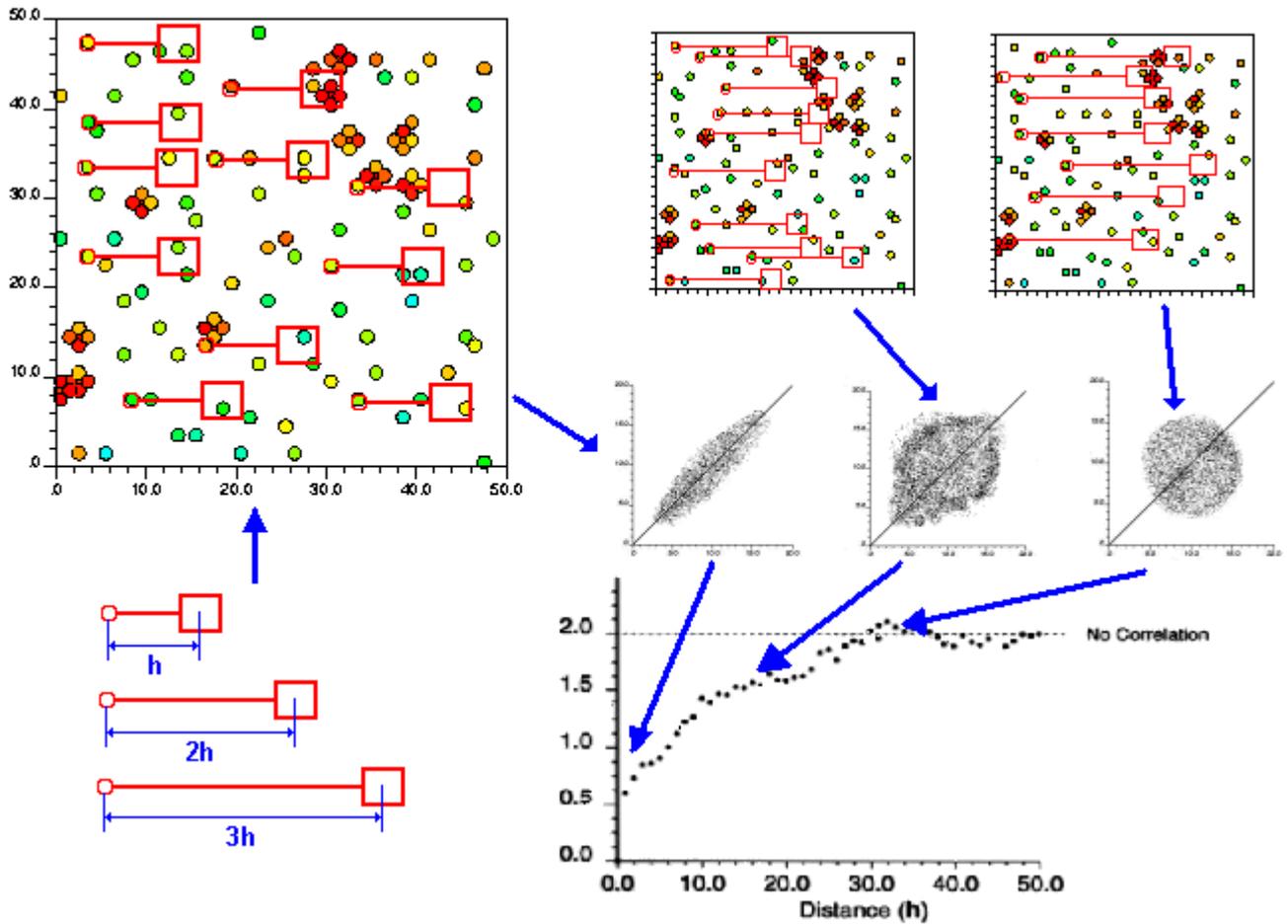


Figure 5.4 The variogram is not calculated from one single point over varying distances  $h$ , rather it moves from point to point and calculates the variogram for each distance  $h$  at each data location.

## Components of the Variogram

There are a few parameters that define some important properties of the variogram:

1. Sill: the sill is equal to the variance of the data (if the data are normal score the sill will be one)
2. Range: the range is the distance at which the variogram reaches the sill
3. Nugget Effect: the nugget is the sum of all of the short scale measurement errors.

Figure 5.5 illustrates the variogram parameters.

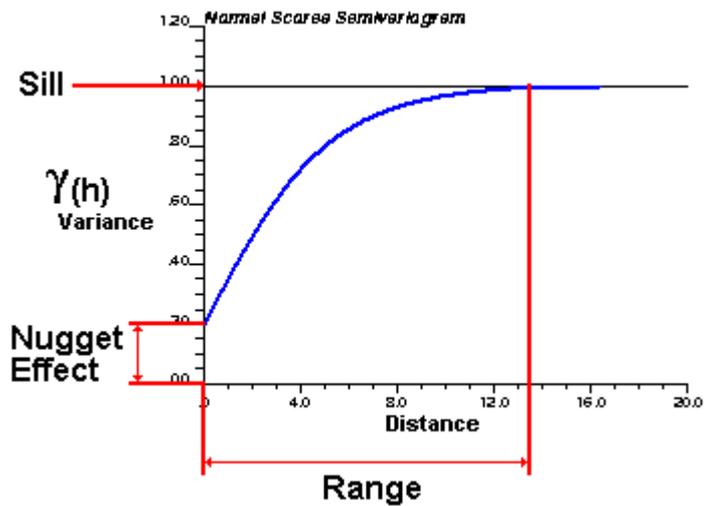


Figure 5.5a The components of the variogram; the sill is the variance of the variable under study, the range is the distance at which the variogram plateaus, the nugget effect is the short scale variability.

The nugget effect is a measure of short scale variability, any error in the measurement value or the location assigned to the measurement contributes to the nugget effect. The range shows the extent of correlation, and the sill indicates the maximum variability, or the variance of the data.. Figure 5.5b shows what happens when we change two parameters the nugget effect and the range. Recall that the sill is a fixed value. It is the variance of the data. Images *a*, *b*, and *c* in Figure 5.5b shows the effect of different ranges. A variogram with no range is shown in image *a*, image *b* has an intermediate range, and image *c* has a long range. Images *d*, *e*, and *f* show the effect of increasing nugget effect. Image *d* shows the effect of no nugget effect, or no short scale variability, image *e* shows an intermediate amount of nugget effect, and image *f* shows pure nugget effect, or complete short scale variability.

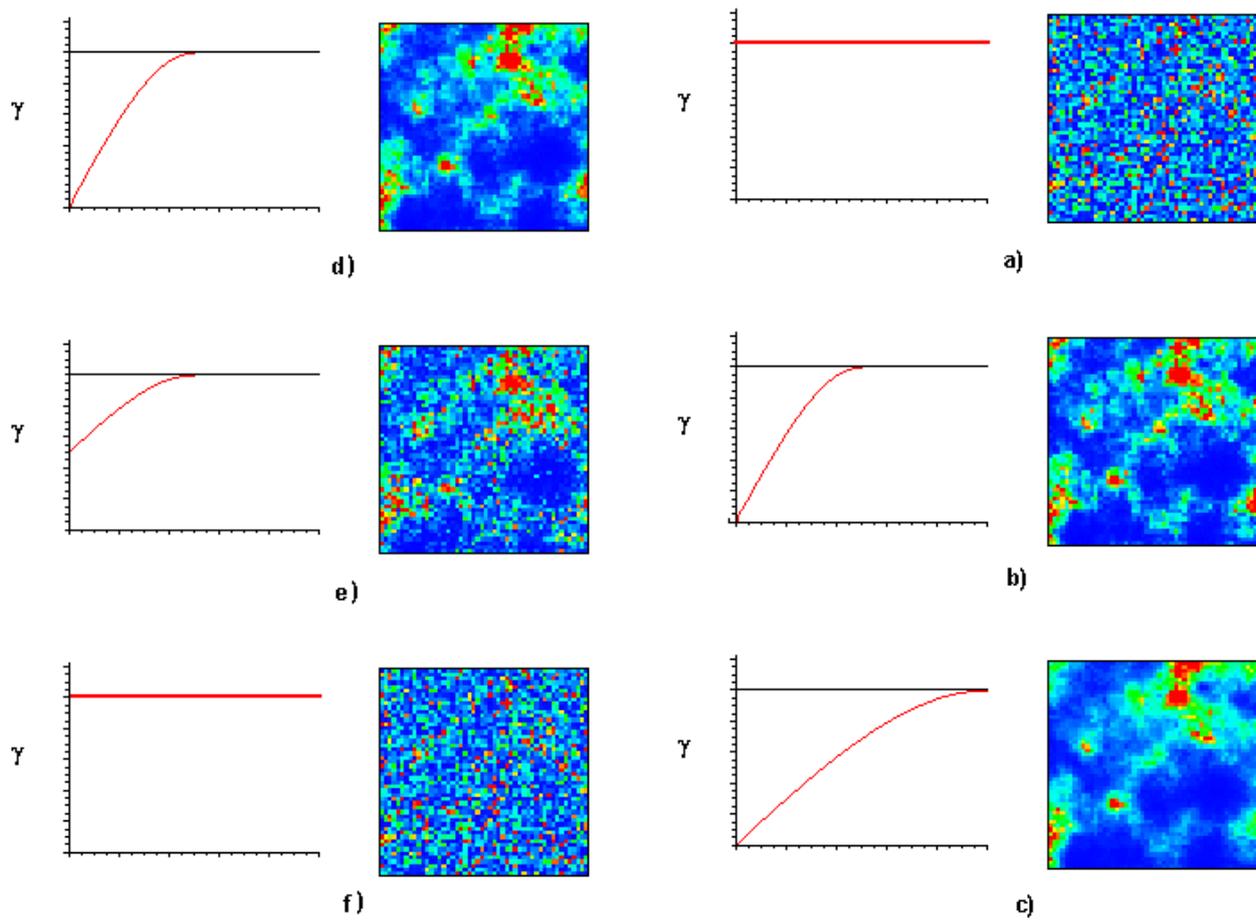


Figure 5.5b Some variogram examples to show the effect of different parameters

## Qualitative Spatial Data Analysis

In probabilistic notation the variogram is written:

$$2\gamma(\mathbf{h}) = E\{[Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})]^2\} \quad (5.1)$$

Which says the variogram is the expected value of the squared difference of  $Z(\mathbf{u})$  and  $Z(\mathbf{u} + \mathbf{h})$ . The semivariogram is defined:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\mathbf{u} \in \mathbf{h}} [Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})]^2 \quad (5.2)$$

To be precise the semivariogram is one half the variogram. In this lecture we will assume that the variogram and the semivariogram are synonymous.

Before the variogram is calculated some data preparation must be performed. Data must be free from outliers and systematic trends, as well, since geostatistical simulation requires normally distributed data, the data must be transformed into normal space. Estimation and simulation of indicator data such as lithofacies requires that data be transformed to indicator space. For convenience and ease of understanding it is useful to transform the coordinate axis to be aligned with the reservoir, or in some cases to be aligned with the direction of maximal continuity.

### Choosing variogram directions and lag distances

Spatial correlation is rarely isotropic. That is, spatial correlation is rarely the same in all directions. When a property changes with direction or distance it is said to be anisotropic. Since geostatistics is performed in 3D we require a definition of the spatial correlation in all three directions, and most reservoirs exhibit 3D anisotropy. For this reason variogram analysis is performed iteratively. The first variogram that should be calculated should be omnidirectional; not considering directions of anisotropy, and in the horizontal plane.

The calculation of the experimental omnidirectional variogram requires a lag distance, a lag tolerance, and number of lags. A good first estimation for the lag distance is a lag distance equal to the average distance between samples. The variogram is loosely defined as the average squared difference between data separated at a distance approximately equal to  $\mathbf{h}$ . It is nearly impossible to calculate the variogram for data separated precisely by the distance  $\mathbf{h}$ , so we include a lag distance tolerance. A good starting point for the lag distance tolerance is between one half of or equal to the lag distance. Figure 5.6 illustrates the concept of lag tolerance. The number of lags should not exceed more than two thirds of the field of study.

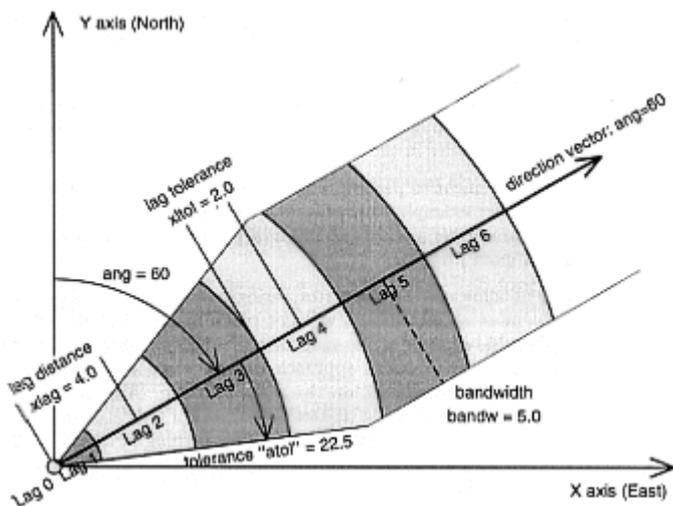


Figure 5.6 An illustration of the the lag, lag tolerance, azimuth, azimuth tolerance and bandwidth parameters for variogram modeling

Even calculating the omnidirectional experimental variogram is not an easy task. An acceptable lag distance for a omnidirectional experimental variogram requires an iterative approach; the lag distance and lag tolerance must be tweaked.

After calculating the omnidirectional experimental variogram we must determine the direction of maximal and minimal continuity so that a 3D definition of the spatial correlation can be found. To define the 3D spatial continuity we require a variograms for three directions the direction of maximal continuity, the direction of minimal continuity and one other direction. We calculate these variograms and combine them to define the 3D spatial correlation. In geostatistics the direction of minimal continuity is defined as perpendicular to the direction of maximal continuity. This defines the spatial for 2D geostatistics. For 3D geostatistics the remaining direction is defined as perpendicular to the 2D plane. Figure 5.7 illustrates this point. This is an adequate means of defining the 3D spatial continuity of the reservoir.

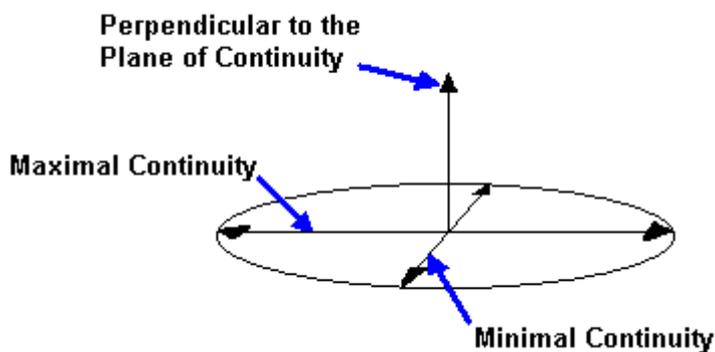


Figure 5.7

There are three parameters required for determining 3D definition of the spatial continuity (1) the direction of spatial continuity, (2) the directions of the variogram, (3) and the azimuth tolerance. One useful tool for determining the direction of maximal and minimal continuity is the variogram map. The variogram map calculates the variogram from the center of the location map and radially outward in a clockwise/counterclockwise direction. The result is a map illustrating directions of minimal and maximal continuity as in Figure 5.8. Additionally the direction of maximal continuity can be found by

searching for the variogram offering the greatest range, or by referring to a contour map of petrophysical properties.

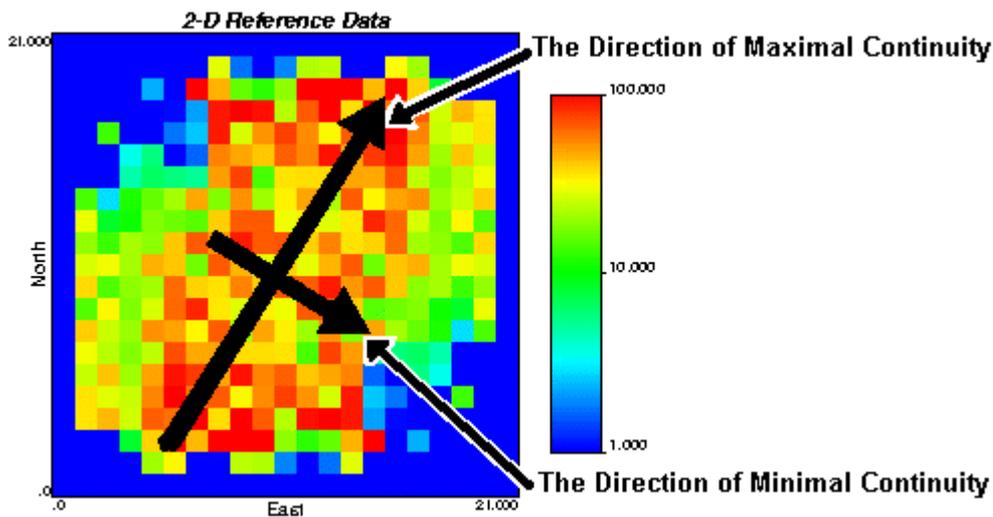


fig 5.8

As with the definition of the lag distance and the lag tolerance it is difficult to calculate the variogram along a single direction, thus we define a directional tolerance, or an azimuth tolerance. Figure 5.6 illustrates the concept of azimuth tolerance. A good starting point is an azimuth tolerance of  $\pm 22.5^\circ$ , giving a total azimuth tolerance of  $45^\circ$ . As with the omnidirectional variogram, a good set of 3D variograms requires an iterative approach; the lag distances and tolerances may be different in each direction, the azimuth tolerance may require tweaking to get a good experimental variogram, and the direction of maximal continuity may require tweaking. Note that the other two directions are fixed. The direction of minimal continuity is always perpendicular to the direction of maximal continuity and the third direction is always perpendicular to the plane of continuity.

## Variogram Interpretation and Modeling

The experimental variogram is only one third of a variogram analysis. The remaining pieces are modeling and interpretation the spatial correlation (the 3D variogram), and combining the models into a licit variogram model that defines the spatial correlation of the reservoir.

### Variogram Interpretation

Variogram interpretation is important. The calculated variogram points are not directly useable since (1) noisy results should be discounted, (2) geological interpretation should be used in the of the final variogram model, and (3) we need licit variogram measure in all directions and distances. For these reasons, the variogram must be understood and then modeled appropriately (Deutsch, 1999). There are some important pointers for modeling the variogram:

1. the sill is the variance  $\sigma^2$ . If the data are normal scores then the sill is 1.0.
2. variogram values below the sill imply positive correlation, variogram values at the sill imply no correlation, and above the sill implies negative correlation.
3. the range is the point where the variogram meets the sill, not the point where the sill appears to flatten out, or plateau.
4. a nugget effect of greater than 30% is unusual and should be investigated.



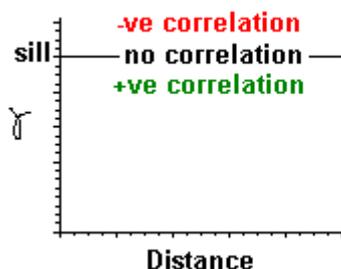


Figure 5.9a

There is some discussion as to whether or not the sill is or is not equal to the variance. The concern is that the variance is susceptible to outliers, this is why it is important weed out outliers before the variogram analysis. another concern is the use of declustering weights. using declustering weights reduces the variance. so which variance do we use? for now we will use the apriori variance. the final issue concerns the dispersion variance. I will leave the issue of dispersion variance for the reader to investigate.

## Anisotropy

If a petrophysical property has a range of correlation that is dependent on direction then the petrophysical property is said to exhibit geometric anisotropy. if the petrophysical property reaches the sill in one direction and not in another it is said to exhibit zonal anisotropy. In other words, a variogram exhibits zonal anisotropy when the variogram does not reach the expected sill. Most reservoir data exhibit both geometric and zonal anisotropy. Figure 5.9 first geometric anisotropy, second zonal anisotropy, and lastly both forms of anisotropy.

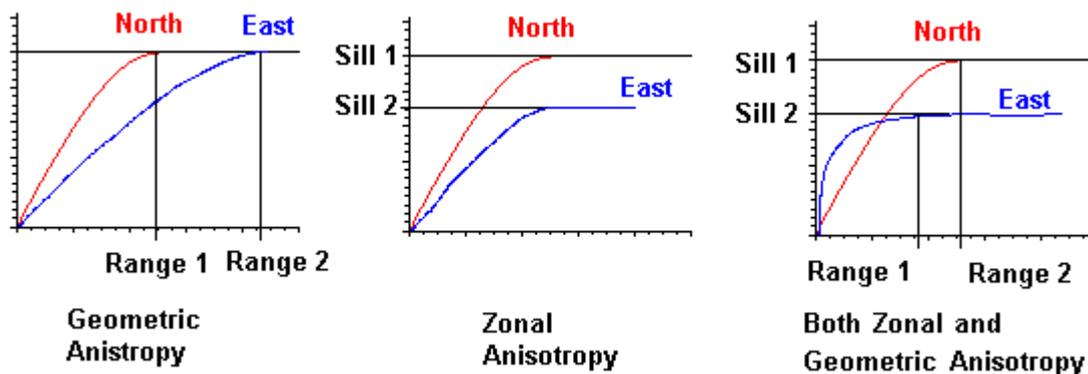


figure 5.9b

Zonal anisotropy can be the result of two different reservoir features: (1) layering, the horizontal variogram does not reach the expected sill because there are layer like trends that exist and variogram is not reaching full variability; and (2) areal trends, the vertical variogram does not reach the expected sill due to a significant difference in the average value in each well.

## Cyclicality

Geological phenomenon often formed in repeating cycles, that is similar depositional environments occurring over and over. A variogram will show this feature as cyclicality. as the variogram measures

the spatial correlation it will pass through regions that bear positive then negative correlation while still trending to no correlation. A cyclic variogram can be seen in Figure 5.10.

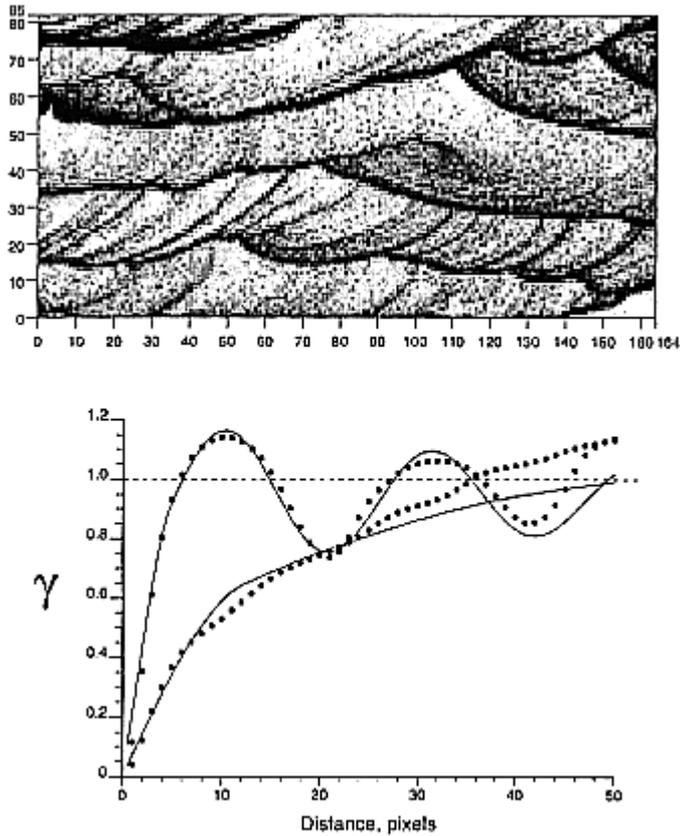
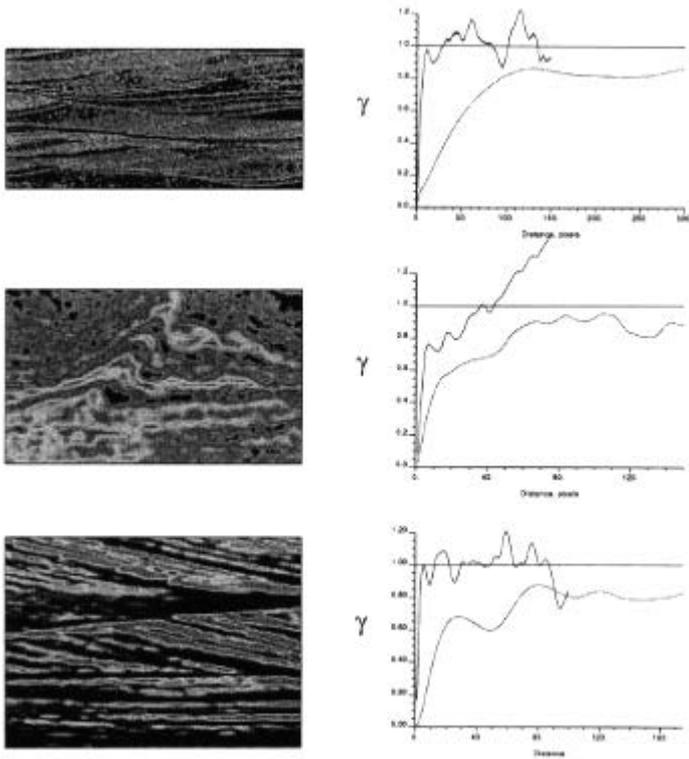
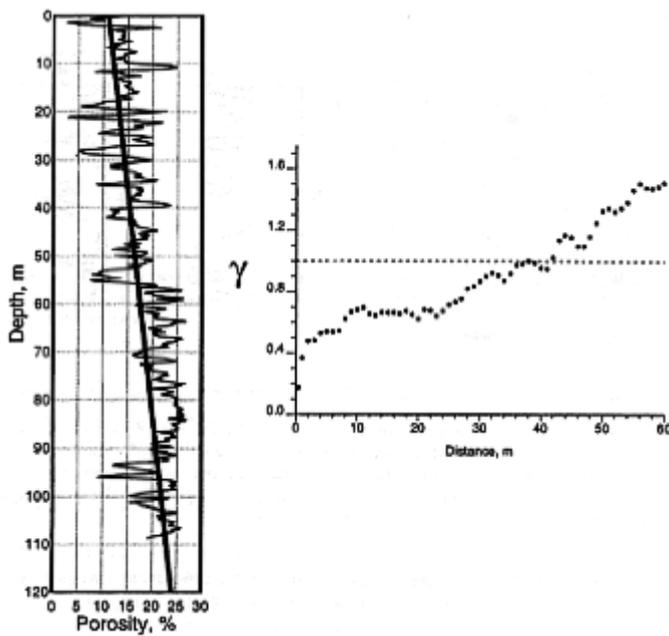


Figure 5.10 Gray scale image of an Eolian sandstone and the corresponding vertical and horizontal semivariograms. The semivariogram was calculated on the normal score transform of the gray scale level (finer grained low permeability sandstone appears darker). Note the cyclic behavior in the vertical direction, and the long range correlation of the horizontal variogram (Deutsch, 1999).



## Large Scale Trends

virtually all geological processes impart a trend in the petrophysical property distribution. Dolomitization is the result of hydrothermal fluid flow, upward fining of clastics, and so on, are large scale trends. Figure 5.11 shows how large scale trends affect the histogram. Trending causes the variogram to climb up and beyond the sill of the variogram.



## Variogram modeling

All directional variograms must be considered simultaneously to understand the 3D spatial correlation.

1. Compute and plot experimental variograms in what are believed to be the principal directions of continuity based on a-priori geological knowledge.
2. Place a horizontal line representing the theoretical sill. Use the value of the experimental (stationary) variance for continuous variables (1 if the data has been transformed to normal score) and  $p(1 - p)$  for categorical variables where  $p$  is the global proportion of the category of interest. In general, variograms are systematically fit to the theoretical sill and the whole variance below the sill must be explained in the following steps.
3. If the experimental variogram clearly rises above the theoretical sill, then it is very likely that there exists a trend in the data. The trend should be removed as detailed above, before proceeding to interpretation of the experimental variogram.
4. Interpretation:
  - o **Short-scale variance:** the nugget effect is a discontinuity in the variogram at the origin corresponding to short scale variability. It must be chosen as to be equal in all directions; pick from the directional experimental variogram exhibiting the smallest nugget. At times, one may chose to lower it or even set it to 0.0.
  - o **Intermediate-scale variance:** geometric anisotropy corresponds to a phenomenon with different correlation ranges in different directions. Each direction encounters the total variability of the structure. There may exist more than one such variance structure.
  - o **Large-scale variance:** (1) zonal anisotropy, characterized by directional variograms reaching a plateau at a variance lower than the theoretical sill, or (2) hole-effects representative of a "periodic" phenomenon (cyclicality) and characterized by undulations on the variogram. The hole-effect does not actually contribute to the total variance of the phenomena, however, its amplitude and frequency must be identified during the interpretation procedure, also, it can only exist in one direction.
5. Once all the variance regions have been explained and each structure has been related to a geological process, one may proceed to variogram modeling by selecting a licit model type (spherical, exponential, Gaussian) and correlation ranges for each structure. This step can be referred to as the parameter estimation part of variogram analysis. Constraining the variogram model by a prior interpretation step with identification of structure types can lead to a reliable automatic fit of the experimental variogram.

### Variogram Models

There are 3 common model types and 4 not so common model types. The 3 common model types are:

1. The nugget effect. The nugget effect should normally only explain up to 30% of the variance. The nugget effect is that portion of the variance that is due to error and small scale variability. The nugget effect is numerically modeled using formula 5.3, and Figure 5.13 shows a nugget effect variogram.

$$\gamma(h) = C \quad (5.3)$$

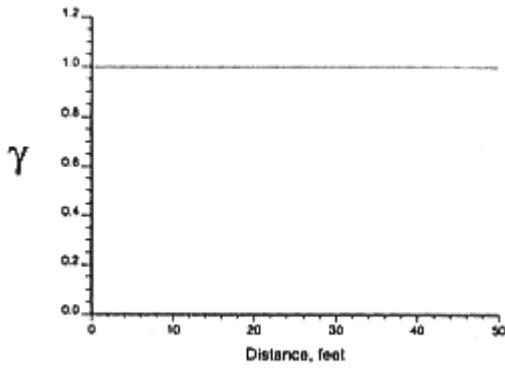


Figure 5.13

2. The spherical model. The spherical model is the most common variogram model type. The spherical model is mathematically defined by formula 5.14, and Figure 5.14 shows a spherical type model.

$$\gamma(h) = C \bullet Sph\left(\frac{h}{a}\right) = \begin{cases} C \bullet \left[1.5\frac{h}{a} - 0.5\left(\frac{h}{a}\right)^3\right], & \text{if } h \leq a \\ C & \text{if } h > a \end{cases} \quad (5.4)$$

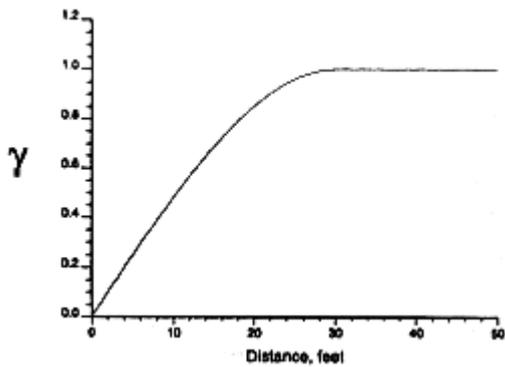


Figure 5.14

3. The exponential model. The exponential model is similar to the spherical model but it approaches the sill asymptotically. It is mathematically defined by formula 5.5 and shown as a variogram in Figure 5.15.

$$\gamma(h) = C \bullet \exp\left(\frac{h}{a}\right) = C \bullet \left[1 - \exp\left(-\frac{h}{a}\right)\right] \quad (5.5)$$

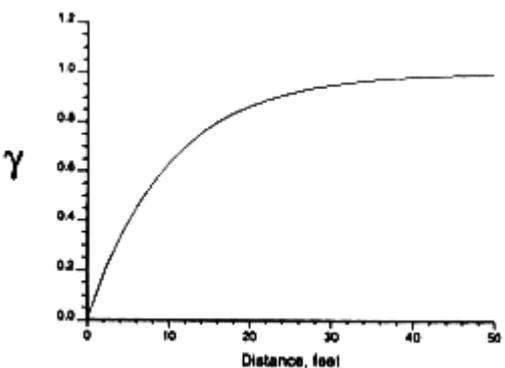


Figure 5.15

- The Gaussian model. The Gaussian model is typically used for very modeling very continuous experiment variograms. It is mathematically defined by the formula 5.6 and shown as a variogram model in Figure 5.16.

$$\gamma(h) = C \cdot \left[ 1 - \exp\left(-\frac{h^2}{a^2}\right) \right] \quad (5.6)$$

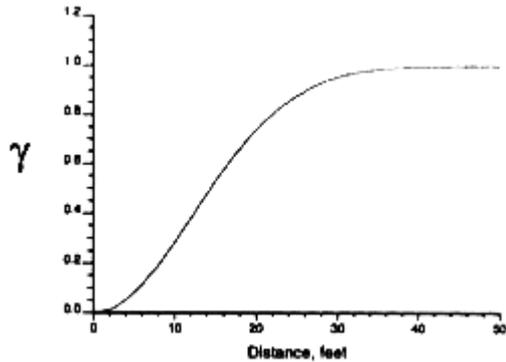


Figure 5.16

There are three remaining not so common variogram models. The first is the power model. The power model is not generally used in geostatistical applications that require stationarity. The power model is defined by formula 5.7 and shown in figure 5.17 as a variogram.

$$\gamma(h) = C \cdot h^\omega \quad \omega \in [0,2] \quad (5.7)$$

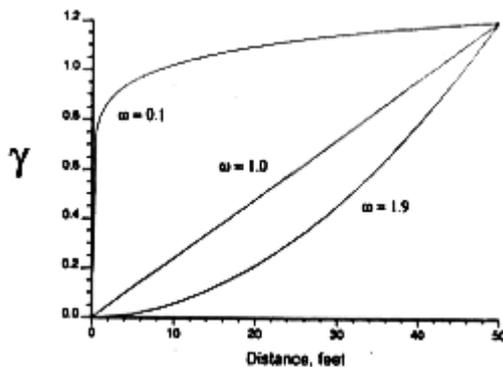


Figure 5.17

- The hole effect model. The whole effect model is used for modeling cyclic experimental variograms. Mathematically it is defined in formula 5.8 and shown as a variogram in Figure 5.8.

$$H_a(h) = 1.0 - \cos(h \cdot a) \quad (5.8)$$

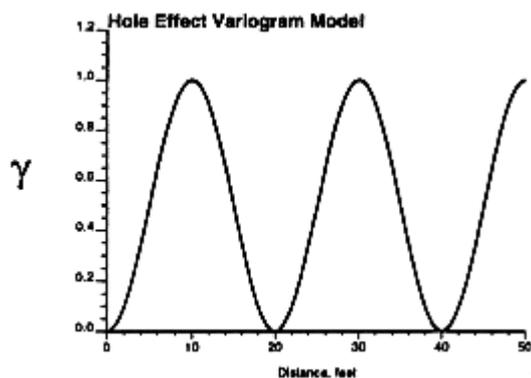


Figure 5.18

6. The last model type is known as the dampened hole effect because it includes a damping function in its mathematical formula (formula 5.9). The model variogram is shown in Figure 5.19.

$$DH_{d,p}(h) = 1.0 - \exp\left(\frac{-h \bullet a}{d}\right) \cos(h \bullet a) \quad (5.9)$$

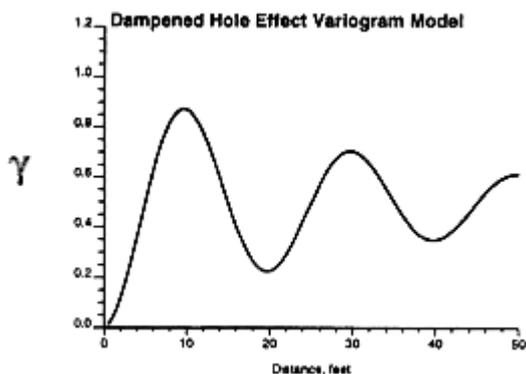


Figure 5.19

## Examples

The variogram models in the principal directions (major horizontal, minor horizontal, and vertical) must be consistent, i.e., same nugget effect and same number and type of structures. This is required so that we can compute variogram values at angles not aligned with the principle axis. in off-diagonal directions and between experimental values. The responsibility for a licit variogram model is left to practitioner, current software does not help very much.

