Grade control based on economic ore/waste classification functions and stochastic simulations: examples, comparisons and applications

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Grade control and ore/waste delineation in open pit mining operations was traditionally based on the comparison of estimated grades with an economic cutoff. In the 1990s, an alternative approach to ore selection was applied and established, taking into account financial indicators through the so-called economic classification functions in combination with grade uncertainty assessment. Grade uncertainty is assessed using multiple grade realisations from geostatistical or stochastic simulations. Ore/waste selection integrates and is supported by the evaluation of economic consequences of sending a block of mined material to a processing facility or to the waste dump, and the related asymmetric financial implications.

The benefits and practical implications of this efficient alternative framework are best illustrated by comparing the performance of three economic functions when combined with three commonly used stochastic simulation methods under different conditions. The latter conditions include a sparse and a dense blasthole sampling patterns and three cutoff grades. A general observation is that the minimum loss classification function combined with the indicator sequential simulation presents the most consistently better performing combination. This observation is reinforced in an application at a gold mine where the above combination outperforms the already well reconciling conventional grade control approach of the mine. The extension of the framework of economic functions to account for geometallurgical properties follows. This extension shows the integration of ore and waste grindability, a key aspect of ore comminution. Finding shows the improvements that could be made over current best practice when grindability is considered, and suggests how other geometallurgical attributes may be further integrated into grade control, as long as economic classification functions and orebody uncertainty models are considered.

Keywords: Grade control, Stochastic simulations, Loss functions, Open pit mining, Uncertainty

Introduction

Grade control or ore control is an operation where blasted material is flagged as either ore- or waste-based on blasthole samples and geological information. The procedure usually includes sampling, geological mapping, grade estimation, ore/waste selection, which may include several categories, blasting and dispatching of truckloads. The efficiency of the grade control is subject to various factors such as sampling errors, conditional bias introduced by grade estimators, and blast movements. In this scenario, the major concern is the misclassification of material. The misclassification of truckloads always results in a net loss, and is largely attributable to the lack of perfect knowledge about real grade distribution. As a result, different modelling frameworks have been developed to address the shortcomings of traditional methods.

The traditional approach to grade control in open pit operations is based on estimation methods. In its simplest form, grade control consists of manually delineating boundaries between ore and waste, or ore blocks, on a map of blasthole grade values. The map produced is equivalent to a map of polygonal estimates. Typically, blocks of an orebody are flagged as ore or waste by comparing the economic cutoff grade of ore with the estimated block grades. Ordinary kriging and inverse square distance are traditional methods of

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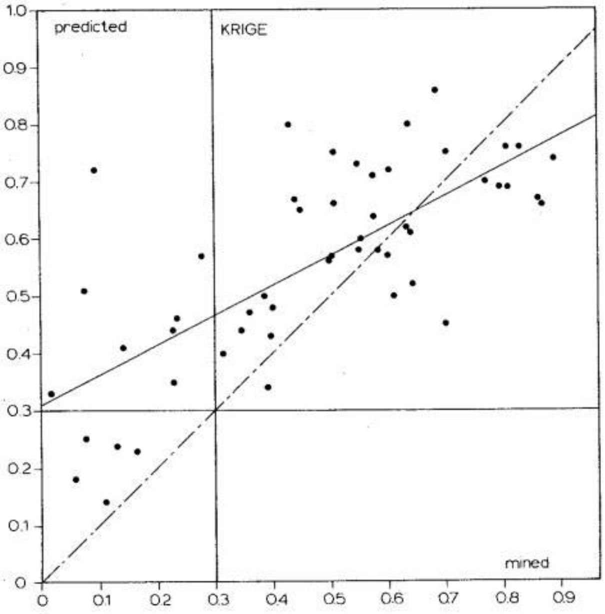
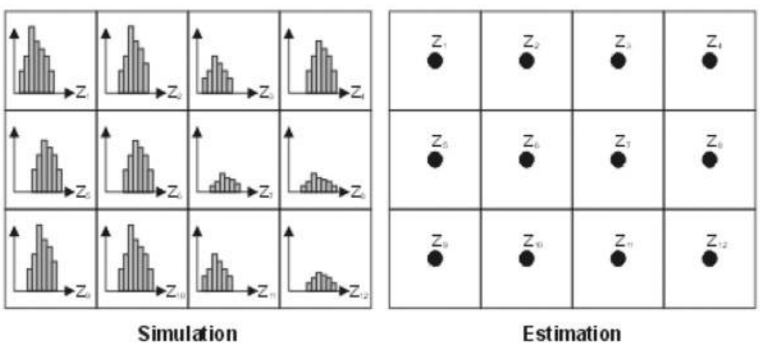


1 Plot of a classification scattergram between true grades versus estimated grades, where grade Z_c (0·3) is the ore/waste cutoff; top right and lower left quadrants represent correct classification (from Krige, 1951)

estimating grades (Journel and Huijbregts, 1978; David, 1977, 1988). Improvements from kriging come through minimising errors in the estimated grades which minimisation, however, still does not account for the financial implications of material misclassification as discussed next. Figure 1 shows true grades versus estimated grades for a certain number of blocks in an example based on Krige's work in the 1950s in the South African Rand gold mines (Krige, 1951). The application of the cutoff grade divides the plot into four quadrants, and traditional estimation methods like kriging minimise the misclassification in the upper left and lower right quadrants through the minimisation of estimation variances. Unfortunately, the traditional methods do not take into account (a) the asymmetry in the financial cost, that is, the economic consequences of misclassification, or (b) consider the uncertainty in the grade estimation. It is important to stress that the relationships between dollar losses and the classification error are asymmetric, as the economic loss resulting from sending ore to the waste dump is not the same as the loss from sending waste to the processing plant. Optimal selection criteria may be used to account for the asymmetric economics of misclassification through the so-called 'loss functions' adapted to the geostatistical context in a study by Journel (1984) and further developed by Srivastava (1987). Several studies have explored this idea and shown its practice, including Isaaks (1990), Srivastava *et al.* (1994), Douglas *et al.* (1994), Glacken (1997), Schofield and Rolley (1997), Shaw and Khosrowshahi (1997), Godoy *et al.* (2001), Collett and Corley (1999), Deutsch *et al.* (2000), Richmond (2004), Verly (2005); and others, leading to making the use of 'loss' or, in a broader context, 'economic' functions an established approach since the 1990s. The application of economic classification functions for grade control requires, by definition, to account for grade uncertainty, thus the concept and methods described as geostatistical or conditional or stochastic simulations (Goovaerts, 1997).

For new approaches based on simulation, the block grades are no longer seen as single values, but as possible distributions of grades, as shown in Fig. 2. Given these distributions, the expected loss or profit associated with each economic scenario of classification can be assessed and used to derive an 'economically-optimal' ore selection or ore selection indicator. In dealing with an unknown distribution of ore grades in a bench of an open pit mine, several models of the bench can be generated using the same data (usually obtained from blasthole samples) and their statistical characteristics. These models are all constrained to reproduce all available information and to represent equally probable models of the actual spatial distribution of grades. A series of conditionally simulated models allow the assessment of the uncertainty in the actual distribution of grades, as discussed for example, Journel (1994), Dimitrakopoulos and Luo (2004), Benndorf and Dimitrakopoulos (2007), and Boucher and Dimitrakopoulos (2012). Several issues may be raised in the implementation of economic functions and stochastic simulations in grade control. Simply put, how different economic functions such as minimum loss or maximum profit or either of those with 'risk coefficients' will perform, and particularly when combined with different and readily available simulation algorithms such as Gaussian or indicator or probability field simulation (P-field). In all above cases, methods and algorithmic implementations are different, and so is their specific use in practice and input parameters needed; thus, is worthwhile to explore



2 Simulated models of block grades in a deposit (left) and a typical estimated model of block grades in a deposit (right)

the options generated by the different possible combinations and assess performance. It should be noted that, in practice, ore selection approaches compared herein while integrating specific financial considerations, are assessed with additional considerations to their 'profit indicator', including the proportion of ore selected, metal produced and so on, as one does in practice through reconciliations.

Economic functions are but a way to integrate more information in the ore\waste selection decision processes and become of particular importance as one attempts to deal with the non-linearity in various additional aspects, such as geometallurgical attributes (from recoveries, multiple elements, to grindability and so on). The framework of economic functions for ore\waste selection facilitates the integration of rock grindability attributes to grade control and is presented herein, given that it is a well appreciated component of delivering ore from a mine to a mill and relates to improving performance of grinding circuits, costs and time efficiencies, and power consumption, among other aspects. The non-linear metal recoveries or any other geometallurgical attribute of interest can be facilitated through adopting the economic functions discussed herein. In addition and for reasons of completeness, notes that the effect of blast movement needs to be in practice incorporated as well, and is a topic presented by Dowd and Dare-Bryan (2007).

All conditional simulation methods return a range of possible values from a conditional cumulative distribution function (cdf) of the type

$$F(x;z|(n)) = \text{Prob}\{Z(x) \le z|(n)\}$$

where $Z(x)$ represents a way to look at modelling the uncertainty in the unknown true value, and (n) represents local conditioning data within the specific neighbourhood of location (x). The steps needed to obtain a sample of the cdf for point or block grades are to: (a) generate a change of support, i.e. obtain the average of point grades over the volume of any block (Peattie and Dimitrakopoulos, 2013), although the most practice is to work with point support models and draw dig-lines based on them; and (b) obtain the conditional distribution of the grades $Z(x)$.

