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The Art and Science of Resource Estimation

A Practical Guide for Geologists and Engineers

Jacqui Coombes

PERTH COOMBES CAPABILITY 2008

“In general, an ounce of geology is worth a pound of geostatistics; this may be disappointing to geostatisticians with no geological background. Tough.”

– Harry Parker

Preface

This book was written for Resource Analysts, especially if you are new to generating resource models. Based on a collection of hints and tips, the purpose here is to help accelerate your learning of the estimation process by focussing on the implications of decisions rather than on the mathematics of geostatistics.

It has been a pleasure to work with so many commodities in a variety of geological settings around the world. Best of all, working in the mining industry has exposed me to the wonderful world of reconciliation. The opportunity to generate a resource model and follow it through mining and reconciliation provides true learning of what works when.

Having presented geostatistical/resource estimation courses since 1990, I have learnt from a diverse range of course participants' questions and perspectives. I owe so much to those of you who have helped direct my learning through your questions and observations. Your questions have always had a practical basis and have spurred my interest in the side of geostatistics that really matters – the practical let's-build-a-model-of-reality side of geostatistics. After all, geostatistics and all the techniques are just tools. How we use them, what we think about before and when we use them, and what we assume and know when we use them really shape our resource models.

This is not a book where you will learn the theory of geostatistics – there are many of those already. You will not find detailed discussions about formulae or debates about the current topical Geostatistical technique. This book is about the practical side of generating a resource model. The focus is on the process and an evidence-based approach to selecting the most appropriate geostatistical tools and parameters. I have attempted to distil the learning my mentors have shared with me.

You will encounter many vagaries during resource estimation. These are in response to the uniqueness of the orebodies we want to model. Like people, each orebody has its own personality, its own history and it is our challenge to describe the richness of these orebodies in as much detail as we can through our geostatistical toolset.

Here is to your learning
and the great opportunity to understand the world a little better.

Jacqui Coombes

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Acknowledgements

Gathering knowledge, skills and experience does not happen in isolation. I owe an immense gratitude to those who have shaped my understanding.

Firstly, thanks to André Journel who inspires parsimony – complexity in itself is not the key to intellectual superiority, nor is it the key to practical usefulness. André speaks of pursuing simplicity and my first exposure to his approach to problem solving fuelled a desire in me to understand and distil the simplicity of geostatistics to help solve complex problems. André has contributed his amazing insight and intelligence to our field, as well as his passion and enthusiasm that we should all know better.

Thanks also to Louis Voortman who opened his door to me at Gencor 20 years ago when I was working without a mentor and in isolation in Cape Town. Louis explained so much of how geostatistics worked and introduced me to the South African Geostatistical Association[1]. Thank you to Louis for guiding me to André, and for insisting I spend some time at Stanford. Louis, your openness and willingness to share was a major step in my learning curve.

Margaret Armstrong played a pivotal role in my career. Her guidance was the catalyst in my decision to pursue an industry career rather than an academic one. Margaret's integration of geostatistics and financial models proved invaluable as a voice for geological risk in the market place. Margaret helped me understand that resource estimation has a context well beyond the grade tonnage curve and that resource estimates have an important, but not dominant, place at the business table.

Viv Snowden has been a mentor many would envy – her practical approach to mining reconciliation helped me learn about the real world, the science of geostatistics and the importance of an evidence-based approach to decision making. Viv's approach to resource estimation was always embedded in developing models that reconcile with reality rather than just beautiful mathematical creations. I am grateful for the time Viv shared with me – in the office working through projects or visiting mine sites around the world (especially the trips through Canada and South Africa). From Viv I learnt so much more than just geostatistics. Thank you.

Isobel Clarke showed me the importance of good training. Isobel presented the first geostatistics course I attended – I loved her wacky, but systematic approach to geostatistics – if you thought "Practical Geostatistics" was the simplest geostatistics book to follow, you should attend one of her courses! Isobel knows her stuff so well she is able to explain it in the simplest of terms. I also adore Isobel for being who she is, as she is, and for never being afraid to ask the questions everyone should be asking. Isobel is never intimidated and her dedication to practical geostatistics has allowed many of us to understand the subject better.

A special thanks to Danie Krige – a South African giant and a gentleman. Danie inspires me because he never stops learning! Spending time with Danie and Mrs Krige reminds one of a more dignified era where love and respect are the foundations to a fulfilled life. Still a competent consultant and learner after nearly 70 years in the industry (yes, he began in 1939!), Danie Krige's thirst for knowledge and competent contribution is reminiscent of the father of management theory Peter Drucker - and Danie deserves at least equal accolades for his contributions to the mining industry.

[1] Louis was also the force behind initiating the Geostatistical Association of Australia

In my brief stints at Stanford, I was fortunate to attend Clayton Deutsch's courses. Clayton's systematic thinking and processes have given our science a stable grounding from which so many practitioners have leveraged. I am convinced Clayton's mind is like a neatly organised series of well-marked filing cabinets! Clayton's approach to training and processes has helped me develop systematic explanations during my own training courses – I owe Clayton special thanks for this approach.

During my visits to Stanford, I also spent time with Pierre Goovaerts whose willingness to share knowledge was a kindness reserved for a true spirited academic. Pierre wants us to understand and his quiet, deliberate explanations cannot hide his passion for geostatistics. Nor did it hide the enjoyment he gets from sharing knowledge. From Pierre I learnt the mechanics of geostatistics and how the parameters worked together to form a well-oiled machine.

Peter Ravenscroft influenced my thinking through his calm collected questioning – not necessarily with an answer in mind, but rather a question presented for pursuit. Peter stirred my curiosity until, after searching for an answer for almost a decade, I was able to implement a technical result I knew Peter would appreciate. Peter's calmness conceals an incredible depth of understanding in geostatistics. I am honoured to be influenced by Peter's rationality.

A special note of appreciation to Harry Parker: Harry's enquiring mind and focus on maintaining geological relevance in all resource models has inspired me more than he could possibly realise. His presence and attentiveness towards practical application has been a force in our industry for which we can all be grateful.

My thanks would be hollow if I did not mention a special mentor – both in work and life: Christine Standing. Christine is a beacon of integrity. To her I owe thanks for her creativity and persistence in problem solving – Christine is a determined data detective, leaving "no stone unturned" to give the data the benefit of true understanding. Christine's approach to incorporating geological context to the data is extraordinary. Working with Christine has given me a great opportunity to understand further the importance of understanding the geology context before making judgments and assumptions. Working on projects with Christine is always a delight – Christine inspires me to do justice for every data set and to never lose sight of geology.

I am so grateful to the people who diligently gave their time, comments and constructive critiques for this book. Jim Hodgkison, Lisa Bascombe, Mick Martin and Phil Dangerfield - your continual commitment to your own professional growth is inspirational. Thank you for taking the time and stepping up to offer great ideas for this book.

A huge acknowledgement to my clients and colleagues and the wonderfully inquisitive people I have mentored and trained – you have all enriched my life and my learning. I have enjoyed working with you as we solved problems, tackled difficult projects and got through the (sometimes!) seemingly impossible. Thank you for the amazing friendships we have developed and the trust that we cultivated along the way.

Finally to my beautiful family: to Brett for his unswerving belief in me – you are my rock and my one true love. Lauryn, you help me believe in my soul and the world's tranquillity and beauty. Hayden, you light up my life and remind me that anything is possible if you set your mind to it.

Jacqui

Contents

Introduction

Resource estimation is the process of creating a three-dimensional reflection of *in situ* mineralisation based on sparse samples, current geological thinking and a truck load of common sense.

The challenge is to combine qualitative understanding of geological processes with the patterns exhibited in the quantitative data to create accurate spatial predictions that ultimately help mining engineers plan, design and extract mineralisation economically.

Geostatistics provides an avenue for exploring and understanding patterns in the sample data. However, without an intelligent handle on the geology, geostatistical analysis is reduced to alchemy in a mathematical fantasyland.

My objective in writing this book is to wrest the process of resource estimation out of the hands of mathematicians (or pseudo-geostatisticians who hide behind the veneer of introduced complexity for the sake of beautiful mathematics – often at the expense of geological common sense) and return the process to its rightful owners – the geologists. The geologists, who have worked with the data collection, looked at and touched the rocks, examined the nuances and linked the patterns between drillholes, are the people who should be building these three-dimensional models. Their experience of the patterns and nuances is invaluable to the process, the result and the mining decisions that follow.

I believe the two common serious weaknesses in the generation of resource models today are insufficient geological support and deficient data integrity. Too often resources are estimated based on assumed data integrity. Frequently the only quality test presented in resource estimation reports are scatterplots between the original assays and laboratory duplicates. However, these say nothing about either the suitability or the quality of the sample collection. I have included a section in this book on sampling and quality control. In particular there is a conceptual exposé of the fundamental sampling error (FSE) followed by an explanation of how statistical tools are used to investigate sampling integrity. My hope is this will bring a more enlightened understanding to the management of drilling programs as well as sample collection prior to laboratory analysis.

The geologist who can understand data quality, include their geological knowledge and combine these with resource estimation skills is in a strong position to create accurate reflections of the *in situ* mineralisation.

No doubt, some geologists will seek to know more than is provided in this book. You are encouraged to read other books such as Isaaks and Srivastava's "Introduction to Geostatistics" or the ultimate geostatistical reference book "Mining Geostatistics" by Journel and Huijbrechts. Other valuable references are listed in the bibliography.

Ultimately, this book is designed to get you to understand the fundamentals of resource estimation and to encourage you to use all opportunities to examine and understand data from both the qualitative (geological) and quantitative (statistical) perspectives. When this happens, you improve the chances of the mined resources reflecting your original estimates and have a chance to experience the thrill of great reconciliation between estimate and actual.

Put the "geo" back into geostatistics

Resource analysts depend on the geological modelling to define volumes (domains) that are stationary (no dominant trends, no extreme grades, no mixed populations and the continuity between samples is the same wherever you are in the volume). In my experience, when the geological domain is well defined, the statistical side of geostatistical modelling is so simple and reconciliation so much closer.

Fundamental to any resource model is the realism used to constrain the estimation parameters. Without due consideration to the geological context, geostatistics becomes a numbers game with, at best, a chance of being somewhere in the ball park of reality. When we inject our geological understanding into a resource model, we create an opportunity to understand, and then use, the patterns we observe in the data.

I believe geological domaining is the most important step in resource estimation. Resource analysts often gloss over the geology input due to time constraints, but this only creates problems and horrendous pressure later in the process. It is the one aspect that resource analysts, particularly with a good geological background, can do well and where they will have the most significant impact on the resource model.

This book emphasises how and when to incorporate geological understanding and interpretation into a resource model and how to validate choices (method and parameters) using statistical tools.

A generic resource estimation process

The flow of this book is structured around the four phases in resource estimation:

1. Preparation
2. Investigation
3. Model Creation
4. Validation

The first phase (Preparation) involves the collection of relevant data and information required for a quality resource estimate. This includes due consideration and assessment of the data quality and the database integrity. All models are conditional to our efforts to secure a firm basis on which to make assessments in the data continuity and connectivity. Central to the quality of a resource model is the quality of the data.

Within the preparation phase are detailed analyses of the geological controls as well as how the sample grades correlate to the controls. A fundamental and inherent assumption in estimation techniques is that the data is stable: from a single grade population within a homogeneous geological unit and with a single orientation of grade continuity. Good geological input results in clear grade domains.

The second phase (Investigation) involves detailed investigation of the grade patterns within domains. Here we investigate the statistical patterns as well as the spatial patterns. Clear analysis of the sample data provides a stable platform for selecting the most appropriate estimation technique and the associated parameters.

The third phase is the model-building phase. Excellent work in the first two phases makes this phase simple. The work reduces to selection of the most appropriate estimation method and setting up correct estimation parameters.

The final phase tests the validity of the model created. There are several basic tests to run to ensure fair representation of the data. In addition to comparing models to the input data, reconciliation of production and grade control against the model is invaluable for detecting inconsistencies and including nuances in model updates.

Reporting guidelines

Reporting guidelines in the western world provide professionals in the mining industry with a technical context within which to perform. Although the codes tend not to be prescriptive, they do emphasize a *spirit* of professionalism. In particular, the codes call on us to report according to principles of Materiality, Transparency and Competence.

Operating within a principle of Materiality ensures we communicate all that is relevant and can influence how decisions are made. It enforces us to have a broader understanding of the context within which each person's technical work is done. This guiding principle encourages us to focus on what matters to the project and to be aware of the scale of impact of our professional actions and decisions.

The guiding principle of Transparency calls us to be honest with data, with the process and with the quality of results, analyses and decisions. The code also charges us to be transparent in our communications regarding our work so that others may make informed decisions.

The third guiding principle of Competence exists for us to take responsibility for our roles and the impact of our decisions. To do this we naturally take due care in our tasks and seek to understand the broader impacts of our work. We are also responsible for continual investment in our own professional development.

The reporting codes encourage us to act professionally at all times. In this way, we uphold our reputations as well as the trust investors have in our industry.

Ensure you have a good understanding of the code under which you report (the JORC code, NI43-101, SAMREC and others).

As you progress through each resource estimation, ensure you make time to reflect on how the data quality, data density as well as your decisions, analyses, parameters and interpretations affect your confidence level in the resource model you will ultimately provide for decision making.

How to use this book

The purpose of this book is to foster resource estimation skills within the mining industry. As such, this book follows the four phases of resource estimation for ease of implementation and reference.

There are several ways to use this book; however, it is primarily designed to help you work through the phases.

The gist of this book is to provide guidelines to understand the purpose within each phase. Since no two orebodies are alike – they may be similar, but are as unique as the people in our industry – each data set you encounter will have its own nuances that need to be understood.

Alternatively, the tips, hints and rules of thumbs included in this guideline may help to enrich a process you already adopt.

Phase 1: Preparation

Purpose

The purpose of the Preparation phase is to ensure the quality of data and information you are working with. When we only assume data quality, without checking its integrity, we fall into the trap of believing too fastidiously in false data. Alternatively, when we operate under constant doubt about the data integrity, it is difficult to extract valuable information to guide our decisions.

In this section, we look at:

- Evaluating the quality of our data,
- Understanding the results from QAQC process measurements, and
- How we interweave the geological information with a deeper understanding of our database to establish estimation domains.

You will need the following information to apply the learning in this section:

- QAQC reports on drilling, sampling and laboratory test work
- A valid database containing
    - Geological logging data
    - Assays
    - Any relevant structural, lithological or weathering information
    - Geochemistry data
- Geological information
    - Geological, geotechnical reports and technical papers
    - Interpreted geological features

Project quality is a function of data quality

Nowhere is the old adage of garbage in, garbage out more appropriate than in the mining industry. We rely on a miniscule sample size to make enormous decisions regarding the set up of a mining project. The quality of these decisions rests most significantly on the quality of the data.

Our purpose here is to introduce some fundamental concepts and tools to ensure excellent quality data collection. The format is practical with a focus on tools and concepts rather than theory. There are plenty of references should you wish to pursue the topics in more detail.

Resources classified according to the reporting codes (such as JORC, SAMREC, NI 43-101 and others) require meticulous attention to data collection, data representativity and database integrity. The data collection is the foundation of all subsequent decisions. The integrity of the data collection is too often treated as an afterthought, only once a project is viewed as commercially interesting.

A positive Quality Assurance and Quality Control (QAQC) is the database's stamp of approval. Without this stamp, all the effort put into complex geological interpretations, mathematical estimations and resource classification is called into question. Poor QAQC practice is equivalent to generating resource estimates in the dark.

When do we put effort into QAQC? Well, it is a process that should be analysed as data is collected. QAQC should accompany every parcel of data. QAQC is most effective if used **during** a drilling campaign to MONITOR data collection and call a halt to a campaign or laboratory that is underperforming. Running a QAQC at the end of the data collection program is an ineffective way to manage project quality.

The objective here is to shed light on the tools and techniques available to assess the various stages of the process of data collection.

Sampling basics

Concepts

Population and representative samples

When it comes to mining, we aim to extract the economic portion of a population (the portion that makes a profit). We do not know our full population. Instead, we rely on a miniscule subset of the population (the samples) to make our decisions. If we are to make reasonable decisions, the samples we rely on must represent the total population.

What makes a sample set representative? In statistical terms, this means the samples we collect of the population provide a fair indication of how the population behaves.

A histogram is a useful plot for understanding the population and for measuring of the population distribution. A histogram is a plot of the count of data points within successive intervals (see an example histogram for copper in Figure 1). This bar chart provides a summary of typical spread of grades in the data set.

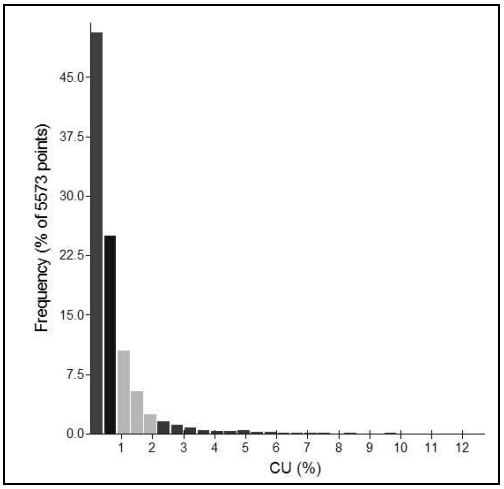


Figure 1 Example of a histogram

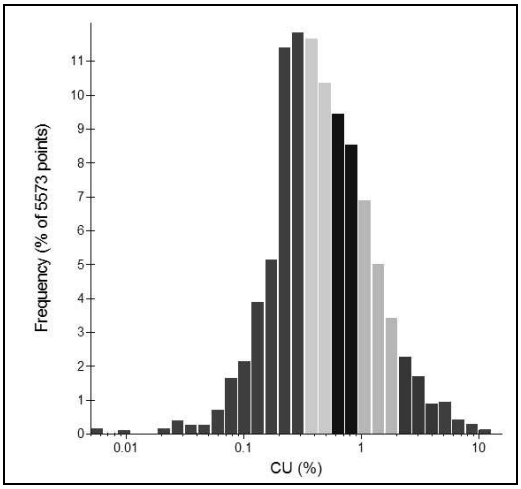


Figure 2 Example of a histogram on log-scale

When a data set has a positive skew, most of the sample values are low grade with a small percentage of more extreme high grades. This means most of the samples occur within a few intervals of the histogram (see the left hand bars of the histogram in Figure 1). A useful option is to change the intervals for the histogram. The easiest way to do this is to apply a log-transform to the data. A log-transform maintains the data order, so the same low grades samples on a normal scale are low grades on a log-scale, and the highest sample on a normal scale is still the highest sample on the log-scale. Figure 2 is a log-scale histogram of the data presented in Figure 1. A log-transformation effectively magnifies the lower grade end of the distribution and contracts the higher grade scale.

A sample histogram is a representative reflection of the population histogram when it accurately reflects the total population (Figure 3).

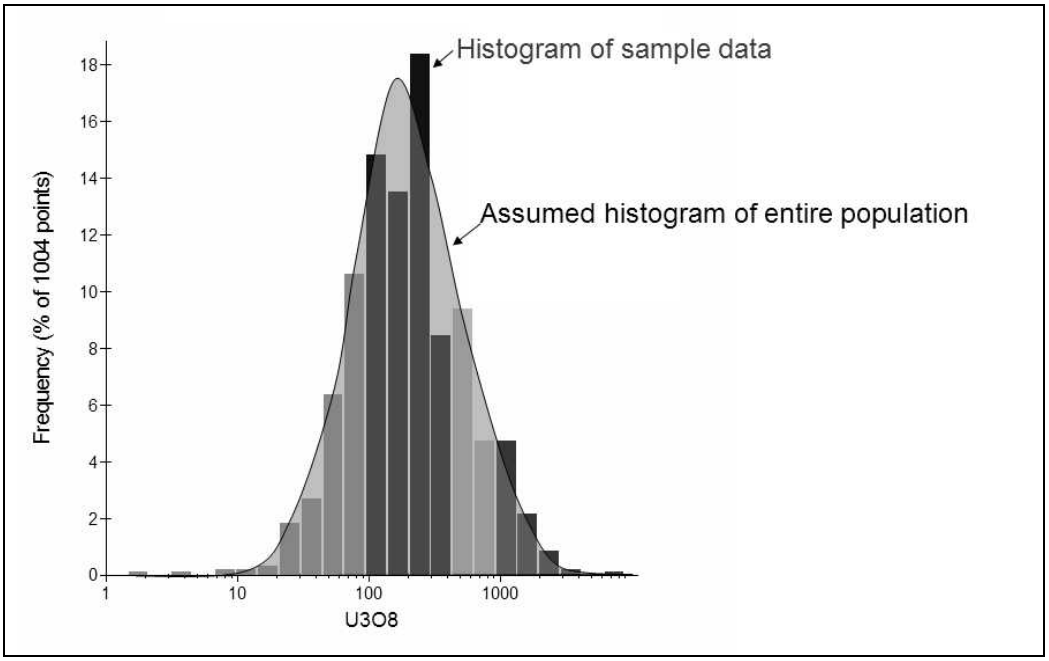


Figure 3 Histogram of samples and population

A good way to ensure a representative data set is to have fair coverage of the population (no bias introduced by clustered drilling) and even sampling within each geologically controlled population (as best as can be achieved – often samples are collected over variable lengths to represent the various geological units or population controls).

In addition, for a sample to be representative the difference between the sample value we obtain and the true value should be as close as possible. If we take numerous repeat samples at the identical location, the difference between them should be small and the average of all of them should be as close to the true value as possible. This means the samples are precise (small overall error) and accurate (close to the true value). The consistent difference between the average of the repeat samples and the true value is called a bias.

Precision, accuracy and bias

In reality, data collection errors lead to a mismatch between what we sample and the population we are trying to represent. This difference can occur in the following ways:

- **Precision** describes our ability to be specific about a grade – the number of decimal places we report describes our ability to be precise. Precision is measured by comparing repeat samples.
- **Accuracy** describes how well the average of the repeat samples targets the true (but unknown) grade.

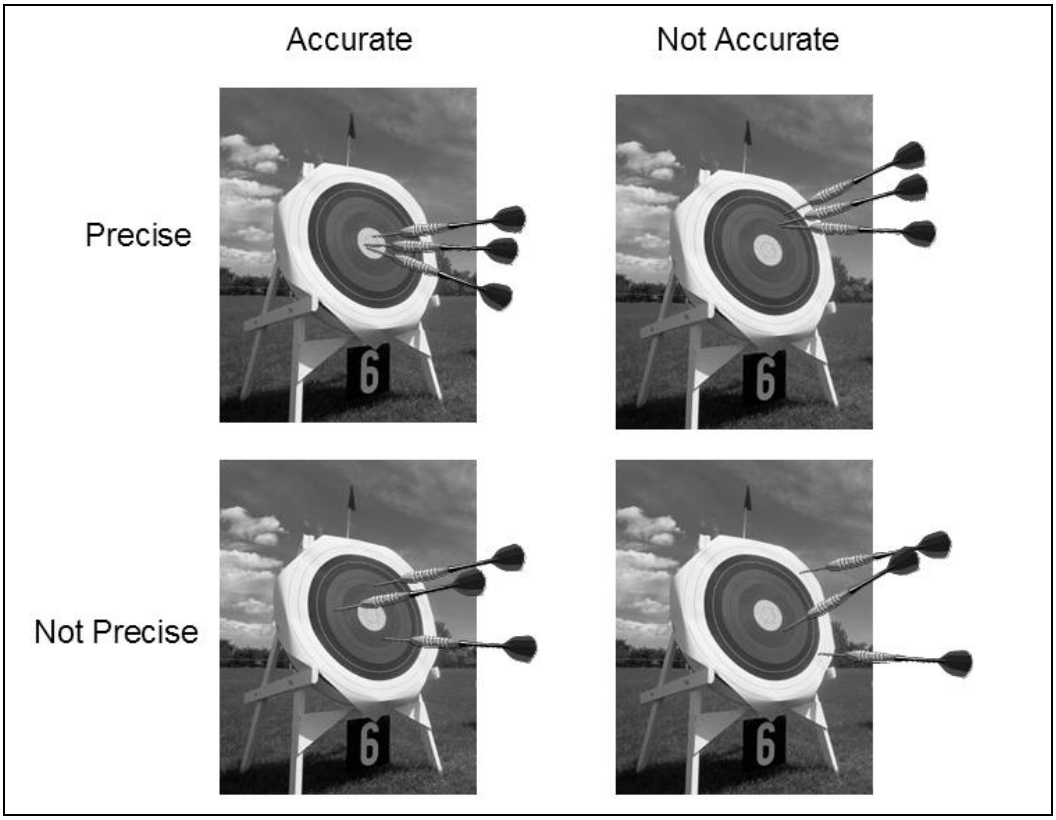


Figure 4 Accuracy and precision – target example

Bias is the measure of the systematic difference between the average of our repeat samples and the true grade.

In statistical terms, the histograms of the repeat samples either reflect the true unknown value or not according to a shift in the average away from the true mean (a bias), or a wider than acceptable spread (lack of precision) as described in Figure 5.

In reality, samples invariably incur a degree of imprecision and inaccuracy. We need to ensure that through proper sampling practices the errors incurred are as small as possible.

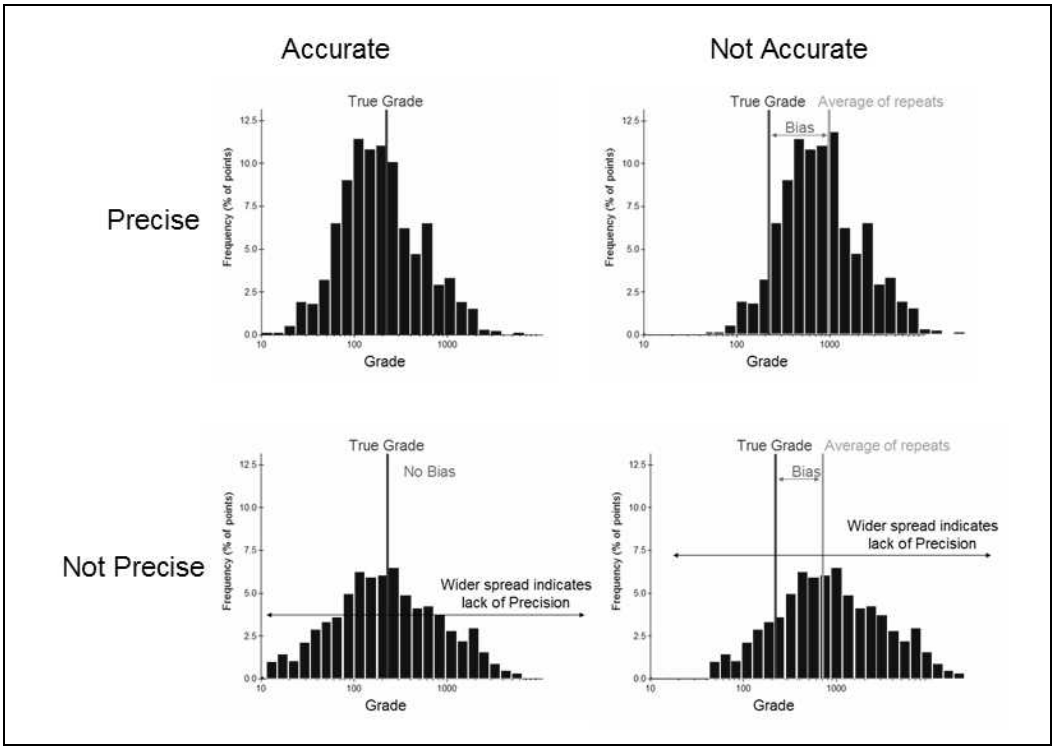


Figure 5 Accuracy and precision in statistical terms

Samples and lots

Samples are collected at different volumes - consider the difference between the volumes of 1m of RC chips compared with the volume of the pulp that is eventually analysed in the laboratory. Pierre Gy describes these differences as the "lot" or the "sample", where the sample is the volume ultimately analysed for grade, while the lot is the volume of material collected for sampling.

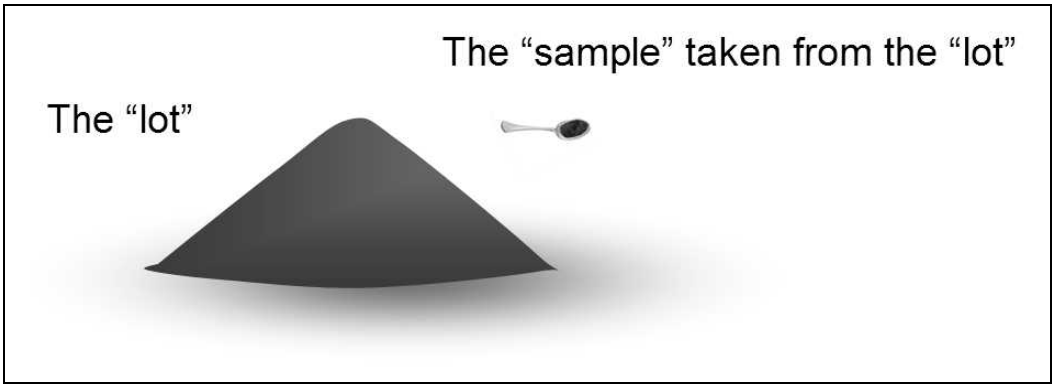


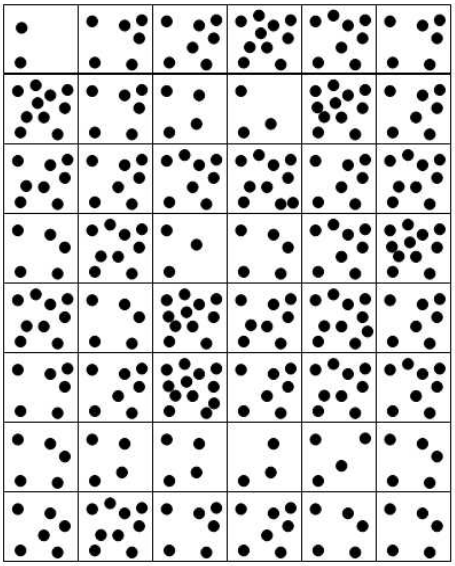
Figure 6 The difference between a "lot" and a "sample"

Other examples of lots are: blasthole cone, diamond core, development face chips and stockpiles. Examples of samples are half diamond core, riffle split sample, mill pulp and fire assay sample.

Activity

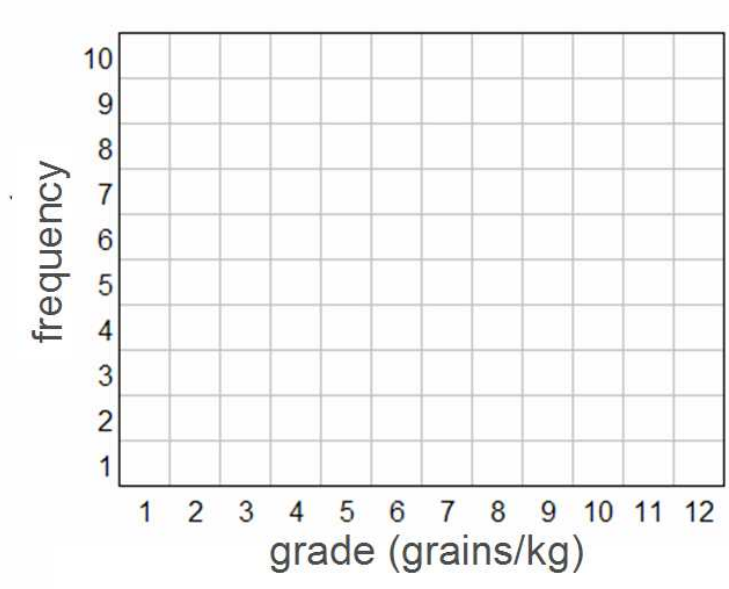
Consider a 48 kg lot that contains precious grains. We are interested in the number of grains per kg. The lot is divided into 48 one-kilogram samples.

1. Count the grains per kilogram within each sample (record in count template).



2. Calculate the overall average of the samples _____. This is the grade of the lot.

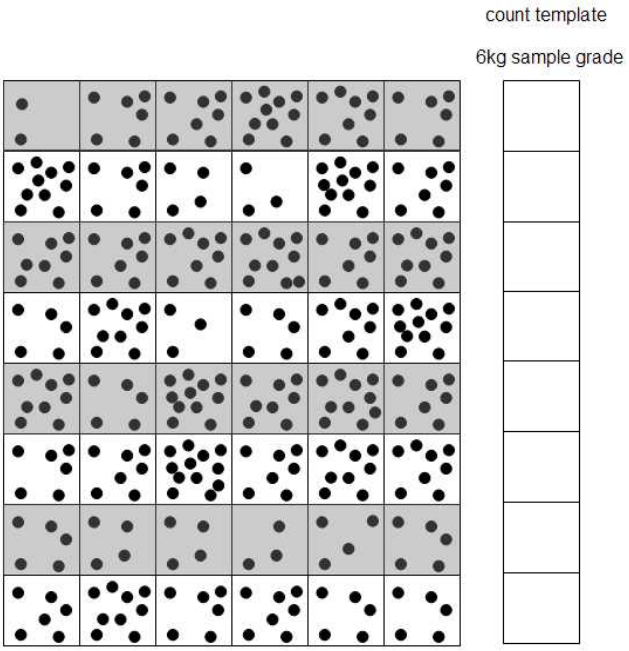
3. How well does each sample reflect the grade of the lot? To answer this question, plot a histogram of the sample grades.



Highlight the lot grade on the histogram. Compare the sample grades to the lot grade.

Calculate the variance and standard deviation.
What does this tell you about the precision of the samples?

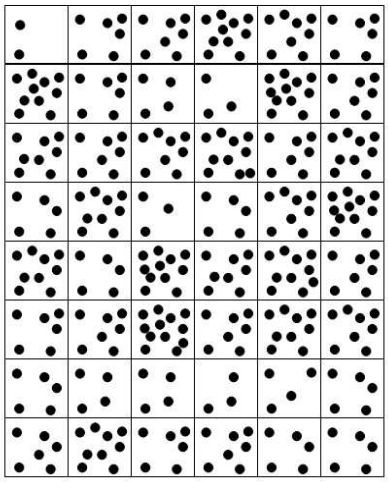
4. Suppose we take bigger samples, say 6kg samples. How precise will these samples be? To answer this, calculate each 6kg sample's grade as grains per kilogram.

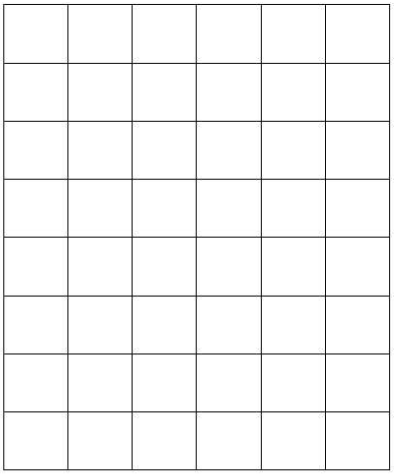


Calculate the overall average. Calculate the standard deviation. How does this compare to the standard deviation of the 1kg samples?

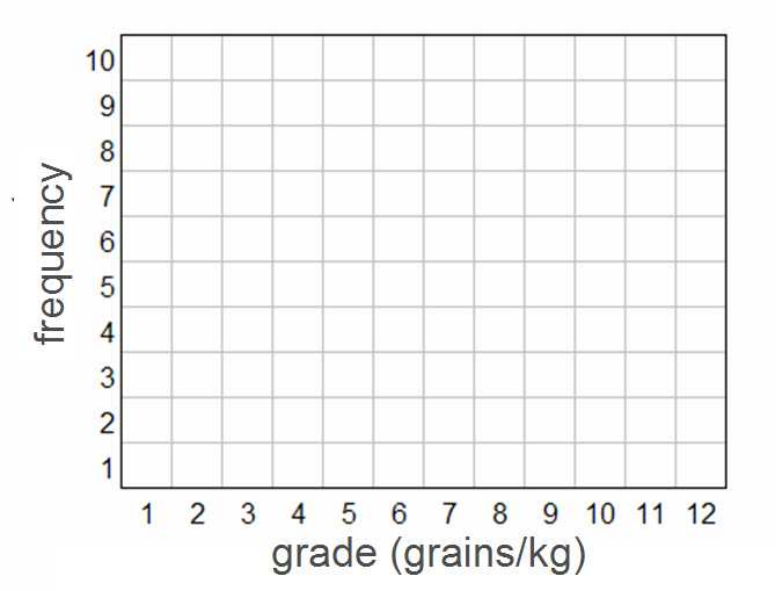
The change in standard deviation or precision can be attributed to the volume-variance effect at the sampling scale (see page 151).

5. Suppose there is a problem with your counting apparatus. For each 1kg sample, a grain is lost. With this adjustment, recalculate the true grade of the lot.





Recalculate the standard deviation of the samples and plot the histogram of the problem data.



What do you observe?

Heterogeneity and sampling

The fragments within the sample will not be uniform units as we had in the previous activity. Instead, particles will have different shapes and sizes to each other (Figure 7) and there will be a preferential distribution of similar particles (Figure 8). These two types of heterogeneity are:

1. Constitution heterogeneity: describes how the fragments vary internally. This heterogeneity increases when the differences in composition between each fragment particle increases.

2. Distribution heterogeneity: describes how the fragments are distributed within the lot.

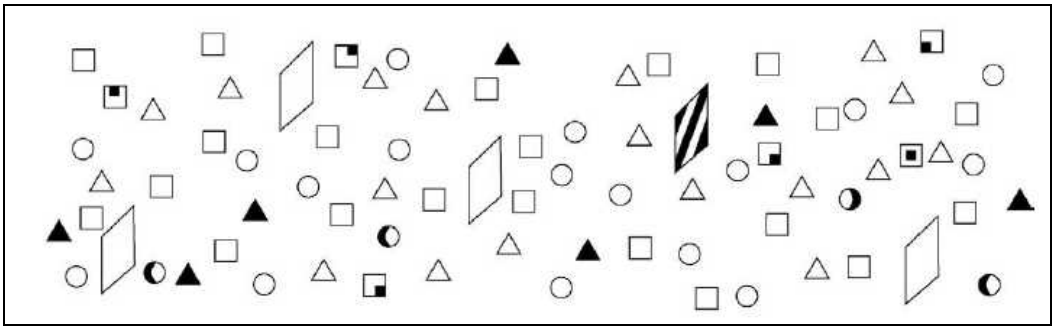


Figure 7 Individual fragments within a lot can be very different to each other
(After Gerlach and Nocerino, 2003)

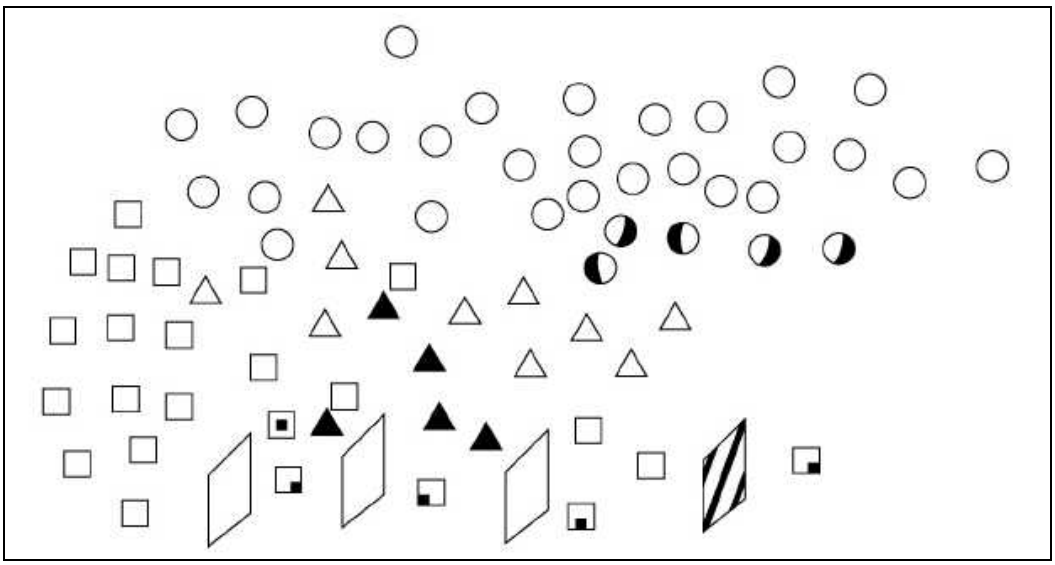


Figure 8 Different types of fragments may distribute differently within a lot
(After Gerlach and Nocerino, 2003)

Constituent Heterogeneity means that within an individual sample, each fragment will influence the sample grade differently. Contrast the sample drawn in Figure 9 with the grain samples in the previous activity.

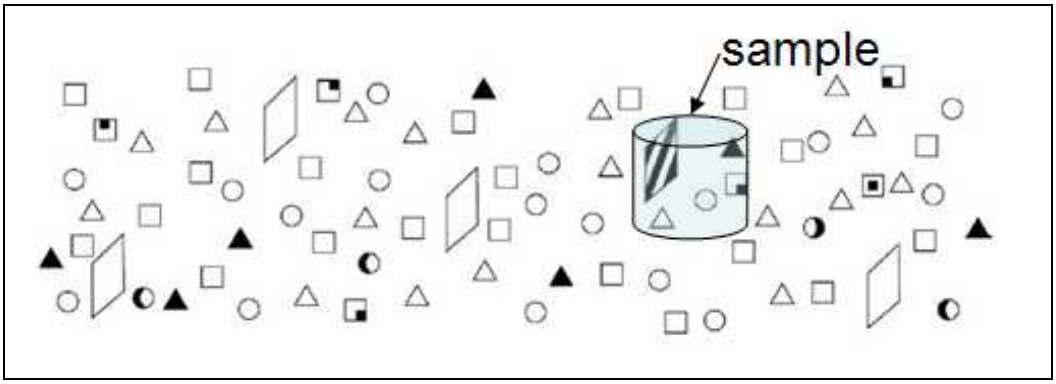


Figure 9 Different types of fragments will influence a sample differently
(After Gerlach and Nocerino, 2003)

Distribution Heterogeneity will mean sample grades will be influenced by where samples are taken from within the lot – compare the components of the three samples in Figure 10.

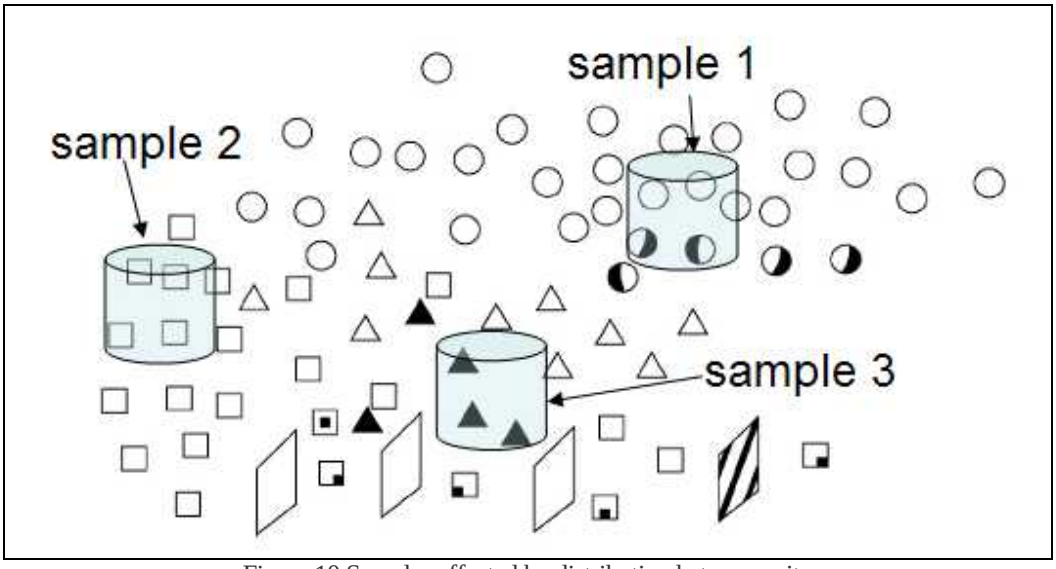


Figure 10 Samples affected by distribution heterogeneity
(After Gerlach and Nocerino, 2003)

In addition, the heterogeneity will affect how fragments move or settle as the lot is extracted from the ground. For example, heavy fragments within a lot from an RC drillhole will land differently to lighter or finer fragments (Figure 11). Merely spearing the lot to collect a sample could result in a sample grade that does not represent the lot.

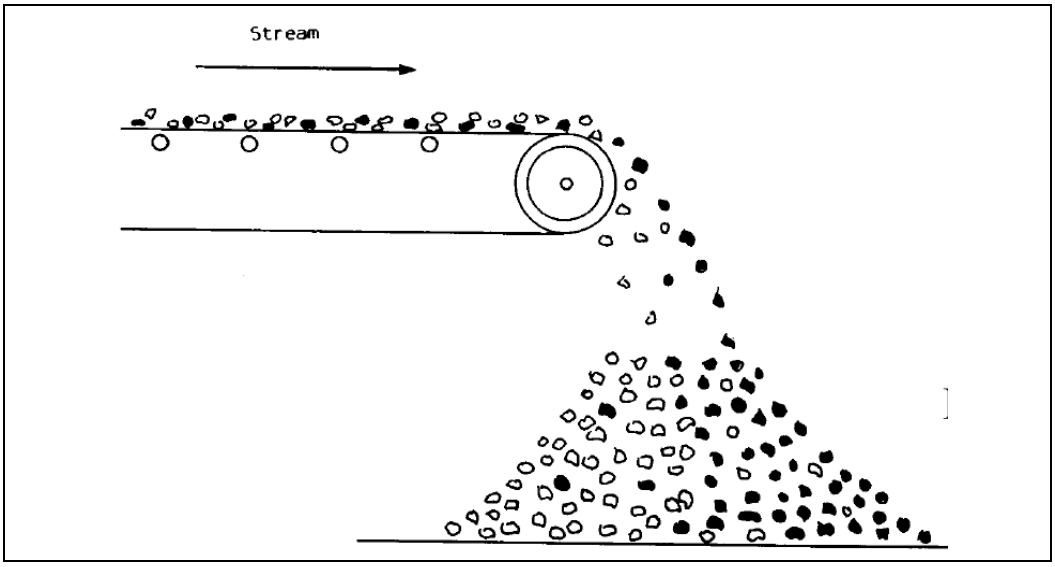


Figure 11 Different samples fall and settle differently (after Pitard 1993)

Remember our objective is to take a sample that will be representative of the lot. We need to ensure we do not introduce more heterogeneity than already exists, since this will increase the bias between the sample and the lot.

Since samples are costly, we want to take as small a sample as is feasible. However, there is a physical lower limit on the number of particles contained within a sample. If the sample is too small, there may not be enough of the mineralised component to represent the lot.

Designing a sampling protocol to suit the geological characteristics is the purpose of a sampling nomogram (see page 28).

Overview of sampling errors

Sampling errors occur at each stage of the sampling process. Consider those listed in Table 1.

Table 1 Pierre Gy's Seven Sampling Errors (after Pitard, 1993)

Type of Error	Description	Mitigation
Fundamental sampling error	The error (or loss in precision) due to the physical composition and structure of the material being sampled. This error includes particle size distribution (Constituent heterogeneity)	Use sampling nomogram to manage FSE for various crush-split protocols
Grouping and segregation error	Attributed to the non-random physical distribution of particles (Distribution heterogeneity)	Managed by homogenising and splitting the sample Increase sample volume
Long-range heterogeneity error	Refers to the non-random differences due to location of sample within the orebody	Use variograms to analyse spatial variability and manage the effect of spatial differences by taking several sub-samples to form a sample
Periodic heterogeneity error	Describes spatial or temporal fluctuations (e.g. phases caused by periodic weathering)	Managed by compositing samples before analysing the grade relationships between samples
Increment delimitation error	The error due to inappropriate sampling design and/or incorrect sampling equipment selection	Care in sample design and equipment selection
Increment extraction error	Occurs when the correct sampling procedure is not followed	Easiest to manage through correct sampling design and adherence
Preparation error	Occurs when some of the sample is lost, contaminated or altered	Strict adherence to field and laboratory protocols is essential

Pitard (2003) discusses both the causes and the mitigation of these and other errors in detail. By way of introduction to the concepts, let us look at some of these from a practical perspective.

Fundamental Sampling Error (FSE)

The Fundamental Sampling Error (FSE) describes the bias between the sample and the lot it is representing due to the constituent heterogeneity. The fundamental sampling error is the "error that remains when the sampling operation is perfect" (Pitard, 1993).

The FSE is the only error that can be estimated ahead of the actual sampling.

The FSE is calculated as

FSE = [variation due to volume] * [variation due to fragmenting] * [mass variation]

where

- The variation due to volume is affected by the following factors:
  - the shape of the fragments (shape factor)
  - the particle size variability (granulometric factor)
  - actual size of the particles (particle size)

- The variation due to fragmentation is controlled by the following factors:
  - the heterogeneity of the particles (mineralogical factor)
  - how well the particles separate (liberation)

- The mass variation is a comparison of the sample mass to the mass of the lot.

The FSE equation in terms of these factors is

$$\sigma^2_{FSE} = f \times g \times d^3 \times c \times l \left[\frac{1}{M_S} - \frac{1}{M_L} \right]$$

For most situations, the mass of the sample is so much smaller than the mass of the lot that the FSE equation is simplified to

$$\sigma^2_{FSE} = \frac{f \times g \times d^3 \times c \times l}{M_S}$$

These terms are described and discussed next.

Shape factor (f)

There are two extremes of shape considered in the FSE. These are the cube and the sphere (Figure 12).

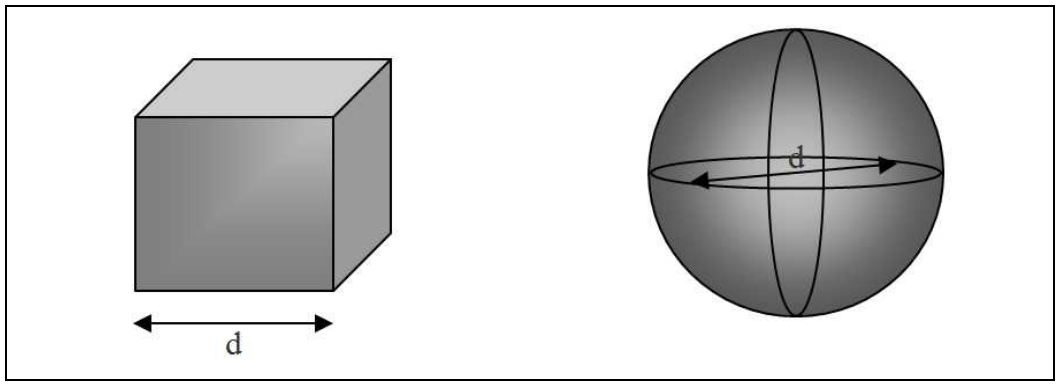


Figure 12 Shape factor as a cube or a sphere

A shape factor is also known as the **coefficient of cubicity** because it describes the shape of a particle relative to a cube**.** The shape factor for a cube is one, while the relative volume of the sphere within the cube gives a shape factor of a sphere. For a cube with a length of one, the volume is 1 x 1 x 1 or a shape factor of one. For a diameter of one, the volume of a sphere is 0.52 and so the shape factor is 0.52.

Most ores have a shape factor of around 0.5. Flaky minerals have volumes that are flatter and more rectangular than a cube and so their volume is reduced to effectively the anisotropy of the rectangle shape (for example 0.1 for a flaky mineral whose height is about 10% of its length). Soft homogeneous minerals (such as gold) have a shape factor of 0.2.

Particle size (d^3)

The particle size describes the volume of the cube within which the particle fits (Figure 13).

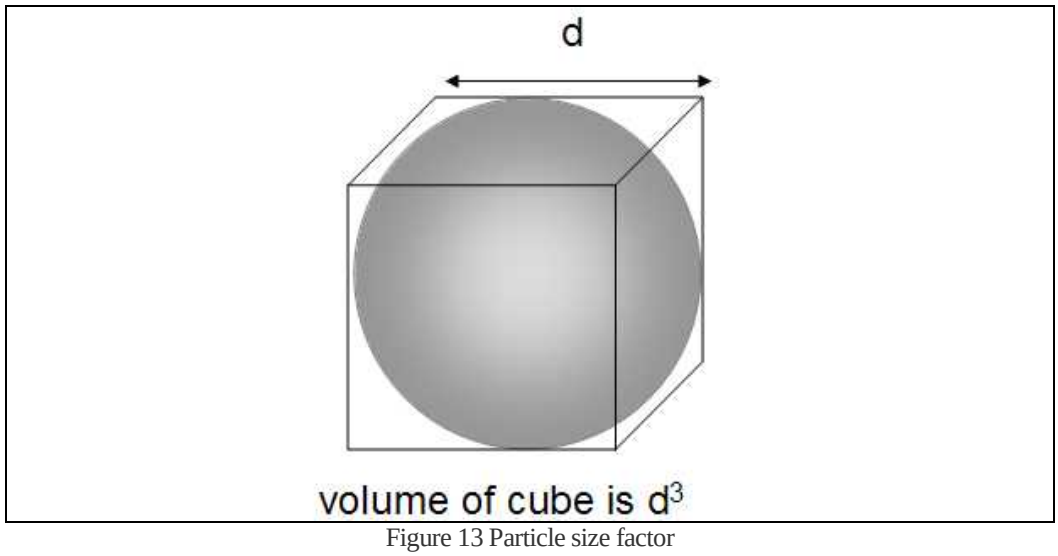


Figure 13 Particle size factor

Granulometric factor (g)

The granulometric factor, or **particle size distribution factor**, describes the particle size distribution (Figure 14). The size of each particle is not a constant. This factor accounts for the varying particle sizes when estimating the fundamental error.

The granulometric factor describes the range of particle sizes in a sample. When particles all have the same size (or diameter), g is set to one. Particles in crushed samples, the g factor is typically around 0.25. For particles retained between screenings, g is 0.55.

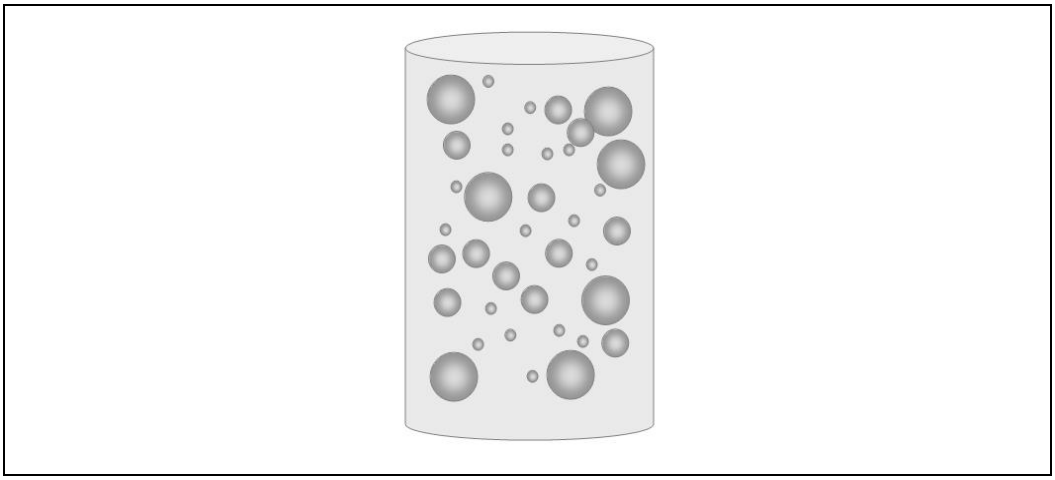


Figure 14 Granulometric factor describes the size distribution

Due to the log-normal distributions used to describe the size distributions, for most cases the P95 (95% mass passing size) samples have a granulometric factor of 0.25.

Mineralogical factor (c)

The mineralogical factor (c) describes the composition and heterogeneity when all the particles are liberated (Figure 15). The mineralogical factor is essentially the proportionally weighted density of the mineralisation and the density of the gangue.

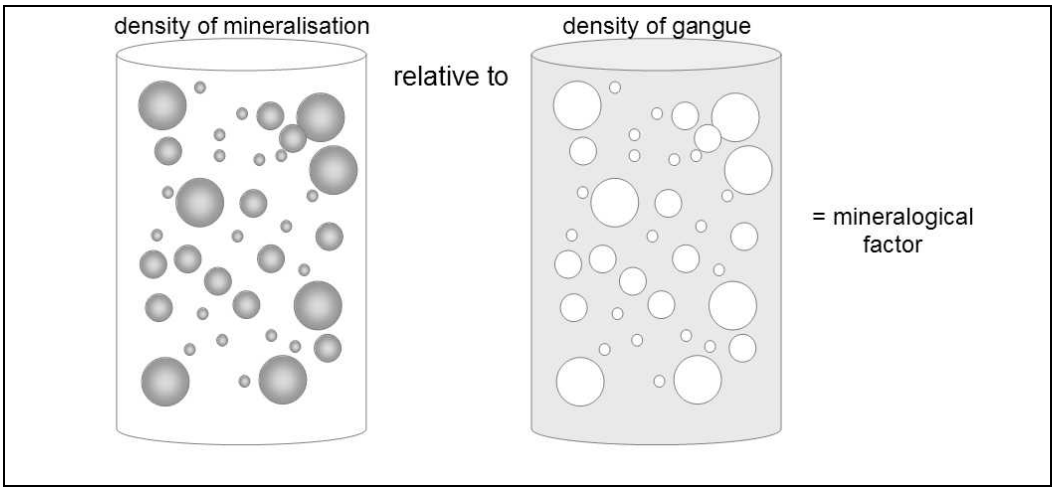


Figure 15 Mineralogical factor concept

Liberation factor (l)

The liberation factor is the fraction of the mineralised particles that can be separated as pure mineralisation from the gangue (Figure 16).