Basic idea is to explain the total variability by a set of nested structures where each nested structure each having different range parameters in different directions:

$$h = \sqrt{\left(\frac{h_x}{a_x}\right)^2 + \left(\frac{h_y}{a_y}\right)^2 + \left(\frac{h_z}{a_z}\right)^2} \quad (5.10)$$

where  $h$  is the distance,  $h_x, h_y, h_z$  are the direction specific distance parameters, and  $a_x, a_y, a_z$ , are the directional range parameters derived from the variogram models. The range parameters  $a_x, a_y, a_z$  can approach 0 or positive infinity.

How do we ensure a legitimate model?

1. pick a single (lowest) isotropic nugget effect
2. choose the same number of variogram structures for all directions based on most complex direction
3. ensure that the same sill parameter is used for all variogram structures in all directions
4. allow a different range parameter in each direction
5. model a zonal anisotropy by setting a very large range parameter in one or more of the principal directions

Figure 5.20 shows some simple 1D experimental variograms, the respective models and parameters. For each example the sill is 1.0.

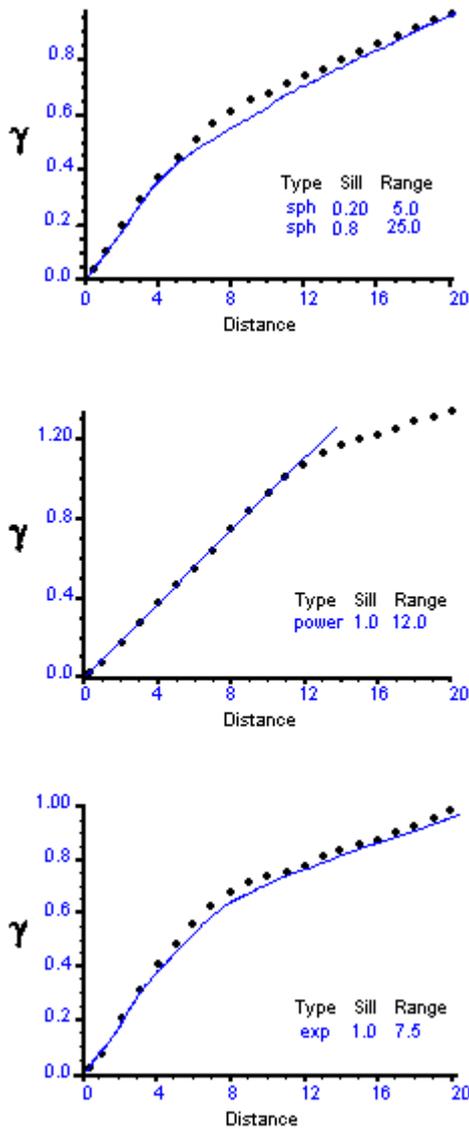


Figure 5.20, The top example shows a 1D example with two nested structures. Points to note are (1) the sum of the sill contributions is equal to one, (2) the total range of the variogram is the sum of the component ranges. The middle example shows a power model variogram, note that neither the experimental or the the model variogram reach a sill. The bottom variogram is a simple exponential model.

Most beginning practitioners over model the experimental variogram. That is most beginners apply to many nested structures and try very hard to catch each point in the model. There should never be more than three nested structure (not including the nugget effect), and the model need only be accurate within the range of its use. Referring to the middle variogram of Figure 5.20, if the variogram is not needed beyond 12 units there is no need to model beyond 12 units. Figure 5.21 shows some more



complex examples. Note that they are all 2D examples. Modeling 3D variograms is a simple extension of the same principles used to model 2D. In short only model what you need to.

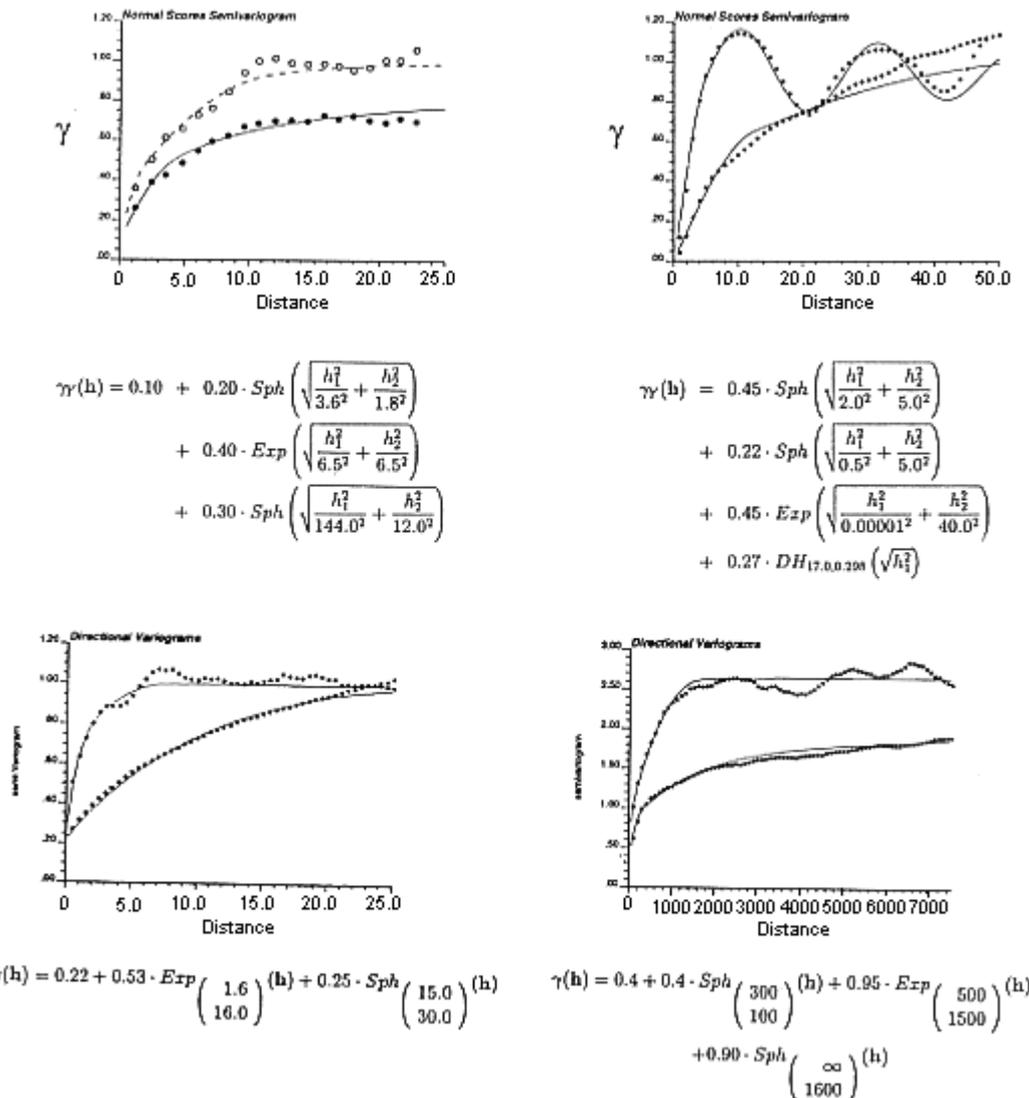
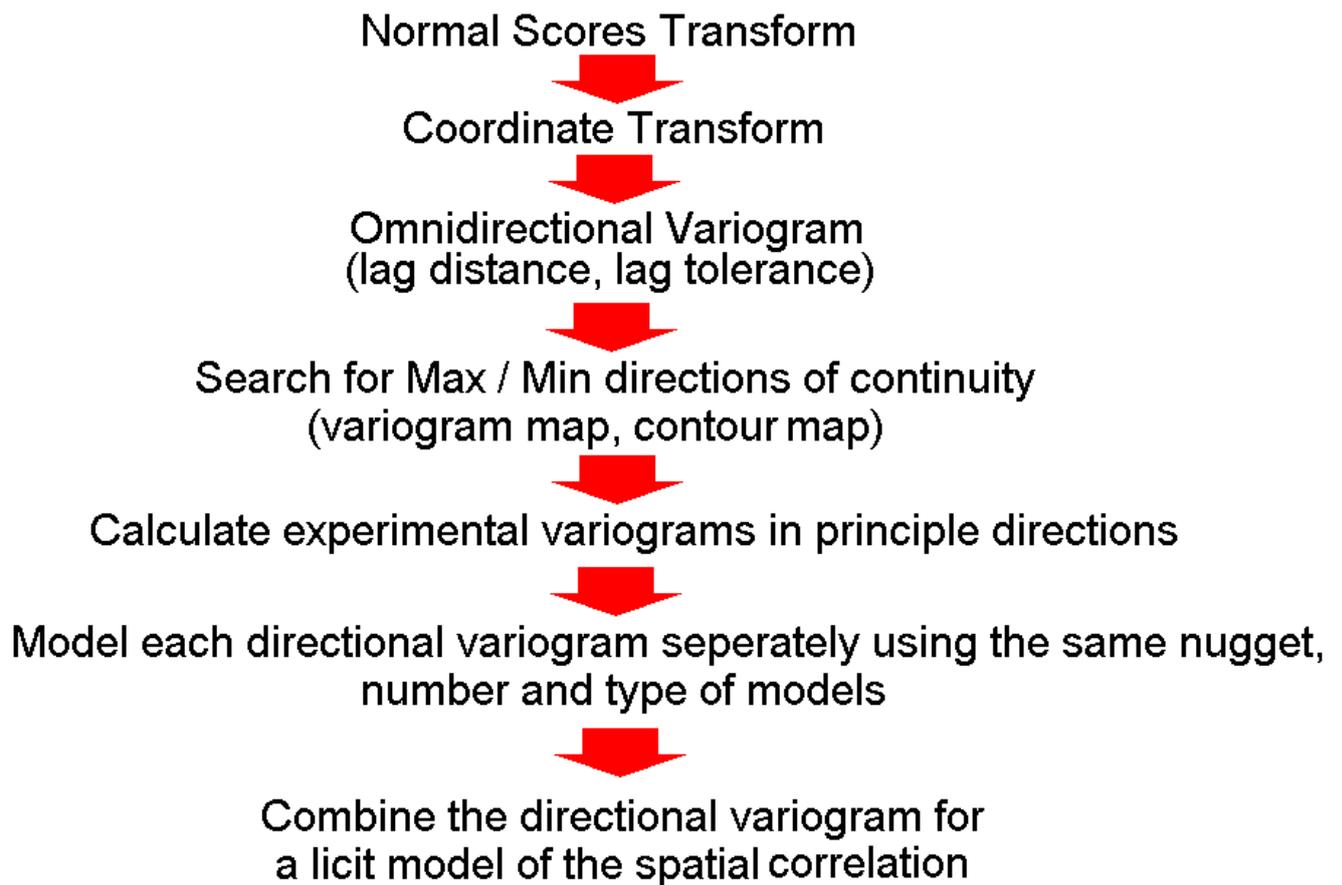


Figure 5.21, The top left and bottom right examples model zonal anisotropy. The bottom left example shows a model of geometric anisotropy. The top right example is quite complex, requiring the need of a hole effect model to correctly model the spatial correlation.

### Work Flow

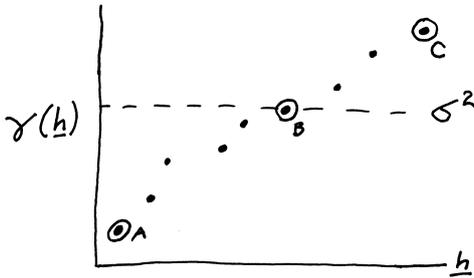
Modeling the spatial correlation is the most difficult and important step in the geostatistical modeling process. Great care should be taken.



June 18, 1999

## Lecture 5: Spatial Data Analysis, The Quiz

### Question 1



The scatterplot is an excellent visual tool to display correlation. The correlation coefficient is an intuitive measure of two point correlation. For every point on a variogram plot ( $\gamma(\mathbf{h})$  versus  $\mathbf{h}$ ) there is a scatterplot of  $z(\mathbf{u})$  and  $z(\mathbf{u}+\mathbf{h})$  values. Draw a scatterplot and give an approximate correlation coefficient for the three points labeled A, B, and C on the above figure.

### Question 2

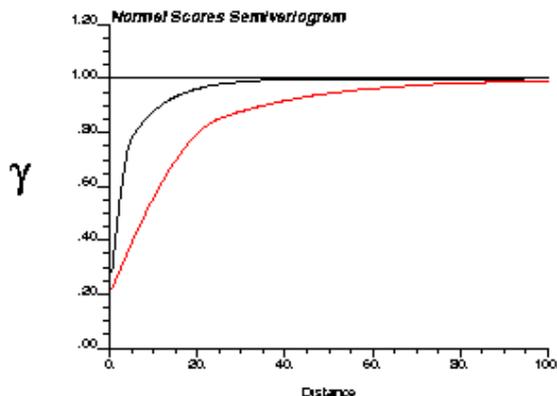
An analytical variogram model is fit to experimental points. Give three reasons why such variogram modeling is necessary.

### Question 3

What is the difference between the *variogram* and *semivariogram*? What is the difference between an experimental and a model semivariogram?

### Question 4

Complete the shorthand notation for the following variogram model:



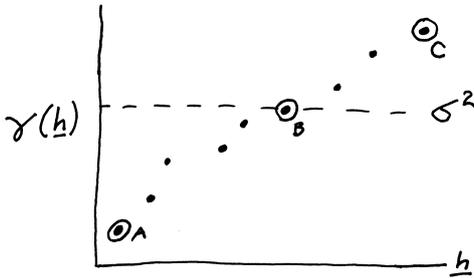
$$\gamma(h) = 0.2 + 0.4 \text{ Sph} \left( \begin{array}{c} \square \\ \square \end{array} \right) + 0.4 \text{ Exp} \left( \begin{array}{c} \square \\ \square \end{array} \right)$$

### Question 5

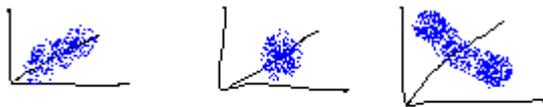
How does geometric anisotropy differ from zonal anisotropy? What is happening in an experimental variogram exhibits zonal anisotropy?

## Lecture 5: Spatial Data Analysis, The Quiz Solutions

### Question 1



The scatterplot is an excellent visual tool to display correlation. The correlation coefficient is an intuitive measure of two point correlation. For every point on a variogram plot ( $\gamma(\mathbf{h})$  versus  $\mathbf{h}$ ) there is a scatterplot of  $z(\mathbf{u})$  and  $z(\mathbf{u}+\mathbf{h})$  values. Draw a scatterplot and give an approximate correlation coefficient for the three points labeled A, B, and C on the above figure.



A) about 1.0   B) about 0.0   C) about -1.0

### Question 2

An analytical variogram model is fit to experimental points in the three principle directions. Give three reasons why such variogram modeling is necessary.

1. We need variogram values between the experimental points
2. We need variogram values at angles other than the principle angles
3. opportunity to incorporate additional geological information

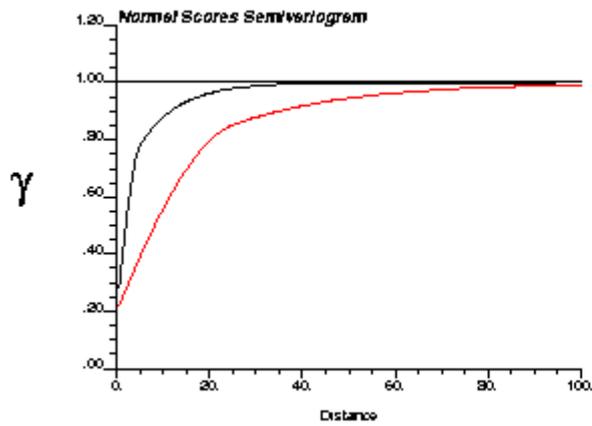
### Question 3

What is the difference between the *variogram* and *semivariogram*? What is the difference between an experimental and a model semivariogram?

A variogram is twice the semivariogram      An experimental variogram is the variogram calculated from the data and a model variogram is an analytically calculated variogram.

### Question 4

Complete the shorthand notation for the following variogram model:



$$\gamma(h) = 0.2 + 0.4 \text{Sph}_{\left(\frac{5}{25}\right)} + 0.4 \text{Exp}_{\left(\frac{25}{75}\right)}$$

### Question 5

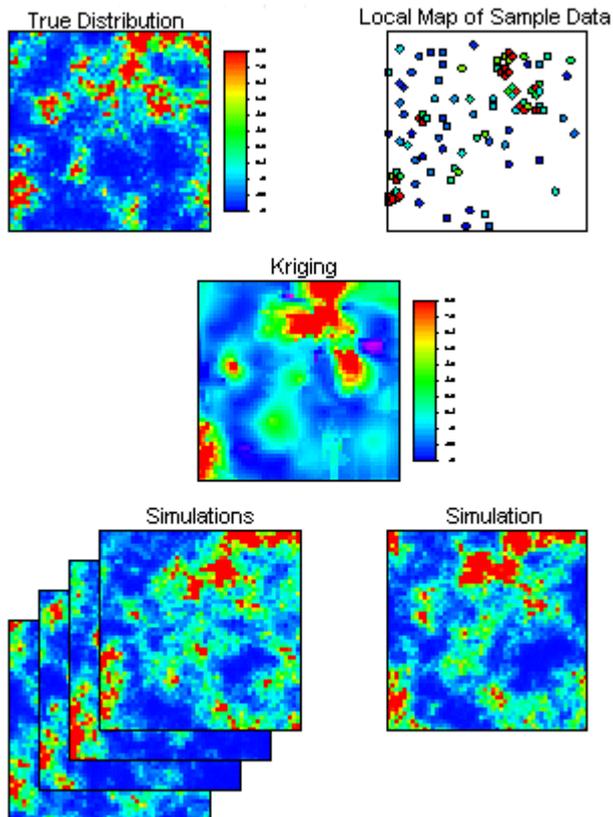
How does geometric anisotropy differ from zonal anisotropy? What is happening in an experimental variogram exhibits zonal anisotropy?

- a) Geometric: the range in one direction differs than the range in another Zonal: Ranges are the same but one direction does not reach the sill.
- b) The experimental variogram in the direction that is reaching the sill is achieving full variability, and the other is not. The could be a result of either an areal trend, or layering in the reservoir.

## Lecture 6: Geostatistical Mapping Concepts

- [Estimation](#)
- [Facies Simulation Distribution](#)
- [Petrophysical Property Simulation](#)
- [Lecture 6 Quiz](#)

### Introduction



At any instant in time there is a single true distribution of a geological attribute. The true distribution is not available, but we do the best we can to map the true distribution given some sample data. From the need to map the true distribution as accurately as possible many interpolating algorithms were developed. The most common, and the most useful is kriging. Kriging is a locally accurate and smooth interpolator, appropriate for visualizing trends, but inappropriate for flow simulation where preservation of heterogeneity in the reservoir is important. An extension of the kriging algorithm is sequential simulation. Simulation is appropriate for simulation, and allows an assessment of uncertainty with alternative realizations. The first part of this Lecture is devoted to the kriging algorithm and kriging variance. The lecture then moves on to simulation of petrophysical properties and concludes with simulation of facies.

## Estimation

Consider the problem of estimating the value of an attribute at any unsampled location  $\mathbf{u}$ , denoted  $z^*(\mathbf{u})$ , using only sample data collected over the study area  $A$ , denoted by  $z(\mathbf{u}_n)$  as illustrated in Figure 6.1.

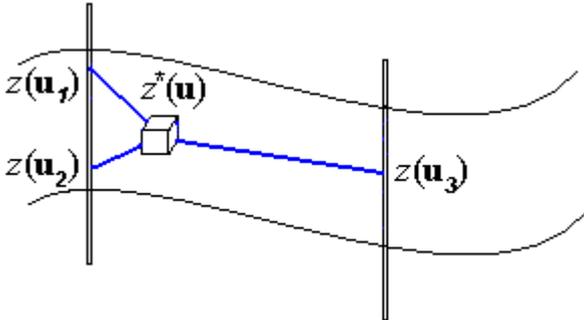


Figure 6.1

The algorithm used to solve this problem was pioneered by Danie Krige and in recognition of his efforts the algorithm is called kriging. The kriging algorithms are a family of generalized least squares regression techniques that estimate  $z^*(\mathbf{u})$  using sample data  $z(\mathbf{u}_n)$ . There are several different flavours of kriging, each addressing different needs. In general the the kriging equations are known as:

$$z^*(\mathbf{u}) - m(\mathbf{u}) = \sum_{\alpha=1}^n \lambda_{\alpha} \cdot [z(\mathbf{u}_{\alpha}) - m(\mathbf{u}_{\alpha})] \quad (6.1)$$

Where  $z^*(\mathbf{u})$  is the estimator at location  $\mathbf{u}$ ,  $m(\mathbf{u})$  is the mean at location  $\mathbf{u}$ ,  $z(\mathbf{u}_{\alpha})$  is one of the  $\alpha$  different data at location  $\mathbf{u}$  used in the estimate,  $m(\mathbf{u}_{\alpha})$  is the mean at location  $\mathbf{u}$ , and  $\lambda_{\alpha}$  are the weights. The kriging equations state that the estimate is a weighted linear combination of the sample data, or more generally:

$$z^*(\mathbf{u}) = \sum_{\alpha=1}^n \lambda_{\alpha} \cdot z(\mathbf{u}_{\alpha}) \quad (6.2)$$

Reconsider figure 6.1 with equation 6.2 in mind. Equation 6.2 indicates that the estimator  $z^*(\mathbf{u})$  is the weighted sum of the data, or mathematically:

$$z^*(\mathbf{u}) = \lambda_1 z(\mathbf{u}_1) + \lambda_2 z(\mathbf{u}_2) + \lambda_3 z(\mathbf{u}_3) \quad (6.3)$$

There are a few goals that we strive for when choosing the weights:

1. closeness to the location being estimated, the estimator is equidistant from both knowns.
2. redundancy between the data values, the knowns lie on either side of the estimator. If the knowns were both on the same side as the estimator then it would be more difficult to make the estimate.
3. anisotropic continuity (preferential direction)
4. magnitude of continuity / variability

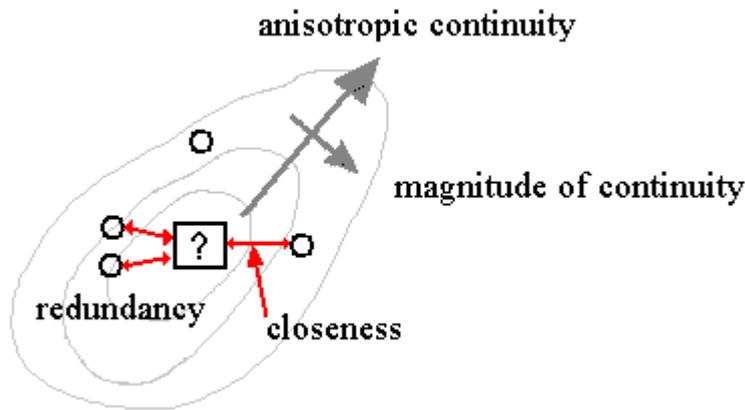


Figure 6.2. The kriging weights must consider redundancy of the data, the closeness of the data, and the direction and magnitude of continuity.

There is one other goal when estimating the unknown attribute: minimize the error variance. If the error variance is minimized then the estimate will be the best estimate. The error variance is the expected value of the difference between the known and the estimate and is defined by:

$$\sigma_{sk}^2(\mathbf{u}) = E\left\{[z^*(\mathbf{u}) - z(\mathbf{u})]^2\right\} \quad (6.4)$$

where  $z^*(\mathbf{u})$  is the estimator, and  $z(\mathbf{u})$  is the true value. One obvious question raised by this equation is how can we determine the error if we do not know the true value? True, we do not know the true value, but we can choose weights that do minimize the error. To minimize the estimation variance take the partial derivative of the error variance (equation 6.4) and set to 0, but before taking the derivative equation 6.4 is expanded:

$$\begin{aligned} &= E\left\{[z^*(\mathbf{u})]^2\right\} - 2 \cdot E\{z^*(\mathbf{u}) \cdot z(\mathbf{u})\} + E\{[z(\mathbf{u})]^2\} \\ &= \sum_{\alpha=1}^N \sum_{\beta=1}^N \lambda_{\alpha} \lambda_{\beta} E\{Z(\mathbf{u}_{\beta}) \cdot Z(\mathbf{u}_{\alpha})\} - 2 \cdot \sum_{\alpha=1}^N \lambda_{\alpha} E\{z(\mathbf{u}) \cdot z(\mathbf{u}_{\alpha})\} + C(0) \\ \sigma_{sk}^2(\mathbf{u}) &= \sum_{\alpha=1}^N \sum_{\beta=1}^N \lambda_{\alpha} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) - 2 \cdot \sum_{\alpha=1}^N \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) + C(0) \end{aligned} \quad (6.5)$$

The result is an equation that refers to the covariance between the data points  $C(\mathbf{u}_{\alpha}, \mathbf{u}_{\beta})$ , and the data and estimator  $C(\mathbf{u}, \mathbf{u}_{\beta})$ . A first this may seem like a problem because we have not discussed the covariance between the data and the estimator, but we did discuss the variogram, and the variogram and the covariance are related. Recall that the variogram is defined by:

$$2\gamma(\mathbf{h}) = E\{[z(\mathbf{u}) - z(\mathbf{u} + \mathbf{h})]^2\}$$

and note that the covariance is defined by (the covariance is not the squared difference whereas the variogram is):

$$C(\mathbf{h}) = E\{z(\mathbf{u}) \cdot z(\mathbf{u} + \mathbf{h})\}$$

The link between the variogram and the covariance is:



$$2\gamma(\mathbf{h}) = [E\{Z^2(\mathbf{u})\}] + [E\{Z^2(\mathbf{u} + \mathbf{h})\}] - 2 \cdot [E\{Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})\}]$$

$$2\gamma(\mathbf{h}) = E\{[Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})]^2\}$$

$$2\gamma(\mathbf{h}) = 2[C(0) - C(\mathbf{h})]$$

so the variogram and the covariance are linked by:

$$\gamma(\mathbf{h}) = C(0) - C(\mathbf{h}) \quad (6.6)$$

where  $\gamma(\mathbf{h})$  is the variogram,  $C(0)$  is the variance of the data, and  $C(\mathbf{h})$  is the covariance.

This makes it possible to perform kriging in terms of the variogram instead of the covariance.

Continuing with the derivation of the kriging equations, we know that formula 6.5 must be minimized by taking the partial derivative with respect to the weights and set to zero:

$$= \sum_{\alpha=1}^n \sum_{\beta=1}^n \lambda_{\alpha} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) - 2 \cdot \sum_{\alpha=1}^n \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) + C(0)$$

$$\frac{\partial [\sigma_{\mathbf{E}}^2(\mathbf{u})]}{\partial \lambda_{\alpha}} = \sum_{\beta} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) - 2 \cdot C(\mathbf{u}, \mathbf{u}_{\alpha}), \quad \alpha = 1, \dots, n$$

setting to zero...

$$\sum_{\beta} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) = C(\mathbf{u}, \mathbf{u}_{\alpha}), \quad \alpha = 1, \dots, n \quad (6.7)$$

The result of the derivation in terms of the variogram is the same because both the variogram and the covariance measure spatial correlation, mathematically:

$$\sum_{\beta} \lambda_{\beta} \gamma(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) = \gamma(\mathbf{u}, \mathbf{u}_{\alpha}), \quad \alpha = 1, \dots, n \quad (6.8)$$

and the system of equations in terms of the variogram is:

$$\begin{bmatrix} \gamma(h_{1,1}) & \gamma(h_{1,2}) & \gamma(h_{1,3}) & \gamma(h_{1,4}) \\ \gamma(h_{2,1}) & \gamma(h_{2,2}) & \gamma(h_{2,3}) & \gamma(h_{2,4}) \\ \gamma(h_{3,1}) & \gamma(h_{3,2}) & \gamma(h_{3,3}) & \gamma(h_{3,4}) \\ \gamma(h_{4,1}) & \gamma(h_{4,2}) & \gamma(h_{4,3}) & \gamma(h_{4,4}) \end{bmatrix} * \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} \gamma(h_{1,0}) \\ \gamma(h_{2,0}) \\ \gamma(h_{3,0}) \\ \gamma(h_{4,0}) \end{bmatrix} \quad (6.9)$$

This is known as simple kriging. There are other types of kriging but they all use the same fundamental concepts derived here.

## Discussion

There are a couple of motivations behind deriving kriging equations in terms of the covariance:

1. Its easier. Solving the kriging equations in terms of the variogram requires that the mean be carried throughout the derivation. It is easier to simplify in terms of covariance.
2. It is possible to have the variance at  $h=0$  be zero with a variogram, this makes the matrix very unstable. The covariance is defined as the expected value of the difference, not the squared difference therefore the value of the covariance at  $h=0$  is always large and hence the main diagonal in the matrix will always be large. Matrices that have small main diagonal elements such as when using the variogram are difficult for solution algorithms to solve due to truncation errors and so on.
3. it is easy to convert the variogram to covariance

## Implimenting Kriging

Once again, consider the problem of estimating the value of an attribute at any unsampled location  $\mathbf{u}$ , denoted  $z^*(\mathbf{u})$ , using only sample data collected over the study area  $\mathbf{A}$ , denoted by  $z(\mathbf{u}_n)$  as illustrated in Figure 6.3. Figure 6.3 shows the estimator (the cube), and the data ( $z(\mathbf{u}_n)$ ). To perform kriging just fill in the matrices. For example, filling in the left hand matrix, entry 1,1, consider the variogram between points 1 and 1. The distance between a point and itself is 0, and thus the first entry would be the nugget effect. entry number 1,2, consider the distance  $h$  between points 1 and 2, read the appropriate variogram measure and enter it into the matrix. repeat for the all of the variogram entreis and solve for the weights  $\lambda_n$ .

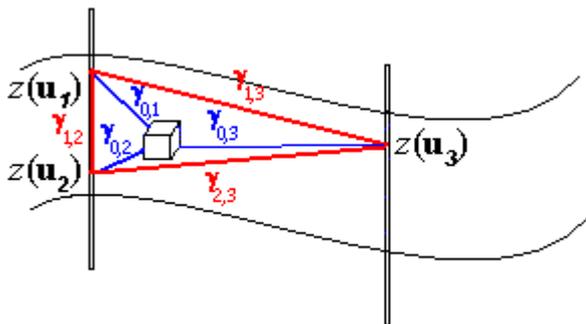


Figure 6.3

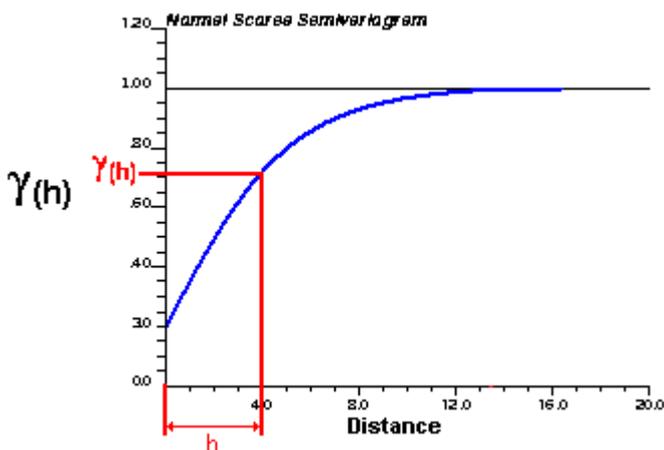


Figure 6.4

$$\begin{bmatrix} \gamma(h_{1,1}) & \gamma(h_{1,2}) & \gamma(h_{1,3}) & \gamma(h_{1,4}) \\ \gamma(h_{2,1}) & \gamma(h_{2,2}) & \gamma(h_{2,3}) & \gamma(h_{2,4}) \\ \gamma(h_{3,1}) & \gamma(h_{3,2}) & \gamma(h_{3,3}) & \gamma(h_{3,4}) \\ \gamma(h_{4,1}) & \gamma(h_{4,2}) & \gamma(h_{4,3}) & \gamma(h_{4,4}) \end{bmatrix} * \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} \gamma(h_{1,0}) \\ \gamma(h_{2,0}) \\ \gamma(h_{3,0}) \\ \gamma(h_{4,0}) \end{bmatrix}$$

Figure 6.5

The estimate is then calculated as:

$$z^*(\mathbf{u}) = \lambda_1 z(\mathbf{u}_1) + \lambda_2 z(\mathbf{u}_2) + \lambda_3 z(\mathbf{u}_3) \quad (6.10)$$

The result is an estimate of the true value and the error associated with the estimate, as Figure 6.6 illustrates.

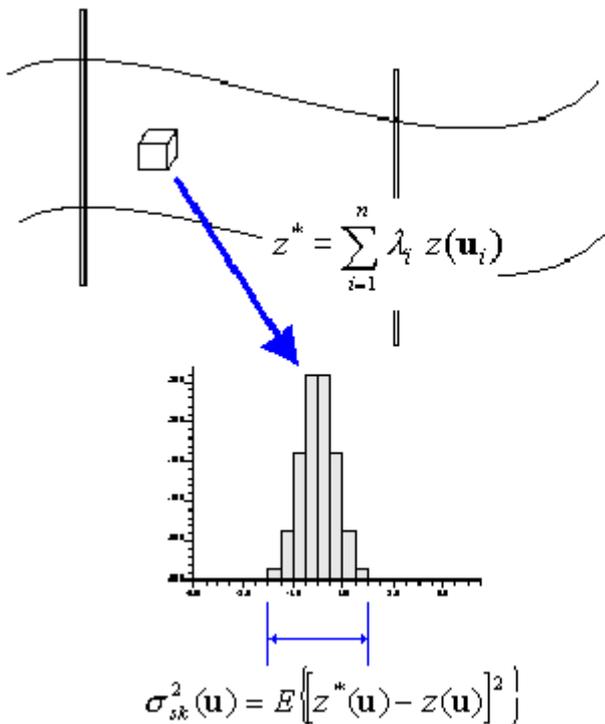


Figure 6.6

Kriging provides the best estimate but there are some issues:

The Pros and Cons of Kriging	
<b>Pros:</b>	<b>Cons:</b>
The "best" linear unbiased estimator	Smooths
Uses the variogram	Does not honor the variogram

Gets the covariance between the data and the estimate correct	Does not honor the histogram
	Does not quantify global uncertainty

Since kriging is a linear estimator it smooths, and thus reduces the heterogeneity in the model. This is acceptable for attributes that are already smooth but in most cases kriged outputs are not acceptable for mapping because the true heterogeneity is removed. Another issue that arises from smoothing is the failure of kriging to honor the histogram and the variogram. This also a result of the smoothing effect of kriging. Kriging offers one solution to the estimation problem, but it offers the mean estimate for all points. There are other possibilities that are expressed in the error variance.