In the following sections, first, economic classification functions for grade control are presented in detail. Subsequently, the comparative performance of the three different economic classification functions when combined with three different commonly used simulation methods is presented for a different blasthole drilling patterns (densities) and cutoff grades; all are benchmarked against the traditional estimation based grade control, as well as compared to the 'completely known' deposit. Third, an application and comparison to the conventional approach at a gold mine follows and benefits are presented. Next, the issue of adding other critical grade control parameters such as grindability in the economic classification functions previously examined is presented and elucidated using an example. Lastly, summary and conclusions follow.

Economic classification functions

In this section, the formal mathematical expressions for the economic classification functions discussed in the previous section are presented. Mining may involve the removal of two categories of material, ore and waste. In more complex situations, the materials mined may be divided into several categories. In both cases, economic classification functions can be used to optimise the ore selection process in economic terms (Godoy, 1998). Once the conditional grade distribution function has been generated, the classification that optimises a specific economic classification function can be used. Specifically, two basic formulations are available in the technical literature: 'loss function' introduced in Isaaks (1990) and 'profit function' also with the additional concept of 'risk coefficients' introduced in Glacken (1996). These are detailed next.

Loss functions

The loss associated with each type of misclassification error can be expressed as a loss function L of the actual, but unknown, grade. This loss is the potential revenue of the block less the actual recovered revenue

$$L(\$) = \text{potential revenue} - \text{recovered revenue}$$

In the scenario of misclassifying a block of ore as waste, the potential revenue corresponds to the metal left unrecovered minus the mining and processing costs

$$\text{potential revenue} = prz - c_t - c_m$$

where p is the unit metal price, r is the metal recovery fraction, z is the metal grade and c_t and c_m correspond to the processing and mining unit costs. Assuming that the cost of mining ore and waste is the same, the recovered revenue is given by

$$\text{recovered revenue} = -c_m$$

Through the conditional distribution function, the probability threshold for the block to be ore P_o and the average grade above the cutoff m^+ can be determined and used to calculate the expected loss resulting from misclassifying this block as waste $E(L_w)$

$$E(L_w) = P_o \times [prm^+ - c_t]$$

Similarly, the expected loss for the case of a block of waste misclassified as ore $E(L_o)$ is

$$E(L_o) = (1 - P_o) \times [c_t - prm^-]$$

where m^- is the average grade below the cutoff. The block will be selected as ore if the expected loss for selecting the block as ore is less than the expected loss for selecting the block as waste $E(L_w)$, i.e.

$$E(L_o) < E(L_w)$$

Profit functions

Profit functions involve specifying the expected profit $E(\text{Pr})$ associated with each classification scenario. The expected profit of classifying a block of ore as waste $E(\text{Pr}_w)$ is given by

$$E(\text{Pr}_w) = -P_o \times [prm^+ - c_t]$$

where $(prm^+ - c_t)$ corresponds to the lost opportunity cost resulting from the misclassification. The expected profit resulting from sending the block to the processing plant $E(\text{Pr}_o)$ is

$$E(\mathrm{Pr_o}) = P_o \times [prm^+ - c_t] + (1 - P_o) \times [prm^- - c_t]$$

Considering that negative profit is equivalent to loss, the only difference between the profit and loss functions is that the former includes the profit corresponding to the correct classification of ore blocks ($prm^+ - c_t$). Note that this definition of profit function does not explicitly include the costs of mining. The block will be selected as ore if the expected profit for selecting this block as ore is greater than the expected profit for selecting the block as waste, i.e.

$$E(\mathrm{Pr_o}) > E(\mathrm{Pr_w})$$

Risk coefficients

Risk coefficients are positive multipliers quantifying the impact of the loss. These coefficients can be applied to both profit and loss functions. They may reflect the mining requirements in terms of production targets by increasing the chance of rejecting ore blocks or sending waste blocks to the processing plant. The real impact of these coefficients is yet to be explained. It has been suggested that calibration studies should be carried out through batch treatment, which would provide true tonnage and grade results (Glacken, 1997). It is not guaranteed that these coefficients will be the same from bench to bench or for different parts of the deposit. In fact, the calibration may be a difficult and expensive process and therefore the use of risk coefficients should be avoided unless their real impact on the production targets is well understood.

The profit–loss approach using risk coefficients as suggested by Glacken (1997) ignores the mining cost c_m and uses only one cutoff grade. This leaves room for interpretation when formulating the profit in a more general case. The following interpretation may not be unique but it handles several cutoff grades and is consistent with Glacken's examples

$$\begin{aligned} \text{Profit} &= \text{recovered revenue} - (\text{weight} \times \text{penalty}) \\ \text{Penalty} &= \text{potential revenue} - \text{recovered revenue} \end{aligned}$$

For a block called ore, the recovered revenue value is ($prz - c_m - c_p$). If the grade is higher than the cutoff grade (correct acceptance)

$$\begin{aligned} \text{Penalty} &= 0 \\ \text{Profit} &= prz - c_m - c_p \end{aligned}$$

If the grade is lower than the cutoff grade (false acceptance)

$$\begin{aligned} \text{Potential value} &= -c_m \\ \text{Penalty} &= (-c_m) - (prz - c_m - c_p) \\ &= -(prz - c_p) \\ \text{Profit} &= (prz - c_m - c_p) - \omega'_1(-(prz - c_p)) \\ &= -c_m + (1 + \omega'_1)(prz - c_p) \end{aligned}$$

where ω'_1 is the weight applied to the penalty. The formulation used by Glacken (1997) is $(1+\omega'_1) = \omega_1$ where he considers ω_1 as a 'coefficient quantifying the impact of loss … related to the risk aversion profile of the company'. The risk coefficient ω_1 is referred to as a risk aversion factor.

For a block called waste, the recovered revenue value is ($-c_m$). If the grade is lower than the cutoff grade (correct rejection)

$$\begin{aligned} \text{Penalty} &= 0 \\ \text{Profit} &= -c_m \end{aligned}$$

If the grade is higher than the cutoff grade (false rejection)

$$\begin{aligned} \text{Potential value} &= (prz - c_m - c_p) \\ \text{Penalty} &= (prz - c_m - c_p) - (-c_m) = (prz - c_p) \\ \text{Profit} &= -c_m - \omega_2(prz - c_p) \end{aligned}$$

where ω_2 is the weight applied to the penalty. ω_2 may be interpreted as a coefficient that quantifies 'the willingness of the company to accept wasting resource through false rejection of ore blocks'. The risk coefficient ω_2 is referred to as a loss of opportunity cost factor

The profit–loss function $g(z)$ of a block called ore and its expectation $\mathrm{PL_{ORE}}$ are

$$g(z) = \begin{cases} -c_m + \omega_1(prz - c_p), & z \le z_c \\ prz - c_m - c_p, & z > z_c \end{cases}$$

$$\begin{aligned} &\mathrm{PL_{ORE}} = E[g(z)] = \\ &E[i(z, z_c) \times \omega_1(prz - c_p)] + E[(1 - i(z, z_c)) \\ &\times \omega_1(prz - c_p)] - c_m] \end{aligned}$$

The profit–loss function $g(z)$ of a block called waste and its expectation $\mathrm{PL^1_{WST}}$ are

$$g(z) = \begin{cases} -c_m, & z \le z_c \\ -c_m - \omega_2(prz - c_p), & z > z_c \end{cases}$$

$$\begin{aligned} &\mathrm{PL_{ORE}} = E[g(z)] = \\ &E[i(z, z_c) \times \omega_1(prz - c_p)] + E[(1 - i(z, z_c)) \\ &\times \omega_1(prz - c_p)] - c_m] \end{aligned}$$

The block is selected as ore if $\mathrm{PL_{ORE}} > \mathrm{PL_{WST}}$, and waste otherwise. Glacken (1996) presents an approach in which he sets the values for the risk aversion and loss of opportunity cost factors to be $\omega_1 = 1$, $\omega_2 = 0$. In this case, the profit function $g(z)$ of a block called ore and its expectation $\mathrm{PL^1_{ORE}}$ are

$$g(z) = prz - c_m - c_p$$

$$\mathrm{PL^1_{ORE}} = E[g(z)] = E[prz] - c_p - c_m$$

The profit–loss function $g(z)$ of a block called waste and its expectation $\mathrm{PL^1_{WST}}$ are

$$g(z) = -c_m$$

$$\mathrm{PL^1_{WST}} = E[g(z)] = -c_m$$

The block is selected as ore if $\mathrm{PL^1_{ORE}} > \mathrm{PL^1_{WST}}$, and waste otherwise.

Deutsch *et al.* (2000) suggest a simplification of Glacken's profit–loss approach that amounts to setting both risk aversion and loss of opportunity cost factors to 1 ($\omega_1 = \omega_2 = 1$). This is equivalent to applying a penalty for underestimation (false rejection), but no penalty for overestimation (false acceptance). Indeed, the false acceptance penalty weight is ω'_1 such that

$$(1+\omega'_1)=\omega_1 \Leftrightarrow \omega'_1=0, \text{ if } \omega_1=1$$

The argument to penalise underestimation is that 'it does cost money to mistakenly put high grade ore on the waste dump' (Deutsch *et al.*, 2000). The profit function $g(z)$ of a block called ore and its expectation PL^2_{ORE} are

$$g(z)=prz-c_m-c_p$$

$$PL^2_{ORE}=E[g(z)]=E\left[prz-c_p\right]-c_m$$

The profit function $g(z)$ of a block called waste and its expectation PL^2_{WST} are

$$g(z)=\begin{cases}-c_m, & z\leq z_c\\ -c_m-(prz-c_p), & z>z_c\end{cases}$$

$$PL^2_{WST}=E[g(z)]=E\left[(1-i(z,z_c))\times\left(-\left(prz-c_p\right)\right]-c_m\right.$$

The block is selected as ore if $PL^2_{ORE}>PL^2_{WST}$, and waste otherwise.