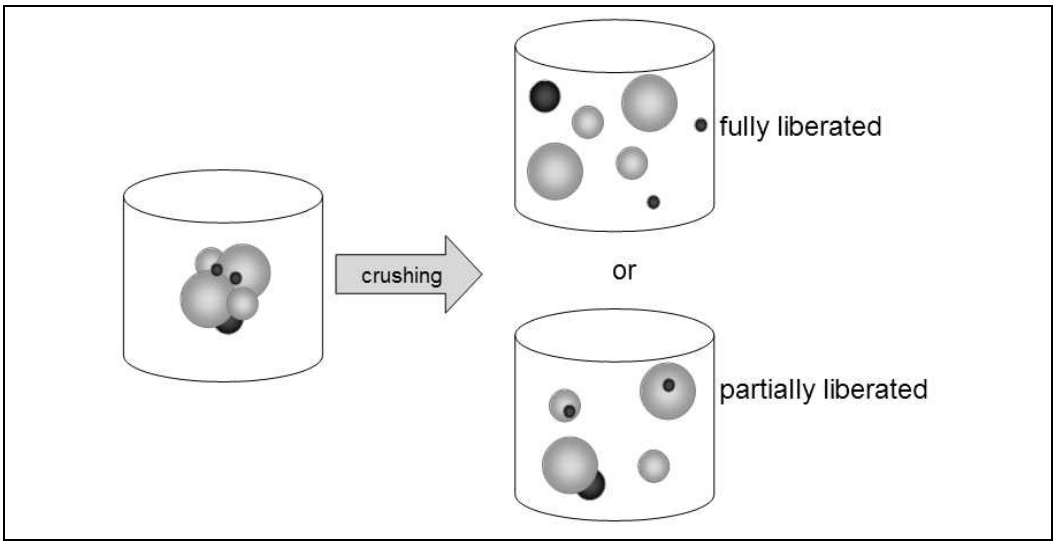


Figure 16 Liberation factor concept

One way to calculate the liberation factor is to compare the diameter of the particles needing liberation to the size of P95 screening diameter. This ratio is typically scaled by raising to a power of one for high-grade concentrates and up to no more than three for other minerals.

Mass factor (M_S and M_L)

The mass factor describes the mass of the sample relative to the mass of the lot. The inverted mass of the lot is subtracted from the inverted mass of the sample. The closer the sample mass to the lot the more similar these values and the less the difference between them. Ultimately for large samples relative to the lot mass, the mass factor is small and then so too is the FSE.

The FSE equation

The factors described above are multiplied together to provide a measure of fundamental error in the sample of the lot due to the nature of the material we are working with:

$$\sigma^2_{FSE} = f \times g \times d^3 \times c \times l \left[\frac{1}{M_S} - \frac{1}{M_L} \right]$$

Or the simpler

$$\sigma^2_{FSE} = \frac{f \times g \times d^3 \times c \times l}{M_S}$$

Understanding the nature of the material, or the Fundamental Sampling Error of the material, allows us to plan a sampling strategy – how big the sample should be relative to the lot size, how much homogenisation is required, when and how we can split a sample and how close we can get to a representative sample.

Solving practical sampling problems

The fundamental sampling error provides a useful guide for solving practical sampling problems, including:

- To what diameter should we be crushing?
- How big should our sample size be if we want to maintain a precision of at least 10%?

Let us tackle each of these by way of an example.

Suppose we have mineralisation in material with the following properties:

- The screen diameter is 1.25cm for 95% of material. This is the particle diameter (d).
- The mineralogical factor (c) is 15000000
- The fragments are spherical (f = 0.52)
- The granulometric factor (g) is 0.25 (typical for gold)
- The liberation factor (l) is 0.000025
- The sample mass is 10kg (10000 g)

The simpler FSE equation gives us an error variance factor of

$$\sigma^2_{FSE} = \frac{f \times g \times d^3 \times c \times l}{M_S}$$

$$\sigma^2_{FSE} = \frac{0.52 \times 0.25 \times 1.25^3 \times 15000000 \times 0.000025}{10000}$$

$$\sigma^2_{FSE} = 0.01333$$

Taking the square root of this value gives us a precision of **11.5%**

What do we need to do to get the precision below 10%?

Either we could crush the sample to pass through a finer screen or we could take a larger sample. Let us investigate these two options.

To investigate the required diameter we need to re-arrange the formula and substitute 10% for the precision (i.e. 0.1^2 for the σ^2_{FSE})

$$d^3 = \frac{\sigma^2_{FSE} \times M_S}{f \times g \times c \times l}$$

$$d^3 = \frac{(0.1^2 \times 10000)}{0.52 \times 0.25 \times 15000000 \times 0.000025}$$

$$d^3 = 1.465$$

$$d = 1.136$$

To achieve a precision of 10%, the material will need to be crushed to 95% passing 1.14 cm.

Alternatively, we could take a larger sample. Re-arranging the formula again gives us:

$$M_S = \frac{f \times g \times d^3 \times c \times l}{\sigma^2_{FSE}}$$

$$M_S = \frac{0.52 \times 0.25 \times 1.25^3 \times 15000000 \times 0.000025}{0.1^2}$$

$$M_S = 13\,330\text{g}$$

By taking a slightly larger sample of 13.33 kg, we can improve the precision to 10%.

In practice, both options need to be evaluated for practicality – what makes more sense: crush more or take a bigger split?

Grouping and segregation errors

Grouping and segregation occurs when a lot or sample is moved (for example in a conveyor or as a sample blasts through to surface in an RC drillhole).

Grouping and segregation is due to differences in fragment density, fragment size or fragment shape. Other causes include air turbulence (for example when a riffle split is fed too quickly or has no door panels to keep the fines within the sample).

Consider the examples described by Pitard (1993) presented in Figure 17 to Figure 21. Note that these errors cannot be undone, fixed up or cancelled out in the rest of the sampling process.

Impact of density variation

Variability in density is the main cause of segregation and grouping - consider the density of gold (19.3 g/cm^3) compared with gangue (2.6 g/cm^3) and the impact of this on the sampling process.

In other deposits the density of the critical component tends to be double that of the gangue (for example: mineral sands the zircon and ilmentite have densities around 4.7 g/cm^3 compared with about 2.6 g/cm^3 for the clay and quartz gangue.

Activity

Consider the projects you work on.

- How different is the density of the critical component to the gangue?
- What are the stages of sampling in your project where density would cause segregation and grouping?
- What do you do or can you do to reduce segregation and grouping on your sampling process?

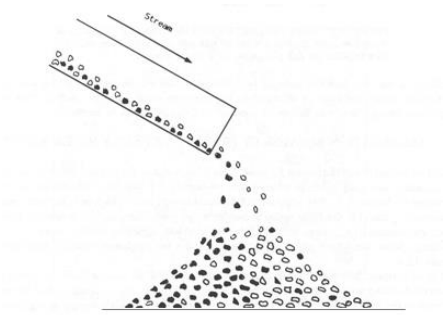


Figure 17 Segregation of dense fragments from lighter ones in the same fraction (Pitard, 1993)

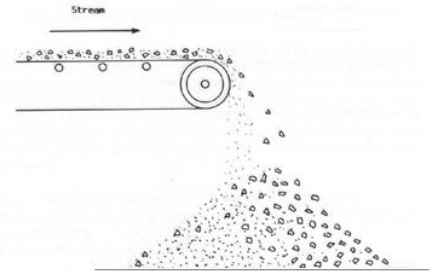


Figure 18 Segregation of fine particles from larger fragments with the same density (Pitard, 1993)

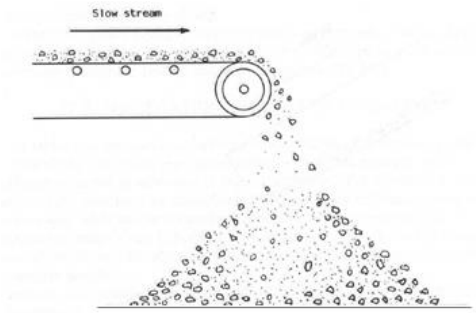


Figure 19 Fines sift inside the pile while coarse fragments role down the outside (Pitard, 1993)

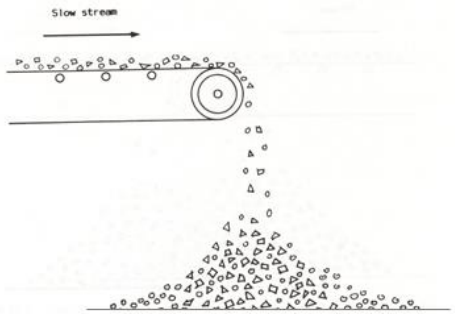


Figure 20 Segregation generated by different angle of repose (Pitard, 1993)

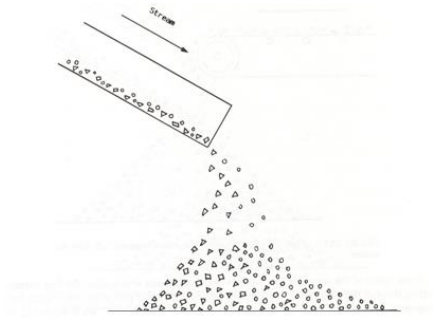


Figure 21 Segregation generated by different friction rate (Pitard, 1993)

Minimising grouping and segregation

Pitard (1993) measures the grouping and segregation error as the product of two factors (the grouping factor and the segregation factor) and the FSE:

Grouping and segregation error (GSE) = grouping factor x segregation factor x FSE

Homogenisation (or mixing) is the ideal process used to minimise the grouping and segregation error. This, however, is costly on large lots and so has limited appeal.

An alternative is to take increments or sample subsets in an attempt to generate a representative sample. When we take increments to create a sample (say spearing an RC sample), the more increments we take the more likely we are to get a representative proportion of the lot.

Consider the example in Figure 22. The overall lot shows a third of the fragments are mineralised (15 out of 45 fragments). This is not what is reflected within each increment (1 out of 8; 2 out of 5 and 4 out of 8). Individually the increments do not represent the lot. A sample made up of increments, however, is representative of the lot (7 out of 21). Pitard (1993) recommends 30 such increments. This is consistent with requirements in statistical theory (for example in the central limit theorem).

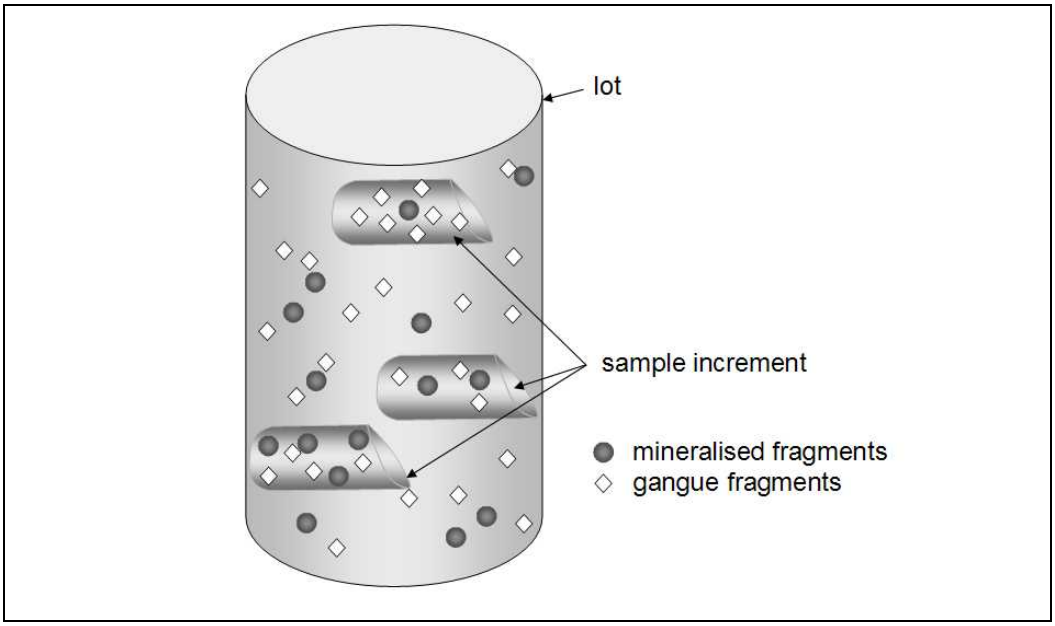


Figure 22 Increments improve representativity when fragments are grouped and segregated

When the grouping and segregation is due to stratified or bedded material, we need to ensure the samples are collected at right angles to the bedding, thereby achieving a sample representative of the bedded lot.

Sampling collection errors

Sample collection errors differ from the FSE since the sample collection errors can be reduced. Consider these three avoidable errors:

1. Poorly delineated sample. Before the sample is even collected, errors can occur because the target geometric volume is incorrectly or poorly defined.

2. Poorly extracted sample. As the sample is extracted, the target volume is not collected (for example poor core recovery) and either too little or too much material is collected.

3. Incorrect preparation occurs when the sample is changed before assaying. Examples include:

    - Contamination (for example downhole smearing or dust contamination)
    - Loss of either part of the critical component or the gangue (for example loss in the high grade mineralised fines to the water used whilst cutting core)
    - Alteration
    - Unintentional faults such as dropping the sample or equipment failure
    - Intentional error (such as fraud or sabotage).

The objective of sampling theory is to understand sampling errors so we can be more aware and manage or minimise them through QAQC processes.

Sampling for density

Lipton (2001) provides a comprehensive summary of the approaches to sampling for density.

Sampling nomograms

Sampling nomograms are used in the design of a sampling protocol. Various scenarios of crushing and splitting can be assessed before a drilling program.

The idea behind the sample nomogram is to evaluate various crushing and splitting options according to the fundamental sampling error, practical and cost considerations.

The nomogram plots the FSE against the sample mass. Both axes are plotted on a log scale. The FSE of the sampling protocol is tracked on the nomogram (Figure 23).

stage	Crush diameter P95 diameter (cm)	Split mass mass (g)	process
initial sample	5	50000	initial lot of 50 kg
first reduced sample	2.5	5000	split to 5kg
first crushing	1	1000	crush + split to 1kg
second crush	0.25	100	crush + split to 100g
pulp for FA	0.05	15	pulverisation and split to 15g

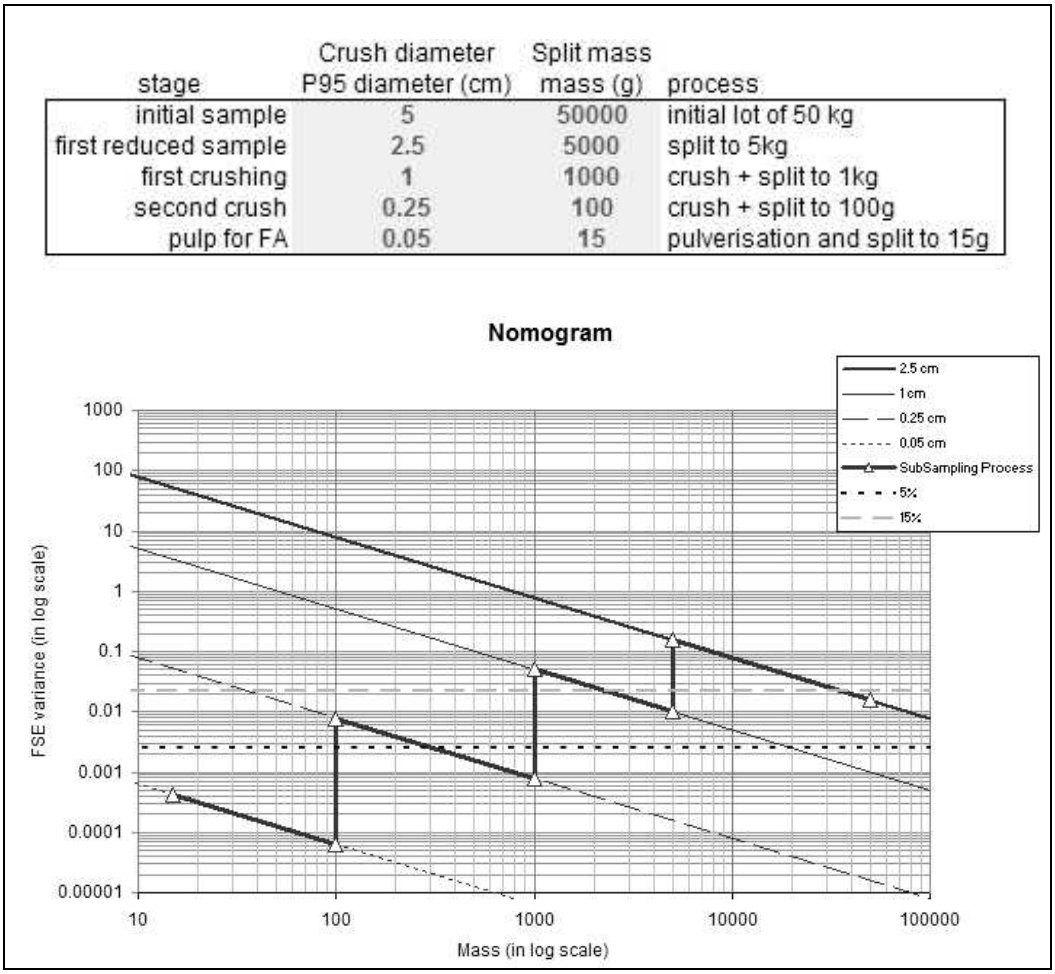


Figure 23 Example of a sampling protocol and a Nomogram

The slopes of the FSE are based on a test of the material to be sampled. In this test, typically 50 points are assessed for the shape, granulometric, mineralogical and liberation factors. Naturally, these factors will vary according to changes in the geological units.

Once these factors are estimated, the FSE equation needs only the fragment size, the lot mass and the sample mass. The guidelines on the nomogram are simply the FSE values for changes in sample mass for a given P95 fragment size.

QAQC

Introduction

The purpose of the Quality Assurance and Quality Control (QAQC) is to ensure the quality of data and information used to make decisions on project value and project progression are sound. The quality of mining and project decisions is fundamentally contingent on data quality.

In this section, we discuss the use of statistical tools to monitor sample quality.

A positive QAQC is the database's stamp of approval. Without this stamp, all the effort on complex geological interpretations, mathematical estimations and resource classification is called into question. Poor QAQC practice is equivalent to generating a resource blindfolded.

When do we put effort into QAQC? Well, it is a process that should be analysed as data is collected. QAQC should accompany every parcel of data. QAQC is most effective if used **during** a drilling campaign to MONITOR data collection and call a halt to a campaign or laboratory that is underperforming. Running a QAQC at the end of the data collection program is an ineffective way to manage data collection.

A good QAQC process is one that is active, ongoing and reviewed as data is collected; it is easy to understand, makes sense and gives you sufficient information to take timely corrective action on your rig, with your sampling procedure or at the laboratory.

Let us look at ways to analyse data quality.

Statistical process control

Process control is the act of checking the quality of a process. In simplest terms, we take a value we know (standards and blanks) and insert them into the assaying process. We then plot the assay values returned by the laboratory for these known values and evaluate how close they are to the actual value. Some variation is expected, however, we want to ensure the returned value as well as the variability of the returned values is reasonable.

Run charts

Run charts are simply a plot of values measured over time. The value measured is plotted on the y-axis. These values are plotted against time on the x-axis.

Run charts are useful for tracking trends. For example, a run chart on turnaround time at the laboratory may indicate a trend of increasing turnaround time (Figure 24). In particular, run charts are useful for spotting non-random patterns (Figure 25).

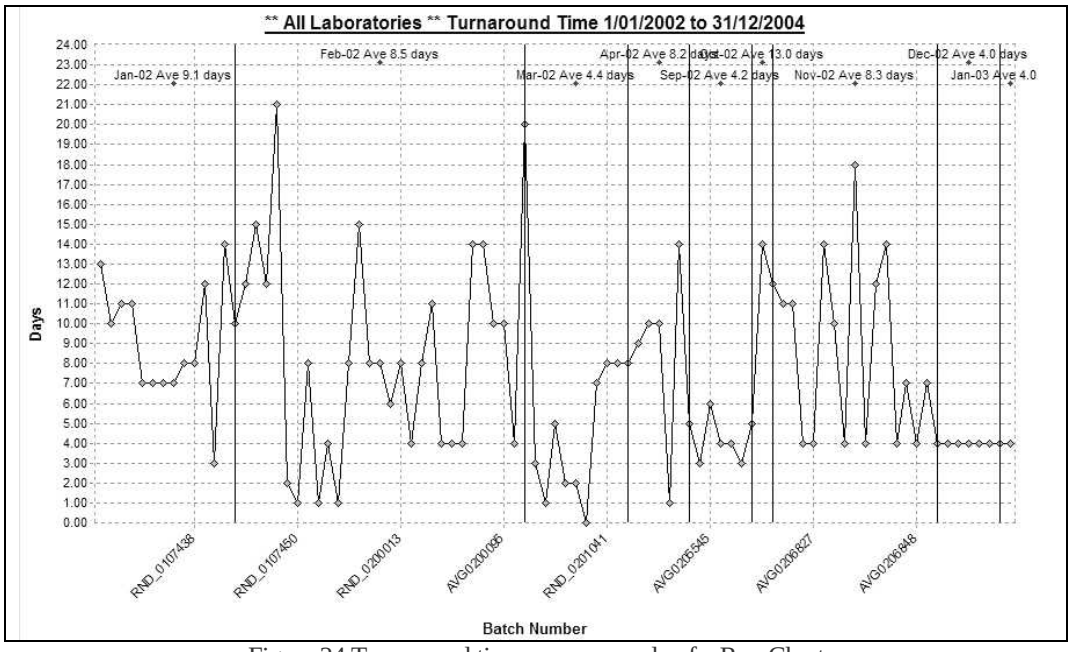


Figure 24 Turnaround time as an example of a Run Chart
(Maxwell's QAQCR program)

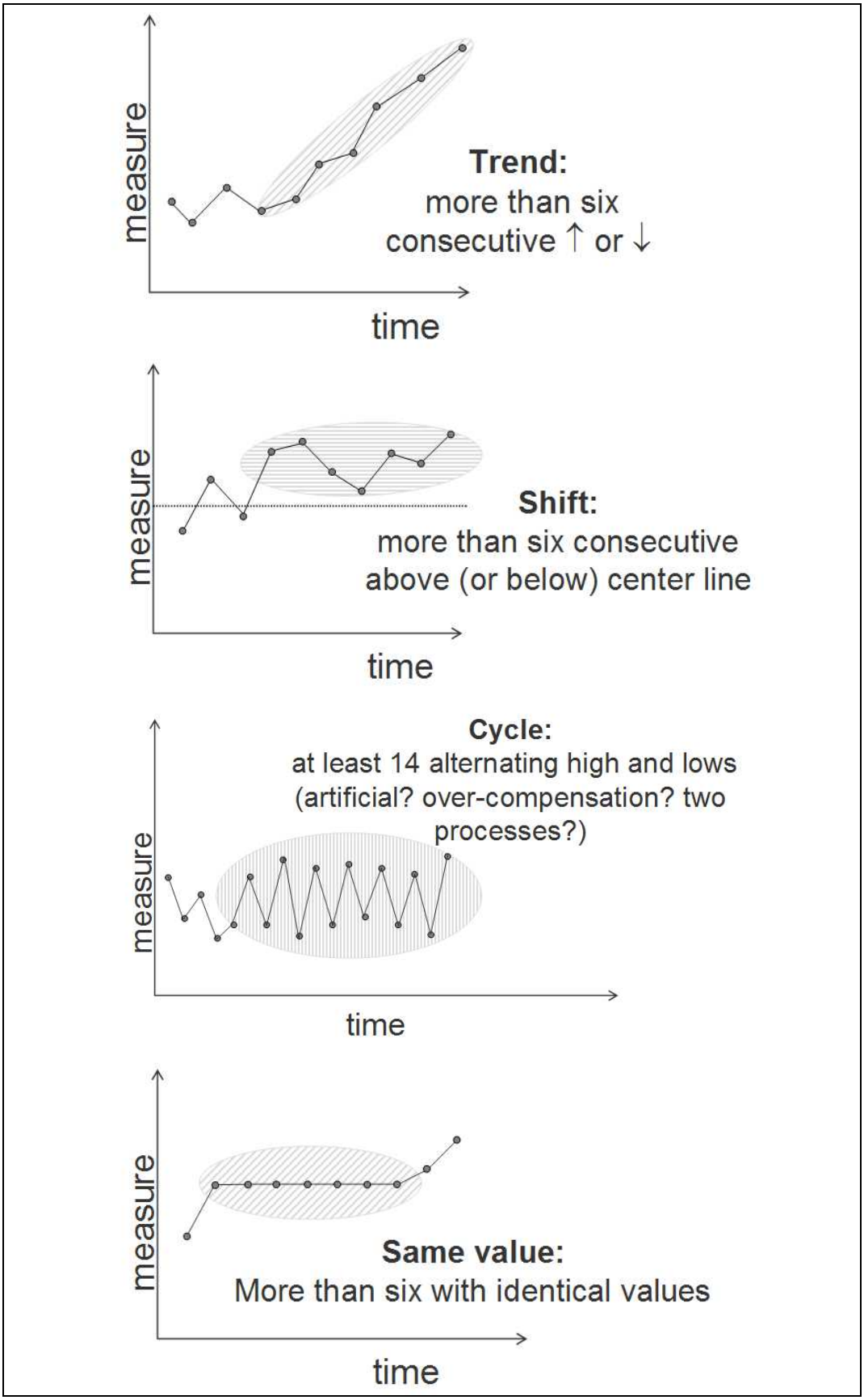


Figure 25 Look for non-random behaviour

Control charts

Control charts are run charts with control limits. Control limits provide a sense of when to start worrying about a process or when the process is out of control. The control limits are predefined (expected standard grade and variability).

In Figure 26 instance 46 is out of the control limits. Notice the erratic values that precede instance 46 and the sudden run of three almost identical values …. mmmmm makes me wonder?!

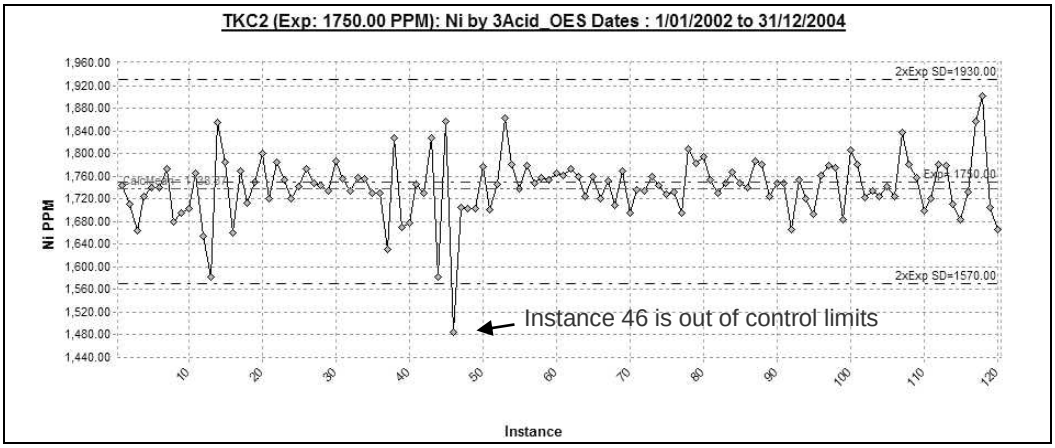


Figure 26 Example of a control chart
(Maxwell's QAQCR program)

A plot with all the standards and blanks on a single graph helps identify consistent patterns in the batches (Figure 27).

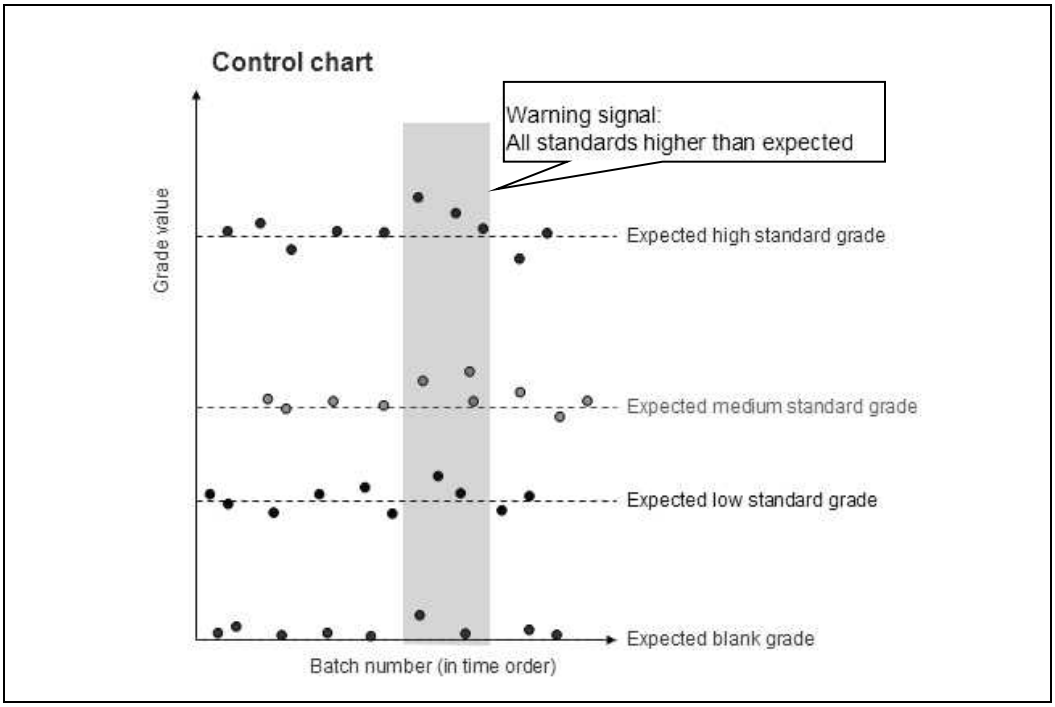


Figure 27 Example of a control chart with simultaneous plotting of results

Another useful grade to plot on the control chart is the grade of the sample just prior to the standard grade analysed as well as the grade returned for the standard sample. This is useful for checking for sample smearing in the laboratory assaying procedure.

Looking for patterns

Bias

Typically, we look out for a consistent bias or patterns in the control charts. This is revealed when the laboratory delivers sample values for the standards that are consistently high or low. However, how many consistently high (or low) samples do we need to be confident that there is a problem?

If the laboratory is producing random high and low grades around the standard, we will expect to see this plotted as a sample value either above or below the expected grade.

Consider what a random variability around the standard value should look like. For one, it should look random. But how do we know when a pattern is random or not? The simplest and best-known random process is flipping a coin. Flip a coin and you have an even chance of either a head or a tail.

Let us simulate the probability of getting random high grades back from the laboratory. In flipping a coin, we will record a head as a grade above the expected standard value and a tail as a grade below the expected standard value.

If we submit only one standard sample we will see either the grade as either above or below the expected standard grade. There is thus a 50% chance the standard value is higher than the expected value.

Now, if we submit two standards we are essentially flipping two coins. The two resulting standard values are one of head-head, head-tail, tail-head or tail-tail. Therefore, there is a 25% (one in four) probability that the two consistently high standard values delivered by the laboratory are due to random errors.

Consider three standard values. The random patterns are associated with the flipped coin are

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

So there is a one in eight (or 12.5%) probability that three consecutive higher than expected standard values is due to random noise. Three consecutive high values do not tell us enough about the presence of a bias in the laboratory.

But what about four consecutive high values? How random is this? The probability of four consistently high grades is one in 16 (or one out of 2 x 2 x 2 x 2), which is a 6.25% probability that four consecutive high values is random.

And five consecutive high grades? … one in 32 or 3.125% probability.

And just for good measure, the probability of six consecutive high grades is one in 64 or 1.5625% probability.

Similarly, there is a one in 128 (or less than 1% probability) that there will be seven consecutive high grades.

A good dose of common sense is invaluable when interpreting control charts of standards and blanks. We expect the laboratory to have a degree of error in the standards we submit. We do not want to see consistent bias in our data (either above or below the expected standard grades).

Non random patterns

We expect to see random fluctuations about the expected standard grade. Any discernable patterns should be a warning to interrogate the batch of data further. Patterns include:

- Consistently higher (or lower) grades than expected
- Cyclical increasing and decreasing grades
- Increasing (or decreasing) variability in the standard values
- Grades trending up (or down) or
- Standards returning identical values.

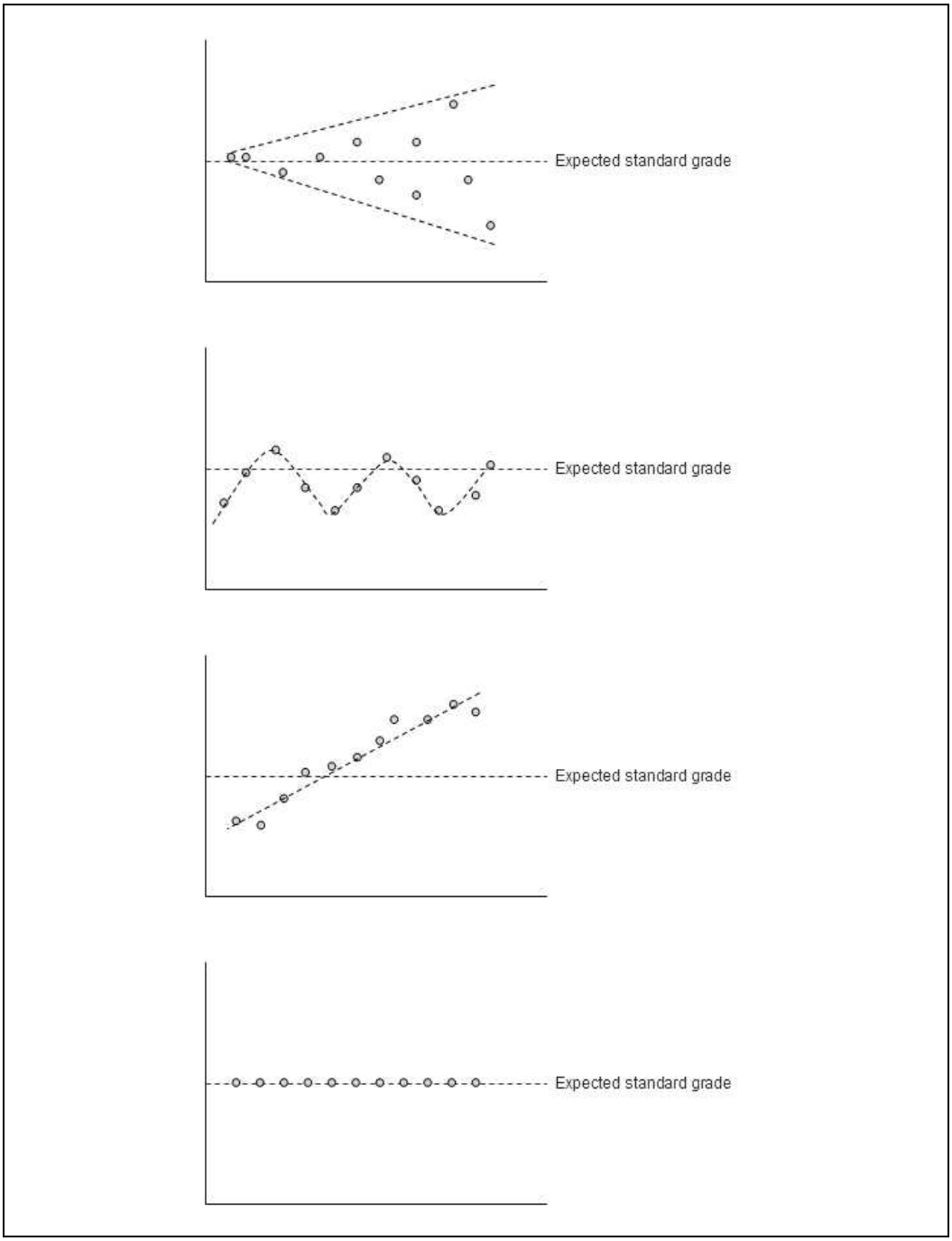


Figure 28 Examples of warning patterns in control charts

Setting control limits

Control limits provide a warning that the laboratory is returning standard grades that are out of bounds of the natural variability expected in the standard data. Useful control limits[2] are typically 10% either side of the expected standard grade. Laboratory supplied standard values outside this range are a warning to check the batch of data provided by the laboratory.

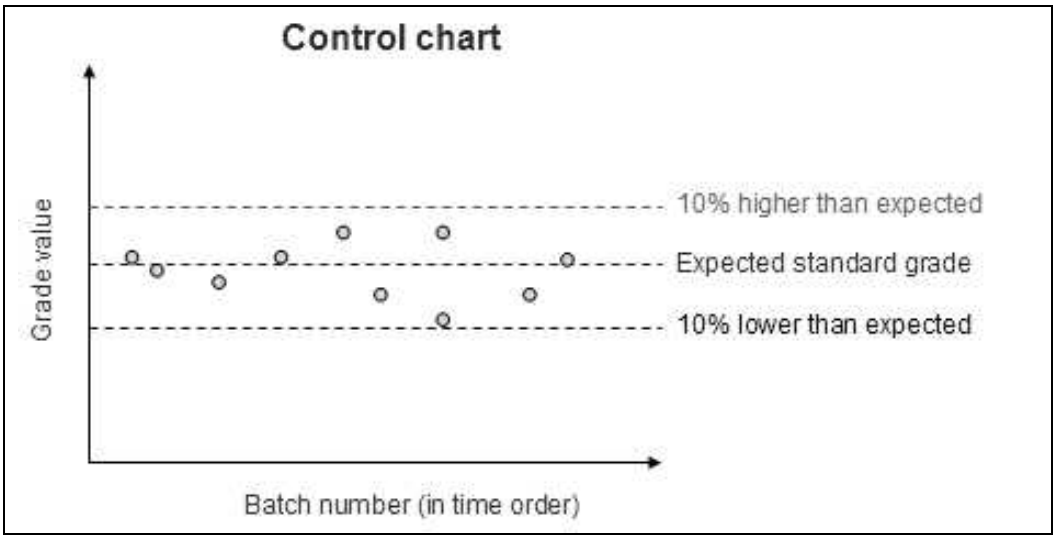


Figure 29 Example of a laboratory in control

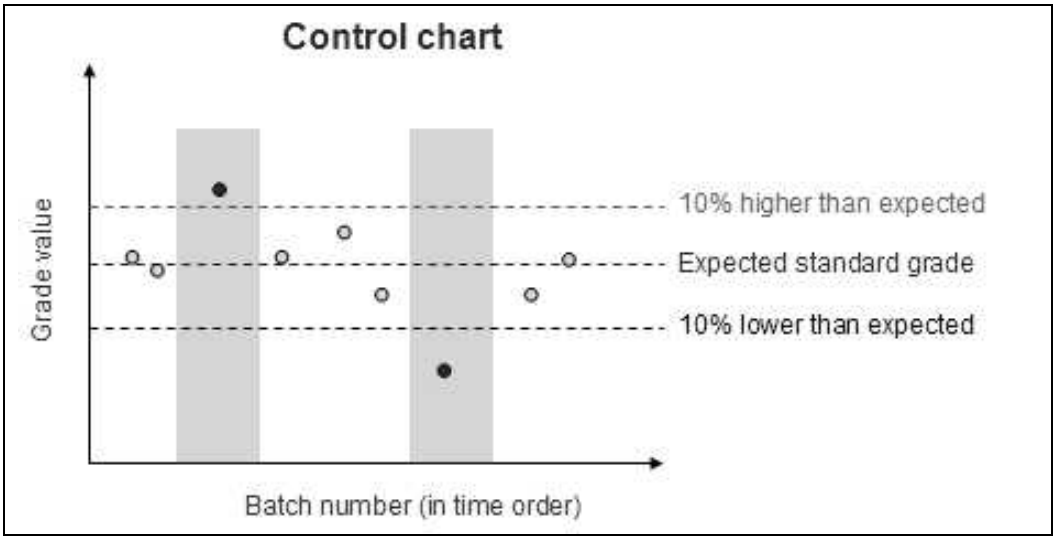


Figure 30 Example of a laboratory with two standards with grade outside the ±10% control interval

[2] Upper and lower control limits should be set at reasonable percentages for each standard. Here we use 10% as an example.

Warning signals used within Minitab software's control charts include:

- One point more than three standard deviations from the centre line
- Nine points in a row on same side of the centre line
- Six points in a row, all increasing or all decreasing
- Fourteen points in a row, alternating up and down
- Two out of three points more than two 2 standard deviations from the centre line (same side)
- Four out of five points more than one standard deviation from the centre line (same side)
- Fifteen points in a row within one standard deviation of the centre line (either side)
- Eight points in a row more than one standard deviation from the centre line (either side)

Activity

Assess the following three control charts. Are there any batches of samples you would question?

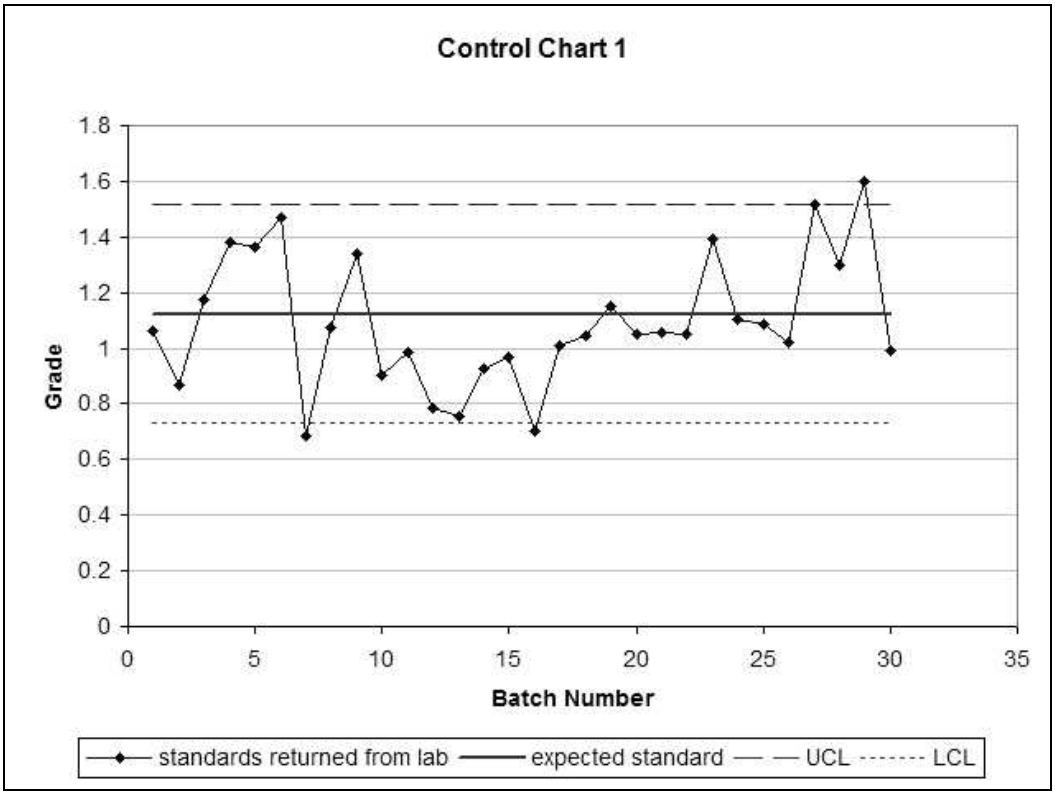


Figure 31 Control Chart Activity – chart 1

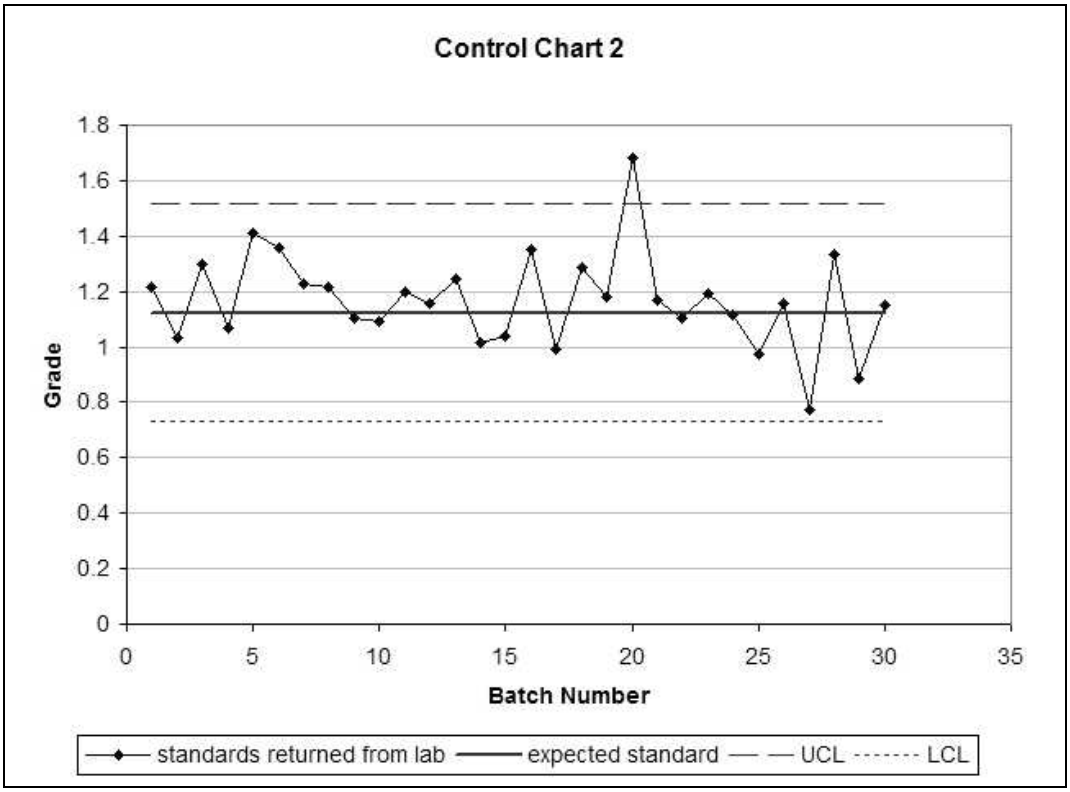


Figure 32 Control Chart Activity – chart 2

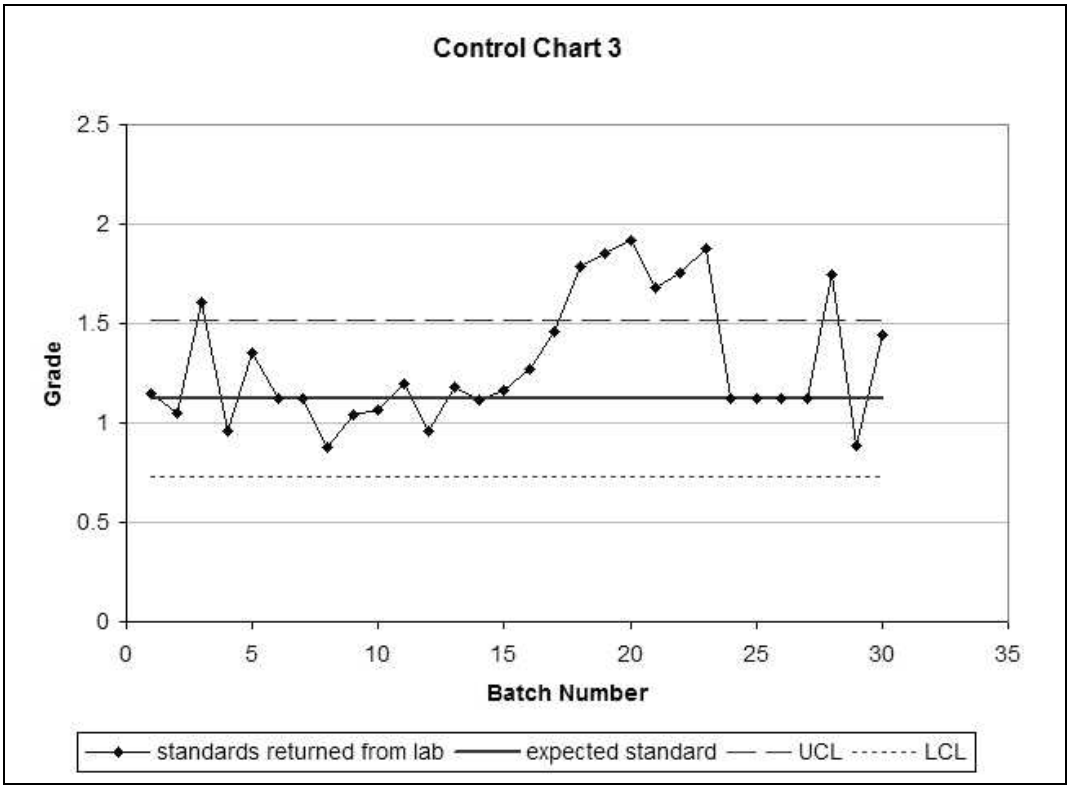


Figure 33 Control Chart Activity – chart 3

Evaluating reproducibility

Scatterplots

QAQC checks include comparisons between duplicate and repeat samples against the original sample grade on a scatterplot[3] (Figure 34).

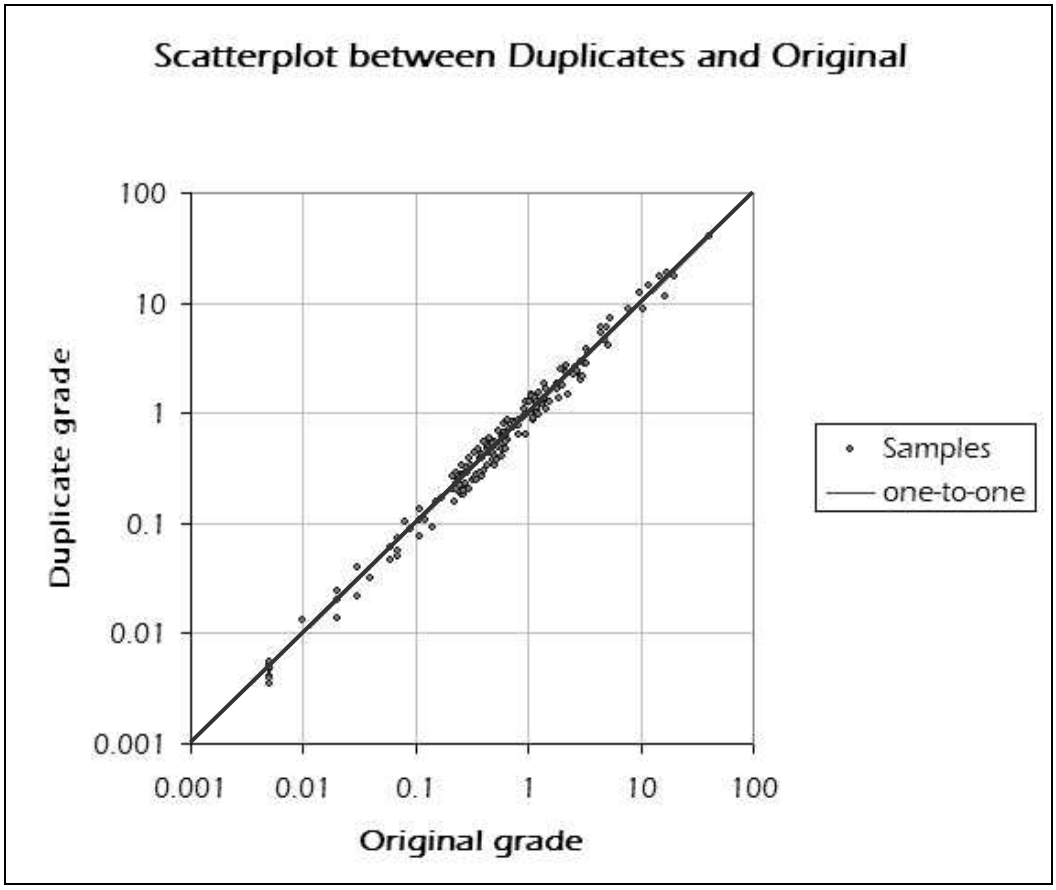


Figure 34 Scatterplot between duplicates and original sample grades

We look for both accuracy (lack of bias) and precision (degree of reproducibility) in a scatterplot.

Firstly, the scatter between the duplicate (or repeat) sample and the original sample should lie roughly along the one-to-one line. This indicates there is no bias in the data. The example in Figure 35 shows duplicates are consistently higher than the original sample grades.

[3] **Quick tip:** The original or oldest data set is plotted on the X-axis of a scatterplot.

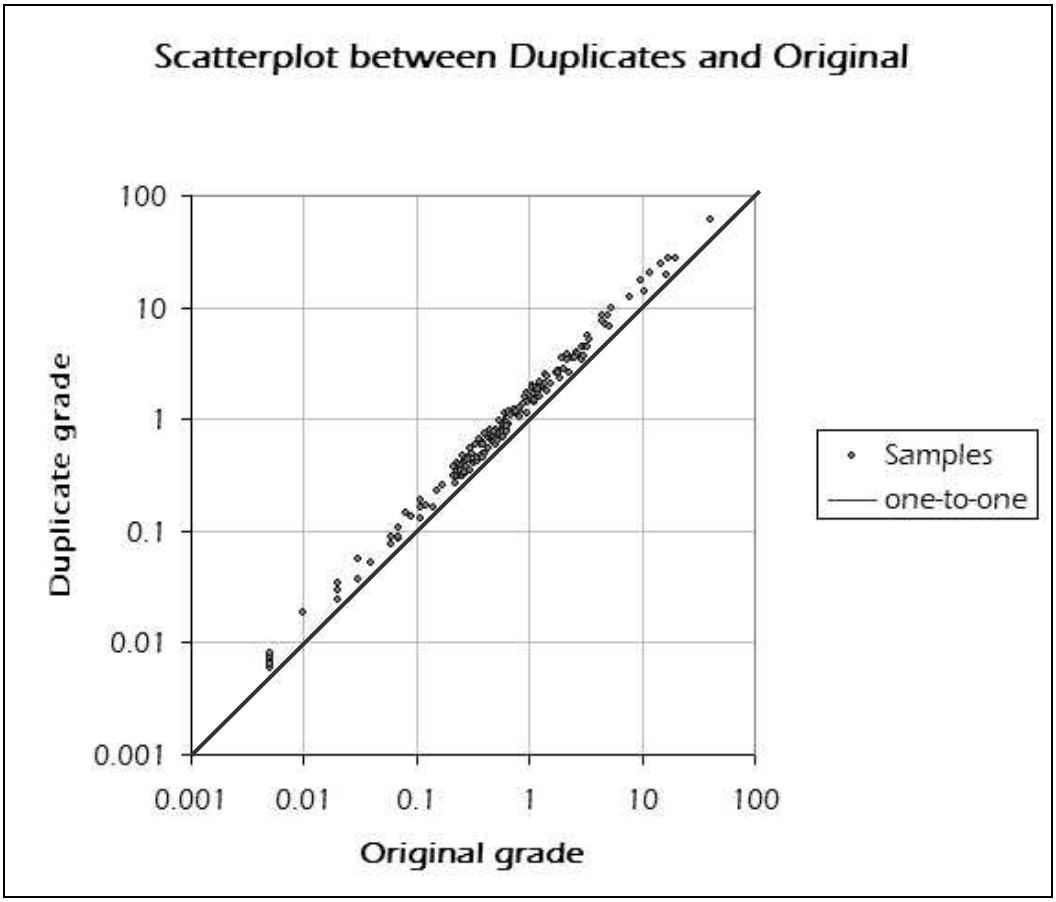


Figure 35 Scatterplot between duplicates and original sample grades with a bias

Secondly, the scatter around the one-to-one line gives us a clue about how different the duplicates (or repeats) are to the original samples. Figure 36 illustrates increasing variability between the original and the duplicate sample grades. Notice that even a 20% typical difference between original and duplicate shows up as a reasonable scatter.

Scatterplots allow a quick assessment of both the bias and possible differences between the duplicate and original samples and they should be generated as data becomes available.