## The Kriging Variance

Recall that the expanded kriging variance is:

$$\sigma_{sk}^2(\mathbf{u}) = \sum_{\alpha=1}^n \sum_{\beta=1}^n \lambda_{\alpha} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) - 2 \cdot \sum_{\alpha=1}^n \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) + C(0) \quad (6.5)$$

and that the kriging equation is:

$$\sum_{\beta} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) = C(\mathbf{u}, \mathbf{u}_{\alpha}), \quad \alpha = 1, \dots, n \quad (6.7)$$

substituting equation 6.7 into equation 6.5:

$$\sigma_{sk}^2(\mathbf{u}) = C(0) - \sum_{\alpha=1}^n \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) \quad (6.11)$$

Equation 6.11 is the kriging variance. Kriging may be considered spatial regression, that is, it creates an estimate that is smooth. There are many interpolation / estimation algorithms that construct smooth estimates. The advantage of kriging over other algorithms is that it provides quantification of how smooth the estimates are. The variance of the kriging estimate may be calculated as:

$$\begin{aligned} \text{Var}\{z^*(\mathbf{u})\} &= E\{[z^*(\mathbf{u})]^2\} - E\{z^*(\mathbf{u})\}^2 \\ \text{Var}\{z^*(\mathbf{u})\} &= \sum_{\alpha=1}^n \sum_{\beta=1}^n \lambda_{\alpha} \lambda_{\beta} C(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) - 0^2 \end{aligned} \quad (6.12)$$

substituting equation 6.7 into equation 6.12,

$$\text{Var}\{z^*(\mathbf{u})\} = \sum_{\alpha=1}^n \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) \quad (6.13)$$

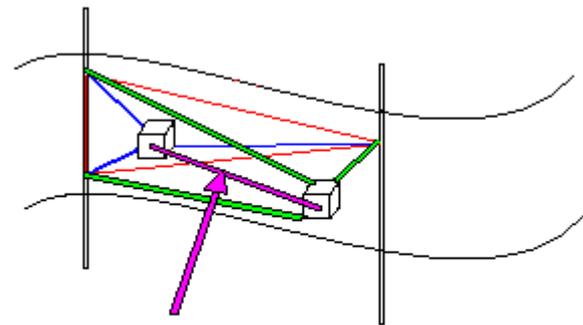
Equation 6.13 tells us the variance of the kriging estimator at location  $\mathbf{u}$ , however, the variance is stationary, that is, the variance should be the same everywhere. Therefore there is a missing variance equal to:

$$\text{missing variance} = C(0) - \sum_{\alpha=1}^n \lambda_{\alpha} C(\mathbf{u}, \mathbf{u}_{\alpha}) \quad (6.14)$$

Which is exactly the kriging variance. Thus the missing variance is the kriging variance. When kriging at a data location, the kriging variance is zero and there is no missing variance. When kriging with no local data the kriging variance is  $C(0)$  and all of the variance is missing.

## Petrophysical Property Simulation

Simulation addresses the cons of kriging and still makes use of the kriging algorithm and all of the good things about kriging. One of the important things that kriging does correctly and provides the motivation for sequential simulation is that it gets the covariance between the data and the estimator right. The issue that leads to sequential simulation is the fact that although kriging gets the covariance correct between the data and the estimate correct but it fails to get the covariance between the estimates. That is, rather than use the estimators as additional data the kriging algorithm simply moves on to the next estimator not including the covariance between the new estimator and the last. Sequential simulation does just that. The first estimator is kriged with only data because that is all that there is, just data, no other estimates. The next estimate is kriged but the previous estimator is used as



**Sequential Simulation includes the Variance Between the Estimators, Kriging Does not.**

data and its variogram is included in the algorithm.

This is sufficient motivation to proceed sequentially with estimation, but there is still the issue of the missing variance. Recall that the missing variance is the kriging variance:

$$\text{Var}\{Z^*(\mathbf{u})\} = C(0) - \sigma_{\text{SK}}^2(\mathbf{u}) \quad (6.15)$$

This missing variance must be added back in without changing the variogram reproduction properties of kriging. This is done by adding an independent component with a zero mean and the correct variance to the kriged estimate:

$$R(\mathbf{u}) = Z^*(\mathbf{u}) - m(\mathbf{u}) \quad (6.16)$$

$$Z_s(\mathbf{u}) = Z^*(\mathbf{u}) + R(\mathbf{u}) \quad (6.17)$$

The sequential simulation workflow is as follows:

1. Transform the original Z data to a standard normal distribution (all work will be done in "normal" space). We will see later why this is necessary.
2. Go to a location  $\mathbf{u}$  and perform kriging to obtain kriged estimate and the corresponding kriging variance:

$$Z^*(\mathbf{u}) = \sum_{z=1}^N \lambda_z \cdot Z(\mathbf{u}_z)$$

$$\sigma_{sk}^2(\mathbf{u}) = C(0) - \sum_{z=1}^N \lambda_z C(\mathbf{u}, \mathbf{u}_z)$$

- Draw a random residual  $R(\mathbf{u})$  that follows a normal distribution with mean of 0.0 and variance of  $\sigma_{sk}^2(\mathbf{u})$ .
- Add the kriged estimate and residual to get simulated value:

$$Z_s(\mathbf{u}) = Z^*(\mathbf{u}) + R(\mathbf{u})$$

Note that  $Z^*(\mathbf{u})$  could be equivalently obtained by drawing from a normal distribution with mean  $Z^*(\mathbf{u})$  and variance  $\sigma_{sk}^2(\mathbf{u})$ .

- Add  $Y_8(\mathbf{u})$  to the set of data to ensure that the covariance with this value and all future predictions is correct. As stated above, this is the key idea of sequential simulation, that is, to consider previously simulated values as data so that we reproduce the covariance between all of the simulated values.
- Visit all locations in random order (to avoid artefacts of limited search).
- Back-transform all data values and simulated values when model is populated
- Create another equiprobable realization by repeating with different random number seed

(Deutsch, 1999)

## Why a Gaussian (Normal) Distribution?

The key mathematical properties that make sequential Gaussian simulation work are not limited to the Gaussian distribution. The covariance reproduction of kriging holds regardless of the data distribution, the correction of variance by adding a random residual works regardless of shape of the residual distribution (the mean must be zero and the variance equal to the variance that must be re-introduced), and the covariance reproduction property of kriging holds when data are added sequentially from any distribution. There is one very good reason why the Gaussian distribution is used: the use of any other distribution does not lead to a correct distribution of simulated values. The mean may be correct, the variance is correct, the variogram of the values taken all together is correct, but the "shape" will not be. In "Gaussian space" all distributions are Gaussian [13]. There is a second, less important, reason why the Gaussian distribution is used: the central limit theorem tells us that the sequential addition of random residuals to obtain simulated values leads to a Gaussian distribution. The construction of kriging estimates is additive. By construction, the residuals are independent and there are many of them. The only caveat of the central limit theorem that we could avoid is the use of the same shape of distribution, that is, we may avoid multivariate Gaussianity if the shape of the residual distribution was

changed at different locations. The challenge would be to determine what the shape should be. (Deutsch, 1999)

<b>The Pros and Cons of Simulation</b>	
<b>Pros:</b>	<b>Cons:</b>
Honors the histogram	Multiple images
Honors the variogram	Conceptually difficult to understand
Quantifies global uncertainty	Not locally accurate
Makes available multiple realizations	

## **Facies Simulation**

Facies are considered an indicator (categorical) variable and simulating facies requires indicator simulation.

## ***Lecture 6: Geostatistical Mapping Concepts, The Quiz***

### **Question 1**

Kriging is often referred to as a *best linear unbiased estimator* at an unsampled location. Explain the essential factors that are considered in establishing the weights of the kriging estimator?

### **Question 2**

Stochastic simulation corrects for the smoothing of kriging by (1) proceeding sequentially, and (2) injecting the missing variance at each step. What property of kriging leads us proceed sequentially? How much variance must be *injected*?

### **Question 3**

What is Monte Carlo simulation? How is Monte Carlo simulation performed from a cdf  $F(x)$ ?

### **Question 4**

Why is *declustering* used prior to stochastic simulation of porosity?

### **Question 5**

What are the features of a simulated realization that make it preferable to a Kriging map for oil reservoir evaluation? What features are not appropriate for Gaussian simulation?

## ***Lecture 6: Geostatistical Mapping Concepts, The Quiz Solutions***

### **Question 1**

Kriging is often referred to as a *best linear unbiased estimator* at an unsampled location. Explain the essential factors that are considered in establishing the weights of the kriging estimator?

1. closeness to the location being estimated
2. redundancy between the data values
3. anisotropic continuity (preferential direction)
4. magnitude of continuity / variability
5. the variogram

### **Question 2**

Stochastic simulation corrects for the smoothing of kriging by (1) proceeding sequentially, and (2) injecting the missing variance at each step. What property of kriging leads us proceed sequentially? How much variance must be *injected*? Exactly how do we "inject" variance in a Kriging estimate?

The covariance is correct between the data and the estimator, the only covariance not accounted for is the covariance between the estimators

the missing variance is the kriging variance

inject a Monte Carlo simulated random residual by addition to the estimate



### **Question 3**

What is Monte Carlo simulation? How is Monte Carlo simulation performed from a cdf  $F(x)$ ?

draw a random number read the appropriate quantile

### **Question 4**

Why is *declustering* used prior to stochastic simulation of porosity?

We use the distribution in sequential Gaussian simulation so the distribution must be as accurate as possible

### **Question 5**

What are the features of a simulated realization that make it preferable to a Kriging map for oil reservoir evaluation? What features are not appropriate for Gaussian simulation?

models the heterogeneity whereas the kriging map smooths out the heterogeneity

features that are connected like permeability are not appropriate for simulation

## **Lecture 7: Structural Modeling**

- 
- [Velocity Uncertainty](#)
- [Surface Based Modeling](#)
- [Surface Flapping Distribution](#)
- [Fault Handling](#)
- [Lecture 7 Quiz](#)

### **Introduction**

It is estimated that hydrocarbon reserves recoverable through improved reservoir management exceed new reserves that can be added through exploration. Increasingly, it is being recognized that 3D seismic data analysis is a critical reservoir management technology and plays a key role in reservoir detection, delineation, characterization, and monitoring. However, 3D seismic alone is inadequate for many applications due to (1) limited resolution, and (2) the indirect and/or weak relationships between critical reservoir parameters such as permeability, porosity, and water saturation. As a result, it is generally recognized by reservoir scientists that proper reservoir description and monitoring require full integration of 3D seismic with engineering, geological (including geochemical and geostatistical), petrophysical, and borehole geophysical methods.

Seismic is very good at many things such as resolving large scale structural features, recovering information from 99.99% of the volume of interest, but it is not good at resolving fine scale features such as resolving petrophysical properties, this task is left to geostatistics. Other more intimate petrophysical property sensing tools such as logs offer fine scale measures of petrophysical properties but offer no insight into what lies beyond the tools range, this task is left to geostatistics. Core data is an even more finely scaled measure of petrophysical properties, but its range of insight is even less than that of log data. Geostatistics

Geostatistics uses the coarse-scale structural information offered by seismic, the mid-scale information offered by electric logs and the fine scale information of core data to generate high resolution models of oil reservoirs. A reservoir model is built starting with the large scale features, the structural features, first. This lecture will discuss the uncertainty in interpreted seismic surfaces, and in the event there is no reliable seismic data, how to simulate the surfaces that would define structure and fault handling.

### **Velocity Uncertainty**

The geophysical branch of the exploration science is primarily concerned with defining subsurface geological features through the use of seismic techniques, or the study of energy wave transmissions through rock. Seismic techniques can be used to make subsurface maps similar to those developed by standard geological methods.

The three main rock properties that the geophysicist studies are 1) elastic characteristics, 2) magnetic properties, and 3) density. Although studies of density and magnetic properties provide useful information, elastic characteristics are considerably more important since they govern the transmission of energy waves through rock. It is this elastic characteristic that is studied in seismic surveys. The word seismic pertains to earth vibrations which result from either earthquakes or artificially induced disturbances.

Reflection seismic surveys record the seismic waves that return or reflect from subsurface formation interfaces after a seismic shock wave has been created on the surface. By measuring the time required

for different waves to be reflected from different formations, the geophysicist can identify structural variations of the formations. Figure 7.1 illustrates this process in a typical land survey operation.

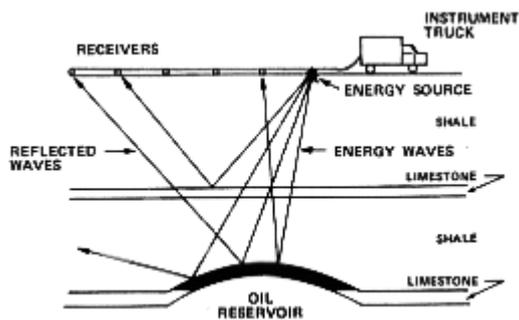


Figure 7.1, a typical land survey operation.

The objective of seismic work is to develop maps that indicate structures which might form traps for oil or gas from the data provided on the record cross sections. The geophysicist makes maps by calibrating the seismic attribute to core data, well log data, and analogue data. Any feature that causes a change in propagation of sound in rock shows up in the seismic survey. Changes in dip, different rock types, possible faults, and other geological features that are some of the features indicated in the sections. These features are not immediately evident in the seismic data. Several steps are necessary to convert seismic data into useful structural and stratigraphic information. Even the best data processing techniques cannot completely filter out all the unwanted "noise" and distortion, so making good, reliable interpretations of seismic data requires judgment and experience. Some sources of uncertainty include:

- **Survey Information:** seismic information measures the time it takes for energy waves to propagate through rock, the velocity of the waves travelling through rock. Velocity is distance divided by the amount of time it takes to traverse the distance in the numerator. The frequency of the energy source is known, as is the distance between the energy source and the geophone on present-day surface. This sufficient information to resolve the location of structures below present-day surface. If the the distance between the source and receiver is not known then there will uncertainty in the location of sub-surface.
- **Density:** one of the rock properties that affects the velocity of the waves is the density. One of the things that seismic is very good at is capturing distinct changes in wave velocity and wave velocity is a function of density. Very dense rock has higher wave velocities than less dense rock. The density of rock is a result of the rock type, the porosity, the contents of the pore spaces (water is incompressible and thus allows energy waves to pass whereas gas is incompressible and will not allow energy waves to pass), and the overburden pressure (the further down you go the greater the overburden pressures are and the greater the velocity of energy waves). The density must be assumed or derived from core data. Hopefully the core information intersects all possible rock types in the area of study. Imagine a shale (of highly variable density) in the middle of the deposit yet unaccounted for in the well data. The results would be detrimental to the interpretation of sub-surface. Oil, water, and gas have very different densities and hence oil / water and oil / gas interfaces are easily distinguished.
- **Human Interpretation:** there are elaborate algorithms for deciphering seismic information, but human interaction is still required. The decisions made by humans have uncertainty imbedded in them, there is no way of knowing the truth.

The uncertainties derived from velocity can permeate every aspect of a geostatistical reservoir study. The result is uncertainty in the structural surfaces used for modeling the reservoir. Some of the repercussions include:

- **Gross Volume Estimates:** there is a need for accurate estimates of original volume of hydrocarbon in the reservoir. These estimates are used for determination of the economic viability of the reservoir, comparison of the economic merits of other ventures, determination of the appropriate size of production facilities, and others. Uncertainty in velocities translates to uncertainty in location, extent and magnitude of the reservoir in question which in turn translates to uncertainty in volume estimates.
- **Flow Simulation:** The prediction of reservoir performance and different production scenarios is a critical step in reservoir evaluation. The optimal number and location of production / injection wells can be significantly affected by even one shale structure (impedance to flow). Velocity uncertainty can make important structures indiscernible and thus significantly impact flow simulation.
- **3-D Connectivity:** Recovery strategies strive to have the greatest possible connectivity between producing structures. Velocity uncertainty can fail to reveal important structures the might impact well location and the value of infill wells.

## Surface Based Modeling

There are a few geostatistical methods for simulating surfaces that use well data for conditioning and seismic data as soft data. Keep in mind the purpose behind surface based modeling is not to get the surface per se, but to define the sediment packages. One method models the geology as simple parametric surfaces. The approach to this method is to simulate construction the reservoir in the same way that it was created; as a series of depositional events. Reservoirs are characterized by large-scale geologic events. Some of these events are depositional and some are erosional, but the interface for each of these events marks a time surface. In the case of a depositional environment the material between the base time surface and the top surface indicate a period of homogenous deposition. That is, the sediment package sandwiched between the top and base surfaces contain material that is homogenous in terms of its genesis and hence its petrophysical features. These large-scale events occur over long periods in time. During the large-scale events small-scale events occur within the time surfaces of the large-scale events. Figure 7.2 shows that a reservoir is constructed in a hierarchical manner; large scale features followed by small scale features,

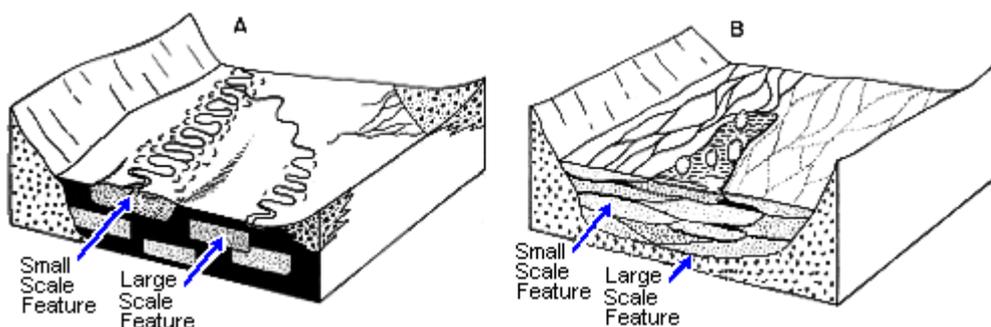
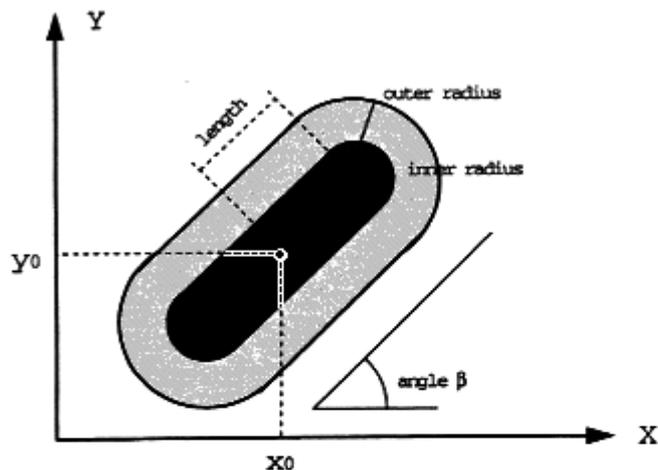


Figure 7.2 An illustration of the hierarchical nature of a typical reservoir.

each bound by a time surface. Large-scale time surfaces are usually visible with seismic data, and sometimes small-scale time surfaces are discernable, but this is not always the case. Sometimes there is no good seismic data and the time surfaces are not available. The goal behind surface modeling is

not to get the surfaces, but to provide constraints for modeling facies and petrophysical properties. Therefore, the constraints that define a surface can include not just time but other properties such as grain size and trends such as fining/coarsening upward. Once these surface parameters have been defined then the properties within the sediment package, such permeability or porosity can be modeled bound by surfaces. The method eluded to earlier, the use of simple parametric surfaces to mimic geologic time surfaces is particularly useful for modeling both large and small scale geologic events. Figure 7.3 illustrates a simple parametric surface and the parameters for its definition.

Top View of Idealized Surface



Cross Section of Idealized Surface



Figure 7.3 A simple parametric surface used for simulation of geologic event surfaces. The surface is parameterized by a center point  $x_0, y_0$ , an inner and outer radius tolerance, a length tolerance, an orientation angle, and a surface height.

The simulated surfaces are parameterized by a center point  $(x_0, y_0)$ , an inner and outer radius tolerance for the surface, a length tolerance, an orientation angle, and a maximum surface height. The center point is used by the algorithm to determine the location of the simulated surface. The inner and outer radius tolerance are user defined tolerances that are used to constrain the aspect of the surface to those that are in the reservoir. The length and height of the surfaces are also entered by the user as a tolerance so that the dimensions of the surfaces may be limited to those found in the reservoir. The angle of orientation allows the user to orient the surface to the direction of deposition indicated by seismic or core data. Geologic surfaces are rarely smooth in reality so another parameter called undulation is added so that the surface better approximates reality. Figure 7.4 illustrates the concept of undulation.

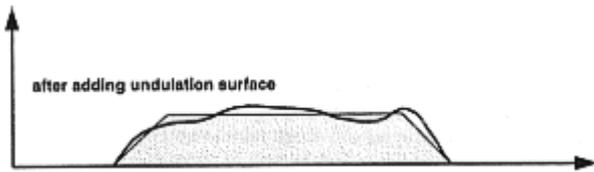


Figure 7.4 The parametric surface after "undulation" has been added.

The model is built surface by surface and each surface is deposited on top of the existing surfaces (if there are any, and there wont be at first) using the following protocol:

1. The central location of the new surface ( $x_0, y_0$ ) is selected stochastically. The distribution used for selection of the location is derived from the distribution of possible locations given the thickness of the reservoir. At the beginning of the simulation all surfaces have the same probability of selection, but as the simulation continues the reservoir builds up thickness and there fewer permissible surfaces that will comply to the selection and conditioning criteria.
2. The length of the surface  $X$  is randomly selected from a triangular pdf with the minimum and maximum parameters being user selected
3. The inner and outer widths, the height of the surface and the orientation of the surface are selected from a triangular distribution with parameters provided by the user.
4. The surface is "dropped" onto the reservoir. Any existing surfaces will truncate the new surface. Figure 7.5 shows the dropping principle in action.

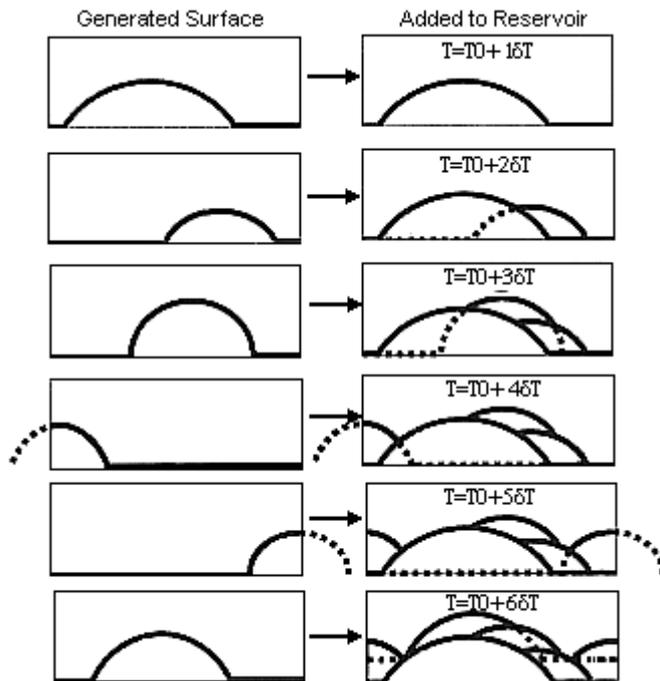


Figure 7.5 The dropping principle used in the simulation of surfaces.

5. Condition the surface to the data. All of the surfaces are dropped to the same datum as indicated in figure 7.5. There are two solutions if the surface does not conform to the intersections provided by the well data (1) raise the surface to meet the intersection, and (2) lower the surface to meet the intersection. If the surface is raised it could be rejected if it too short and will not be truncated by existing surfaces, instead, the surface is lowered to the intersection as in Figure 7.6.

Current Thickness of Reservoir With Single Conditioning Data

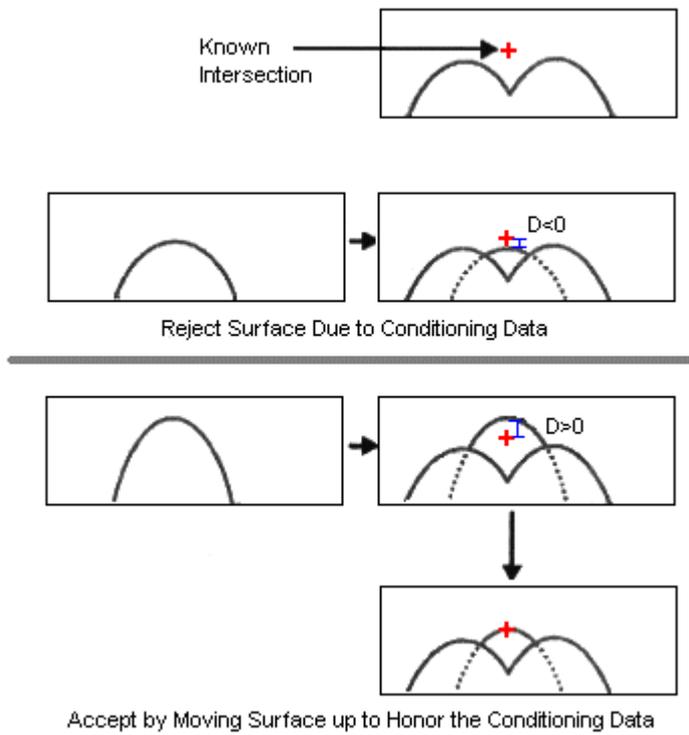
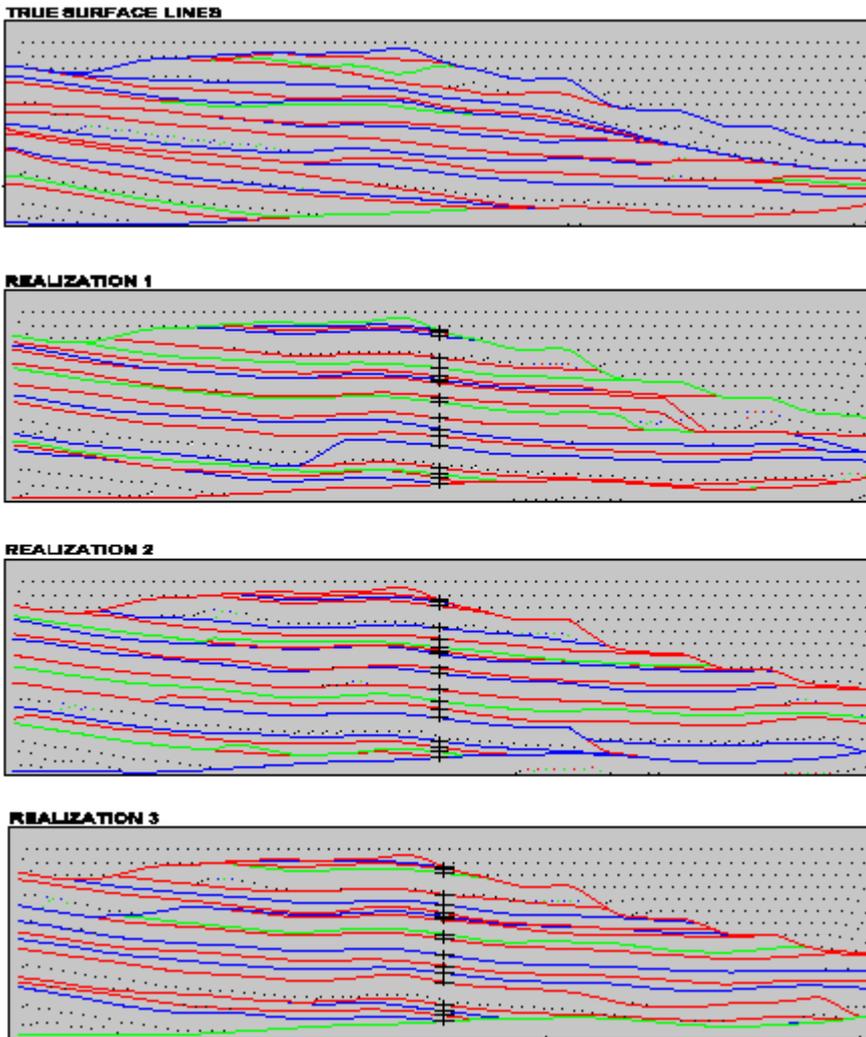


Figure 7.6 Conditioning of the surface to a single well data.

6. Repeat until the reservoir is fully populated

Figure 7.7 shows an example of simple parametric surface simulation.



## Surface Flapping

Surface flapping is an acronym for surface uncertainty. Figure 7.8 shows surface flapping. The pink vertical lines are well data that the surface must be conditioned to. The light blue horizontal line is the gas oil contact and the pink horizontal line is the oil water contact. The dark blue lines are top surface realizations and the green lines are bottom surface realizations. There is uncertainty in the true location of the top surface and the bottom surfaces everywhere except at the wells. The blue and green lines illustrate the extent of uncertainty about these surfaces. The green lines do not flap as wildly as the blue lines. There is sound reasoning for this. Surface uncertainty cannot be assessed independently, once the uncertainty in the top surface with respect to the present-day surface has been established all other remaining surfaces will have less uncertainty. The uncertainty in remaining surfaces is accounted for in the uncertainty with respect to the distance between layers. One could imagine considering each of the surface uncertainties modeled independently but in doing so negative volumes could be created (the surface lines could cross). Also the distribution of thickness would be ignored. In those locations where the top and bottom surfaces cross there might be at minimum zero thickness, but the seismic derived distribution of thickness might suggest that there is a very low possibility of zero thickness. This is why we model the top surface uncertainty first conditioned to the uncertainty of the surface with respect to the present-day surface and all remaining surfaces are modeled conditional to the uncertainty of the thickness between surfaces. An important point that must be pointed out is velocity uncertainty and how this uncertainty relates to depth / surface determination is not considered here, this is a simplified model meant only to illustrate surface uncertainty with respect to the well data. The uncertainty in the surfaces can be modeled using sequential Gaussian simulation



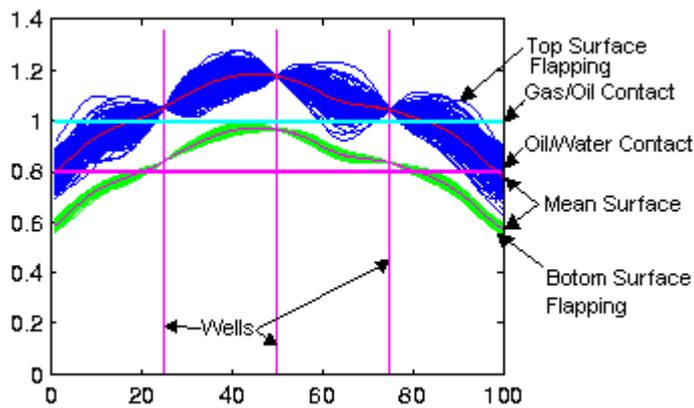


Figure 7.8 A diagram of surface uncertainty.

Assessing the uncertainty in surfaces is important for the determination of pore volume and hence predicted oil in place volumes. For example consider the calculation of the gross pore volume:

$$\text{Pore Volume} = \text{Gross Rock Volume} \cdot \text{Net-to-gross Ratio} \cdot \text{Net Porosity}$$

The net-to-gross ratio and net porosity are inferred from the well data, available seismic data and geological interpretations. There are uncertainties existing in the determination of the net-to-gross ratio and the net porosity due to limited well data and uncertainty in the calibration of soft seismic and geological data. Uncertainties in all factors propagate to uncertainty in the final calculation of pore volume. The uncertainty in pore volume is a function of the multivariate distribution of the three contributing factors: GRV, net-to-gross ratio, and net porosity. Inference of this multivariate distribution is difficult due to the poorly known dependencies such as the relationship between porosity and surface interpretation. A particular model of this multivariate distribution can be built assuming that the three factors are independent. We will adopt such a model. The distributions of uncertainty in the three controlling variables must be determined.

The top and bottom surfaces will be stochastically modeled to quantify the distribution of uncertainty in the GRV. This modeling is guided by well data and the best estimate of the surface from seismic. The uncertainty of the average net-to-gross ratio and the net porosity are determined by bootstrap resampling from the best distribution that can be inferred from limited well data and supplementary seismic and geologic data. The *gross rock volume* is the reservoir volume above the oil/water contact (OWC) constrained by the top and bottom surfaces of reservoir. A gas-oil contact is needed for reservoirs with gas. Figure 7.9 shows a cross section view of a reservoir.

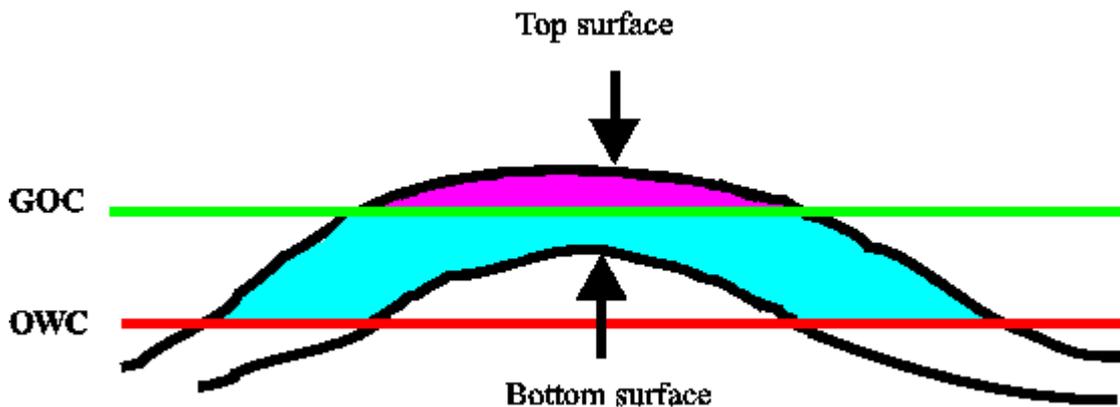


Figure 7.9. A cross-section of a hypothetical oil reservoir.

The reservoir is constrained by a top and bottom surfaces (black curves). The OWC is represented by a red horizontal line and the gas-oil contact *GOC* is denoted by a green horizontal line. The oil-containing volume of the reservoir is the portion of the reservoir constrained by both top/bottom surfaces and the OWC/GOC levels (light blue shaded area), whereas the gas-containing volume is the portion of reservoir constrained by top/bottom surfaces and above the GOC level (pink shaded area). The oil and gas-containing volumes of the reservoir are of economic significance. The volume below the hydrocarbon contacts is also of importance in accounting for historical production data and in the prediction of aquifer drive. Once the distributions of the three factors are available, the uncertainty of *pore volume* is estimated by Monte Carlo simulation. Values of gross rock volume, net-to-gross ratio and net porosity are sampled by Monte Carlo, and the value of pore volume is calculated. The procedure may be repeated many times and the distribution of *pore volume* is thus estimated. Figure 7.10 shows the surface uncertainty given three wells.

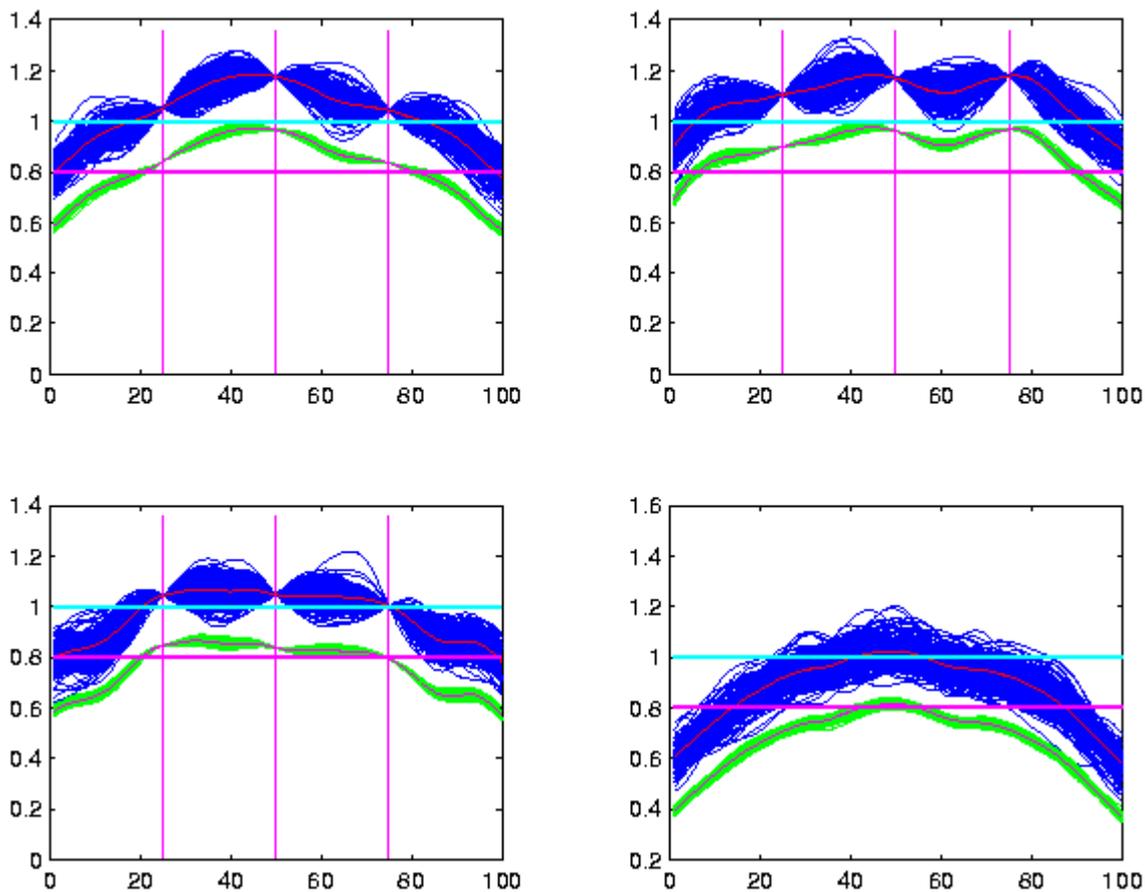


Figure 7.10 The bottom right hand image is an unconditioned surface section of a hypothetical oil reservoir. All other images are the result of multiple simulations with conditioning data.