Another simplification consists of penalising both underestimation and overestimation, which could make sense since money is lost in both cases. This simplification amounts to setting the weights to be $\omega'_1=\omega_2=1$, or $\omega_1=2$ and $\omega_2=1$. The profit–loss function $g(z)$ of a block called ore and its expectation PL^3_{ORE} are

$$g(z)=\begin{cases}-c_m+2(prz-c_p), & z\leq z_c\\ prz-c_m-c_p, & z>z_c\end{cases}$$

$$PL^3_{ORE}=E[g(z)]=$$
$$E\left[i(z,z_c)\times 2\left(prz-c_p\right)\right]+$$
$$E\left[(1-i(z,z_c))\times\left(prz-c_p\right)\right]-c_m$$

The profit–loss function $g(z)$ of a block called waste and its expectation PL^3_{WST} are

$$g(z)=\begin{cases}-c_m, & z\leq z_c\\ -c_m-(prz-c_p), & z>z_c\end{cases}$$

$$PL^3_{WST}=E[g(z)]=E[(1-i(z,z_c))\times-\left(prz-c_p\right)]-c_m$$

The block is selected as ore if $PL^3_{ORE}>PL^3_{WST}$, and waste otherwise.

Comparative performance and benchmarking

This section uses an exhaustive dataset representing a bench to be mined and combinations of different grade control sampling patterns, three different economic classification functions and three simulation methods to select mining blocks as either ore or waste. The profit returned and the quantity of ore selected are compared for each combination and this is then compared with a benchmark kriging estimate representing the traditional grade control practices, as well as the results from knowing the original exhaustive dataset and applying ore and waste classification.

Case study procedure

The following case study compares and contrasts simulation algorithms for grade control and illustrates the use of economic classification functions for grade control. This case study uses the exhaustively sampled public domain Walker Lake data set (Isaaks and Srivastava, 1989) consisting of 78 000 values on a regular 260×300 grid, as shown in Fig. 3*a*, modified to represent a bench in a typical gold mine.

The procedure followed for the case study is summarised by the following steps:

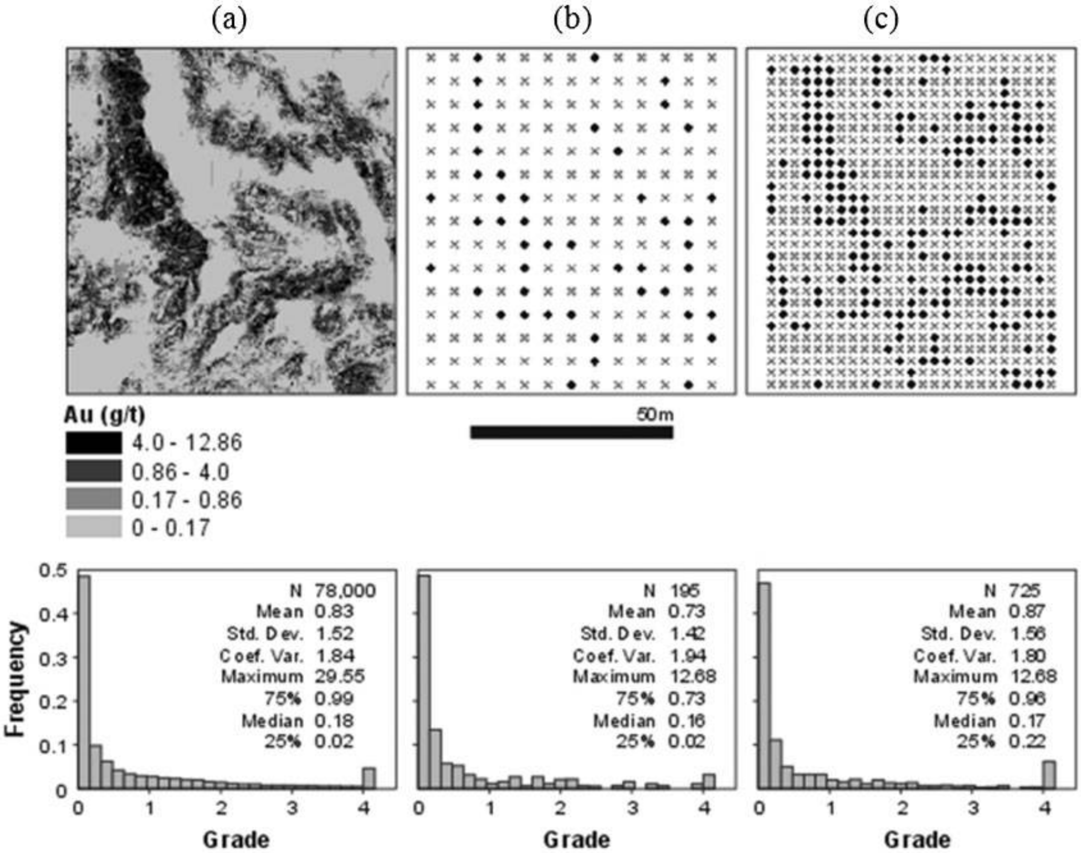
1. Sample the deposit (bench) on a 6×6 m grid to mimic a typical blasthole grade control pattern, collecting 195 samples for the bench (Fig. 3*b*).
2. Re-block (average) the known deposit data set to mining selection blocks of 10×10 m, representing the case for perfect ore selection.
3. Conditionally simulate the bench from the 195 samples using three conditional simulation techniques, and generating 100 realisations for each technique.
4. Re-block the conditionally simulated nodes to mining selection blocks of 10×10 m, and classify each block using each of the economic classification functions.
5. Compare the value and quantity of selected ore with the case for perfect selection from the known data set.
6. Repeat steps 3–5 using 725 data points on a 3×3 m grid (Fig. 3*c*).

Grade simulation using SGS, SIS and P-field algorithms

Three commonly used conditional simulation algorithms, sequential Gaussian simulation (SGS), sequential indicator simulation (SIS) and P-field (Goovaerts, 1997), are compared with each other and with a benchmark kriging approach. Two different sampling patterns, 3×3 and 6×6 m, are used. To facilitate the comparison of results, mining blocks are assumed to be 10×10 m and it is also assumed that free access is possible for every block.

Using the procedure in steps 1–6 above gives the following results. For each sample data subset, and for each conditional simulation algorithm, a single realisation is shown as a pixel plot in Fig. 5, with a plot of the actual known data for Walker Lake in Fig. 4*d*. From Fig. 4, it can be seen that the different conditional simulations all reproduce the general spatial distribution of grades for Walker Lake. The 725 sample data set appears to reproduce the true grade patterns better than the 195 sample data set, as may be expected. The SGS, shown in Fig. 4*a* for 195 data points and Fig. 4*e* for 725 data points, display greater disorder than the other simulation methods, with less similarity between adjacent pixels.

The conditional simulations are then re-blocked (averaged) into minable blocks. For comparison with the conditional simulation methods using economic classification functions, an ordinary kriged block grade estimate has also been developed with the data set for Walker Lake re-blocked to 10×10 m. The selectivity of ore is based upon a mining block of 10×10 m, and the cost of mining ore and waste is assumed to be the same. Three cutoff grades are considered, to highlight differences in performance between simulation methods and economic classification functions for different prevailing economic conditions. Gold recovery is assumed to be

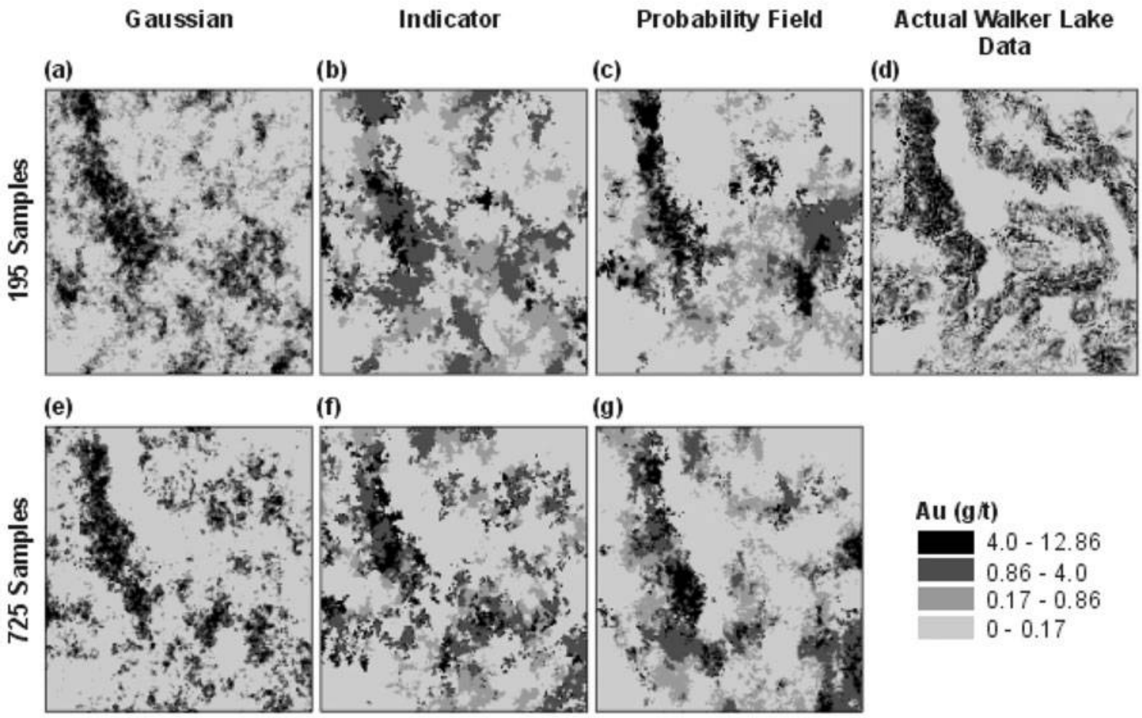


3 A bench of grade control data from the Walker Lake 'deposit', please see explanations in the text. The histograms for each of datasets (a), (b) and (c) are shown below each plot of data values