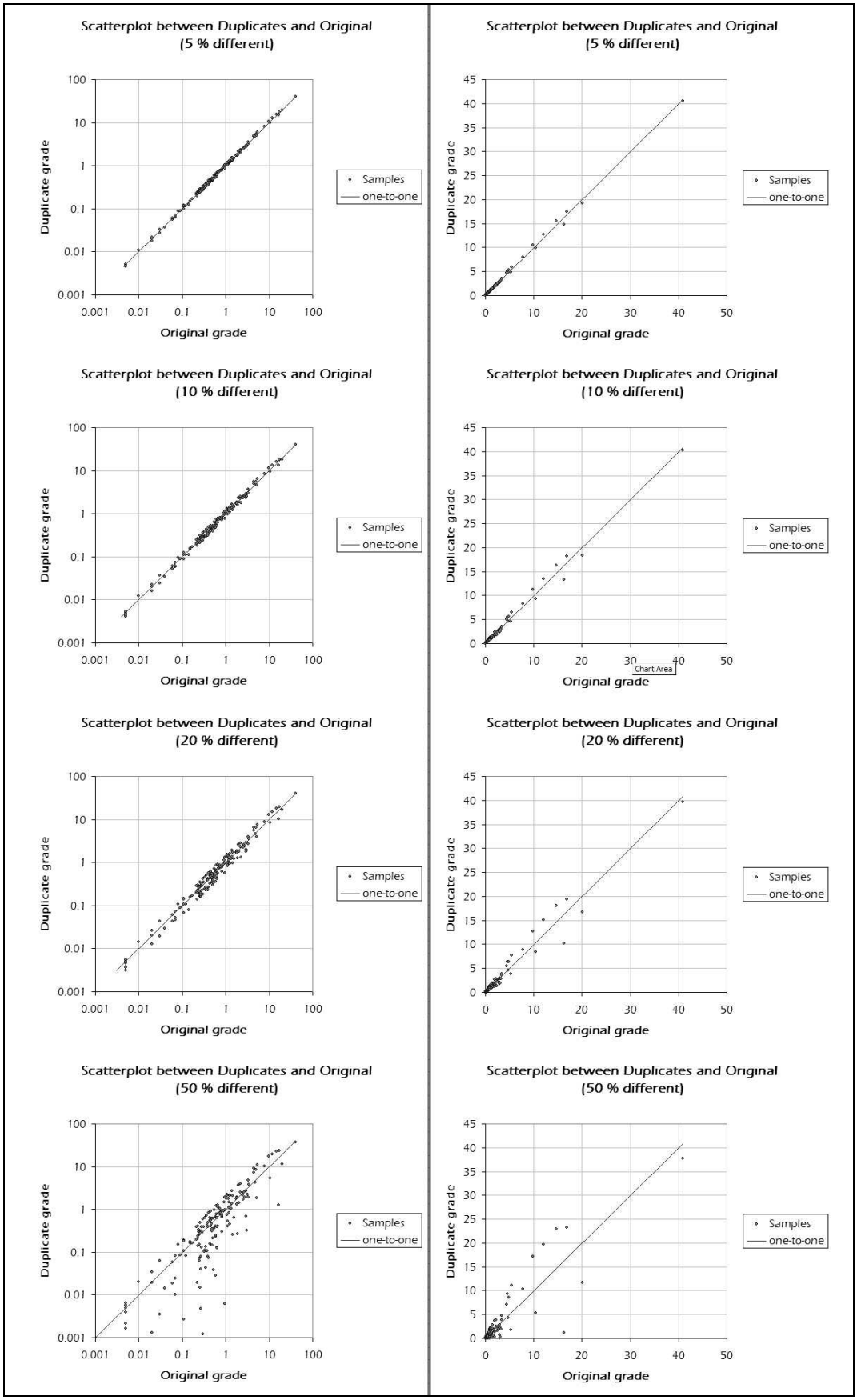


Figure 36 Scatterplots with decreasing precision (log-scale and grade scale)

Comparing histograms (Q-Q plots)

Q-Q plots (or quantile-quantile plots) compare two data sets by plotting the respective percentile grades against each other. This is equivalent of creating a scatterplot between the histograms (Figure 37). Practically, Q-Q plots are generated as follows:

1) Sort each data set in ascending order
2) Calculate the percentiles for each data set
3) Plot the percentiles from the two data sets against each other

Examples are presented in Figure 38 and Figure 39.

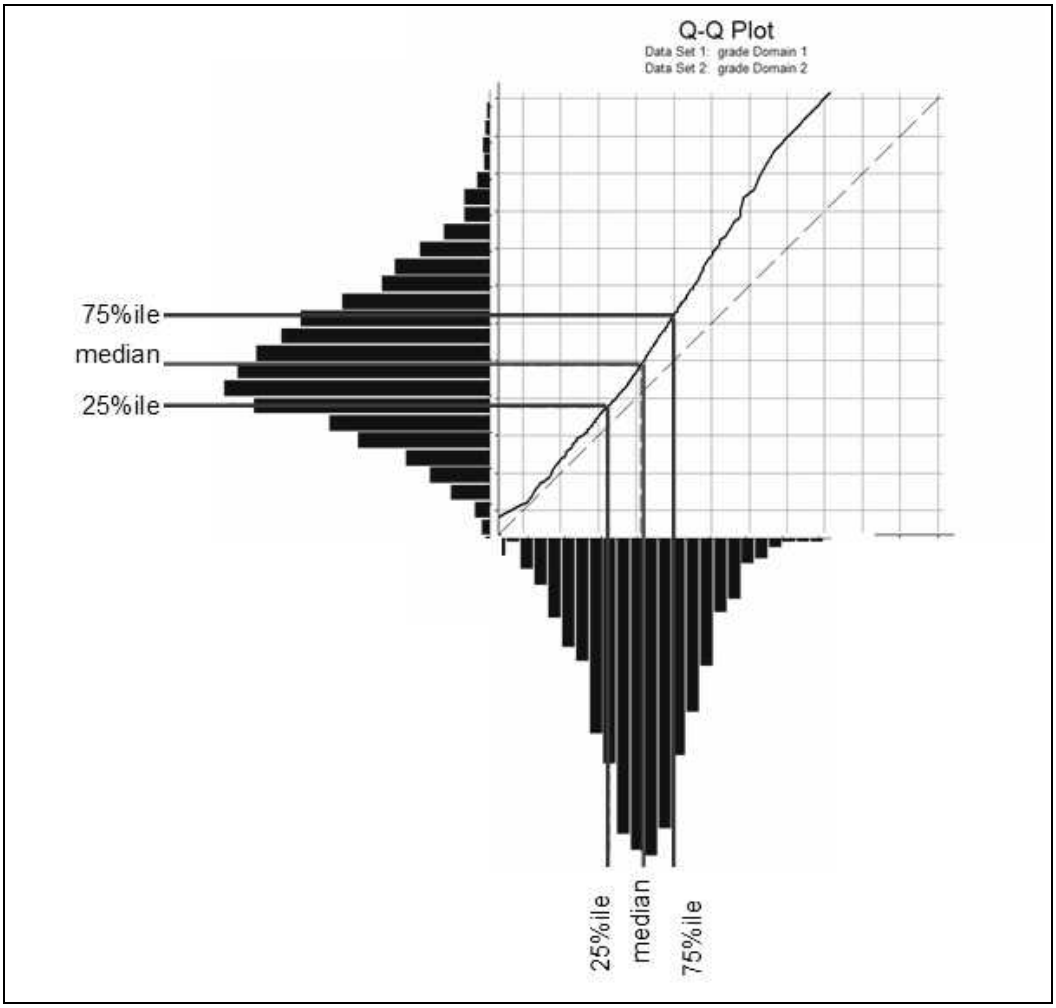


Figure 37 A Q-Q plot provides a comparison of two histograms

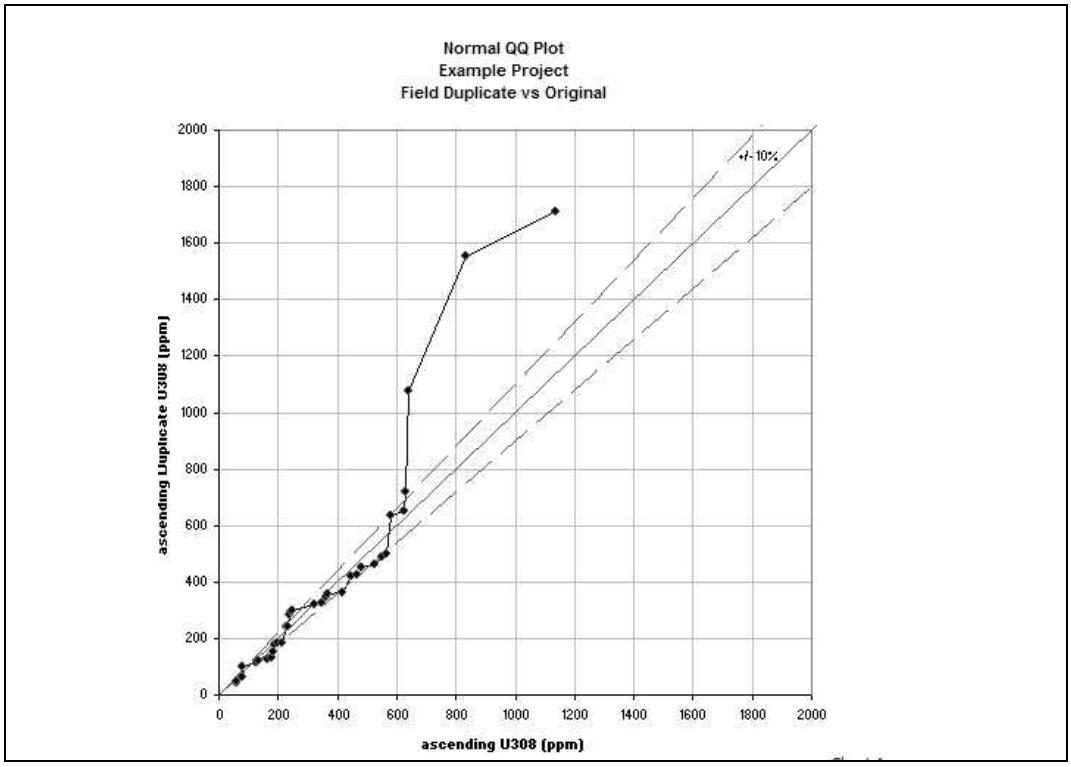


Figure 38 Example of a Q-Q plot (grade scale)

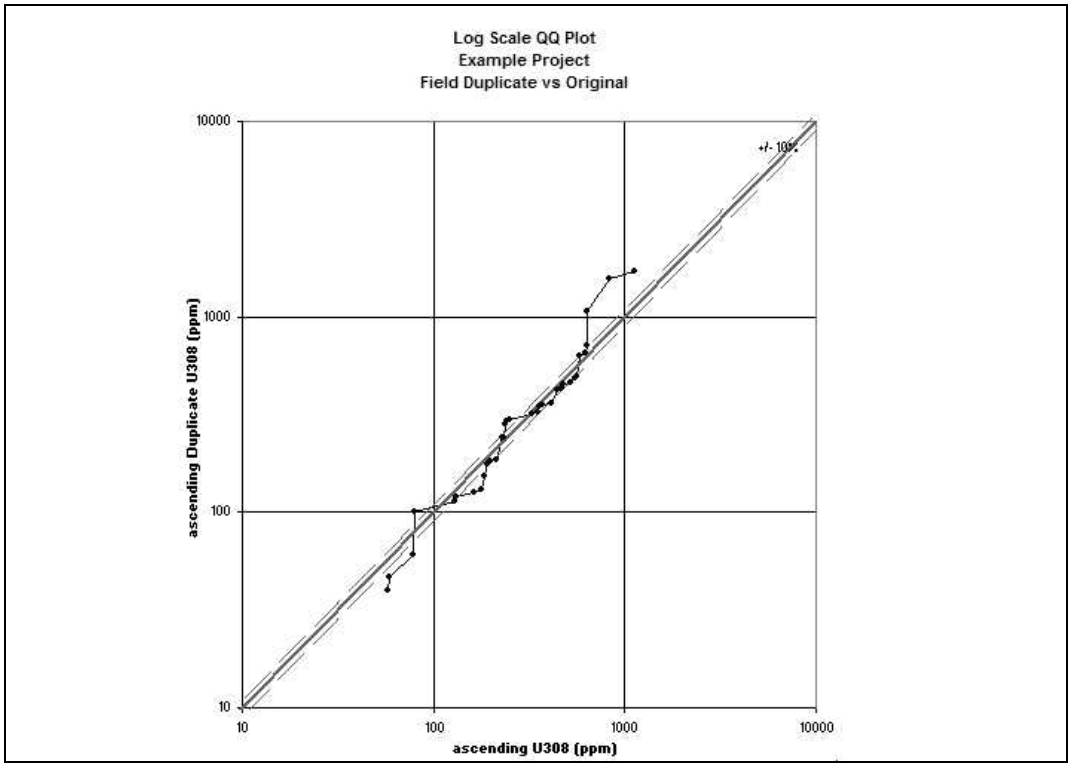
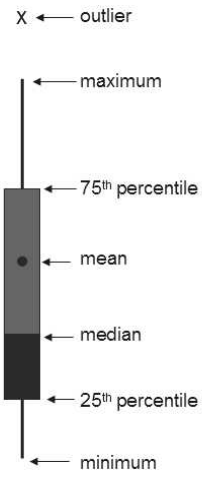


Figure 39 Example of a Q-Q plot (log scale)

Box-and-whisker plot

A box-and-whisker plot provides a quick way to compare the spread of data from several sources.

The box describes the grade of the 25th percentile, the median and the 75th percentile (the quartiles). This is equivalent to the middle half of the data. The whiskers usually describe how far the minimum and maximum extend beyond the quartiles. Some packages plot the whiskers to the 10th and 90th percentiles and the remaining extremes at the actual values.



Box-and-whisker plots are useful for comparing several data sets – those with similar boxes and whiskers are likely to have similar histograms and statistics. A Q-Q plot between the similar data sets should be used to confirm this assumption.

Figure 40 compares the spread from four laboratories. Notice the range of grades from laboratory 3 is lower than the other laboratories. We need to question whether the laboratory results are based on the same source of data, or why results from laboratories 2 and 3 are so much lower than from laboratories 1 and 4.

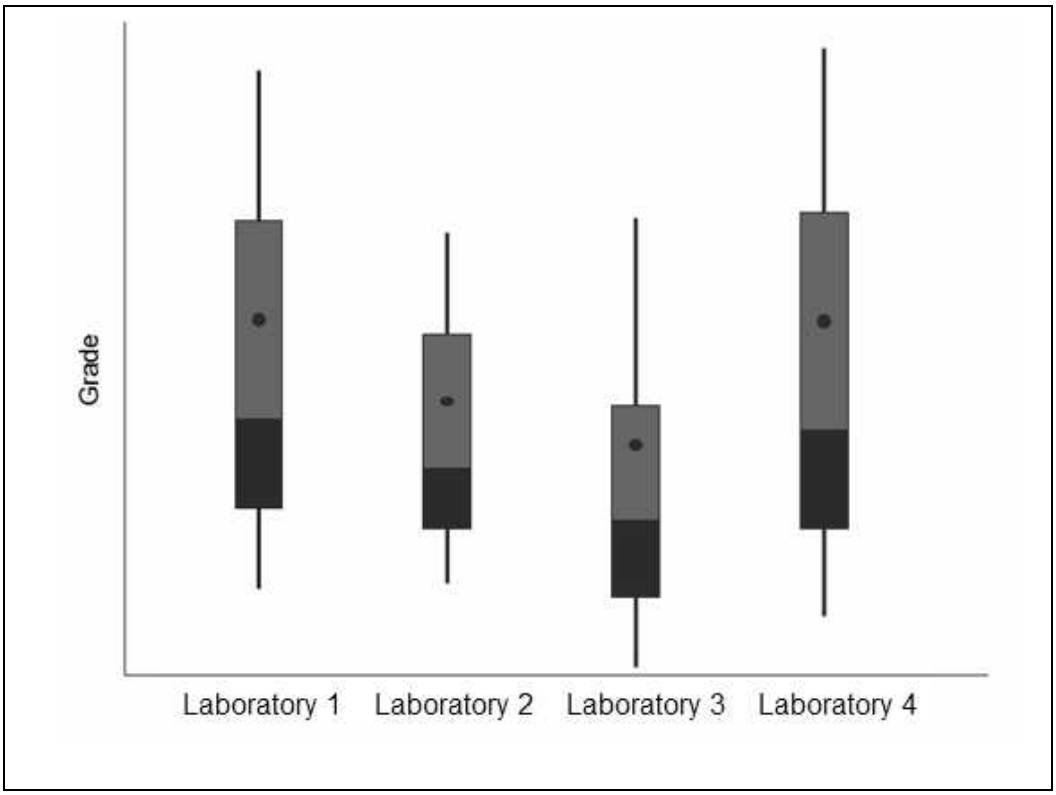


Figure 40 Example of a box-and-whisker plot

Measuring precision

The scatterplots tell us something of the bias as well as something of the repeatability, but how precise are the duplicates or repeat samples? How do we quantify precision?

Relative difference plot

An easy way to look at precision is to plot the difference between the duplicate and the original samples (scaled by the original sample grade) plotted against the original sample grade.

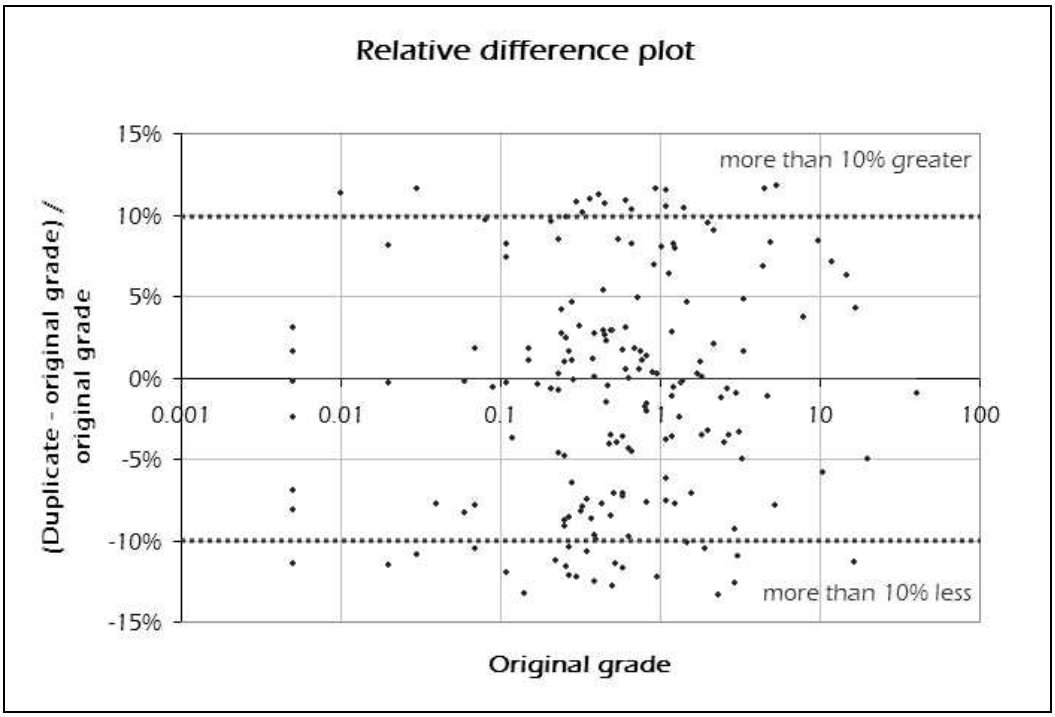


Figure 41 Relative difference plot

Ideally, relative differences should be within 10% of the sample grades. High relative difference could indicate poor sampling practice, poor assaying or a high inherent nugget effect in the mineralisation. If the reason for high relative differences is an inherent nugget effect, this could be an indication that the sample volume is too small to generate representative sample grades.

Precision plot

A precision plot is similar to a relative difference plot. Instead of a relative difference, we calculate the absolute difference, halved, between each original-duplicate pair of samples and plot this against the original sample grade.

Note that the precision lines (5%, 10% and 15% limits) are created as the respective percentages of the original grades.

The precision plot in Figure 42 indicates the duplicate data tends to have a precision of no more than 10% of the original samples. So, duplicate samples are expected to have values within 10% of the original sample values.

Higher precision is expected in a laboratory assaying process than in a field duplicate – the material is expected to be more homogenised by the time it reaches a laboratory.

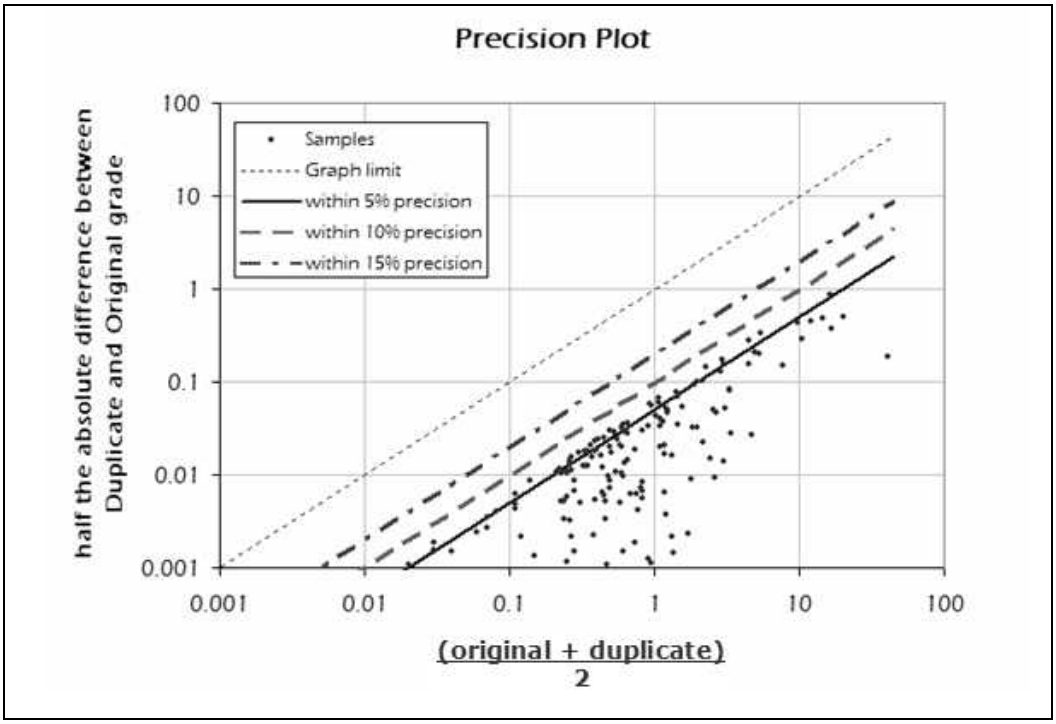


Figure 42 Precision plot

Activity

Assess the quality of the Fe and SiO_2 data using the following.

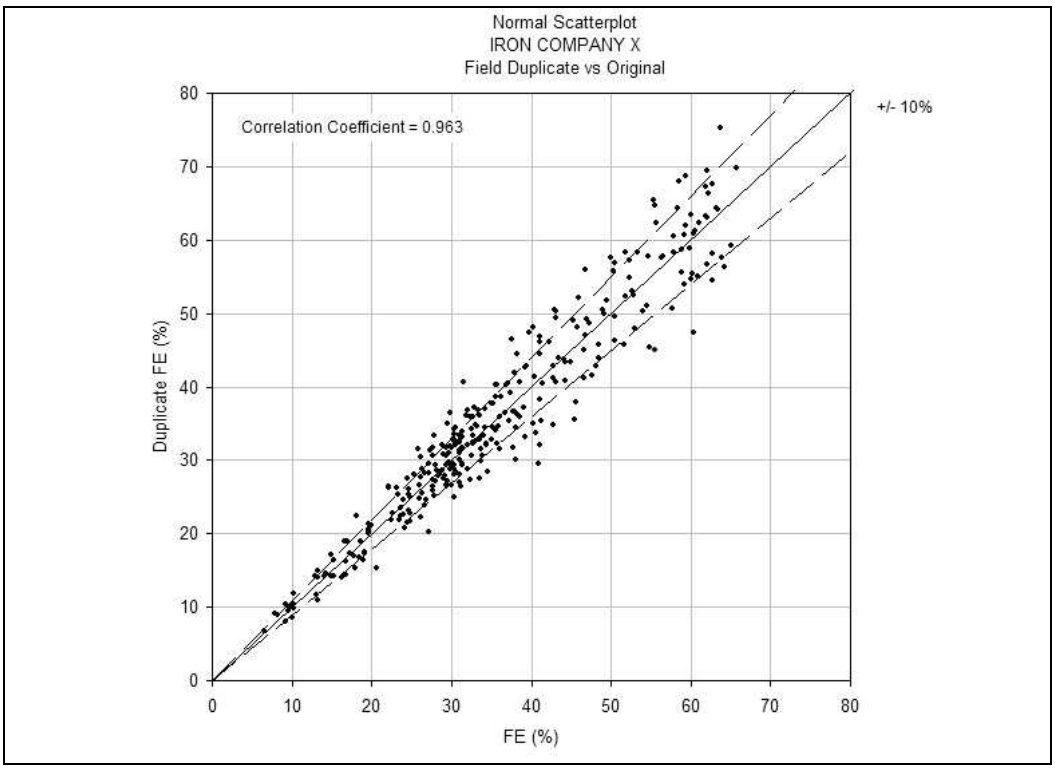


Figure 43 Scatterplot between duplicate and original

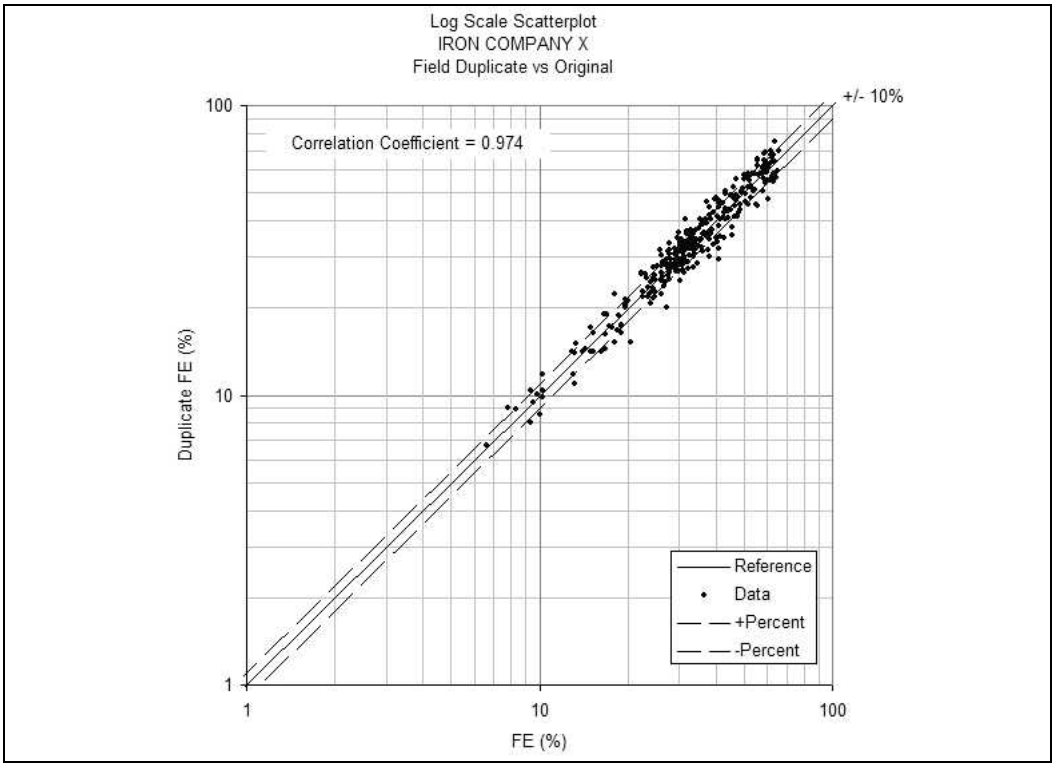


Figure 44 Log-Scale Scatterplot between duplicate and original

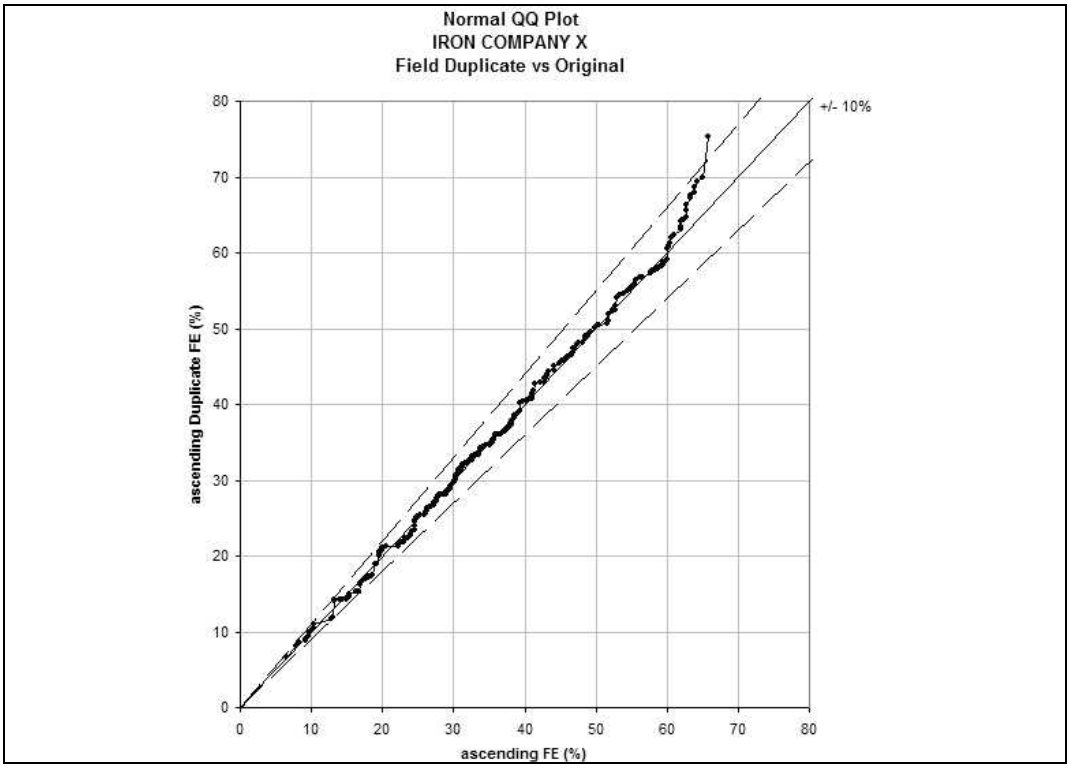


Figure 45 Normal scale Q-Q plot between duplicate and original

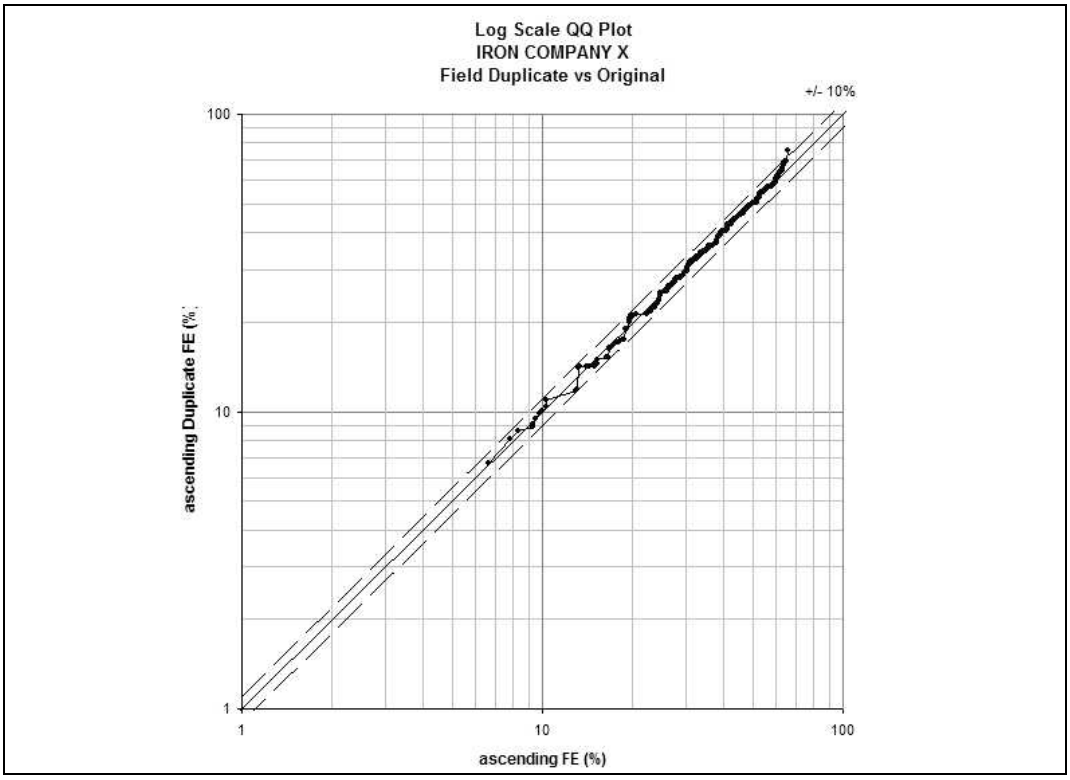


Figure 46 Log-scale Q-Q plot between duplicate and original

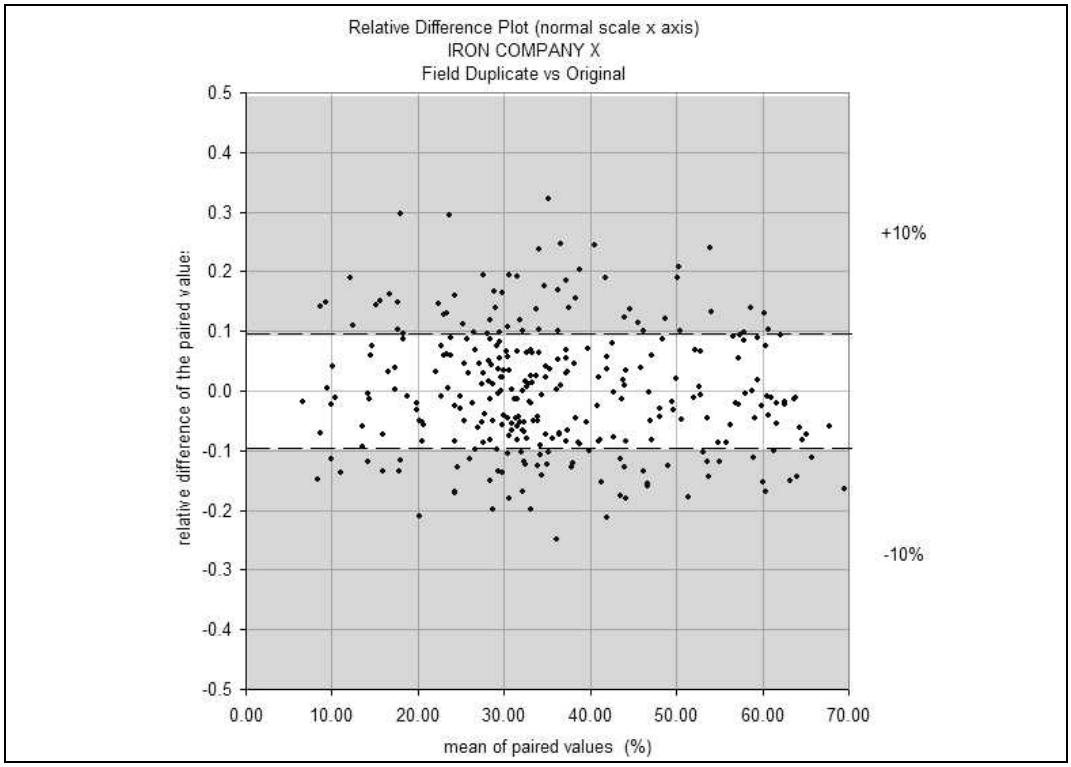


Figure 47 Relative difference plot (relative difference against original grade)

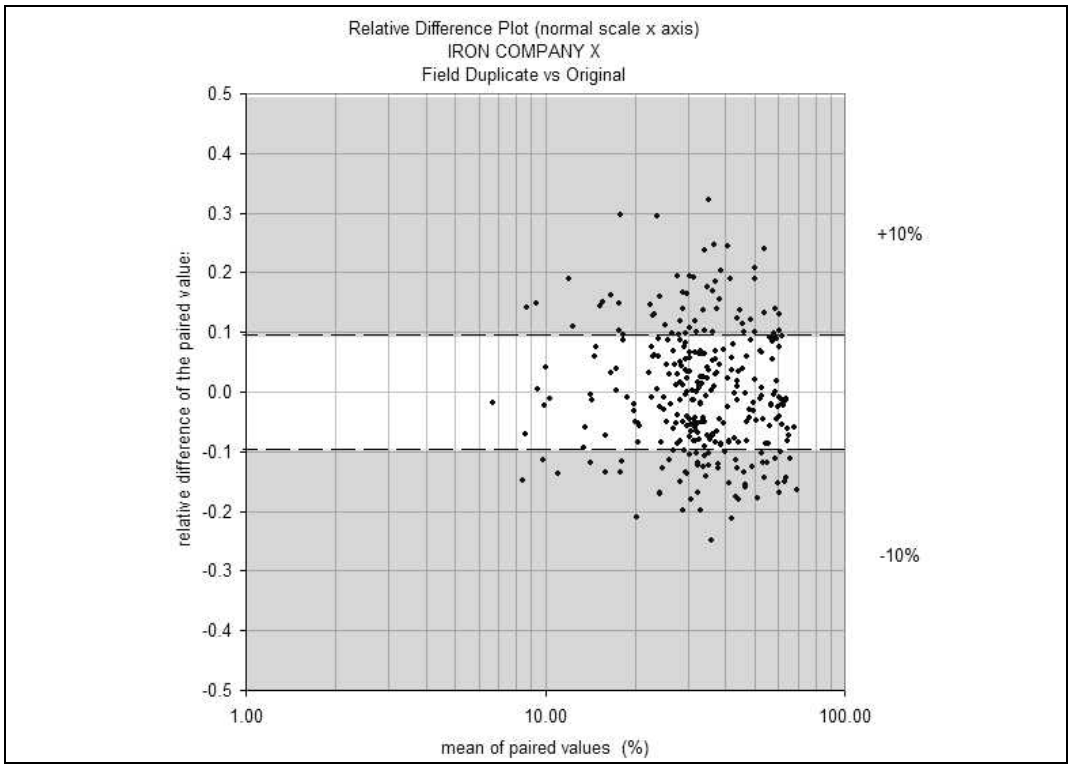


Figure 48 Log-scale relative difference plot (relative difference against original grade)

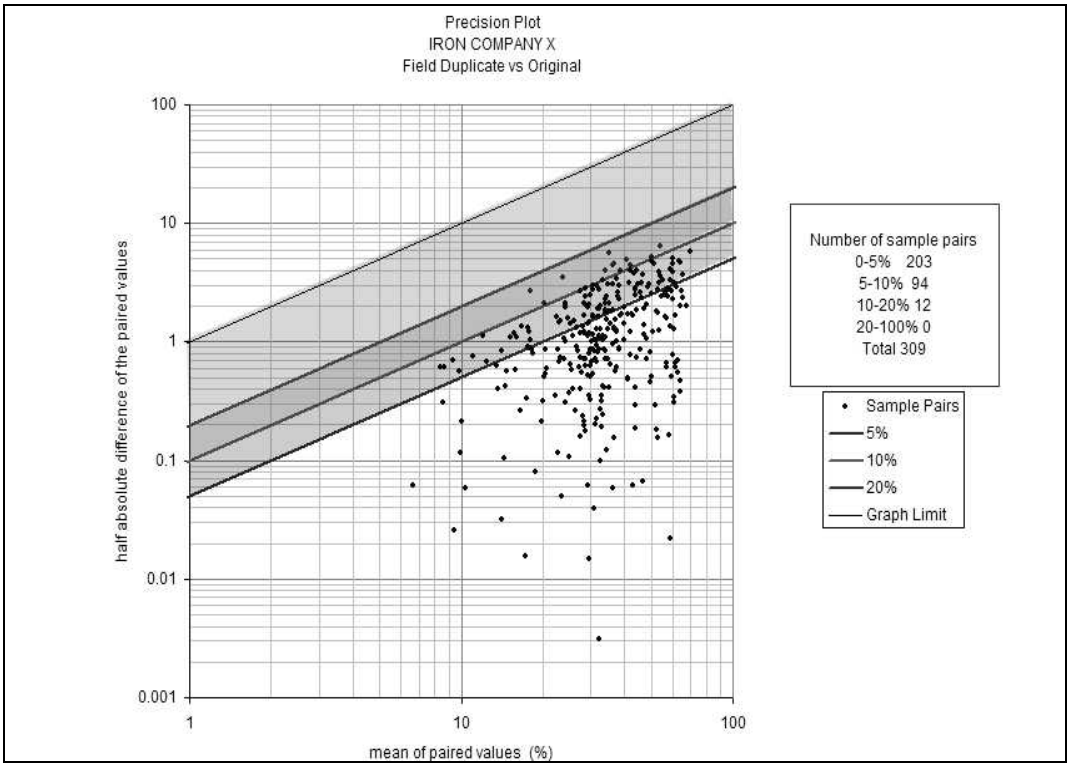


Figure 49 Precision plot (1/2 absolute difference against average of original and duplicate)

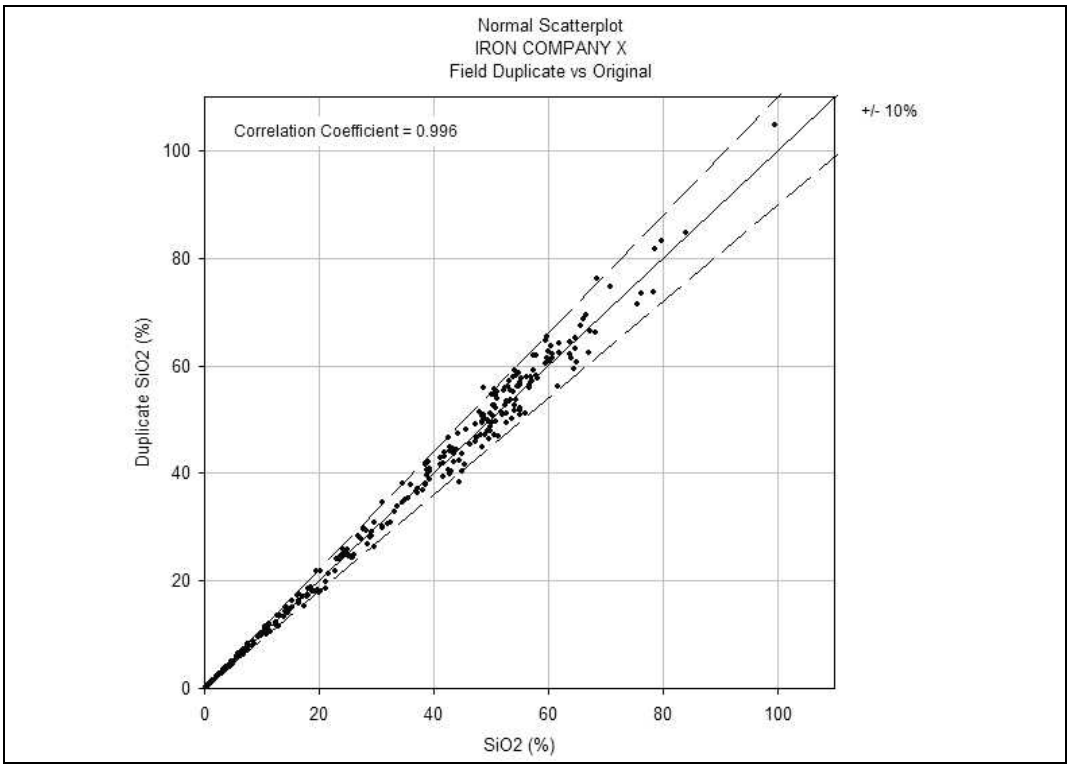


Figure 50 Scatterplot between duplicate and original

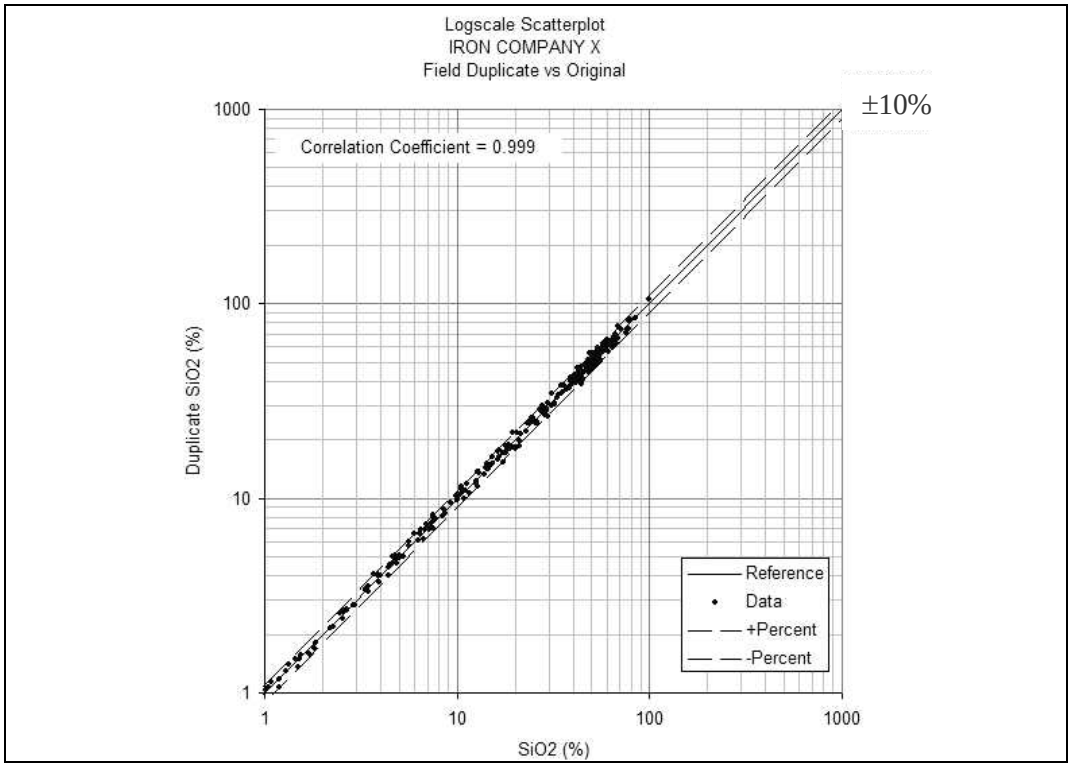


Figure 51 Log-scale scatterplot between duplicate and original

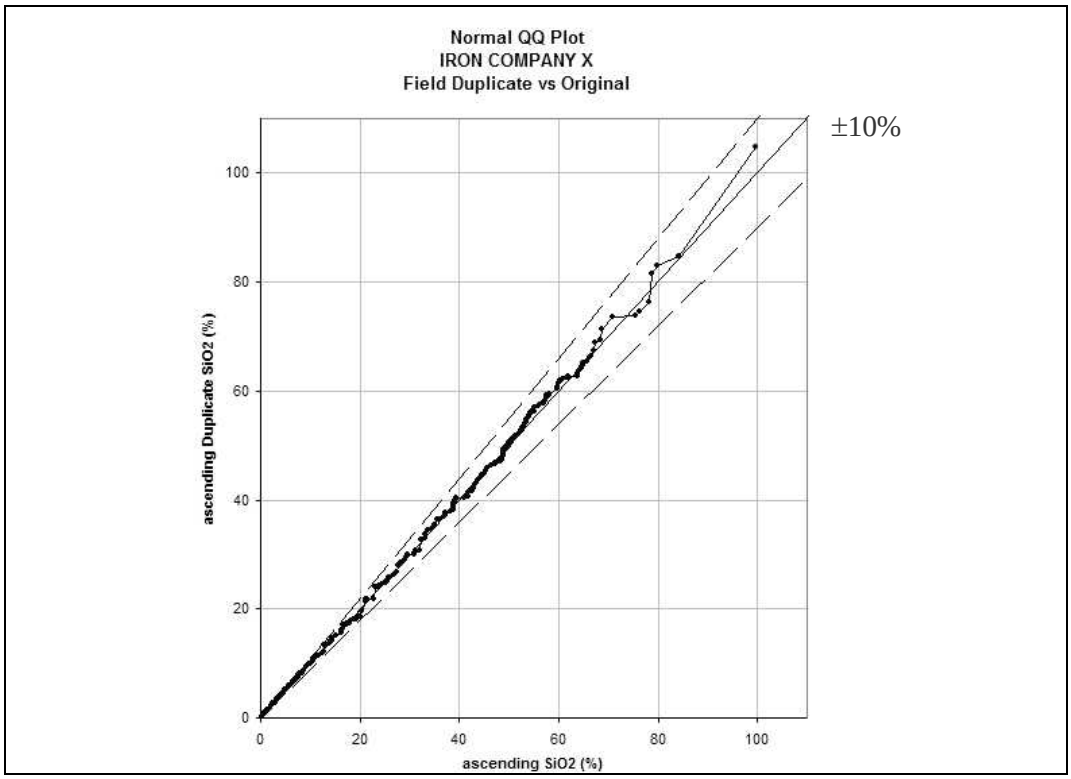


Figure 52 Normal scale Q-Q plot between duplicate and original

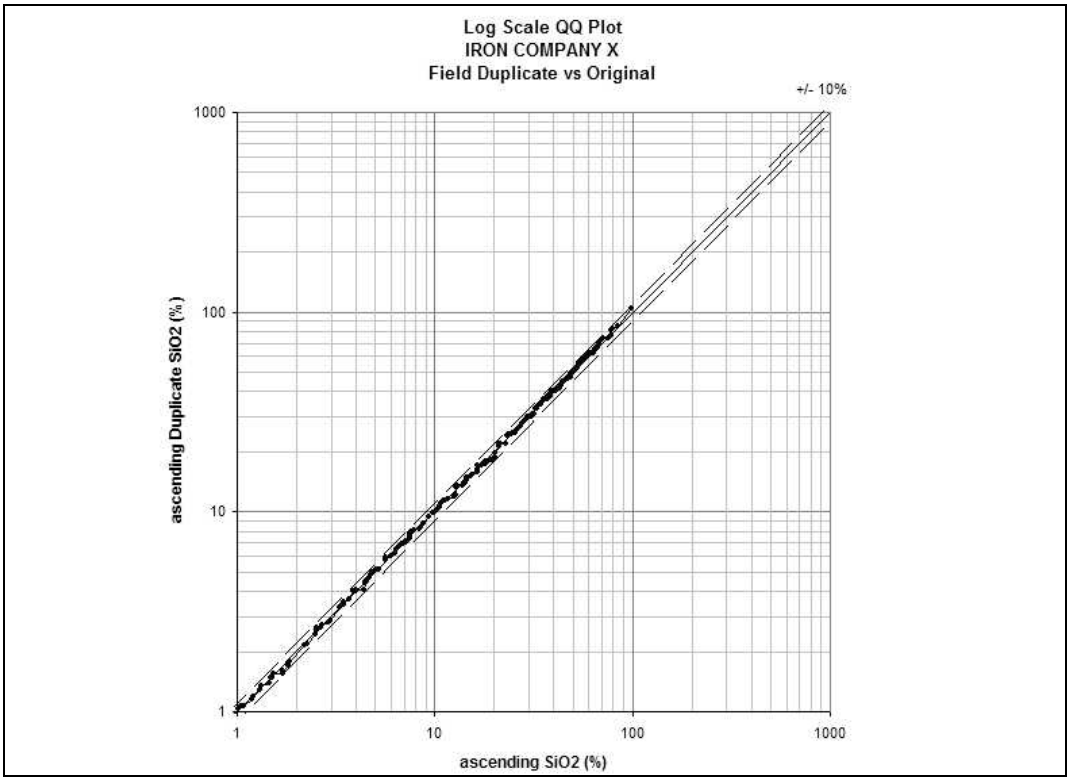


Figure 53 Log-scale Q-Q plot between duplicate and original

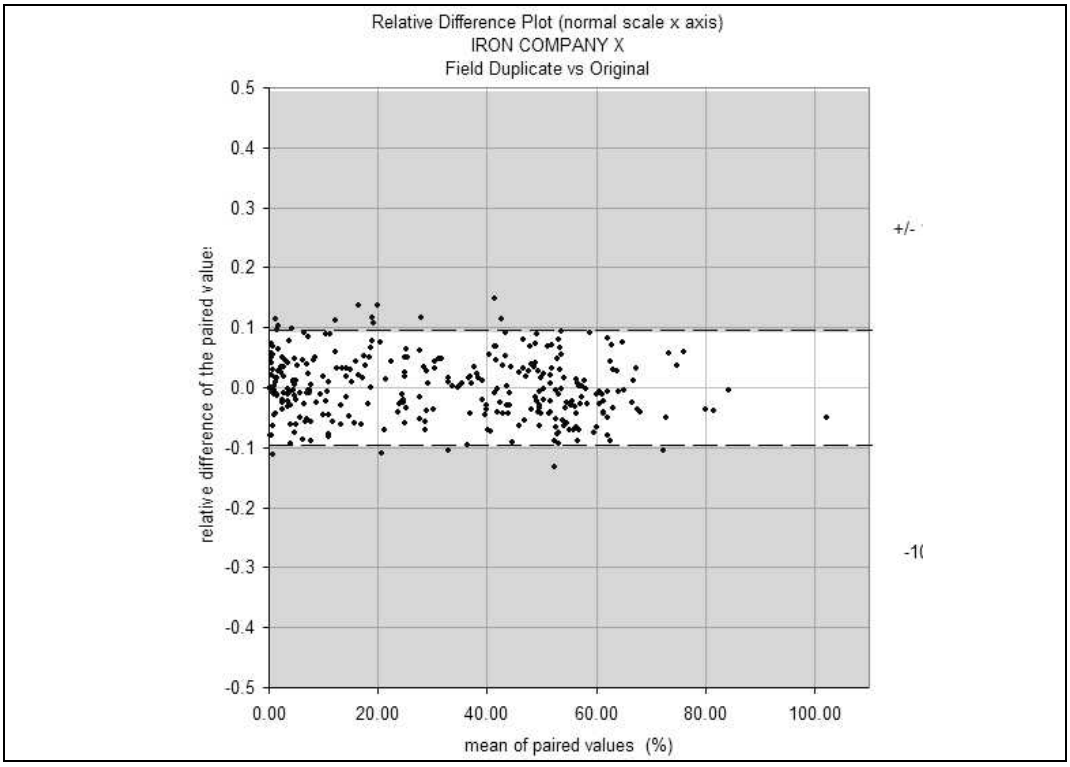


Figure 54 Relative difference plot for SiO_2 (relative difference against original grade)

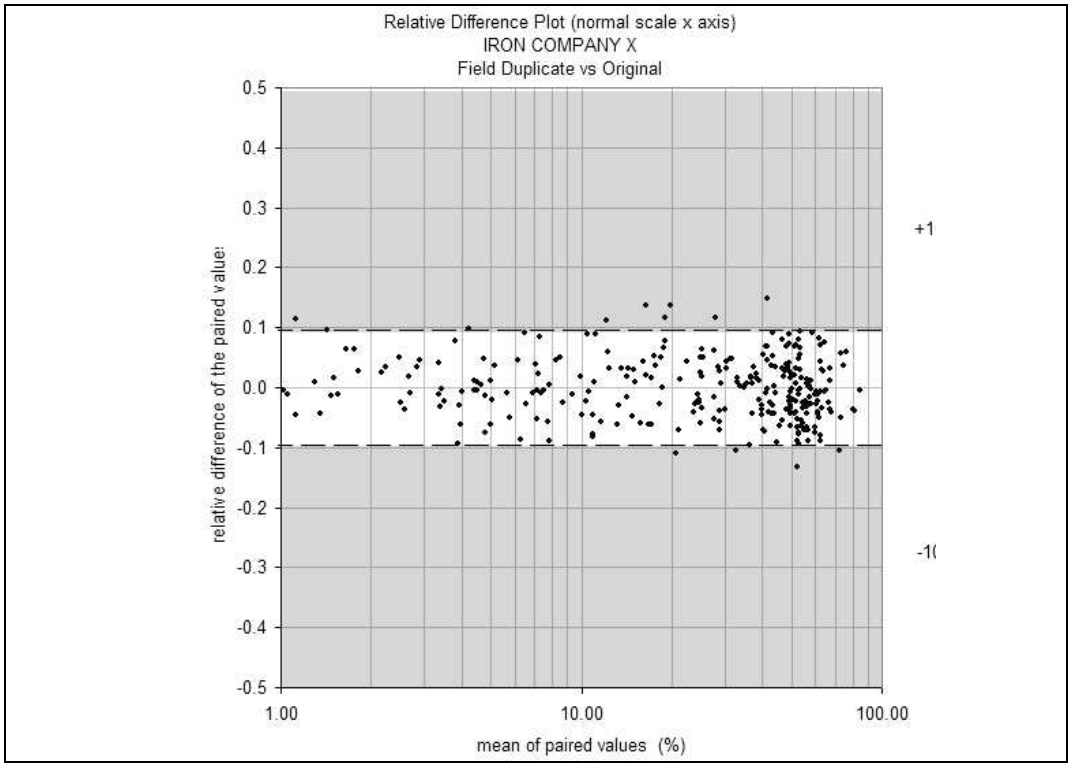


Figure 55 Log-scale relative difference plot for SiO_2 (relative difference against original grade)

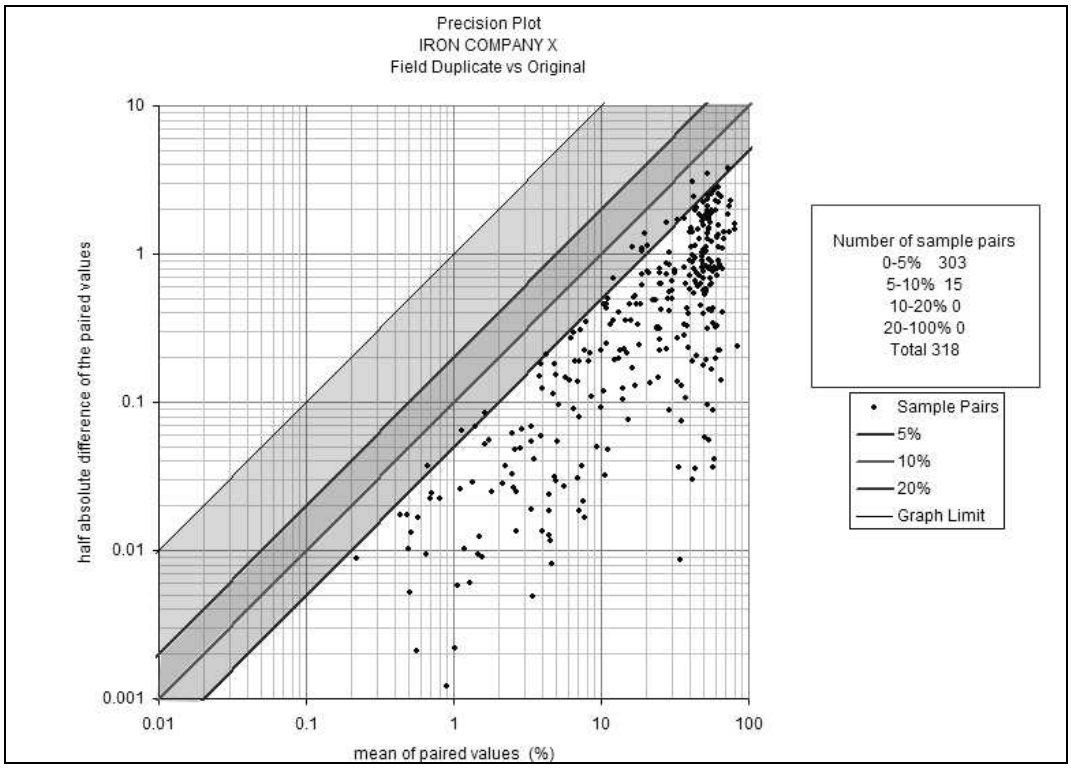
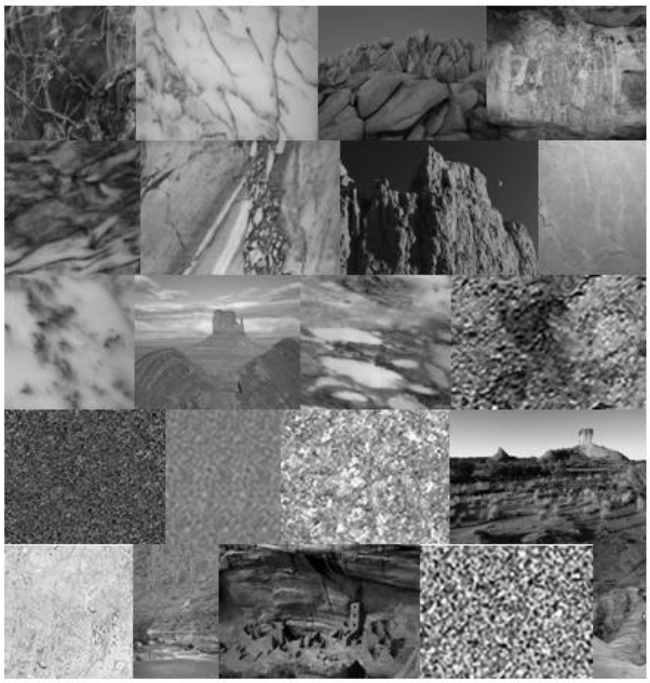


Figure 56 Precision plot for SiO_2 (1/2 absolute difference against average of original and duplicate)

Building domains from geology

Geology is the cornerstone of any accurate resource model. In my opinion, geology contributes 90% of the accuracy of a resource estimate. The time spent on the geology rarely reflects this importance.