## Fault Handling

Faults can be considered as surfaces. Faults, in comparison to the surrounding rock, have no appreciable thickness and are thus appropriately modeled as surfaces. One can imagine modeling faults in the same fashion as indicated above; model as a surface conditioned to well data and to the seismic distribution of uncertainty regarding the location of the fault.

As with the aforementioned surfaces, fault surface uncertainty bears significant importance to reservoir characterization / flow simulation. This especially the case with faults because all reservoirs have undergone structural deformation and have faults/joints/deformation bands that are stress-release features which can (1) enhance permeability, (2) decrease permeability as a result of cements filling fractures and and disconnecting once connected structures, and (3) occur at all scales. Small scale faults or fault sets are usually of low priority because (1) big faults are most important and are identified by seismic (2), poorly understood, input statistics (spacing, density, size, characteristics) are very difficult to infer, and (3) small scale fractures can be approximately handled by effective flow properties.

In general faults are difficult to work with as they present problems that are beyond the scope of this lecture. Faults tend to disappear and reappear, and have significant discontinuities. These issues are difficult to deal with in geostatistics. Normal faults are generally coped with by simple coordinate transformation, but this may not be an appropriate solution in all cases, the model may have to be broken up into smaller pieces where this assumption is valid.

## **Lecture 7: Structural Modeling, The Quiz**

### **Question 1**

We are not explicitly trying to model surfaces that define geological structures. What are we trying to model? Why?

### **Question 2**

What kind of data is geophysics useful for (in the context of this lecture)? What is the place of geostatistics, and how does it integrate with geophysical data to provide accurate models?

### **Question 3**

Why model the surface closest to present-day surface first? Why not model each surface with the same uncertainty? What does this technique prevent?

### **Question 4**

Name two sources of velocity uncertainty, and suggest how these sources can impact the reservoir model.

### **Question 5**

Small scale faults are often ignored in the reservoir model, why?

[solutions](#)

July 31, 1999

## **Lecture 7: Structural Modeling, The Quiz Solutions**

### **Question 1**

We are not explicitly trying to model surfaces that define geological structures. What are we trying to model? Why?

We are trying to model the geologic event interfaces, and hence, the sediment packages housed between the surfaces.

Sediment packages constrain the distribution of properties to genetically homogeneous environments

### **Question 2**

What kind of data is geophysics useful for? What is the place of geostatistics, and how does it integrate with geophysical data to provide accurate models?

Large scale features, density, elastic properties of rock.

Geostatistics quantifies uncertainty, and models heterogeneity, and it can do this at greater resolution than geophysical data.

Geophysical samples, specifically seismic samples, sample nearly the entire volume of interest. Core and log data only sample small fractions of the volume of interest.

### **Question 3**

Why model the surface closest to present-day surface first? Why not model each surface with the same uncertainty? What does this technique prevent?

The first surface captures the uncertainty from the present-day surface down to the depth of the reservoir; subsequent surface uncertainty is the incremental uncertainty in the distance between layers.

Once one surface is determined the uncertainty in the location of others is reduced

Estimating / simulating each surface can create negative volumes, an impossibility. It is better to model first the uncertainty in the top surface then model the thickness between layers.

### **Question 4**

Name two sources of velocity uncertainty, and suggest how these sources can impact the reservoir model.

X, Y, Z, errors can increase the uncertainty in almost all volumetric measures.

Human interpretation can lead to the addition / negation of important geological features that can impact flow simulator results.

## **Question 5**

Small scale fractures are often ignored in the reservoir model, why?

It is assumed that small scale fractures can be approximately handled by effective flow properties.

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## **Lecture 8: Cell Based Facies Modeling**

- [Methodology](#)
- [Sequential Indicator Methods](#)
- [Truncated Gaussian Methods](#)
- [Cleaning Cell Based Realizations](#)
- [Lecture 8 Quiz](#)

### **Introduction**

Reservoir simulation and decision making requires 3D distributions of petrophysical attribute like porosity, permeability and saturation functions. There is no direct need for lithofacies models. Nevertheless lithofacies are considered important because petrophysical properties are often highly correlated within the lithofacies types. Knowledge of the lithofacies types serves to constrain the range of variability to the distribution of properties within the lithofacies. Lithofacies are distinguished by different grain size diagenetic alteration, or any other distinguishing feature. For example shale is different than sandstone which is different than limestone which is in turn different than dolomite. It should be born in mind however that we are first and foremost concerned with making realistic distributions for the purpose of decision making, not making pretty pictures

### **Methodology**

Before even beginning to model lithofacies one should ask if it is a worthwhile venture. Modeling lithofacies may not always yield improved prediction of reservoir performance. To make this decision easier consider the following:

1. The lithofacies types must have significant control over the petrophysical properties. Significant control is a contentious issue, however a useful guide would be to consider a difference in the mean, variance, and shape of at least 30%. As well the saturation function should not overlap between lithofacies, and here there should also be a difference in the overall average of 30% between lithofacies.
2. Distinct lithofacies must be easily discerned in well log data and core data.
3. the lithofacies must be at least as easy to model as the petrophysical properties. Simplicity rules here. Overly elaborate models will be detrimental to the model.

How many lithofacies should be modeled? The number of lithofacies to be modeled is a decision that must be made at the time of modeling, however, there are some pieces of advice to be offered. Two "net" and one "non-net" (net means oil bearing) lithofacies often provide sufficient detail for most reservoirs. The limit of workability is three net and three non-net lithofacies. Beyond this limit models become nearly unworkable.

With the lithofacies distinguished select the modeling technique:

1. cell based modeling (purely stochastic)
2. object based modeling (partly stochastic and partly deterministic)
3. deterministic modeling (purely deterministic)

Note that deterministic modeling is always preferred. Why leave things to chance when know the response?

Cell based modeling is by far the most preferred method for modeling lithofacies. Some of the reasons include:

1. Local data are honored in the model.
2. The required statistics can be inferred from local data (well and log data).
3. Soft data and large scale trends (small scale trends are handled by the data) are incorporated into the model in a straight forward fashion.
4. The results appear realistic especially where there are no discernable geologic formations.

It should be noted that while most reservoirs fall into a single systems tract (depositional environment) such as fluvial, deltaic and so on, it is possible that the reservoir may give way to some other systems tract. For example, a fluvial reservoir may change into a deltaic reservoir and a deltaic reservoir may turn into a deep water reservoir. Large scale changes in facies, such as tract changes, must be accounted for in the lithofacies model, despite the modeling technique. In cell based modeling systems tract changes are modeled as trends, both vertically and horizontally. In object modeling of facies, objects can be constructed so as to include the tract change, i.e. change in dimension relative to the direction of change (in a way, kind of like including a trend).

## Sequential Indicator Simulation

Sequential indicator simulation is a widely used because it generates indicator models that account for the proportions of each indicator (histogram), and accounts for measures of spatial correlation (the indicator variogram) and trends. Lithofacies are categorical (indicator) variables and are amenable to indicator techniques. As a review, recall that continuous variables and indicator variables must be handled a little differently, consequently the summary statistics are a little different for indicator variables. Let  $\{s(\alpha), \alpha = 1, \dots, n\}$  be the set of observations of the categorical attribute  $s$  ( $s$  is used to indicate the state of the variable) measured on  $n$  individuals  $\alpha$ . The set of possible states  $s_k$  that any value can take is denoted by  $\{s_1, \dots, s_k\}$ . The  $K$  states are exhaustive and mutually exclusive in that each individual of the entire set can belong to one and only one category. In other words:

$$i(\mathbf{u}, s_k) = \begin{cases} 1 & \text{if } (\mathbf{u}) \text{ is in category } s_k \\ 0 & \text{otherwise} \end{cases} \quad k = 1, \dots, K \quad (8.1)$$

A little decoder for the terminology:

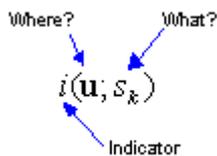


Figure 8.1

Comparing categorical statistics to continuous statistics:



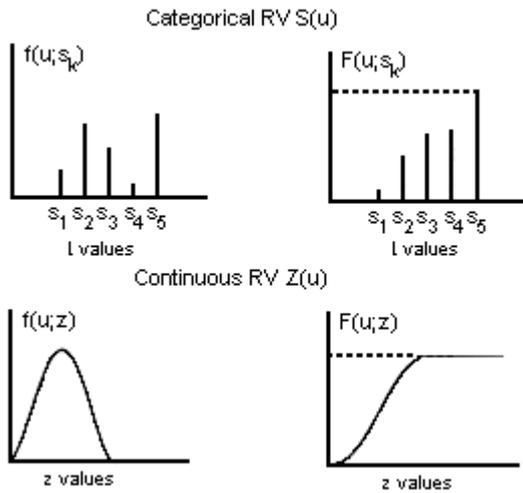


Figure 8.2

Note that for each estimated / simulated node there is a probability that the node could be any of  $K$  indicators, unless it is a conditioning data. For example At node  $x, y, z$  table 8.1 defines the proportions for each indicator. The proportions can be interpreted as probabilities, and as such the sum of the proportions at any node must be one.

Rock Type	Estimate
RT1	13%
RT2	57%
RT3	30%

Table 8.1 A table indicating the proportions for lithofacies at a single node. The sum of all of the proportions is 1.

### Indicator Variograms

Indicator variograms are calculated in the same way as continuous variable variograms, the only difference is the variable for the variogram is use of indicator formalism:

$$\gamma_I(\mathbf{h}; z_k) = \frac{1}{2N(h)} \sum_{z=-}^N [Z(\mathbf{u}; z_k) - Z(\mathbf{u} + h; z_k)]^2 \quad (8.2)$$

For example, suppose that we are calculating the variogram for the category  $z_k$ , and that 4 out the 5 data that fall into the variogram window tolerance fall into the the category , then the variogram calculation would be:

$$\gamma_I(\mathbf{h}; z_k) = \frac{1}{2(5)} [(1-1)^2 + (1-1)^2 + (1-1)^2 + (1-1)^2 + (1-0)^2] = \frac{1}{10}$$

Alternatively, suppose that the data were not very continuous and that only 1 out of the 5 fell into category  $z_k$ :

$$\gamma_I(\mathbf{h}; z_k) = \frac{1}{2(5)} [(1-1)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2 + (1-0)^2] = \frac{2}{5}$$

Note that there would be a variogram for each of the indicators. The spatial correlation for each lithofacies must be known. Yet another reason to minimize the number lithofacies in a reservoir model. Why model more variograms than needed?

### Sequential Indicator Simulation

Just as in sequential Gaussian simulation each node is visited in a random path, and but the indicator algorithm is:

1. at each grid node find nearby original and simulated data (preserve the covariance)
2. construct the conditional distribution using indicator kriging (this is where the multiple variograms come in)
3. using Monte Carlo simulation draw a simulated value from the conditional distribution.
4. Check the results

Note that both simulated and original data must be included in the estimate so that the spatial structure between the original, the simulated and original, and the simulated values is correct. Both sequential Gaussian simulation and indicator simulation use kriging, but different flavors of kriging. Indicator simulation uses indicator kriging. The difference is the estimate provided by indicator kriging is a conditional probability distribution instead of a mean result. For example, consider an estimation of 3 lithofacies: rock types 1, 2, and 3. The hypothetical estimate for a single node could be:

Rock Type	Estimate
RT1	13%
RT2	57%
RT3	30%

Chart 8.1

The corresponding cumulative distribution function would be

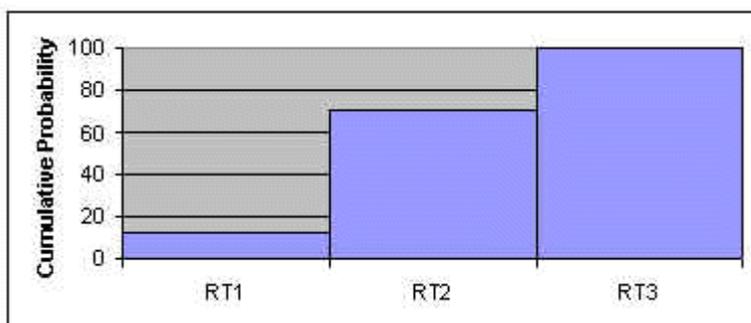


Figure 8.3

An interesting result of kriging is that it does not always give positive weights. This is a serious problem in indicator kriging because negative weights imply that the probability of occurrence is less than not possible, which is impossible. This issue is known as order relations. To remedy this all

kriging weights that are less than zero are equated to zero, and all of the remaining weights are standardized to sum to 1 with:

$$p_k = \frac{p_k^*}{\sum p_k^*} \quad (8.3)$$

Indicator simulation adds one additional step: a random number is drawn to read a quantile from the ccdf (conditional cumulative distribution function) and a assign a rock type. In this way simulation honors the proportions of the original data and the variogram.

## Truncated Gaussian Methods

Truncated Gaussian methods are widely used due to the versatility that they offer, their simplicity and ease of data integration. There are a couple of easily over come obstacles:

1. the conditioning data must be transformed into continuous Gaussian conditioning data for the purposes of simulation. This is done using the following formula:

$$y_k = G^{-1} \left( \frac{cp_{k-1} + cp_k}{p_k} \right) \quad (8.4)$$

where  $y_k$  is the transformed value,  $G^{-1}$  is an operator that extracts the normal scores value for the indicator given its proportion,  $cp_{k-1}$  and  $cp_k$  are the cumulative proportions for the  $k-1$  and the  $k^{th}$  states, and  $p_k$  is the number of states. For example given two indicators calculate the pseudo-Gaussian scores;  $cp_{k-1}=0$  (not in the  $k$  state) and  $cp_k=1$  (in the  $k^{th}$  state), and  $p_k=2$ , now read the 0.50 quantile from the standard normal Gaussian cdf, which is 0, therefore  $y_k=0$  (recall that standard normal Gaussian distributions have a mean, the 0.5 quantile, of 0 and a variance of 1)

2. the variogram must be calculated for the conditioning data. Using the pseudo-Gaussian data leads to inappropriately high nugget effects in the reproduced variogram. The recommended approach is to transform the indicator variogram of the most important variogram to a correspondingly appropriate normal scores variogram. This is not an easy task. Fortunately there is software to perform the transform with little user intervention.

Truncated Gaussian techniques require only one variogram. This is both an advantage and a disadvantage to the technique. It is an advantage because there is no need to model the variogram for all indicators, just the most important one. It is a disadvantage because there is no explicit definition of continuity for each indicator. Trends are injected as a locally varying threshold that is exhaustively defined throughout the area of study. Figure 8.4 illustrates, conceptually, how truncated Gaussian simulation works. The variable is simulated throughout the area of study, and at each node is compared to the locally varying threshold. If the simulated value falls below the lowest ( $k^{th}$ ) threshold then the node is labeled as indicator 1, if the value falls between the lowest and next highest threshold ( $k^{th}+1$ ), then it is labeled as indicator 2. This is repeated for each threshold.

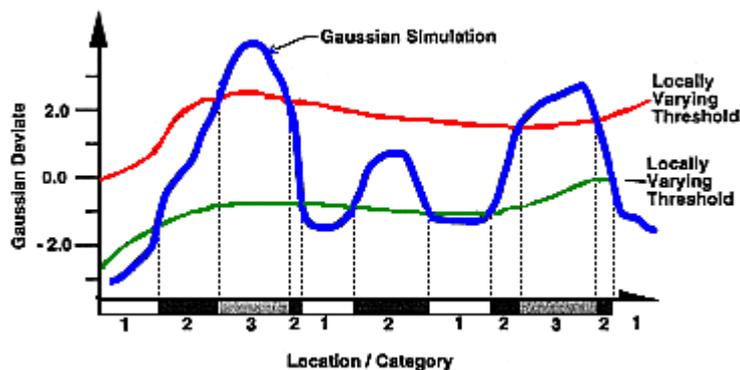


Figure 8.4

## Cleaning Cell Based Realizations

One concern with cell based lithofacies modeling is the presence of unavoidable short-scale lithofacies variations / noise. Noise makes the realizations appear unrealistic and make them less credible. In some cases the noise will make an impact on the flow simulation and predicted reserves; a more justifiable reason for considering cleaning of lithofacies realizations. Another concern is that the lithofacies proportions deviate from the original proportions. Usually the smaller proportions suffer. This is a product of the order relations carried out in the indicator kriging algorithm (correcting for negative kriging weights).

The easiest way out of this dilemma is to post process the realizations to honor the target proportions. One useful method for cleaning is quantile transformation. Quantile transformation works well when there is a natural nesting order to the lithofacies categories, such as a continuous variable transformed to a categorical variable, however, artifacts can occur when dealing with more than two unordered lithofacies.

## Maximum A Posteriori Selection

Maximum a posteriori selection (MAPS) replaces the lithofacies type at each location  $\mathbf{u}$  by the most probable lithofacies type based on a local neighborhood. The probability of each lithofacies type is based on the following criteria:

1. Closeness of the data in the window to the location  $\mathbf{u}$
2. whether the data is a conditioning data
3. mismatch from the target proportion

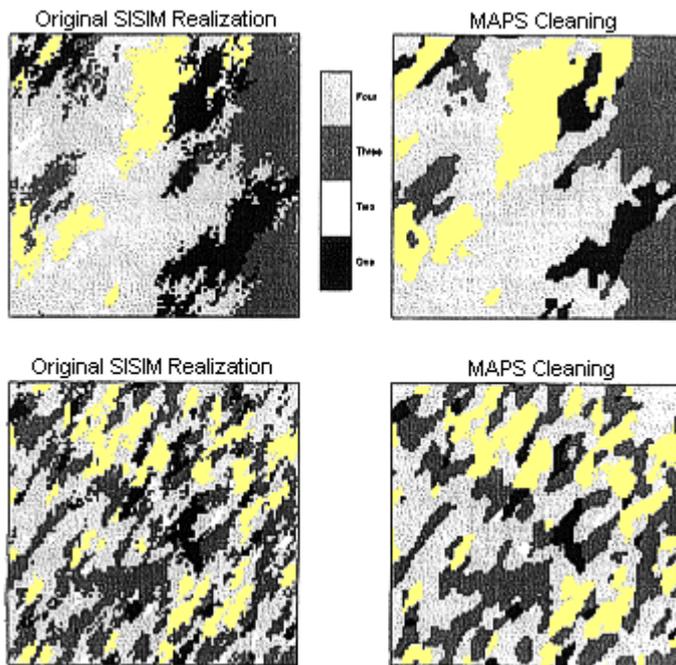


Figure 8.5 The MAPS algorithm at work...

Formally, consider an indicator realization,

$$i_k^{(0)}(\mathbf{u}), \quad k = 1, \dots, K$$

where the proportions of each lithofacies type in the realization are

$$p_k^{(0)}(\mathbf{u}) = \text{prob}\{I_k^{(0)}\} = p_k, \quad k = 1, \dots, K \in (0,1)$$

(the probability of the indicator being state  $k$  is between 0 and 1) with the sum of all proportions being equal to one, and the target proportions of each lithofacies bound by the same properties

Now consider these steps to cleaning the realization

$$i_k^{(0)}(\mathbf{u}), \quad k = 1, \dots, K \quad \mathbf{u} \in A$$

and bringing the the proportions

$$p_k^{(0)}(\mathbf{u}) = \text{prob}\{I_k^{(0)}\} = p_k, \quad k = 1, \dots, K$$

closer to the target proportions

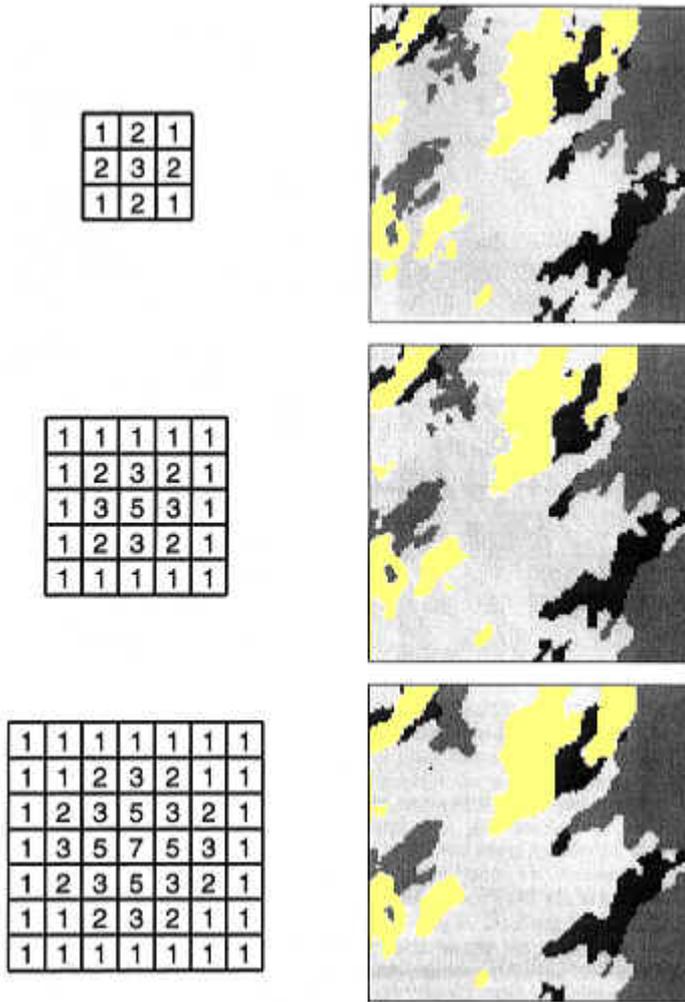
$$p_k(\mathbf{u}), \quad k = 1, \dots, K$$

At each of the  $N$  locations for all of the locations within the area of study, calculate the probability  $q_k(\mathbf{u})$  for each indicator based on a weighted combination of surrounding indicator values:

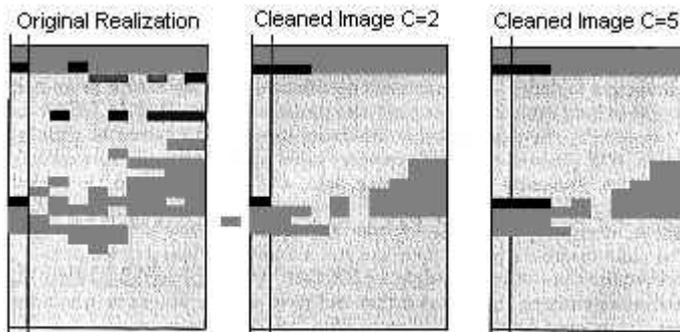
$$q_k(\mathbf{u}) = \frac{1}{\sum_{\mathbf{u}' \in W(\mathbf{u})} W(\mathbf{u}') \cdot c(\mathbf{u}') \cdot g_k} \cdot i_k^{(0)}(\mathbf{u}') \quad k = 1, \dots, K$$

where  $s$  is a constant to ensure that  $\sum q_k=1$ ,  $W(\mathbf{u}')$ ,  $c(\mathbf{u}')$ , and  $g_k$ ,  $k=1,\dots,K$  are weights accounting for closeness to  $\mathbf{u}$ , conditioning data and mismatch from global proportions, that is,

- $W(\mathbf{u}')$ , a weight to account for the closeness to  $\mathbf{u}$ : to make the best decision about what state the node should be it is necessary to establish what other lithofacies are present in the local neighborhood of  $\mathbf{u}$ . The definition of the local neighborhood, which could be defined by trends or anisotropy, and the nature of the weights (linear, quadratic, or some other function), determine the extent of cleaning. The extent of cleaning and the weighting function  $W(\mathbf{u}')$  must be determined by experiment and on an individual basis, therefore there are no rules for the magnitude or weighting functions; these decisions are left to the practitioner.



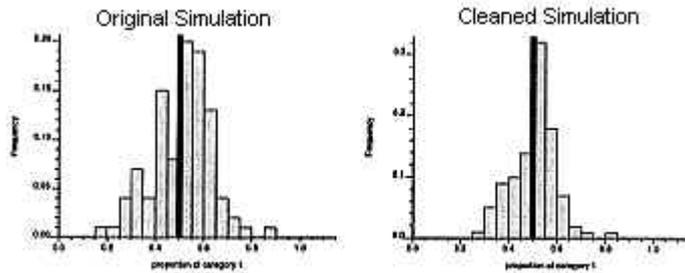
- $c(\mathbf{u}')$ , a weight to ensure the representitiveness of the conditioning data.  $c(\mathbf{u}')=1.0$  for all non-conditioning data locations, and equal to  $C$ , some user defined constant where  $C>10$  at data locations  
 $\mathbf{u}=\mathbf{u}_\alpha,$   $\alpha=1,\dots,n$



- $g_k$ =weight to ensure that the new lithofacies are closer to the target global proportions; specifically, increase the proportion if it is less than the target proportion, and decrease it if it is too high:

$$g_k(\mathbf{u}) = \frac{p_k}{p_k^{(0)}}, \quad k = 1, \dots, K$$

where  $p_k$  are the target proportions and  $p_k^{(0)}$  are the realization proportions.



As one would expect, the size of the window  $W(\mathbf{u}')$  and the distance weighting have significant impact on the "cleanliness" of the result.  $c(\mathbf{u}')$  has the effect of cleaning in favour of conditioning data.  $g_k$ =does not impose the target proportions, it merely moves the simulated proportions closer to the target proportions. The typical result is a reduction in the proportion variance.

Further improvement in the lithofacies reproduction could be achieved with user defined weights that would preferentially select lithofacies. Another method could be iterative cleaning. There are other methods for cleaning lithofacies simulation, but MAPS is easily understood and easy to implement. In MAPS the most probable lithofacies category is retained at each location. All categories are considered, whether the data is conditioning data, and the target properties are all considered in the cleaning algorithm. The only drawback is that short scale variation may be removed for the sake of visually acceptable maps.

## Work Flow

1. Assemble relevant well data with vertical and horizontal trend information for each lithofacies modeled.
2. Calibrate secondary data if possible. Soft data may or may not be useful beyond structural information. The applicability must be resolved by analysis.
3. Decluster. Spatially representative global proportions of lithofacies must be known (recall that lithofacies are categorical variables and the parameters that describe the distribution are not the same as those used for continuous variables. There is a review later in this section, but in the mean time, proportion refers to the fraction of each lithofacies in the reservoir).
4. Spatial analysis. The spatial correlation (indicator variogram) or size distribution (object based modeling) must be known.
5. Construct and validate the model.

## ***Lecture 8: Cell Based Facies Modeling, The Quiz***

### **Question 1**

Models are cleaned to the detriment of what reservoir property? How would flow simulator results be affected?

### **Question 2**

What motivating factors would promote modeling of lithofacies?

### **Question 3**

What distinguishes simple indicator kriging from simple kriging. In what way are the estimated values different?

### **Question 4**

In your own words explain how truncated Gaussian simulation simulates lithofacies.

### **Question 5**

What advantages are there to modeling lithofacies as cells?

[solutions](#)

August 2, 1999



## **Lecture 8: Cell Based Facies Modeling, The Quiz Solutions**

### **Question 1**

Models are cleaned to the detriment of what reservoir property? How would flow simulator results be affected?

Small scale heterogeneity can be lost

reduced measure of uncertainty

### **Question 2**

What motivating factors would promote modeling of lithofacies?

Petrophysical properties that differ by 30% in the mean, variance and shape, and non-overlapping saturation function in the sense that the mean saturation differs by at least 30%

### **Question 3**

What distinguishes simple indicator kriging from simple kriging. In what way are the estimated values different?

The use of indicators and an indicator variogram

The estimated values are proportions of each of the indicators

### **Question 4**

In your own words explain how truncated Gaussian simulation simulates lithofacies.

Generate a continuous variable and at each of the local thresholds classify the indicator

### **Question 5**

What advantages are there to modeling lithofacies as cells?

There are fewer parameters to be concerned with in comparison the object based modeling

August 2, 1999

## Lecture 9: Object Based Modeling

- [Methodology](#)
- [Object Based Fluvial Modeling](#)
- [Object Based Deep Water Systems](#)
- [Lecture 9 Quiz](#)

### Introduction

As in the previous lecture, there may or may not be a need to model the lithofacies. Two motivations for modeling lithofacies is to improve the accuracy of the reservoir model by injecting as much deterministic information as possible and for crisp geometric representations of geologic objects. Object based modeling injects more information than cell based modeling. The cost of having improved accuracy comes by way of time and effort. Object based models require much more effort because there are many more parameters to consider. Object based models are constructed in a hierarchical fashion; that is, the large scale features are modeled first, the large scale features are then disassembled and then small scale features are modeled within each of the large scale components, the small scale features are then populated with the desired petrophysical properties, the model is completed by reassembling the small scale components within the large and then large scale reassembly. This lecture will discuss some current aspects of cell based modeling.

### Methodology

Object-based stochastic 3-D modeling of well defined geometric objects pseudo genetically simulates depositional history of a reservoir. Two common objects that are simulated are fluvial channel complexes and fluvial channel and deep water lobes. Statistical control is obtained from cores, logs, seismic and outcrops. Specifically, object-based modeling techniques require information on the size, shape, and relationship between the different objects for statistical control. For braided fluvial reservoirs, some of the needed information includes:

- fraction of channel sand (could vary areally and vertically)
- width and thickness of channel sands (could vary vertically and follow a distribution of possible sizes)
- measures of channel sinuosity (depend on size of channel and vertical position)
- geometry of channel "families" or multi-story channels.

For example, the channel size could change with depth. Figure 9.1a illustrates the size of three different channels with respect to depth. This trend must be included in the model.

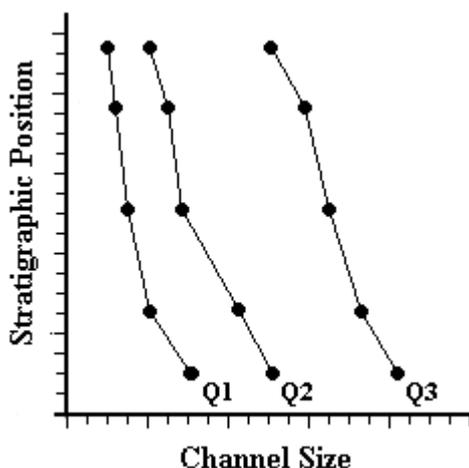


Figure 9.1a Shows the change in size of three different channels with respect to depth (stratigraphic position). (*Deutsch, 1999*)

The vertical axis on this plot represents restored stratigraphic position and the horizontal axis is channel thickness. The Q1, Q2, and Q3 lines represent the quartiles (25%, 50% and 75%) values of the distribution. Note how the channels are smaller at the top of the zone. Simulate the deposition of the reservoir by stochastically positioning geometric shapes

The methodology is hierarchical: start from the bottom and alternately lay down floodplain sediments and channel fill. Specify the distribution of channel sizes, spacing, and so on. The methodology presented here is simplistic, it is easy to imagine model crevasse deposits and point bar sands. Deep water systems are modeled in the same hierarchical fashion: start from the bottom and alternately lay down sediment lobes at the end of channels. As with all simulation algorithms honoring data is the goal. The need to honor the conditioning data can also be a problem when there is too much data as there will not be enough degrees of freedom for the algorithm.

## **Object Based Fluvial Modeling**

Object based Fluvial modeling is performed in a hierarchical manner. Figure 9.1 illustrates the hierarchy that will be adopted in this lecture. Each of the steps has a label. Each of the steps will be discussed below.

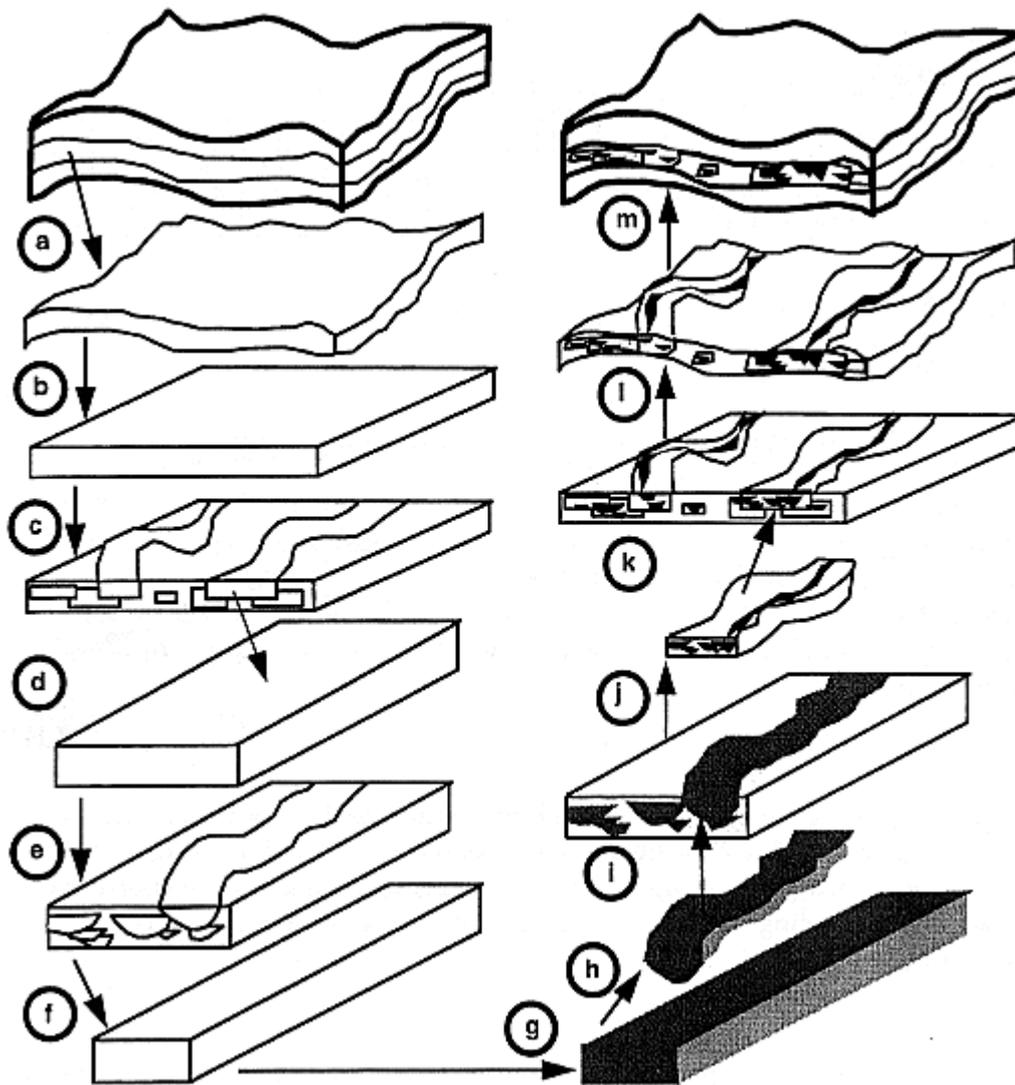


Figure 9.1b Illustration of the hierarchical object modeling method. (Deutsch, 1999)

The modeling approach in figure 9.1b continually adapts the coordinate system to the appropriate principle directions of continuity. This approach, while appearing quite complex, is quite simple to follow. In general the goal is to separate each of the components in the model, convert them to a regular volume, populate the model, and reassemble the model into its original format.