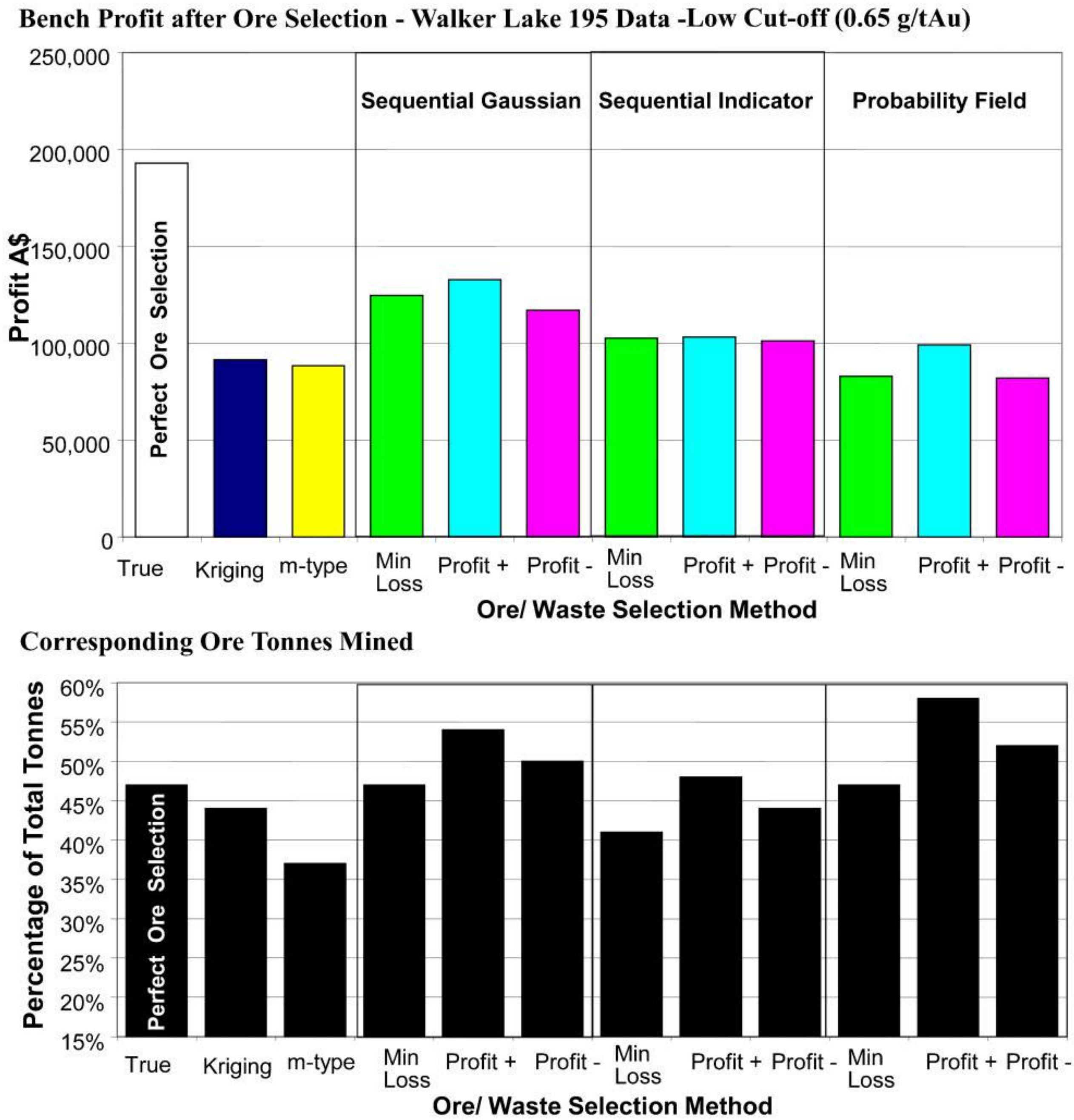
constant and independent of ore grade. No allowance is made for mining dilution and free access is assumed to all mining blocks. Mining and processing costs, metal price, recovery, and cutoff grade for three different cost–price scenarios are summarised in Table 1.

Results and analysis for the sparser blasthole spacing

Figures 6–8 present the results of classifying the Walker Lake deposit for the case where a 6×6 m sampling



4 Pixel plots of a realisation for each conditional simulation method, and for both the 195 data set and the 725 data set



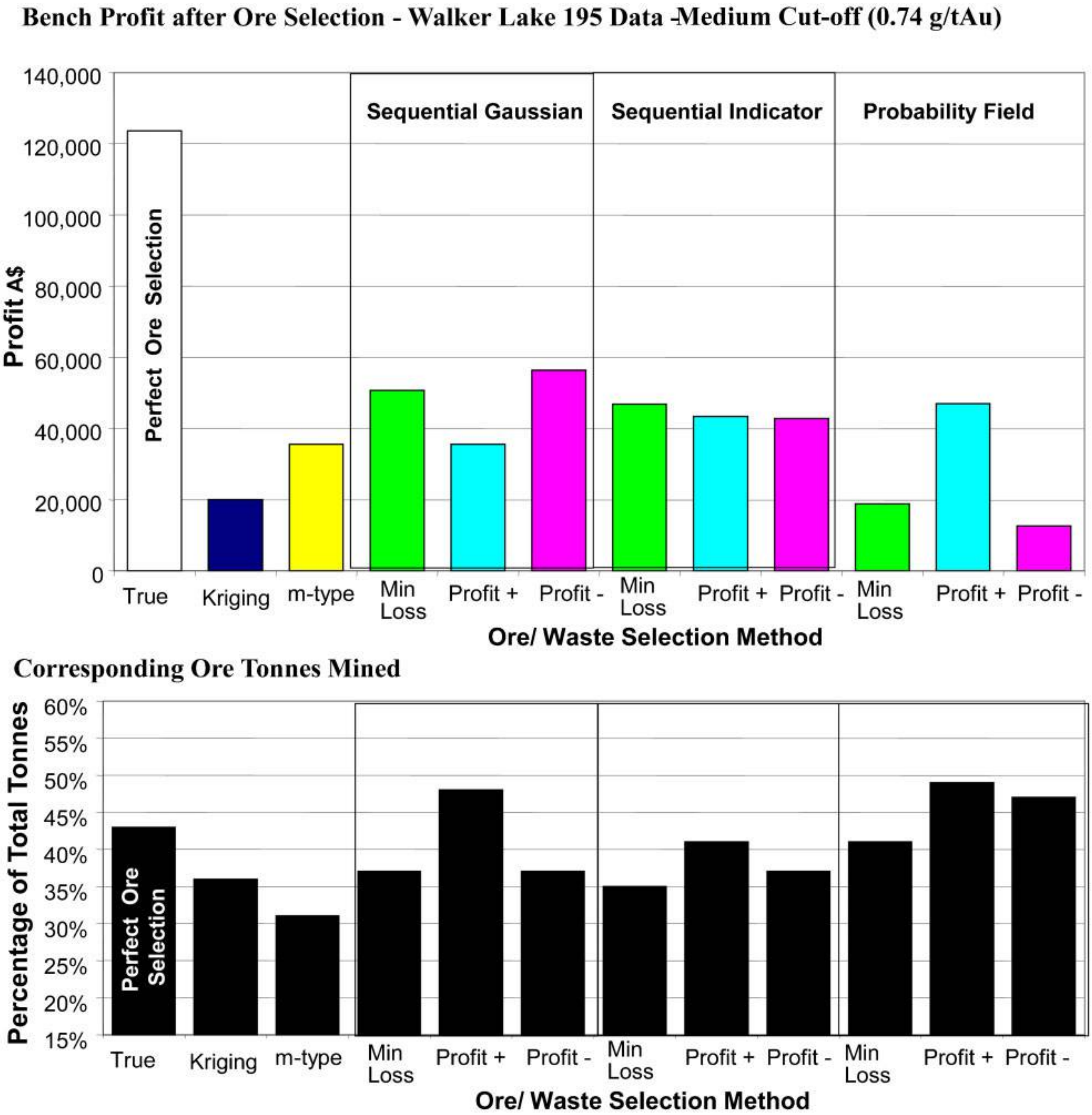
5 Summary of outcomes from classifying the Walker Lake deposit sampled on a 6×6 m pattern for a low cutoff grade relative to the sample mean

pattern is used. Each combination of deposit/sample pattern/cutoff grade is presented as one scenario, with a value for profit and a value for ore tonnage in the case of perfect selection for selection based on deterministic ordinary kriging estimates. Additionally, for each realisation, the median of 100 SGS realisations has been re-blocked. Selection for this m-type model was based on the single median grade estimate for each block. This case removes conditional simulation as a factor, and allows a comparison of the results of either using or not using economic classification functions for selection. Each summary figure also shows the histogram of sample data, with the mean and median grades indicated, for comparison with the cutoff grade.