However, the better the geological model, the simpler the resource estimation (and the simpler the mathematics required to generate a representative resource model!)



What is so important about geology?

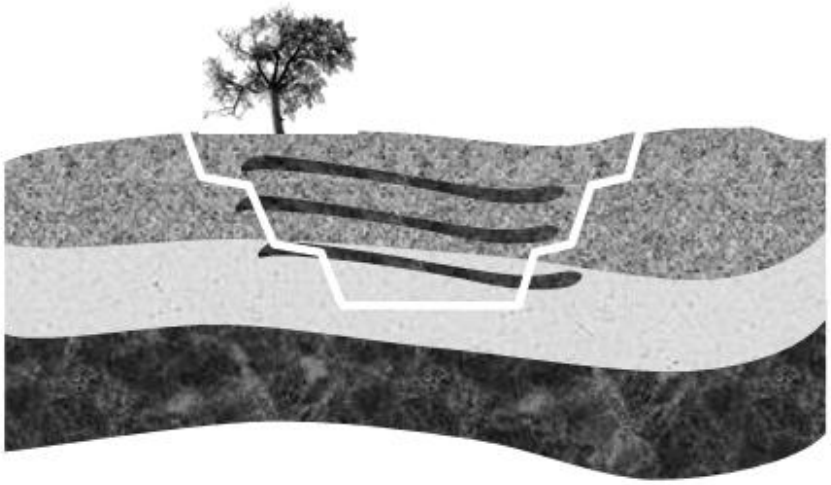
In resource estimation, we create blocks of grade based on the nearest sample data. If the block grade is to be relevant, then the samples it is based on must be relevant. In other words, we need to identify the populations of interest that are relevant to the block we want to estimate. This means we need to understand the boundaries of the populations to constrain the relevant samples. But what makes a sample relevant? And how can we know we have sub-divided the data into groups of relevant samples? Building domains is about:

- using our understanding of the geological controls on mineralisation to create the limits of each mineralisation population
- using statistical tools to validate our interpretation of the mineralisation populations
- understanding and defining domains before creating three dimensional envelopes of the populations

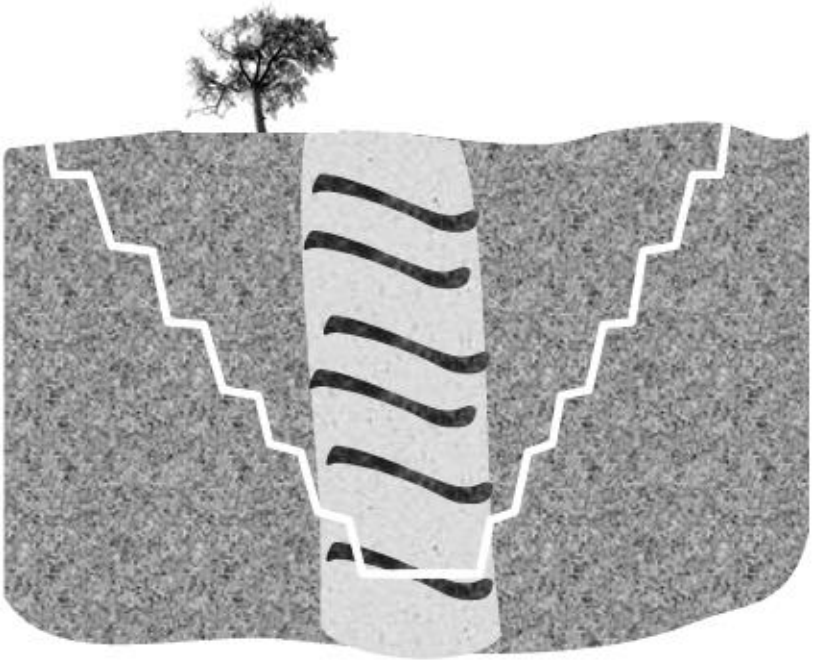
Let us look at some examples of geological interpretations and the effect on the mining.

Example 1

Consider two interpretations of geological controls on mineralisation that are based on identical data: lithological control or structural control.



(a) lithological model



(b) structural model

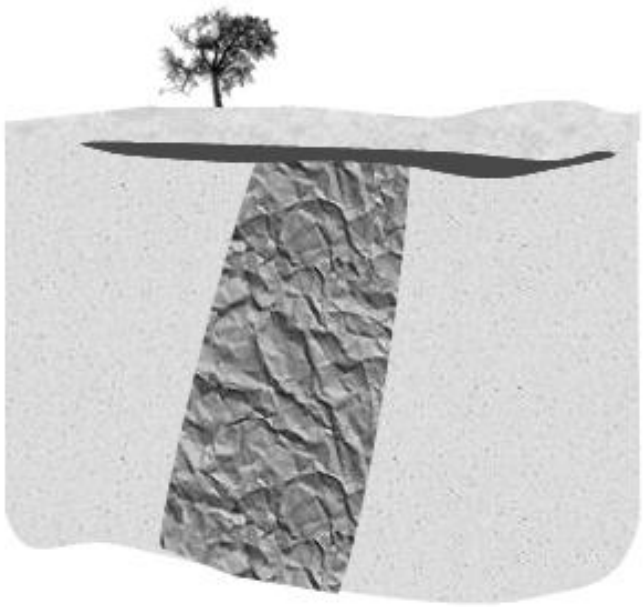
Figure 57 Lithological versus structural interpretation

Either interpretation of the geology is plausible given the available data. However, the lithologically interpreted model results in a pit optimisation that is shallow with a low strip ratio. The optimum pit on the structural model extends deeper and has a higher strip ratio. The structural model also results in a significantly higher Net Present Value (NPV).

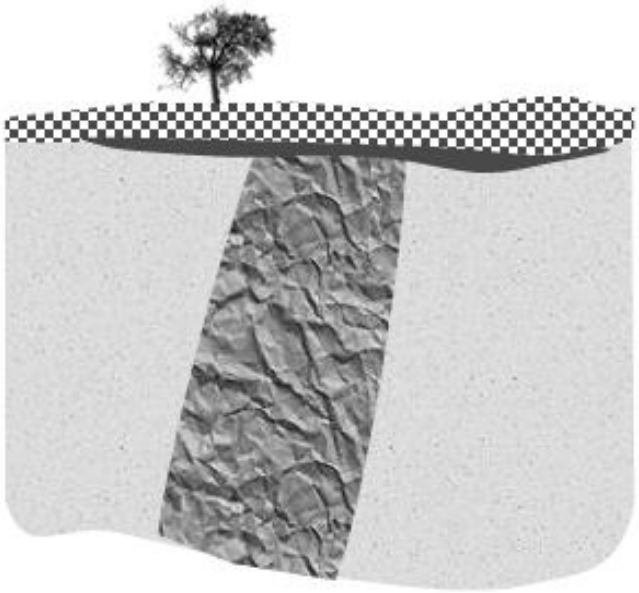
So, the geological interpretation and its role in constraining the mineralisation affects the economic expectation of a project.

Example 2

Consider the situation of a high-grade supergene zone. If this zone is not recognised nor interpreted as a separate high-grade zone, then the high grades will be smeared either into the oxide or into the fresh material, thereby over-estimating both grade and tonnes in these zones.



(a) Constrained estimation of high grades within supergene



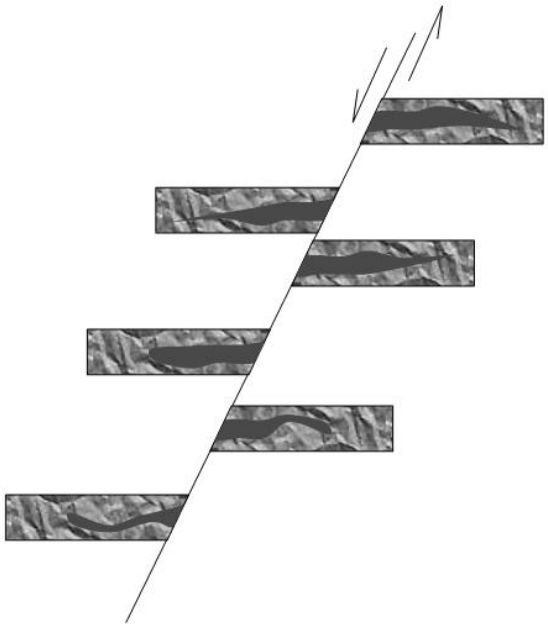
(b) Unconstrained high grade smeared into oxide profile

Figure 58 Constraining supergene enrichment

Example 3

Another situation to consider is whether mineralisation pre- or post-dates the structural events. In the first instance, mineralisation is interpreted to occur only within the lithological layers, whilst the interpretation of mineralisation post-dating the faulting allows additional ribbons of mineralisation to be interpreted.

(a) Mineralisation pre-dates faulting



(b) Mineralisation post-dates faulting

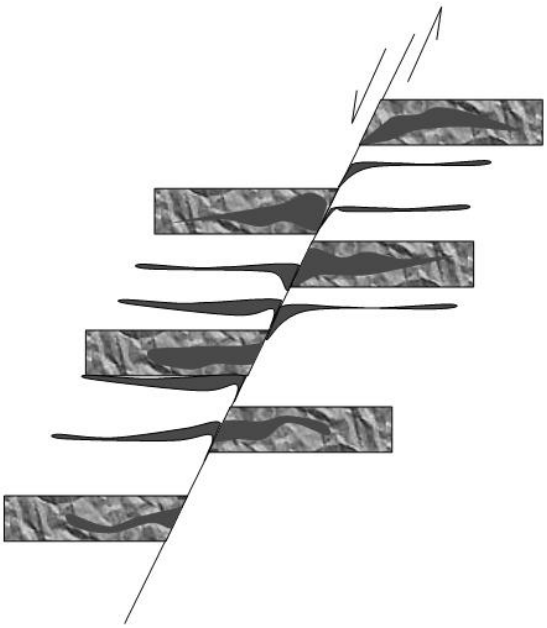


Figure 59 Mineralisation and structural events

Geological models

Understanding mineralisation populations requires us to understand genesis, lithology, deposition, structural controls, weathering, mineralogy and any other factors that may influence our understanding, definition and delineation of the mineralisation volumes.

Take a moment to describe the regional controls on your deposit.

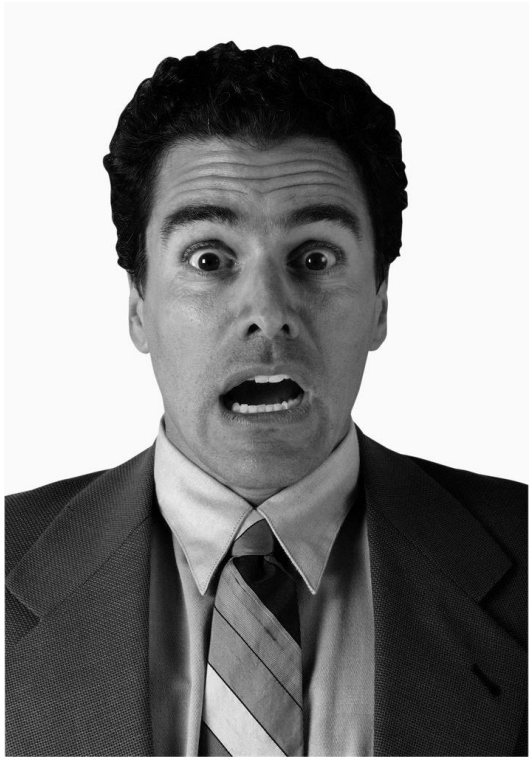
Regional controls …

Now focus in a bit more locally. Describe the significant geological structures, lithologies, impact of weathering or any other relevant features.

Local controls …

Using your geological understanding, you should be able to build a three-dimensional model containing significant geological structures and controls on mineralisation. The big secret here is to be open to alternatives and to distance yourself emotionally from your interpretation.

So take time to explain your model, ask for other people's insights and suggestions that may improve your geological interpretation. Update your model to reflect a wide range of experience.



The purpose of a geological interpretation is to synthesize all information and experience to

minimise the element of surprise

when it comes to mining an orebody

Mineralisation models

The geological model forms a context within which to build a mineralisation model. Take a moment to describe the mineralising events of the deposit and how these relate to the geological interpretation(s).

Mineralising event(s) and relation to geological context …

Next, we use our understanding of the geological controls and the mineralising events to interpret mineralisation envelopes. Here we are looking for the relationship between geology and economic mineralisation, and aim to describe the spatial envelope of the *in situ* mineralisation population.

Ideally, each sample in your database is logged for the factors affecting mineralisation. We can use this information to build a statistical image of the mineralisation populations and define domains.

Building domains

What is a domain? For resource estimation, the domain is a spatial volume where:

1. the geology is consistent (homogeneous)
2. contains a single grade population
3. has a single orientation for search

One expects different geological units to mineralise differently. It naturally follows that the homogeneity of the geology lends support to the mineralisation envelope – when we can understand the origin of the mineralisation, we will be more confident in how far to extrapolate the mineralisation continuity (Figure 60).

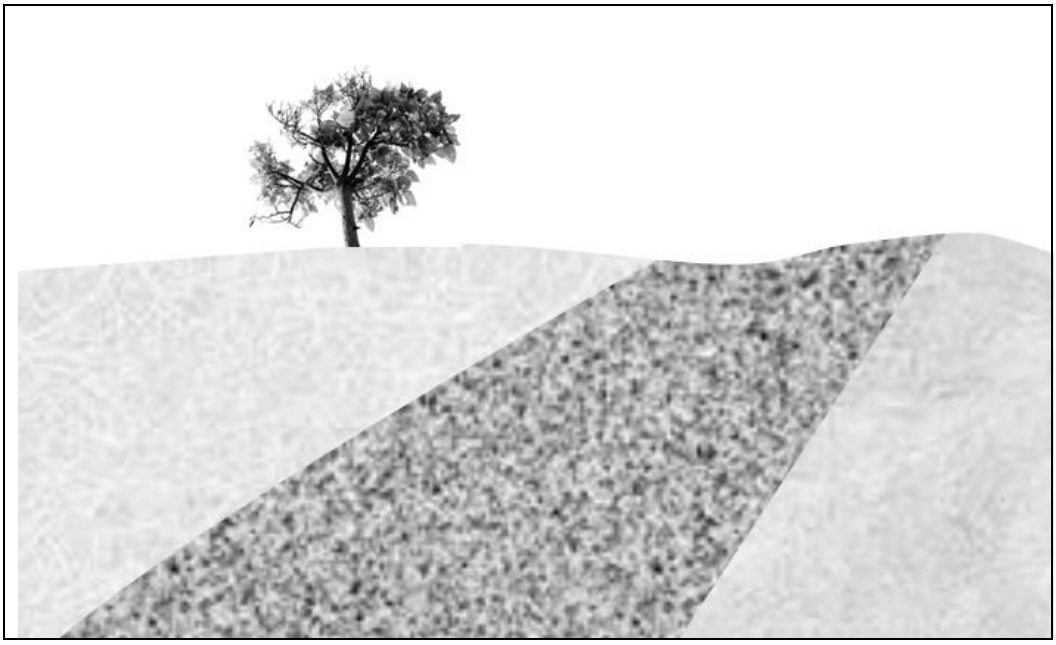


Figure 60 Domains contain consistent (homogeneous) geology

The second requirement of a domain is that it contains a single grade population. When we fail to meet this criterion, our estimates reflect neither the lower nor the higher-grade population. The estimates we produce tend to be over-smoothed between the two underlying populations (Figure 61). When the histogram shows evidence of two populations, we need to return to our data and attempt to separate the populations spatially (Figure 62and Figure 63).

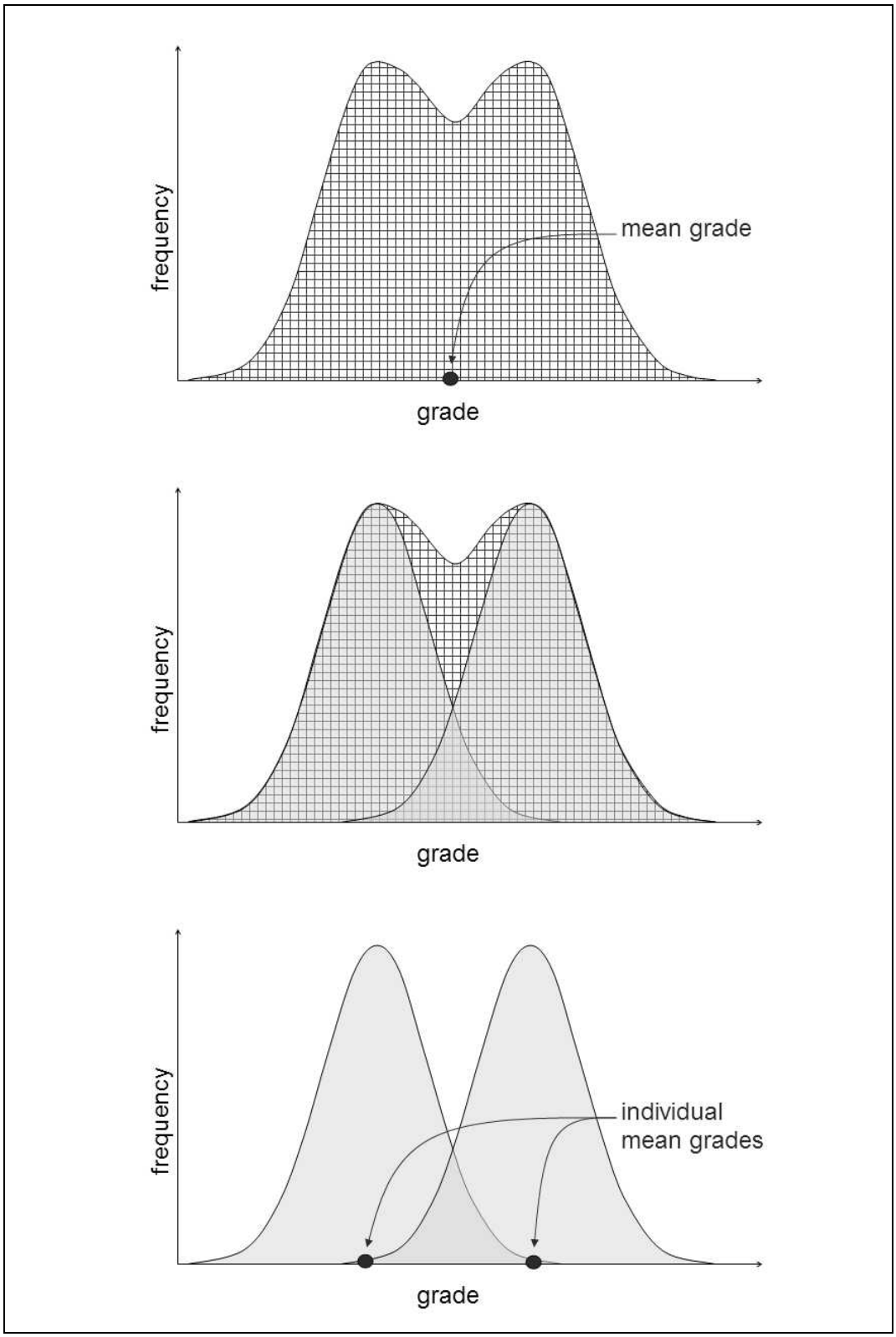


Figure 61 Mixed populations

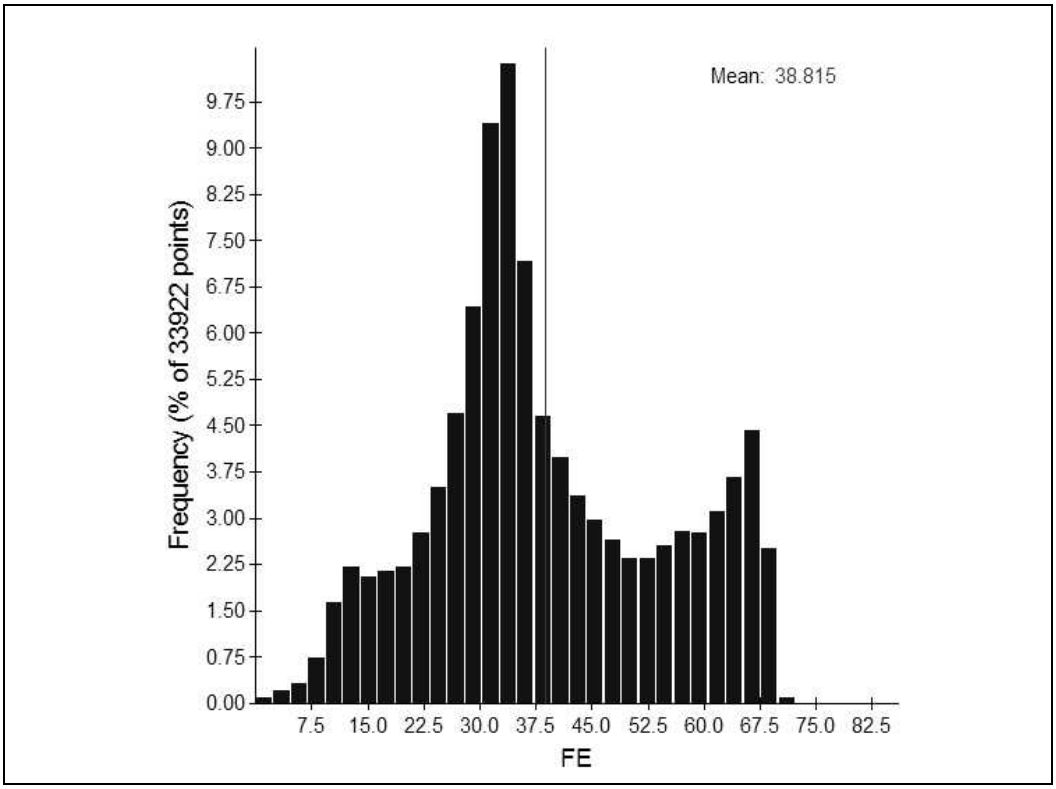


Figure 62 Histogram of data with mixed populations

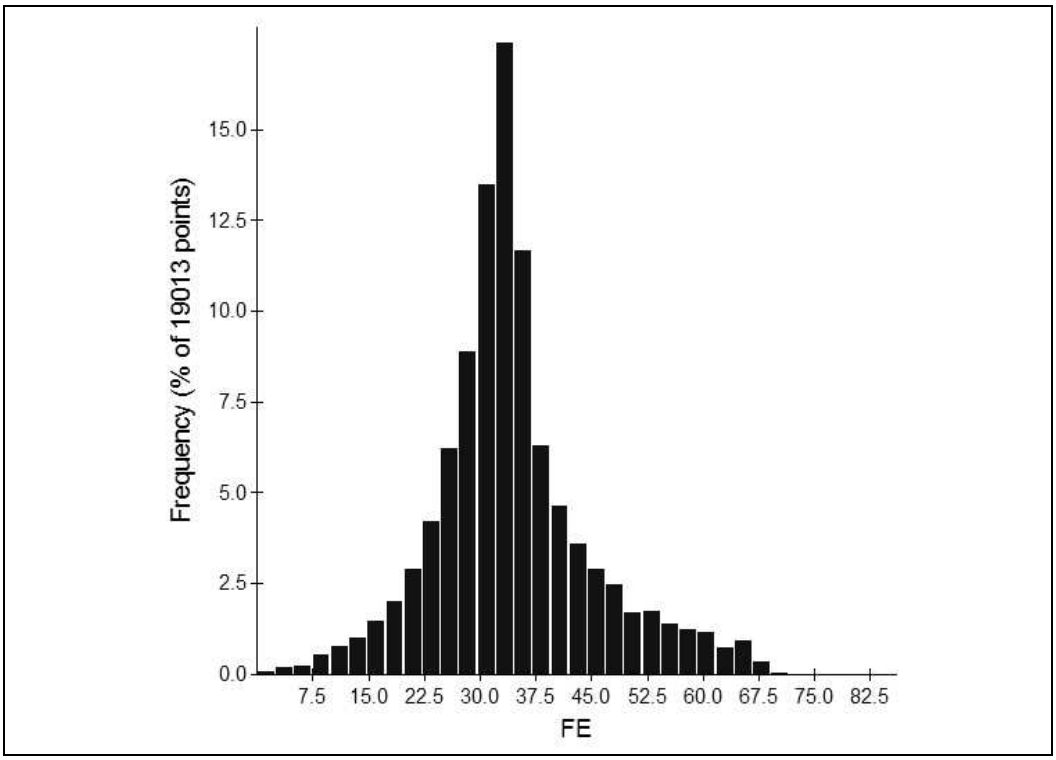


Figure 63 Domain with a single grade population

The third requirement of a single orientation of continuity ensures the search ellipsoid and variograms are orientated in the best alignment for the grade continuity during estimation. Mineralisation volumes that have undergone folding post mineralisation should be unfolded prior to data analysis. This way the appropriate samples can be aligned for analysis and estimation (Figure 64).

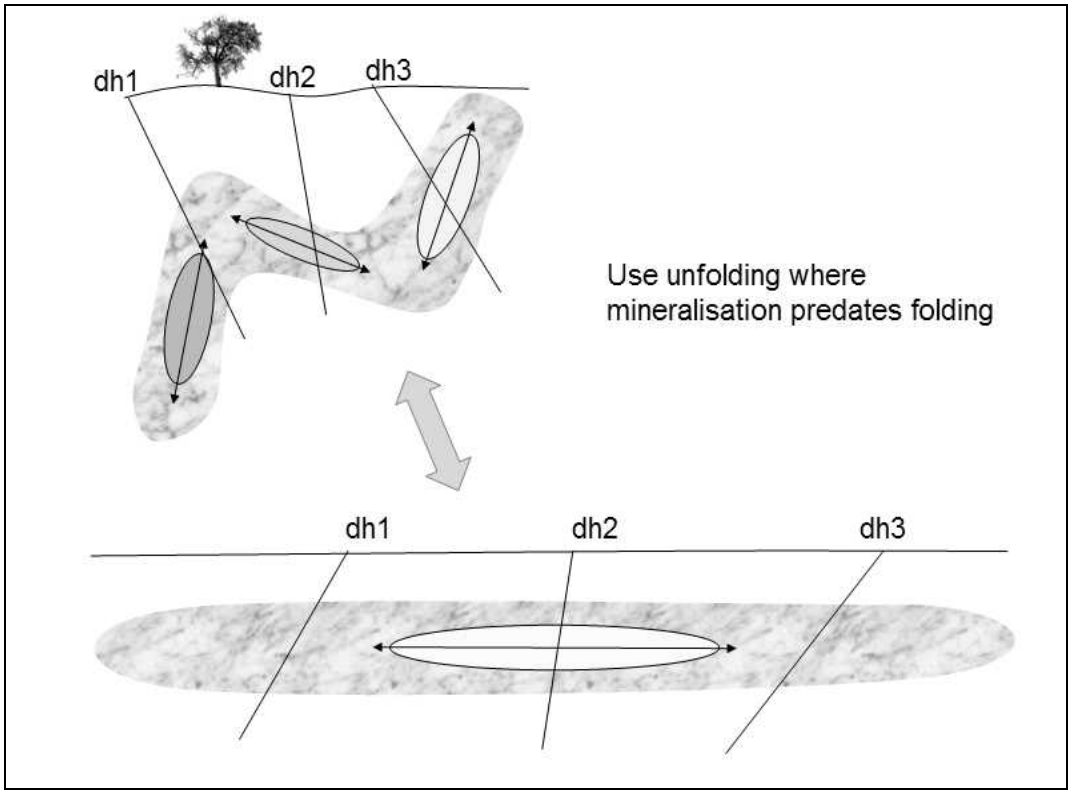


Figure 64 Domains are restricted to a single orientation of continuity
and can sometimes be unfolded to create a single orientation of continuity

The purpose of building and defining domains is to control the use of sample data during estimation – we only want to use what is relevant to the volume we are estimating. Domains highlight changes in geological texture, statistical population and/or orientation of continuity.

Consider the geological context below. Can you identify the domains? Consider structural, lithological and weathering controls, as well as changes in orientation and anisotropy.

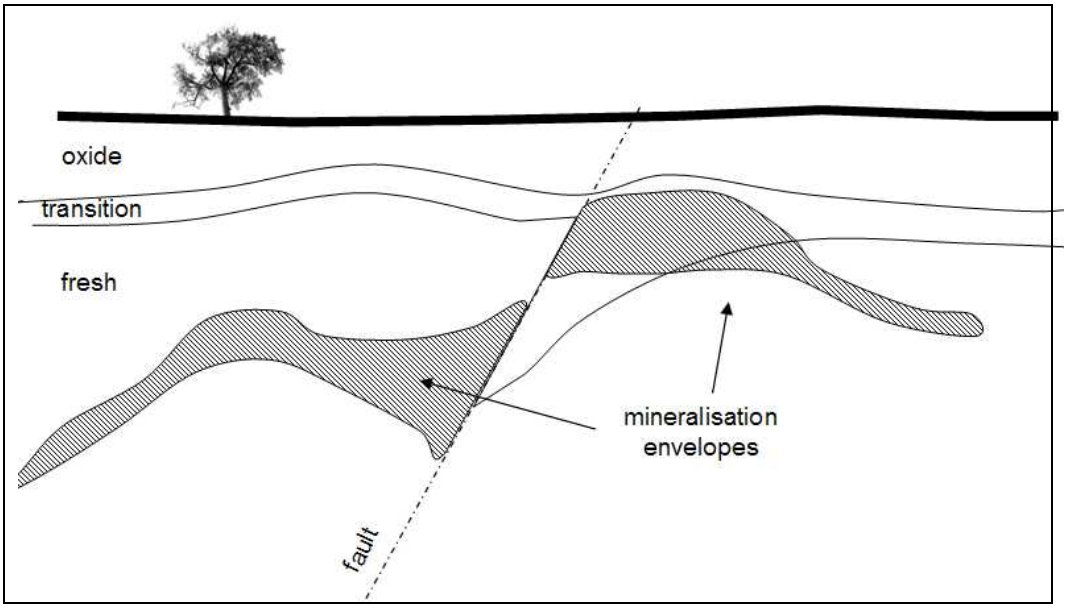


Figure 65 Weathering profiles, structure and minerlisation example

Processes for investigating domain control

Breakdown and concatenate approach

One process for exploring controls is to breakdown the mineralisation to the lowest common collection of data and then to build these up by comparing the statistics of the various groups. Let us look at an example of this:

Consider a data set where each assay interval in the database is coded according to lithology, structural logging and weathering.

1. The first step in the process is to create a new code constructed as a concatenation of the three codes (LITH:STRUC:WEATH)[4].

2. Secondly, we plot histograms of the data subdivided by concatenated codes[5].

3. Next, we group typically lower, medium and higher grade subdomains. This grouping requires us to compare the mean, mode and medians of the subdomains.

4. Once we have a broad categorisation of the data, we then overlay the histograms on a light-table to establish how similar each sub-domain is. New groups are created to reflect similar typical grade and similar grade distributions (as reflected by the peak and possibly the spread of in the histograms).

5. The groups should then each be given a group code. This may require manual entry[6].

6. Then visualise the groupings in 3D, colour coded by group. Identify the controlling mechanisms on the grade populations. Sometimes these are easily reflected by a single aspect of the geological logging. Most times, however, there is an interplay between geological controls.

7. Once you have confirmed the controls, apply the combination to define the domains to reflect single grade populations and single orientation controls.

[4] Deciphering codes later is made easier if codes within each field are the same length

[5] Plot the histogram about twice the size of a business card to facilitate quick comparisons

[6] The Filter option in Microsoft's Excel program is useful for this.

Cluster Analysis approach

Another approach is to apply the statistical technique of cluster analysis[7]. This approach works well for multiple elements.

Let us look at a zinc project as an example. This particular dataset has zinc, lead, silver and sulphide. The geological controls for this project are difficult to discern and limits are required to identify the single core high-grade population.

The total zinc population shows mixed populations, with a higher-grade population evident above about 1% zinc (Figure 66). Note the low-grade spikes reflect the detection limits for various ages of data. The true population is more like the dotted line.

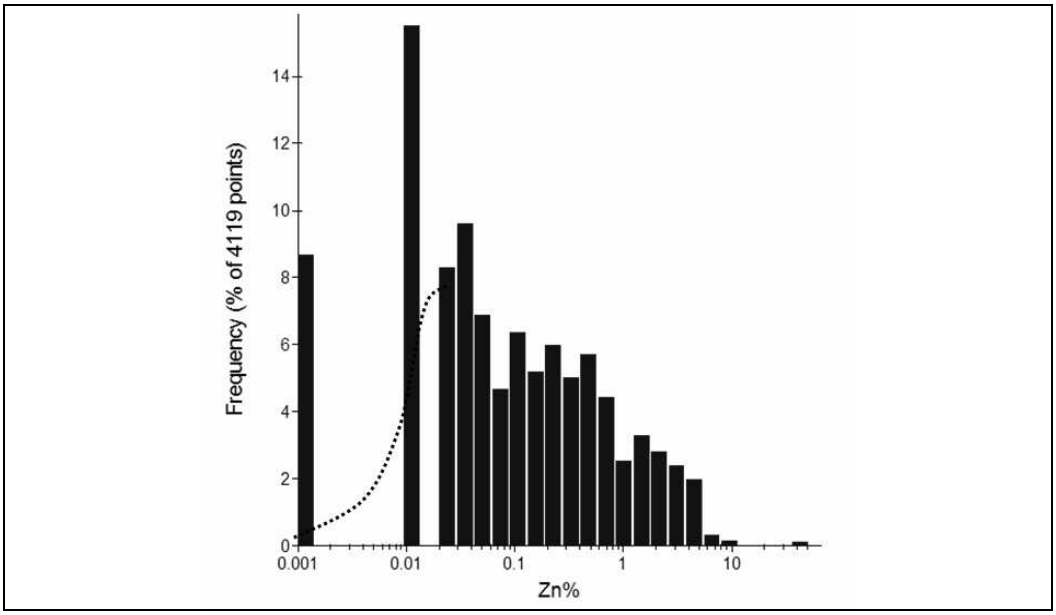


Figure 66 Example of mixed zinc population

Cluster Analysis groups data according to a measure of similarity. The process goes something like this:

1. Firstly, you nominate a number of groups (in this zinc example we nominate three: one for the high grade, one for a possible medium grade and a third for the waste).
2. Next, the first samples (three in our example) are taken to represent the groups (you have to start somewhere!) and the difference between each and every sample grade and these samples is calculated. This difference will incorporate the difference between all the other nominated elements as well (in this case lead, silver and sulphide).
3. Samples are separated according to how close the grades across all elements are to these three initial groups.
4. The group averages are updated.
5. The difference between each and every sample and the three group averages is recalculated and the grouping updated to reflect which samples are closest to the group averages.

[7] I use the Cluster Analysis option in the Minitab software program.

6. This is repeated until the differences and the averages settle down.
7. The final groups then contain those samples that are most simular to the sample average.

The cluster analysis approach for the zinc data produced three populations:

- a typically higher grade population (Figure 67)
- a lower grade or waste population, and
- a data set with too few points to define a domain.

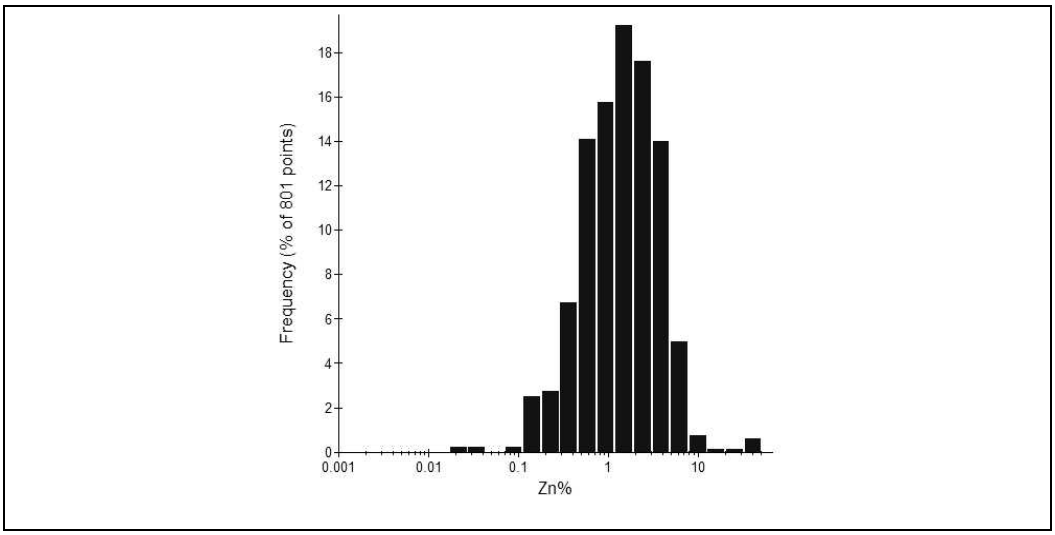


Figure 67 High-grade Zinc population identified using Cluster Analysis

The three groups also tend to be spatially separate (Figure 68).

The next step is to investigate the groupings relative to the geological logs, any available core and the grade trends. This will help define the domain controls and form a basis to develop the domain boundaries by creating a digital solid (also called wireframing).

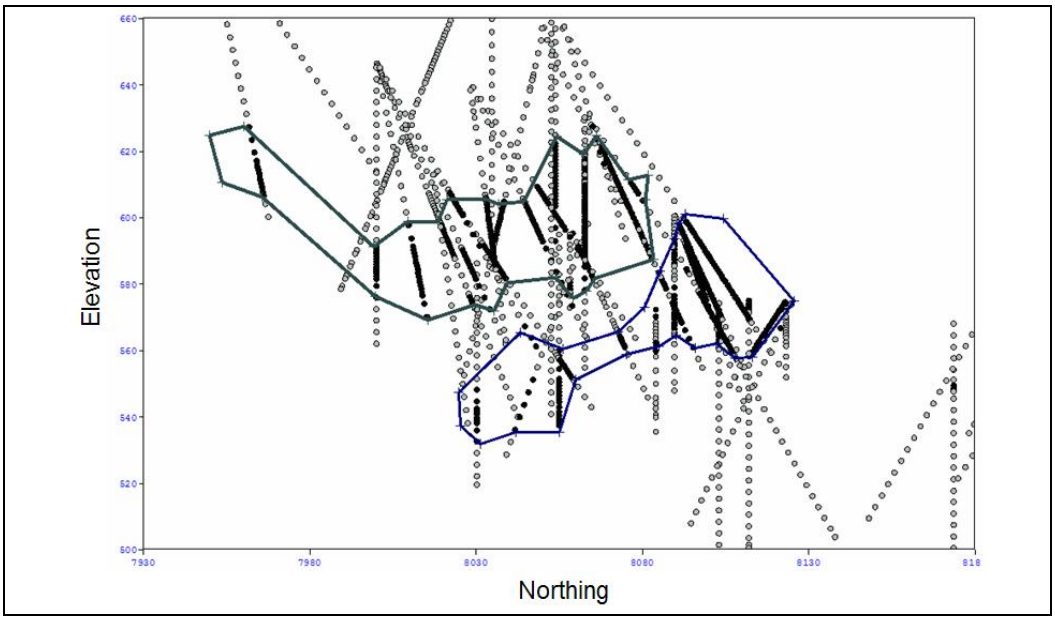


Figure 68 Sectional projection of domain codes with outlines of groups from Cluster Analysis

Validating mineralisation domains

Validation of domains is best done before detailed wireframing (to save valuable time if the interpretation is not valid).

To check whether we have domained our mineralisation well, we must check against the three criteria:

1. geological homogeneity
2. single grade population
3. single orientation.

Geological homogeneity

We test for geological homogeneity by checking whether there is geological evidence for the population and its spatial context. This involves checking the logs and any information that exists to build a reasonable explanation for the population.

If there is no evidence to support the definition of the domain, it is likely that the domain will be called into question at some stage of the project.

Figure 69 Domains should be supported by geological evidence

Single grade population

Statistical tools are particularly helpful when validating domains. Remember domains ideally contain a single grade population. To validate this simply plot the histogram of all composite grades within the interpreted domain. A single population on the histogram indicates a single population within the interpretation. However, sometimes mixed populations can be masked in a histogram. In these cases, the probability plot more clearly exposes mixed populations. Below are two created examples to demonstrate how useful probability plots are.

Sample set one is a single population following a normal distribution with a mean of 2.0 and a variance of 0.7. Notice the single bell-shaped histogram and the straight line in the probability plot (Figure 70).

Sample set two is a mix of two normal distributions: one half of the data follows a normal distribution with a mean of 2.0 and a variance of 0.7, while the other half follows a normal distribution with a mean of 3.0 and a variance of 1.0. Notice the histogram suggests a single population with a slight evidence of additional high grades (Figure 71). The mixed populations are easier to pick on a probability plot. An attempt to fit a straight line to the probability plot proves quite difficult (Figure 72a). Instead, two lines fit better, suggesting the presence of two populations (Figure 72b).

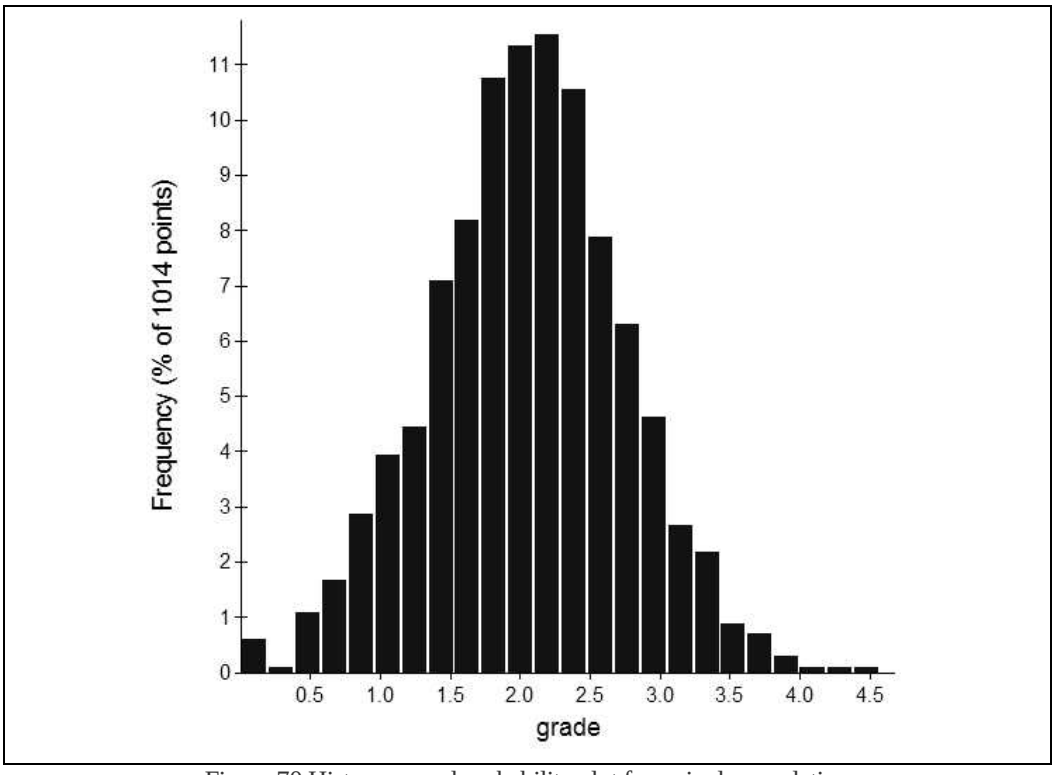


Figure 70 Histogram and probability plot for a single population

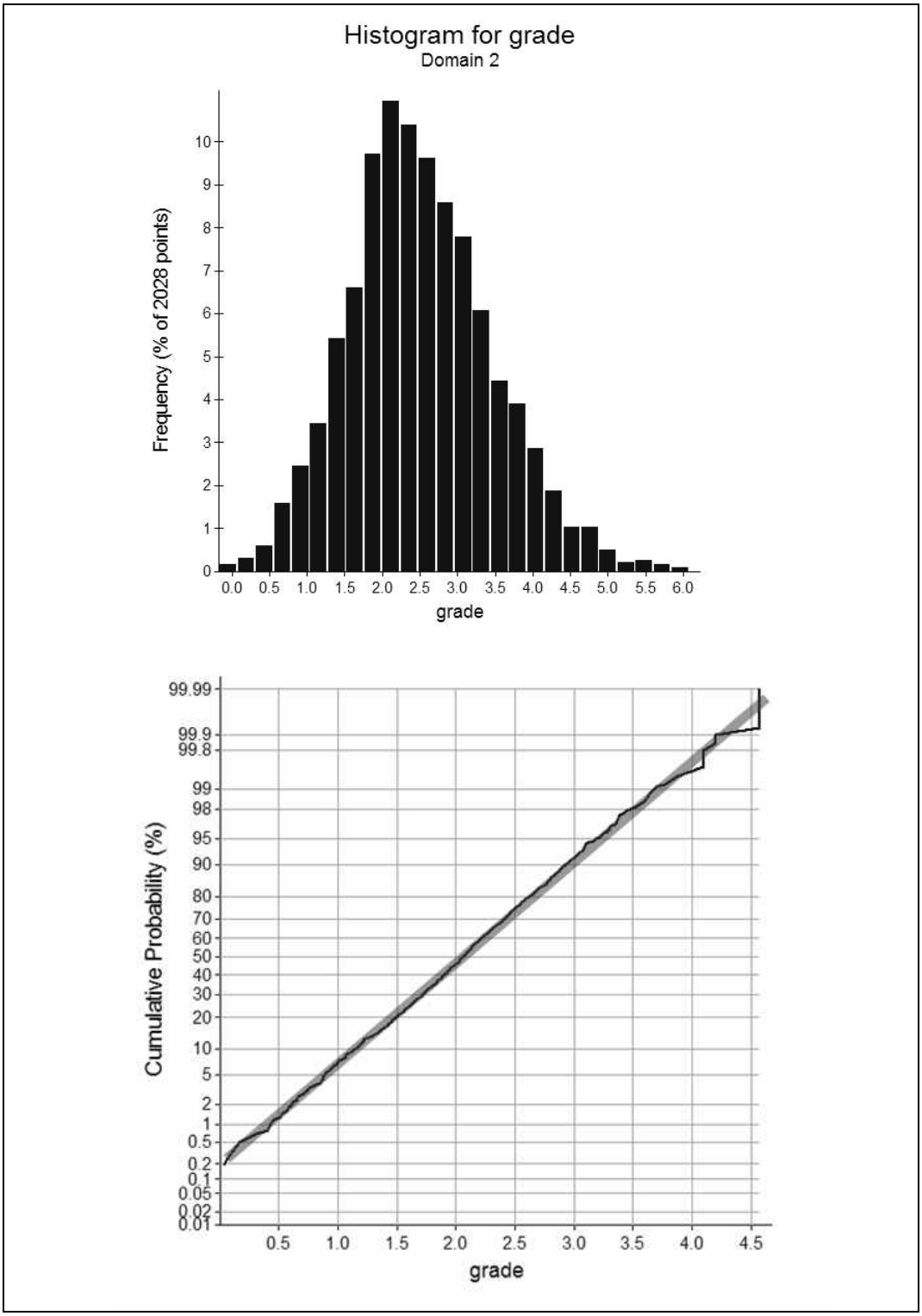
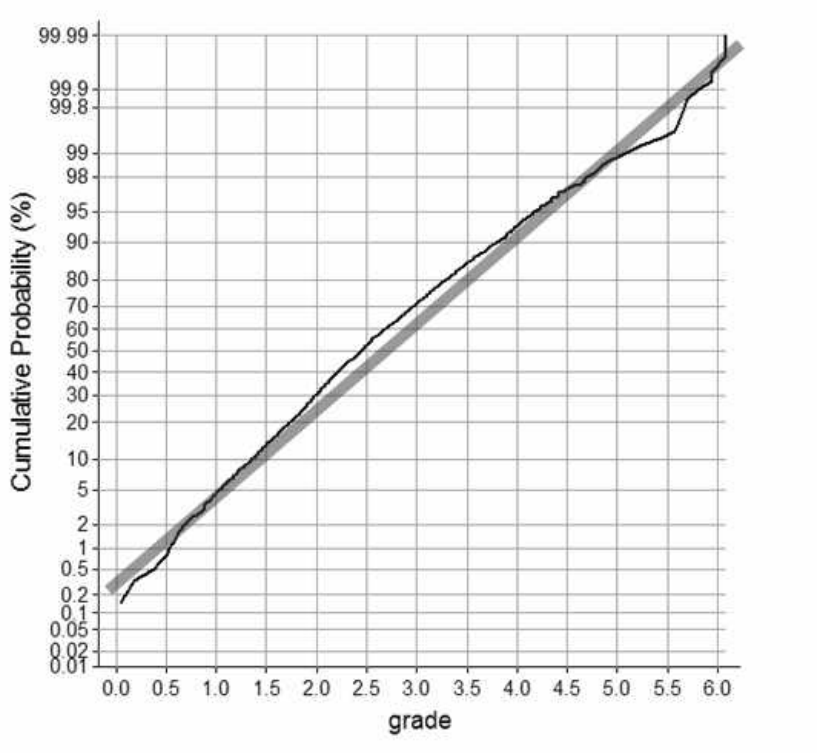


Figure 71 Histogram for a created mixed population

(a) inadequate fit of straight line



(b) better fit with two lines

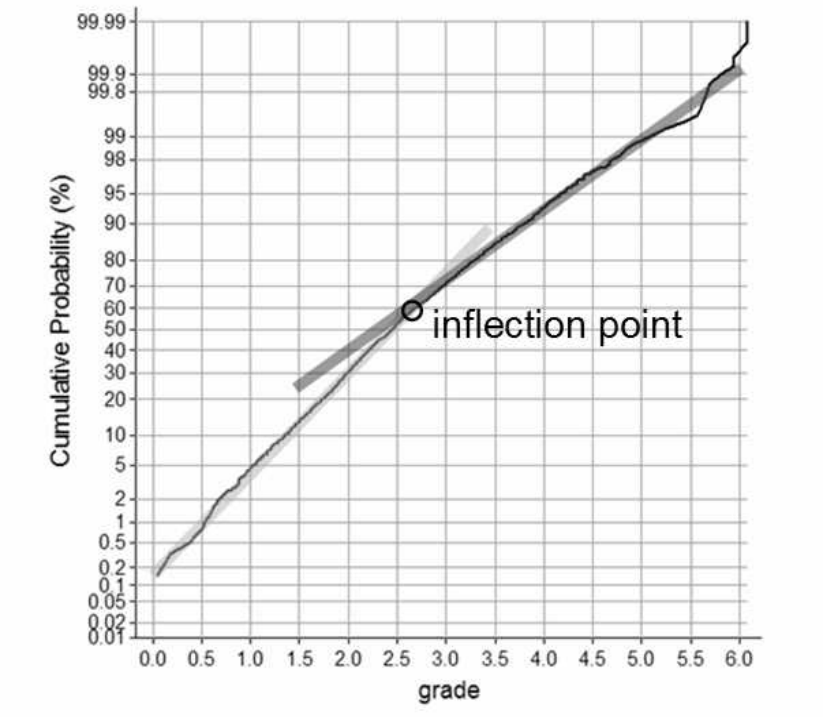


Figure 72 Probability plot with a mixed populations

When data is positively skewed, it is more useful to view these plots with the grade axis on a log scale (Figure 73).

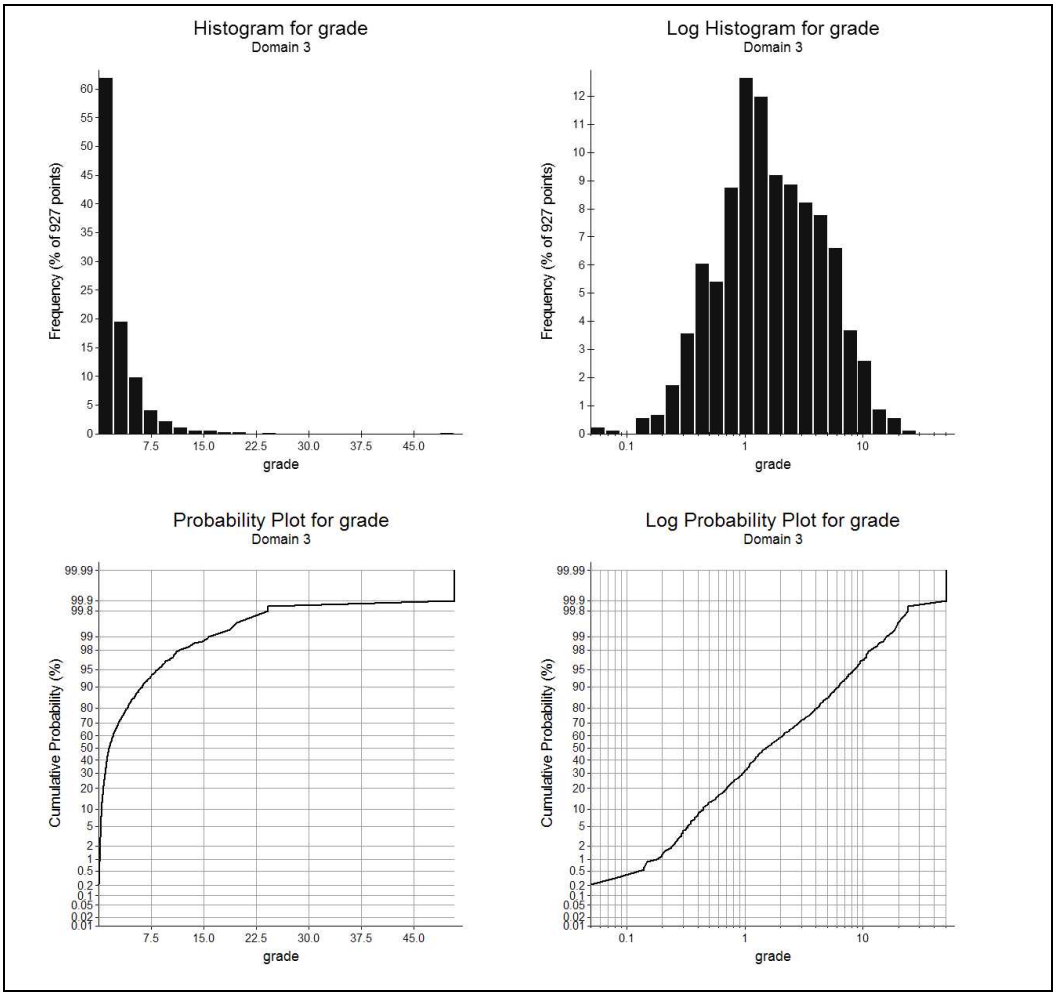


Figure 73 Comparison of original and log-scale grade axes

Single orientation

We check whether a domain has a single orientation of continuity by visualising the data in three dimensions and identifying any significant changes in orientation. Sub domains may need to be created to allow the search parameters to follow continuity of the overall domain in a way that best suits the data.