## STEP A

After modeling the geologic surfaces separate them for independent modeling. Recall that the picture of the depositional environment will never be complete for petroleum reservoir, thus the goal of geologic modeling is to characterize the reservoir. Since geologic parameters tend to be more uniform within the lithofacies the reservoir is broken into time zones that can be modeled independently (chronostratigraphic layers)

## STEP B

### Vertical Transform

There are several transforms in step b. The first is to transform the stratigraphic layer into a regular grid. This is achieved by using a  $z$  coordinate transform:

$$z_2 = \frac{z_1 - z_{rt}}{z_{rt} - z_{rb}} \quad (9.1)$$

where  $z_2$  is the transformed coordinate,  $z_1$  is the surface coordinate  $z_{rt}$  is the restored top and  $z_{rb}$  is the restored base. Note that, as in figure 9.2 the restored top and base need not conform to any specific surface, they only need house the layer of stratigraphic continuity. The transform may be reversed by:

$$z_1 = z_{rb} + z_2 \cdot (z_{rt} - z_{rb}) \quad (9.2)$$

Note that it is assumed that faults must be removed before the transform is performed.

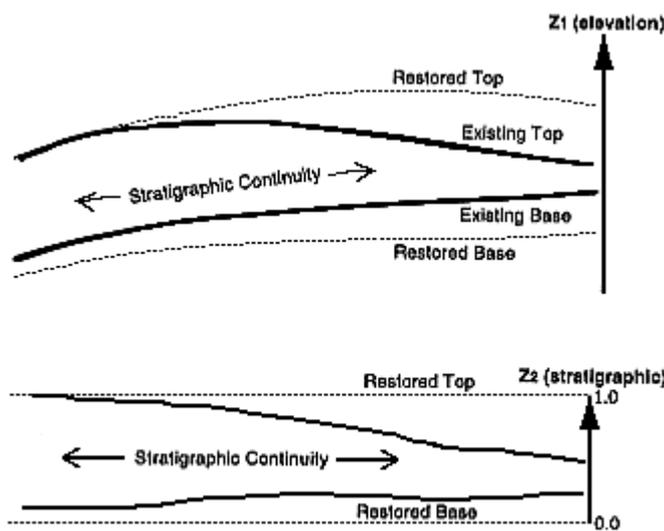


Figure 9.2 The  $z$  transform which transforms the stratigraphic surface into a regular volume.

### Areal Translation and Rotation

The reservoir coordinate system is based on some pre-existing or arbitrary coordinate system not aligned with the direction of maximal continuity. For ease of understanding, it is wise to translate and rotate the coordinate system to a coordinate system aligned with the maximal and minimal directions of continuity. Recall coordinate transforms:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x_1 - x_1^0 \\ y_1 - y_1^0 \end{bmatrix} \quad (9.3)$$

and the reverse transform:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} x_1^0 \\ y_1^0 \end{bmatrix} \quad (9.4)$$

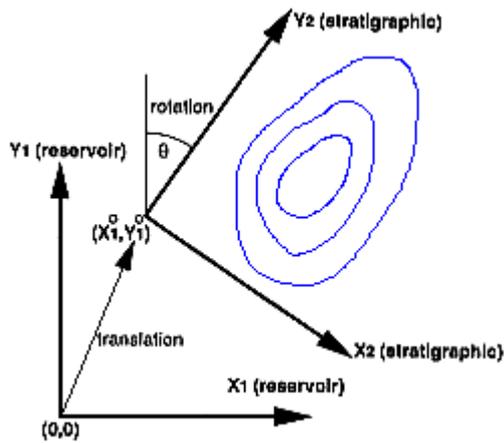


Figure 9.3 The coordinate system is often translated to align with the reservoir. The translation and rotation are performed using equation 9.3, and reversed using equation 9.4. (Deutsch, 1999)

### STEP C

At this step the channel complexes are modeled and separated from the stratigraphic layer. A channel complex can be defined as a large scale geologic structure.

### STEP D

Fluvial channels often cluster together in channel complexes and at times the channel may not be aligned with the direction of maximal continuity. Another rotation and translation is used to bring the in line with the center line of the structure:

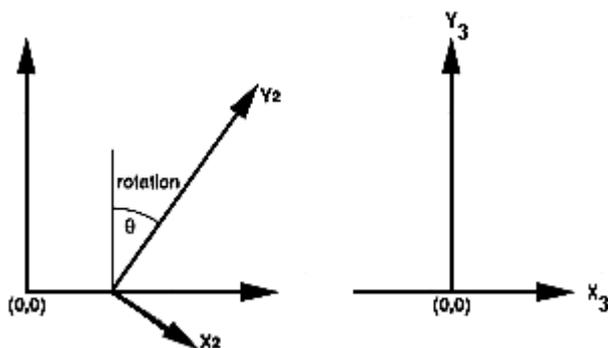


Figure 9.4 (Deutsch, 1999)

The transform uses the same matrices that we are familiar with for coordinate translation and rotation, however, there is no y translation:

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 - x_2^s \\ y_2 \end{bmatrix} \quad (9.5)$$

and the translation rotation can be reversed with:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} + \begin{bmatrix} x_2^j \\ 0 \end{bmatrix} \quad (9.6)$$

The channel is likely to be sinuous and of non-uniform width. To prepare the channel for modeling the sinuosity must be removed, transforming the channel into a uniform width. Before the channel is straightened one other issue must be dealt with: the width of the channel is measured perpendicular to the  $y_3$  axis and this measure of width is incorrect; the true measure of width is perpendicular to the tangent, at an angle to the  $y_3$  axis as shown in figure 9.5. The following correction is used to increase the apparent width based on the effective width:

$$W_a(y) = \frac{W(y)}{\cos\beta(y)} \quad (9.7)$$

where  $\beta(y)$  is the angle between the  $y_3$  axis and the local tangent to the channel center line.

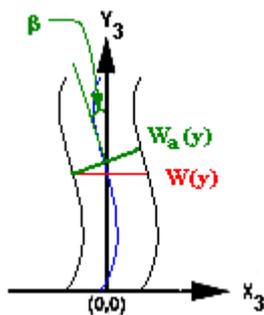


Figure 9.5 The addition of apparent width to repair the effect of sinuosity.

The channel still undulates; it must be straightened. The function  $x_4 = x_3 - f^{cc}(y_3)$  (9.8) measures the of the complex from the  $y_3$  axis, and translates the center line to the  $y_3$  axis as in Figure 9.6, and is reversed by  $x_3 = x_4 + f^{cc}(y_3)$  (9.9).

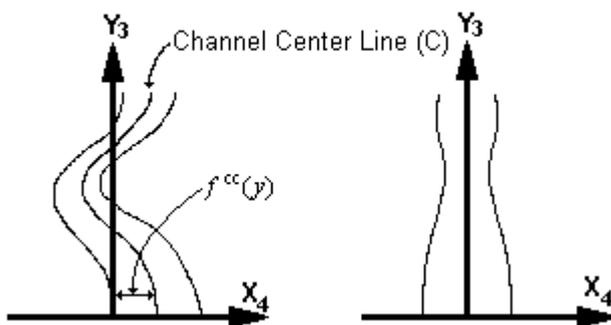


Figure 9.6 Transforming the sinuous complex structure into a straight structure using equation 9.8. (Deutsch, 1999)

The last transform required to make the complex a regular volume is a straightening function:

$$x_5 = \frac{2}{W^{cc}(y_3)} \cdot x_4 \quad (9.10)$$

Note that  $x_4$  is about one half as wide as  $W^{cc}$ . The result is shown in figure 9.7.

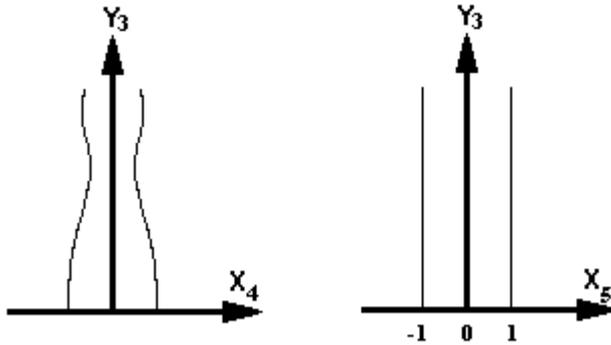


Figure 9.7 (Deutsch, 1999)

### STEP E

The channels are created within the channel complexes. These are the small scale geologic features.

### STEP F

As with the the channel complex, the channel must be transformed into a regular volume. The process is the same as the transform in STEP D except there is no need to rotate the channel; it is already oriented in the same direction as the complex ( $\alpha=0$ ). We need only translate the channel to have the  $y_3$  axis aligned with the center line of the channel using:

$$\begin{bmatrix} x_6 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_5 - x_5^{cs} \\ y_3 \end{bmatrix} \quad (9.11)$$

which can be reversed using:

$$\begin{bmatrix} x_5 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_6 \\ y_3 \end{bmatrix} + \begin{bmatrix} x_5^{cs} \\ 0 \end{bmatrix} \quad (9.12)$$

and using the same straightening transform to make the limits of the channel parallel with  $y_3$  axis:

$$x_8 = \frac{2}{W_a^c(y_3)} \cdot x_7 \quad (9.13)$$

### STEP F

Now the channel is populated with geostatistical derived petrophysical properties.



## STEPS H TO M

The remaining steps are simply the reverse of the disassembly steps a through f. This will restore the reservoir to its original configuration

### Conditioning Data

There is a need for conditioning data, the channels and channel complexes can not be haphazardly simulated. The conditioning data can be derived from well and seismic data. Specifically what is required for conditioning data are the facies proportions; the size distributions for the channels and channel complexes; and well data for identification of the facies, channel complexes, and channels.

The facies proportions are required areally, vertically and globally. Areal proportions are required for horizontal trends and anisotropy. Vertical proportions are required for vertical trends such as fining up or down, effect of percolating hydrothermal fluids. Conforming to global distributions is one of the hallmarks of simulation, therefore the global proportion is required.

### Simulation Procedure

each channel complex is a volume within which a number of channels will be placed. The complex is simulated using indicators:

$$i_{cc}(\mathbf{u}) = \begin{cases} 1 & \text{if } \mathbf{u} \text{ is within a channel} \\ 0 & \text{if not} \end{cases}$$

Knowing the parameters that define the channel and channel complex it is possible to define within the area of interest the locations of the channel and channel complex. During simulation, channels that overlap are truncated with the simulated channel of highest elevation taking precedence.

There is obvious similarity between simulation of channels and channel complexes, as such, the simulation procedure is the same for both, just at different scales.

### Object Based Deep Water Systems

Deep water depositional environments differ from fluvial environments in many aspects, but the feature that will be discussed here are the lobes at the ends of the channels. The lobes are the result of discharge from a fluvial depositional environment. they are created when the moving water from a fluvial system hits the relatively still water off a deep water system. The sudden drop in energy allows particles to drop out and deposit at the mouth of a fluvial system.

The parameters of a lobe are inferred from seismic, well, and outcrop data. Each lobe is the result of the same stochastic process used to simulate channels and channel complexes differing only in the shape function. Some of the parameters used to simulate the lobes are indicated in Figure 9.9. The parameters illustrated in Figure 9.9 are:

- **sw** = starting width; typically set to the final channel width. The user, however, could set this arbitrarily.
- **ll** = lobe length; total length of the lobe from the channel to terminus.
- **rm** = relative position of maximum width; the lobe reaches maximum width a distance of  $rm \cdot ll$  from the start of the lobe.

- **lwlr** = lobe width / length ratio; maximum width of the lobe is given by  $lwlr * ll$  at location specified by relative position **rm**.
- **st** = start thickness; start thickness of the lobe next to channel (could be associated to the channel thickness at that point). This is the thickness at the lobe center line.
- **ft** = final thickness; thickness of the lobe at the terminus (at the lobe center line) relative to starting thickness.
- **ct** = cross section thickness correction; amount that the thickness reduces from the center line to edge of lobe.

Not only are the parameters defining the shape of the lobe stochastically derived but its position is, as indicated in Figure 9.10, also. The lobe must be positioned so that it conforms to the conditioning data and the "physics" of the already in place channel or channel complex, but there is some room for uncertainty.

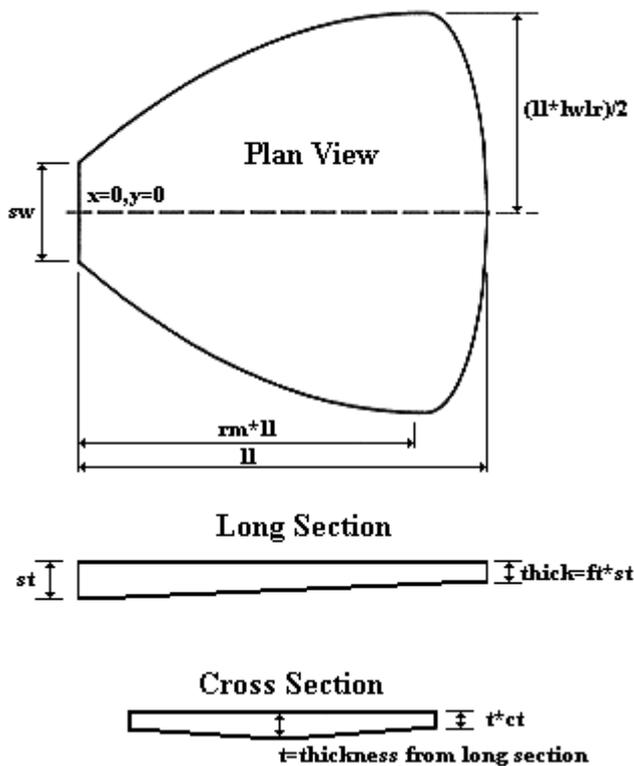


Figure 9.9 Some of the parameters used for simple lobe simulation. (Deutsch, 1999)

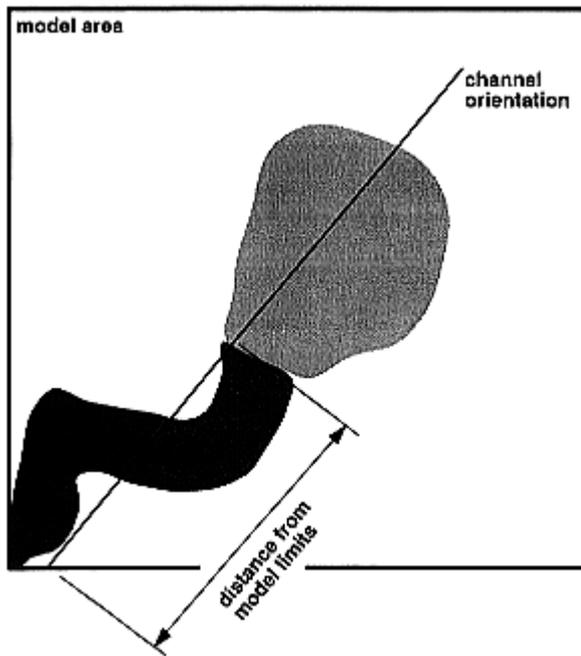


Figure 9.10 An assembled object including a channel and a lobe.

## Simulation

We have discussed the creation of the objects and the parameters of objects. What is missing is the algorithm that populates the model with objects. There are a large number of different ways to populate the model, but only a few are applicable to this lecture: the bombing model or marked point, bombing model with conditioning data, and simulated annealing.

### The Bombing Model or Marked Point Process

The bombing model is a very simplistic simulation algorithm. Stochastically derived objects are dropped into the model space until some stopping criteria is met. Figure 9.11 illustrates a model where circles are dropped onto the area of study with the goal of having 20% of the area enclosed by a red circle. The algorithm randomly selects a location for a circle and places the object. The algorithm continues until the stopping criteria (20% enclosed volume) is met. In this example only one parameter is stochastically derived; the location. Any number of parameters describing the object could be stochastically derived.

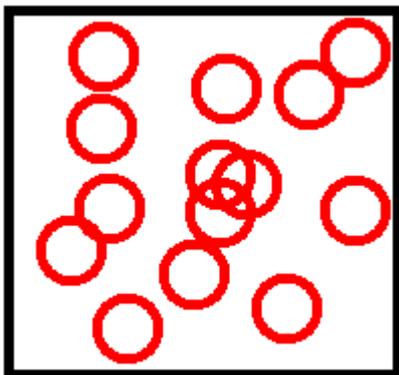


Figure 9.11 the bombing model in action. The algorithm continues to drop circles at random

The disadvantage to this method of population is that it does not conform to any conditioning data, the objects are just dropped until the stopping criteria are met.

## Bombing Model With Conditioning Data

One way of improving the bombing model is to place objects, by hand, to accommodate the conditioning data and allow the bombing model to fill in the remainder of the model. Using the same example as above, consider three conditioning data as shown in Figure 9.12. At these three locations a circle is known to reside so they are added, and the rest of the model is populated with circles with the bombing model until the stopping criteria is met (20% enclosed volume) as shown in Figure 9.13.

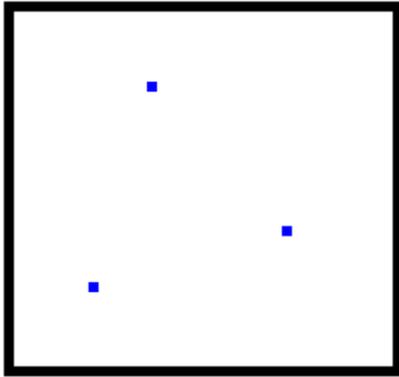


Figure 9.12

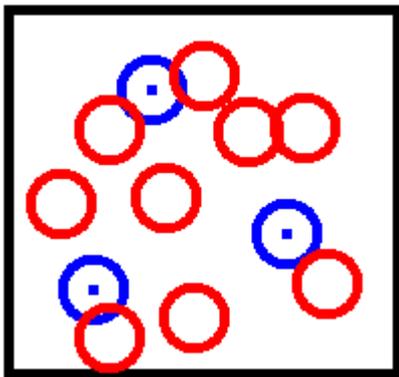


Figure 9.13

The problem with this methodology is that it is biased.

## Simulated Annealing

Simulated annealing is now becoming a popular algorithm for modeling. The algorithm starts by randomly populating the model with sufficient objects to meet some user defined criteria and perturbs the objects until some objective is met. Using the example from above, the area is filled with enough circles to have 20% enclosed volume. The locations of the circles are perturbed until an objective is reached; suppose there were two objectives: (1) the conditioning data are honored, and (2) a minimum overlapping of circles. In Figure 9.14 the circles are randomly placed in the area of study. in Figure 9.15 one circle is perturbed (moved from grey location to blue location) and the objective function is calculated: are the data honored? The objective is not met but at least one of the circles are closer to the objective. Figure 9.16 shows the result of several perturbations: all of the data are honored and

there is a minimum of overlapping circles. Note that each of the circles are perturbed randomly, and at each perturbation the objective function is calculated. The algorithm continues until either the objective function is met or a user criteria (too many perturbations for example) stops the algorithm.

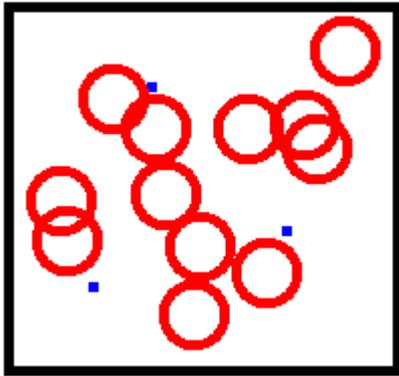


Figure 9.14 First the annealing algorithm adds an appropriate number of objects.

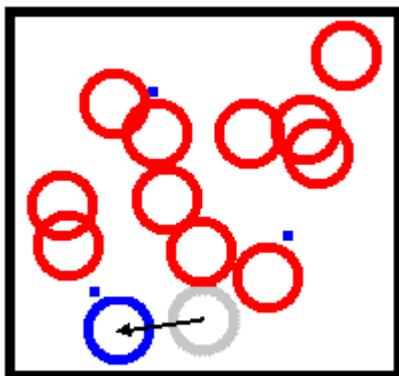


Figure 9.15 Then the location of objects is perturbed until the objective criteria is met.

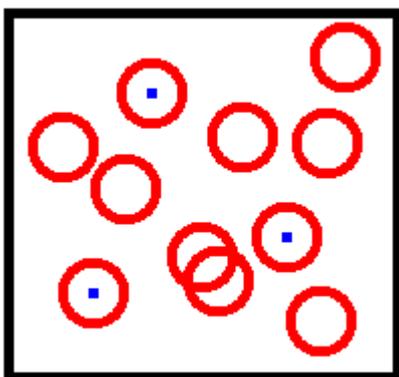
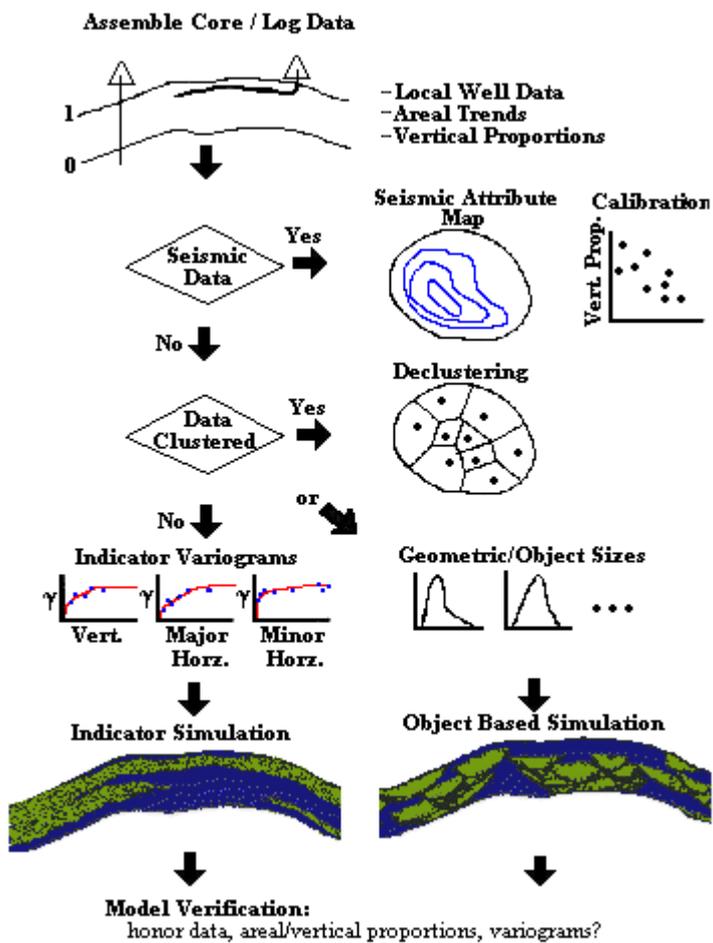


Figure 9.16 The algorithm stops when either the objective criteria is met or some other stopping criteria is met.

## Work Flow:



## Procedure for modeling:

1. geological zonation, layering, conceptual model
2. statistics: declustering, modeling, variograms
3. rock type modeling (indicator, object-based, hybrid)
4. porosity modeling
5. permeability modeling
6. model validation

August 8, 1999

## ***Lecture 9: Object Based Modeling, The Quiz***

### **Question 1**

An advantage to modeling reservoirs using objects is:

- a. it is easy to infer required parameter distributions
- b. crisp geometric representation of geology
- c. easy to honor conditioning data
- d. more accurate representation of geology
- e. faster than cell based modeling

### **Question 2**

When should you model a reservoir using object based modeling:

- a. in a carbonate structure
- b. when geologic entities can be represented by geometric objects
- c. when net-to-gross ratio is low
- d. when net-to-gross ratio is high

### **Question 3**

Where do object parameters come from?

### **Question 4**

When do object based techniques run into problems?

### **Question 5**

What parameters defining an object can be simulated?

[solutions](#)

August 8, 1999

## Lecture 10: Petrophysical Properties

- [Modeling petrophysical Properties](#)
- [Sequential Gaussian Simulation](#)
- [P-field / Cloud](#)
- [Porosity Permeability Transforms](#)
- [Indicator Methods](#)
- [Lecture 10 Quiz](#)

### Introduction

This lecture focuses on populating the model with geostatistically derived petrophysical properties. We aim to work within "homogeneous" lithofacies/rock-type classification when we populate the model, and this, as previously noted, may require a first step to model lithology. There are several methods for populating the cells, however we must constrain ourselves to sequential Gaussian simulation, sequential indicator simulation, p-field simulation, and porosity-permeability transforms.

### Modeling Petrophysical Properties

After all of the effort expended to get to this point, the modeling of the petrophysical properties, one would expect a break in the geostatistical modeling process. Well there are no breaks. There are problems here also. Some of the problems encountered are:

- Simplicity vs. flexibility to integrate different data types
- Vertical trends such as fining upward or coarsening upward
- Regional trends imparted by original deposition or diagenetic alteration
- local directions of continuity

Geostatistics has the ability to integrate data from different sources and types. This is an important feature because good decision making and appropriate measures of uncertainty require consideration of all available data. Good data integration techniques must be therefore simple to use and flexible in application. This can sometimes be a difficult task to achieve because of varying data quality and availability. Small scale trends can often be neglected because the data themselves will take care of the trend, but this is not the case for large scale trends such as directional fining or coarsening of sediment. Other trends such as dolomitization fall under this specification also. Trends must be identified, known throughout the area of study, and explicitly injected into the model. Of course the variogram, or the spatial correlation of the data must also be known.

### Sequential Gaussian Simulation

Recall the kriging estimator:

$$z^*(\mathbf{u}) = \sum_{\alpha=1}^n \lambda_{\alpha} \cdot z(\mathbf{u}_{\alpha})$$

and the corresponding kriging system:

$$\sum_{\beta} \lambda_{\beta} \gamma(\mathbf{u}_{\beta}, \mathbf{u}_{\alpha}) = \gamma(\mathbf{u}, \mathbf{u}_{\alpha}), \quad \alpha = 1, \dots, n$$

The kriging system forces the variogram between the kriged estimate and the data values to be correct:



$$\gamma\{Z^*(u), Z(u_\alpha)\} = \sum_{\beta=1}^n \lambda_\beta \gamma(u_\alpha, u_\beta) = \gamma(u, u_\alpha) = \gamma\{Z(u), Z(u_\alpha)\}$$

Although the variogram between the estimates and the data is correct, the variance is too small:

$$\text{missing variance} = C(0) - \sum_{\alpha=1}^n \lambda_\alpha \gamma(\mathbf{u}, \mathbf{u}_\alpha)$$

correct the variance without changing the covariance by adding an independent (random) component with the correct variance:

$$Z_s(\mathbf{u}) = Z^*(\mathbf{u}) + R(\mathbf{u})$$

where  $R(\mathbf{u})$  corrects for the missing variance.

The variogram between kriged/simulated values is not correct:

$$\gamma\{Z^*(u), Z^*(u_\alpha)\} \neq \gamma\{Z(u), Z(u_\alpha)\}$$

This is because kriging does not use the variogram between the estimates and previously estimated values, and this leads us to proceed sequentially.

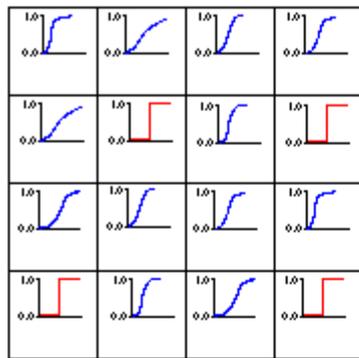
The idea of sequential simulation is to use previously kriged/simulated values as data and reproduce the covariance between all of the simulated values. A Gaussian distribution is used because it is extraordinarily straightforward to establish conditional distributions: the shape of all conditional distributions is Gaussian (normal) and the mean and variance are given by kriging. Thus, all distributions within the algorithm are Gaussian and have a known variance and mean that is provided by kriging.

## Steps in SGSIM :

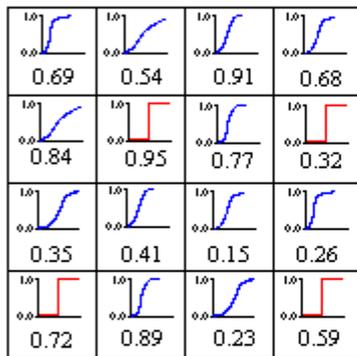
1. Transform data to "normal space"
2. Establish grid network and coordinate system ( $Z_{rel}$ -space)
3. Decide whether to assign data to the nearest grid node or keep separate
4. Determine a random path through all of the grid nodes
  - o search for nearby data and previously simulated grid nodes
  - o construct the conditional distribution by kriging
  - o draw simulated value from conditional distribution
5. Back transform and check results

## P-field / Cloud

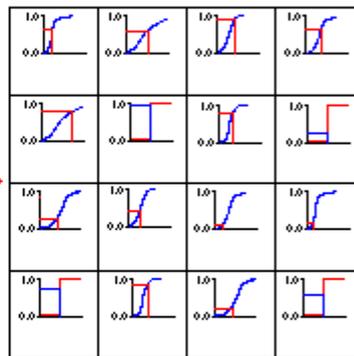
Sequential simulation is successfully reproduces the variogram between the data and the simulated values, and injecting the variance that kriging forgets, but the algorithm is slow because for each realization it recreates the local cdfs (uses kriging, simple or indicator, to find the mean and estimation variance) for each realization. P-field creates the local cdfs once and for each realization generates correlated random numbers at each node. The correlated random numbers are drawn using the normal scores variogram (sill=1). This is a much faster way of creating multiple realizations. Kriging is only one way of creating the local cdfs. Cdfs can be taken from any source, seismic for instance. Figure 10.1 illustrates the idea behind pfield simulation.



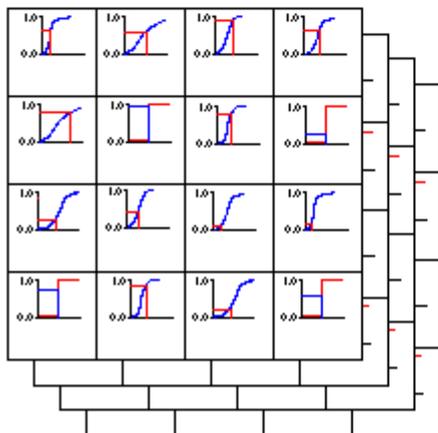
Determine the exhaustive cdfs



Draw a correlated random number



Read the quantile  
Create a realization



Repeat n times

Figure 10.1 Illustrates the idea behind p-field simulation. Establish the local cdfs, generate a correlated random number at each node and draw the appropriate quantile. Repeat for  $n$  realizations.

## Porosity Permeability Transforms

Porosity and permeability transforms are essentially calibration tables that relate a porosity to a permeability. This is a simplistic approach, which assumes a direct correlation between the two properties, thus porosity permeability transforms are inappropriate for populating a model with permeability. Often a linear regression is used between  $\log(\text{permeability})$  and porosity, and sometimes conditional class averages are used. Some attempts have been made to preserve the scatter

but no success has been found. The typical results using porosity permeability transforms have smoothed out highs and lows, and the final permeability values do not honor the variogram.

## Indicator Methods

Indicator methods differ from continuous methods in that with indicators one estimates/simulates the probability of occurrence for each indicator and with non-indicator methods the magnitude of the property is estimated/simulated. Indicator methods have the advantage of being able to specify different continuity for high and low values. This is particularly important for properties such as permeability where high and low values do display different continuities. Using indicator techniques for continuous variables requires that the continuous variable be transformed to a categorical variable using the following transform:

$$i(\mathbf{u}; z_k) = \begin{cases} 1 & \text{if } z(\mathbf{u}) \leq z_k \\ 0 & \text{if not} \end{cases} \quad (10.1)$$

where  $z_k$  is the threshold of interest. The range of  $Z$  variability is discretized with a series of  $K$  threshold values  $z_k, k=1, \dots, K$ . The transform can be interpreted as a cumulative distribution function in that it is a non decreasing function and the probability of  $z(\mathbf{u})$  being  $z_k$  is equal to the proportion of the  $k$ th state. For conditioning data the proportion can only be one or zero.

The estimate of the proportion of each category is best done using kriging with the estimate of the proportion written as:

$$\begin{aligned} [i(\mathbf{u}; z_k)]^* &= E\{I(\mathbf{u}; z_k | (n))\}^* \\ &= Prob^*\{Z(\mathbf{u}) \leq z_k | (n)\} \\ &= \sum_{\alpha=1}^n \lambda_{\alpha} \cdot i(\mathbf{u}_{\alpha}; z_k) \end{aligned}$$

which says that the estimated indicator is equal the expected value of the indicator  $I$  for the local conditional cumulative proportions for the categorical variable  $z_k$  at location  $\mathbf{u}$ , which is in turn equal to the probability of the random variable  $Z$  being less than or equal to the conditional cumulative proportion of category  $k$ , which is equal to the sum of the weighted indicators within the search radius. In this way the indicator transform is constructed directly by kriging (estimating) the proportion of each indicator.

## Work Flow

August 10, 1999

## **Lecture 10: Petrophysical Properties Simulation, The Quiz**

### **Question 1**

What type of data do you use to calculate the variogram for use with sequential Gaussian simulation?

1. raw data
2. indicator data
3. declustered data
4. uniform score data
5. normal scores data

### **Question 2**

Sequential Gaussian simulation is good for modeling high permeability streaks, true or false?

### **Question 3**

Where do the ccdfs come from for p-field simulation?

### **Question 4**

Why is p-field simulation fast?

### **Question 5**

What are the advantages and disadvantages of og indicator estimation/simulation?

[solutions](#)

Aug 10, 1999

## **Lecture 10: Petrophysical Properties Simulation, The Quiz Solutions**

### **Question 1**

What type of data do you use to calculate the variogram for use with sequential Gaussian simulation?

1. raw data
2. indicator data
3. declustered data
4. uniform score data
5. normal scores data

### **Question 2**

Sequential Gaussian simulation is good for modeling high permeability streaks, true or false?

False, sequential Gaussian permits discontinuity of extremes and this is not appropriate for petrophysical properties like permeability where highs and lows are often continuous.

### **Question 3**

Where do the cdfs come from for p-field simulation?

Any source that offers exhaustive cdfs

### **Question 4**

Why is p-field simulation fast?

The cdfs are constructed once not each time a simulation is run.

### **Question 5**

What are the advantages and disadvantages of og indicator estimation/simulation?

Advantages: explicitly capture the continuity of highs and lows and anisotropy

Disadvantages: many variograms to calculate, not enough data to infer important parameters.

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## Lecture 11: Seismic Data Integration

- [Calibration of Data](#)
- [Cross spatial variability](#)
- [Cokriging](#)
- [Simulation Alternatives](#)
- [Lecture 11 Quiz](#)

### Introduction

Data Integration is a fundamental principle of geostatistics / reservoir modeling; the goal is to explicitly account for all of the available data. Seismic data provides abundant data compared to well data that is sparse. Seismic data however does not give any directly useable information and must be calibrated to the limited well data. There are numerous techniques that permit the direct use of seismic data such as cokriging and collocated cokriging. There are also many methods for determining the uncertainty in estimated maps such as cosimulation, collocated cosimulation. Other simulation alternatives include annealing.

### Calibration of Data

Usually the most difficult step in the geostatistical study is finding a relationship between the well parameter of interest and some aspect of the seismic data (Wolf et al., 1994). A relationship between seismic data to well data is often quite difficult to infer and once found summarized as a simplistic linear regression; none of the scatter is preserved. Figure 11.1 shows a calibration between sand thickness and a seismic attribute. This is a simplistic approach and is not always appropriate for some petrophysical properties such as permeability.