Table 1 Three different cost/price scenarios are considered, each having a different cutoff grade

	Scenario A	Scenario B	Scenario C
Processing cost/$/t	8·00	8·64	8·85
Mining cost/$/t	2·00	2·00	2·00
Recovery/%	80	95	80
Cutoff grade/g/t	0·65	0·74	0·86
Gold price/$/oz	$480	$380	$400

The results presented in Figs. 6–8 may be further summarised as an aid to drawing conclusions, as shown in Fig. 8. As none of the combinations achieved exact perfect selection, all scenarios return less revenue than the perfect case. The performance generally decreases as the cutoff grade increases. While no single method is best in all cases, SGS with either a minimum loss or a maximum profit plus mining costs function performs



6 Summary of outcomes from classifying the Walker Lake deposit sampled on a 6×6 m pattern for a cutoff approximately equal to the sample mean

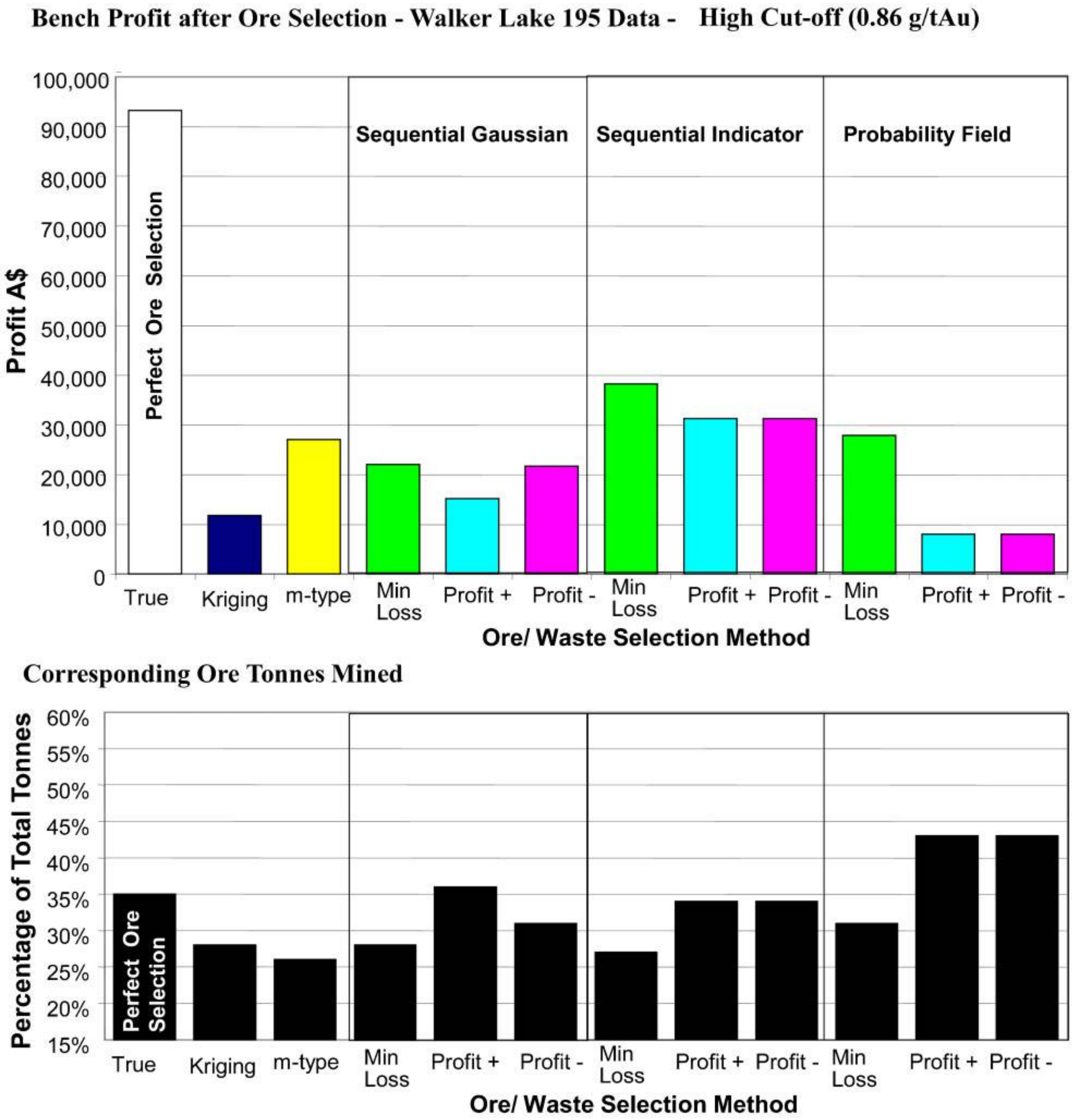
best for the two lower cutoffs. For the highest cutoff, SIS performs best. The P-field method combined with any economic classification function performs poorly in all cases for the 6×6 m sample spacing. Ore selection from estimation or simulation yields between 10% and approximately 65% of the possible revenue when using 6×6 m sampling.

Results and analysis for the dense blasthole spacing

Figures 9–11 present the results of classifying the Walker Lake deposit according to three economic classification functions applied to models from three conditional simulation methods, for the case where a 3×3 m sampling pattern is used.

Figure 12 shows the relative performance in terms of revenue for each combination of conditional simulation method and economic classification function, based on a sample spacing of 3×3 m. Overall, the range of outcomes is between 55 and 90% of the possible revenue. This smaller range of outcomes and generally better performance compared with the 6×6 m sample pattern illustrates the economic value of increased drilling. However, while four times more drilling has doubled the relative profit, the benefit from this needs to be offset against the cost of additional drilling. The cost of drilling a 3×3 m pattern compared to a 6×6 m pattern has not been included in this study.

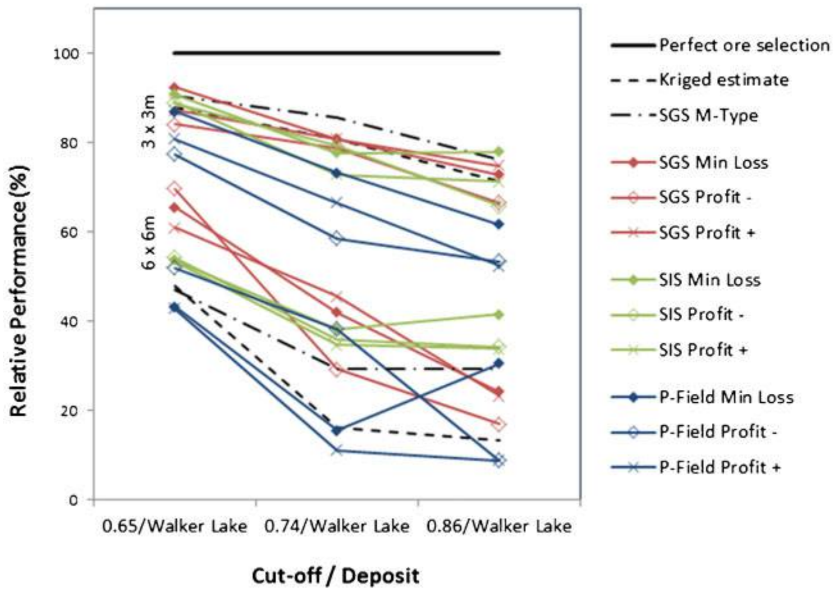
For the Walker Lake deposit and a 3×3 m drilling pattern, the P-field method performs very poorly, while the other simulation methods with any economic classification function are quite similar in performance in terms of relative profit. In general, profit does not appear to be particularly sensitive to the method



7 Summary of outcomes from classifying the Walker Lake deposit sampled on a 6 × 6 m pattern for a high cutoff relative to the sample mean

employed for selecting mining blocks of 10 × 10 m as ore when sufficient data are available, as is the case using the 3 × 3 m sampling pattern. This is not to say some improvement may not be possible. The actual example and comparison from a gold mine presented in the next section shows that while the requirement for closely spaced blastholes (~3 × 3 m) stems from rock breakage characteristics, the implementation of conditional simulation and economic classification functions still contributes to an increase in profit.

The 6 × 6 m drilling case shown in Fig. 12 exhibits large variations in tons of material sent for processing. For the Walker Lake deposit, the range of variation is approximately 20% relative to the case for perfect selection. As a measure of accuracy, the tonnage of material selected as ore should be considered in conjunction with the relative economic performance. In Fig. 8, SGS with a minimum loss function selects approximately 50% more material as ore, than the perfect selection case. At best, if the SGS ore tonnage includes all perfect selection ore, then the SGS plus minimum loss approach misclassifies half as much ore again. However, Fig. 8 shows that in terms of relative economic performance, the SGS plus minimum loss approach outperforms most other methods. Clearly, the selection of a conditional simulation plus economic classification function combination for grade control needs to consider not only the accuracy of the method but also the precision. In contrast, the sequential Gaussian m-type estimate from 6 × 6 m sampling selects only 5–20% more tons as ore compared with the perfect selection case (Fig. 12), and has nearly the same



8 Revenue earned by each combination of simulation method and economic classification function for each cutoff grade relative to perfect selection for the 6×6 m 195 data points sampling pattern

economic performance as the SGS plus minimum loss (Fig. 8) but with at least 40% less ore tonnage. The P-field method generally selects more material as ore than other methods, with a notable exception being the SGS case discussed in the last paragraph. In particular, the probability field approach performs poorly in terms of ore tonnage compared to other methods using a 3×3 m sampling pattern for the Walker Lake deposit, as shown in Fig. 12. Table 2 summarises results, showing the best and worst performing combinations of economic classification functions and conditional simulation methods for example presented. If there is a generalisation to be made, then the prevailing better performing combination, particularly when relatively high cutoffs are used, seems to be that of SIS with the minimum loss economic function. This is not surprising and is largely due to the fact that for high ore/waste cutoffs Gaussian methods and their maximum entropy aspect destroy the spatial connectivity of the high grade values, no matter what the mineralisation is. On the other hand, indicator simulations cater to the spatial connectivity of high and very high grade/metal values. This is why the present trends on simulation focus on advancing the so-called multiple-point or high-order simulations (e.g. Straubhaar *et al.*, 2013; Jones *et al.*, 2013; Mustapha and Dimitrakopoulos, 2010).