Alternatively, a domain may need to be unfolded (Figure 74).

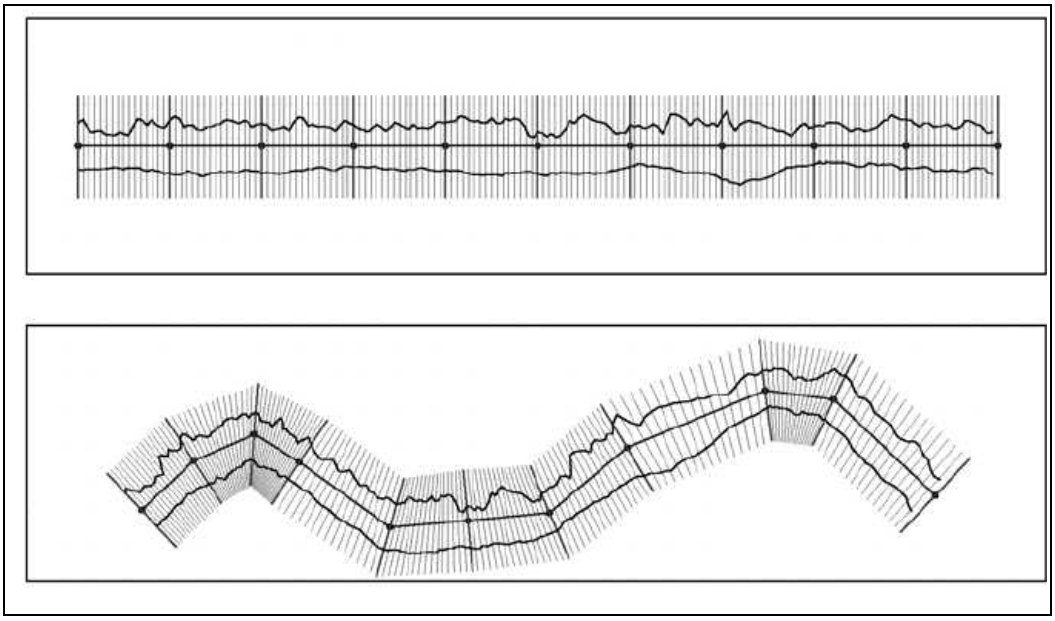


Figure 74 Example surfaces in unfolded (top) and original co-ordinates (bottom) (After Deutsch, 2005)

Note in the discussions here about validating domains, equal sample lengths are assumed. If samples are not equal in length, it may be necessary to composite the data to get representative statistics (see page 77).

Phase 2: Investigation

Purpose

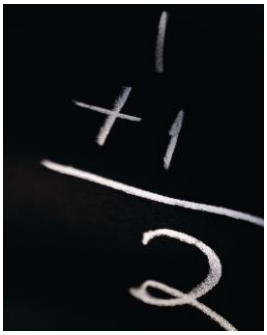
The purpose of this phase is to build our understanding of the data set we are working with: What are the typical values we are coming across? How different are samples? Are there any patterns we should take note of? How nuggety is the data? Are there any relationships between samples in space?

So, this is all about grabbing hold of the data and twisting and squeezing it until you get the clearest picture you can from the available information. Our tools here are primarily statistical; however, these are meaningless unless we bring our geological knowledge and understanding to the analysis.

It is common for people to shy away from statistics: this is unnecessary. When dealing with long streams of data we need some way of synthesizing and consolidating the information - and this is where statistics can be useful.

Here we will focus on the concepts and their relevance to resource estimation.

For resource estimation, we use statistics to:



- Monitor and validate QAQC data
- Check whether we can combine different data sources (say from different drilling campaigns)
- Validate domains
- Understand the grade patterns within domains
- Validate the final resource model

We will address two aspects of investigation here. These are:

1. Descriptive data analysis

   and

2. Variography.

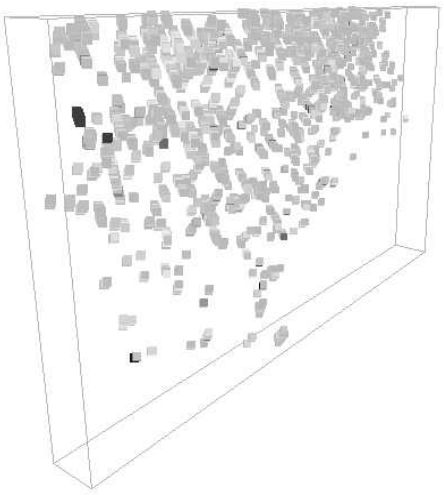
Descriptive data analysis

Once we understand the geological controls, we need to describe, compare and contrast the data in the various domains. Statistics is a simple way of distilling the streams of sample data into summary numbers.

Layout of the information

Before embarking on any data analysis it is wise to understand where data is coming from, what it represents and how the data has been collected.

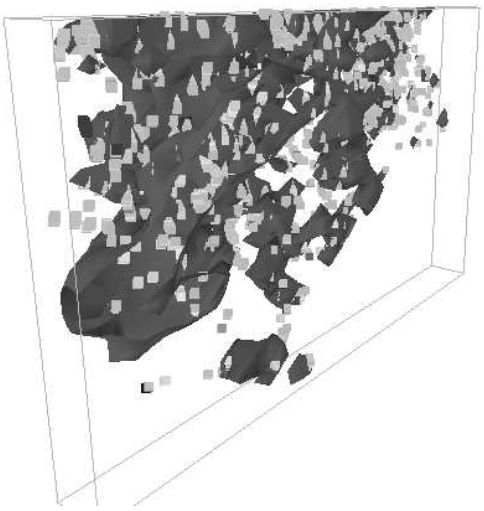
Next, we should view the data in 3D to build our understanding of where typically high grades lie relative to lower grades in the data set. Here our geological understanding of the orebody can play a major role in helping build our perceptions of the data and our expectations of a resource model.



Specifically, we want to understand whether there are any trends in the data – along strike? with depth? and how do these relate to the geological model?

We would also want to have an idea of which domains tend to have the higher grades and how the domains compare with each other.

Viewing boundary conditions also helps us define how the data is used during estimation: are the grades transitioning gradationally or is there an abrupt difference in grade at boundaries?



A location map is useful for conveying drill spacing and coverage. By looking at the drilling coverage, we also get an idea of whether the data may need to be composited and/or declustered.

Compositing and declustering

For our statistics to be representative, we need to ensure even coverage of the sampling (and drilling). For example: any over-sampling of high grades (through more intense drilling at known high grade locations or short intercepts of high grades mixed with long intercepts of low grades) will result in high grading the statistics.

Declustering

Consider a regularly sampled data set with a single grade population (Figure 75 and Figure 76).

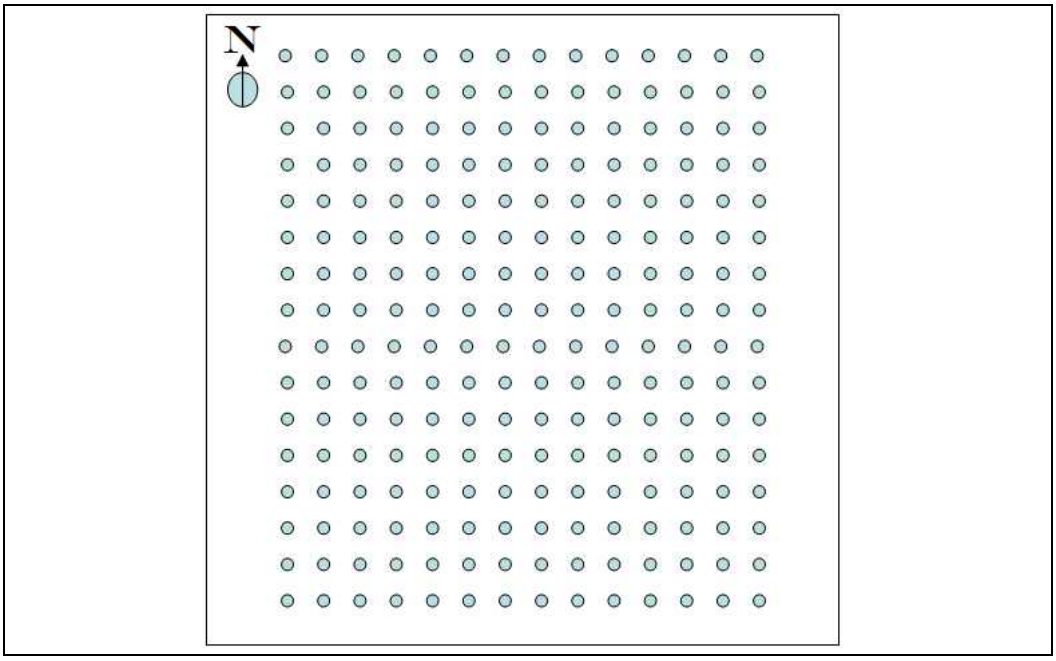


Figure 75 Regularly spaced data

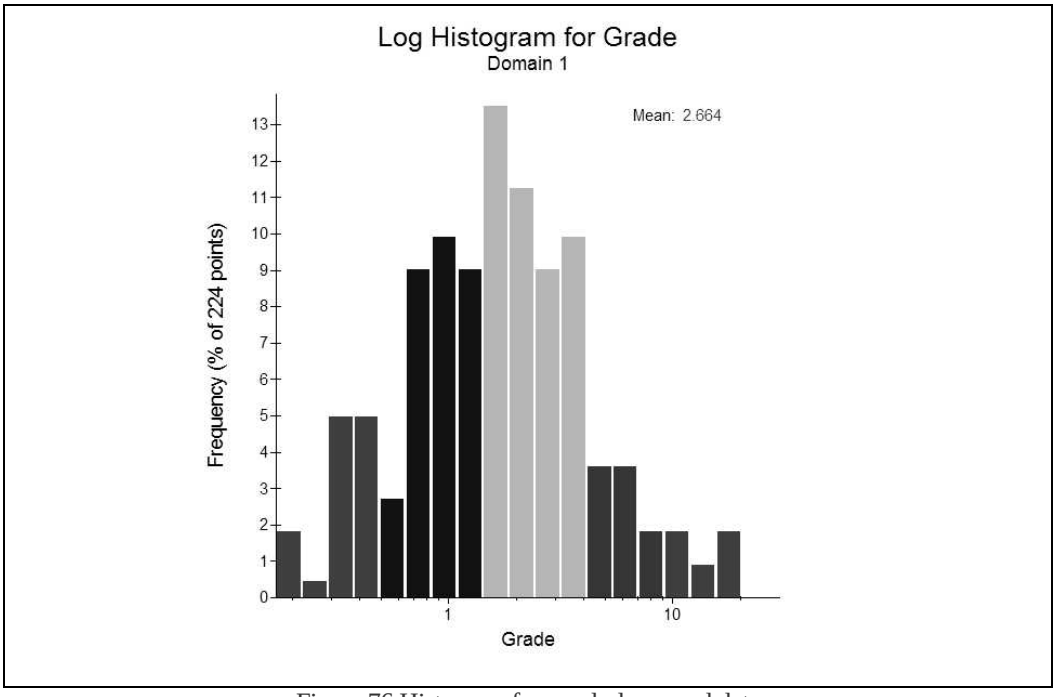


Figure 76 Histogram for regularly spaced data

Now suppose an additional 18 samples are taken in the high-grade area (Figure 77). This is reflected as additional high grades in the histogram as well as an increase in the overall mean grade (Figure 78).

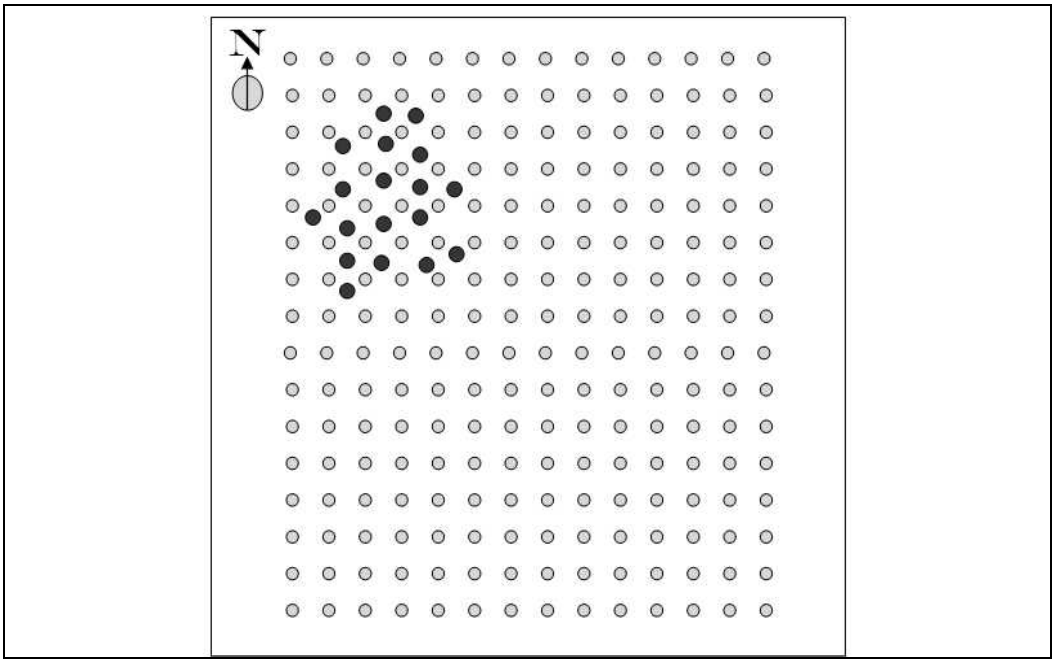


Figure 77 Clustered data

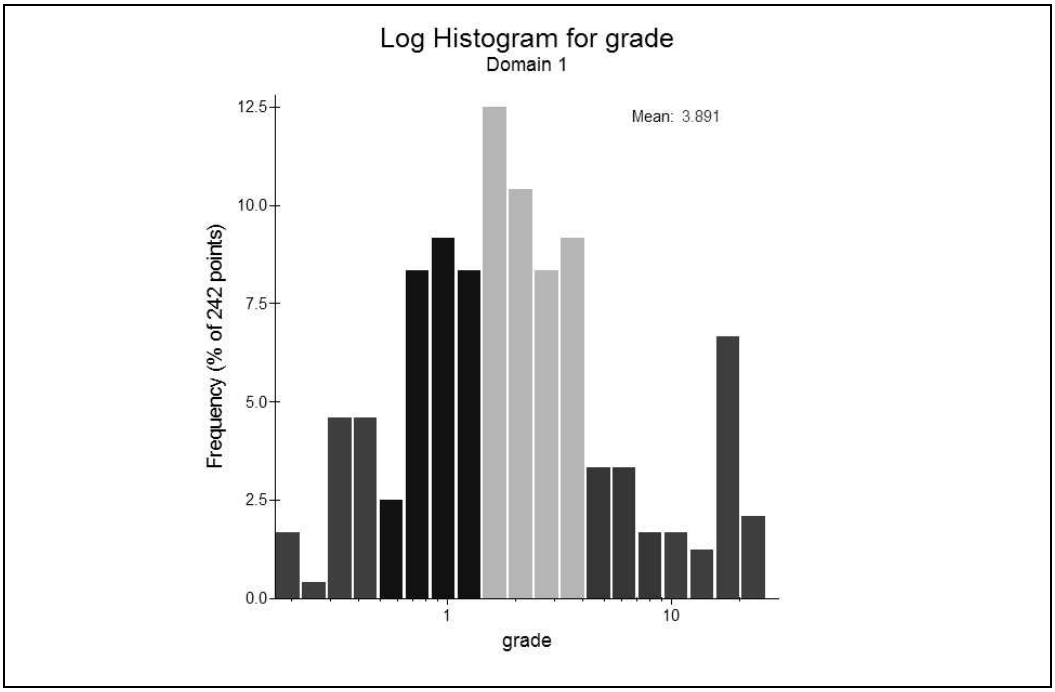


Figure 78 Histogram for clustered data

Since clustering can have an effect on the statistics, which describe our data and help us select estimation parameters, we need to temper the impact of the clustering through declustering techniques. Three different declustering techniques are presented below:

1. declustering by removal of specific drillholes,
2. keep a single sample per grid cell, and
3. weight samples by the number of samples within a grid cell.

Decluster by specific removal

Physically remove the cluster from the data being analysed (Figure 79). For example, a drillhole oriented down plunge can be excluded from the analysis by filtering out sample data with the drillhole's hole-id.

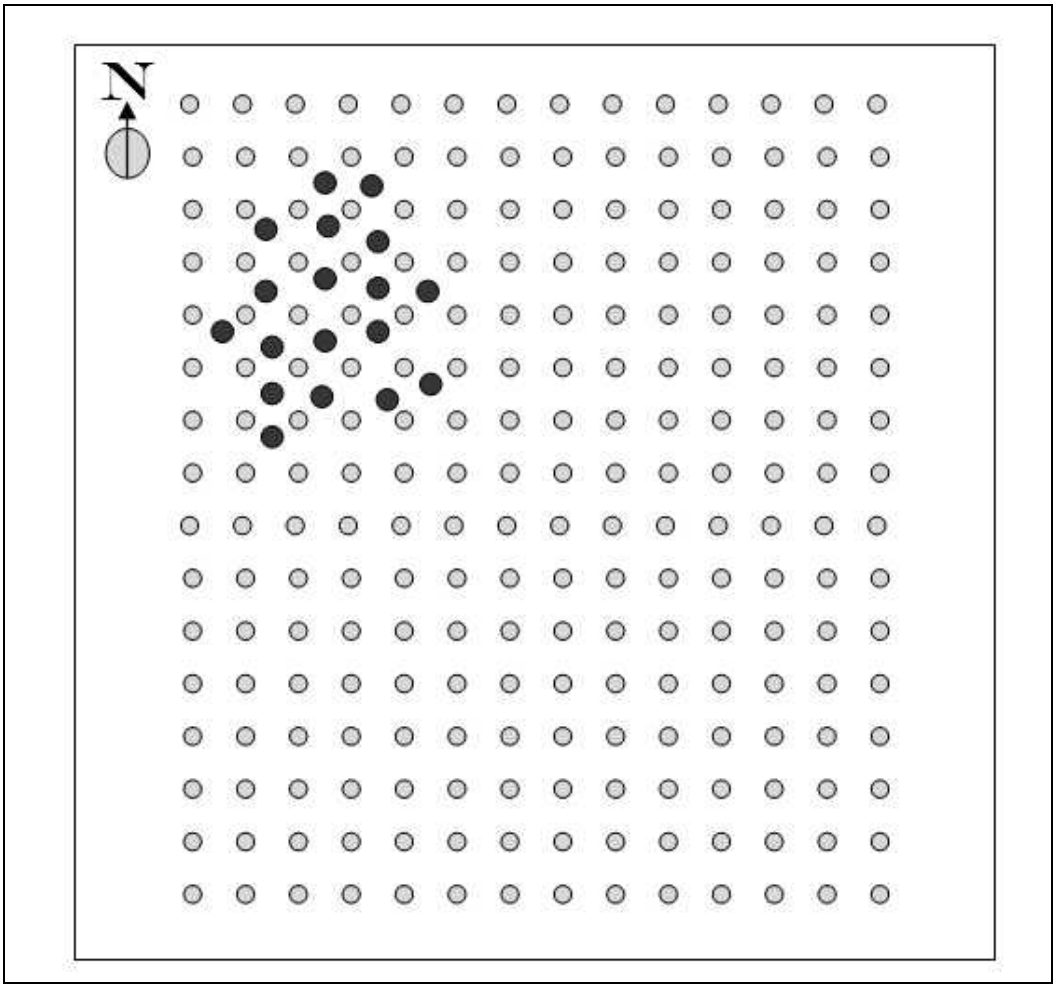


Figure 79 Decluster approach 1: Physically remove samples that create the cluster (shown in bold)

Decluster by thinning

Cover the data with a grid at the declustered scale and select only a single sample (or drillhole) within each cell of the grid (typically the sample closest to the centroid of the cell).

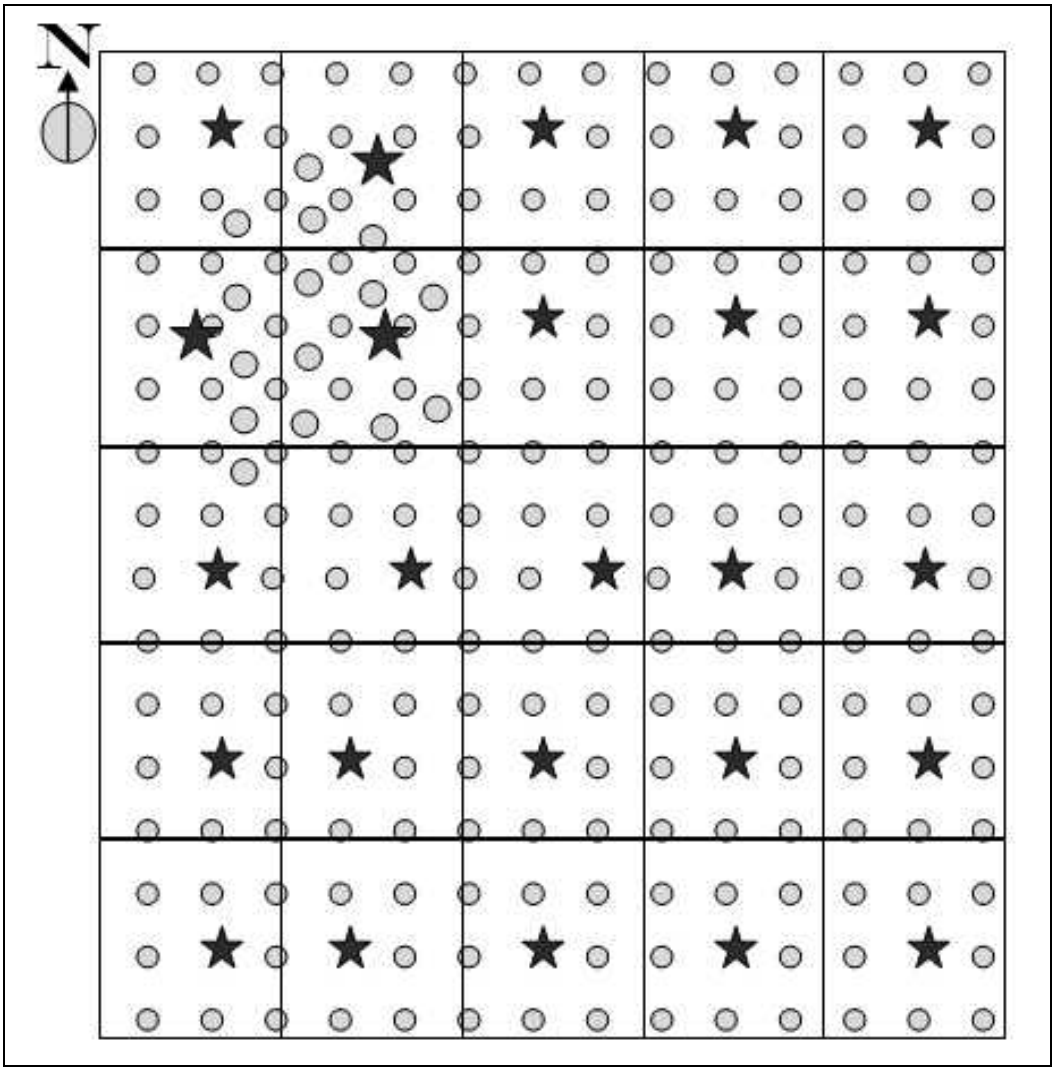


Figure 80 Decluster approach 2: Select only the sample closest to the grid centroid (shown as stars)

Decluster by cell weighting

Cover the data with a grid at the declustered scale and weight each sample according to the number of samples within the cell. For example, samples within a cell with three samples would each be weighted by a third, while samples within a cell with ten samples would each be weighted by a tenth. So, a high grade in a cell of many samples will have a smaller impact on the mean grade than a high-grade sample in an area of sparse data.

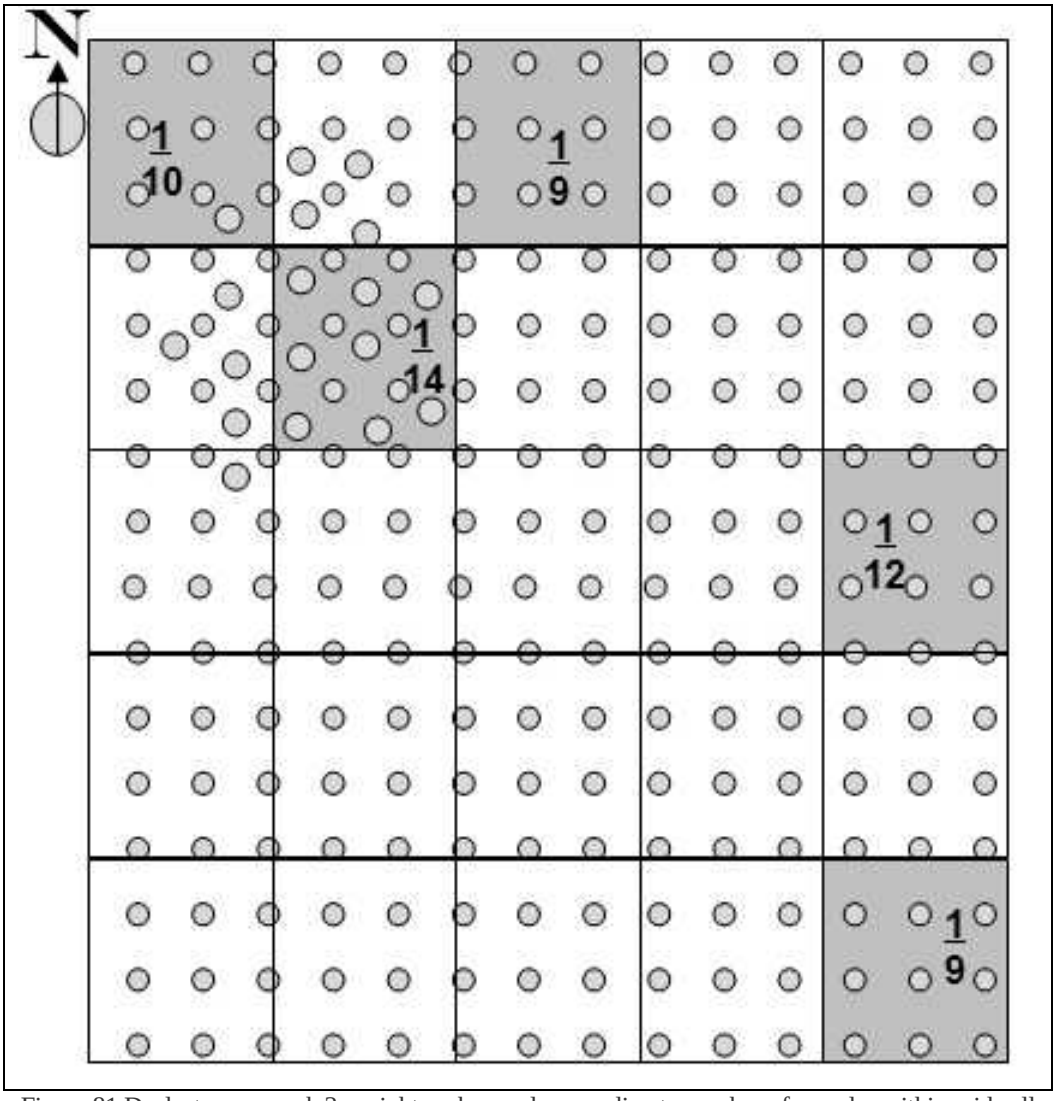


Figure 81 Decluster approach 3: weight each sample according to number of samples within grid cell (five example cells with weights shown)

Be careful, however, of the impact that this third approach can have on perfectly regular data with curved boundaries. Notice how the peripheral cells have fewer samples and so receive a higher weight than samples in a central cell (Figure 82).

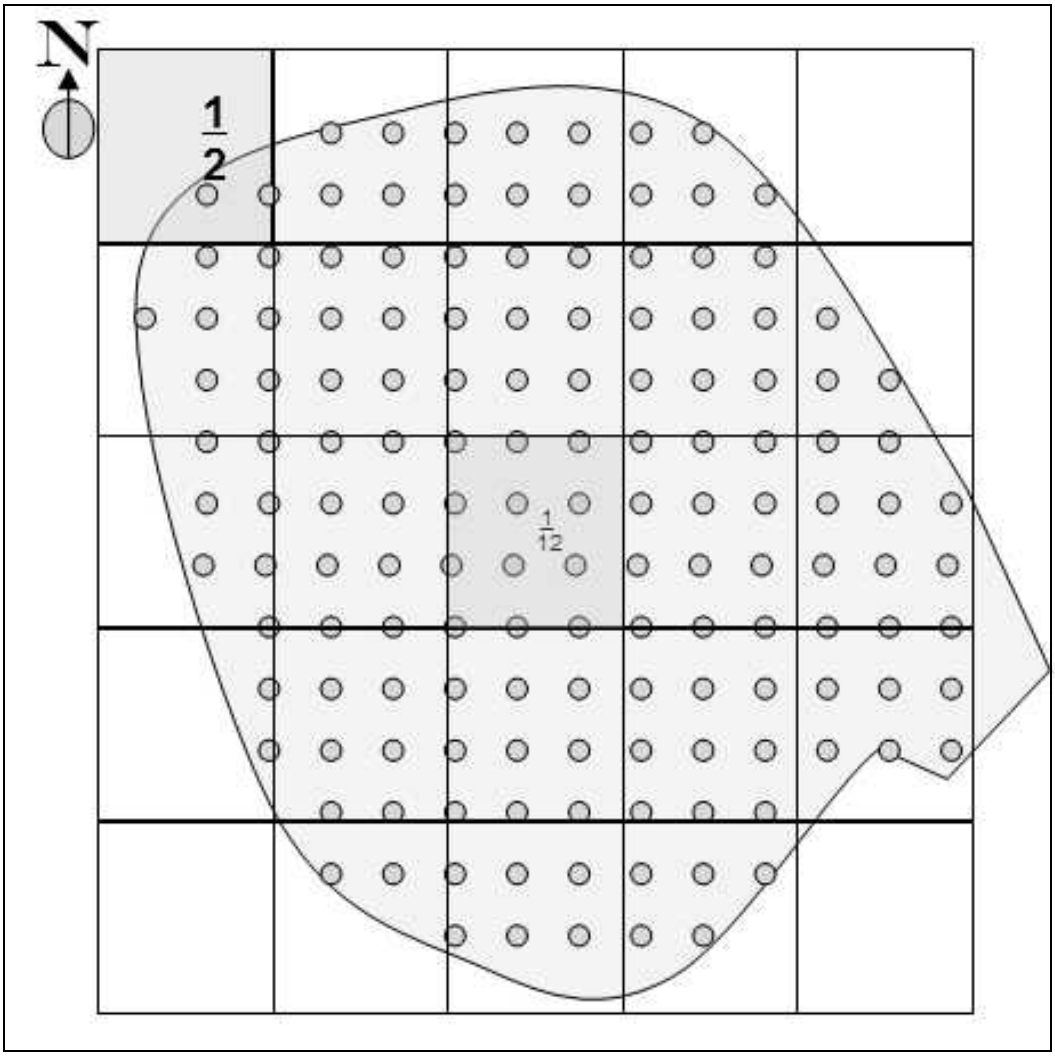


Figure 82 Impact of curved edge on declustered weights

In my experience, this approach works well for declustering underground fan drilling. A good way to evaluate the impact is to view the weights in 3D, colour coded from low to high weights.

Compositing

Compositing is essentially a downhole declustering technique to ensure samples have comparable influence on the statistics. If the samples are collected over variable lengths, then there is a risk of introducing a bias into the data analysis. The effect of compositing is to weight sample grades according to the corresponding interval length.

Composite intervals should be as close to the original sampling interval as possible. As such, a histogram of sample length is useful for deciding on a composite interval. A scatterplot between the grades and sample length provides an indication of the selectivity for sampling interval.

Sample data can be composited according to many different controls in the software you use. The main considerations are:

- the boundary conditions, and
- whether the high grades are situated on the hanging wall or footwall.

If the interpreted domain boundary is abrupt, then the compositing should be constrained within the domain. In contrast, if the boundary is gradational it is usually advisable to composite across the boundary.

It is also preferable to composite in a direction that allows the range of grades within the envelope to be represented fairly. So if high grades are situated on a footwall and the compositing regime dictates only keeping a composite interval if at least 50% of the composite length is captured, it may be advisable to composite upwards in each drillhole.

If samples are all at the same sample length, there is no reason to composite.

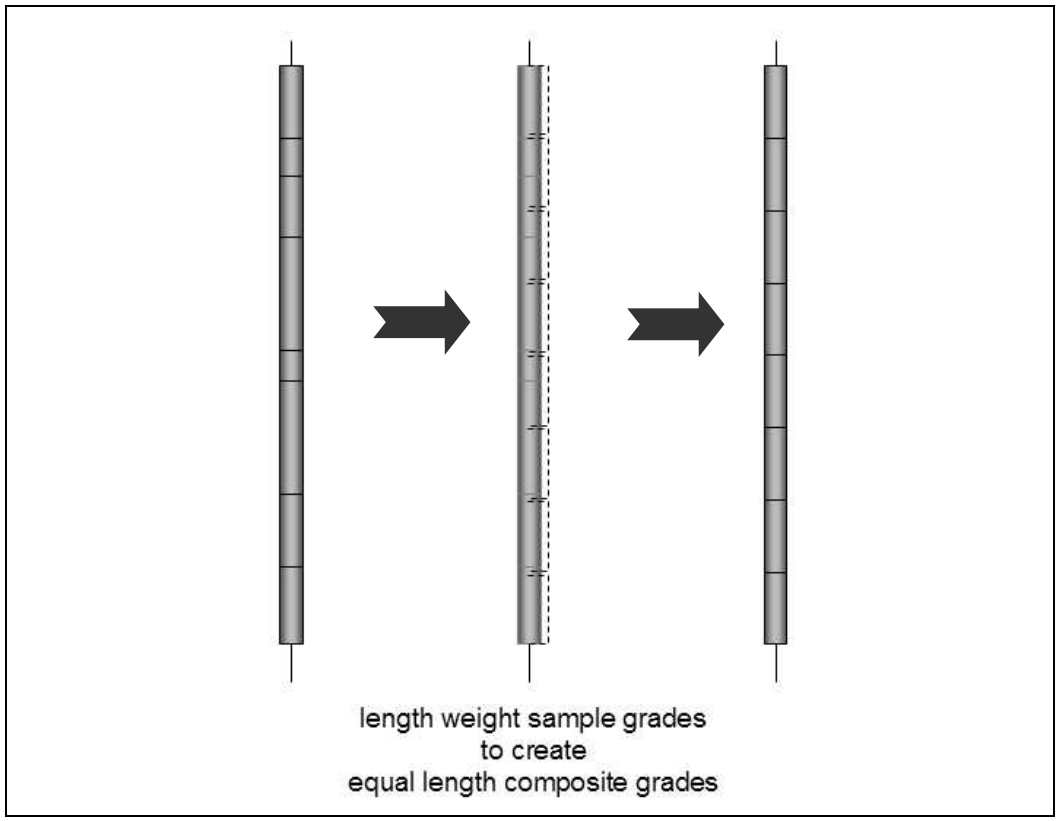


Figure 83 Composite grades created by length weighting sample grades

Statistics to describe a data set

Once we have made sure our data provides us with a representative sample (through compositing and declustering), we need to understand the data behaviour. Questions we may consider include:

- What grades are typical for each domain?
- How variable is the data – are we dealing with mostly homogenous grade, or is our sample set littered with extreme grades?
- Is our data so skew that the extreme grades will bias our estimation?
- Are there any similarities or significant differences between domains?

When we understand the typical grade values, the variability and can compare domains, we position ourselves to more accurately reflect these patterns when we build a resource model.

There are two sets of statistics used to describe a data set:

1. measures of what is typical, and
2. measures of how different values are from the typical

Let us look at these two sets of statistics:

Measures of what is typical

The mean is the sum of the sample values divided by the number of samples.

$$mean = \frac{sum\ of\ values}{number\ of\ values}$$

The median is the middle value when the samples are sorted in grade order.

Sort samples in increasing order of grade
Pick the middle sample
This is the median

The mode is the most typical sample value and is usually read as the peak of the histogram. When a histogram has two peaks, we describe it as "bi-modal"

Examine a histogram
The interval with the most samples is the mode

Measures of difference

The range is the difference between the maximum and minimum sample value.

Find the maximum
Find the minimum
Range = maximum - minimum

The inter-quartile range is the difference between the top and bottom quarter values when the data is sorted in grade order.

Sort samples in increasing order of grade
Pick the middle sample
This is the median

Look at the samples from lowest to median
Pick the middle sample in this lower half of data
This is the lower quartile value

Look at the samples from the median to highest
Pick the middle sample in this upper half of data
This is the upper quartile value

Inter-quartile range = upper quartile – lower quartile

The variance is the typical difference between each sample value and the mean grade. We measure this difference by subtracting the mean from each sample grade. Next, we square the differences to ensure the positives and negatives do not cancel out. Once we have summed all these squared differences we divide by the number of samples less one. This "less one" accounts for the degrees of freedom we lose because instead of comparing the sample grades to the true (unknown) population mean, we compare them to the sample mean. We lose a degree of freedom when we use one statistic to calculate another.

$$\text{var}\,iance = \frac{sum\,of\,\left(each\ \ sample\ value - mean\right)^2}{\left(number\ of\ samples - 1\right)}$$

The **standard deviation** is the square root of the variance and brings the number back into a grade sense rather than a grade-squared sense.

$$standard\ deviation = \sqrt{variance}$$

Other statistics used in resource estimation

Other statistics that are useful for understanding data during resource estimation include:

- coefficient of variation,
- Sichel's mean, and
- percentiles.

Let's take a quick look at these.

The Coefficient of Variation (also called the COV or CV) is the variability relative to the mean grade.

$$coefficient\ of\ \text{var}iance = \frac{standard\ deviation}{mean}$$

The COV is useful for comparing the variability between data sets whose typical grades may be quite different. The COV is also a useful guide for deciding whether a data set needs to be top cut (see pages 99 and 100).

Sichel's mean (also called the log-estimated mean) is the unbiased mean when a data set is log-normally distributed. The assumption of log-normality is absolutely necessary for the Sichel's mean to be meaningful. So while calculating the Sichel's mean is not difficult, it is only valid under the strict condition of a data set having a log-normal distribution (see page 88). The Sichel's mean is calculated as

$$Sichel's\ mean = \exp^{(\log\ mean)} \times \exp^{\left(\frac{\log\ \text{var}iance}{2}\right)}$$

The Sichel's mean is a useful guide when selecting a top cut for log-normally distributed data (see page 99).

Percentiles describe the grade at which the percentage of data lies below. A tenth percentile grade has 10% of the sample data less than the value and 90% greater than the value. The percentiles are generated by sorting the data from lowest to highest grade, then selecting the sample values for which the corresponding percent of the data set lies below. A series of percentile grades provides an assessment of the relative grade distributions.

Statistics in action

Consider these examples of statistical analysis used to describe a data set.

Table 2 Example of gold data statistics for five domains

Statistic		Domain				
		A	**B**	**C**	**D**	**E**
Samples		309	276	4921	567	242
Minimum		0.02	0.01	0.005	0.03	0.005
Maximum		37.8	25.37	201	39.5	165
Mean		**1.21**	**1.17**	**2.12**	**1.72**	**2.52**
Standard deviation		2.53	1.96	5.23	2.92	5.81
Coefficient of variation		2.08	1.68	2.46	1.70	2.30
Variance		6.39	3.85	27.38	8.55	33.71
Log mean		-0.44	-0.53	-0.09	-0.11	0.01
Log variance		1.20	1.59	1.70	1.29	2.06
Sichel's mean		**1.17**	**1.30**	**2.14**	**1.72**	**2.83**
percentile	10%	0.17	0.12	0.2	0.21	0.18
	20%	0.29	0.25	0.4	0.40	0.40
	30%	0.44	0.4	0.52	0.54	0.54
	40%	0.51	0.49	0.69	0.71	0.76
	50%	**0.62**	**0.63**	**0.90**	**0.95**	**1.04**
	60%	0.79	0.81	1.20	1.20	1.48
	70%	1.08	1.1	1.69	1.55	2.12
	80%	1.52	1.51	2.53	2.17	3.14
	90%	2.31	2.51	4.53	3.49	5.71
	95%	3.45	4.28	7.21	5.38	9.42
	97.5%	5.48	5.29	11.30	9.00	12.80
	99%	8.73	8.2	20.74	14.23	20.75

Statistical observations from the above table include:

- Domain E is the highest grade domain. Domain E also has the highest coefficient of variation, which suggests a high degree of variability in the data. The percentiles for Domain E reflect the a high percentage of higher grades than the other domains.

- The Sichel's mean is typically higher than the raw mean, suggesting the data set is NOT log-normal (the high degree of variability is observed in the log-variance) and impacts the Sichel's mean calculation.

Typical data distributions in the mining industry

A representative sample data set should reflect the underlying grades for the entire domain. The distributions of these populations, though never truly or completely known to us, do tend to follow common patterns for different commodities.

A key difference between populations of various commodities is the relative distribution of lower and higher grades within the rock mass (Figure 84). Commodities that take up a large proportion of the rock matter, for example iron, tend to reflect negative skewness (most of the grades are high with a smaller proportion of lower grades). At the other extreme precious metals, measured in parts per million or grams per tonne, take up a miniscule proportion of the rock mass compared with the gangue. Distributions of precious metals are always positively skewed - most of the samples report low grades with a small percentage of extreme high grades. Moderate to high grade base metals (in the range 7.5 to 15%) tend to have no dominant skew. Base metals from domains with lower grades tend to have a small positive skewness.

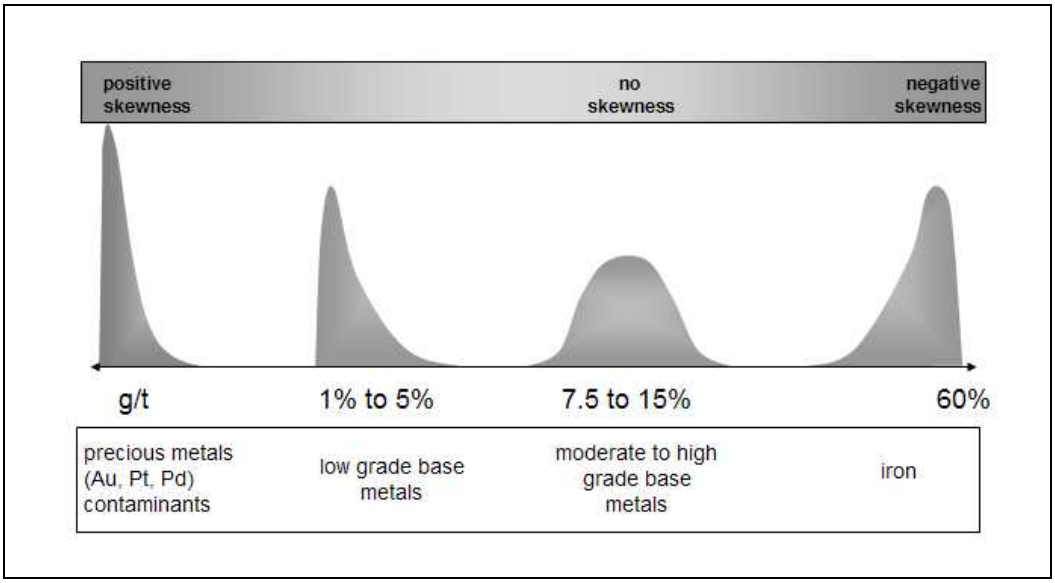


Figure 84 Typical distribution shapes by commodity type

Other useful graphs

Cumulative probability plot

A useful plot for examining the distribution of grades is a cumulative probability plot. This plot is created by sorting the data from lowest to highest grade and then plotting the order of grade (or percentile) against the grade (Figure 85).

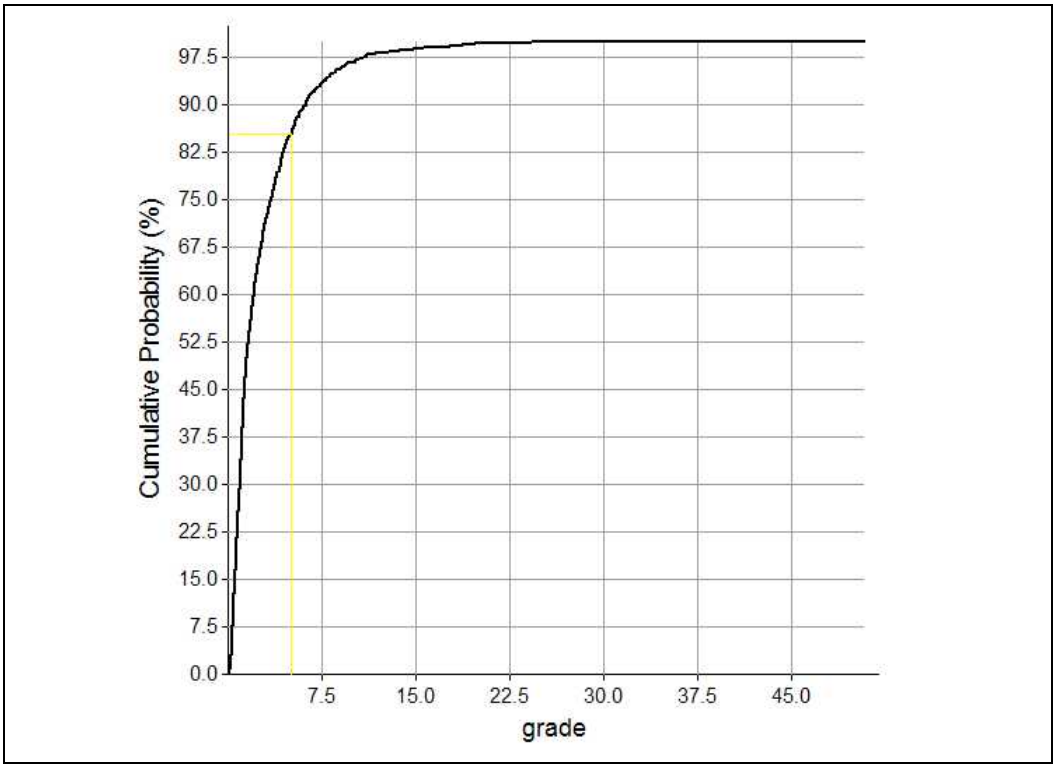


Figure 85 Example of a cumulative probability plot

This cumulative probability plot is a summary of the proportion of samples that occur below each grade. For example, 85% of the samples have a grade less than 5% (Figure 85). The grades associated with the steep part of the cumulative probability plot are the more frequent grades. So with the data presented in Figure 85, the lower grades are more frequent, indicating a positively skewed data set.

Probability plot

An alternative plot used for analysing the population distribution is the probability plot. The probability plot is a cumulative probability plot, but with the probability axis adjusted in such a way that if the probability plot produces a straight line, the distribution follows a Normal bell-shaped curve.

A probability plot is essentially a Q-Q plot of the data set against a Normal distribution, where the scale of the Normal distribution is a probability scale. If the probability plot presents as a single straight line then the data set reflects a Normal distribution (Figure 86). Since most of the available statistical techniques are based on an assumption of Normality (that the data has a Normal distribution), any data set that does have a Normal distribution is likely to be easier to model. Normal distributed data tends to be well behaved – there are no extreme grades and the variability relative to the mean is stable. High-grade base metals can have Normal distributions.

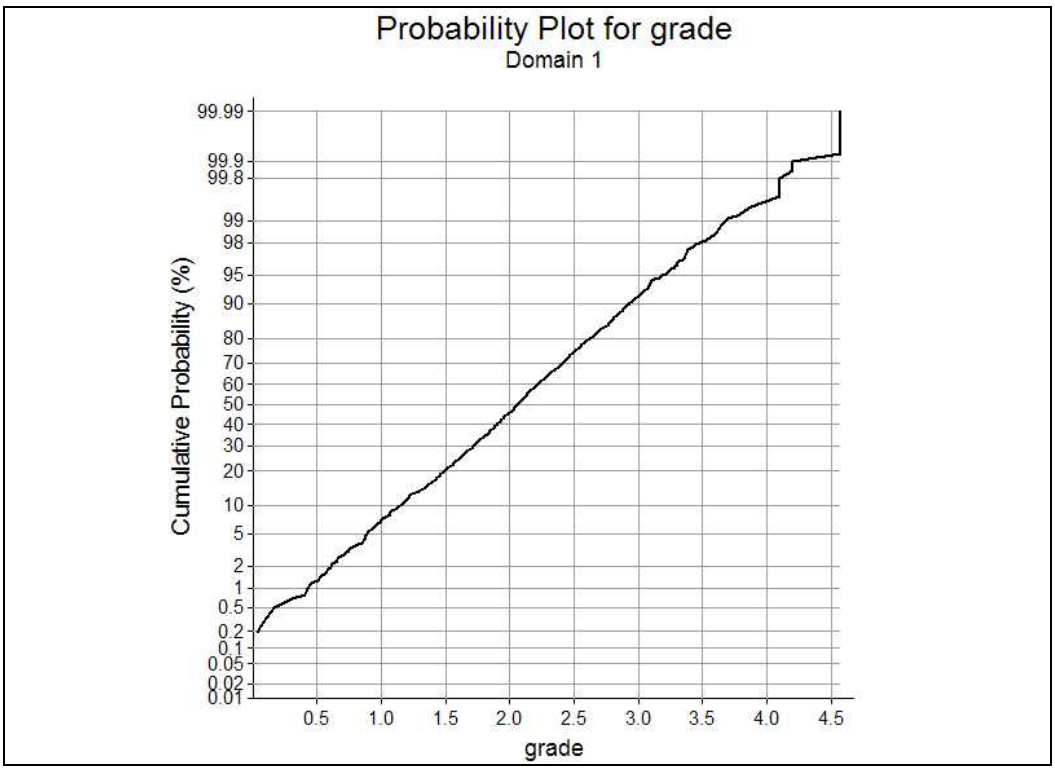


Figure 86 Example of a probability plot with a single straight line

When the grade scale is converted to a log-scale and the probability plot presents as a single straight line then the data set reflects a log-normal distribution (Figure 87). Precious metals and low-grade base metals often have log-normal distributions. This occurs so often that when I see a gold distribution that is not following a log-normal distribution I immediately check the domaining protocol. In most cases where a grade boundary has been used to delineate gold mineralisation, the presenting distribution is not log-normal. When the domaining protocol on the same data is reviewed to reflect geological controls, log-normal gold distributions result. This is not always easy to achieve and often a compromise is needed. However, whenever geology is injected into the process, there is a greater chance that the distributions are clear.

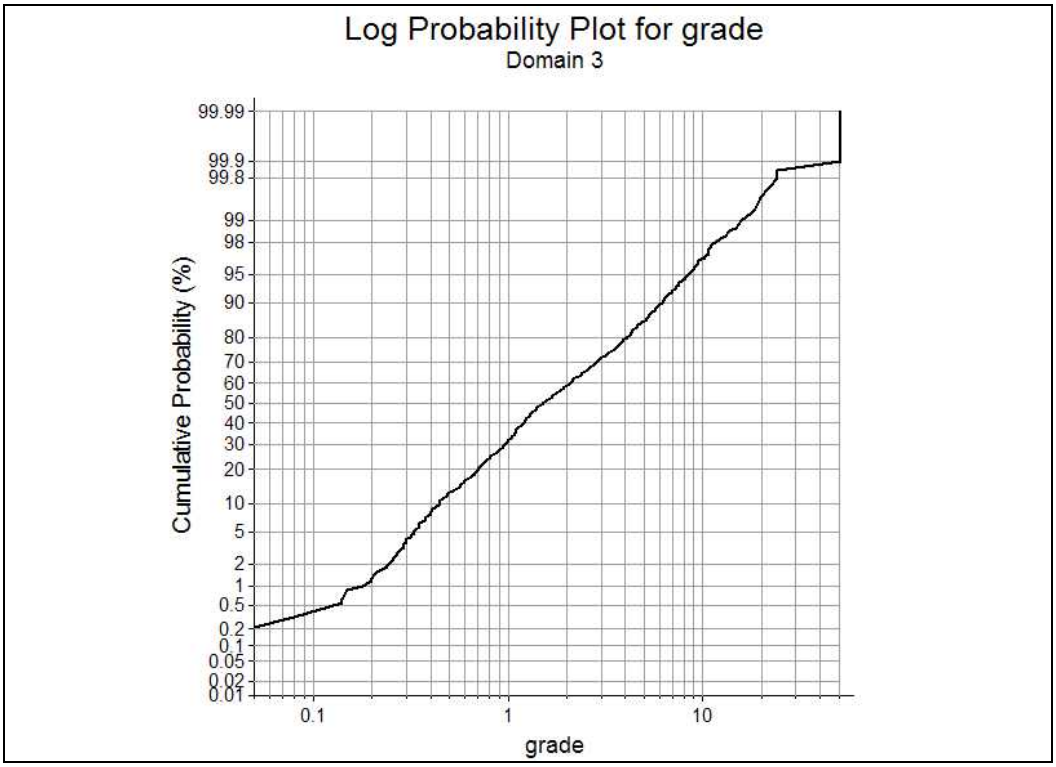


Figure 87 Example of a log-scale probability plot with a single straight line\

When a data set follows a log-normal distribution, then the over-bearing influence of the extreme grades can be managed by transforming the data. In addition, the Sichel's mean is likely to be a representative unbiased estimate of the true mean and can be used to guide selection of a top cut value (see page 99).

(Note the discussion on page 70 regarding mixed populations and domains.)

On the opposite extreme to positive skew is negative skew. A histogram of higher grade iron, for example, is dominated by high grade samples with a few low grade samples. Given the scale of the grades (typically 50 to 60% Fe), there is no extreme influence of the lower grades (typically 20 to 30%) since these lower grade values are still in a similar order of magnitude to the typical grades. There is no reason, therefore, to transform the data to obtain representative statistics.

Other graphs that are useful for understanding the data patterns include:

- Trend plots
- Boundary plots
- Q-Q plots
- Box-and-whisker plots.

Each of these is examined next.

Trend plot

Trend plots provide a summary of the grade trends in the data. There are at least two approaches to investigating the trends in the data:

- Trend in a three dimensional model, and
- Trend along the northing, easting and elevation axes.

A simple way to generate a three-dimensional trend model is to run a nearest neighbour or an unconstrained estimation model.

Some sort of simple 3D modelling (nearest neighbour or a quick inverse distance) is a good way to get a feel for any spatial trends – in particular are any of the domains showing regions of higher grades? Remember one of the fundamental assumptions is that the sample data within the domain is "stationary" or stable – i.e. there are no trends!

Another useful trend plot can be generated by calculating the average grade in slices (say by bench or by northing) and plotting the slice-averages against the bench or northing slice value (Figure 88). Notice the number of samples for each slice is presented as bars. This provides a sense of how much support each slice average has. We are less likely to respond to variations with low support than we are to changes in trend where there is more data.