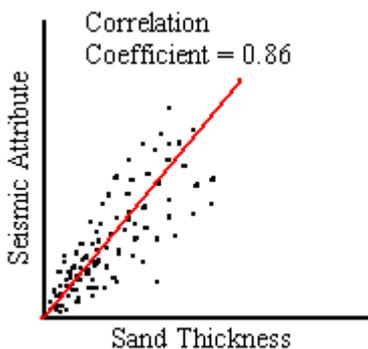


Figure 11.1

At all stages the user must justify the calibration.

Sometimes there are too few data to infer a relationship and the user must infer a relationship based on previous experience or analogue data. Figure 11.2 shows a scenario where there are too few data to infer a useful relation between porosity and depth. The distribution of depth throughout the reservoir is exhaustively known as a result of seismic interpretation. The depth information is exhaustively known, yet there are only five well data to infer a relation between porosity and depth.

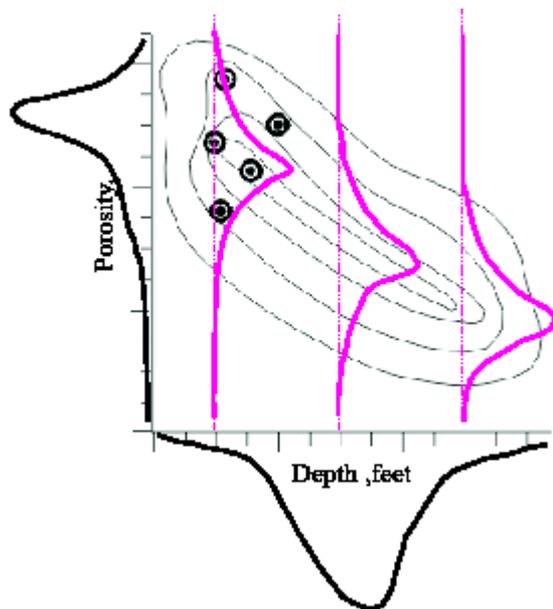


Figure 11.2

Instead of abandoning the calibration a relation has been manually inferred (the grey ellipses) from analogue data retrieved from a reservoir that is similar. The proposed methodology is to sum the inferred distributions of porosity conditional to the depth yielding an inferred global distribution for porosity. In terms of probabilistic notation:

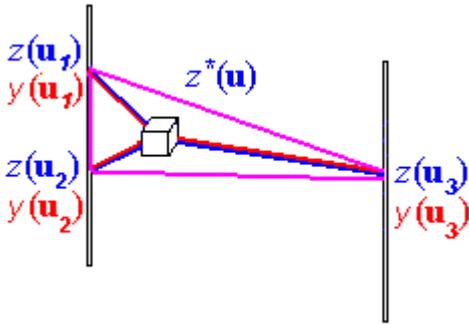
$$f_Y^*(y) = \sum_{\substack{\mathbf{u} \in A \\ c}} \frac{1}{c} f_{Y|X}(y | X = x(\mathbf{u})) \quad (11.1)$$

where  $c$  is a normalizing constant.

In summary the calibrations is performed: (1) map the secondary variable  $X$  at all locations, (2) develop a bivariate relationship between  $X$  and the  $Y$  variable interest, and (3) generate a distribution of  $Y$  by combining the conditional distributions. Beware that the user must always justify the calibration routine.

### Cross Spatial Variability

In the univariate case (one variable) the model variogram must be a compilation of different variogram models that are known to allow the solution to the system of kriging equations to be unique. As well, the model variogram that defines the spatial correlation in 3-D must be modeled such a way that each variogram structure be the same and have the same contribution in all directions; only the range may change for each structure in each direction. In the multivariate case, where there is a primary variable of interest and a secondary correlated variable, and an equivalent requirement called the linear model of coregionalization. Just as the univariate variogram is constructed as a set of intercorrelated random functions. The linear model of coregionalization, provides a method for modeling the auto- and cross-variogram of two or more variables so that the variance of any possible combination of these variograms is always positive. This ensures a unique solution to the cokriging system of equations. The criteria are: (1) the determinant must be greater than zero, and (2) all diagonals must be greater than zero.



The linear model of coregionalization is a technique that ensures that estimates derived from cokriging have a positive or zero variance. To ensure this, the sill matrices for each basic structure must be positive definite. As an example, for the two variable case, we can check whether the linear model of coregionalization is honored:

- $\text{Gamma}_x(h) = 10 + 20 \text{ Sph} + 40 \text{ Exp}$
- $\text{Gamma}_y(h) = 20 + 35 \text{ Sph} + 55 \text{ Exp}$
- $\text{Gamma}_{xy}(h) = 7 + 10 \text{ Sph} + 30 \text{ Exp}$  (cross-variogram)

For the nugget structure:

10	7
7	20

- determinant =  $(10 \times 20) - (7 \times 7) = 151 > \text{zero}$  --- OK
- all diagonals are positive (10 and 20) --- OK

For the spherical structure:

20	10
10	35

- determinant =  $(20 \times 35) - (10 \times 10) = 600 > \text{zero}$  --- OK
- all diagonals are positive (20 and 35) --- OK

For the exponential structure:

40	30
30	55

- determinant =  $(40 \times 55) - (30 \times 30) = 1300 > \text{zero}$  --- OK
- all diagonals are positive (40 and 55) --- OK

Since for all structures the determinants are greater than zero and the diagonals are positive, then the linear model of coregionalization is honored. (*AI-Geostats*)



## Cokriging

Often there is more than one type of available data, and often these data are correlated. Also, the available data are not always sampled at the same density; core samples and seismic data are good examples because core samples only sample a small volume of the population whereas seismic can sample nearly the entire volume. When there is more than one type of available data of interest is called the primary data and all other data is called secondary data. In kriging the spatial correlation of a single data type is used to make the best estimate at locations where there is no data. Cokriging differs from kriging in that it uses the spatial correlation of the primary data and the spatial correlation of primary data to secondary data to fortify the estimate of the primary variable. For example, we know it is hot and dry in Texas and that there is a good chance of finding cactus whereas it is cold and dry in Alberta and there is poor chance of finding cactus. Cokriging is particularly useful when there is fewer primary data than secondary data. The cokriging estimate, like the kriging, estimate is the linear combination of data. Cokriging however is the linear estimate of the primary and secondary data as shown in equation 11.20. The cokriging system of equations is derived in the same way as the kriging system of equations and will not be covered here.

$$z_{cok}^*(\mathbf{u}) = \sum_{\alpha=1}^{N_1} \lambda_{\alpha} \cdot z(\mathbf{u}_{\alpha}) + \sum_{\beta=1}^{N_2} \lambda'_{\beta} \cdot y(\mathbf{u}'_{\beta}) \quad (11.2)$$

and the system of equations given two primary and two secondary data is:

$$\begin{matrix} & z_1 & z_2 & y_1 & y_2 & & z \\ \begin{matrix} z_1 \\ z_2 \\ y_1 \\ y_2 \end{matrix} & \begin{bmatrix} \gamma(z_1, z_1) & \gamma(z_1, z_2) & \gamma(z_1, y_1) & \gamma(z_1, y_2) \\ \gamma(z_2, z_1) & \gamma(z_2, z_2) & \gamma(z_2, y_1) & \gamma(z_2, y_2) \\ \gamma(y_1, z_1) & \gamma(y_1, z_2) & \gamma(y_1, y_1) & \gamma(y_1, y_2) \\ \gamma(y_2, z_1) & \gamma(y_2, z_2) & \gamma(y_2, y_1) & \gamma(y_2, y_2) \end{bmatrix} & * & \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda'_1 \\ \lambda'_2 \end{bmatrix} & = & \begin{bmatrix} \gamma(z, z_1) \\ \gamma(z, z_2) \\ \gamma(z, y_1) \\ \gamma(z, y_2) \end{bmatrix} \end{matrix} \quad (11.3)$$

## Cokriging Discussion

Cokriging, in addition to considering the proximity of data and the location of the data, also must consider:

- the correlation between the primary and the secondary data
- the spatial continuity of the primary data to the secondary data
- the spatial configuration of the primary and secondary data
- the sampling density of the primary and secondary data

Cokriging and kriging estimates are likely to be similar when variables are weakly correlated, when the data are equally sampled or when the cross variogram is the same shape as the direct variograms (the variables have the similar spatial correlation). Cokriging and kriging estimates are equivalent when there is no correlation between the primary and secondary. This makes sense when one considers the system of equations in 11.21. When there is no correlation between the primary variable and secondary the variogram goes to zero and the cokriging portion of the system of equations vanish. The cokriging estimate is the same as the kriging estimate when the correlation between the primary and secondary data are proportional. Note that when the location of the estimate is beyond the correlation range of both the primary and secondary data then, as with kriging, the estimate becomes the mean.

## Simple Kriging Versus Simple Cokriging

Cokriging is much more demanding than kriging since:

- there are  $N_v(N_v+1)/2$  variograms required; one direct variogram for each data, and a cross-variogram for each data.
- A linear model of coregionalization is required
- a large cokriging system must be solved

The overwhelming advantage to cokriging is the fact that the cokriging estimation is always equal to or less than that of kriging.

## Colocated Cokriging

The cokriging system of equations becomes unstable when the secondary variable is much more sampled than the primary variable because the secondary data is much more correlated to the closer secondary data than the primary is to the further primary data. Colocated cokriging is reduced version of cokriging that consists of retaining only the colocated secondary variable which must be available at all locations  $\mathbf{u}$  being estimated. The colocated cokriging estimator is written:

$$z_{cokok}^*(\mathbf{u}) = \sum_{\alpha=1}^{N_1} \lambda_{\alpha} \cdot z(\mathbf{u}_{\alpha}) + \lambda'_{\beta} \cdot y(\mathbf{u}'_{\beta}) \quad (11.4)$$

The requirement of the secondary variable be known at all locations is not as limiting as one might think. Typically secondary data is seismic and seismic data is often available for all locations required and if not it is easy enough to estimate/simulate the attribute at all locations.

## Simulation Alternatives

### Matrix Approach (LU Decomposition):

Lu decomposition is a fast technique for solving small matrices. Its advantage is that it is very fast for simulations that have small areal and vertical extent and few conditioning data. It is not used because the size of the matrix in most cases exceeds the usefulness of the algorithm (an  $N \times N$  matrix must be solved where  $N$  could be in the millions).

### Turning Bands:

The first large-scale 3-D Gaussian simulation algorithm actually implemented (Deutsch, 1998). Simulate the variable on 1-D lines and combine in 3-D. Not used because of artifacts and difficulty in conditioning to local data tb3d

### Fractals:

By using a true fractal model, we are assuming that we can determine the variance of the property at one scale based on the variance at any other scale (statistical self-similarity) (Shibli, 1999). True fractal simulation is not extensively used because self-similarity is only valid for a limited range of scales and difficulty in conditioning to local data (Deutsch, 1992)

### Annealing:

Annealing is an optimization algorithm based on an analogy with the physical process of annealing. The central idea behind simulated annealing is an analogy with thermodynamics, specifically with the way liquids freeze and crystallize, or metals cool and anneal. At high temperatures the molecules can move freely. As the temperature is slowly lowered the molecules line up in crystals which represent the minimum energy state for the system (Deutsch, 1999). Here, the objective of the system is to get the energy as low as possible; the molecules all being optimally oriented so that the energy of the system is as small as possible. In simulated annealing the goal is to optimize some objective function. Instead of temperature, simulated annealing uses probability of acceptance. For example, before a liquid it freezes the molecules are free to move anywhere. As the liquid cools to freezing the probability of the crystal accepting a molecule moving decreases. When the liquid becomes solid there is an even lesser probability of the crystal accepting a move, but it is never zero. In simulated annealing nodes are perturbed, that is moved to a different location in space. This is the analogue to molecules mobilizing to prepare for solidification. The simulated annealing algorithm calculates the energy of the system including the newly moved node and compares the value of the new objective function to the value of the objective function before the old. If the newly perturbed node leads the model toward optimality the perturbation is accepted. If the perturbation moves the model away from optimality the perturbation may or may not be rejected based on the probability of acceptance (the temperature of the model). If the temperature is high there is a high probability of acceptance, if it is low there is a low probability of acceptance. Perturbations are repeated until some stopping criteria is met such as the objective function being met or too many perturbations. The algorithm chooses the nodes for perturbation randomly so it is possible to create multiple realizations. The algorithm can be summarized by:

1. Establish an initial guess that honors the data by assigning a value to each cell by drawing from the conditional distribution of petrophysical property
2. Calculate the initial objective function. Numerical measure of mismatch between the desired objective and of the initial guess
3. Consider a change to the model. Randomly choose a non-data cell and then consider a new value for the petrophysical property from the conditional distribution.
4. Evaluate new objective function
  - a. better? - accept change
  - b. worse? - consider the temperature and possibly reject the change
5. Is objective function close enough to zero? Have there been too many swaps?
  - a. yes - done
  - b. no - go to 3

Annealing seems like a very attractive technique but it does have issues. It is difficult to implement because there are so many parameters. Sometimes the choice of too many or the wrong objectives can make the problem intractable. It can take a significant amount of time to converge. Despite these issues simulated annealing is a robust simulation algorithm that has a variety of applications.

An example showing simulated annealing in action is shown in figure 11.3. The figure shows a starting image and the object, the variogram. Initially, the starting image is a random array of the lithofacies in the correct proportions and the variogram shows randomness. Halfway through the algorithm the image starts to take shape and the variogram starts to be honored. The final image shows the the simulated structure and the honored variogram after the objective, honoring the variogram, has been met.

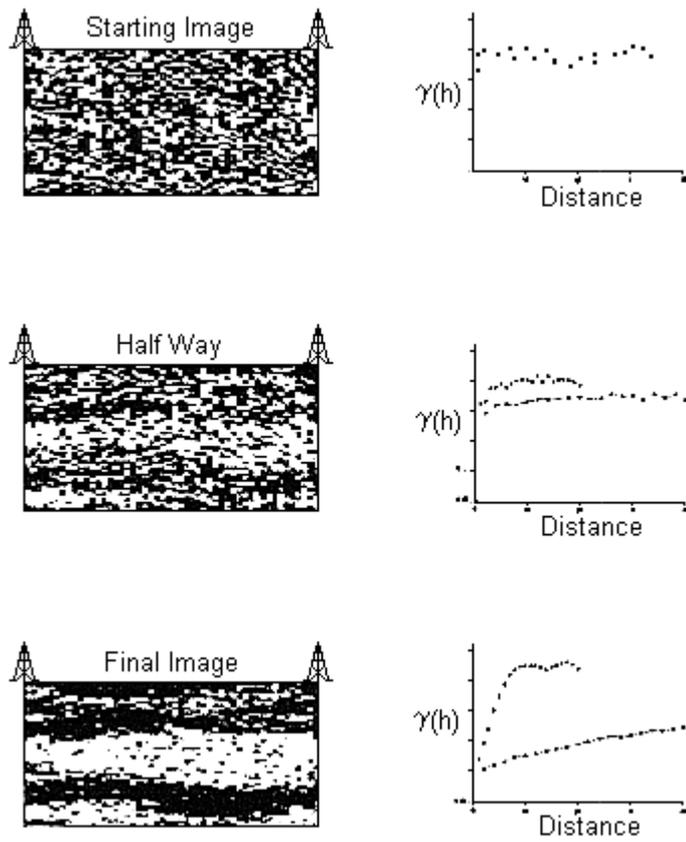


Figure 11.3 The figure shows the results from simulated annealing at three different stages: initial, halfway, and the final image. The objective is to honor the variogram shown at the bottom.

Aug 15 1999

## **Lecture 11: Seismic Data Integration, The Quiz**

### **Question 1**

Circle the correct answers:

Seismic information has *high low* resolution

Seismic information has *high low* coverage

Seismic information has *high low* reliability

Well information *high low* resolution

Well information *high low* coverage

Well information *high low* reliability

### **Question 2**

The linear model of coregionalization ensures that the 3-D model of spatial correlation for use in cokriging is?

### **Question 3**

The advantages and disadvantages of colocated cokriging over more stringent cokriging are:

### **Question 4**

When the primary data is not correlated with the secondary data what happens to the cokriging matrix?

### **Question 5**

Annealing has many advantages and disadvantages, list two of each.

[solutions](#)

Aug. 16, 1999

## ***Lecture 1: Purpose / Motivation for Geostatistics, The Quiz***

### **Question 1**

List three major benefits that geostatistics offers, and describe what each mean and explain why they are important.

Quantification of uncertainty: summarizes our lack of knowledge for better decision making

Rigorous mathematics: means that there are sound mathematical laws applied for repeatability

Data Integration: data of many types can be integrated using geostatistical tools

### **Question 2**

Decision making in presence of uncertainty is important. What are the two steps for risk-qualified decision making?

Quantification of uncertainty and then quantification of risk.

### **Question 3**

In general terms, explain the link between spatial variability (heterogeneity) and uncertainty.

As spatial variability increases heterogeneity increases and hence uncertainty increases

### **Question 4**

In your own words describe the information effect and how it relates to uncertainty.

The information effect is the result of increased available information which leads to less uncertainty

### **Question 5**

Geostatistics is useful at every point in the life cycle of a reservoir, but where is it most useful and why?

Geostatistics is most important in the early stages of the life cycle because it makes intelligent use of limited data and allows for decision making that is tempered with a knowledge and understanding of the uncertainty inherent in the numerical-geological model.





## Lecture 12: Post Processing

- [Model Validation](#)
- [Understanding Uncertainty](#)
- [Ranking and Selecting Realizations](#)
- [Decision Making](#)
- [Lecture 12 Quiz](#)

### Model Validation

Model validation checks the results of the models we build. Some of the techniques include histogram and variogram validation, cross validation, the jackknife, and the bootstrap. These techniques are designed to make sure that parameters that we make the model honor are honored. Short of sampling the area to exhaustion there are few ways to check the validity of the models we construct.

### Histogram and Variogram Validation

If the algorithm honors the histogram and the variogram why bother checking it? The algorithm only honors the histogram that it is provided and only does so on average. If the histogram is incorrect then the wrong histogram will be reflected in the results. Validating the histogram is essential because it checks for blunders. The same is true for the variogram. It is essential for the model variogram to be reproduced. Checking the validity of the modeled histogram and variogram can be performed in the same manner as calculating the a priori statistics and the experimental variogram. Note that the variogram is only reproduced on average.

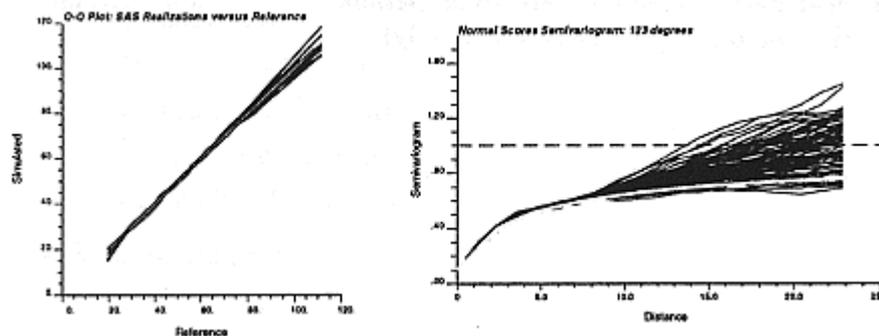


Figure 12.1

### Cross Validation and the Jackknife

Cross validation is a good way of checking the model parameters before modeling proceeds, although it can not be limited to this use. Cross validation validates the model by considering a single known value as unknown and estimates the pseudo-unknown value and compares it to the true value. This sample is replaced, and the process is repeated for all of the known values. Plotting the errors on a map illustrates areas of concern by exposing those areas which are plagued by over or under estimation. The modeling parameters, such as the variogram, should be updated appropriately or the model broken into more appropriate chunks.

The jackknife is similar to cross validation but in the jackknife the data set is split into non-overlapping sub-data sets. The idea is to estimate the one sub-set of data using another and compare the estimated to the true.

Some useful tools for exposing a problem include:

- A scatterplot of the true values versus the estimated shows where the estimated values deviate from the true without considering location.
- A location map of the errors reveals trouble spots. The errors should be spatially independent of each other.
- The distribution of errors  $\{e(\mathbf{u}), i=1, \dots, N\}$  should be symmetric and centered on a zero mean with minimum spread. Deviation of the mean from 0 indicates a bias.
- Summary statistics for the error distribution, i.e. the mean error and the mean squared error.

## Bootstrap

The bootstrap is a statistical resampling procedure used to determine the uncertainty in a statistic by using the data itself. Monte Carlo simulation is used to sample the data with replacement as though it were the population. The result is a distribution of uncertainty for the data.

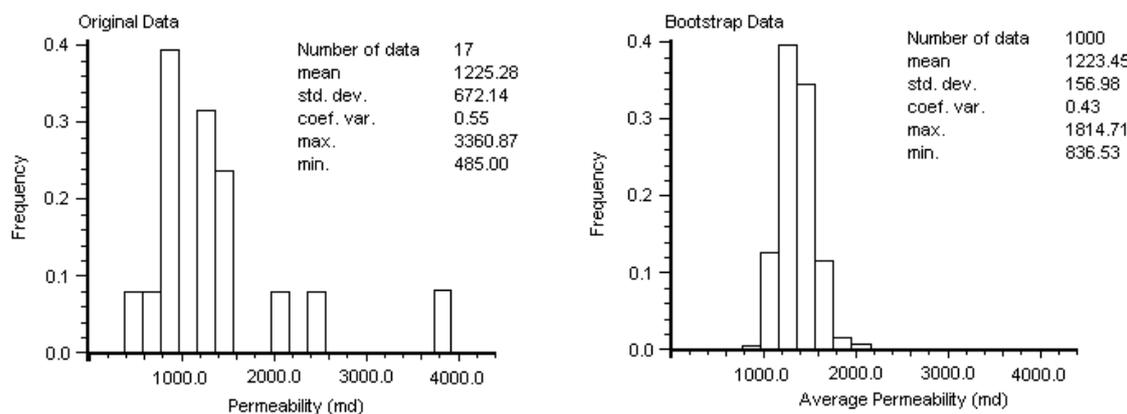


Figure 12.2

## Understanding Uncertainty

### Ranking and Selecting Realizations

Multiple realizations are boasted as equiprobable and representative of the histogram and variogram, so why would there be a need for ranking and selecting a realization? Running each realization through a flow simulator or considering the viability of every realization is prohibitively expensive in terms of man power and cpu time. Instead the realizations are ranked and only the worst, most likely and the best scenarios are run in a simulator. The ranking can be a result of any criteria the user desires. For example one could rank the realizations by oil-in-place, or by connectivity, or by combinations of any criteria. Why then is it essential to run many simulations only to remove a few candidates for consideration? Only a few realizations are required to assess the average of an attribute, but many are required to assess the extremes of a distribution. It is the average value and the extremes that we are interested in, and in order to get a reliable representation of the extremes many realizations are required.

## Risk Qualified Decision Making

Risk has two components: (1) probability of loss, and (2) quantity of loss. Sound decision making entails assessing the risk associated with the gross generalizations made from sample data; the errors made in the model. Geostatistics provides all of the components required for risk qualified decision making. Simulation provides the uncertainty, or the probability of loss. If we knew the truth there would be no loss, the probability of loss would be zero. Greater uncertainty invites greater potential for loss. What remains is an assessment of loss, the loss function. The loss function administers loss commensurate to the magnitude of error. The greater the error, the greater the loss. The loss function is derived from the model. For example over-estimating the oil in place in a reservoir is more punishable than under-estimating because of the investment required to exploit the reservoir. Likewise, it is punishable to under-estimate oil in place as a potential reservoir may be over-looked. The risk for each of these scenarios is different, there is more risk in over-estimating than in under-estimating, therefore, the loss function need not administer the same loss for over-estimation as for under-estimation. Figure 12.3 shows three loss functions. The first shows a loss function that heavily penalizes for under-estimation, the middle loss function administers loss equally, and the last shows a loss function that penalizes over-estimation heavily.

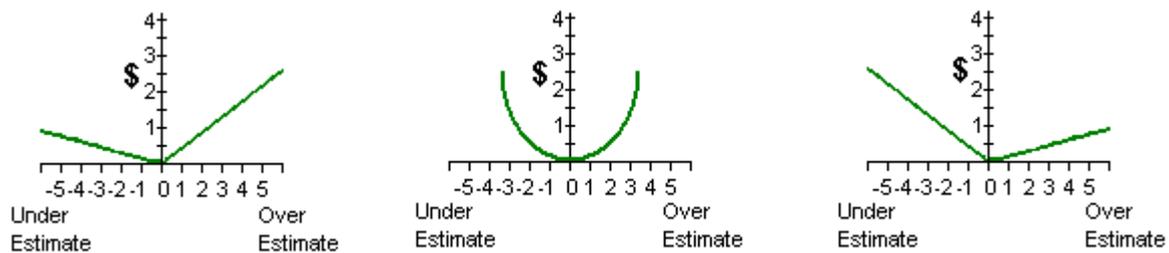
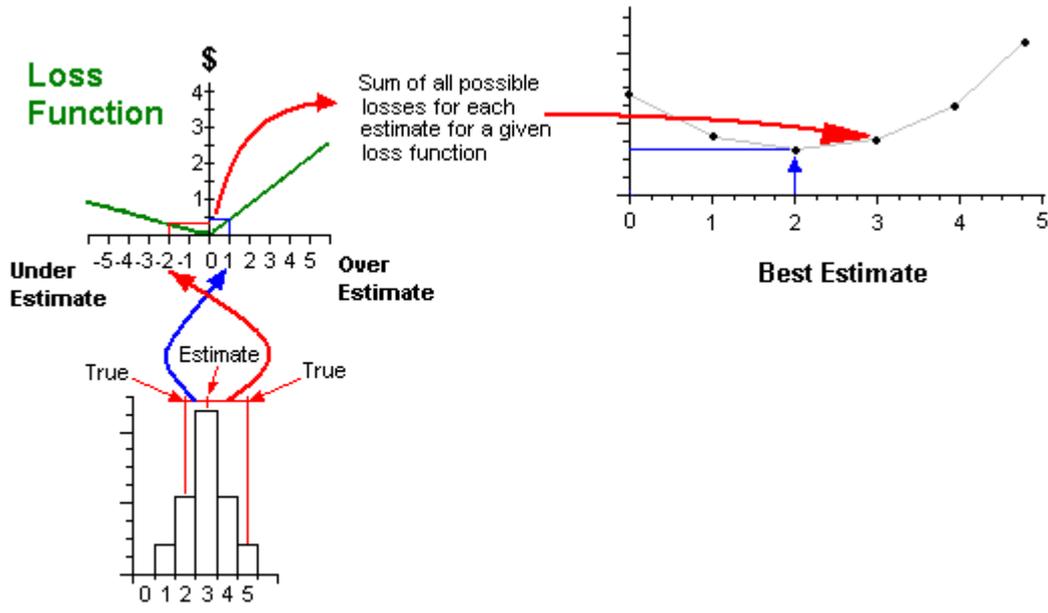


Figure 12.3

The decision making part of risk qualified decision making does not decide whether or not to drill, the user decides that. What it does is decide what the optimal attribute value is based on criteria set forth by the loss function. The criteria for optimality is minimum loss. The selected attribute value represents the "safest" value, or the value that would incur the least loss. This is the attribute value that decisions are made with. The algorithm is best explained by a complete example, the link below is an example using an Excel spread sheet.

### [Loss Function Example](#)

The algorithm starts by considering one of the possible values within the distribution of uncertainty as the pseudo best estimate and considers the consequences of that decision. If the pseudo best estimate is an over-estimate of the true value then the loss due to over-estimation is tabulated. The loss is considered for all possible truths. This is repeated for all possible values. The value with the lowest loss is the best estimate. The philosophy here is to choose the value that offers the least loss, and hence the greatest profitability. For example a best estimate might suggest  $X$  barrels of in situ oil for a particular reservoir. Corporate philosophy may dictate that  $Y$  barrels must be present for a reservoir to be considered a viable project.  $X$  does not equal  $Y$ , therefore the project is turned down.



**Histogram of Possible Scenarios**

# A

## **abandon:**

To cease producing oil or gas from a well when it becomes unprofitable. A wildcat may be abandoned after it has been proved nonproductive. Sometimes, before a well is abandoned, some of the casing is removed and salvaged. Usually, one or more cement plugs are placed in the borehole to prevent migration of fluids between the various formations. In many states, abandonment must be approved by an official regulatory agency before being undertaken. (spwla)

## **absolute permeability:**

The permeability of a rock measured when only one fluid phase is present in the rock. Usually measured in millidarcies or Darcies. See permeability. Compare effective permeability and relative permeability. (spwla)

## **affine correction:**

The simplest of the various support effect correction procedures. The basic idea is to reduce the variance of the distribution by moving all of the data values closer to the mean. Use of this support effect technique requires the variance reduction factor. (reference the big book of geostats)

## **anisotropy:**

Having different physical properties in different directions. Variograms or other measures of spatial correlation often exhibit different ranges of correlation in different directions. anisotropy. The property of a rock which allows it to show different responses or measurements when measured along different axes. Microscopic anisotropy relates to the variation of a property measured perpendicular to the bedding from that measured parallel to the bedding because plate-like mineral grains and interstices tend to orient themselves parallel to the bedding. Macroscopic anisotropy relates to the variation resulting from thin beds and fracture planes whose properties differ appreciably from those of the remaining volume over which a measurement is made.

## **a priori:**

Involving reasoning, but before investigation.

## **aquifer:**

- (1) A water-bearing stratum of permeable rock, sand, or gravel capable of producing water.
- (2) In a petroleum reservoir with a natural water drive, the aquifer is that part of the reservoir containing water.

## **azimuth:**

- (1) In well logging. In a horizontal plane, it is the clockwise angle of departure from magnetic north. See also magnetic declination.
- (3) In land surveying. It is the clockwise angle of departure in a horizontal plane from true north.

**arithmetic mean:**

The arithmetic mean (average) is the sample estimate of the of the expected value. Mathematically, it is the unweighted sum of all of the samples divided by the number of samples. (BBG)

# B

**bed:**

A subdivision of a stratified sequence of rocks, lower in rank than a member or formation, internally composed of relatively homogeneous material exhibiting some degree of lithologic unity, and separated from the rocks above and below by visually or physically more or less well defined boundary planes.

**bedding planes:**

In sedimentary or stratified rocks, the division planes that separate the individual layers, beds, or strata.

**bedrock:**

A general term for the rock, usually solid, that underlies soil or other unconsolidated, superficial material.

**bed thickness:**

(1) True bed thickness is the thickness of the stratigraphic unit measured along a line normal to the direction of extension of the unit. True thickness can be derived from information determined by the dipmeter. (2) The bed thickness determined from some well logs is an apparent bed thickness corresponding to the distance the borehole remained in the bed. The borehole may not have penetrated the bed normal to its upper or lower boundary surface because of hole deviation and formation dip. See also true vertical depth.

**bootstrap:**

A statistical resampling procedure (with replacement) whereby the uncertainty in a calculated statistic is derived from the data itself. Monte Carlo simulation is used for sampling from the data distribution. (*Deutsch, 1999*)

**bombing model:**

A particular indicator variogram model based on the result of of randomly dropping (or dropping ellipsoids, coded as "0,s" into a background matrix, coded as "1's" (*Deutsch, 1999*)

**boundary conditions:**

In the context of determining the effective properties of a grid block, boundary conditions are the pressure and flow rate conditions surrounding the block of interest.

## **bulk density:**

It is the value of the density of rock as it occurs in nature. In well logging, it is the density of the rock with the pore volume filled with fluid. The equation commonly used to compute porosity from well log derived bulk density is:

$$\phi = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$$

where  $f$  is pore volume filled with a fluid of density  $\rho_f$ ,  $\rho_b$  is the well-log-derived bulk density, and  $\rho_{ma}$  is the density of the rock framework. See density log and Z/A effect.

# C

## **cokriging:**

A bivariate extension of the idea of kriging where an estimate of a given variable is made which takes into account not only the *variogram* of that variable, but also the relevant cross-variograms between the primary and secondary variables.

## **conditional bias:**

In resource estimation, if insufficient *smoothing* occurs, the high-grades are overstated and the low-grades are understated. Minimalisation of conditional bias is an important objective in good estimation.

## **conditional simulation:**

A geostatistical method of producing a number of equally-likely images of the deposit. Conditional simulations can be considered as extrapolations of the data as opposed to interpolations like kriging. Simulations are not estimations, their goal being to characterise variability (or risk).

## **core:**

Cylindrical sample recovered from *diamond drilling*.

## **cross variogram:**

A statistical measure used in *geostatistics* to characterise the spatial covariability of two variables. A bivariate analogue of the *variogram*.

## **calcite:**

See calcium carbonate.

**calcium carbonate:**

A chemical combination of calcium, carbon, and oxygen. the main constituent of limestone. It forms a tenacious scale in water-handling facilities and is a cause of water hardness. Chemical symbol is  $\text{CaCO}_3$ . carbonate A compound containing the radical  $\text{CO}_3^{+2}$  or rock composed of carbonate minerals. See carbonate rock.

**calibration:**

In the context of geostatistical modeling, calibration is the exercise where the relationship of soft secondary data to the hard primary data is established. The calibration relationship is almost always probabilistic in the sense that there is no one-to-one relationship.

**carbonate:**

A geological reservoir type consisting of calcareous deposits that are the result of shells deposited by reef-dwelling organisms. A compound containing the radical  $\text{CO}_3^{+2}$  or rock composed of carbonate minerals. See carbonate rock.

**carbonate rock:**

A sedimentary rock primarily composed of calcium carbonate (limestone) or calcium magnesium carbonate (dolomite).

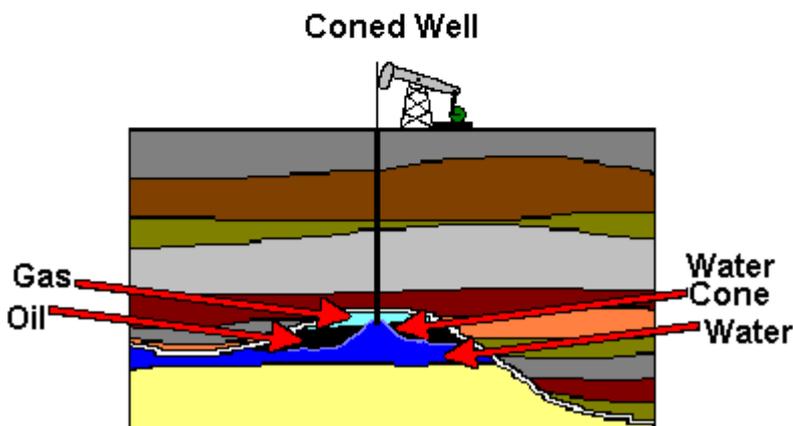
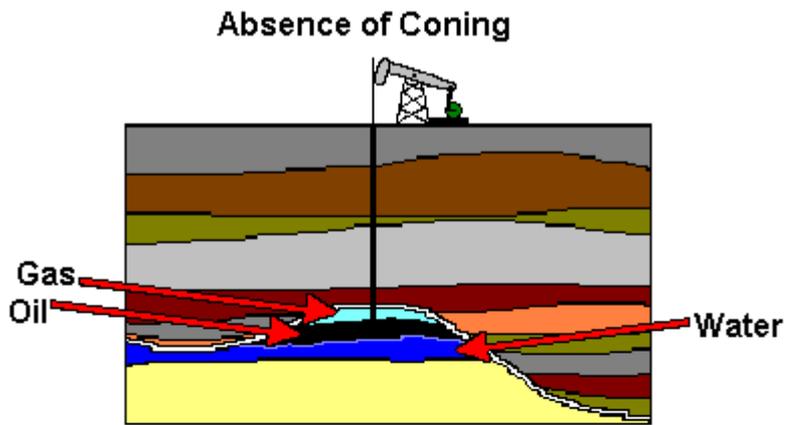
**categorical variable:**

Categorical variables are also known as discrete variables and indicator variables. A categorical variable has only two possible outcomes: 0 and 1. Either the data falls in the category or it does not.

**coning:**

Coning can occur in a oil reservoir for both gas and water. Because all oil reservoirs have water in them, and most have gas coning is a serious concern. Water and gas have a lower viscosity than oil and therefore flow preferentially instead of oil if allowed. If a preferential path of flow for water or gas opens then the well may become unsalvageable because the water or gas may form a cone about the well displacing the oil as in the figure below:





**continuous random variable:**

If an attribute has a continuous, no breaks in the distribution, range of possible outcomes with natural ordering then it is called a continuous variable.