Finally, this comparative study is designed to contrast different economic classification functions and conditional simulation methods, and the considerable additional costs expected when blasthole drilling is increased from 6×6 to 3×3 m have not been included.

Applications of grade control using economic indicators

This section presents a comparison of traditional grade control as practices in a gold mine and the mine's comparison and production reconciliation of the minimum loss classification function based on the SIS method.

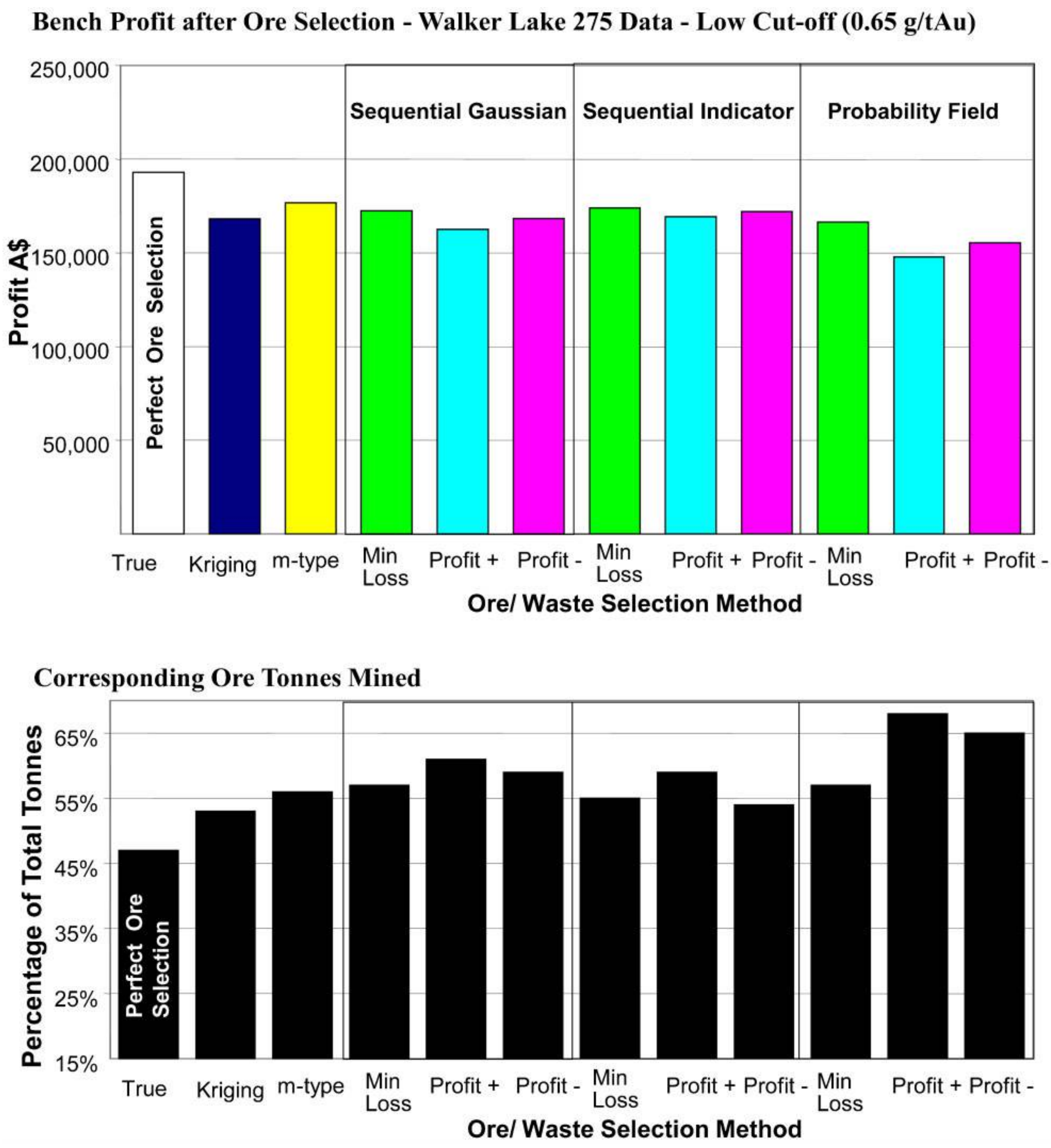
Traditional, well-performing grade control at a gold mine

The example of a well-performing grade control process at an operating gold mine is presented here, the mine uses blasthole patterns of about 3×3 m, as required to facilitate blasting and rock fragmentation. The mine's grade control is based on a combination of grade control classification based on ordinary kriging and polygons based on drilling data. The results of these two methods are superimposed and dig-lines are drawn. Reconciliations of grade control forecasts and production are excellent and an example of that is shown in Fig. 13. The figure shows the reconciliation of mill grade and grade control grade over 3 years and represents one of the various reconciliation graphs that document the well performing traditional grade control approach at the mine. The question to be addressed here is, whether the application of grade control based on economic classification functions and simulations would further improve performance, and is addressed next.

The improvement from minimum loss classification and indicator sequential simulation

The application of the minimum loss economic classification function and indicator sequential simulation shows improvement in the mines grade control performance. Figure 14 shows the comparison between the mine's grade control dig-lines (top) and the local classification from the minimum loss approach (bottom) so as to demonstrate differences, while the metal content between the two approaches is also compared (middle).

The question 'how do we know we are any better off?' from the use of the minimum loss based classification is addressed with the results reported in Table 3. The table summarises one of the mine's 2-month long reconciliations which conclude with the value of the new grade control approach valued at an additional 12 226 tonnes at 1·02 (400·9 oz of gold) or an additional $102 755 PBIT increase. This is not an insignificant increase for a



9 Summary of outcomes from classifying the Walker Lake deposit sampled on a 3×3 m pattern for a low cutoff relative to the sample mean

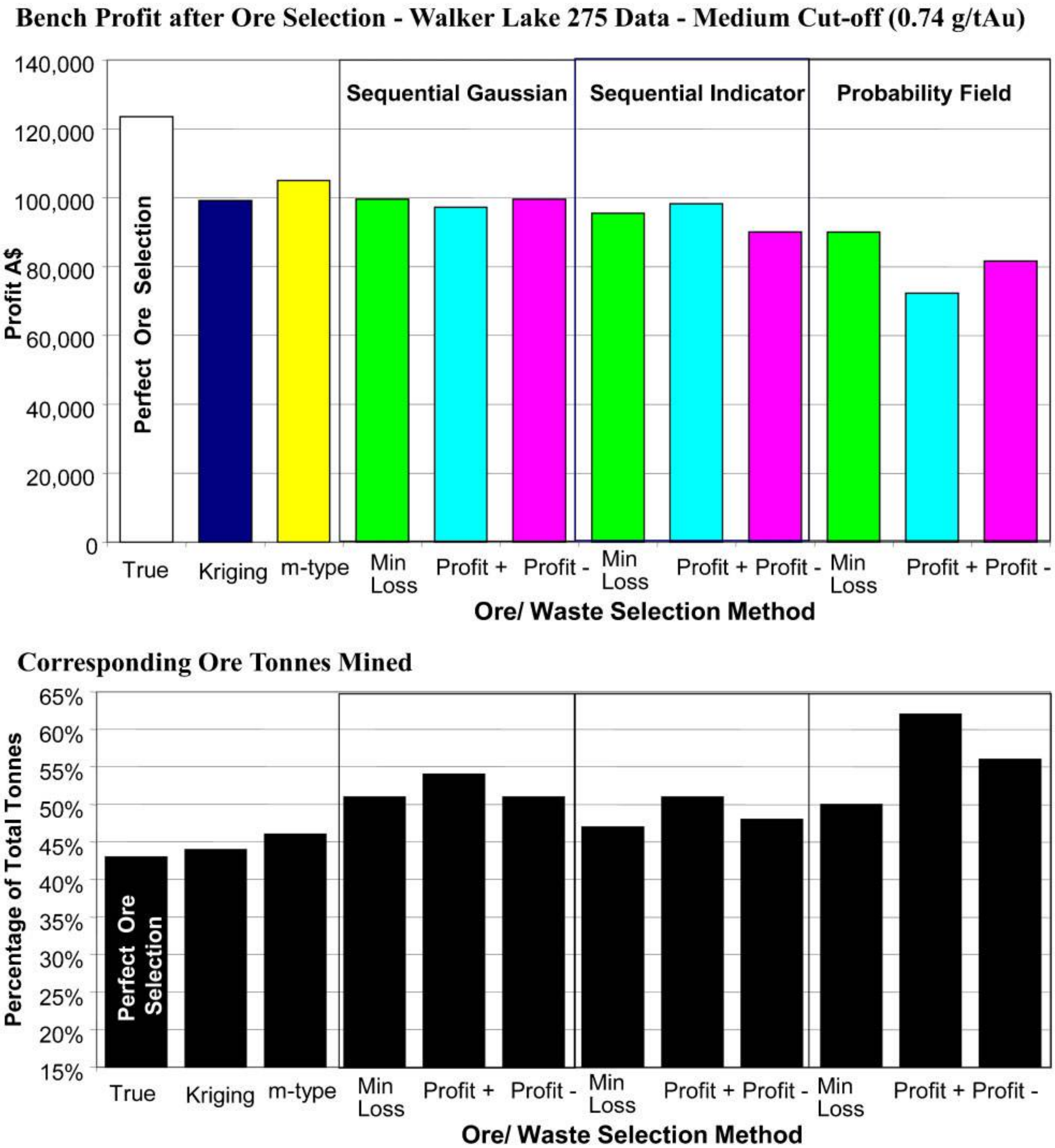
mine where the traditional grade control practice already performs very well (Collett and Corley, 1999).