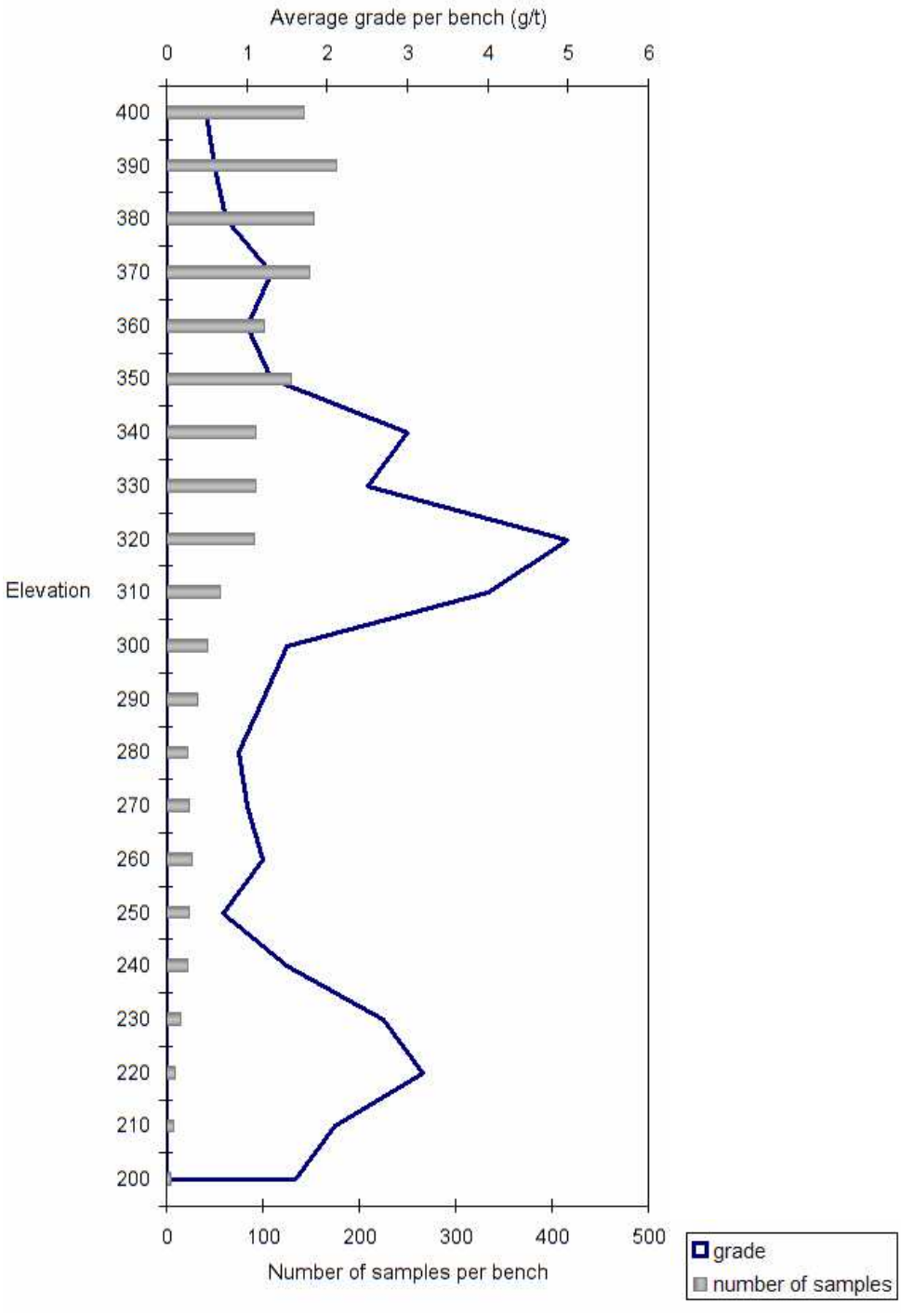


Figure 88 Vertical trend plot

Boundary plot

Boundary plots offer us a deeper insight to the grade behaviour at the boundary of our interpretation. This is useful later when we have to decide whether to access samples across the boundary during estimation.

A boundary plot is generated by:

1. Identifying the point in each drillhole where the drillhole intersects the boundary.
2. Collecting and averaging:
    a. all the samples one composite interval before the intersection
    b. all the samples two composite intervals before the intersection
    c. all the samples three composite intervals before the intersection
    d. all the samples one composite interval after the intersection
    e. all the samples two composite after before the intersection
    f. all the samples three composite intervals after the intersection.
3. Graphing these averages against the number of composite intervals from the boundary.

Figure 89 illustrates two boundaries – one gradational and another abrupt.

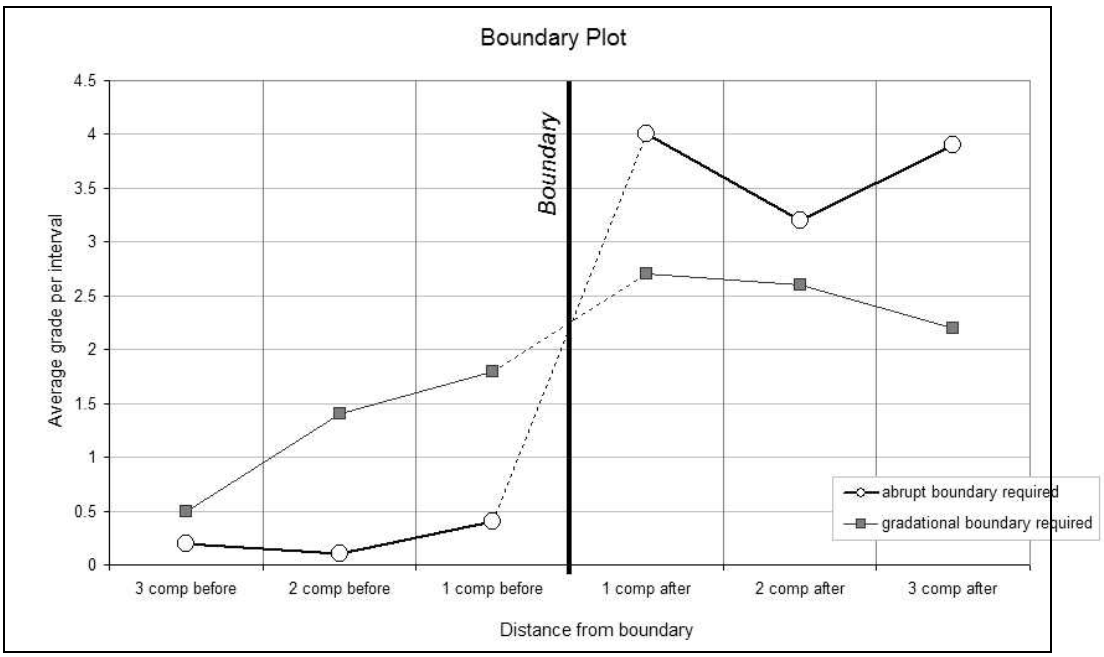


Figure 89 Boundary plot (single element)

Q-Q plot

Q-Q plots provide valuable comparisons between domains, vintages of data and types of drilling. When each data source is attempting to sample the same population, it is reasonable to assume the resulting statistics will be similar (i.e. a straight one-to-one line on the Q-Q plot).

Remember to constrain data comparisons within volumes you expect to see similar statistics (Figure 90 and Figure 91). Comparing deep diamond samples to shallow RC samples does not provide a fair platform for comparison.

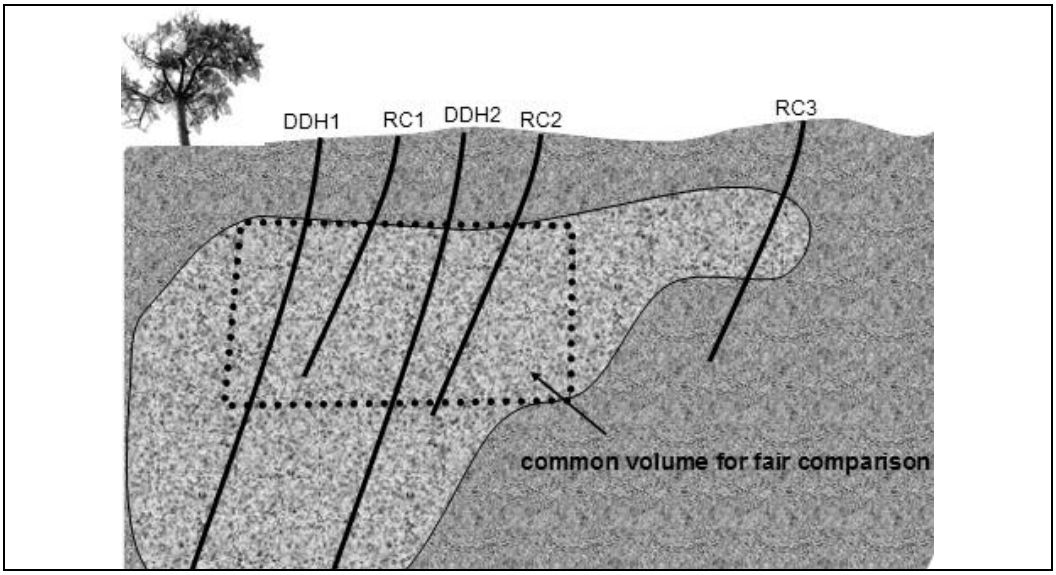


Figure 90 Comparisons should be made within a common volume

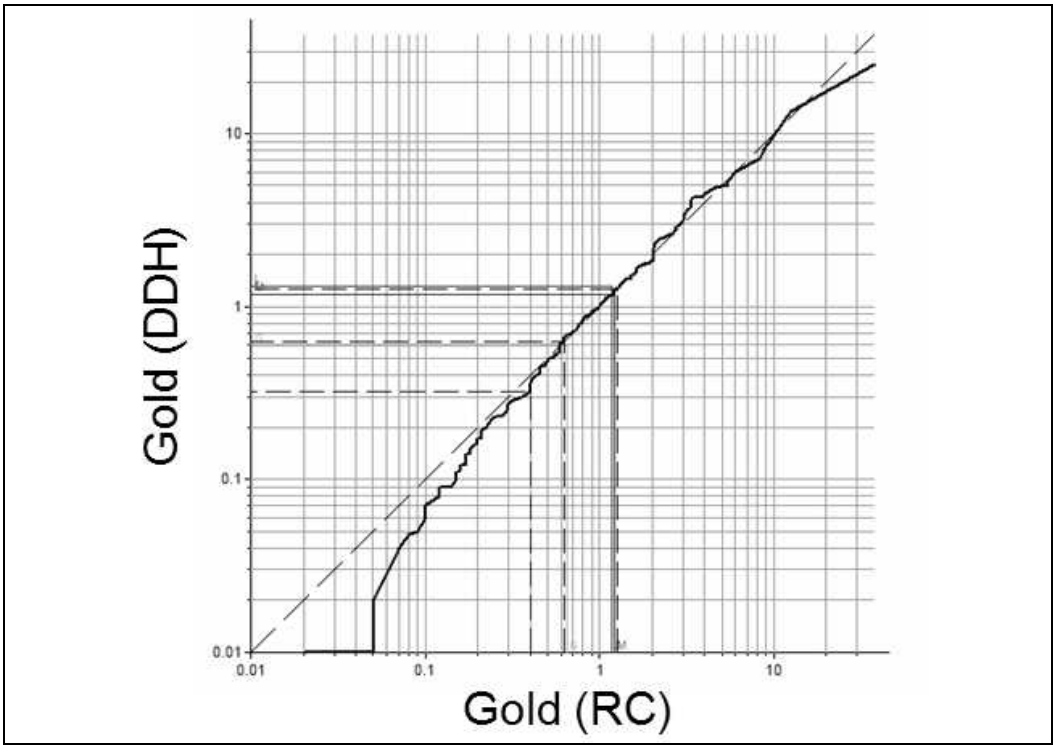


Figure 91 Q-Q plot between gold samples from two sources (DDH and RC) within common volume

Box-and-whisker plot

Box-and-whisker plots are useful for comparing domains (see section on page 43 for more details on constructing a box-and-whisker plot). The Box-and-Whisker plot in Figure 92 shows Domain 2 is consistently higher than Domain 1.

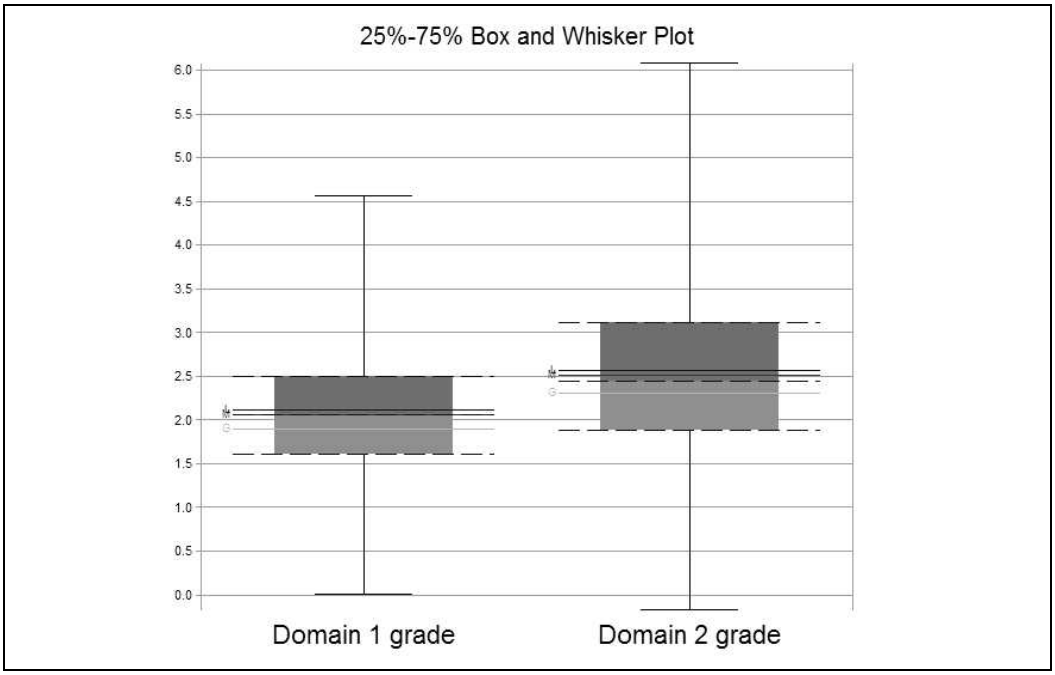


Figure 92 Example of a box-and-whisker plot for two domains

Box-and-Whisker plots help quickly identify the grade patterns across several domains (Figure 93).

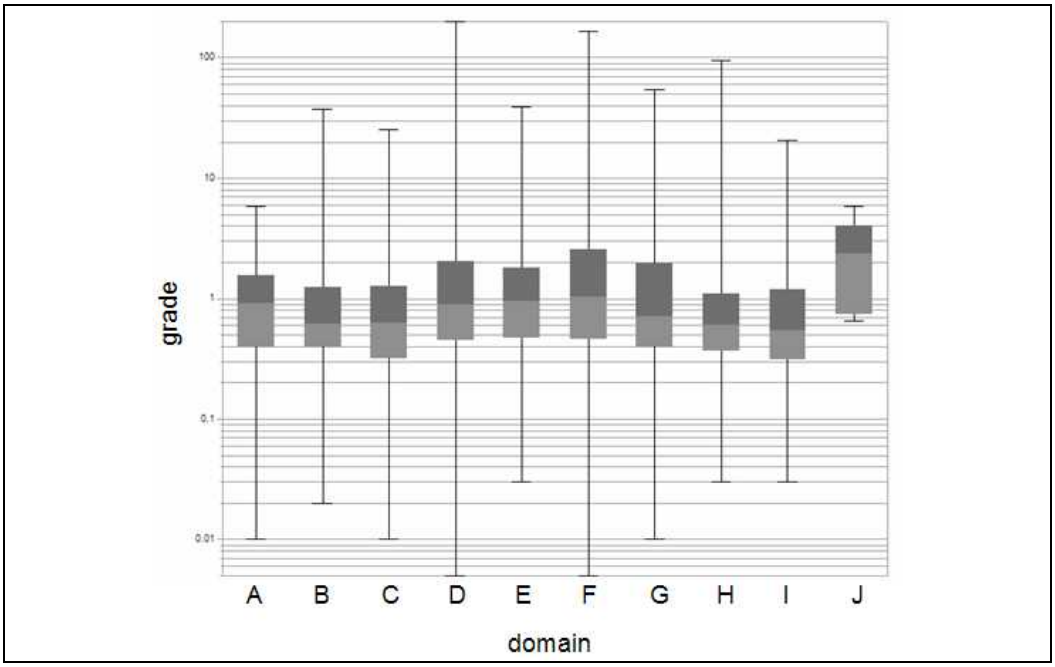


Figure 93 Example of a box-and-whisker plot for ten domains

Selecting top cuts

Extreme grades, or outliers, can bias the calculation of the average towards the extreme value, particularly when there are only a few samples. Table 3 illustrates the effect the maximum grade of a data set has on both the mean and the standard deviation.

Table 3 Effect of maximum grade on coefficient of variation

	Data set				
sample number	1	2	3	4	5
1	1.1	1.1	1.1	1.1	1.1
2	1.2	1.2	1.2	1.2	1.2
3	1.4	1.4	1.4	1.4	1.4
4	2	2	2	2	2
5	3	3	3	3	3
6	3.5	3.5	3.5	3.5	3.5
7	5	5	5	5	5
8	6	6	6	6	6
9	8	8	8	8	8
10	12	25	50	100	150
average	4.3	5.6	8.1	13.1	18.1
standard deviation	3.5	7.2	14.9	30.6	46.4
COV	0.8	1.3	1.8	2.3	2.6

This effect is greater when the distribution is positively skewed. The statistic we use to understand the degree of skewness and the need to apply a top cut is the coefficient of variation (COV):

$$\textbf{Coefficient of variation} = \frac{\textbf{standard deviation}}{\textbf{mean}}$$

The COV value increases for more positively skewed data (Table 3).

Wellmer (1998) illustrates COV values expected for typical grades (Figure 94). Note the x-axis is presented in percent. This means minerals measured in grams per tonne or parts per million will correspond to percentages less than one.

According to Wellmer's guide in Figure 94, extreme grades for low intensity mineralisation (typical grades less than 1%) exerts a significant influence when the COV is higher than 1.5. When the COV is between 0.9 and 1.5, normality is not possible.

Rules of thumb you may find useful include:

- Data sets with COV greater than 1.2 generally need to be top cut.
- Data sets with COV less than 1.0 generally do not need to be top cut.

Note: an inordinately high COV (say greater than 3) is indicative of mixed distributions and more domaining may be necessary.

To top cut a data set means to re-set any composite grades that are higher than the top cut value to the top cut value. So for data set 3 in Table 3, a top cut of 30 would mean the sample grade of 50 is re-set to a value of 30 with no change to the other sample values. If, however, the top cut was 7, then the top two samples would be cut back to 7 since they both have grades higher than 7. This ensures the high grades are still included in subsequent analyses, but the extremity of their grades does not overly influence the statistics.

NOTE: I have come across the practice of evaluating the suitability of a top cut by examining the COV of the top cut data. There is absolutely no scientific reason or rationale to look at the top cut COV – it is meaningless. The COV of the raw data provides a guide as to whether there is a need to top cut or not. There is a risk of top cutting too severely if one attempts to achieve a suitable COV of top cut data. The COV can be used to decide whether a top cut is necessary and not used to judge the suitability of a top cut value.

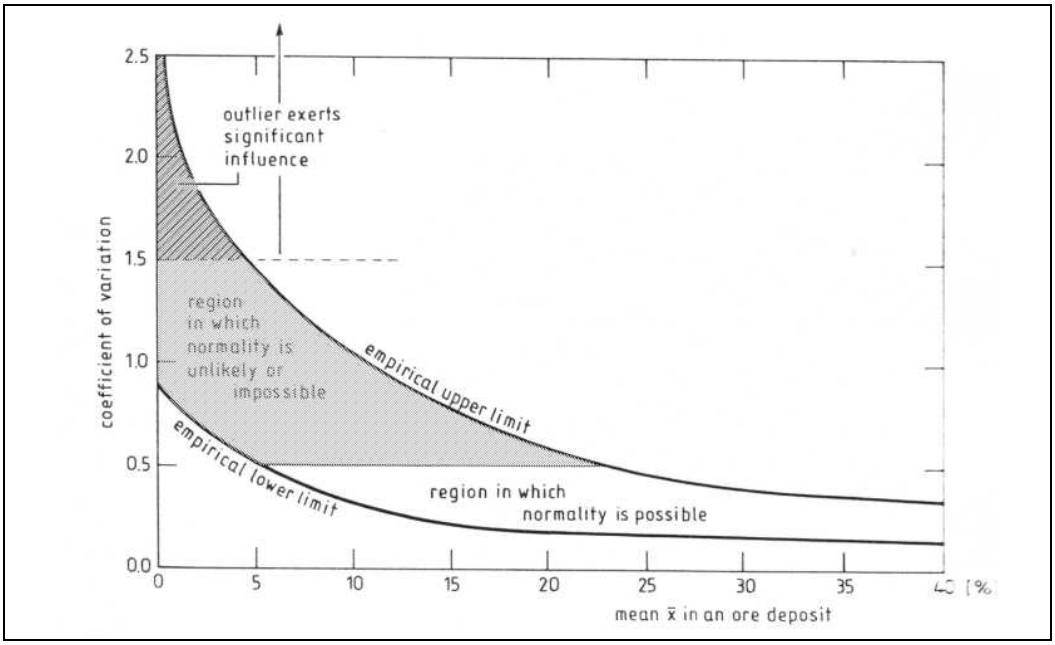


Figure 94 Coefficient of variation as a guide to the need to top cut
(after Wellmer 1998)

All approaches to selecting a top cut are subjective. An evidence-based approach to selecting top cuts considers the following:

1. Top cuts are evaluated independently for each domain. Where sufficient geology has been used to guide the delineation of a single grade population, the top cut is more likely to produce a fair representation of the underlying population mean.
2. Where necessary, data should be declustered before selecting a top cut value. This ensures the data set you are working with is representative of the mineralisation population.
3. Ideally, reconciliation information provides the best guide to the top cut value. A series of resource models are generated based on a range of potential top cut values. The resulting models are compared within the reconciliation volume to both the grade control and the production grade. The top cut value that results in a resource estimate closest to the production grade is selected as the top cut for the domain. This approach is of course only helpful once mining has commenced.
4. If the data is log-normal, use the Sichel's' mean as a guide to selecting a top cut value. See explanation below.
5. If the data set is not log-normal, then the top cut is typically based on the point of disintegration of the high-grade tail in a log-scale histogram. See explanation below.
6. Note that if the high-grade samples are scattered within a domain, they are likely to have more of a biasing influence than if they are grouped close together. Top cutting matters more when high grades scattered throughout a domain, so take a moment to visualise the data in 3D – where are the typically high grades relative to the rest of the data? What influence are these high-grade samples likely to have on local estimates?

Sichel's mean to guide top cutting

The Sichel's mean is an unbiased mean for a log-normally distributed data set. For a confirmed log-normal distribution, the Sichel's mean provides a more stable estimate of the true mean than the average of the data values.

The Sichel's mean is calculated as follows:

- Calculate the natural log-transform[8] for each sample grade. If the data follows a log-normal distribution, then the log-transformed data follows a normal distribution (bell-shaped curve).
- Calculate the average of the natural log-transformed values. Since the data is log-normal, this transformed data set will be normal and the mean, mode and median will coincide.
- The back-transformed[9] average of the log-values is called the geometric mean. Since the data order is preserved and the mean and median coincide, the geometric mean corresponds to the median, rather than the mean. This value, however, is too low an estimate for the population mean.
- Herbert Sichel is credited as providing the empirical work to establish the Sichel's factor which, when applied to the geometric mean, provides the unbiased Sichel's mean. The Sichel's factor is the back-transform of half the variance of the log-transformed data.

$$Sichel's\ mean = \left(\exp^{mean\ of\ \log-transformed\ grades}\right) \times \left(\exp^{\frac{\text{var}\,iance\ of\ \log transformed\ grades}{2}}\right)$$ The

Sichel's mean is often called the log-estimated mean.

Since the Sichel's mean is the unbiased mean for a data set, we can use this as a guide for testing a range of top cuts. The process is:

- Iteratively top cut the sample grades and calculate the average of the top cut data.
- The top cut value that gives a top cut mean close to the original Sichel's mean is selected as most suitable.

[8] Microsoft Excel's function for the natural log transform is "=LN( )"

[9] Microsoft Excel's function for back-transforming the natural log is "=EXP( )"

Disintegration approach to top cutting

When a data set is not log-normally distributed, the usual approach to selecting a top cut is to identify the point at which the number of samples supporting a high-grade tail diminishes.

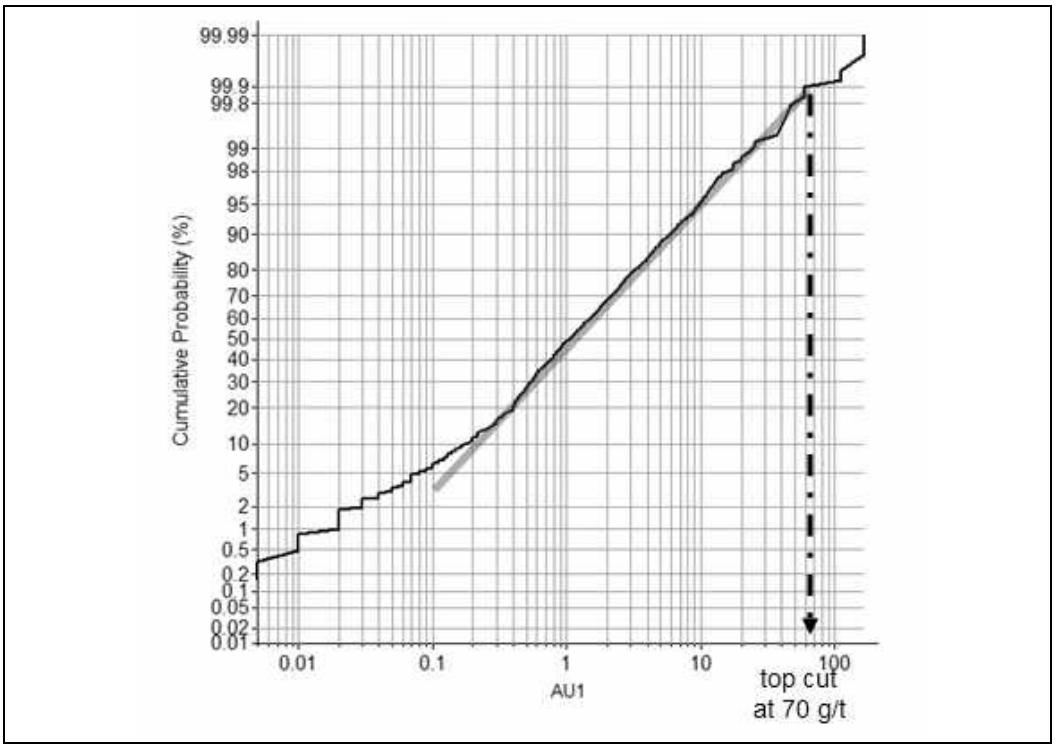


Figure 95 Disintegration approach to top cutting (log-scale probability plot)

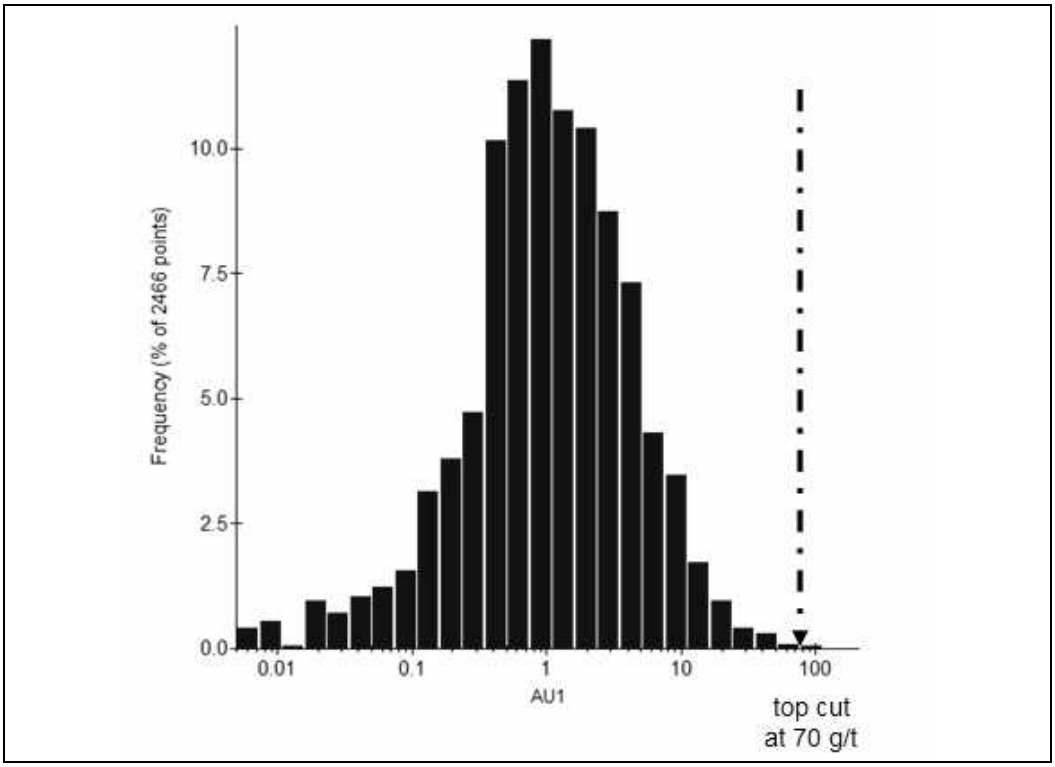


Figure 96 Disintegration approach to top cutting (log-scale histogram plot)

Multi-variate data analysis

The first step in any multi-variate data analysis is to compare each element against every other element – preferably as scatterplots or as a matrix of scatterplots (Figure 97). This provides a quick assessment of whether there are obvious relationships between the elements, and whether these relationships are positive (as one element increases so does the other) or negative (as one element increases in grade the other decreases in grade).

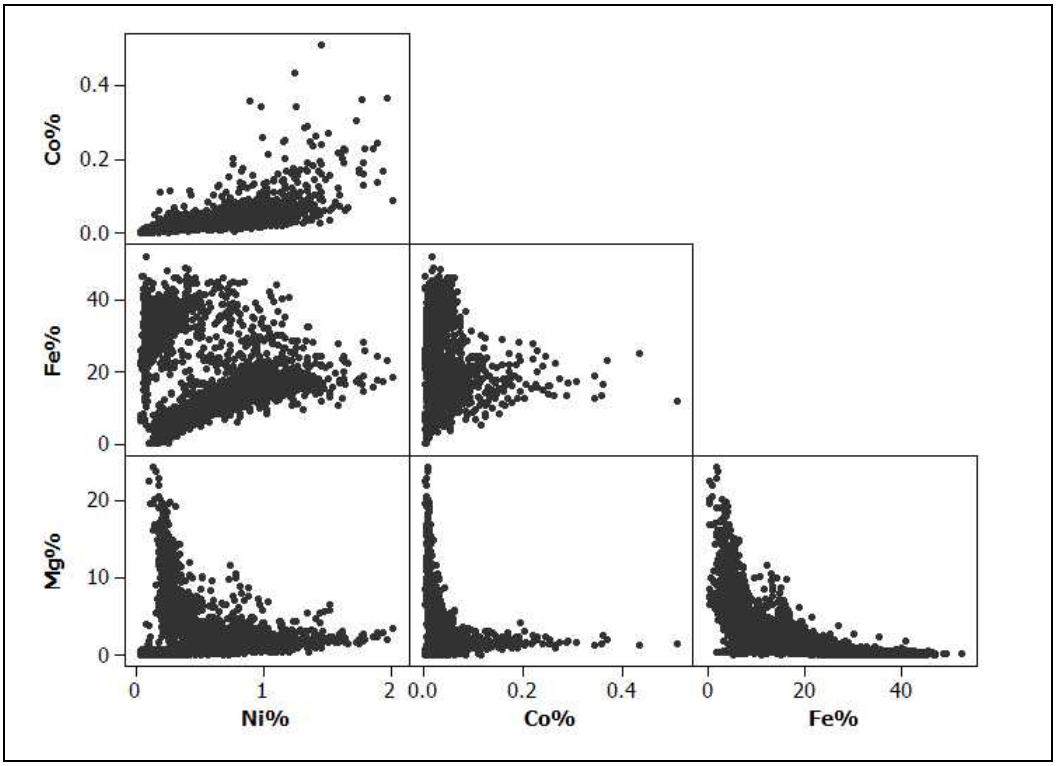


Figure 97 Matrix scatterplot between Ni%, Co%, Mg% and Fe%

Sometimes it may be necessary to differentiate the relationships according to domain (Figure 98).

The correlation coefficient provides a statistic to summarise the strength of the relationships (Table 4). The closer the correlation coefficient is to +1.0, the stronger the positive relationship. Conversely the closer the correlation is to -1.0 the stronger the negative relationship. Correlation coefficients close to zero imply no relationship exists between elements.

Typically, in the Mining Industry, an absolute correlation coefficient greater than about 0.6 is a signal of a strong relationship.

High absolute correlations also indicate the variability in one variable is closely related to the variability in the other variable.

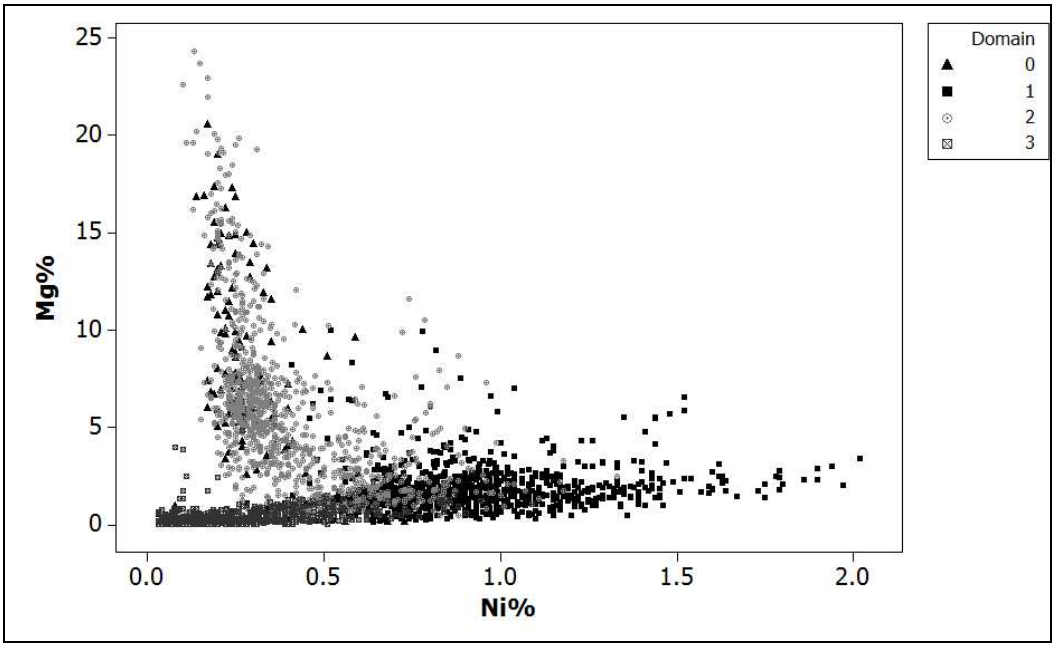


Figure 98 Scatterplot coloured by domain (Mg% and Ni%)

Table 4 Example of correlation between elements

	Ni	Co	Mg	Al
Co	0.736			
Mg	-0.518	-0.463		
Al	0.198	0.156	-0.266	
Fe	0.814	0.618	-0.532	0.468

It is possible to model the relationship and to predict one element from another for those variables that have a pattern in the scatterplot and have strong correlations. This modelling is called regression. Regression is the process of fitting a line to data and providing the best-fit equation.

Do be careful though, just because your computer program can generate a regression does not mean the regression is appropriate. For example, a single point that is very different from the rest of the data can sway a regression significantly. These points are called leverage points.

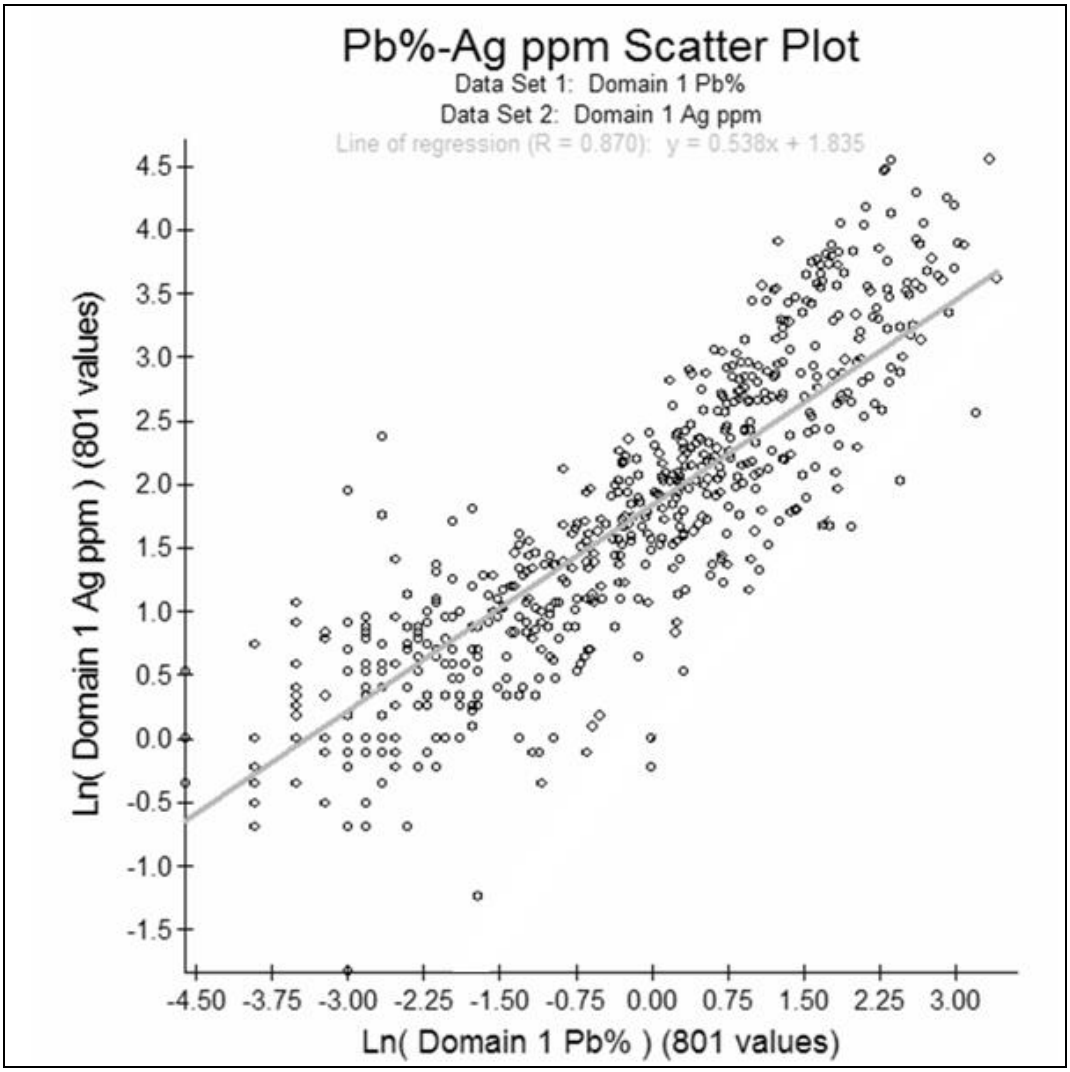


Figure 99 Regression line fit between elements

Statistical data analysis - summary

The main reason for conducting a data analysis is to understand the data we are working with. Summary statistics help create an idea of typical grades and variability as well as the need to top cut. As a minimum, your data analysis should include summary statistics, histogram (log-scaled if necessary), probability plots and comments contrasting the domains and characteristics of the data. In addition, and this is important, the statistical analysis should be linked backed to the geological context. If one domain is reporting higher typical grades, it is your responsibility to understand that enrichment. Observe and then seek to understand.

Analysis of continuity

Essentially, analysis of grade continuity (or variography) is about comparing samples according to the distance and orientation between the samples. The premise is: If we can understand how samples relate to each other in space, we can use this information to build an expectation of a block grade based on weighting the surrounding samples according to the variogram.

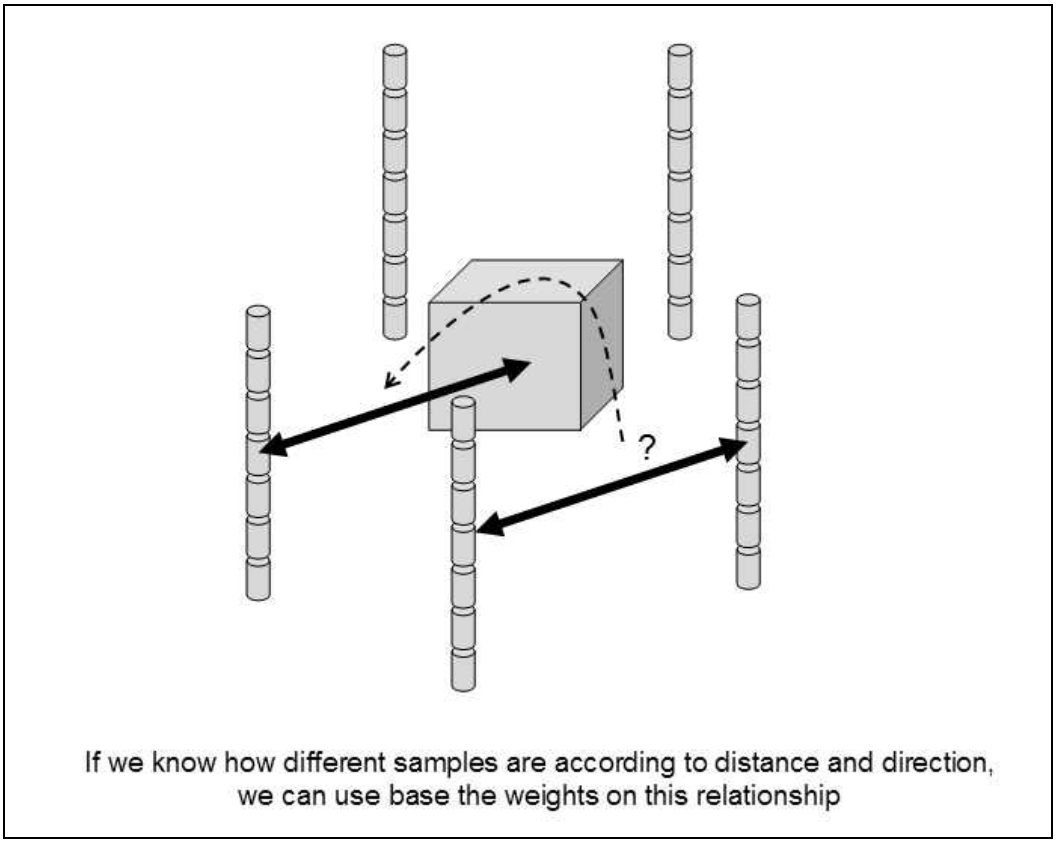


Figure 100 Understanding the relationship between samples helps weight samples

There are four stages to a variogram analysis:

1. **Preparation for variography** – check due consideration of factors influencing calculation and interpretation of variograms.

2. **Variogram calculation** – purposeful use of parameters to generate a representative variogram.

3. **Variogram modelling** – describe the likely underlying population continuity through curve fitting.

4. **Interpretation** – explain the likely patterns of continuity and possible consequences for estimation, mining and future data collection.

Preparation for variography

Preparation for variography relates to understanding the context of the data you are working with. These aspects include the data collection/quality, geological context and statistical properties of the domain.

The following assumptions are inherent when calculating variograms:

1. the sample grades are sourced from a single grade population (see domain discussions on page 70), and
2. the difference in grades between pairs is consistent everywhere in the domain (see domain discussions on page 69 and 74).

It is worth validating these assumptions prior to calculating a variogram on the data.

Data integrity

Nothing can replace decent data collection. Errors in the data collection affect the nugget effect and the perceived continuity of grades.

In addition, the variogram analysis assumes representative samples. However, if the sampling volume is too small (narrow drilling diameter), or there is insufficient homogenising in the sampling process, or the assaying technique is inappropriate, then the calculated variogram will be distorted.

Compositing the data goes some way to ensuring comparable sample support. However, if compositing involves splitting too many samples, the variogram on the resulting data set can present as a lower than expected nugget effect.

Geological context

Building an understanding of the controls on sampled mineralisation is central to preparation for variography. For example: understanding structural controls on mineralisation is useful in complex ore bodies with multiple, possibly cross-cutting structures.

Understanding the perceived grade continuity or the structural elements on a stereonet provides background for the analysis of the grade continuity.

Awareness of multiple phases of mineralisation (or enrichment, remobilisation, faulting, folding or whether structural events pre- or post-date mineralisation) all provide a basis for subdividing the data or the requirement for unfolding (or unwrinkling).

Domains and statistical analysis

Understanding the variability within a domain, as well as the expected differences between domains, helps govern your expectations of the data set when comparing samples spatially. For example a highly variable data set (say with a high coefficient of variation) is likely to have extreme grade samples that will dominate the variogram calculations. In this scenario, it may be necessary to transform the data prior to calculating the variogram. Another domain may be less variable and require less effort to understand the spatial relationships.

Misinterpreted domains can also lead to mixed patterns in the variograms. These show up as nonsensical continuity or higher than expected nugget effects.

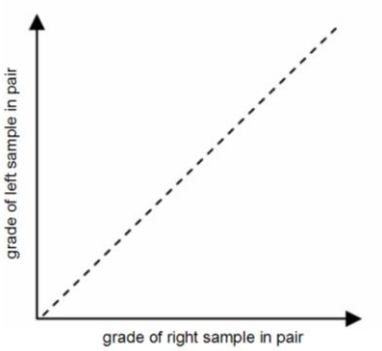
The types of transformations we typically look at are the log-transformations, normal score transformations or the Hermite polynomials. These transformations are an attempt to convert the data to a data set with a more normal distribution.

Other variogram types also exist to dampen the effect of extreme grades.

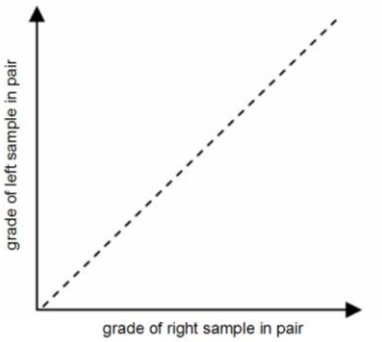
Variograms – concepts behind the calculations

Consider collecting all pairs in a data set that are separated by a given distance (say 20m). Let us tell the samples apart by calling the left-most sample of the pair the "left sample" and the right-most sample of the pair the "right sample".

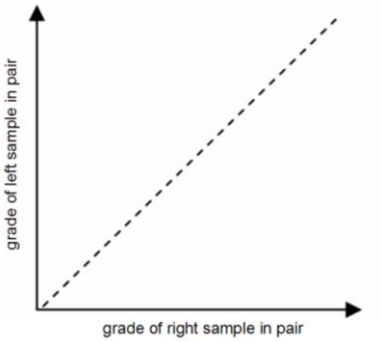
We can then plot the grade of the pairs of samples in a scatterplot with the left sample plotted against the right sample (called an h-scatterplot)



Now consider the h-scatterplot of samples 2 x 20m apart.



…. And 3 x 20m apart?



What difference do you expect to see in the cloud of points?

As the separation between pairs of samples increases, the grades of the pairs become more different, and the cloud of points becomes broader or less correlated.

One measure of the broadness of the cloud at each separation is the correlation coefficient. When this is expressed as a function of the separation distance, the measure becomes the correlation function or the correlogram.

The variation expressed as a function of separation distance is the covariance function.

Another way to express this variation is the "moment of inertia" about the one-to-one line. The "moment of inertia" is probably best understood when imagining an ice-skater doing a pirouette. At first, the ice-skater begins with her arms outstretched. As she pulls her arms in, she reduces her "moment of inertia" and spins faster. This is also referred to as "conservation of momentum". Think of the one-to-one line as the ice-skater's spine and the cloud of points as the amount of space taken up by the ice-skater. The formula for the "moment of inertia" is the semi-variogram formula, where the ½ reflects our interest in the perpendicular difference from the one-to-one line:

Half average square difference between pairs.

Note that any "odd" sample pair will have leverage in all three approaches to calculating the variability according to separation.

Calculating variograms

A variogram[10] is a plot of the typical differences in grade plotted against the distance separating samples.

A variogram may be orientated so the typical differences between samples may be based only on the samples that are in a specific orientation (give or take a few degrees).

To calculate a point on a variogram plot:

- collect all the samples pairs in the database that are separated by a specific distance,
- calculate the difference between the sample grades,
- square the differences (to prevent positives and negatives cancelling out),
- add up all the square differences,
- divide by the number of pairs,
- scale the variogram by a half so that the population variance and the plateau of the variogram (also called the sill) are the same, and
- plot the point on the graph of variogram values against the separation distance.

The variogram formula for a specific distance is

$$variogram(\textit{for a specific separation}) = \frac{sum(\textit{sample value} - \textit{sample value}(\textit{specific distance away}))^2}{2 \times (\textit{number of pairs being compared})}$$

[10] Note: here I call this the "variogram". To be theoretically correct I should call this the "semi-variogram". Since I tend to use standardised variograms – all the semi-variogram values are scaled by dividing each semi-variogram point by the overall population variance – the variogram to me is more about the relative change between the calculated points than the actual points themselves.

Variogram terms

Features used to describe a variogram are:

The **nugget effect**: the typical difference between the samples where we to take samples almost adjacent to each other. Imagine splitting core and submitting both halves. The difference between the grade of the core halves would be the nugget effect. Precious metals are expected to have a higher nugget effect than based metals.

The **sill**: as the separation between samples increases, so the difference between them increases, until we reach a distance beyond which the difference between sample grades is not dependent on their separation, but is the same as the background variability or the population variance. This plateau in the variogram value is called the sill.

The **range**: the range is the distance beyond which the samples are no longer spatially correlated. The range is the distance at which the total sill is reached.

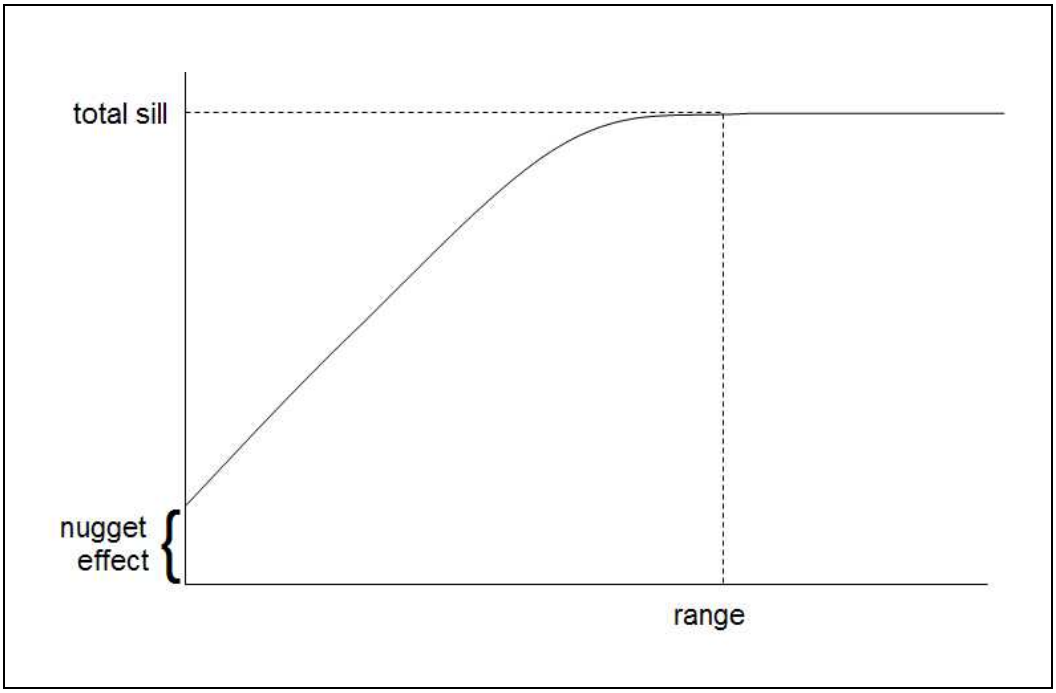


Figure 101 Variogram terms

Directional variograms

By comparing variograms for pairs restricted to directions, we can evaluate the direction of lowest difference for longest range (or the orientation of mineralisation).

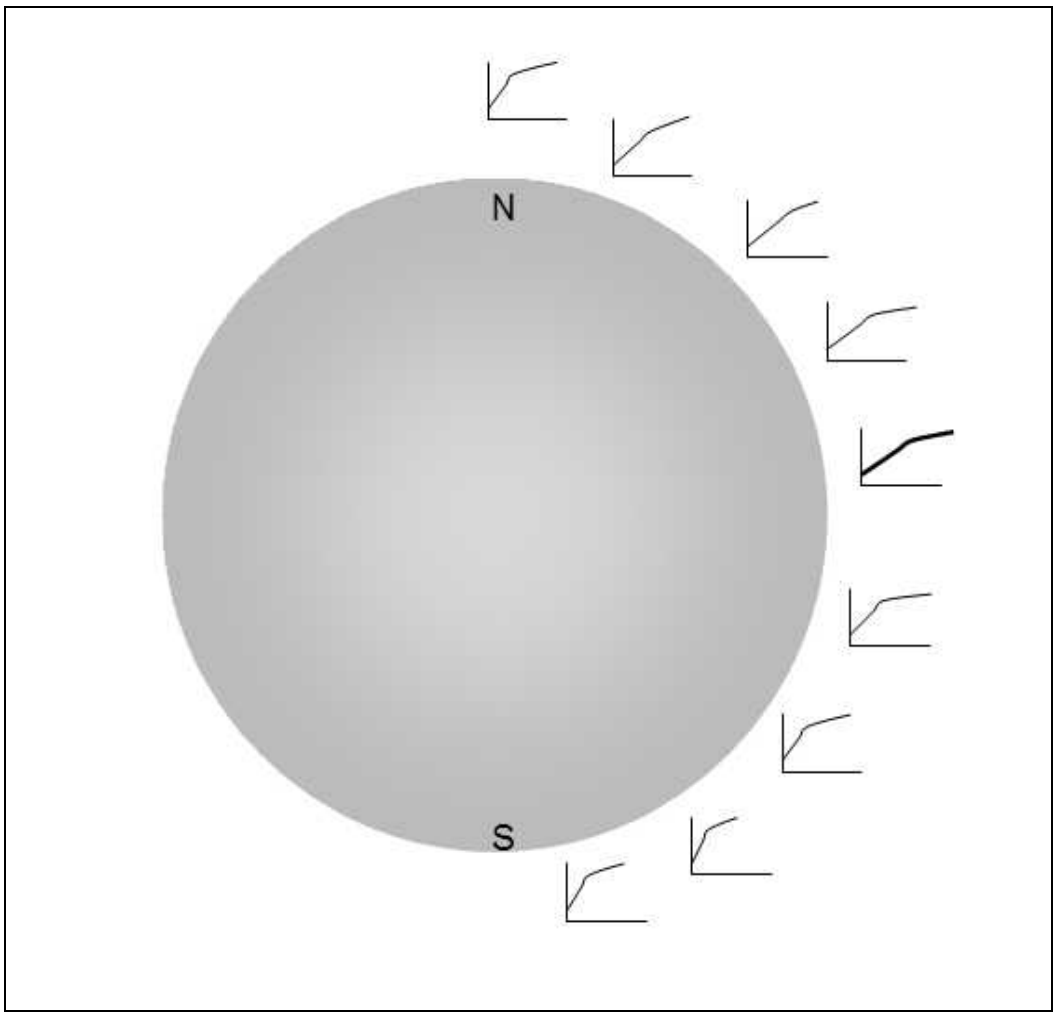


Figure 102 Calculating directional variograms

An easier way to view and interpret variograms for direction is to plot the variogram values according to distance within a direction and then to contour the variogram values. The direction with the lowest variability for the longest range is the direction of maximum continuity.

Study this progression to understand how a horizontal fan can be calculated and interpreted.