**correlation:**

The characteristic of having linear interdependence between random variables or between sets of numbers.

**covariance:**

The covariance is a measure of the joint variation of two random variables about their respective means. The formula is:

$$\sigma_{ij} = \frac{1}{n} \sum_{\alpha=1}^n ((z_i(\alpha) - m_i) \cdot ((z_j(\alpha) - m_j))$$

**D**

**Darcy's equation:**

Sometimes referred to as Darcy's law. A relationship for the fluid flow rate  $q$  through a porous medium:

$$q = \frac{kA}{\mu} \cdot \frac{\Delta p}{\Delta x}$$

where:  $k$  = permeability.  $A$  = cross-sectional area,  $\mu$  = viscosity and  $\Delta p$  = pressure difference across the distance  $\Delta x$ .

**delineate:**

To show or illustrate by drawing, description or sampling

**density:**

Mass per unit volume (often expressed as specific gravity). Well-logging units are  $\text{g/cm}^3$ , often written  $\text{g/cc}$ .

**deterministic:**

A deterministic model is a model with properties that are so well and completely known that it can be modeled with no uncertainty. An example would be dropping a ball of known physical properties from a known height, velocity, and direction. Given these parameters we can model the path of the ball. Unfortunately the physics involved in most geological processes are not completely known so deterministic modeling is not often an option. Usually it is possible to inject some deterministic knowledge into the model such as faults and so on.

**development well:**

A well drilled in an area in a field to complete a pattern of production. An exploitation well.

**diagenesis:**

The chemical, physical, and biological changes that a sediment undergoes after initial deposition and burial that convert the sediment to consolidated rock and/or result in the creation of some forms of porosity. Such changes might result from compaction, cementation, recrystallization or replacement, but exclude metamorphism and fracturing resulting from tectonic stresses.

**diagenetic porosity:**

Porosity developed in a sediment after its initial deposition as a result of chemical and biological changes and burial. A form of secondary porosity as opposed to primary porosity. Fracture porosity is not thought of as diagenetic porosity.

**digitize:**

Convert data from analog trace records to digital, machine-useable numbers.

**dilatancy:**

The property of rock to expand. (1) Consolidated rock can expand as a result of the creation of microfractures.

(2) Unconsolidated rocks, which have a large number of grain contacts, have the property to expand by readjusting grain positions to those positions resulting in fewer contacts.

**dip:**

The angle that a structural surface (e.g., a bedding or fault plane) makes with the horizontal, measured perpendicular to the strike of the structure. Observe dip patterns on idealized log in illustration. Also see illustration of dip at strike.

**directional drilling:**

Slant hole drilling. The technique of intentional, controlled drilling at an angle from the vertical by deflecting the drill bit. Although wellbores are normally drilled vertically, it is sometimes necessary or advantageous to drill at an angle from the vertical. Controlled directional drilling makes it possible to reach subsurface areas laterally remote from the point where the bit enters the earth. It involves the use of turbodrill, whipstocks, or other deflecting tools.

**discovery well:**

The first oil or gas well drilled in a new field. The well that reveals the presence of a petroleum-bearing reservoir. Subsequent wells are development wells.

**dispersed:**

A term used in well-logging to refer to solid particles distributed within the interstices of the rock framework. A form of distribution of clays and other small particles and crystals. Detrital and/or authigenic materials often are found dispersed in the form of fines or crystal growths occupying pore space and lining the walls surrounding the pores.

**dissolved-gas drive:**

A solution-gas drive. See also reservoir drive mechanism and bubble point.

**distillate:**

Liquid hydrocarbons, usually water-white or pale straw color, and high API gravity (above 60 degrees) recovered from wet gas by a separator that condenses the liquid out of the gas stream. (Distillate is an older term for the liquid. More commonly it is called condensate or natural gasoline.) The product obtained from the condensation of vapors in distillation.

**dolomite:**

A type of sedimentary rock similar to limestone but rich in magnesium carbonate. sometimes a reservoir rock for petroleum.  $\text{CaMg}(\text{CO}_3)_2$ .

**drainage:**

The migration of oil or gas in a reservoir toward a wellbore due to pressure reduction caused by production of reservoir fluids by the well. A drainage point is a wellbore (or in some cases several wellbores) which drains the reservoir.

**drainage radius:**

The radius, measured from a wellbore, of a circular area of a reservoir which is drained by a single well drawdown (1) The difference between static and flowing bottom-hole pressures. (2) The distance between the static level and the pumping level of the liquid in the annulus of a pumping well.

# E

**effective permeability:**

The ability of the rock to conduct a fluid in the presence of another fluid, immiscible with the first, is called its effective permeability to that fluid. It not only depends on the permeability of the rock itself, but also upon the relative amounts of the two (or more) different fluids in the pores. Usually measured in millidarcies, or darcies. Compare relative permeability.

**effective porosity:**

(1) Interconnected pore volume occupied by free fluids. Hydrodynamically effective pore volume. Compare  $\frac{\text{total porosity}}{\text{porosity}}$ . See  $\frac{\text{porosity}}{\text{porosity}}$ .  
(2) Also electrically effective porosity. May differ from hydrodynamically effective pore volume.

**electric log:**

A generic term used to mean electric well log or electrical well log, without specific reference to type. Compare electrical survey.

**elevation:**

It is the distance measured above or below, a specific depth reference. In well logging, it is the vertical distance between the depth datum used for depth measurements in the well bore and sea level (with appropriate sign). See depth datum.

**ergodicity:**

Ergodicity implies that the expected value can be approximated by the mean of an actual realization over a sufficiently large domain. (big book)

**equiprobable:**

Of the same probability as any other result or realization.

## experimental variogram:

See variogram, experimental

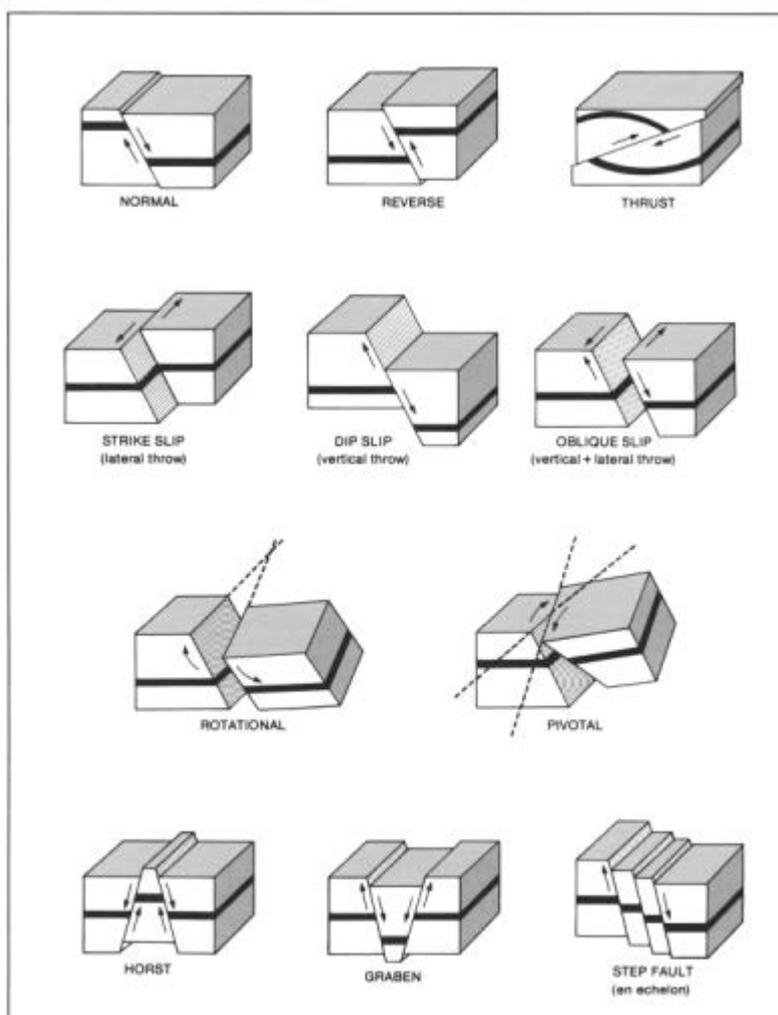
## exploratory well:

Well drilled to find the limits of a hydrocarbon-bearing formation only partly developed.

# F

## fault:

Fracture or break in subsurface strata. Strata on one side of the fault line have been displaced (upward, downward, or laterally) relative to their original positions.



Fault and Fault Block Movements  
(Courtesy of Schumberger)

**fault block:**

A mass bounded on at least two opposite sides by faults. It may be elevated or depressed relative to the adjoining regions, or it may be elevated relative to the region on one side and depressed relative to that on the other. See illustration in fault definition.

**fault trap:**

A subsurface hydrocarbon trap created by faulting, which causes an impermeable rock layer to be moved to a location opposite the reservoir bed.

**finer:**

Fragments or particles of rock or mineral which are too minute to be treated as ordinary coarse material. When found in pore spaces of reservoir rock, fines sometimes can be moved by produced fluids to cause formation damage. See skin effect.

**fissure:**

A crack or fracture in a subsurface formation.

**flood:**

To drive oil from a reservoir into a well by injecting water under pressure into the reservoir formation. See water flood.

**flowing pressure:**

The pressure registered at the wellhead of a flowing well.

**flowing well:**

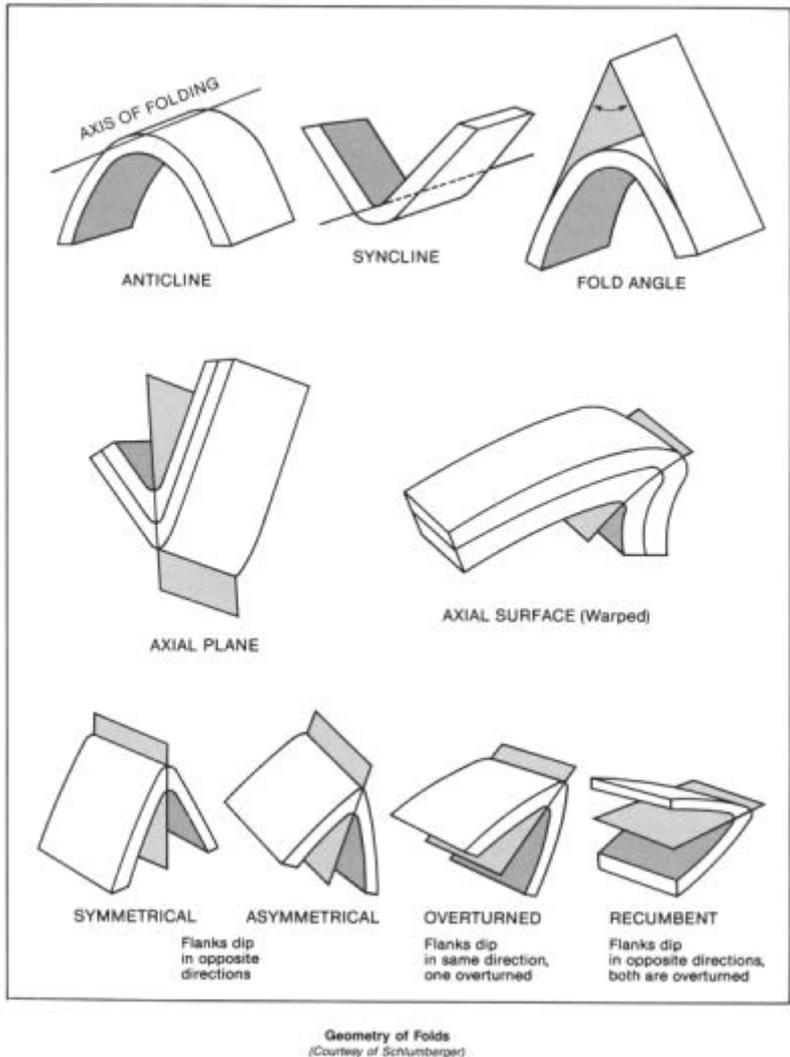
A well that produces oil or gas by its own reservoir pressure rather than by use of artificial means (as pumps).

**fluid:**

Any substance which will undergo continuous deformation when subjected to shear stress. Liquids and gases are fluids.

**fold:**

A flexure of rock strata into arches and troughs, produced by earth movements. See anticline and



syncline.

**formation:**

- (1) A general term applied in the well-logging industry to the external environment of the drilled well bore without stratigraphic connotation.
- (2) The basic or fundamental rock-stratigraphic unit in the local classification of rocks, consisting of a body of rock (usually a sedimentary stratum of strata, but also igneous and metamorphic rocks) generally characterized by some degree of internal lithologic homogeneity or distinctive lithologic features (such as chemical composition, structures, textures, or gross aspect of fossils). Formations may be combined in groups or subdivided into members and beds. A formation name should preferably consist of a geographic name followed by a descriptive lithologic term (usually the dominant rock type) or by the word formation if the lithology is so variable that no single lithologic distinction is appropriate

**formation dip:**

The angle at which a formation bed inclines away from the horizontal. See dip. See illustration at strike.

**formation evaluation:**

The analysis and interpretation of well-log data, drill-stem tests, etc. in terms of the nature of the formations and their fluid content. The objectives of formation evaluation are (1) to ascertain if commercially producible hydrocarbons (or other forms of energy and minerals) are present, (2) to determine the best means for their recovery, and (3) to derive lithology and other information on formation characteristics for use in further exploration and development.

**formation pressure:**

formation pressure The pore pressure existing within reservoir rock or non-reservoir rock at a specified time. The pressure exerted by fluids in a formation, recorded in the hole at the level of the formation with the well shut in. It is also called reservoir pressure or shut-in bottom-hole pressure. See reservoir pressure. Compare geopressure.

**formation strike:**

See description and illustration under strike.

**formation water:**

See interstitial water. Compare connate water.

**fossil fuel:**

A deposit of organic material containing stored solar energy that can be used as fuel. The most important are coal, natural gas, and petroleum.

**fracture:**

A break, parting, or separation in brittle rock.

**fracture porosity:**

Porosity resulting from the presence of openings produced by the breaking or shattering of brittle rocks.

**free interstitial water:**

Mobile interstitial water. Compare bound water.

# G

**geophysics:**

1. Branch of Earth Science which applies the methods of physics.
2. Often used to describe the techniques of geophysical prospecting.



**geostatistics:**

A branch of applied statistics which deals with spatially located data.

**global estimation:**

Estimation relating to a whole deposit or a significant zone in a deposit. Contrast with *local estimation*.

**Gaussian transform:**

A mathematical transformation of grades to a normal (Gaussian) distribution.

**gas:**

A fluid, compressible substance that completely fills any container in which it is confined. Its volume is dependent of the size of the container.

**gas cap:**

A free-gas phase overlying an oil zone and occurring within the same reservoir as the oil. See reservoir.

**gas cap drive:**

Drive energy supplied naturally (as a reservoir is produced) by the expansion of gas in a cap overlying the oil in the reservoir. See reservoir-drive mechanism.

**gas condensate:**

Liquid hydrocarbons present in casinghead gas that condense upon being brought to the surface; formerly distillate, now condensate. Also casinghead gasoline; white oil.

**gas cone:**

See cone and coning.

**gas drive:**

The use of the energy that arises from gas compressed in a reservoir to move crude oil to a well bore. Gas drive is also used in a form of secondary recovery, in which gas is injected into input wells to sweep remaining oil to a producing well.

**gas in solution:**

- (1) Gas dissolved in water or crude oil.
- (2) Gas dissolved in crude oil decreases the specific gravity and viscosity of the oil in a reservoir. Dissolved-gas energy is freed by the expansion of the gas released from solution as the pressure drops along the gradient from the reservoir, through the well, and to the tank. As gas expands, it moves along the pressure gradient carrying or driving oil along with it.

**gas lift:**

The process of raising or lifting fluid from a well by injecting gas down the well through tubing or through the tubing-casing annulus. Injected gas aerates the fluid to make it exert less pressure than formation pressure; consequently, the higher formation pressure forces the fluid out of the wellbore. Gas may be injected continuously or intermittently, depending on the producing characteristics of the well and the arrangement of the gas-lift equipment.

**gas-oil contact:**

The lowest depth (deepest depth in a well) opposite a formation at which virtually 100% gas can be produced. This depth is at the top of the gas-oil transition zone.

**gas-oil interface:**

gas-oil interface. See gas-oil contact.

**gas-oil ratio:**

GOR. A measure of the volume of gas produced with oil, expressed in cubic feet per barrel or cubic meters per metric ton.

**geology:**

The science that relates to the study of the structure, origin, history, and development of the earth as revealed in the study of rocks, formations, and fossils

**geopressure:**

- (1) Pressure pertaining to the earth, ground, or crust of the earth. A specific pressure regime, as geothermal pertains to a specific temperature regime.
- (2) Undisturbed formation pressure. The pore pressure existing within a body of interest prior to disturbance.
- (3) Historically misused to indicate high abnormal pore pressure.

**geopressure gradient:**

The change in pore pressure observed per unit change in depth. If the pore pressures are normal, then the geopressure gradient pertains to the normal pressure gradient (on the Gulf Coast about 0.465 psi/foot depth).

**graben:**

A block of the earth's crust that has slid downward between two faults; the opposite of a horst. See illustration in fault.

**gravity:**

- (1) The force of attraction between bodies of finite mass. Usually expressed in milligals.
- (2) API gravity. A method for specifying the density of crude petroleum.
- (3) That force that causes pain when you fall off of something.

**gusher:**

An oil well which has come in with such great pressure that the oil jets out of the well like a geyser. In reality, a gusher is a blowout and is extremely wasteful of reservoir fluids and drive energy. In the early days of the oil industry, gushers were common, and many times were the only indications that a large reservoir of oil and gas had been found.

# H

**hard data:**

That data with precisely known properties. Not remote sensed data.

**heterogeneity:**

Having differing or dissimilar properties.

**histogram:**

A histogram is a frequency diagram constructed by dividing the data into categories and tallying the frequency of occurrence of data into each category. The categories or bins need to be of the same range of values. The bin frequencies are usually indicated by rectangles, hence the area of rectangles are proportional to the frequency

**horst:**

A block of the earth's crust that has been raised between two faults. The opposite of a graben. See illustration in fault.

**huff-and-puff:**

Cyclic steam injection. A form of enhanced oil recovery.

**hydraulic fracturing:**

The breaking or parting of reservoir rock through the use of injected fluids. Hydraulic fracturing is a method of stimulating production or injection at a specific depth in a formation of low permeability by inducing fractures and fissures in the formation by applying high fluid pressure to its face. Fluids (liquids, gases, foams, emulsions) are injected into reservoir rock at pressures which exceed the strength of the rock and overcome internal stresses of the rock. The fluid enters the formation and parts or fractures it. Sand grains, aluminum pellets, glass beads, or similar materials are carried in suspension by the fluid into the fractures. These are called propping agents or proppants. When the pressure is released at the surface, the fracturing fluid returns to the wellbore as the fractures partially close on the proppants, leaving paths with increased permeability for fluid flow.

**hydrocarbon saturation:**

Fraction of the pore volume filled with hydrocarbons (oil or gas).

# I

## **indicator kriging:**

See kriging, indicator.

## **indicator variable:**

A binary transformation of data to either 1 or 0, depending on exceedence of a cut off.

## **interpolation:**

Resource estimation techniques in which samples falling within a specified *search neighbourhood* are weighted to form an estimate, e.g. *kriging* and *inverse distance weighting*.

## **inverse distance weighting:**

Non-geostatistical method of interpolation which assumes that grades vary in a deposit according to the inverse of their separation (raised to some power). This method does not account for *nugget variance* or other aspects of the *variogram* (such as anisotropy, short scale structure, etc.).

## **impedance:**

The opposition that an electrical circuit presents to the flow of an alternating current (AC). The ratio of the maximum voltage in an AC circuit to the maximum current. Compare resistance.

## **impermeable:**

Preventing the passage of fluid. A formation may be porous yet impermeable if there is an absence of connecting passages between the voids within it. See permeability.

## **immiscible:**

Not capable of mixing or being permanently mixed (as oil and water).

## **inclination:**

Deviation angle. Measured in directional surveys and used in calculating true vertical depths. It is the angle between the axis of the measuring tool (hence, borehole axis) and true vertical. The inclination can also be measured with respect to true horizontal.

## **injection well:**

A well into which fluids have been pumped, and from which fluids have been injected into an underground stratum to increase or maintain reservoir pressure.

**in situ:**

In place. In its natural location.

**interface:**

The contact surface between two boundaries of immiscible fluids, dissimilar solids, etc.

**interstitial water:**

Water occupying interstices or pore volumes in rock.

**inverse distance squared:**

A method estimation that uses known values weighed according their distance from the known value using the following formula:

$$\lambda_i = \frac{\frac{1}{d(u - u_i)^2}}{\sum_{i=1}^n \frac{1}{d(u - u_i)^2}}$$

**irreducible saturation:**

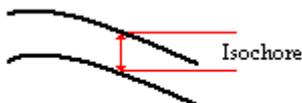
The irreducible saturation of a fluid is the minimum saturation of that fluid attainable when that fluid is displaced from a porous medium by another fluid immiscible with the first.

**irreducible water saturation:**

The fraction of the pore volume occupied by water in a reservoir at maximum hydrocarbon saturation. In water-wet rock, it represents the layer of adsorbed water coating solid surfaces and the pendular water around grain contacts and at pore throats. Irreducible water saturation is an equilibrium situation. It differs from "residual water saturation," measured by core analysis because of filtrate invasion and the gas expansion that occurs when the core is removed from the bottom of the hole and brought to the surface.

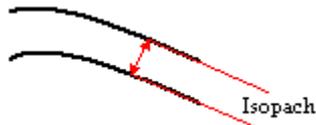
**isochore map:**

A map showing thickness of a unit measured vertically



### **isopach map:**

A geological map of subsurface strata showing the various thicknesses of a given formation normal to the stratigraphic thickness. It is widely used in calculating reserves and in planning secondary-recovery projects.



### **isosaturation surface:**

In a zone where saturations are changing, as in a transition zone, it is an imaginary surface which passes through all saturations of a specific fluid which have the same value.

### **isotropy:**

The property of homogeneity or uniformity of a rock which allows it to show the same responses or characteristics when measured along different axes. Compare anisotropy.

### **iterative:**

Describes a procedure which repeats until some condition is satisfied. Successive approximations, each based on the preceding approximations, are processed in such a way as to converge onto the desired solution.

# J

# K

### **Krige's relationship:**

see volume-variance

### **kriging:**

Geostatistical method of interpolation. Kriging is a weighted average where the weights are a function of the variogram.

### **kriging\_neighborhood:**

The volume surrounding a block to be estimated which is searched for samples during interpolation by kriging. Also called "search neighbourhood."

**kriging variance:**

The variance of the error associated with a given block estimate, where the block has been estimated by kriging. The kriging variance is useful in determination of the kriging neighbourhood and can be a useful guide to resource classification.

**kriging, indicator:**

Ordinary Kriging of indicator variables. A non-linear estimation method.

**kriging, ordinary:**

Kriging technique where interpolation weights are only applied to samples found within the search neighbourhood. The generic term "kriging" usually implies Ordinary Kriging.

**kriging, simple:**

Kriging technique where a proportion of the interpolation weight is applied to the mean grade of a deposit (or zone). Simple Kriging is assuming a higher degree of spatial homogeneity (or stationarity) than Ordinary Kriging.

# L

**lag distance:**

Separation distance (in 3D space). The x-axis of the *variogram* is in units of lag. Denoted "h" by convention.

**lithology:**

1. The physical character of a rock generally determined without the aid of a microscope. 2. Sometimes used with the general meaning of "rock type".

**local estimation:**

Estimation relating to individual blocks or panels within a deposit. Contrast with *global estimation*.

**linear estimator:**

Estimation method based on a linear weighted average of grades, e.g. *kriging*, *inverse distance weighting* etc.

**laminar:**

In the form of a thin layer(s) of sedimentary rock which is of different mineralization or lithology and is visually separable from the host rock or other laminae. Laminae of sand or other permeable sediment sometime show unusually high productivity. Laminae of clay, sandwiched between layers of a host rock, not only support overburden but also take the place of some of the host rock and whatever porosity the host rock would contain.

**lease:**

- (1) A legal document executed between a landowner, or lessor, and a company or individual, as lessee, that grants the right to exploit the premises for minerals or other products.
- (2) The area where production wells, stock tanks, separators, and other production equipment are located.

**least-squares fit:**

An analytic function which approximates a set of data such that the sum of the squares of the distances from the observed points to the curve is a minimum. One must determine the functional form of the fit (whether linear, quadratic, etc.) in order to define the problem.

**lens:**

- (1) A porous, permeable, irregularly shaped sedimentary deposit surrounded by impervious rock.
- (2) A lenticular sedimentary bed that pinches out, or comes to an end, in all directions.

**limestone:**

A bedded sedimentary rock consisting chiefly of calcium carbonate ( $\text{CaCO}_3$ ) yielding lime when burned. Limestone is the most important and widely distributed of carbonate rocks and is the consolidated equivalent of limy mud, calcareous sand, or shell fragments.

**liquefied natural gas:**

LNG. A liquid composed chiefly of natural gas (i.e., mostly methane). Natural gas is liquefied to make it easier to transport if a pipeline is not feasible (as across a body of water). Not as easily liquefied as liquefied petroleum gas, LNG must be put under low temperature and high pressure to become liquefied.

**liquefied petroleum gas:**

LPG. A mixture of heavier, gaseous, paraffinic hydrocarbons, principally butane and propane. These gases, easily liquefied at moderate pressures, may be transported as liquids but converted to gases on release of the pressure. Thus, liquefied petroleum gas is a portable source of thermal energy that finds wide application in areas where it is impracticable to distribute natural gas. It is also used as a fuel for internal-combustion engines and has many industrial and domestic uses. Principal sources are natural and refinery gas, from which the liquefied petroleum gases are separated by fractionation.

**liquid:**

A state of matter in which the shape of the given mass depends on the containing vessel, but the volume of the mass is independent of the vessel. A liquid is practically incompressible fluid.

**lithification:**

The conversion of unconsolidated deposits into solid rock.



**lithology:**

- (1) The physical character and composition of the rock. Refers to the different rock strata within the formations penetrated by the borehole.
- (2) The study of rocks, usually macroscopic. lithostatic load The weight of the overlying rock column without the fluids contained in the pore volumes of the rock.

**log:**

- (1) Well log. A record containing one or more curves related to properties in the well bore or some property in the formations surrounding the well bore.
- (2) To run a survey or well-logging operation.

**logging tool:**

An openhole or cased-hole tool for performing downhole well log data gathering services for determining properties of the formation, or characteristics of the well bore environment.

**loss function:**

A function that measures the impact of any particular level of error in the profitability of an operation.

# M

**macroscopic anisotropy:**

See anisotropy.

**matrix:**

- (1) The solid framework of rock which surrounds pore volume.
- (2) In a rock in which certain grains are distinctly larger than the others, the grains of smaller size comprise the matrix.
- (3) The natural material in which any metal, fossil, pebble, or crystal is imbedded.
- (4) In mathematics, a rectangular array of numbers which obey certain rules.

**md:**

Millidarcy. 1/1000 darcy.

**measured depth:**

MD. Depth measured along the drilled hole. Reported in drilling records, measured by well-logging cables, and shown on well logs. This depth has not been corrected for hole deviation. Compare true vertical depth.

**merged data:**

Refers to well-log data derived from different well logging measurements which have been adjusted for differences in depths of the measurements.

**metamorphic rock:**

An igneous or sedimentary rock which has partially or completely recrystallized in response to elevated temperature, pressure, and chemical environment. The change the metamorphic rocks have undergone generally occurs in the solid state and at depth in the earth.

**metric ton:**

A measurement equal to 1000 kg or 2204.6 lb avoirdupois. In many oil-producing countries, production is reported in metric tons. One metric ton is equivalent to about 7.4 barrels (42 U.S. gal = 1 bbl) of crude oil with specific gravity of 0.84, or 36 °API gravity.

**migration:**

The movement of oil from the area in which it formed to a reservoir rock where it can accumulate.

**millidarcy:**

md. Equal to 1/1000 of a darcy.

**mineral:**

A naturally occurring material having a definite chemical composition and, usually, a characteristic crystal form. May be disseminated in some other mineral or rock. Most mineralogists include the requirements that a mineral have inorganic origin and internal crystalline structure. In accord with the latter requirement, an amorphous compound such as opal would be considered to be mineraloid.

**mineral composition of rocks:**

The crust of the earth contains only eight elements (oxygen, silicon, aluminum, iron, calcium, sodium, potassium, and magnesium) with concentrations greater than 1% by weight. Minerals are compounds of these elements. each mineral having a specified crystalline structure. The most abundant minerals represent only five types of chemical compounds: silicates, carbonates, sulfates, halides, and oxides. Of these minerals, the silicates are more abundant than all the rest combined, comprising 95% of the rest.

**miscible drive:**

A method of enhanced recovery in which various hydrocarbon solvents or gases (as propane, LPG, natural gas, carbon dioxide, or a mixture thereof) are injected into the reservoir to reduce interfacial forces between oil and water in the pore channels and thus displace oil from the reservoir rock.

**N**

## **natural gas:**

A highly compressible, highly expansible mixture of hydrocarbons having a low specific gravity and occurring naturally in a gaseous form. The principal component gases of natural gas, with typical percentages, are

methane		80.0%
ethane		7.0%
propane		6.0%
isobutane		1.5%
butane		2.5%
pentane	plus	3.0%

In addition to these gases, natural gas may contain appreciable quantities of nitrogen, helium, carbon dioxide, and contaminants (as hydrogen sulfide and water vapor). Although gaseous at normal temperatures and pressures, certain of the gases comprising the mixture that is natural gas are variable in form and may be found either as gases or as liquids under suitable conditions of temperature and pressure.

## **nested variogram:**

See Variogram, Nested.

## **non-linear estimation method:**

Estimation method based on a non-linear transformation of data, eg *Uniform Conditioning* or *Indicator Kriging*. Such methods are employed to make *local estimates of recoverable resources*.

## **nugget variance:**

The y-intercept of a *variogram*. The nugget variance represents the chaotic or random component of grade variability. Also referred to as "nugget effect". Denoted  $C_0$  by convention.

## **net overburden pressure:**

That part of the overburden (geostatic load) which is supported by grain to-grain contact of the rock. Net overburden usually is less than (total) overburden because of two reasons: (1) Pore pressure within the bed of interest supports the weight of the column of formation fluid above it. If the pore pressure is higher than normal, then it supports part of the lithostatic load. (2) The stress on the rock concentrated at the grain-to-grain contact may be less than that caused by the weight of overlying dry rock, because of the buoyant effect on the porous rock column produced by the liquid in the pore volumes. Effective stress exerted by porous rock. That support by the grain-to-grain contact of the rock which when combined with the formation pressure is assumed to equal the stress due to overburden.

## **net present value:**

The sum of all revenues discounted to current dollars

**net to gross ratio:**

the amount of pay sand in the reservoir. If the lithofacies is quoted as having 20% sand and 80% shale (nonproducing material) then the net to gross ratio is 20%.

**net pay**

Within the limitations of given cutoffs for porosity, water saturation, etc., it is that portion of reservoir rock which will produce commercial quantities of hydrocarbon.

**net sand thickness**

The accumulated thickness of sandstone of a specified quality which is found within a specific interval of formation.

**noise**

- (1) Sudden spurious readings on a curve. These may be random events or caused by instrumentation problems.
- (2) A sound. In well logging, a sound related to some source in the wellbore environment, e.g., fluid flow, the production of formation fluids through perforations, leakage of fluid through apertures, etc.

**nonconformity**

Where the older rocks were metamorphosed and exposed by profound erosion before the strata were laid down on them, there is a major unconformity, representing a hiatus of great duration. To distinguish unconformities of this significance, the term "nonconformity" is used.

**normalize**

- (1) To adjust two log curves (or any other pairs of data) for environmental differences in order that one value may be compared with others.
- (2) To adjust two log curves or similar data to the same, or equivalent, units so that the data values can be compared.

**normal pore pressure**

In a reservoir rock it is the hydrostatic pressure resulting from the head of water solution (formation water) filling the pores of the rock in communication with the mean water table or sea surface.

# O

**oil field**

The surface area overlying an oil reservoir or reservoirs. Commonly, the term includes not only the surface area but may include the reservoir, the wells, and production equipment as well.

## **oil in place**

The amount of crude oil that is estimated to exist in a reservoir and which has not been produced.

## **oil shale**

The term applied to several kinds of organic and bituminous shales, most of which consist of varying mixtures of organic matter with marlstones, shale, and clay. The organic matter is chiefly in the form of a mineraloid, called kerogen. Oil shales are widely distributed throughout the world and become of economic interest because of the large amounts of oil which can be obtained from them. See kerogen.

## **oil-water contact**

The highest depth (shallowest depth in a well) opposite a formation at which virtually 100% water can be produced. This depth is at the bottom of the oil-water transition zone.

## **oil-water interface**

See oil-water contact.

## **oil wet**

Oleophilic. A condition in which oil wets the rock surfaces. Often described by the angle of contact of an oil droplet on the solid surface. The lower the angle (measured inside the oil phase) the greater the adhesion and the greater the degree of wettability for oil. If the nonwetting aqueous phase should be forced to move, it would advance over the adhesive layer of the oil.

## **ordinary kriging**

See Kriging, Ordinary.

## **oleophilic**

oleophilic Having a strong affinity for oils. Preferentially oil wet.

## **outcrop**

- (1) The exposed portion of a buried layer of rock.
- (2) To appear on the earth's surface (as a rock).

## **overburden**

Geostatic load. The aggregate of overlying rock column including the fluids contained within the pores of the rock.

## **overlay**

- (1) To place one recorded curve over another. See also normalize.
- (2) A well log in which one curve has been drafted or recorded over another and the relationship between the curves can be observed. The position of one curve with respect to the other and the

amount of separation between the two curves provides specific information with regard to rock properties, lithology, mineralogy, and fluid saturations.

## P

### **pay**

The part of a formation which produces or is capable of producing oil or gas, or other economic product.

### **permeability**

Absolute permeability. A measure of the ability of a rock to conduct a fluid through its interconnected pores when that fluid is at 100% saturation. Measured in darcies or millidarcies. See also effective permeability.

### **petroleum**

Oil or gas obtained from the rocks of the earth.

### **petroliferous**

Containing petroleum (of rocks).

### **pinch out**

The progressive thinning of a stratum to its point of disappearance.

### **pore**

An opening or void within a rock or mass of rock, usually small and filled with fluid (water oil, gas, or all three). See porosity. Compare vug.

### **pore pressure**

Pressure exerted by fluids contained within the pores of rock. See formation pressure.

### **porosity**

The ratio of void space to the bulk volume of rock containing that void space. Porosity can be expressed as a fraction or percentage of pore volume in a volume of rock. (1) Primary porosity refers to the porosity remaining after the sediments have been compacted but without considering changes resulting from subsequent chemical action or flow of waters through the sediments. See primary porosity.

(2) Secondary porosity is the additional porosity created by chemical changes, dissolution, dolomitization, fissures, and fractures. See secondary porosity.

(3) Effective porosity is the interconnected pore volume available to free fluids, excluding isolated pores and pore volume occupied by adsorbed water. In petroleum engineering practices, the term porosity usually means effective porosity.

(4) Total porosity is all void space in a rock and matrix whether effective or noneffective. Total porosity includes that porosity in isolated pores, adsorbed water on grain or particle surfaces, and associated with clays. It does not include water of crystallization wherein the water molecule becomes part of the crystal structure.

### **pressure maintenance**

A method for increasing ultimate oil recovery by injecting gas, water, or other fluids into the reservoir before reservoir pressure has dropped appreciably, usually early in the life of the field, to reduce or prevent a decline in pressure.

### **primary porosity**

Porosity which is present in sediment at the time of deposition or formed during sedimentation. Compare secondary porosity.