Integrating 'grindability' to improve grade control and economic performance

Ore comminution, grindability, costs, time and definition of ore in grade control

The performance of economic classification functions can be further improved if issues related to enhancing the efficiency of ore comminution are integrated into the corresponding grade control process. This improvement is in a way complementary, in the sense that conventional grade control methods cannot integrate in a unified way attributes affecting ore comminution. Improved grinding circuit performance translates to reducing costs and time, including energy consumption. The power consumption of a grinding machine is determined by the 'grindability' of the feed material and considerable research has been carried out into the characterisation of rock grinding characteristics and grinding circuit design (e.g. Deniz, 2003). Blast fragmentation also contributes to the grinding characteristics of a rock mass, and research has successfully integrated simulation models for blast fragmentation and grindability.

Ore/waste classification through grade control directly relates to grindability; different grinding characteristics

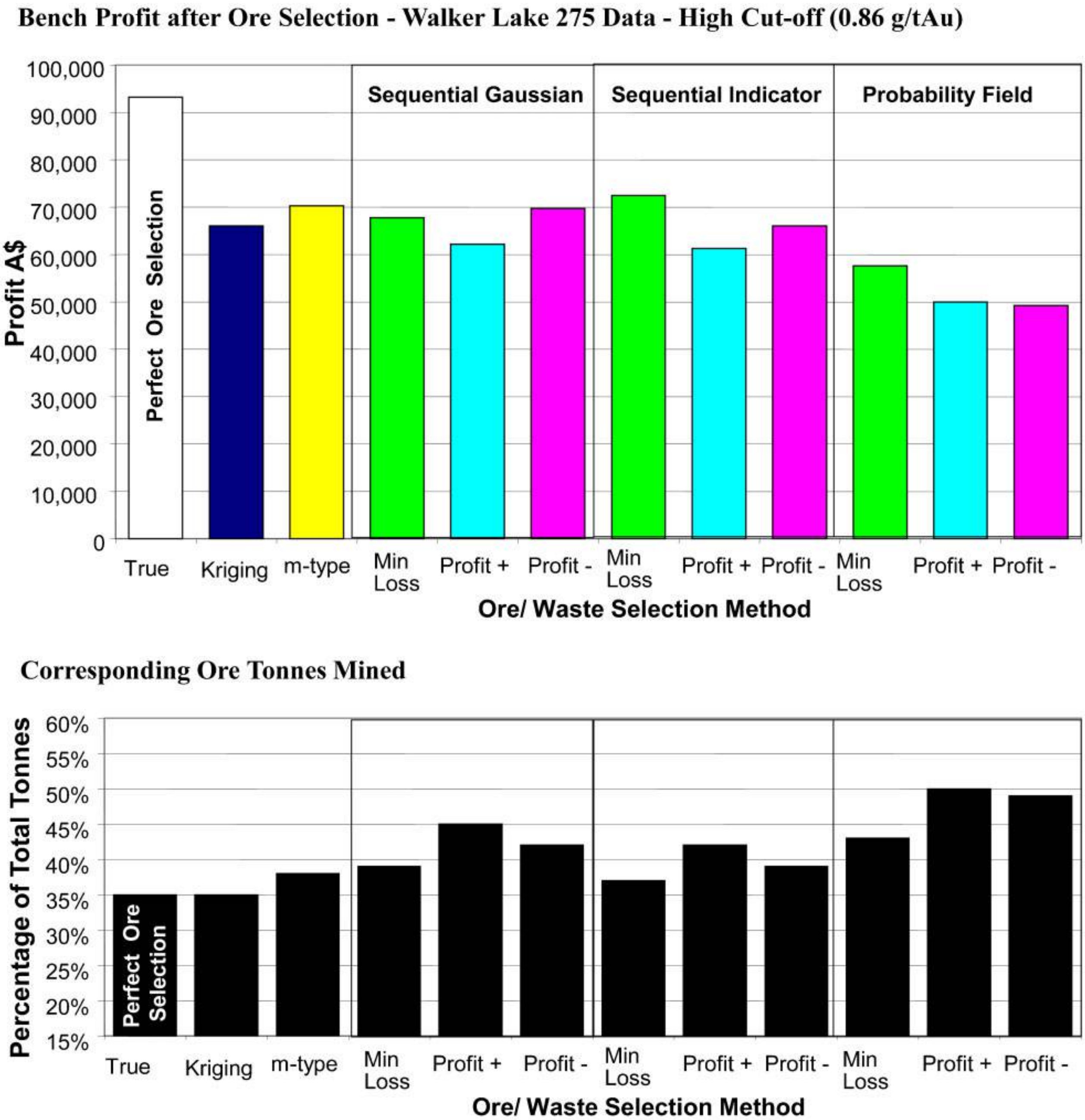


10 Summary of outcomes from classifying the Walker Lake deposit sampled on a 3×3 m pattern for a middle cutoff relative to the sample mean

have different processing costs, for example, harder ores have increased comminution time and large recycling loads within comminution circuits. Changes in processing costs affect the economic definition of ore (Lane, 1988; Asad and Dimitrakopoulos, 2012, 2013; Rendu, 2014) and directly impact ore/waste definition. Grade control can utilises grindability at a smaller scale than the scale of blast domains to directly impact ore block classification, using the full variability of the distribution of grindability. Such an approach draws on a correlation between point load value and semi-autogenous grind (SAG) throughput, and is shown to be plausible, achievable, and effective in improving the optimisation and efficiency of ore mining operations. This leads to economic improvements through reducing costs by avoiding processing of uneconomic material misclassified and processed as ore; reducing financial losses due to ore being sent to waste even though it is easy to grind; and potentially improved milling efficiency through better management of SAG feed.

Minimum loss classification accounting for grindability: an example

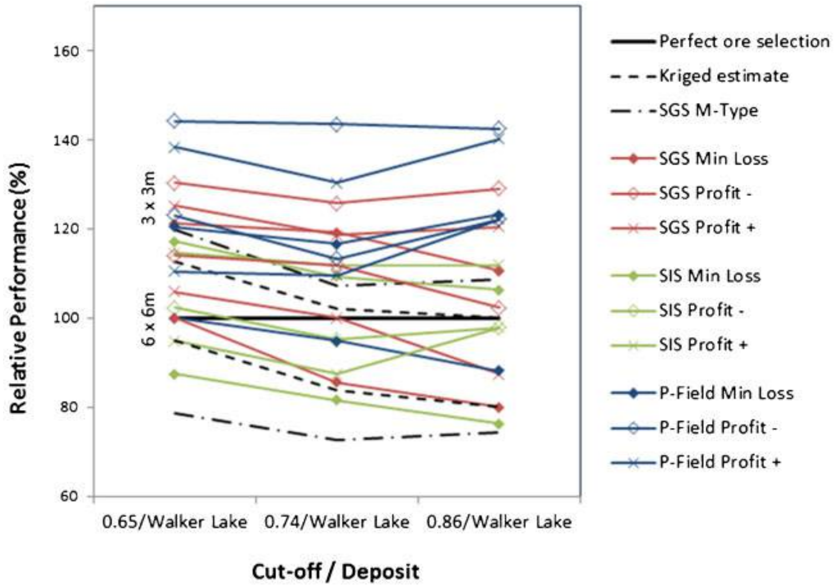
The minimum loss classification described in Section 2 is used here to include a model of grindability, which changes according to local ore variations within mining



11 Summary of outcomes from classifying the Walker Lake deposit sampled on a 3×3 m pattern for a high cutoff relative to the sample mean

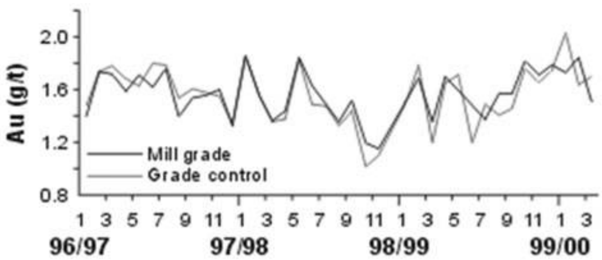
blocks (smu's) for part of a bench in a gold mine. To the extent that grindability affects the cost of processing, it also impacts on the cutoff grade between ore and waste. When the grindability of each block is considered, the effect of the cost of processing on the classification as ore or waste may (a) upgrade a block from waste to ore; or (b) downgrade a block from ore to waste; or (c) leave the classification unchanged, when compared with the base-case scenario.

In the example part of a bench at a gold mine presented herein, for each 5×5 m minable block (or smu), the classification according to a grindability model is compared with that of the base case, and a map of differences between these two models is shown in Fig. 15, along with original minimum loss classification (Fig. 15*a*) and a model of the grinding throughput used to update the parameters of the minimum loss classification function. Differences between the two ore classifications arise from the inclusion of grindability data, and the classification of 41 blocks change from ore in the base case to waste after considering grindability (red blocks in Fig. 15*c*) and 15 blocks changed from waste in the base case to ore after considering grindability (cyan blocks in Fig. 15*c*). This represents misclassification of 6% of smus classified as ore without considering variable grindability, incorrectly classified, representing an avoidable cost (loss) to the operation. Two per cent of ore blocks were incorrectly classified as waste and also represent an avoidable loss to the operation. In total, the blocks that would be classified differently with knowledge of grindability represent 8% of the smus.