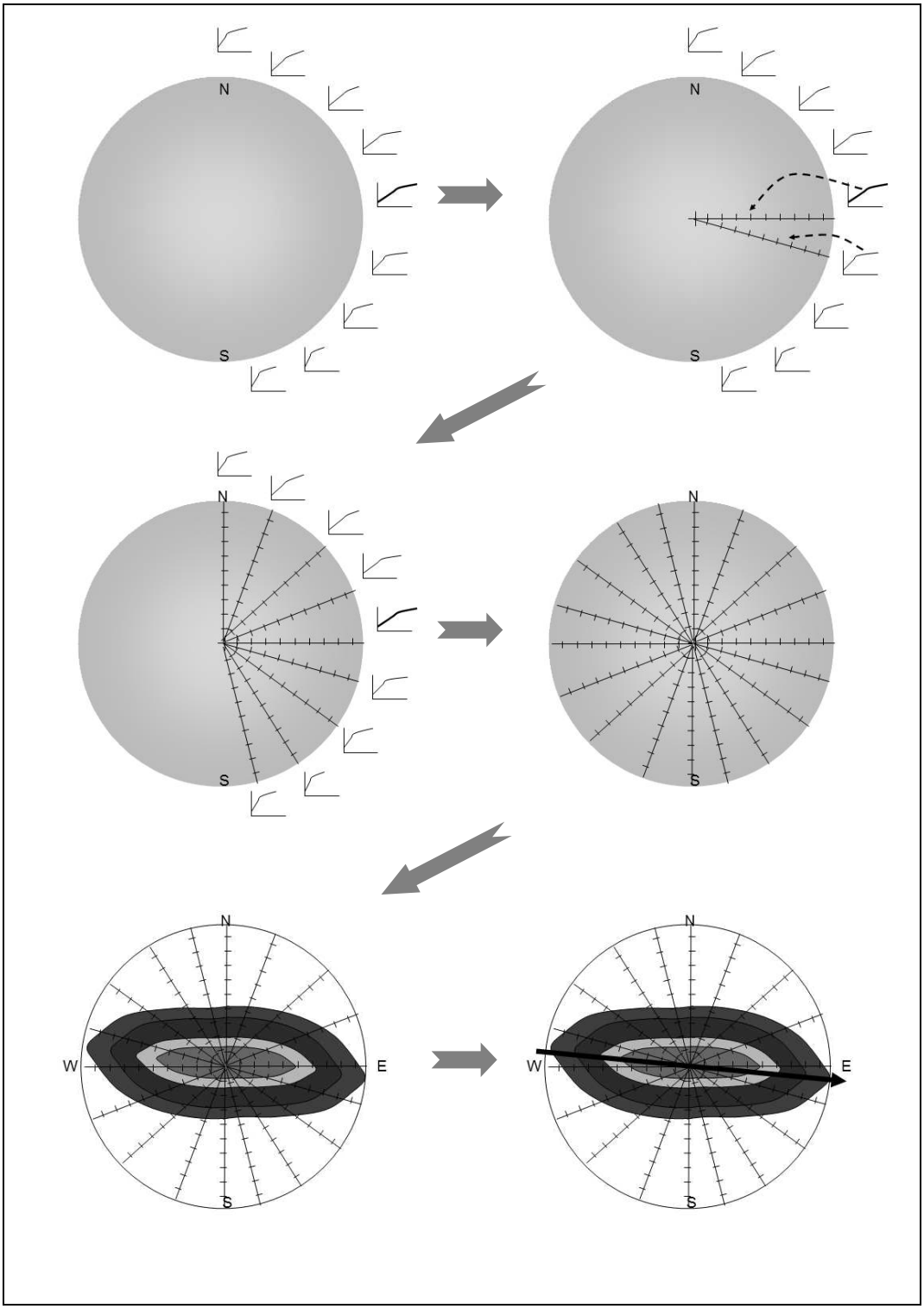
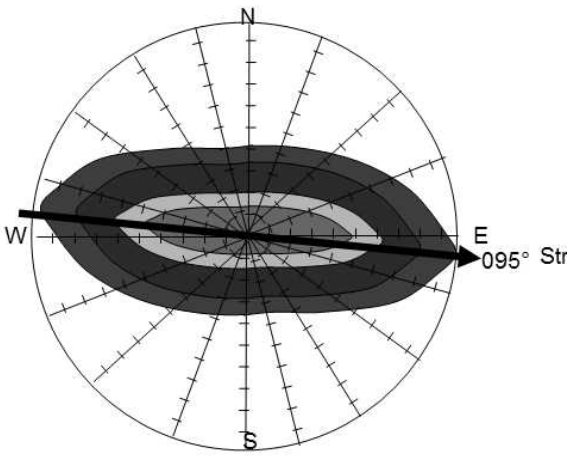
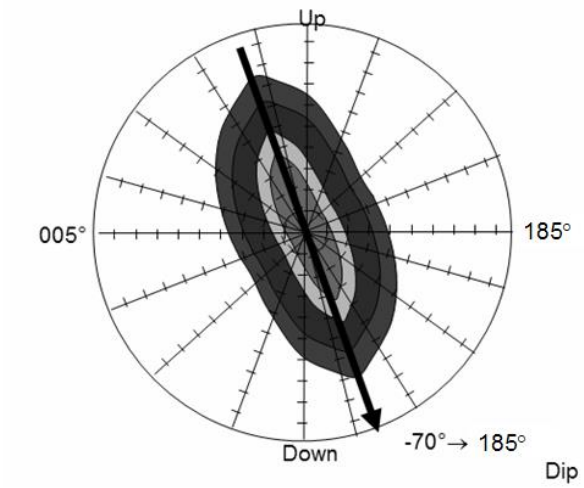


Figure 103 Build up of a variogram map

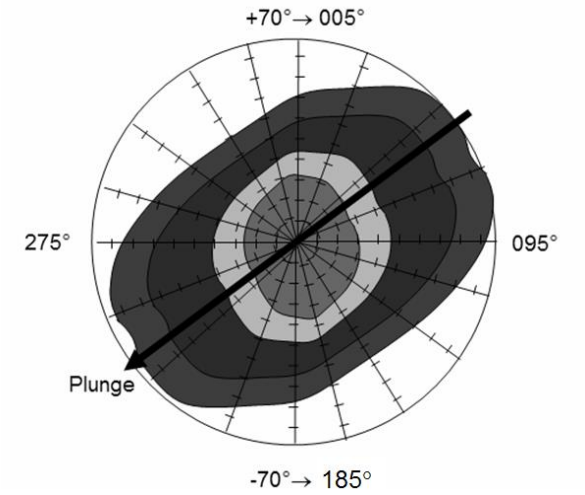
Once we have interpreted a mineralisation strike, we can investigate the dip (which will have an azimuth at 90° to the strike) by repeating the process above, except we restrict the direction analysis to all possible dip directions with an azimuth of the strike + 90°. Once we have the strike and dip, we have the dip plane. The plunge of the mineralisation should be evident as the direction of greatest continuity on the dip plane.



Establish strike:
Variograms with zero dip to establish mineralisation strike (direction with strongest correlation for longest separation)

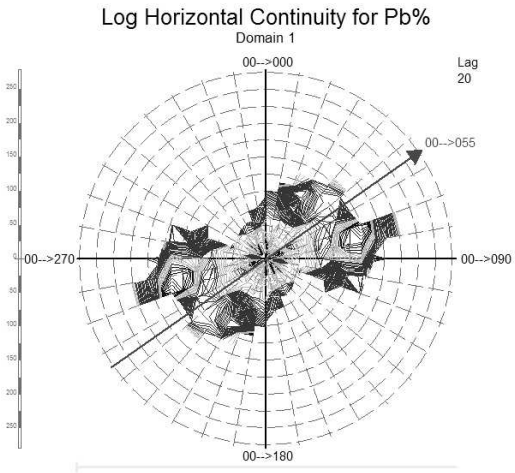


Establish dip:
Variograms with azimuth ±90° to strike and dip components between 0° and -90° to establish mineralisation dip (direction with strongest correlation for longest separation at 90° to strike)



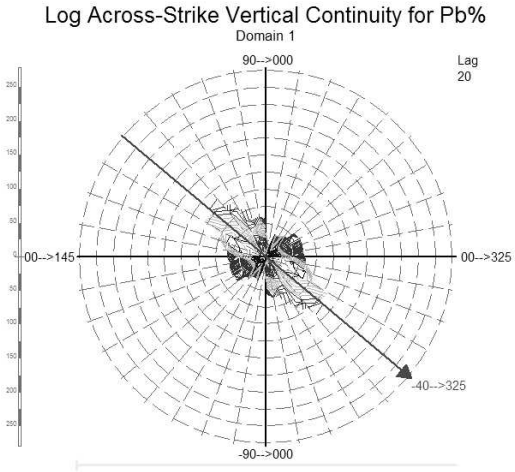
Establish plunge:
Variograms within the dip plane (joining strike to dip) to establish the plunge of mineralisation (direction with strongest correlation for longest separation within the dip plane)

Figure 104 Process for interpreting orientation of mineralisation continuity



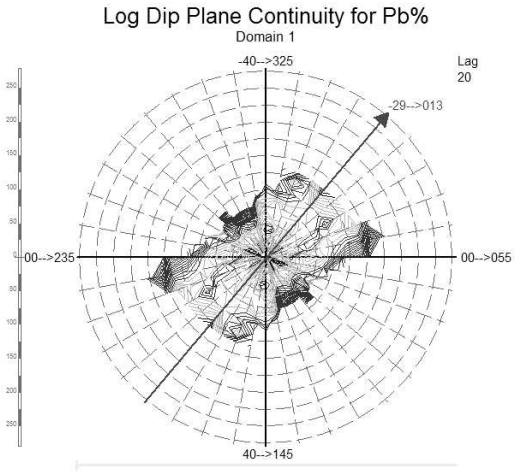
Establish strike:

Variograms with zero dip to establish mineralisation strike (direction with strongest correlation for longest separation)



Establish dip:

Variograms with azimuth ±90° to strike and dip components between 0° and -90° to establish mineralisation dip (direction with strongest correlation for longest separation at 90° to strike)



Establish plunge:

Variograms within the dip plane (joining strike to dip) to establish the plunge of mineralisation (direction with strongest correlation for longest separation within the dip plane)

Figure 105 Real example of interpreting mineralisation orientations

Consider how these three planes fit together to describe an ellipsoid of continuity.

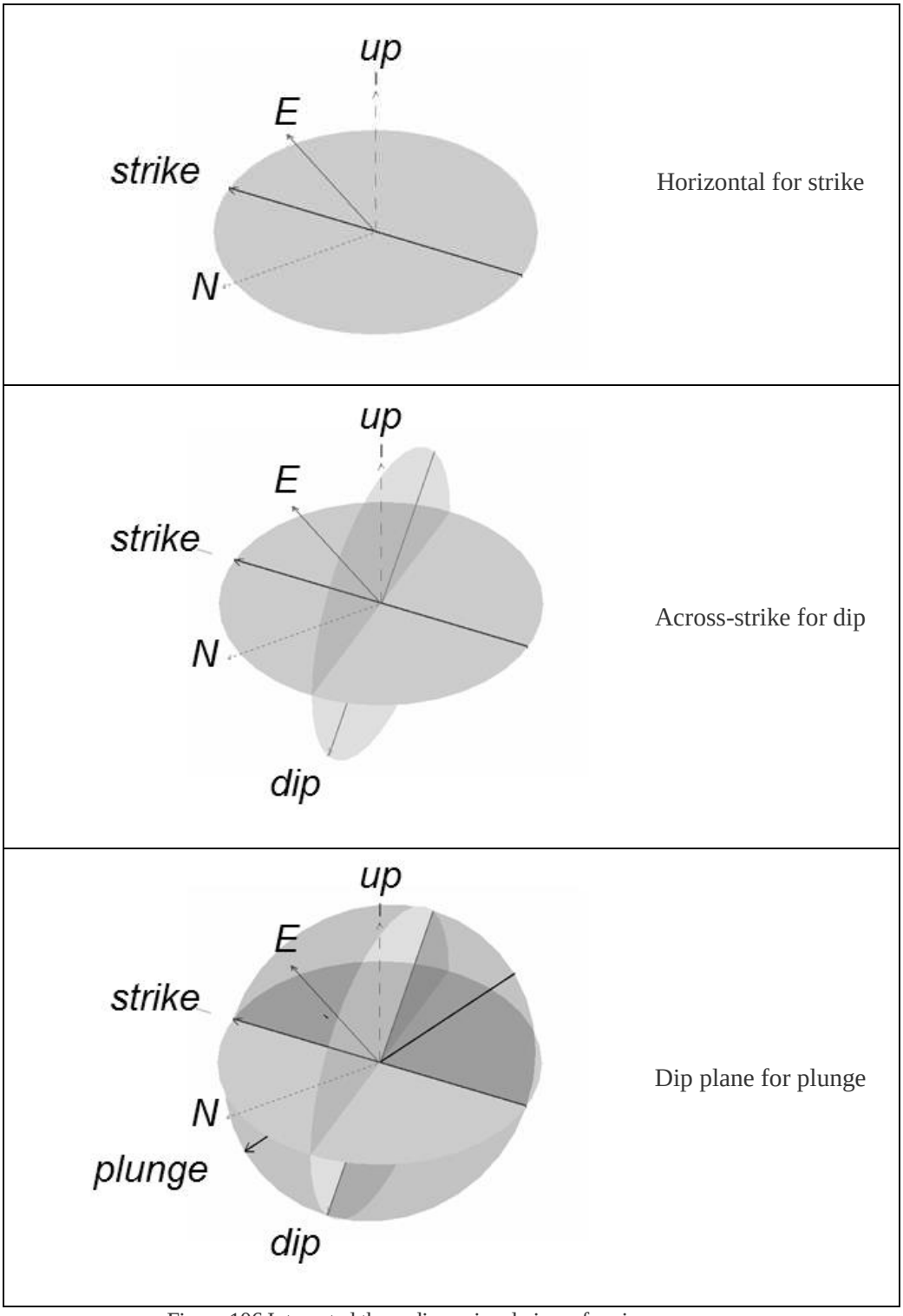


Figure 106 Integrated three-dimensional view of variogram maps

Selecting directions for modelling

Once we have determined the orientation of mineralisation continuity, we need to model the variability. The purpose here is to establish the grade continuity for all possible distances and orientations. This will help during estimation, particularly when the distance and orientation between a block to be estimated and a sample is very different to the configuration of the sample data. Variogram models also make the estimation process more efficient – entering an equation into the process provides a quick calculation of expected variability for any distance and orientation and it is more than possible that the separation between a sample and the block is not available in the sample-to-sample comparison. We build a three-dimensional weighting relationship by modelling the variogram in three-dimensions to mimic the patterns we see in the data.

The easiest way to get to the directions of continuity for variogram modelling is (see the stereonet in Figure 107):

1. Analyse all possible strike directions (all variograms with zero dip) for the direction with lowest variability for the longest range (see "A: strike")
2. The dip direction is at 90° to strike, so analyse all possible dip directions (variograms with an azimuth of strike + 90° and all possible dip components) and determine the dip direction as the direction with lowest variability for the longest range (see "B: dip")
3. The strike and dip directions define the dip plane, which will contain the plunge component[11]. Analyse the variograms in the dip plane to determine the direction with lowest variability for the longest range (see "C: plunge")
4. We model the variogram in the plunge direction, which is also called: the major direction, the primary direction or direction 1.
5. The second direction for modelling is at 90° to the plunge within the dip plane is the direction with the shortest continuity within the dip plane (see direction 2 in Figure 107). This direction is also called the semi-major or secondary direction.
6. The third direction for modelling is at 90° to both direction 1 and direction 2 (see direction 3 in Figure 107). This direction is the shortest overall direction and is also called the minor or tertiary direction.

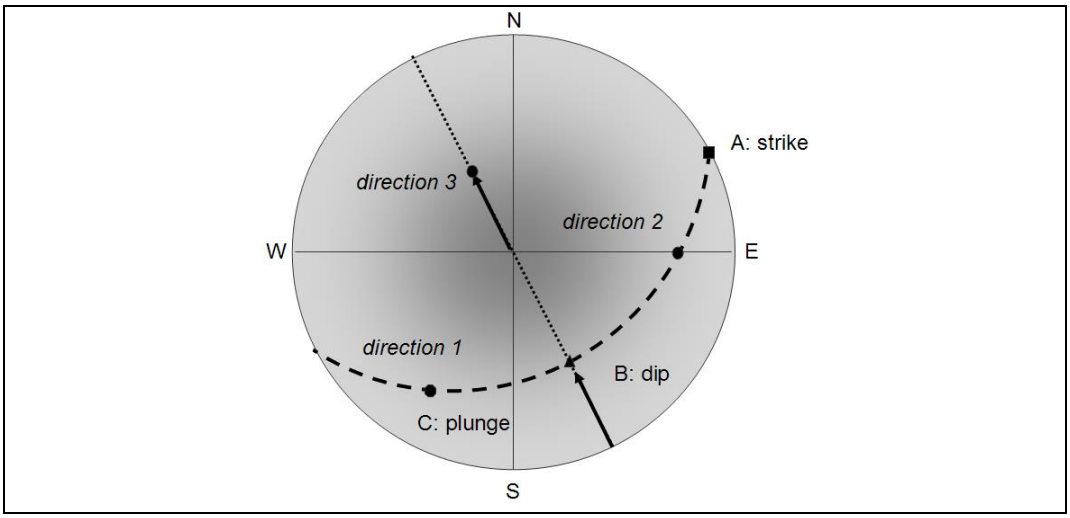


Figure 107 Schematic stereonet of significant variogram directions

[11] If there is no plunge the direction of greatest continuity will be either the strike or the dip direction.

Calculation parameters

The parameters used to calculate variograms include:

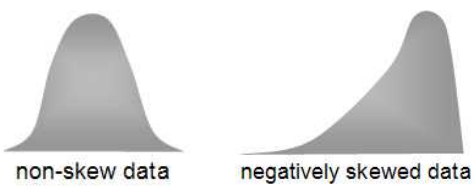
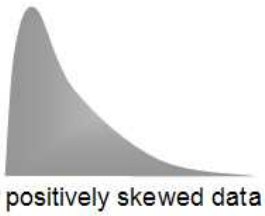
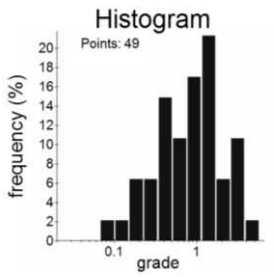
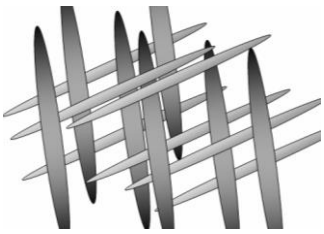
- the lag spacing – the increment in separation distance to calculate each point on the variogram,
- the lag tolerance – typically set at ½ the lag spacing,
- the number of directions to calculate the directional variograms,
- the angular tolerance – the tolerance in angle either side of the direction used for collecting pairs of samples, and
- the maximum separation distance for calculating variograms.

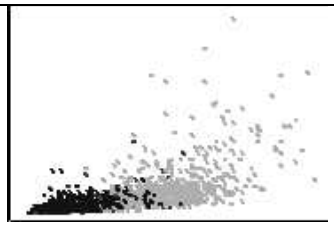
Types of variograms

There are a variety of types of variograms – all designed to overcome some effect in the data (Table 5). Each type of variogram is based on the variogram calculation presented above. The types of variograms differ in the way data is transformed before the calculation, or on the way the variogram is scaled or standardised once the differences between the sample values are calculated.

In this section, the same data set is used for the each of the variograms described in this section to help you gauge the impact of various transformations (Figure 109 to Figure 117). Typically, you can expect a difference in the interpreted nugget effect (relative to the total sill) with the model ranges remaining close to stable.

Table 5 Selection of variogram type

Situation	Variogram type
Non-skew or negative skew 	Traditional variogram Normal Score variogram Indicator variogram Covariance Correlogram
Positive skew 	Log-normal variogram Normal Score variogram Indicator variogram
Low number of samples 	Pairwise relative variogram
Variograms designed to pacify mathematicians ΓΔΩΘλδβμστυ	Semi-rodogram Semi-madogram
Describe variability in cross-cutting, rotating or multi-phase/mixed mineralisation 	Indicator variograms
Describe relationship between elements	Cross-variograms



Variograms for non-skew data

Traditional variogram

The traditional variogram is the basis for describing the relationship between samples behind the kriging algorithm. A key assumption is that the grades compared come from the same population and that the difference between the grades depends only on their relative separation.

A variogram value for a given separation distance is calculated as: one half the average square difference in grade between pairs of samples separated by a given distance.

Let us look at a simple example (Figure 108).

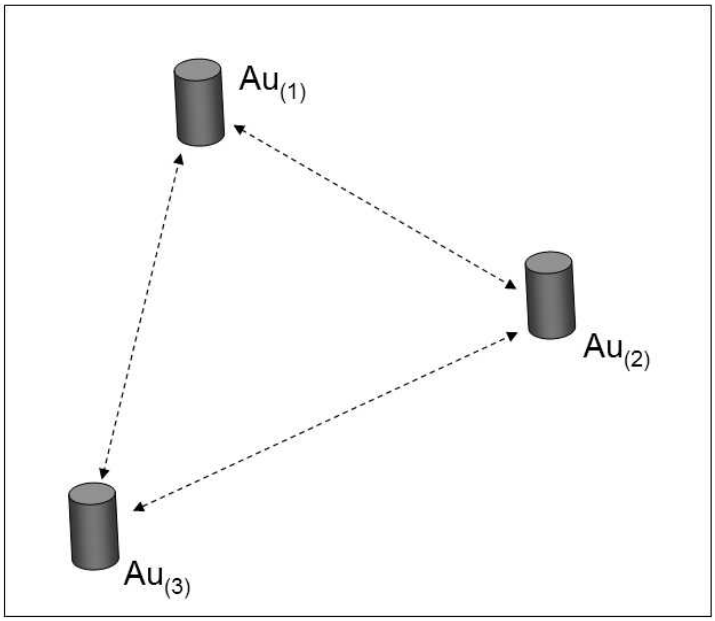


Figure 108 Example of three equidistant samples to compare

The traditional variogram value for the distance separating these three samples is:

$$\frac{(Au_{(1)} - Au_{(2)})^2 + (Au_{(1)} - Au_{(3)}))^2 + (Au_{(2)} - Au_{(3)})^2}{2 \times 3}$$

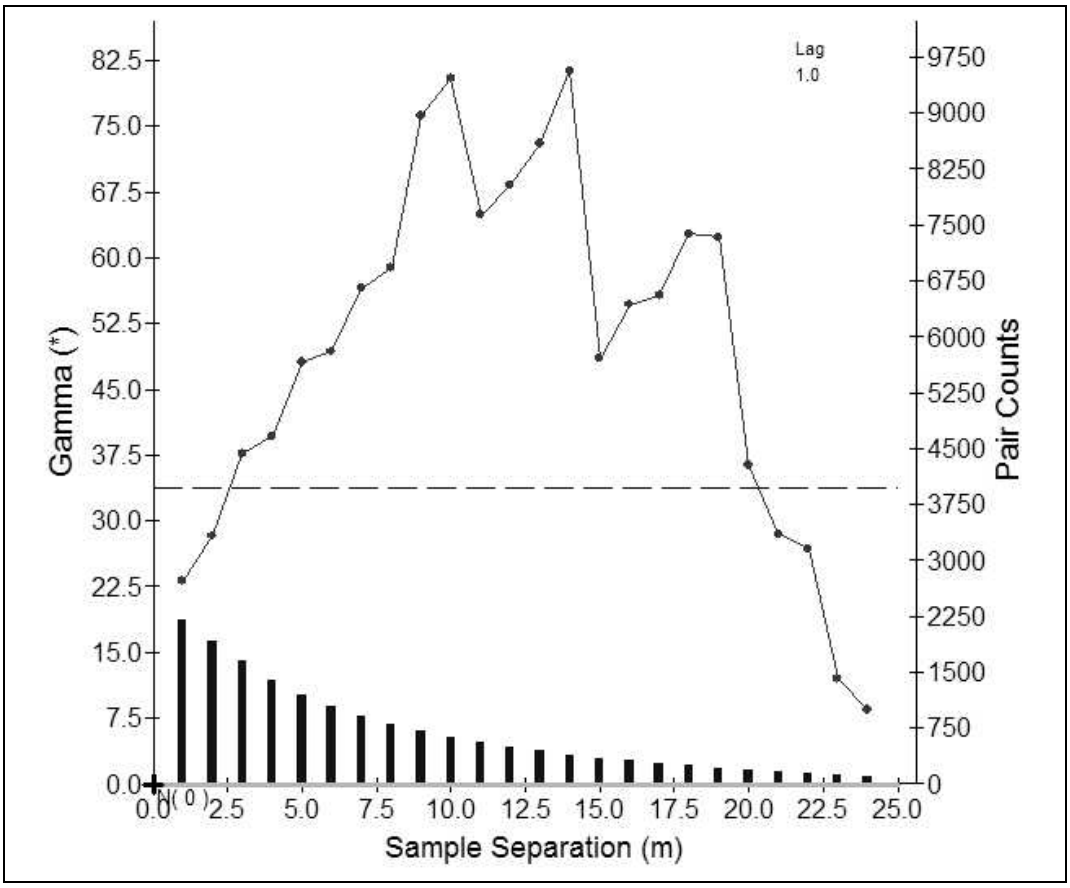


Figure 109 Example of traditional variogram

Covariance

The covariance is calculated by averaging the product of the sample grades from the two samples separated a given distance apart and then subtracting the product. This gives us:

[average(sample grade 1 * sample grade 2)]
Minus
[the average grade from the first samples * the average grade from the second samples]

For the example in Figure 108, this is the same as:

$$[(Au_{(1)} \times Au_{(2)}) + (Au_{(1)} \times Au_{(3)})) + (Au_{(2)} \times Au_{(3)})] \div 3$$

$$- [\text{average } (Au_{(1)}, Au_{(1)}, Au_{(2)}) \times \text{average } (Au_{(2)}, Au_{(3)}, Au_{(3)})]$$

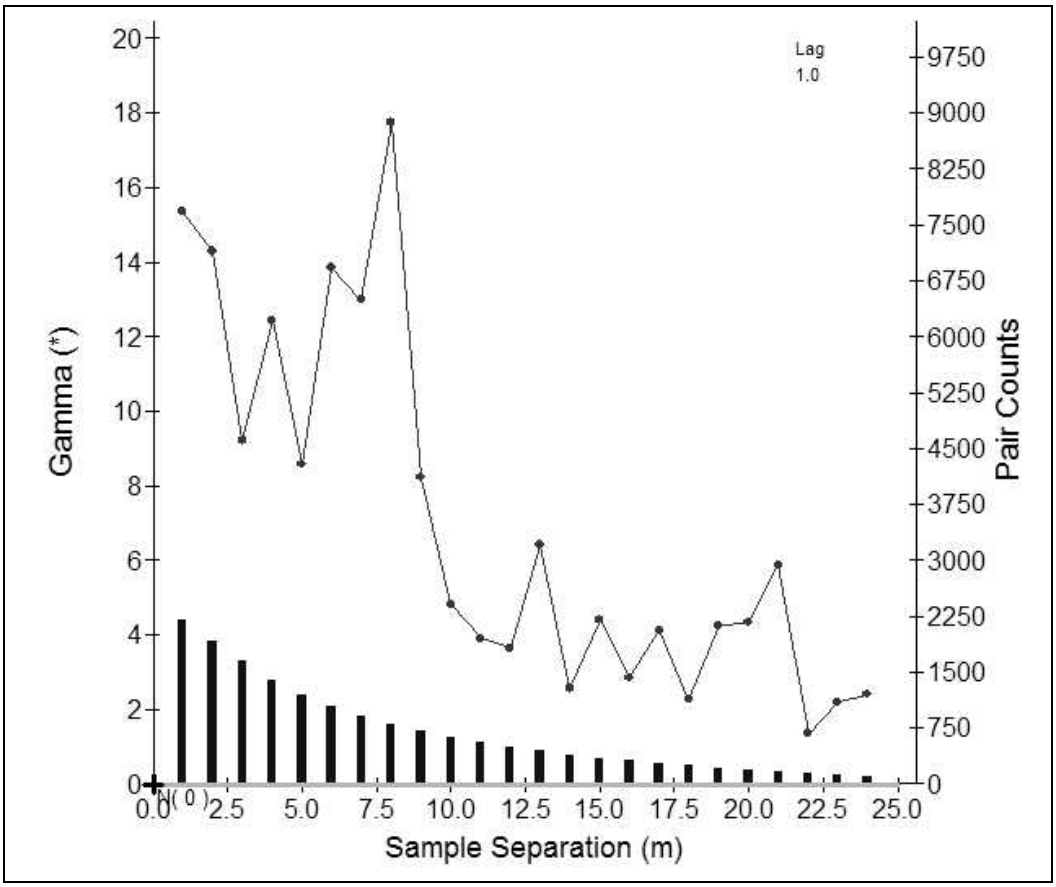


Figure 110 Example of covariance variogram

The first and second set of samples could be from the same set of data, or from two different elements. When the two samples represent different elements, the covariance is the sample cross-covariance.

Correlogram

The correlogram is the covariance standardised by:

(Standard deviations of all the first sample grades)
x (Standard deviations of all the second sample grades)

For the example in Figure 108, this is the same as:

$$\frac{\left\{ \begin{array}{l} [(Au_{(1)} \times Au_{(2)}) + (Au_{(1)} \times Au_{(3)})) + (Au_{(2)} \times Au_{(3)})] \div 3 \\ - \ [\text{average}\ (Au_{(1)}, Au_{(1)}, Au_{(2)}) \times \text{average}\ (Au_{(2)}, Au_{(3)}, Au_{(3)})] \end{array} \right\}}{[\text{std dev}\ (Au_{(1)}, Au_{(1)}, Au_{(2)}) \times \text{std dev}\ (Au_{(2)}, Au_{(3)}, Au_{(3)})]}$$

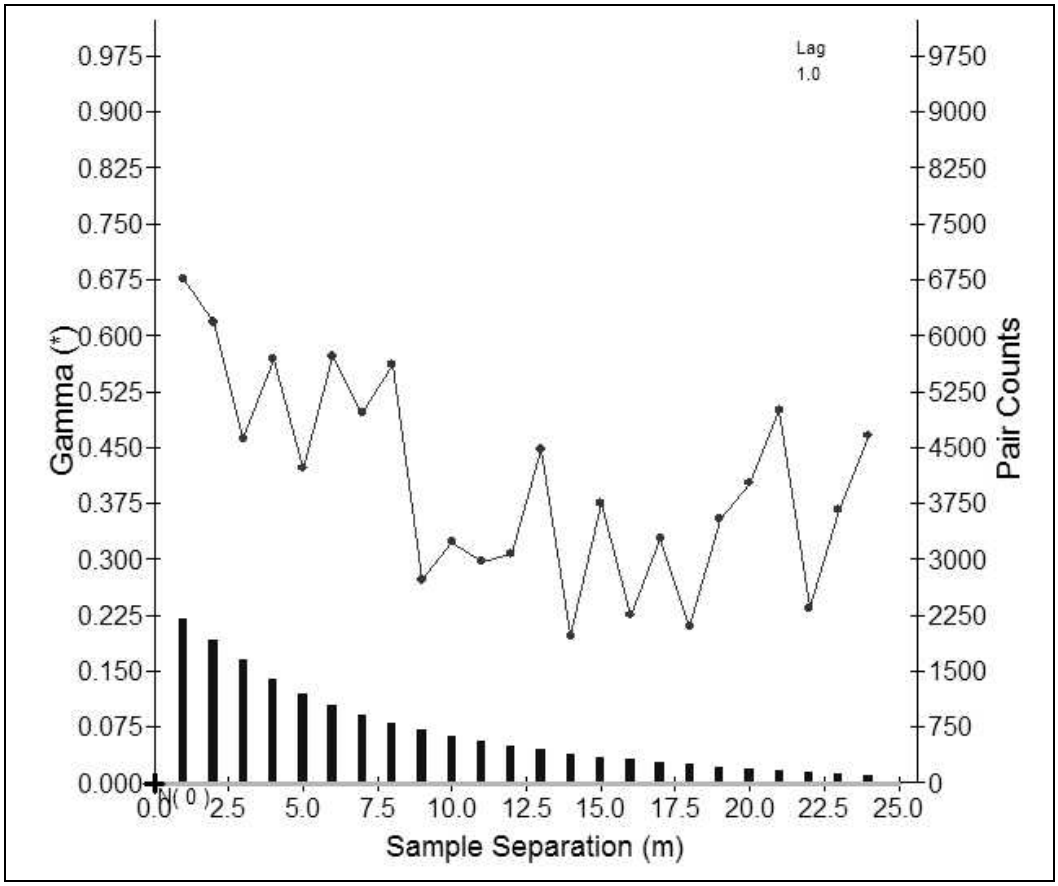


Figure 111 Example of correlogram variogram

When the two sample sets refer to different elements, then the nugget effect should correspond to the correlation between the two elements.

Variograms to limit the influence of extreme grades (skewed data)

General relative

The general relative variogram is the traditional variogram standardised by the square root of the average between the mean grade for the first set of samples and the average of the second set of samples.

$$\left\{ \frac{(Au_{(1)} - Au_{(2)})^2 + (Au_{(1)} - Au_{(3)}))^2 + (Au_{(2)} - Au_{(3)})^2}{2 \times 3} \right\}$$

$$\text{Square root}\{ [\text{average } (Au_{(1)}, Au_{(1)}, Au_{(2)}) \times \text{average } (Au_{(2)}, Au_{(3)}, Au_{(3)})]\}$$

In practice, I have not found this variogram to be particularly useful for mining data sets.

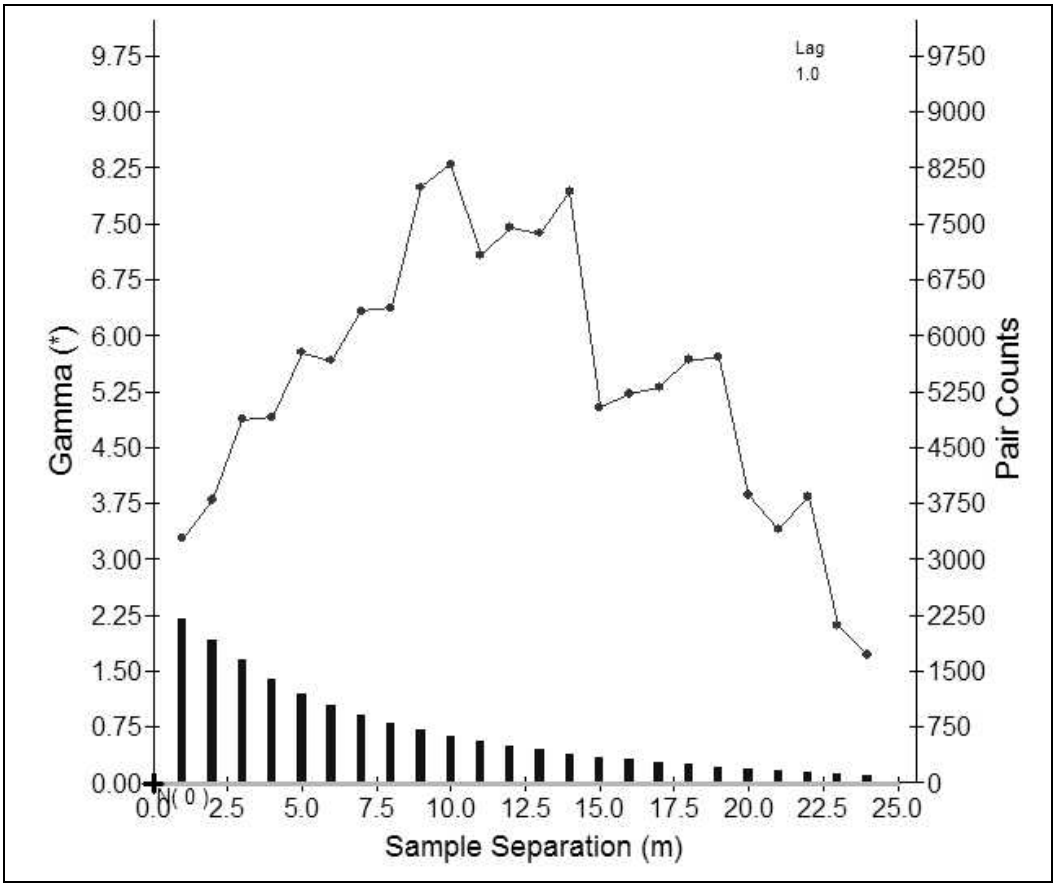


Figure 112 Example of general relative variogram

Log-variogram

The log-variogram is useful for revealing structures when the underlying data is positively skewed. The log-variogram is simply the variogram calculated on the logarithmic transformation of the grades.

For the example presented in Figure 108, the log-variogram is:

$$\frac{(\mathrm{Ln}(Au_{(1)}) - \mathrm{Ln}(Au_{(2)}))^2 + (\mathrm{Ln}(Au_{(1)}) - \mathrm{Ln}(Au_{(3)}))^2 + (\mathrm{Ln}(Au_{(2)}) - \mathrm{Ln}(Au_{(3)}))^2}{2 \times 3}$$

The modelled log-normal variogram parameters need to be re-scaled to reflect the variability of the data. One way to do this is to rescale the nugget affect according to the log-variance and then to distribute the remaining variability according to the sills (David, 1977).

The formula to do this is …

$$\text{Nugget \%} = \frac{(\text{log-variance} * \text{nugget})}{\text{total sill}}$$

$$\text{Transformed Nugget} = 1 \left\{ \frac{[\exp^{(\text{Logvariance} - \text{Nugget \%})} - 1]}{[\exp^{(\text{Logvariance})} - 1]} \right\}$$

The sills for the various model structures can be calculated by distributing the remaining variability (total sill – transformed nugget) according to the relative proportions of sills modelled in the log-variograms. The ranges remain unaltered.

Here is an example of log-variogram transformation in a spreadsheet:

	B	C	D	E	F	G	H	I	J	K	L	M	N
2	**Converting log to normal variograms:**												
3													
4		**1. Enter** log variance :			**2. Enter**				**3. The transformed parameters are**				
5			1.36		log nugget	0.27				nugget:	0.32		
6					log sill 1:	0.12				sill 1:	0.08		
7					log sill 2:	0.94				sill 2:	0.60		
8					log sill 3:	0.00				sill 3:	0.00		
9					Total Sill	1.33				Total Sill	1.00		
10													

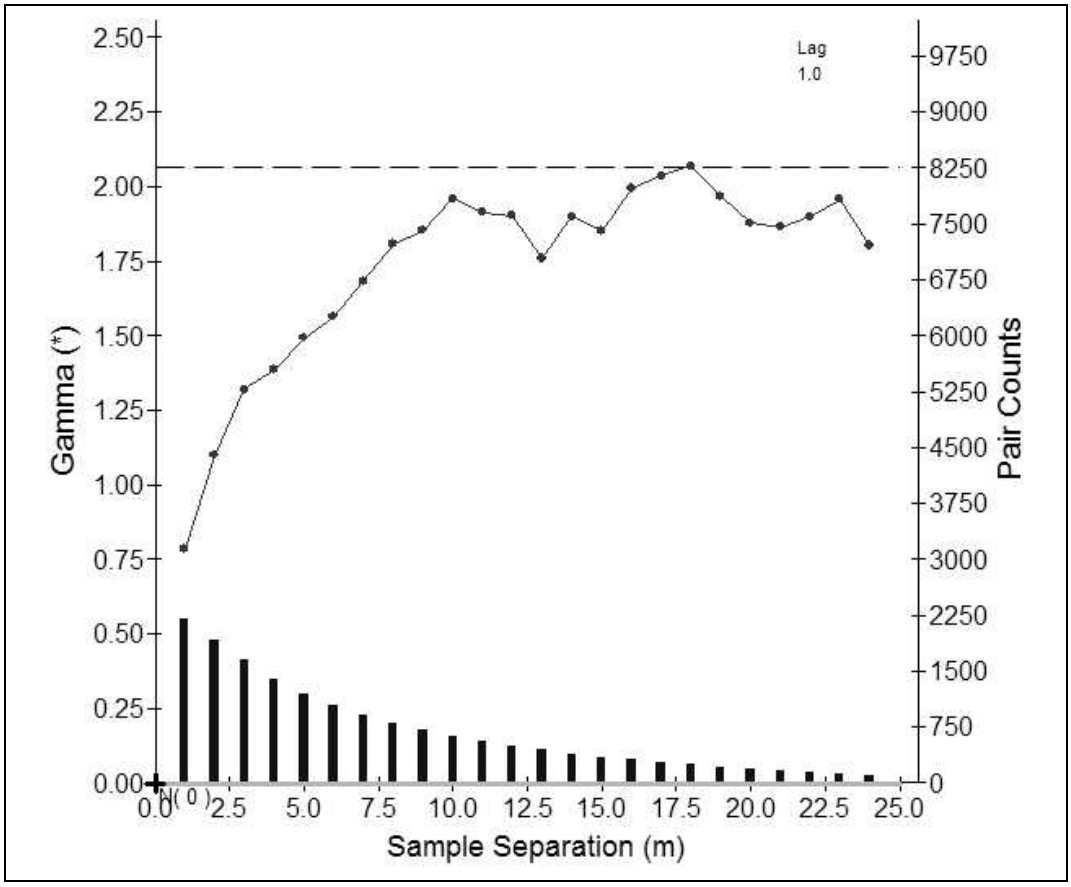


Figure 113 Example of log-variogram

Normal score variogram

The normal score variogram controls the influence of extreme grades by converting the grade distribution to a standard normal distribution before calculating the variogram.

The normal score transform works by calculating the percentile of each sample and reading off the value for a standard normal distribution corresponding to the same percentile (Figure 114).

The normal score variogram is a required variogram for Sequential Gaussian Simulation (SGS). The normal score variogram model parameters are used directly in the SGS algorithms and, in this case, do not require any transformation.

Use of the normal score variogram model parameters to be used in direct grade estimation requires the parameters to be transformed back to reflect grade variability rather than normal score variability. The Hermite Polynomial approach is useful for this.

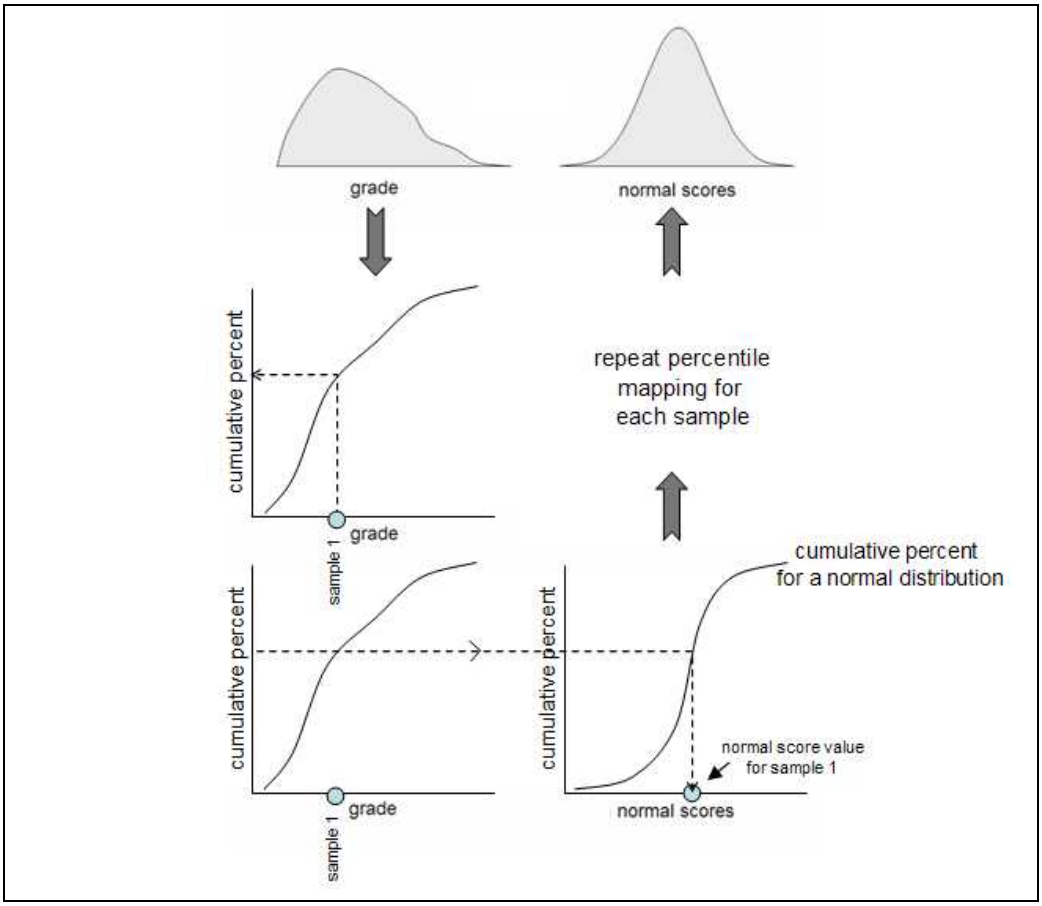


Figure 114 Normal score transformation

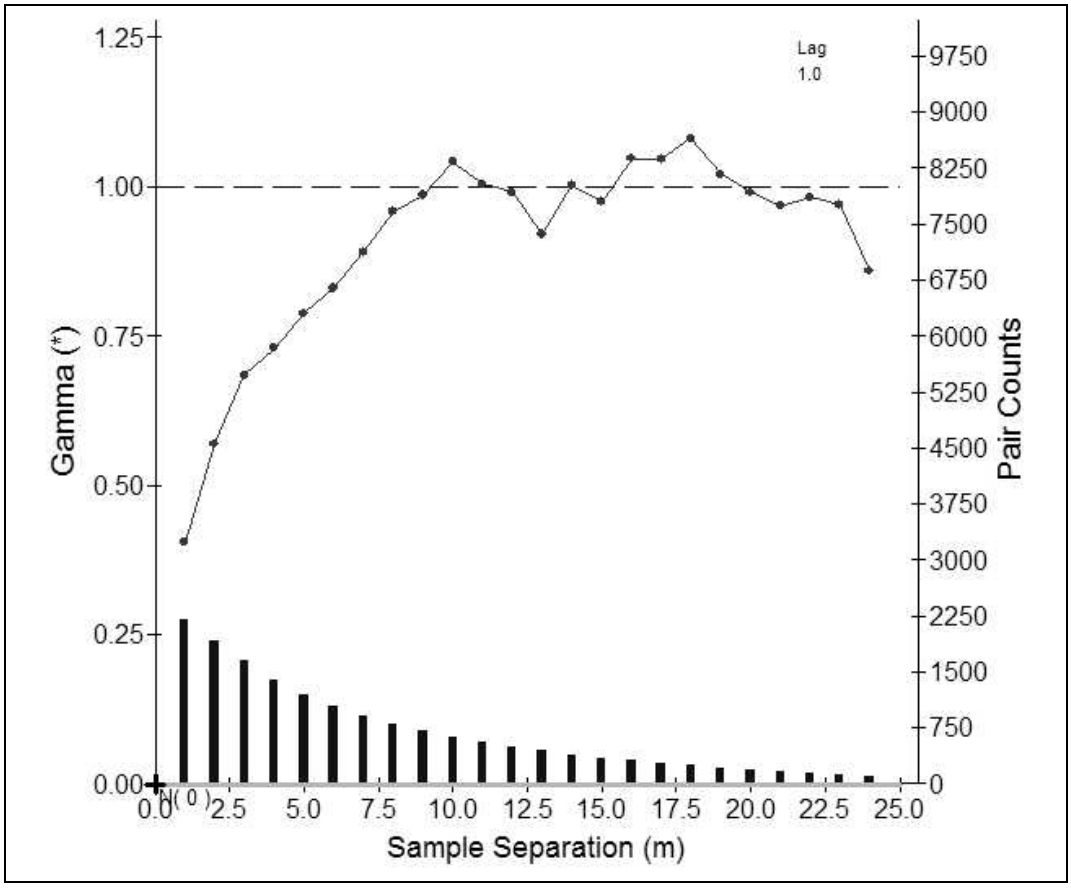


Figure 115 Example of normal score variogram

Variograms for low number of samples

Pairwise relative

The pairwise relative variogram is generated by standardising each square difference between pairs by the square average of the pair of grades.

Following the example in Figure 108, the pairwise relative variogram is calculated as

$$\frac{\left\{ \begin{array}{c} \dfrac{(Au_{(1)} - Au_{(2)})^2}{[(Au_{(1)} - Au_{(2)})] \div 2} \\ + \\ \dfrac{(Au_{(1)} - Au_{(3)}))^2}{[(Au_{(1)} - Au_{(3)})] \div 2} \\ + \\ \dfrac{(Au_{(2)} - Au_{(3)})^2}{[(Au_{(2)} - Au_{(3)})] \div 2} \end{array} \right\}}{2 \times 3}$$

The pairwise relative variogram is stable when there are a limited number of samples and the underlying data distribution is positively skewed. In these circumstances, the pairwise variogram can reveal ranges that are not clear in other variogram types.

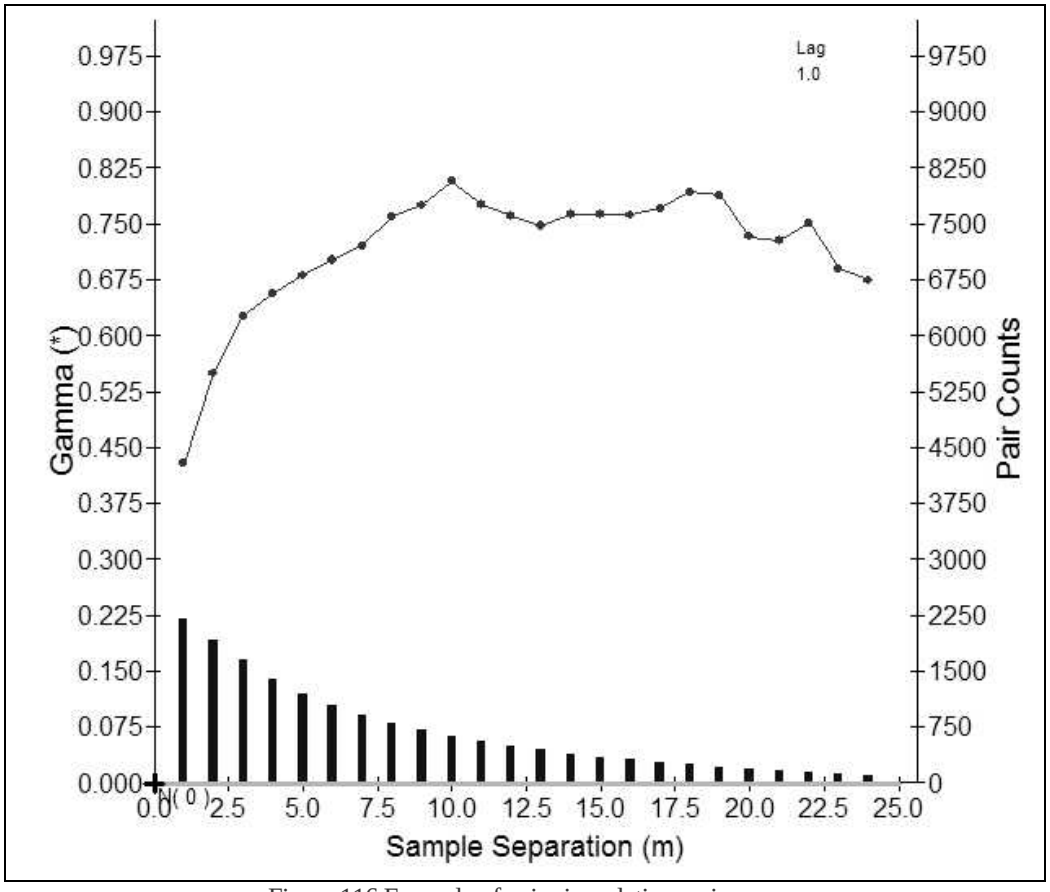


Figure 116 Example of pairwise relative variogram

Variograms designed to pacify mathematicians

The following variograms are usually included in software for completeness rather than significant practical application.

Semi-rodogram

This type of variogram is calculated as: half the average square root of the absolute difference between each pair of the sample grades.

Semi-madogram

This type of variogram is calculated as: half the average absolute difference between each pair of the sample grades.

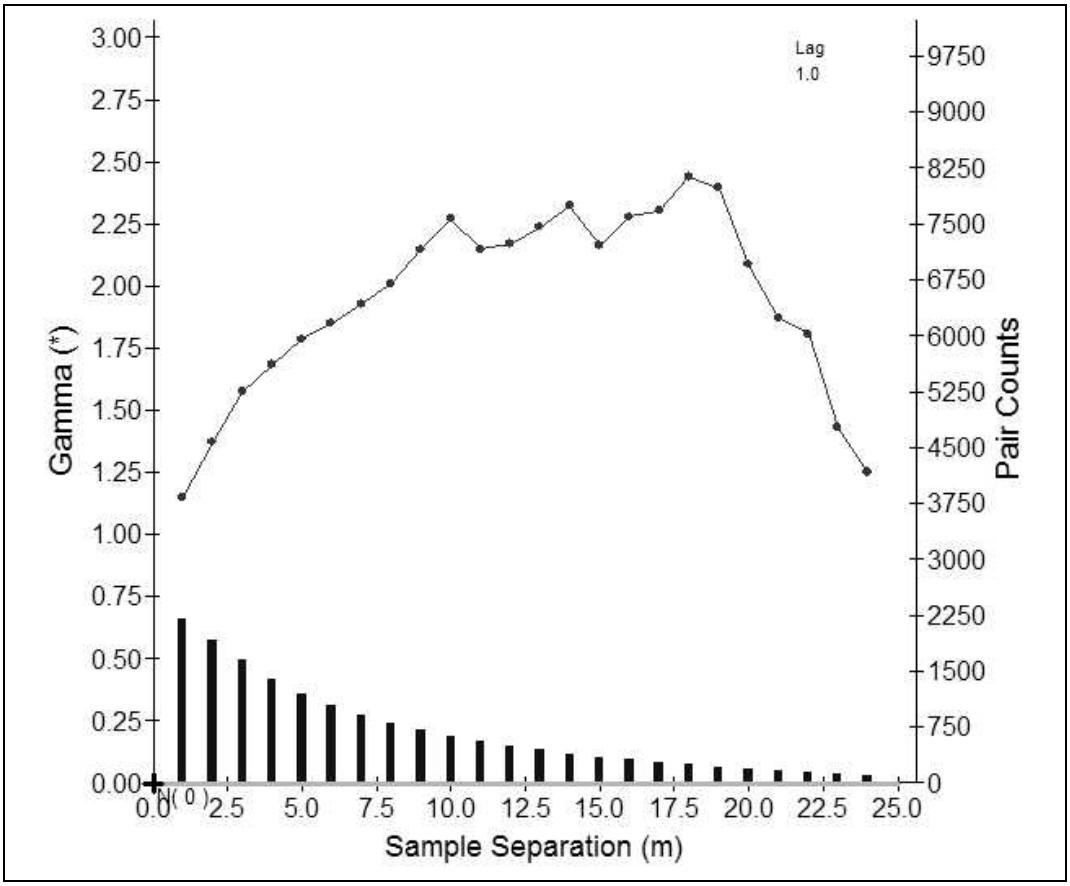


Figure 117 Example of madogram variogram

Variograms to describe variability in cross-cutting, rotating or multi-phase/mixed mineralisation

Indicator variogram

An indicator variogram is the traditional variogram calculated on the indicator codes rather than the grades.

Indicator variograms are useful for describing rotating anisotropy, for example, where low grades are orientated differently to higher grades or in cross-cutting structures.

More information and details on indicator variograms are presented on page 167.

Variograms to describe relationship between elements

Cross-variogram

The cross-variogram is calculated as: half the average
[Difference in grades for element 1] *[Difference in grades for element 2]

The example presented in Figure 108 becomes Figure 118 when considering multi-elements.

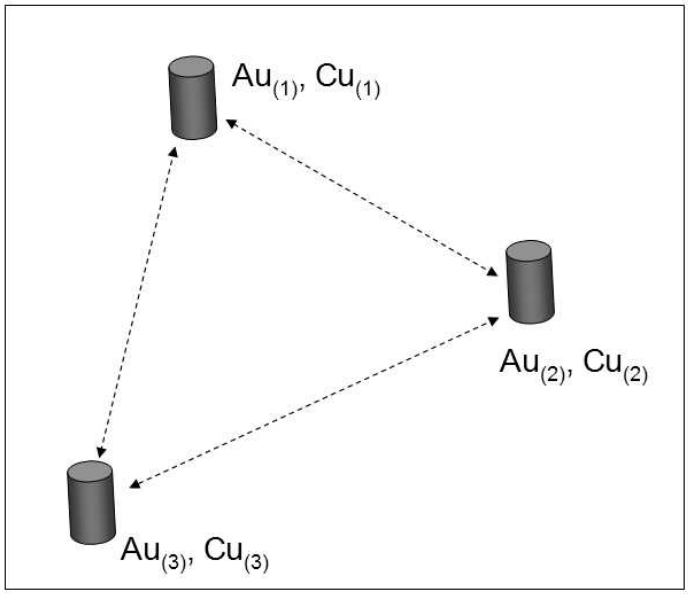


Figure 118 Example of three equidistant multi-element samples to compare

The corresponding cross-variogram calculation is then:

$$\frac{\left\{ \begin{array}{c} [(Au_{(1)} - Au_{(2)}) \times (Cu_{(1)} - Cu_{(2)})] \\ + \\ [(Au_{(1)} - Au_{(3)}) \times (Cu_{(1)} - Cu_{(3)})] \\ + \\ [(Au_{(2)} - Au_{(3)}) \times (Cu_{(2)} - Cu_{(3)})] \end{array} \right\}}{2 \times 3}$$

The cross-variogram results in negative values when the elements are negatively correlated, for example iron and silica (Figure 119).

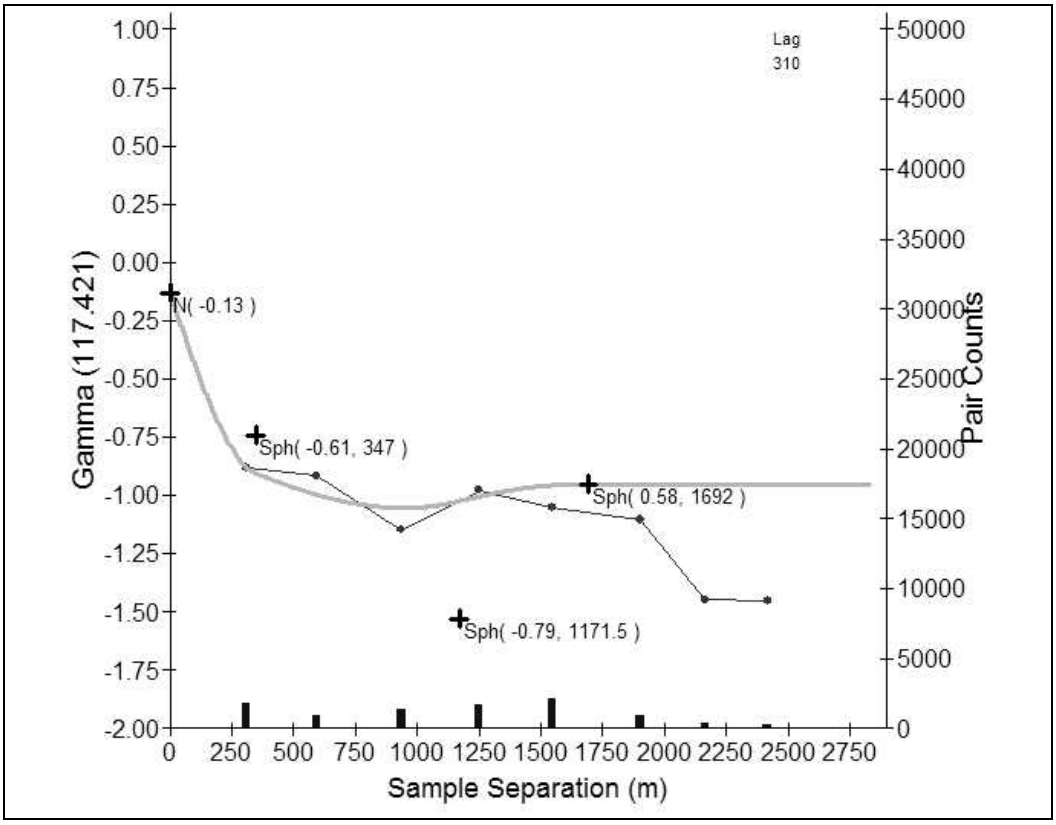


Figure 119 Example of cross variogram between iron and silica

Variogram modelling

Before considering the variogram equations, it is worth noting that modelling the variograms is no more complicated than fitting a line/curve to a series of points.

