HOME

Stochastic Mine Planning – Methods, Examples and Value in an Uncertain World

R Dimitrakopoulos[1]

ABSTRACT

Conventional approaches to estimating reserves, optimising mine planning and production forecasting result in single, often biased, forecasts. This is largely due to the non-linear propagation of errors in understanding orebodies throughout the chain of mining. A new mine planning paradigm is considered herein, integrating two elements: stochastic simulation and stochastic optimisation. These elements provide an extended mathematical framework that allows modelling and direct integration of orebody uncertainty to mine design, production planning, and valuation of mining projects and operations. This stochastic framework increases the value of production schedules by 25 per cent. Case studies also show that stochastic optimal pit limits:

- can be about 15 per cent larger in terms of total tonnage when compared to the conventional optimal pit limits, while
- adding about 10 per cent of net present value (NPV) to that reported above for stochastic production scheduling within the conventionally optimal pit limits.

Results suggest a potential new contribution to the sustainable utilisation of natural resources.

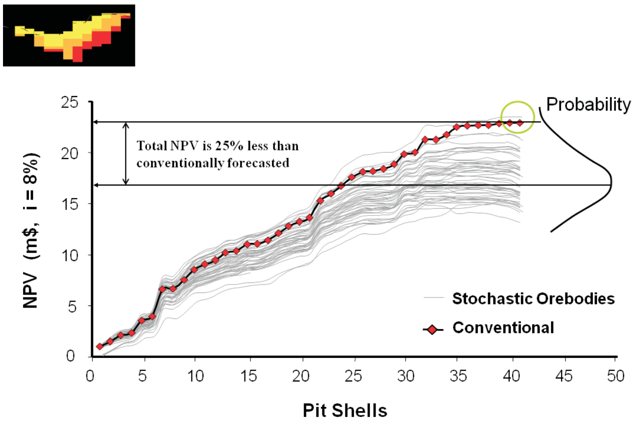

FIG 1 - Optimisation of mine design in an open pit gold mine, net present value versus 'pit shells' and risk profile of the conventionally optimal design.

INTRODUCTION

Optimisation is a key aspect of mine design and production scheduling for both open pit and underground mines. It deals with the forecasting, maximisation, and management of cash flows from a mining operation and is the key to the financial aspects of mining ventures. A starting point for optimisation in the above context is the representation of a mineral deposit in three-dimensional space through an orebody model and the mining blocks representing it; this is used to optimise designs and production schedules (eg Whittle, 1999). Geostatistical estimation methods have long been used to model the spatial distribution of grades and other attributes of interest within the mining blocks representing a deposit (David, 1988). The main drawback of estimation techniques, be they geostatistical or not, is that they are unable to reproduce the *in situ* variability of the deposit grades, as inferred from the available data. Ignoring such a consequential source of risk and uncertainty may lead to unrealistic production expectations (eg Dimitrakopoulos, Farrelly and Godoy, 2002). Figure 1 shows an example of unrealistic expectations in a relatively small gold deposit. In this example (Dimitrakopoulos, Farrelly and Godoy, 2002), the smoothing effect of estimation methods generates unrealistic expectations of net present value in the mine's design, along with ore production performance, pit limits, and so on. The figure shows that if the conventionally constructed open pit design is tested against equally probable simulated scenarios of the orebody, its performance will probably not meet expectations. The conventionally expected NPV of the mine has a 2 - 4 per cent chance to materialise, while it is expected to be about 25 per cent less than forecasted. Note that in a different example, the opposite could be the case.

For over a decade now, a traditional framework has been used when dealing with uncertainty in the spatial distribution of attributes of a mineral deposit, as well as its implications to downstream studies, planning, valuation, and decision-making. Now, a different framework than the traditional has been suggested and is outlined in Figure 2. Instead of a single orebody model as an input to optimisation for mine design and a 'correct' assessment of individual key project indicators, a set of models of the deposit can be used. These models are conditional to the same available data and their statistical characteristics, and all are constrained to reproduce all available information and represent equally probable models of the actual spatial distribution of grades (Journel, 1994). The availability of multiple equally probable models of a deposit enables mine planners to assess the sensitivity of pit design and long-term production scheduling to geological uncertainty (eg Kent, Peattie and Chamberlain, 2007; Godoy, 2010, in this volume) and, more importantly, empower mine planners to produce mine designs and production schedules with substantially higher NPV assessments through stochastic optimisation. Figure 3 shows an example from a major gold mine presented in Godoy and Dimitrakopoulos (2004), where a stochastic approach leads to a marked improvement of 28 per cent in NPV over the life of the mine, compared to the standard best practices employed at the mine; note that the pit limits used are the same in both cases and are conventionally derived through commercial optimisers (Whittle, 1999). The same study also shows that the stochastic approach leads to substantially lower potential deviation from production targets, that is, reduced risk. A key contributor to substantial differences is that the stochastic or risk-integrating approach can distinguish between the 'upside potential' of the metal content, and thus economic value of a mining block, from its 'downside risk', and then treat them accordingly, as further discussed herein.