12 Revenue earned by each combination of simulation method and economic classification function for each cutoff grade relative to perfect selection for the 3×3 m 725 data points sampling pattern

It seems interesting to note that the smu blocks that are most likely to change classification are those that have grades marginal to the cutoff grade. In the example presented herein, three times as many blocks are downgraded to waste as are upgraded to ore, indicating it is more frequently the case that rock has a grindability lower than the SAG throughput budget. This is to be expected given the typical skewness of the grindability distribution, and highlights the sensitivity of the approach to the distribution of grindability. Given the high cost associated with ore treatment compared with waste disposal, the reduction of 'waste sent to processing' errors is a direct and tangible cost saving for a

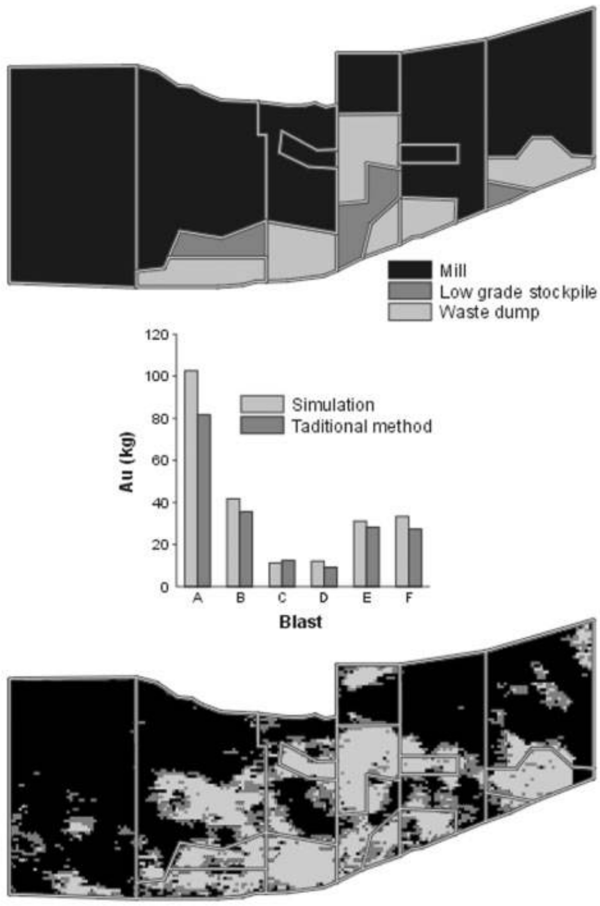


13 Reconciliations (mill grades) of the mine's traditional grade control (Collett and Corley, 1999)

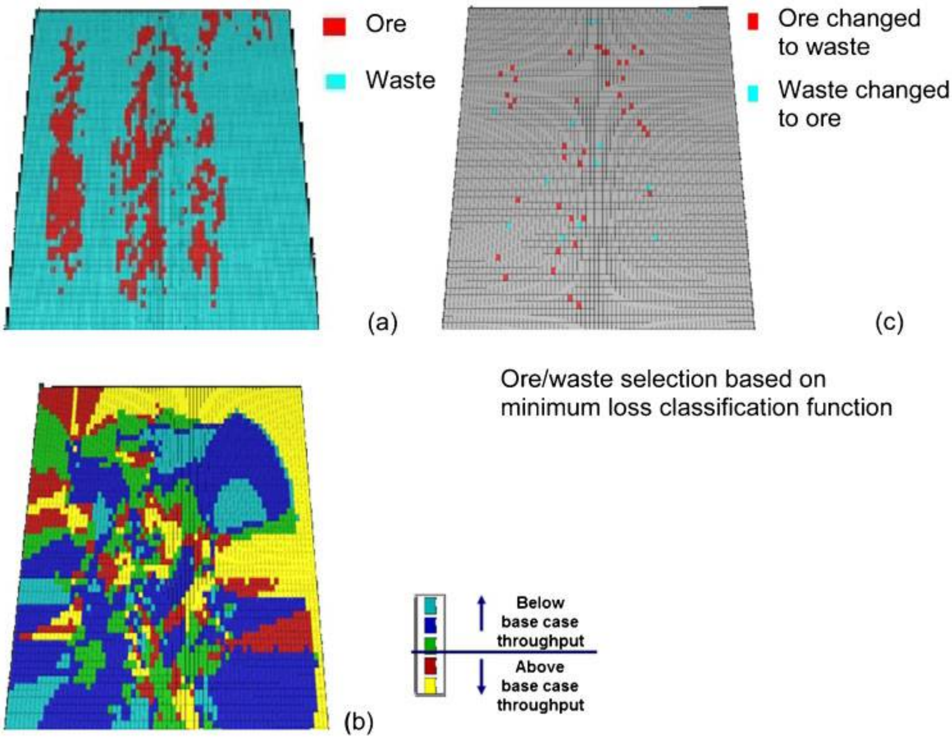
Table 2 Summary 'best' and 'worse' performing combinations of economic classification functions and conditional simulation methods for example presented

Spacing/no. samples	Cutoff	Best	Worst
6×6 m/195	Low	SGS prof+	PFS prof−
	Medium	SGS prof−	PFS prof−
	High	SIS min loss	PFS prof+
3×3 m/725	Low	SIS min loss	PFS prof+
	Medium	SGS min loss	PFS prof+
	High	SIS min loss	PFS prof−

SGS: sequential Gaussian simulation; SIS: sequential indicator simulation; PFS: P-field.



14 Six blasts and dig-lines from the mine's traditional grade control classification (top); the same blasts and grade control classification based on minimum loss and SIS (bottom); and comparison of expected gold content from the two grade control classification, per blast (middle)



15 *a* Ore/waste classification from minimum loss classification without accounting for grindability; *b* map of grinding throughput; and *c* blocks changing from ore to waste (red) and from waste to ore (cyan)

mining operation, which also leads to increased availability of the plant to treat 'true' ore. Reducing the instances of 'ore sent to waste' errors promotes mine life and sustainability.

These findings present an example of improvements that could be made over current best practice when grindability is considered. The impact of ore misclassification will be more significant in regions of marginal grade ore, particularly where such ore also tends to have longer grind circuit residence. In addition, other geometallurgical attributes may be further integrated into grade control, as long as economic classification functions and orebody uncertainty models are considered.

Summary and conclusions

Ore/waste classification in grade control based on economic classification functions as an indicator of the economic impact of different classification decisions was presented in this paper. Two basic approaches, minimising loss or maximising profit were presented along with the classification process involving the combination of conditional simulation of the orebody and calculation of the economic classification function for each orebody block or smu. An advantage of these functions is that the asymmetric relationship between monetary loss and misclassification can be taken into account. However, it is noted that these functions are only indicators of the monetary implications of different classification decisions, and do not represent actual loss or profit figures (for example, discounting is not considered, and there is no link to a mining sequence).

A comparative study using an exhaustive dataset representing the actual grades of a bench to be mined is presented to elucidate the potential performance of three economic functions, namely minimum loss, maximum profit with mining costs and maximum profit excluding mining costs, with three commonly used stochastic simulation methods, namely, SGS, SIS, and P-field. Related combinations are compared for two different blasthole sampling patterns, a sparse one (6×6 m spacing) and a dense one (3×3 m spacing), and three cutoff grades, one below the data mean, one about the mean and one above the mean.

At larger sample spacing, the SIS approach appears more dependable for a range of cutoff scenarios and deposits. Although other methods perform better for some cases, by and large the sequential indicator approach combined with a minimum loss economic classification function returns the highest profit for this case study. By comparison, the probability field method for conditional simulation generally performs poorly, and shows a loss of possible revenue in comparison with other simulation methods. Even more noteworthy is the poor performance of the probability field approach

Table 3 Example of a comparison of performance between the traditional grade control of a gold mine and the performance of grade control classification with minimum loss with sequential indicator simulation (SIS) over a 2-month period

A 2 month long comparison		
		Ounces
Reserves	380 812 at 1·73	21 180
Traditional grade control	385 220 at 1·67	20 682
Min loss/simulation (reconciled)	397 446 at 1·65	21 083
Result from min loss and SIS:		
Additional 12 226 tonnes at 1·02 (400·9 oz)=$102 755 PBIT increase or 2·8%		

SIS: sequential indicator simulation.

compared to ordinary kriging when few samples are available and the cutoff grade is high.

The comparative example presented in Section 3 includes several assumptions, to be kept in mind when considering the results. First, the use of a fixed size block of 10×10 m for selectivity is convenient but unrealistic in practice. In addition, the assumption of free access is not generally the case. In practice, an economic classification function would be used in combination with geological mapping, grade control data from previous benches and reconciliation data, and the economic classification function selections would provide a basis for drafting smoothed ore selection outlines. Such smoothed ore selection outlines introduce further misclassification. Ore selection polygons need not be, and usually are not, squares or rectangles of constant size. Finally, this study is designed to contrast different economic classification functions and conditional simulation methods, and the considerable additional costs expected when blasthole frequency is increased from 6×6 to 3×3 m have not been included.

Comparisons of various combinations of economic classification function and simulation algorithm show that SIS with a minimum loss economic classification function is the most dependable in terms of returning the highest profit, particularly when relatively high cutoffs are used. To follow this observation further, an example is presented at a gold mine where there is a history of excellent reconciliations between the mine's traditional grade control practices and production. Despite this, the loss function approach shows additional improvements in ore production as economic value that stress the contribution of the grade control approaches presented herein.

Last but not least, the present paper presents how the economic classification function approach can uniquely be adapted to further reflect operational complexities such as aspects of ore comminution in the grade control process, thus improving the performance of grinding circuits, reducing costs and time as well as power consumption, in addition advancing the performance of the ore classification process. Significant changes in ore classification have been shown, in the context of the following additive effects of: (1) reducing costs by avoiding processing of uneconomical material as ore; (2) reducing financial losses from ore lost to waste that is easy to grind; and (3) potentially improved milling efficiency through better management of SAG feed.

Acknowledgement

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