### **primary recovery**

Recovery of petroleum oil from underground reservoirs by taking advantage of the natural energy (pressures) in these reservoirs. The most common types of these sources of reservoir pressures are solution-gas drive, gas-cap-expansion drive, and water (aquifer) drive. More than one of these natural drives may occur in a single reservoir. See also secondary recovery and tertiary recovery.

### **production**

- (1) The amount of oil or gas produced in a given period.
- (2) That phase of an energy related industry which deals with bringing the formation fluids to the surface and preparing the product for pipeline or other transportation.

### **production log**

A well log run in a production or injection well. Small diameter tools are used so that they can be lowered through tubing. Services and devices include continuous flowmeter, packer flowmeter, gradiomanometer, manometer, densimeter, watercutmeter, thermometer, radioactive-tracer logs, temperature logs, calipers, casing collar locator, fluid sampler, water entry survey, etc.

### **P-wave**

compression wave.

## **Q**

### **q**

A term representing the fraction of clay occupying pore space. Sometimes calculated by

$$q = \frac{\phi_x - \phi_e}{\phi_x}$$

where:  $\phi_x$  represents the porosity of the framework of the rock and  $\phi_e$  represents effective porosity.

### **quartz**

A hard mineral composed of silicon dioxide. A common component in igneous, metamorphic, and sedimentary rocks.

### **quartzite**

A metamorphic rock composed of quartz which has been formed by the metamorphism of sandstone.

## **R**

### **radial flow**

The flow of fluids into a wellbore from the surrounding drainage area. Also, could be descriptive of the flow from an injection well to the surrounding area.

### **range**

The distance read from the *variogram* at which correlation ceases. Denoted "a" by convention.

### **random function**

A variable which takes a certain number of numerical values according to a certain probability distribution. For instance, the result of casting an unbiased die can be considered as a random variable which can take one of six equiprobable values. If one result is 5 then we say that this value is a particular realization of the random variable "result of casting the die"

### **ratio**

A mathematical relationship between two values, where one value is divided by the other, commonly expressed as a fraction.

### **random variable**

A function  $Z$  assigning one and only one real number  $Z(\mathbf{u}) = z$  to each element  $z$  of a sample space. It is understood that a probability function is also defined to account for the possibility that  $Z$  takes on certain values.

### **regionalized variable**

A single valued function defined over space. In practice a regionalized variable is preferentially used to describe natural phenomenon, such as the elevation of the bottom of the sea, that are characterized by fluctuations that are smooth at a global scale but erratic enough at a local scale to



preclude their analytical modeling. Geostatistics models regionalized variables as realizations of random functions.

### **realization**

A set of values [  $z(\mathbf{u}_1)$ ,  $z(\mathbf{u}_2)$ ,  $z(\mathbf{u}_3)$ , ...,  $z(\mathbf{u}_n)$  ] that may arise from a random function  $Z(\mathbf{u})$ . This realization may be regarded as a member of the random function in the same way as that an individual observation is regarded as a member of a population.

### **recovery**

- (1) The amount of core recovered compared to the amount cut.
- (2) The height of fluid in the drill pipe on a drill-stem test which did not produce enough fluid to rise to the surface.
- (3) The total volume of hydrocarbons that has been or is anticipated to be produced from a well or field.

### **recovery factor**

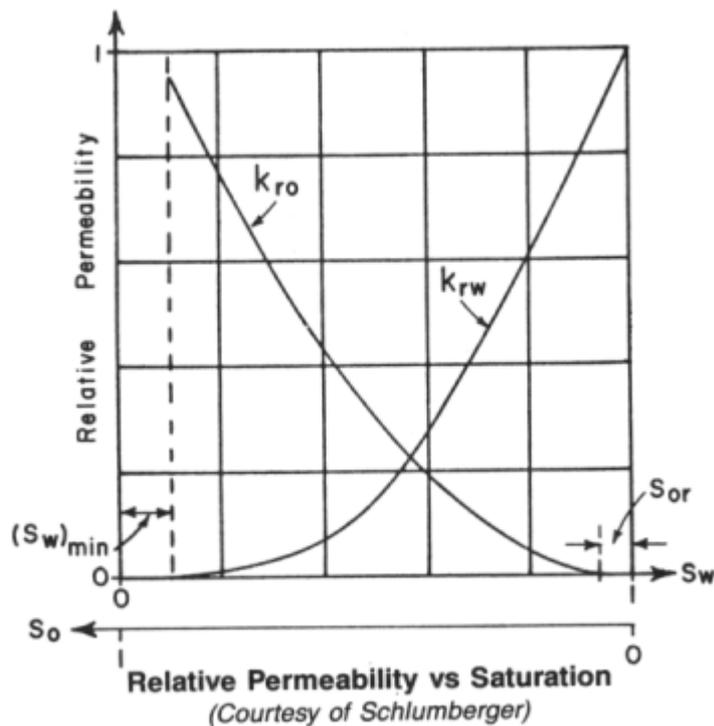
The percentage of oil or gas in place in a reservoir that ultimately can be withdrawn by primary and/or secondary techniques. The percentage of the oil or gas in place (expressed in stock tank barrels or in cubic feet) that will ultimately be recovered.

### **reef**

A type of reservoir composed of rock (usually limestone) formed from the skeletal remains of marine plants and animals. reference point (1) measure point. A mark or position on a tool to which all measurements are related. A tool zero. See measure point. (2) A depth datum.

### **reinjection**

The process of pumping produced water back into a porous and permeable formation by means of an injection well.



## relative Frequency

The number events in a bin divided by the total number of events.

## reserves

The unproduced but recoverable oil or gas in place, in a formation, that has been proved by production.

## reservoir

A subsurface, porous, permeable rock body in which oil or gas or both can be stored. Most reservoir rocks are limestones, dolomites, sandstones, or a combination of these. The three basic types of hydrocarbon reservoirs are oil, gas, and condensate. An oil reservoir generally contains three fluids: gas, oil, and water, with oil the dominant product. In the typical oil reservoir, these fluids occur in different phases as a result of the variation in their specific gravities. Gas, the lightest, occupies the upper part of the reservoir rocks; water, the lower part; and oil, the intermediate section. In addition to occurring as a cap or in solution, gas may accumulate independently of the oil. If so, the reservoir is called a gas reservoir. Associated with the gas, in most instances, are salt water and some oil. In a condensate reservoir, the hydrocarbons may exist as a gas, but, when brought to the surface, some of the heavier gases condense to a liquid or condensate. At the surface, the hydrocarbons from a condensate reservoir consist of gas and a high-gravity crude (i.e., the condensate). Condensate wells are sometimes called gas-condensate reservoirs.

reservoir-drive mechanism The natural energy by which reservoir fluids are caused to flow out of the reservoir rock and into a wellbore. Solution-gas drives depend on the fact that, as the reservoir is produced, pressure is reduced, allowing the gas to expand and provide the driving energy. Water-drive reservoirs depend on water pressure to force the hydrocarbons out of the reservoir and into the wellbore.

reservoir pressure Formation pressure. The pressure found within a reservoir at a specific point in time. Sometimes reservoir pressure is thought of as original pressure or geopressure (prior to disturbance) but at other times is thought of as pressure existing after disturbance. Reservoir or formation pressure should be qualified as to time, condition, and place.

## residual

That which remains after a removal or displacement process. residual oil Oil remaining in the reservoir rock after the flushing or invasion process, or at the end of a specific recovery process or escape process.

## rose diagram

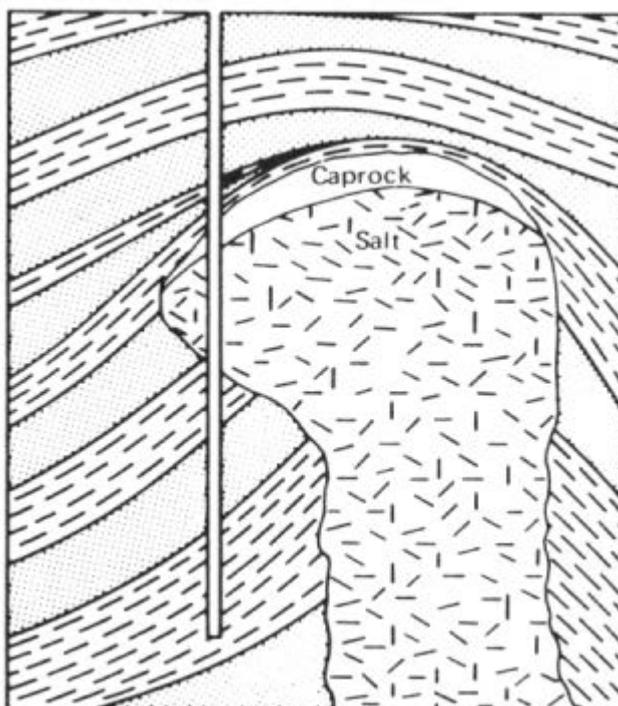
A polar plot or diagram in which radial distance indicates the relative frequency of an observation at a certain azimuth. Used in dipmeter interpretation. Compare azimuth frequency diagram.

# S

salinity The concentration of ions in solution (sodium, potassium, chloride, sulfate, etc.) See total dissolved solids and water analysis.

## salt dome

A dome that is formed by the intrusion of salt into overlying sediments. A piercement salt dome is one that has pushed up so that it penetrates the overlying sediments, leaving them truncated. Formations above the salt plug are usually arched so that they dip in all directions away from the center of the dome. See also diapir.



**Salt Dome**  
(Courtesy of Dresser Atlas)

## sampling nomogram

A graphical representation of sampling error as a function of particle size and sample mass used to design correct sampling protocols.

## sill

The level at which a *variogram* or some component of the variogram model (in the case of nested variograms) stabilises or plateaus. >Simple Kriging See Kriging, Simple.

## smoothing

Reduction of variability. To avoid *conditional bias*, it is necessary to smooth when *interpolating*. Contrast with *smearing*.

## stationarity:

Spatial statistical homogeneity.; In essence, the assumption of stationarity usually adopted for estimation (ie "intrinsic" stationarity) means that it is considered appropriate to characterise the overall spatial variability of a zone by a given *variogram model*.

## stereoplot:

A two-dimensional, graphical means of portraying three-dimensional orientation data, widely used by structural geologists.; Also called "stereographic projection". Support Size, shape and geometry of volumes upon which we either estimate or measure grade, e.g. 2 m half NQ core support; 12.5 m x 12.5 m x 9 m mining block support, etc. Support Effect Grades measured on small *support* are more variable than grades measured on large supports.; Support effect describes the impact on the grade distribution of considering different support.

## sand:

A detrital sediment composed of individual grains of sand (commonly quartz) whether or not the sediment is consolidated.

## sand count:

- (1) The total effective thickness of a permeable section excluding shale streaks or other impermeable zones. Often determined from electrical logs (SP curves and microlog).
- (2) The number of separate permeable sands separated by impermeable zones.

## sandstone

A detrital, sedimentary rock composed of individual grains of sand (commonly quartz) which have been bound together by natural cementing materials precipitating in the interstices. Common cements are silica, carbonates, and iron oxides.

- (1) Containing as much as it can contain under given conditions of temperature and pressure, as in:
- a. solid dissolved in liquid.
  - b. gas dissolved in liquid.
  - c. liquid dissolved in gas
- (2) Filled to capacity, as fluid (liquid or gas) in formation.

(3) Reached the limit of its measuring capacity, as in electrical instruments. The capability of an instrument to detect variations is decreased as the measuring instrument nears saturation. saturation (1) The fraction or percentage of the pore volume occupied by a specific fluid (e.g., oil, gas, water, etc.). The fluids in the pore spaces may be wetting or nonwetting. In most reservoirs, water is the wetting phase, but a few reservoirs are known to be oil wet. The wetting phase exists as an adhesive film on the solid surfaces. At irreducible saturation of the wetting phase, the nonwetting phase is usually continuous and is producible under a pressure gradient to the well bore.

(2) The occupation of fluids in a pore may take different forms:

a. Funicular saturation. A form of saturation in which the nonwetting phase exists as a continuous web throughout the interstices. The nonwetting phase is mobile under the influence of a hydrodynamic pressure gradient. The wetting phase might or might not be at irreducible saturation. In the illustration, the oil in the "A" figure is funicular

### **scale:**

(1) Depth scale. Depth scales vary with locale and requirements. Most single logs are recorded on two different films at once, and the two films may optionally be on different depth scales. One film may be for correlation and the other for detailed interpretation, or one may be for normal use and the other for quick-look interpretation by the overlay technique. In North America the depth scales for correlation use are: 1 or 2 in. of film per 100 ft. of hole (i.e., on a scale of 1:1200 or 1:600 in terms of feet of film per feet of hole). The usual scale for detail use is: 5 in. of film per 100 ft. of hole (i.e., 1:240). An expanded scale of 25 in. per 100 ft. (1:48) is available for close scrutiny of short sections, and a special 60 in. per 100 ft. scale (1:20) is used for dipmeter logs. Most of the rest of the world uses decimal scaling (1:1000, 1:500, 1:200, 1:40, 1:20) based either on the foot or the meter. Metric logs are easily identified by the fact that depth numbers appear at each 50-m. level, while logs scaled in feet have depth numbers at each 100-ft. level.

(2) Grid scale type. Usually linear, logarithmic, or split. Observe illustration. The hybrid scale, which is not shown, was used before the logarithmic scale for use with laterolog type surveys.

(3) Grid scale sensitivity. The scale on which the magnitude of the measured signal is recorded. It corresponds to the value given to the full-scale deflection of the galvanometer or pen, or the width of one track (100 small divisions) on linear scales; center or midscale (one-half width of one track = 50 small divisions) on hybrid scales

(4) Chemical scale. A deposit that accumulates on solid surfaces (e.g., casing, tubing, rock, etc.) or in perforations and other apertures. It reduces fluid production from the formation.

### **sealed reservoir:**

A reservoir of limited size. One from which the formation fluids cannot escape because of a permeability barrier.

### **secondary porosity:**

Post depositional porosity. Such porosity results from fractures, vugs, solution channels, diagenesis, dolomitization, etc. Three common types of secondary porosity are: fracture porosity, shrinkage porosity, and dissolution porosity. Compare primary porosity.

### **secondary porosity index:**

SPI. An estimate of the secondary porosity, calculated from sonic log values in conjunction with either density or neutron log values, or porosity resulting from a crossplot of density and neutron porosities.

If  $f_D$  is the porosity calculated from a density (or neutron) log and  $f_S$  is the porosity calculated from a sonic log, SPI is sometimes defined as  $(f_D - f_S)$  or  $(f_{\text{crossplot}} - f_S)$

### **secondary recovery:**

Recovery of petroleum oil from underground reservoirs by using secondary sources of energy, such as injected natural gas or water to increase producing rates and to improve ultimate oil recovery. Water injection, commonly known as water flooding, usually affords higher recovery than gas injection. Gas injection is generally limited to those reservoirs which have a gas cap and in which gas cap expansion is expected to be an efficient natural drive mechanism. Although the terms "primary" and "secondary" imply a sequence of use, both mechanisms might work concurrently once secondary recovery is implemented. See also primary recovery and tertiary recovery.

### **sediment:**

- (1) Solid matter which has been transported from its source by wind or water and then deposited.
- (2) Solid matter which has been precipitated from solutions or formed from skeletal remains or secretions of organisms.

### **sedimentary:**

Descriptive term for rock formed of sediment, especially:  
a. clastic rocks such as conglomerate, sandstone, and shales formed of fragments of other rock transported from their sources and deposited in water;  
b. rocks formed by precipitation from solution, such as rock salt and gypsum, or from secretions of organisms, such as most limestone.

### **sedimentary basin**

sedimentary basin A geologically depressed area with thick sediments in the interior and thinner sediments at the outer boundaries.

### **seismic**

Pertaining to an earthquake or earth vibration, including those which are artificially induced.

### **seismic attenuation:**

That portion of the decrease in seismic signal strength with distance not dependent on

### **geometrical spreading:**

The decrease depends on the physical characteristics of the transmitting media, involving reflection, scattering, and absorption.

### **seismic discontinuity:**

seismic discontinuity Physical discontinuity within the earth separating materials in which seismic waves travel at significantly different velocities

## **seismograph**

A device which records vibrations in the earth, detected by geophones, used in prospecting for probable oilbearing structures. Vibrations are created by discharging explosives in shallow boreholes or by striking the ground surface with a heavy blow. The type and velocity of the vibrations (as recorded by the seismograph) indicate the general characteristics of the section of earth through which the vibrations pass.

## **separation:**

- (1) The difference observed between two different well-logging measurements of the same or related parameters recorded or plotted on the same log (e.g., porosities, formation factor curves, etc.)
- (2) The differences observed between two similar well logging measurements, made at different depths of investigation, recorded or plotted on the same log (e.g., resistivities, movable oil plot, etc.).

## **shale**

A fine grained, thinly laminated or fissile, detrital sedimentary rock formed by the compaction and consolidation of clay, silt, or mud. The composition is characterized by an appreciable content of clay minerals, or derivatives from clay minerals, and a high content of detrital quartz.

## **shale base line:**

- shale base line (1) A line drawn through the deflections characteristic of shale on an SP curve, which is used as the reference in making measurements to determine the characteristics of permeable rocks and their formation waters.
- (2) The characteristic of thick shale on the gamma-ray log or other well logs.

## **shale potential:**

Shale-membrane potential. See electrochemical potential.

## **shaly:**

Pertaining to, composed of, containing, or having the properties of shale, particularly the property to readily split along close-spaced bedding planes.

## **shape factor**

See porosity exponent.

## **shear modulus**

See elastic properties of rocks.

## **shear\_wave**

S-wave. In acoustics, a transverse wave. Direction of propagation is perpendicular to direction of particle displacement. For transmission of a shear wave, particles in lateral motion must drag neighboring particles in similar lateral motion. Substances which tend to oppose shear can support

propagation of a shear wave (i.e., rigid substances, solids). Compare compression wave. See also acoustic wave.

### **skin:**

A zone of reduced permeability around the well bore, resulting from damage due to the drilling, completion, and/or production practices. See also skin effect part (2).

### **skin effect:**

(1) In well logging. Sometimes called propagation effect. As applied to induction logging, a reduction in the conductivity reading (increase in the resistivity reading) observed in very conductive media. A thick-bed correction for the effect is usually provided in the instrument. Residual corrections, when needed in thin beds, may be made by means of charts. Simply stated, skin effect results from the interactions between adjacent ground loops when, because of their high conductivity, they carry induced currents of appreciable magnitudes. The magnetic fields of the ground-loop currents induce additional eddy currents in adjacent ground loops which are superimposed on those induced by the transmitter coil field. The net result is to nullify somewhat the magnetic field of the transmitter coil, and the resultant field may be shifted in phase. The conductivity signal seen at the receiver coils and the depth of investigation are thereby decreased. The term "skin effect" originated from the tendency in metallic conductors for high-frequency alternating-current flow to be concentrated near the surface or "in the skin" of the conductor.

(2) In pressure transient testing and analysis. Skin effect is the additional resistance to fluid flow which causes an additional pressure drop at the well bore and/or in the formation near the well bore. The additional pressure drop is the result of some form of damage occurring during the drilling, completion, and/or production operations. Some factors which can cause this alteration are: invasion by mud filtrate or whole mud; cement; partially plugged perforations; movable fines; introduced contaminants such as rust, pipe dope, etc. The zone where permeability has been reduced by these factors creates an additional pressure drop during production or injection and is referred to as skin. The resulting effect is skin effect. The skin factor reflects the effects of the additional pressure drop. A positive value of the skin factor denotes formation damage, or damage at the well bore. A negative value of skin effect can result from man-made fractures or any form of stimulation which increases the ease with which fluid can flow in the region immediately surrounding the well bore.

## **Society of Professional Well Log Analysts**

SPWLA. A society of 3800 persons (as of 1984) in over 60 countries which was organized to advance the science of formation evaluation through well-logging techniques. It was organized on January 29, 1959 at Tulsa, Oklahoma; incorporated December 14, 1959 in the State of Oklahoma; and has its business office in Houston, Texas. The SPWLA conducts annual symposia and publishes "The Log Analyst."

### **solution**

A uniform mixture of two or more substances. The dissolved substance is the solute. The substance in which the solute is dissolved is the solvent.

### **solution gas:**

Lighter hydrocarbons that exist as a liquid under reservoir conditions, but become a gas when pressure is reduced as the reservoir is produced. See solution-gas drive.



**solution-gas drive:**

A source of natural reservoir energy, in which the solution gas coming out of the oil expands to force the oil into the wellbore.

**spatial correlation**

the property of having linear interdependence between random variables in space. The random variables may or may not be different attributes.

**specific\_gravity:**

(1) Of solids or liquids, it is the ratio of the density of the material at standard conditions of temperature and pressure to the density of pure water under standard conditions.  
(2) Of a gas, it is the ratio of the density of the specific gas at standard conditions of temperature and pressure to the density of dry air standard conditions.

**spike\_noise:**

A spurious, unwanted event recorded on a curve of a well log.

**standard deviation:**

In statistics. The positive square root of the expected value of the square of the difference between a random variable and its mean. With a normal distribution of data, 68.3% of the data fall within one standard deviation from the mean.

**statistics:**

Statistics is the science of collecting, processing, analyzing and interpreting numerical data. Statistics dilutes numerical information to provide more easily (sometimes) understood insights to the population.;

**step-out well:**

1 A well drilled adjacent to a proven well, but located in an unproven area, in order to determine the extent and boundaries of a producing formation.

**stochastic:**

Stochastic refers to the use of statistics to develop the properties that the model will be populated with.

**stratum**

strata Plural of stratum. See stratum.

**stratification:**

Refers to the vertical succession of unlike strata (layering of the beds) penetrated by the well bore. The layers of strata can be differentiated from one another by the difference in their well log-derived properties caused by differences in composition, porosity, resistivity, etc.

**stratigraphic trap:**

A type of reservoir capable of trapping oil or gas due to changes in porosity and permeability or to the termination of the reservoir bed.

**stratigraphy:**

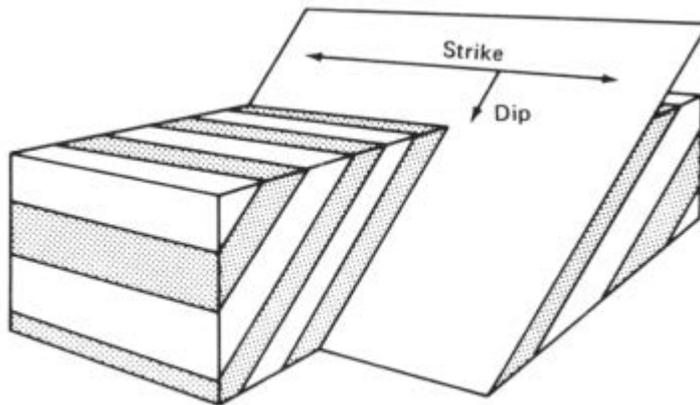
A branch of geology concerned with the study of the origin, composition, distribution, and succession of rock strata.

**stratum:**

A section of a formation that consists throughout of approximately the same kind of rock material. A single sedimentary bed or layer regardless of thickness.

**strike:**

The direction or bearing of a horizontal line drawn on the plane of a structural surface; e.g., inclined stratum, fault plane, etc. The strike is perpendicular to the direction of dip. The bearing can be referred to south or north; e.g., N 30° E or S 30° W.



**Dip and Strike**  
*(Courtesy of Dresser Atlas)*

**structural:**

(1) In the microscopic sense, it pertains to the framework of rock. It refers to that rock material (called matrix by some log analysts) which creates the load-bearing structure. The interstices and other voids in the structure constitute porosity.

(2) In the megascopic sense, it pertains to large visible features of rock.

**structural trap:**

A trap in reservoir rock which has been formed by the deformation (folding or faulting) of the rock layer. Anticlines, salt domes, and faults of different kinds result in barriers which form traps. See fault and fold.

**super-saturated:**

A solution which contains more of the solute than is normally present when equilibrium is established between the saturated solution and undissolved solute.

**syncline:**

A downwarded, trough-shaped configuration of folded, stratified rocks. The reverse of an anticline. See illustration in fold.

# T

**tertiary recovery:**

Recovery methods which increase ultimate oil production beyond that achievable with primary and secondary methods. These methods are usually employed in the latter stages of conventional secondary flooding applications, but may be implemented early in the secondary application or in combination with the secondary method. These oil recovery methods enhance the production of oil by increasing the proportion of the reservoir affected, reducing the amount of residual oil in the swept zones and reducing the viscosity of thick oils. The tertiary methods usually are divided into three broad groups: thermal, miscible and chemical. See also primary and secondary recovery.

**time-average\_relationship:**

An empirical expression used for calculating porosity from interval transit time determined from acoustic logs:

$$\phi = \frac{\Delta t - \Delta t_{ma}}{\Delta t_f - \Delta t_{ma}}$$

where  $t$  = observed interval transit time,  $t_f$  = transit time in the pore fluid, and  $t_{ma}$  = transit time in the rock matrix. This relation works well in clean consolidated formations with uniformly distributed pores. In vuggy formations, the sonic log may not reflect the secondary porosity; and in unconsolidated formations, this relationship may overestimate porosity. In such cases, the formula may be empirically modified to give better values.

**tortuosity:**

The crookedness of the pore pattern. The ratio of the distance between two points by way of the connected pores to the straight-line distance.

## **total\_porosity:**

The total pore volume occupied by fluid in a rock. Includes isolated nonconnecting pores and volume occupied by adsorbed, immobile fluid. See porosity. Compare effective porosity.

## **trap:**

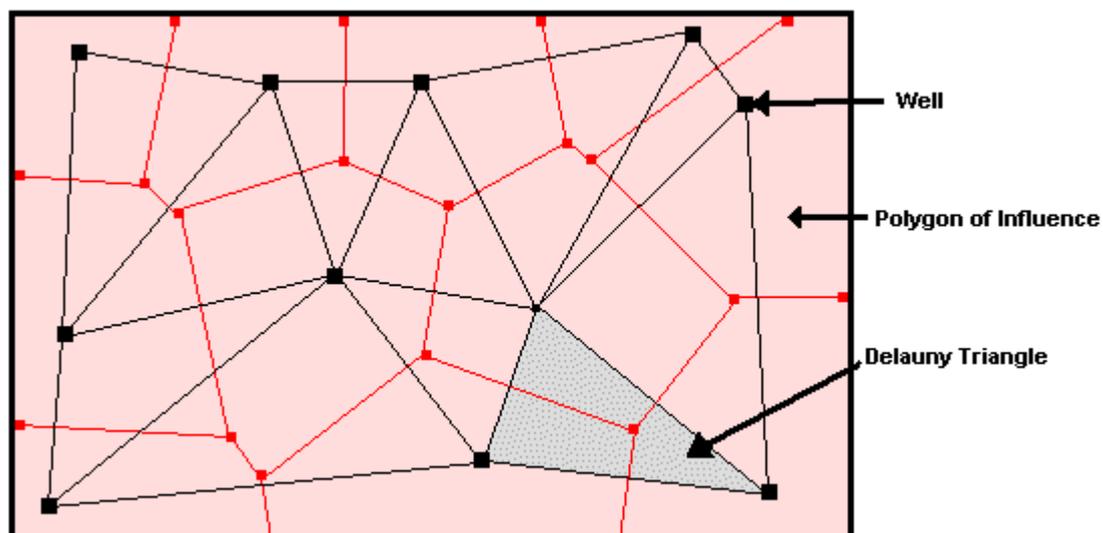
Any feature or characteristic of a formation which will allow the accumulation, but not the escape, of oil or gas. See structural trap and stratigraphic trap.

## **turning band method**

(TBM) A geostatistical technique for *conditional simulation*.; TBM is the best known method of simulation and has a long history of use and development.

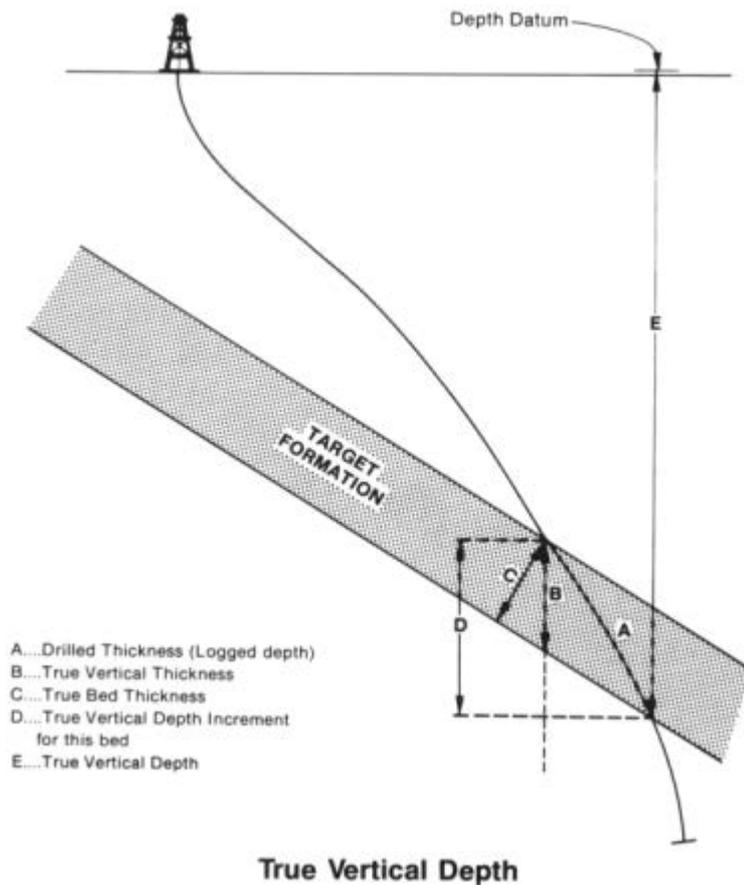
## **triangulation**

Also known as Delaunay triangulation. A 2D subdivision based on the perpendicular bisector of a line joining two points. The polygons defined by the perpendicular take on the same attribute value as the enclosed point, and are called polygons of influence. The diagram below illustrates triangulation:



## **true vertical depth**

TVD. The vertical distance between a specific location in a borehole and a horizontal plane passing through the depth datum. It is determined from directional surveys.



**turbidite:**

A sediment deposited by a turbidity current. See turbidity current. **U**

**Uncertainty**

**uncertainty:**

The lack of assuredness about the truth of a statement or about the exact magnitude of an unknown measurement of an unknown number. Uncertainty is the central concept in the decision making that follows any geostatistical study, in which case it is related to the risk of making an incorrect decision because the estimates do not agree with reality. Uncertainty is often measured by parameters such as confidence, probability intervals, and standard errors.

**unconformity:**

A surface of erosion which separates older sediment from younger sediment. Compare nonconformity.

**unconsolidated:**

Pertains to a rock framework which lacks rigidity. Cohesiveness is lacking because of the absence of cement or other binding means. Compare consolidated.

## **uniform conditioning**

*Non-linear estimation method based on ordinary kriging and the Discrete Gaussian Model.*; Uniform Conditioning is a very robust method for *local estimation of recoverable resources*.

# V

## **variogram:**

A statistical measure used in *geostatistics* to characterise the spatial variability of grades.

## **variogram model:**

A function fitted to the *experimental variogram*.; It is the variogram model that is used in "kriging".

## **variogram experimental:**

The *variogram* calculated from data and upon which a *variogram model* is fitted.

## **variogram nested:**

*variogram* with more than one *range*, i.e. with shorter and longer scales of spatial variability.

## **volume-variance relationship**

A mathematical relationship used in *geostatistics* to predict variance reduction inherent in a change of *support*. (Also known as "Kriging's Relationship").

## **vertical:**

(1) an imaginary line perpendicular (at an angle of 90°) to the plane of the horizon.  
(2) Said of a borehole which is straight. Not deviated vertical resolution The capability to resolve thin beds. Often expressed as the minimum thickness of formation that can be distinguished by a tool under operating conditions. vertical seismic profile VSP. A collection of seismic traces made from one-, two-, or three-dimensional geophones mounted in the same downhole tool which is anchored to the borehole wall. These traces are taken at sequential depths in a well and record the energy over a period of several seconds received from one or more sources located at the surface. The resulting profile displays the direct, reflected, refracted, and diffracted waves in both compressional and shear modes from all interfaces in the sedimentary column. It is the most accurate correlation tool for relating the well logs and lithologic logs to the field seismograms.

## **viscosity:**

Resistance of a fluid to flow. Internal friction caused by molecular cohesion in fluids. The internal properties of a fluid that offers resistance to flow. Viscosity of drilling muds may be reported in different ways:

**vug:**

Solution cavity. See vugular porosity. vugular porosity A form of secondary porosity resulting from the dissolution of more soluble portions of rock or solution enlargement of pores or fractures. Common in carbonates. See also porosity.

## W

**water block:**

A reduction in the relative permeability to oil or gas due to the increase in water saturation near the borehole caused by the invasion of water from a water-base mud. water cone See cone and coning. water cut The volume fraction of water in the total volume of liquid produced from a well.

**water-drive:**

The reservoir-drive mechanism in which oil is produced by the expansion of the volume of the underlying water, which forces the oil into the wellbore. In general, there are two types of water drive: bottom-water drive, in which the oil is totally underlain by water, and edgewater drive, in which only the edge of the oil is in contact with the water. Bottom-water drive is more efficient. watered-out Of a well, having gone to water. water encroachment The movement of water into a producing formation as the formation is depleted of oil and gas by production.

**water flooding:**

A method of secondary recovery in which water is injected into a reservoir in order to move additional quantities of oil toward producing wells.

**water saturation:**

The fraction or percentage of the pore volume of a rock occupied by water. The occupation may take different forms; i.e., funicular, insular, or pendular saturation See saturation. water table The undistorted upper surface of the saturated zone. The pressure everywhere on this surface is at atmospheric pressure. water wet hydrophilic. A solid surface is water wet when the adhesive attraction of water molecules for the solid substance is greater than the attraction between water molecules. The angle of contact of a water droplet on a water-wet solid surface will be less than 90° (measured inside the water phase). A mobile nonwetting oil phase would advance over the adhesive layer of water.

**well completion:**

The activities and methods necessary to prepare a well for the production of oil and gas; the method by which a flow line for hydrocarbons is established between the reservoir and the surface. The method of well completion used by the operator depends on the individual characteristics of the producing formation or formations. These techniques include open-hole completions, conventional perforated completions, sand-exclusion completions, tubingless completions, multiple completions, and miniaturized completions.

### **well log wireline log:**

The product of a survey operation, also called a survey, consisting of one or more curves. Provides a permanent record of one or more physical measurements as a function of depth in a well bore. Well logs are used to identify and correlate underground rocks, and to determine the mineralogy and physical properties of potential reservoir rocks and the nature of the fluids they contain. (1) A well log is recorded during a survey operation in which a sonde is lowered into the well bore by a survey cable. The measurement made by the downhole instrument will be of a physical nature (i.e., electrical, acoustical, nuclear, thermal, dimensional, etc.) pertaining to some part of the wellbore environment or the well bore itself. (2) Other types of well logs are made of data collected at the surface; examples are core logs, drilling-time logs, mud sample logs, hydrocarbon well logs, etc. (3) Still other logs show quantities calculated from other measurements; examples are movable oil plots, computed logs. etc.

### **well spacing:**

The regulation of the number and location of wells over a reservoir as a conservation measure.

# X

### **X-Y plane:**

A plane described by the X and Y axes. A convention used to describe an acoustic wave train recording in which the amplitude of the signal voltage is shown on the Y axis, and time in microseconds is shown on the X axis. See amplitude-time. X-Z plane A plane described by the X and Z axes. A convention used to describe an acoustic wave train display in which the amplitude of the signal voltage is used to modulate the intensity of a photographic beam of light to produce varying photographic density on the Z axis, and time in microseconds on the X axis. See intensity modulated-time and Micro-Seismogram

# Y

# Z

### **Z-axis:**

(1) A third dimension added to a crossplot of two parameters in an X-Y plane. The z-axis is perpendicular to both x- and y-axes. (2) In well logging, variation of the intensity of the recording spot is called z-axis variation, as in Micro-Seismogram.



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