Figure 2 represents an extended mine planning framework that is stochastic (that is, integrates uncertainty) and encompasses the spatial stochastic model of geostatistics with that of stochastic optimisation for mine design and production scheduling. Simply put, in a stochastic mathematical programming model developed for mine optimisation, the related coefficients are correlated random variables that represent the economic value of each block being mined in a deposit, which are in turn generated from considering different realisations of metal content. Note that the second key element of the risk-integrating approaches is stochastic simulation; the reader is referred to Mustapha and Dimitrakopoulos (2010, in this volume) for the description of a new general method for high-order simulation of complex

1. FAusIMM, Professor and Director, COSMO – Stochastic Mine Planning Laboratory, Department of Mining and Materials Engineering, McGill University, Montreal QC H3A 2A7, Canada. Email: roussos.dimitrakopoulos@mcgill.ca

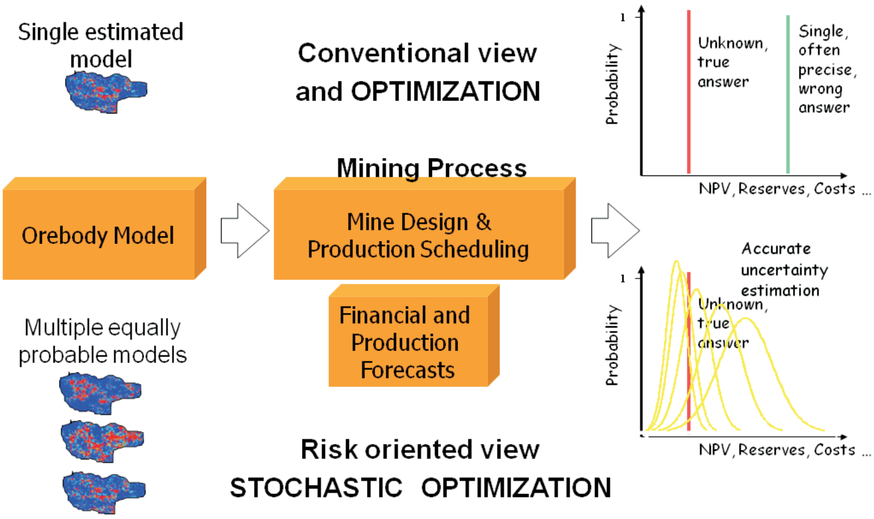

FIG 2 - Traditional (deterministic or single model) view and practice versus risk-integrating (or stochastic) approach to mine modelling, from reserves to production planning and life-of-mine scheduling, and assessment of key project indicators.

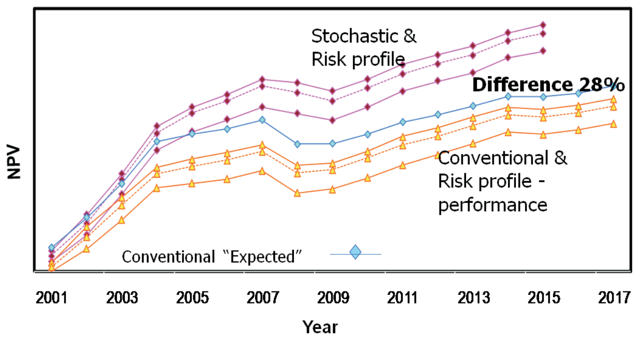

FIG 3 - The stochastic life-of-mine schedule in this large gold mine has a 28 per cent higher value than the best conventional (deterministic) one. All schedules are feasible.

geological phenomena. The further integration of market uncertainties in terms of commodity prices and exchange rates is discussed elsewhere (Abdel Sabour and Dimitrakopoulos, 2010, in this volume; Meagher, Abdel Sabour and Dimitrakopoulos, 2010, in this volume).

The key idea in production scheduling that accounts for grade uncertainty is relatively simple. A conventional optimiser (any one of them) is deterministic by construction and evaluates a cluster of blocks, such as that in Figure 4a, so as to decide when to stop mining, which blocks to extract when, and so on, assuming that the economic values of the mining blocks considered (inputs to the optimiser) are the actual/real values. A stochastic optimiser, also by construction, evaluates a cluster of blocks, but as in Figure 4b, by simultaneously using all possible combinations of economic values of the mining blocks in the cluster being considered. As a result, substantially more local information on joint local uncertainty is utilised, leading to much more robust schedules that also can maximise the upside potential of the deposit (eg higher NPV and metal production) and at the same time minimise downsides (eg not meeting production targets and related losses).

To elaborate on the above, the next sections examine a key element in the risk-integrating framework shown in Figure 2, that of stochastic optimisation. The latter optimisation is presented in two approaches, one based on the technique of simulated annealing, and a second based on stochastic integer programming. Examples follow that demonstrate the practical aspects of stochastic mine modelling, including the monetary benefits.

STOCHASTIC OPTIMISATION IN MINE DESIGN AND PRODUCTION SCHEDULING

Mine design and production scheduling for open pit mines is an intricate, complex, and difficult problem to address due to its large-scale and uncertainty in the key parameters involved. The objective of the related optimisation process is to maximise the total net present value of the mine plan. One of the most significant parameters affecting the optimisation is the uncertainty in the mineralised materials (resources) available in the ground, which constitutes an uncertain supply for mine production scheduling. A set of simulated orebodies provides a quantified description of the uncertain supply. Two stochastic optimisation methods are summarised in this section. The first is based on simulated annealing (Godoy and Dimitrakopoulos, 2004; Leite and Dimitrakopoulos, 2007; Albor Consequega and Dimitrakopoulos, 2009); and the second on stochastic integer programming (Ramazan and Dimitrakopoulos, 2007, 2008; Menabde *et al*, 2007; Leite and Dimitrakopoulos, 2010, in this volume).

Production scheduling with simulated annealing

Simulated annealing is a heuristic optimisation method that integrates the iterative improvement philosophy of the so-called Metropolis algorithm with an adaptive 'divide and conquer' strategy for problem solving (Geman and Geman, 1984). When several mine production schedules are under study, there is always a set of blocks that are assigned to the same production period throughout all production schedules; these are referred to as the certain or 100 per cent probability blocks. To handle the uncertainty in the blocks that do not have 100 per cent probability, simulated annealing swaps these blocks between candidate production periods so as to minimise the average deviation from the production targets for N mining periods, and for a series of S simulated orebody models, that is:

$$MinO = \sum_{n=1}^{N}\left(\sum_{s=1}^{S}\left|\theta_n^*(s)-\theta_n(s)\right| + \sum_{s=1}^{S}\left|\omega_n^*(s)-\omega_n(s)\right|\right), \qquad (1)$$

where $\theta_n^*(s)$ and $\omega_n^*(s)$ are the ore and waste production targets, respectively, $\theta_n(s)$ and $\omega_n(s)$ represent the actual ore and waste production of the perturbed mining sequence. Each swap of a block is referred to as a perturbation.

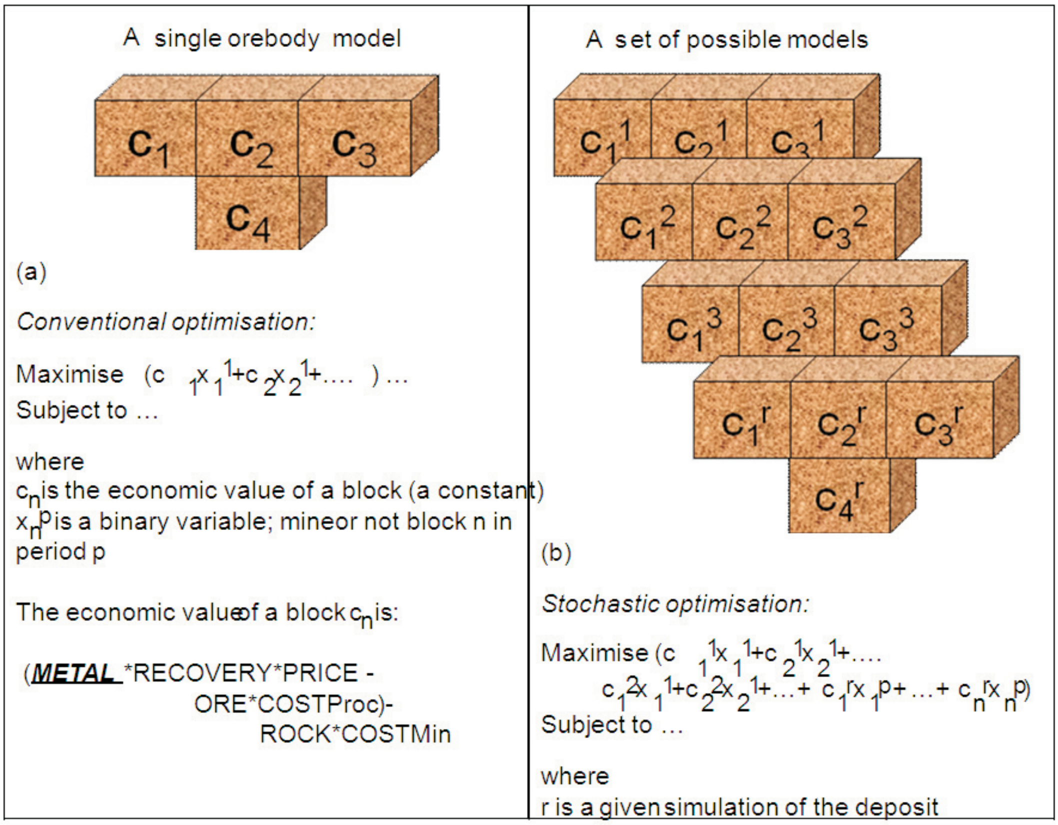

FIG 4 - Production scheduling optimisation with conventional versus stochastic optimisers: (a) single representation of a cluster of mining blocks in a mineral deposit as considered for scheduling by a conventional optimiser; and (b) a set of models of the same cluster of blocks with multiple possible values considered simultaneously for scheduling by a stochastic optimiser.

The probability of acceptance or rejection of a perturbation is given by:

$$Prob\{accept\} = \begin{cases} 1, & if\ O_{new} \le O_{old} \\ e^{\frac{O_{old} - O_{new}}{T}}, & otherwise \end{cases}.$$

This implies all favourable perturbations ($O_{new} \le O_{old}$) are accepted with probability 1 and unfavourable perturbations are accepted based on an exponential probability distribution, where T represents the annealing temperature.

The steps of this approach, as depicted in Figure 5 are as follows:

1. define ore and waste mining rates;
2. define a set of nested pits as per the Whittle implementation (Whittle, 1999) of the Lerchs-Grossmann (1965) algorithm, or any pit parameterisation;
3. use a commercial scheduler to schedule a number of simulated realisations of the orebody given 1 and 2;
4. employ simulated annealing as in Equation 1 using the results from 3 and a set of simulated orebodies; and
5. quantify the risk in the resulting schedule and key project indicators using simulations of the related orebody.

Stochastic integer programming for mine production scheduling

Stochastic integer programming (SIP) provides a framework for optimising mine production scheduling considering uncertainty (Dimitrakopoulos and Ramazan, 2008). A specific SIP formulation is briefly shown here that generates the optimal production schedule using equally probable simulated orebody models as input, without averaging the related grades. The optimal production schedule is then the schedule that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed SIP model allows the management of geological risk in terms of not meeting planned targets during actual operation. This is unlike the traditional scheduling methods that use a single orebody model, and where risk is randomly distributed between production periods while there is no control over the magnitude of the risks on the schedule.

The general form of the objective function is expressed as:

$$\mathrm{MAX} \sum_{t=1}^{p} \left[\sum_{i=1}^{n} E\left\{ (NPV)_i^t \right\} b_i^t - \sum_{s=1}^{m} \left(c_u^{to} d_{su}^{to} + c_l^{to} d_{sl}^{to} + c_u^{tg} d_{su}^{tg} + c_l^{tg} d_{sl}^{tg} \right) \right], \quad (2)$$

where:

p is the total production periods

n is the number of blocks

b_i^t is the decision variable for when to mine block i (if mined in period t, b_i^t is 1 and otherwise b_i^t is 0)

The c variables are the unit costs of deviation (represented by the d variables) from production targets for grades and ore tonnes. The subscripts u and l correspond to the deviations and costs from excess production (upper bound) and shortage in production (lower bound), respectively, while s is the simulated orebody model number, and g and o are grade and ore production targets. Figure 6 graphically shows the second term in Equation 2.

Note that the cost parameters in Equation 2 are discounted by time using the geological risk discount factor developed in

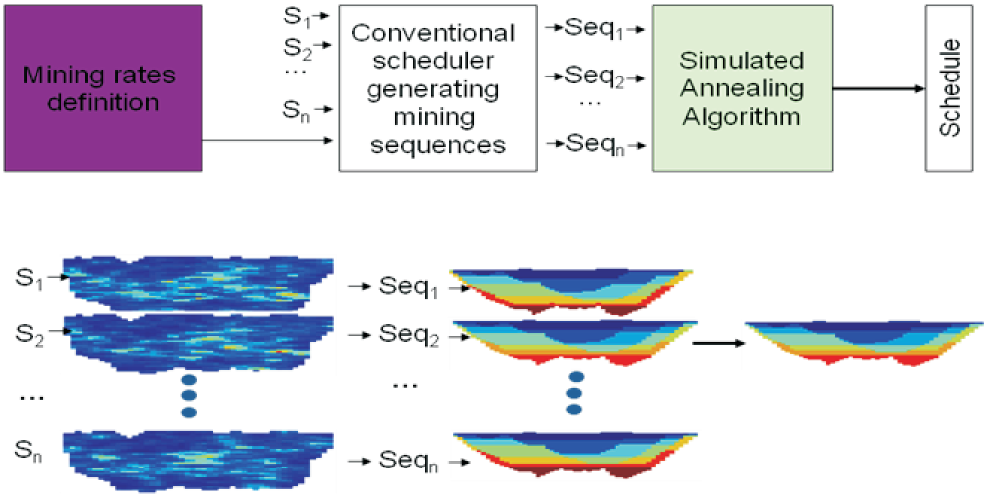

FIG 5 - Steps needed for the stochastic production scheduling with simulated annealing. S_1... S_n are realisations of the orebody grade through a sequential simulation algorithm. Seq_1…Seq_n are the mining sequences for each of S_1…S_n. Mining rates are input to the process.

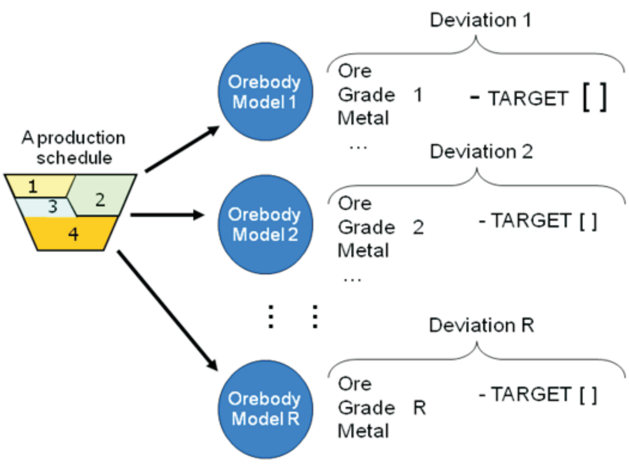

FIG 6 - Graphic representation of the way the second component of the objective function in Equation 2 minimises the deviations from production targets while optimising scheduling. This leads to schedules where the potential deviations from production targets are minimised, leading to schedules that seek to mine first not only for high-grade mining blocks, but also with high probability to be ore.

Dimitrakopoulos and Ramazan (2004). The geological risk discount rate (GRD) allows the management of risk to be distributed between periods. If a very high GRD is used, the lowest risk areas in terms of meeting production targets will be mined earlier and the most risky parts will be left for later periods. If a very small GRD or a GRD of zero is used, the risk will be distributed at a more balanced rate among production periods depending on the distribution of uncertainty within the mineralised deposit. The 'c' variables in the objective function (Equation 2) are used to define a risk profile for the production, and NPV produced is the optimum for the defined risk profile. It is considered that if the expected deviations from the planned amount of ore tonnage having planned grade and quality in a schedule are high in actual mining operations, it is unlikely to achieve the resultant NPV of the planned schedule. Therefore, the SIP model contains the minimisation of the deviations together with the NPV maximisation to generate practical and feasible schedules and achievable cash flows. For details, please see Ramazan and Dimitrakopoulos (2008) and Dimitrakopoulos and Ramazan (2008).

EXAMPLES AND VALUE OF THE STOCHASTIC FRAMEWORK

The example discussed herein shows long-range production scheduling with both the simulating annealing approach in Section 3.1 and SIP model in Section 3.2. Section 3.3 focuses on the topic of stochastically optimal pit limits. The application used is at a copper deposit comprising 14 480 mining blocks. The scheduling considers an ore capacity of 7.5 M tonnes per year and a maximum mining capacity of 28 M tonnes. All results are compared to the industry's 'best practice': a conventional schedule using a single estimated orebody model and Whittle's approach (Whittle, 1999).

Simulated annealing and production schedules

The results for simulated annealing and the method in Equation 1 are summarised in Figures 7 to 10. The risk profiles for NPV, ore tonnages, and waste production are respectively shown in Figures 7, 8, and 9. Figure 10 compares with the equivalent best conventional practice and reports a difference of 25 per cent in terms of higher NPV for the stochastic approach.

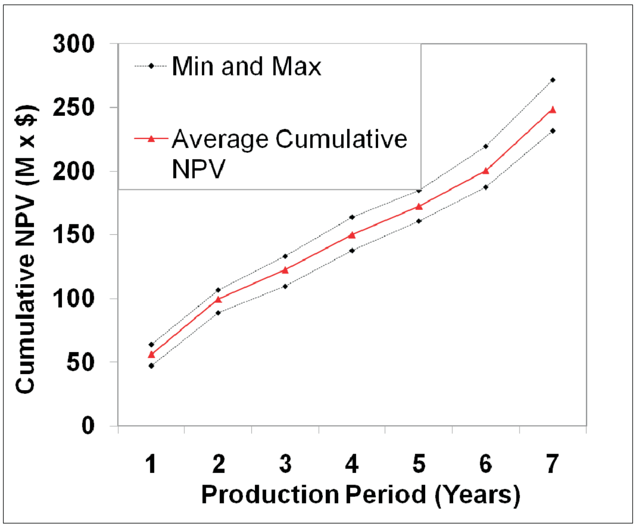

FIG 7 - Risk based life-of-mine production schedule (cumulative net present value risk profile).

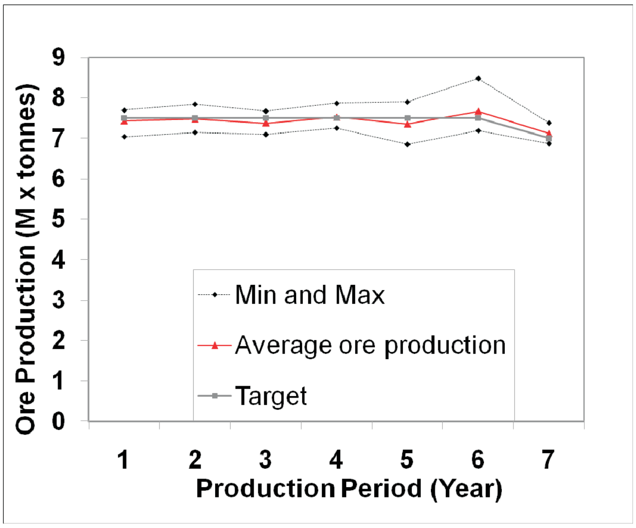

FIG 8 - Risk based life-of-mine production schedule (ore risk profile).

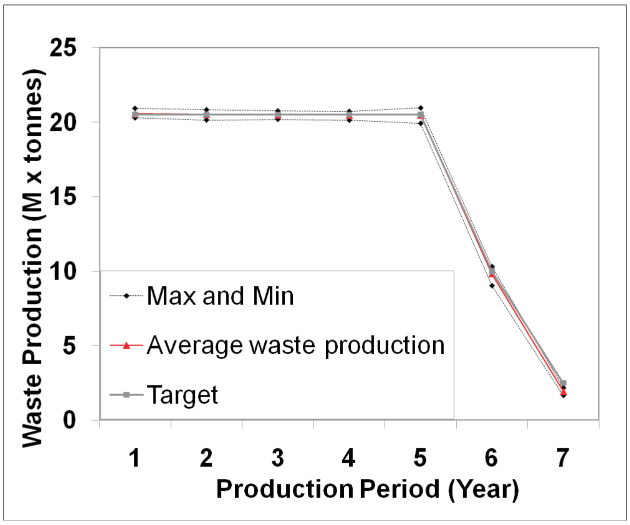

FIG 9 - Risk based life-of-mine production schedule (waste risk profile).

Stochastic integer programming and production schedules

The application of the SIP model in Equation 2, using pit limits derived from the conventional optimisation approach, forecasts an expected NPV at about $238 M. When compared to the equivalent traditional approach and related forecast, the value of the stochastic framework is $60 M, or a contribution of about 25 per cent additional NPV to the project. Note that unlike simulated annealing, the scheduler decides the optimal waste removal strategy, which is the same as the one used in the conventional optimisation with which we compare.

Figure 11 shows a cross-section of the two schedules from the copper deposit: one obtained using the SIP model (bottom) and the other generated by a traditional method (top) using a single estimated orebody model. Both schedules shown are the raw outputs and need to be smoothed to become practical. It is important to note that:

- the results in the second case study are similar in a percentage improvement when compared to other stochastic approaches such as simulated annealing; and
- although the schedules compared in the studies herein are not smoothed out, other existing SIP applications show that the effect of generating smooth and practical schedules has marginal impact on the forecasted performance of the related schedules, thus the order of improvements in SIP schedules reported here remains.

Stochastically optimal pit limits

The previous comparisons were based on the same pit limits deemed optimal using best industry practice (Whittle, 1999). This section focuses on the value of the proposed approaches with respect to stochastically optimal pit limits. Both methods described above consider larger pit limits and stop when discounted cash flows are no longer positive. Figures 12 and 13 show some of the results. The stochastically generated optimal pit limits contain an additional 15 per cent of tonnage when compared to the traditional (deterministic) 'optimal' pit limits, add about 10 per cent in NPV to the NPV reported above from stochastic production scheduling within the conventionally optimal pit limits, and extend the life-of-mine. These are substantial differences for a mine of a relatively small size and short life-of-mine. Further work shows that there are additional improvements on all aspects when a stochastic framework is used for mine design and production scheduling.

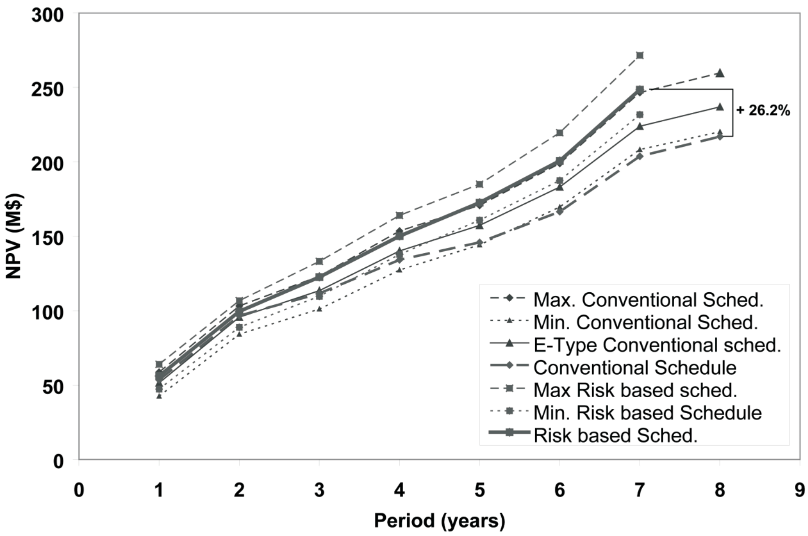

FIG 10 - Net present value of conventional and stochastic (risk based) schedules and corresponding risk profiles.

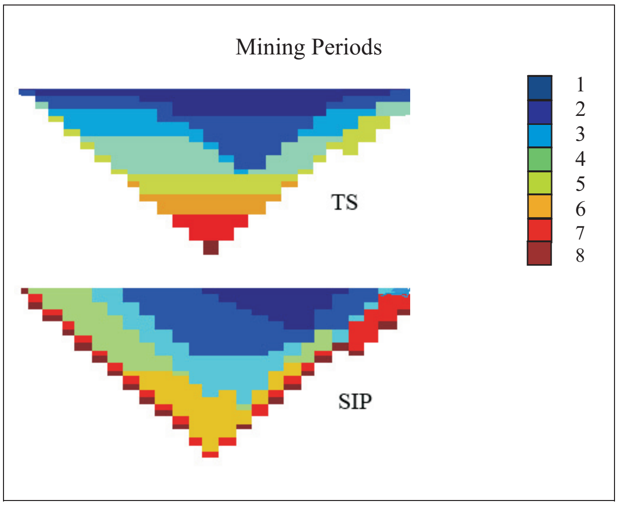

FIG 11 - Cross-sectional views of the Stochastic integer programming (bottom) and traditional schedule (top) for a copper deposit.

The new approach yielded an increment of ~30 per cent in the NPV when compared to the conventional approach. The differences reported are due to the different scheduling patterns, the waste mining rate, and an extension of the pit limits which yielded an additional ~5.5 thousand tonnes of metal.

CONCLUSIONS

Starting from the limits of the current orebody modelling and life-of-mine planning optimisation paradigm, an integrated risk-based framework has been presented. This framework extends the common approaches in order to integrate both stochastic modelling of orebodies and stochastic optimisation in a complementary manner. The main drawback of estimation techniques and traditional approaches to planning is that they are unable to account for the *in situ* spatial variability of the deposit grades; in fact, conventional optimisers assume perfect knowledge of the orebody being considered. Ignoring this key source of risk and uncertainty can lead to unrealistic production expectations as well as suboptimal mine designs.

The work presented herein shows that the stochastic framework adds higher value in production schedules in the order of 25 per cent, and will be achieved regardless of which method from the two presented is used. Furthermore, stochastic optimal pit limits are shown to be about 15 per cent larger in terms of total tonnage, compared to the traditional (deterministic) optimal pit limits. This difference extends the life-of-mine and adds approximately ten per cent of net present value (NPV) to the NPV reported above from stochastic production scheduling within the conventionally optimal pit limits.

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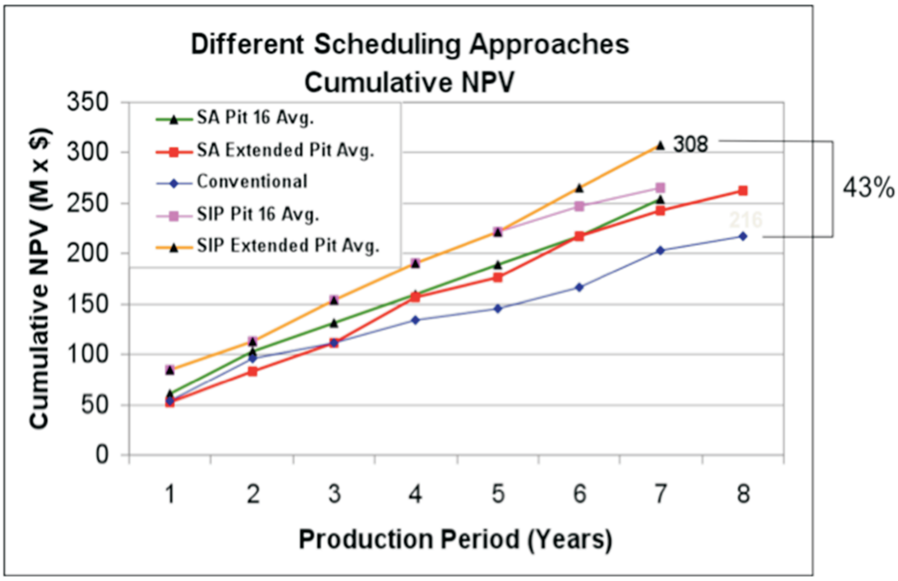

FIG 12 - Life-of-mine cumulative cash flows for the conventional approach, simulated annealing and SIP, compared to results from conventionally derived optimal pit limits.

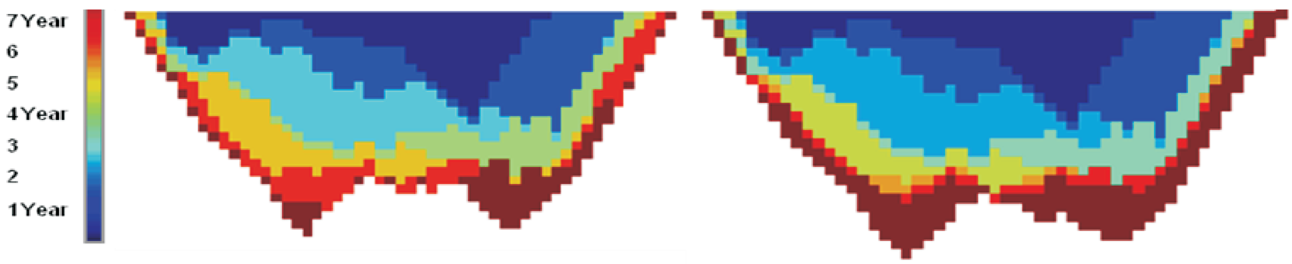

FIG 13 - Stochastic pit limits are larger than the conventional ones; physical scheduling differences are expected when bigger pits are generated.

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