Activity

Try fitting curves to the line plots in Figure 120.

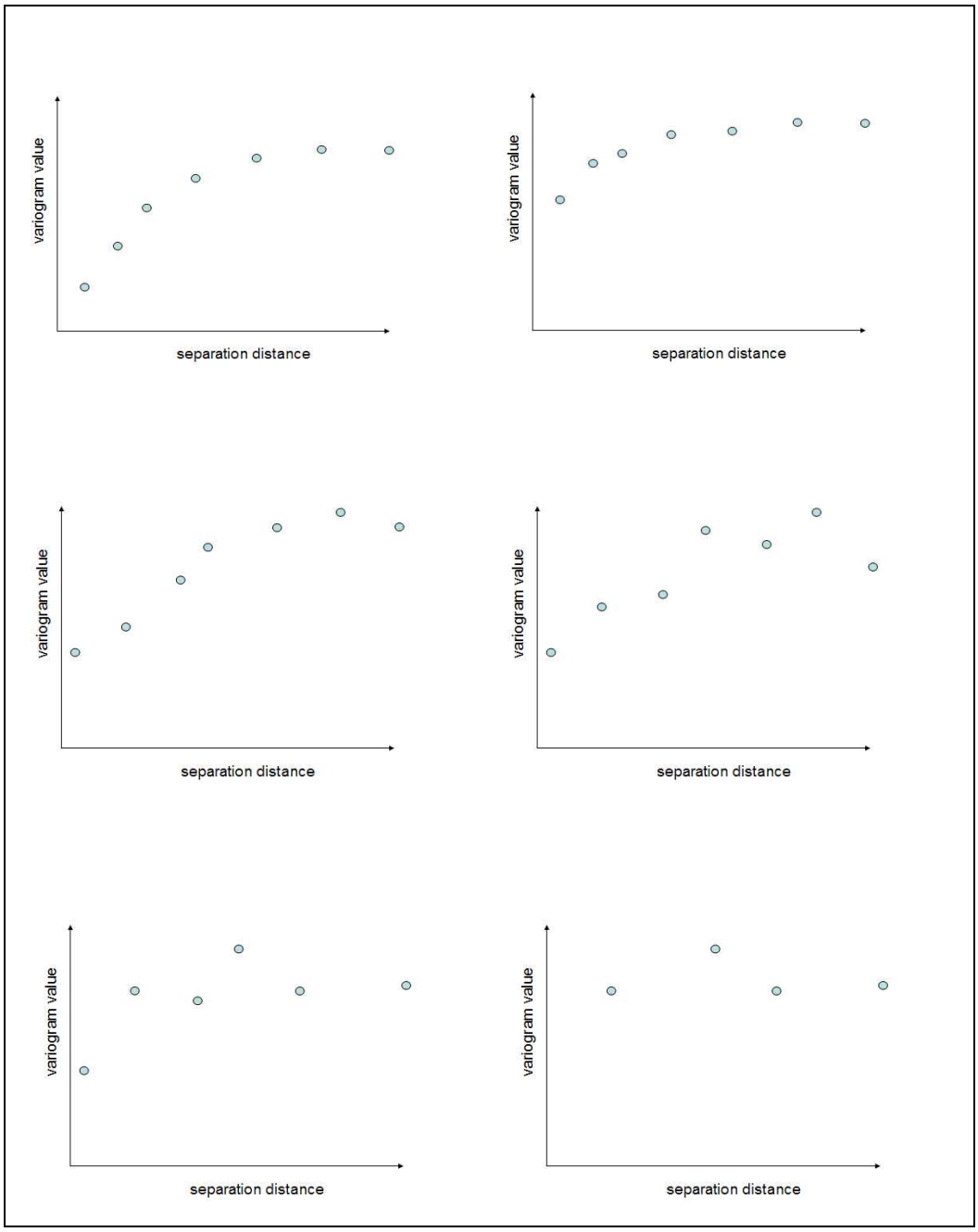


Figure 120 Calculated variograms to model

Special shapes

There are several pre-defined shapes that are known to work well within the kriging system. These include:

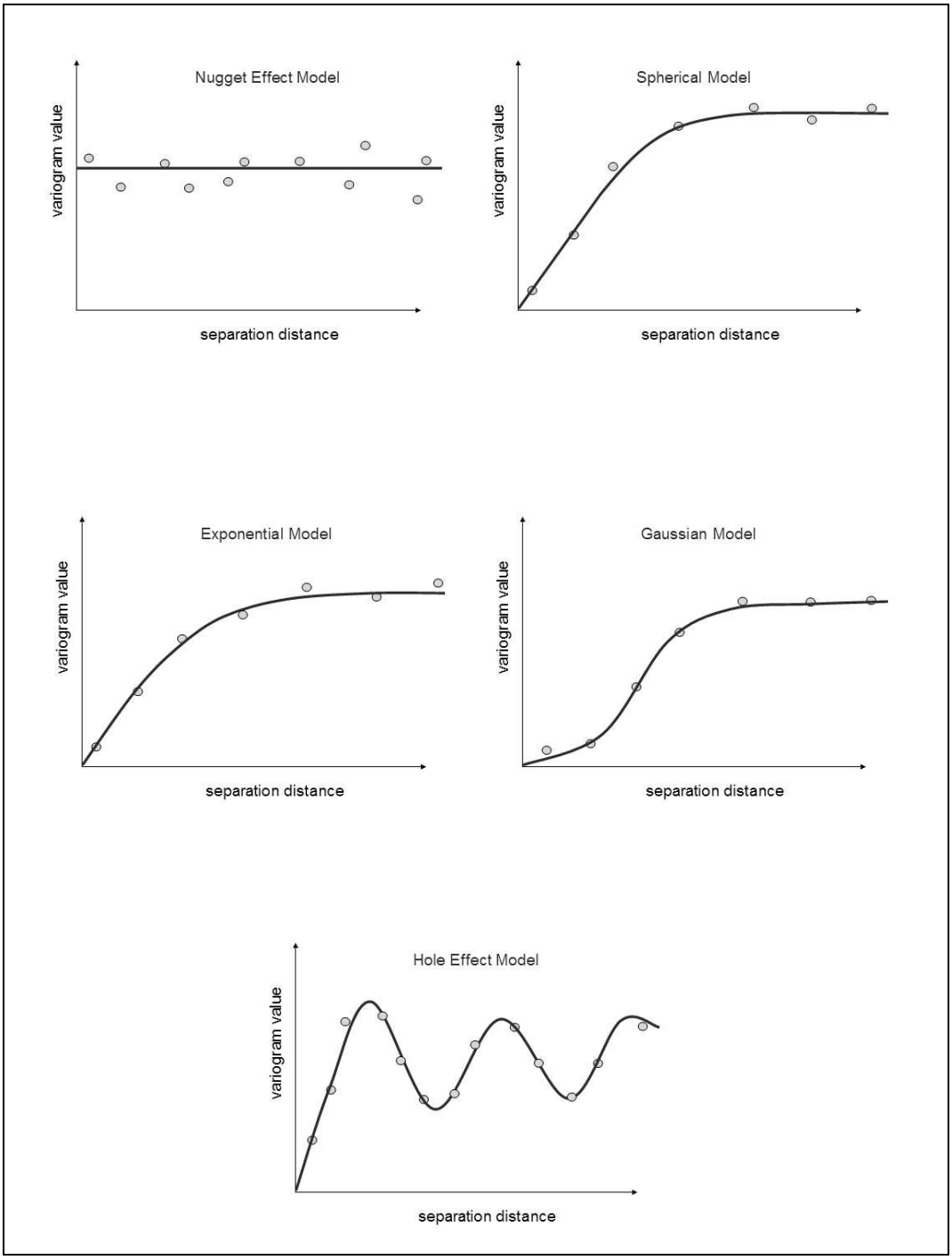


Figure 121 Special shape variogram models

More complex shapes can be constructed by adding (or "nesting") these pre-defined shapes.

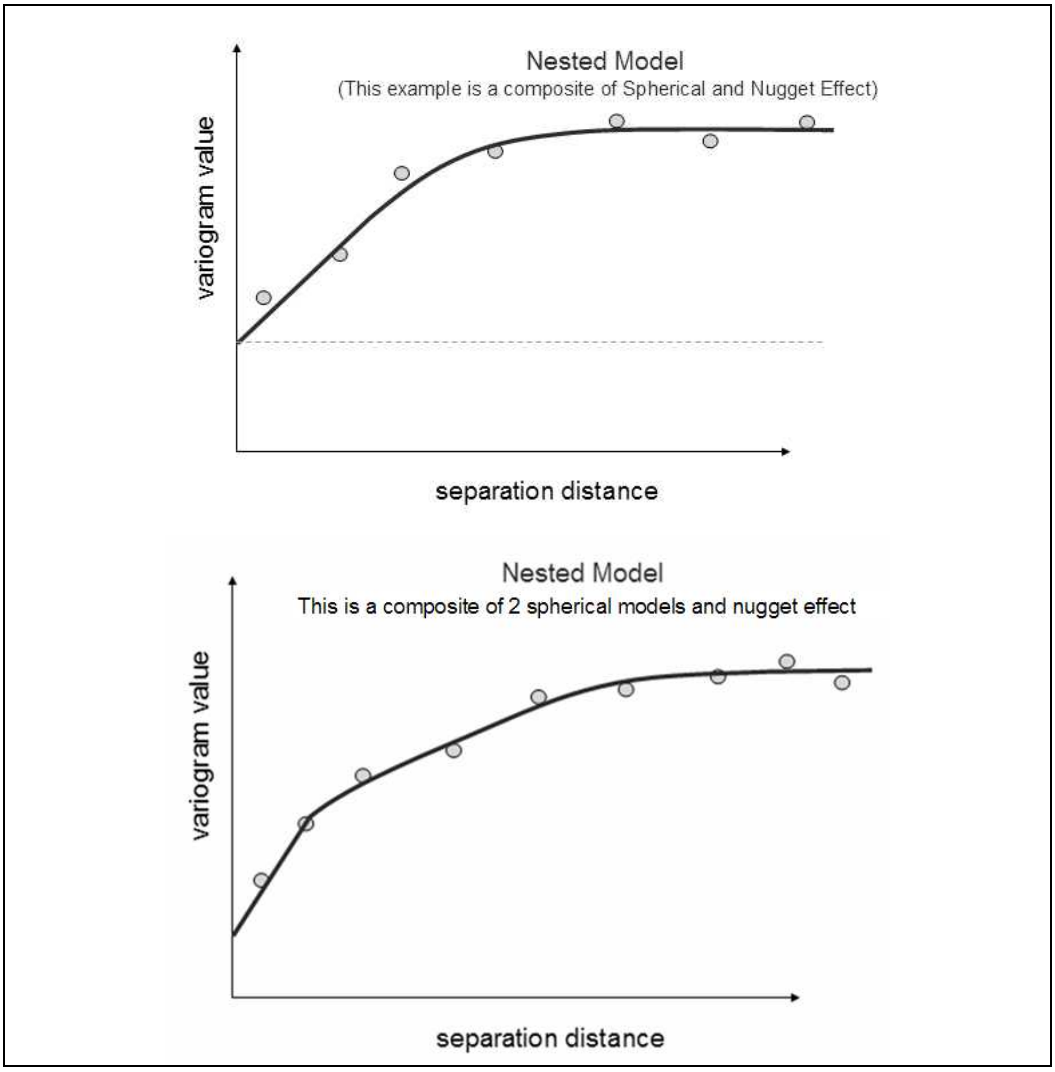


Figure 122 Construction of nested structures for variogram models

Practical process for variogram modelling

There is no one right way to model variograms. This lack of structure and guidance can be daunting to a first-time (or even a several time!) modeller.

Contemplate the exercise above where the purpose was to fit a line/curve to the points. This is essentially all that is required of the modelling process. However, we need to ensure the variograms in three orthogonal directions are modelled – each with the same set of nested variogram types and each with the same sills. The ranges, however, should reflect the anisotropy we expect in the mineralisation.

The approach I find convenient (although there is a degree of iteration involved) is:

1. Use the variogram in the downhole direction (or specifically a downhole variogram) with lags at the sample length to model the nugget effect
2. Apply this nugget effect to the three directions
3. Model the variogram in direction 1 (aim for a good fit; use nested variogram models if necessary)
4. Copy this variogram model to direction 2 and adjust the ranges of the variogram to get the best fit
5. If you need to adjust the sills to get a reasonable fit, do so
6. Apply the updated model for direction 2 on the variogram for direction 1 and adjust the ranges for the model in direction 1 you get a good fit
7. Model direction 3 in the same way as steps 4 to 6 above – now ensuring the sills for directions 1 and 2 and 3 are consistent

Remember, each variogram point is actually the average of a collection of differences. This means each point is surrounded by a set of differences that are less than and greater than the point plotted. The pair differences could be plotted as a box-and-whisker plot for each location and used to temper the variogram model fit.

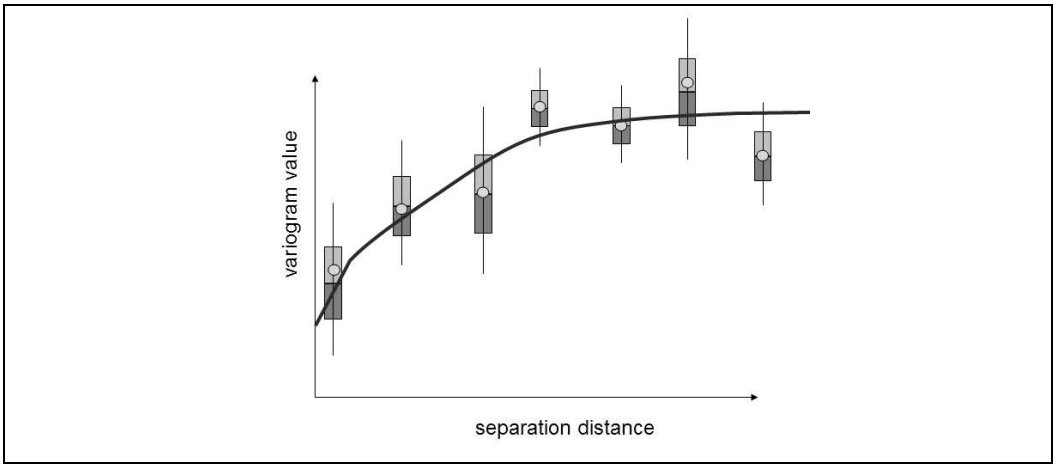


Figure 123 Each variogram point corresponds to an average of many differences

Since the points plotted are essentially averages based on the available pairs of samples, the variogram may not present as a perfect variogram shape often presented in textbooks. As more data becomes available (for example grade control) , the variograms become smoother and better defined.

Modelling "difficult" variograms

There is no such thing as a "difficult" variogram. Rather there are situations where we do not have enough data to enable us to model variograms easily. This is an important differentiation because once we get over it we can use the available geological information and understanding to help fill the gaps. A more challenging variogram to model is the equivalent of a low-pixel digital image. We just do not have the resolution of data to see what the picture is and we need to squint and take a best guess.

Of course, if you know the commodity and the geological setting, you immediately have a sense of the nugget effect – gold is likely to have a moderate to high nugget, while sulphide nickel is likely to have a low nugget with a short to moderate range (depending on the degree of undulation in the mineralisation envelopes). Iron typically has a lower nugget with long range, but the phosphorus contaminants in iron will be more nuggety with a shorter range. Within the context of our expectation, we stand a better chance of taking a best guess when we squint and model the variogram.

The fewer samples we have, the more likely the variogram will vary point to point. Some adjustments I commonly make to the calculation parameters include:

- A revisit of the domains to ensure the criteria are met. Adjustments may include sub-dividing the data according to a change in orientation of the mineralisation envelope.

- If the data is positively skewed and the variogram is challenging, I will apply a log-normal or normal score transform.

- If the number of samples is insufficient, I will test increases in the lag interval and tolerance angle (one parameter at a time) until a get enough statistical mass to produce a meaningful variogram. Be careful of the modelled ranges when the expected anisotropy is high. By increasing the tolerance angle one risks smearing the anisotropy ratio.

- Check whether the data should be and can be unfolded (especially if the unit is thin because subtle variations in the thin wireframe can lead to hanging wall samples being compared with footwall samples when this is not the intention). Sometimes the unfolding I apply is more of an unwrinkling where the unfolding involves calculating a new orientation (say the elevation) as a relative distance from the hanging wall. This simple straightening out of the envelope often "cleans" the variogram.

Examples of dealing with challenging variograms

Example 1: "I can't get a variogram even though I have thousands of samples"

This is common when data is positively skewed. This data set has 2466 samples, a single grade population and a high coefficient of variation (2.3).

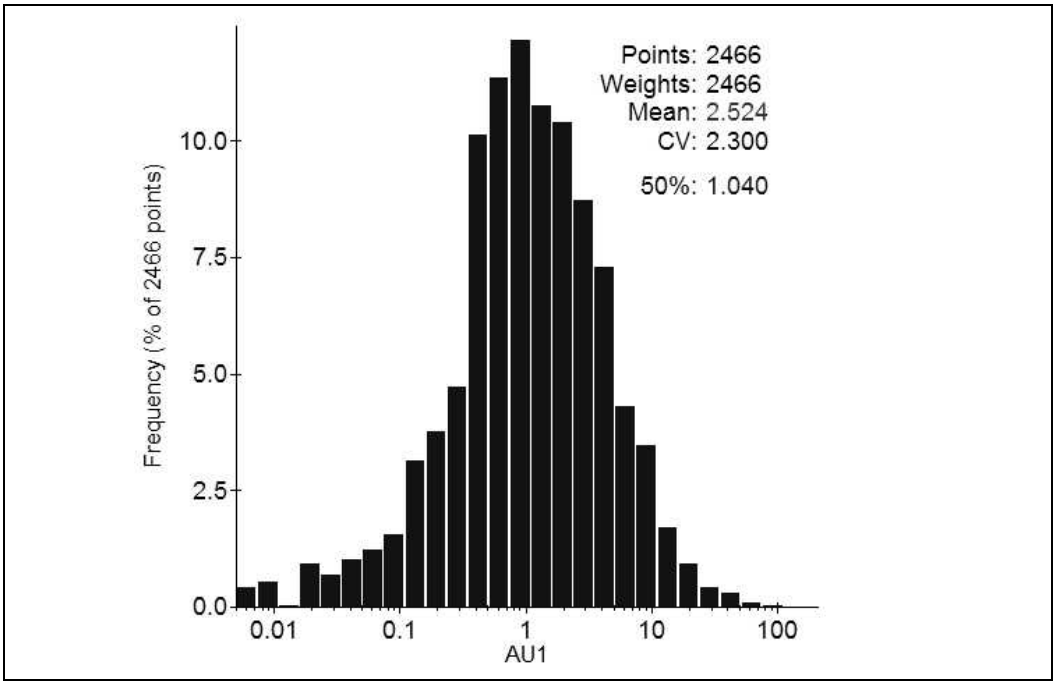


Figure 124 Difficult variograms: data set 1 histogram

The down dip traditional variogram for this data set is erratic – it only has one point below the overall sill (standardised variogram value of 1.0) and all other points are 50% higher than the total sill (Figure 125). A log-variogram, however, produces a variogram that is easier to model (Figure 126).

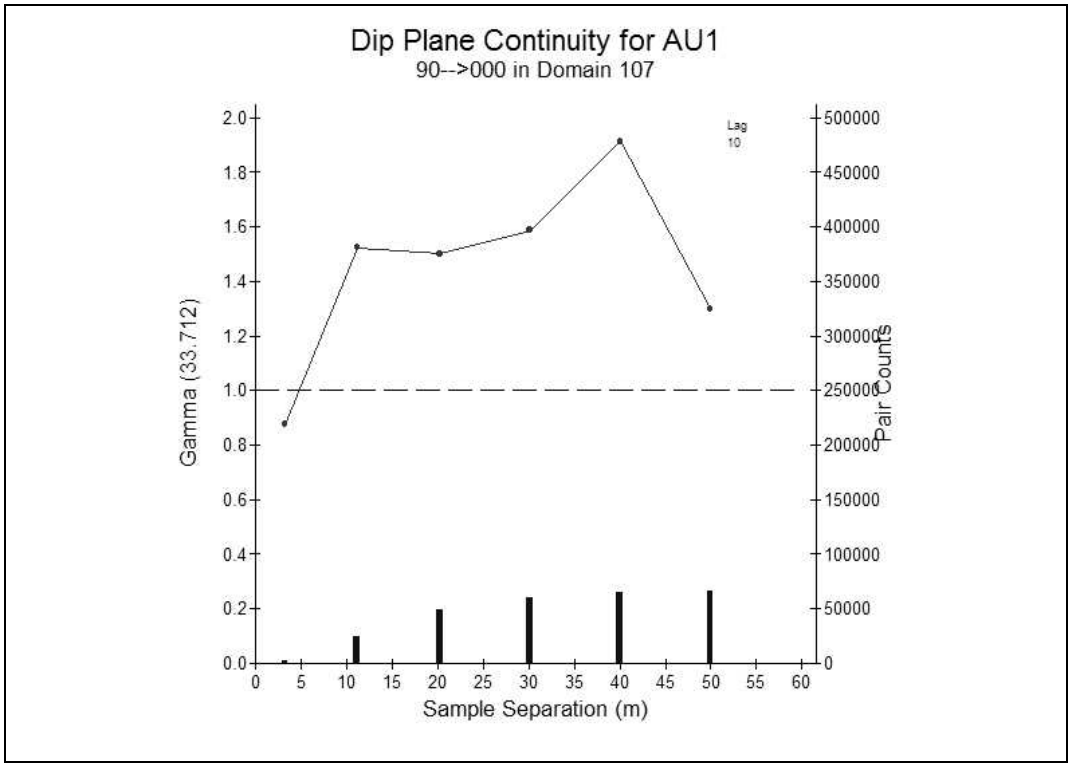


Figure 125 Traditional variogram (difficult data 1)

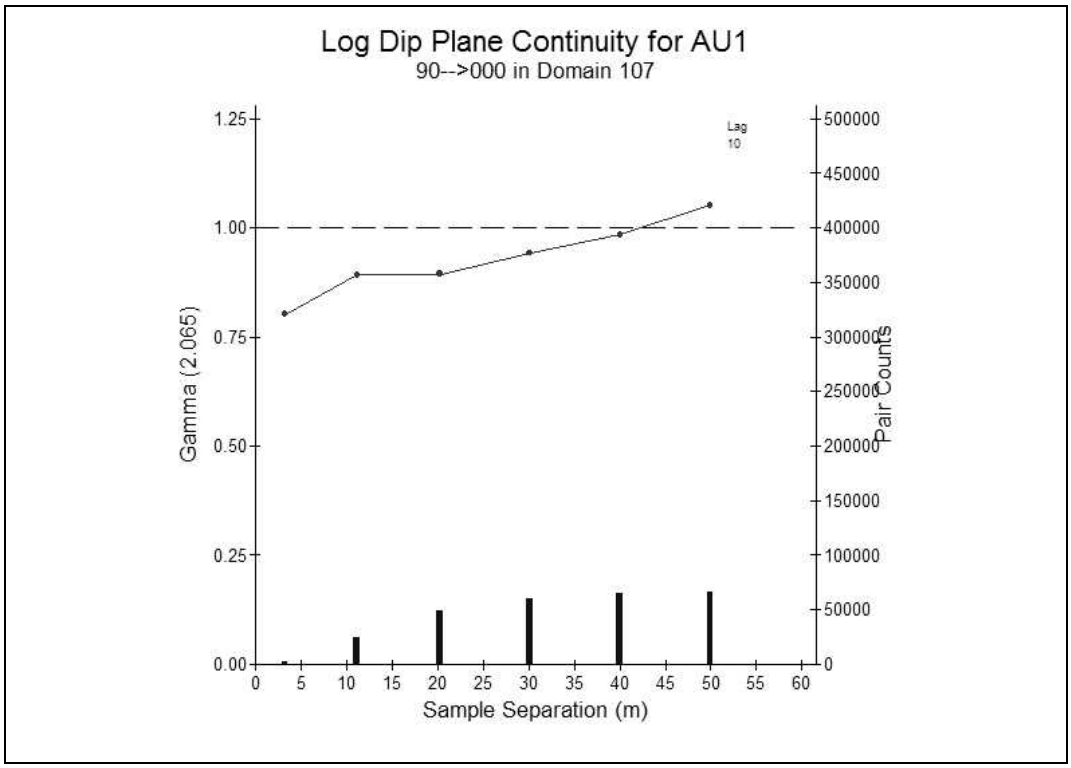


Figure 126 Log-variogram (difficult data 1)

Example 2: "The variogram is too variable"

Figure 127 illustrates a variogram that is erratic and looks too difficult to model.

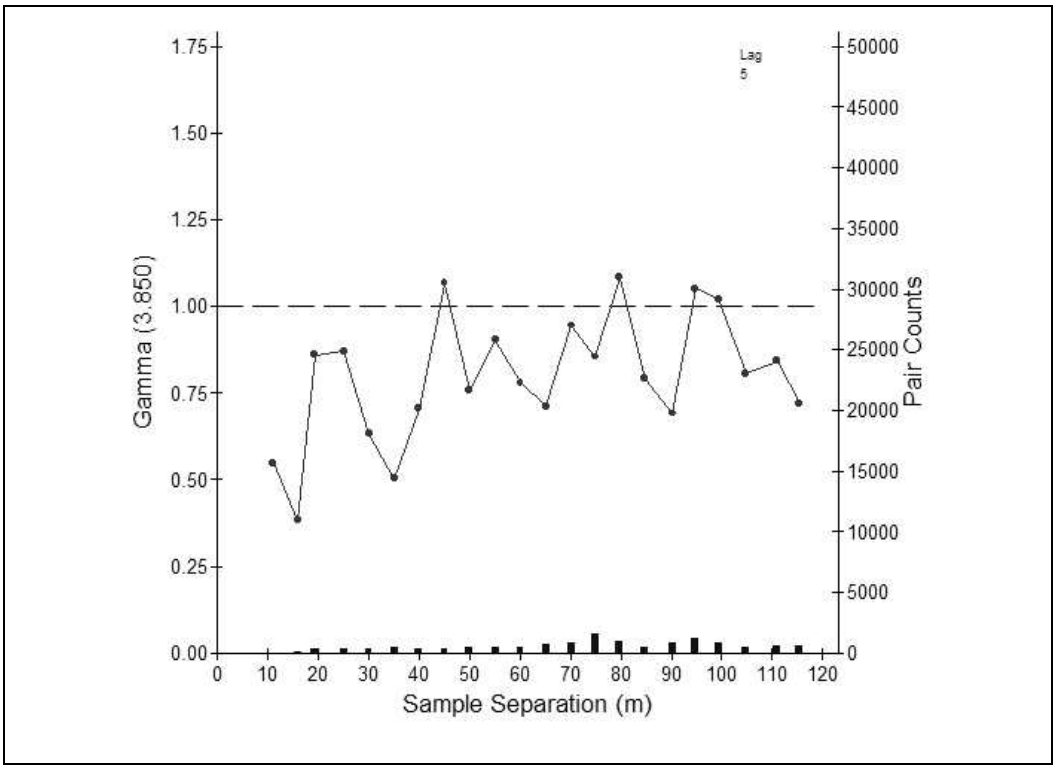


Figure 127 Highly variable variogram (difficult data 2)

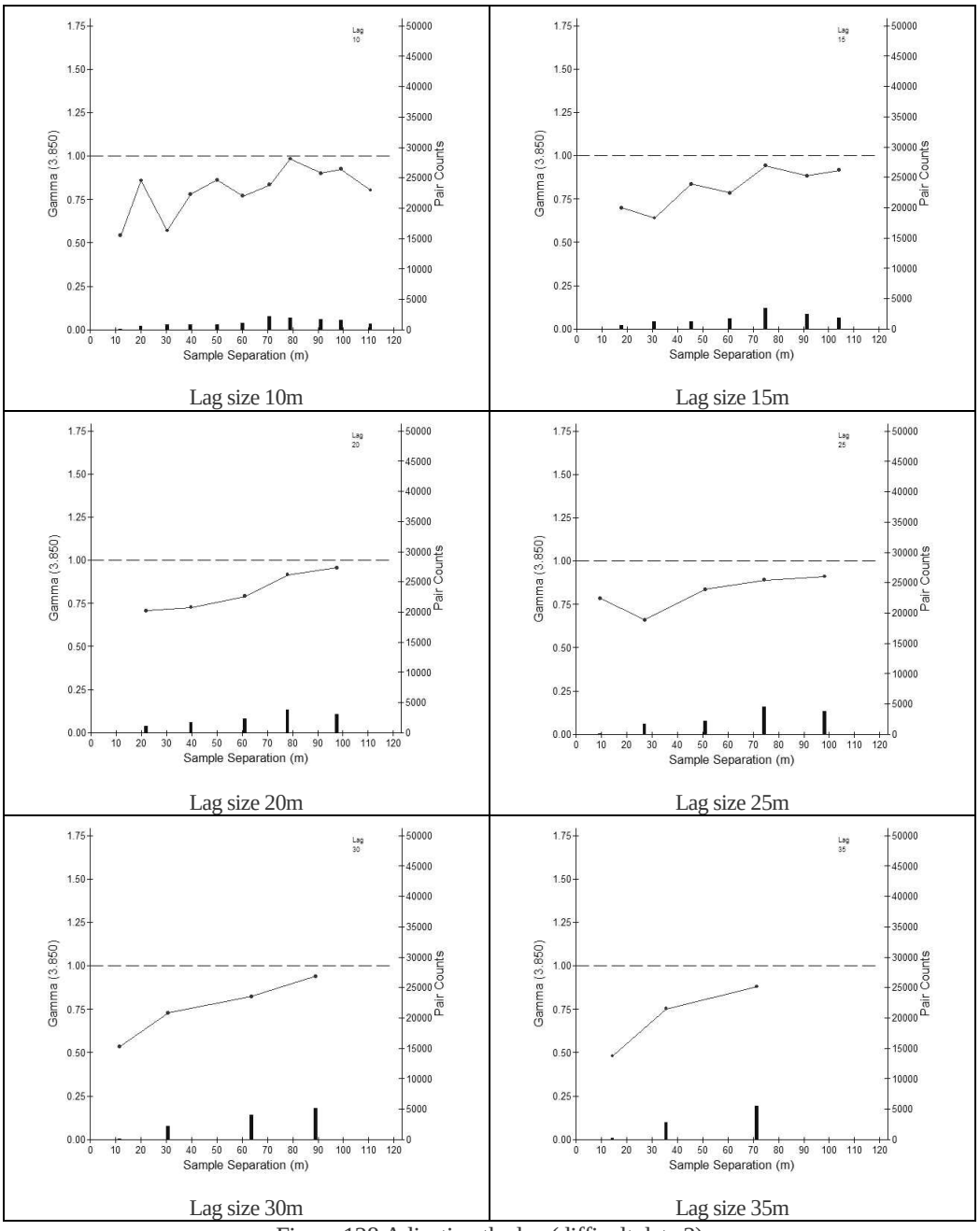
Figure 128 shows the variograms at different lag sized using the same data set.

The variogram smooths out as the lag size increases. The variogram that is now easiest to model is at lag size 35m (Figure 128).

The real test is whether this model is reasonable and robust across the different lag sized. To test this I replot the model against the variogram for the different lag sizes (Figure 130).

The model appears to fit the highly variable data set well (Figure 131).

If the calculated variogram does not smooth out when the lag size is increased, I test a series of tolerance angles.



Lag size 10m | Lag size 15m

Lag size 20m | Lag size 25m

Lag size 30m | Lag size 35m

Figure 128 Adjusting the lag (difficult data 2)

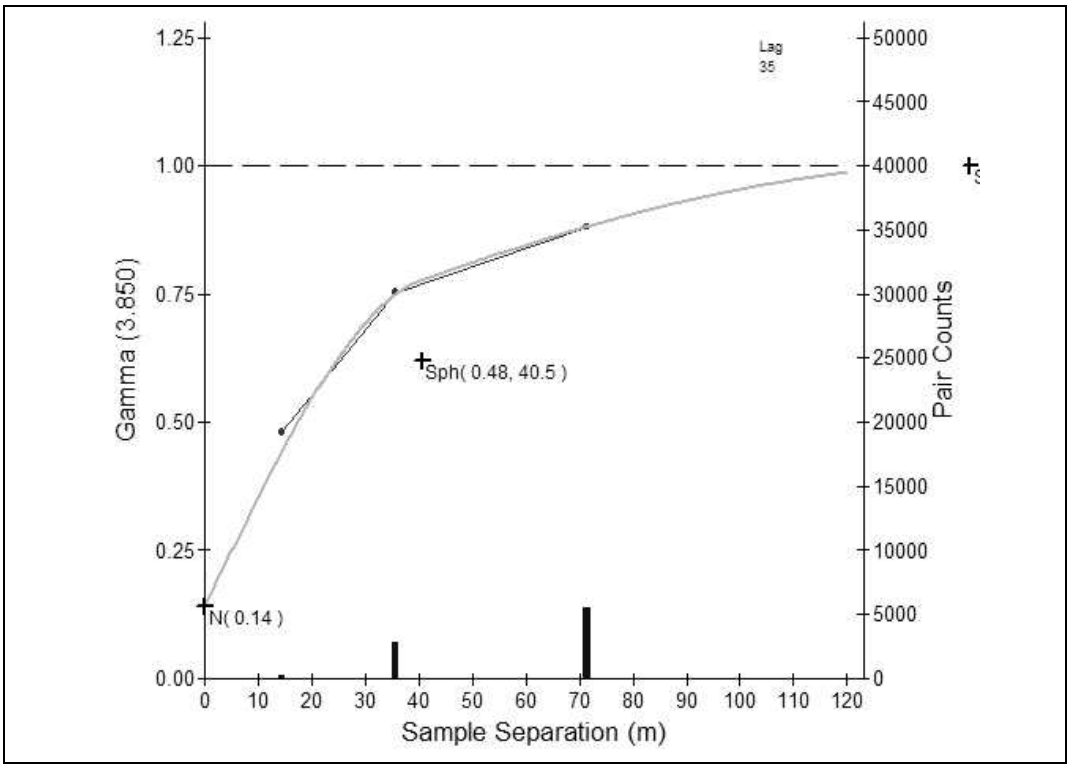
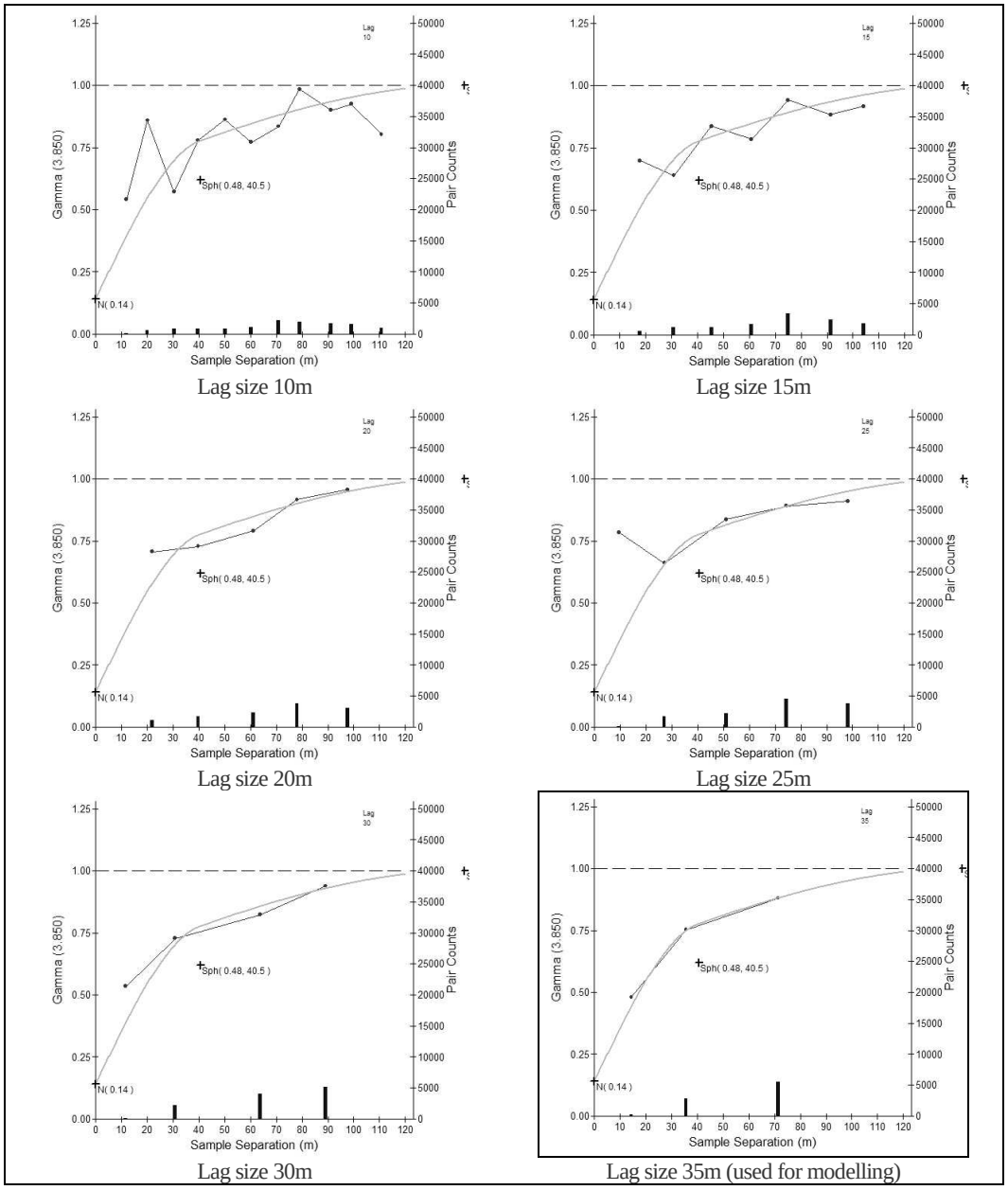


Figure 129 Variogram modelled at 35m (difficult data 2)



Lag size 10m | Lag size 15m

Lag size 20m | Lag size 25m

Lag size 30m | Lag size 35m (used for modelling)

Figure 130 Model against various lags (difficult data 2)

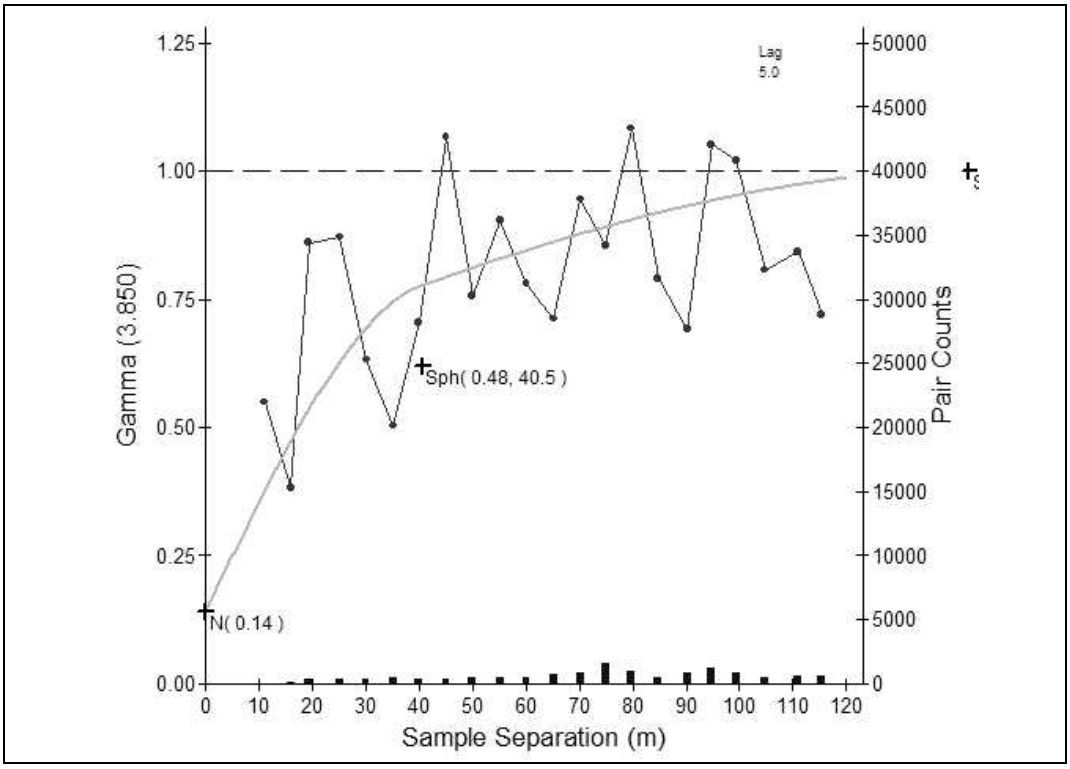


Figure 131 Model against highly variable variogram (difficult data 2)

Phase 3: Model Creation

Purpose

Up until this stage, the work involved in resource estimation is primarily to analyse and understand the context of the mineralisation:

1. Interpreting geological controls
2. Assessing data quality
3. Identifying and constructing mineralisation volumes or domains
4. Understanding the statistical properties of the data within domains
5. Interpreting the orientation of grade continuity within domains

and then

6. Modelling the variability or spatial connectivity (variograms).

These tasks all involve understanding the geological context and patterns in the grade.

The next phase is to develop a three dimensional representation of grade within the domains that accurately reflects our understanding of the domains and the geological controls. The steps to do this are:

1. Create a three-dimensional model of the domains (wireframes)
2. Select the most suitable estimation method
3. Select the most appropriate estimation parameters
4. Run the estimation

Here we discuss the information required to achieve these four steps.

Wireframing

A wireframe is a three-dimensional solid that encompasses a domain. This can be constructed in numerous ways, but typically the process is to interpret the domain boundaries on section and then to tie the interpreted polygons together.

The specifics of the process are software dependant. Ultimately, the volume defined by the blocks within the wireframe should closely reflect the volume of the wireframe.

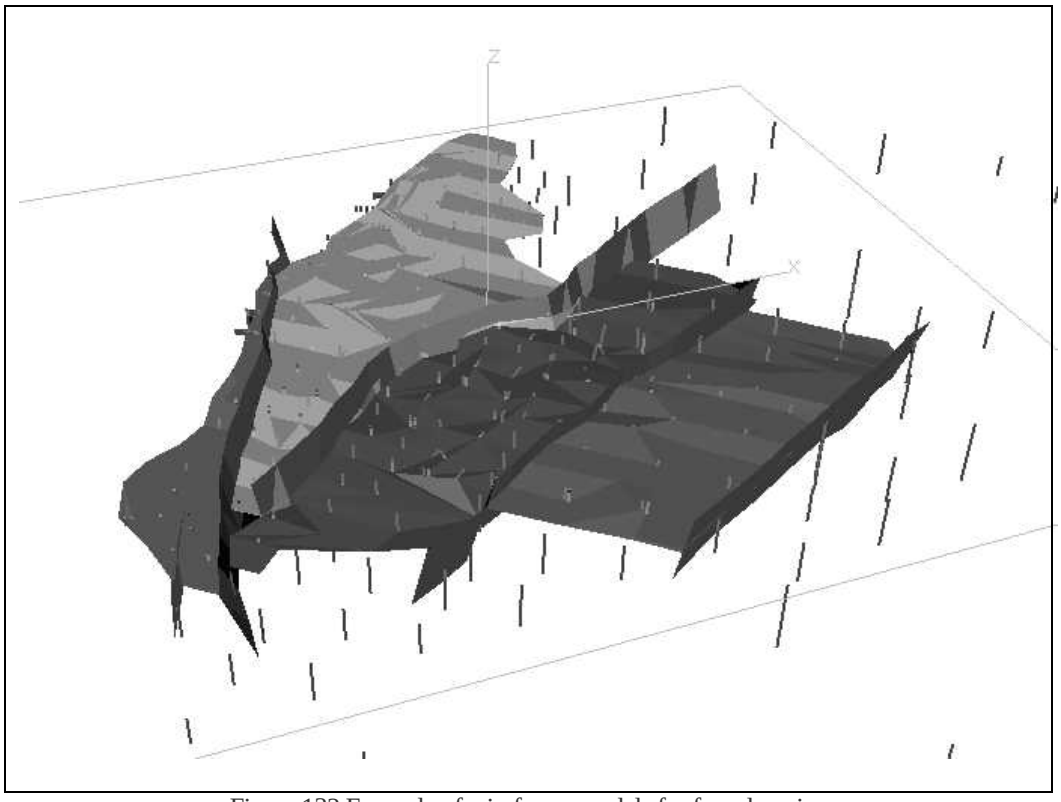


Figure 132 Example of wireframe models for four domains

Estimation

Central to building the three-dimensional model is selection of the estimation method and the estimation parameters to control the estimation process.

Let us look at three of the more common techniques before getting to grips with selecting parameters:

1. Inverse distance
2. Ordinary Kriging and
3. Indicator Kriging

Inverse distance

Inverse distance is the simplest estimation technique to implement. This is because there are only a few parameters to select. The remaining controls are implicit in the estimation technique.

The rationale behind inverse distance is that closer samples are more like the block grade than samples further away.

The inverse distance method presumes samples closer to the point of estimation are more likely to be similar to the sample at the estimation point than samples further away. So closer samples should get more weight and the easiest way to control this is simply invert the distance.

The inverse of the separation distances are rescaled so they sum to one. This ensures the estimated grade is unbiased when compared with the sample grades.

Selecting a power

The inverse distance weights can also be raised to a power. Typically, the power is selected arbitrarily. However, we can consider the impact the power has on an estimate:

One way to think about the power parameter is to consider its effect on a weight. Consider two samples near a point to be estimated, say one is 5m away and the other is 15m away. The sample weights are proportional to 1/5 and 1/15 respectively. Now, if we raise these inverse distances to a power of two, the sample weights become proportional to $(1/5)^2$ and $(1/15)^2$, or 1/25 compared with 1/225. The closer sample gets significantly greater emphasis when the power is increased.

So in selecting a power for inverse distance we should consider how reproducible close samples are. For situations where we have no faith in the data reproducibility, for example alluvial gold, we may be more comfortable with an extremely low power. Ultimately, a power of zero gives samples weights proportional to $(1/\text{distance})^0$, and any value raised to zero is equal to one so all samples receive the same weight. The estimate becomes an average of the samples selected.

The nugget effect on the variogram is a useful guide for selecting the power. A low nugget effect indicates good reproducibility in the sample data. This means samples close to an estimation point can be trusted to be similar to the grade at the estimation point, implying the use of a high power. On the other hand, a high nugget effect suggests poor reproducibility and hence the use of a low power for inverse distance.

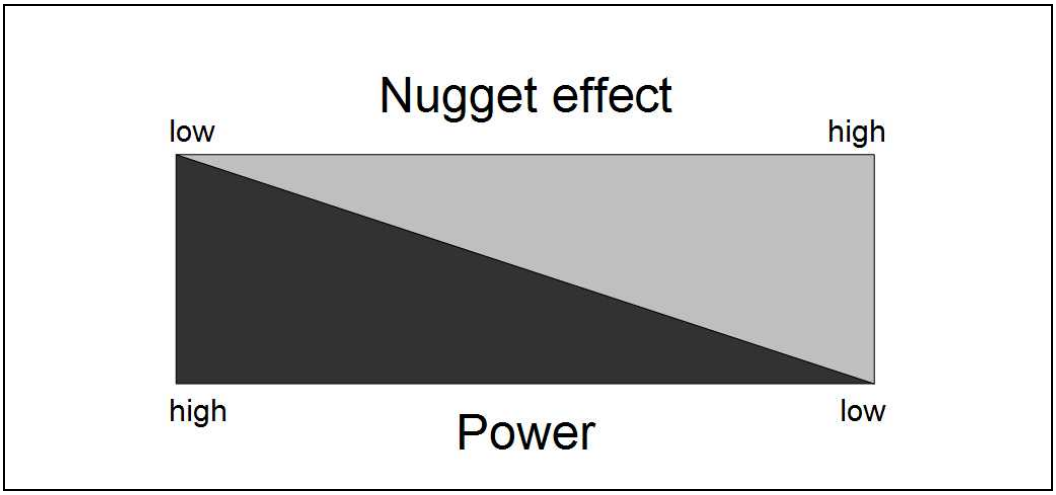


Figure 133 Relationship between nugget effect and selection of power

Defining the search neighbourhood

Most estimation methods require a maximum limit on the number of samples to be used for estimating any one point. This is mainly to speed up the estimation process rather than a theoretical requirement. Since there is no theoretical basis, we need to invoke a bit of logic and common sense.

Typical rules of thumb are:

- At least 30 to 40 samples are usually sufficient to generate robust estimates, however, these estimates may be too smooth for your understanding of the grade continuity, especially when the mineralisation is expected to be more selective.
- The search ellipsoid used for collecting samples should be orientated in the direction of mineralisation continuity.
- The ranges of the variogram in the down plunge, across plunge and across dip directions are good starting points for selecting the search ellipsoid ranges in the respective directions.
- Select a power commensurate with the nugget effect.

Defining the search neighbourhood

Be wary of octants. Octants were introduced to control weight distribution to clustered samples. However, many software packages do not align the octants with the search and/or represent eight segments in the shape of orange segments. When a dip plane cuts through this segment-style octant with a steep dip, the volume represented by each segment is no longer equal, thereby INTRODUCING clustering rather than reducing the effects of clustering.

Ordinary kriging

A brief history

Danie Krige, a South African Mining Engineer, ran empirical comparisons between estimated and actual grades from gold mines in the Witwatersrand, South Africa. Interestingly, he discovered a consistent pattern – estimated high grades were lower than predicted and estimated low grades were higher than predicted. This prompted Krige's (along with French engineer Georges Matheron) discovery of the volume-variance effect, which is the cornerstone for understanding mining reconciliations and resource estimation.

Krige refers to his observation as the "Regression Effect". The implications are that panels or volumes estimated as having high grade were over predicted. So for example, a panel or volume estimated as 7.5 g/t could return a grade of 6.9 g/t, while a panel estimated at 2.9 g/t returned a grade of 3.2 g/t (see diagram). This effect is observed repeatedly – low-grade stockpiles often return grades higher than anticipated, while high grades running through the mill continue to disappoint.

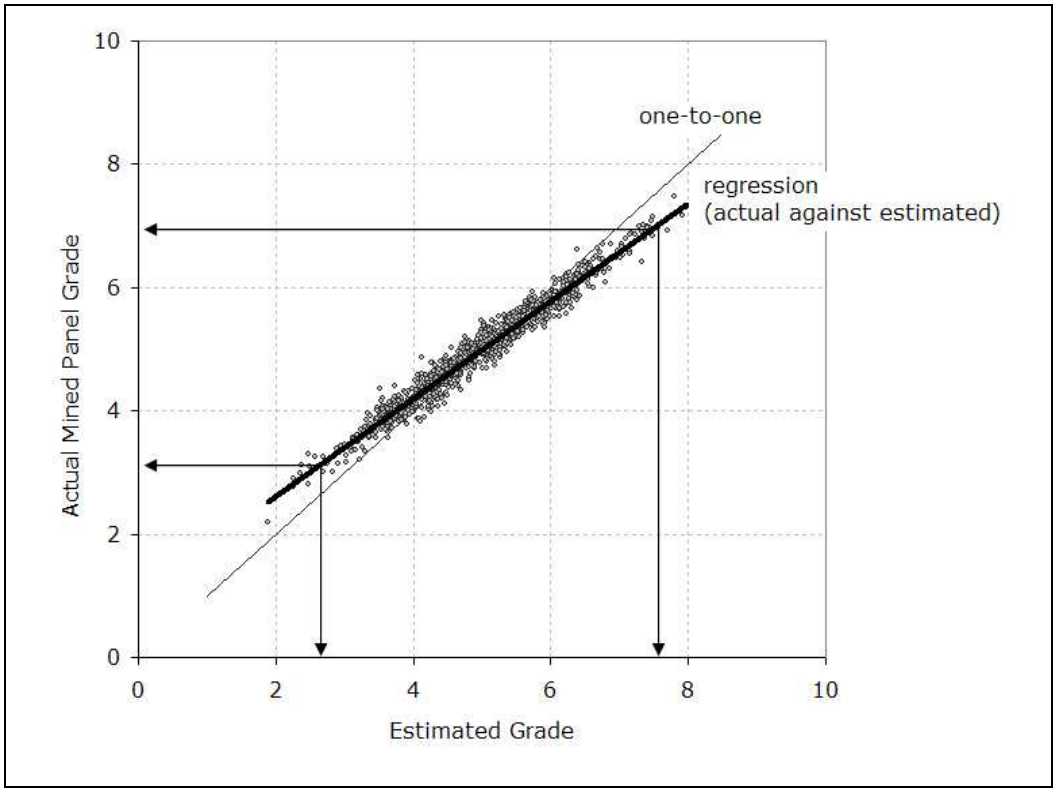


Figure 134 Krige's observation between actual and estimated grades

So why does this happen? It is to do with the volume-variance effect.

The volume-variance effect

The volume-variance effect describes the increase in grade dilution as we select larger volumes.

To understand the volume-variance effect, think about a coarse gold orebody represented by a 44-gallon drum.

If we mine this "orebody" with teaspoons, some teaspoons will contain pure gold nuggets while others will be completely barren. The sampled grades range from extremes of completely barren to pure grade.

Now, if we mine the exact "orebody" with buckets, each bucket may contain some grade, but it is extremely unlikely there will be a beach bucket full of nuggets. The surrounding barren material dilutes the grade nuggets. Similarly, beach bucket samples are unlikely to be completely barren. So, the range in beach bucket grades is narrower than the range in teaspoon grades.

If we increase our sample volume to a third of drum, then we have even more dilution and an even narrower range in grades. Ultimately, a sample volume equivalent to the entire drum gives us a single value and, hence, no range in grades.

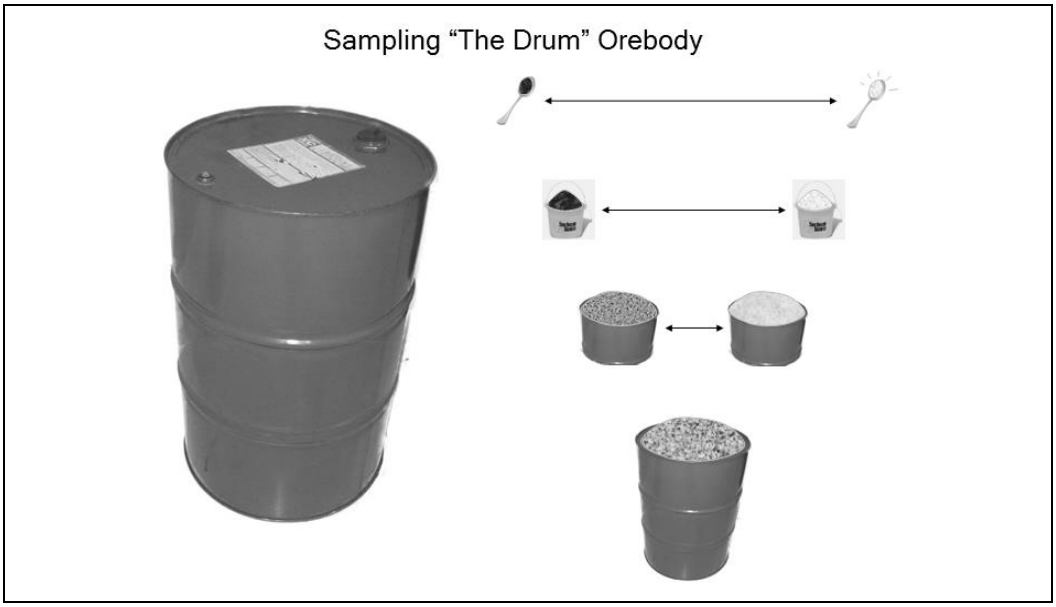


Figure 135 The volume-variance effect

In summary, the larger the volume, the lower the variability in grades. This is purely due to the diluting effect of decreased selectivity.

In a mining sense, if we use sample grades to represent mining units, say in a polygonal estimate when we define dig lines using grade control samples without accommodating for the volume-variance effect, we will continue to over-call high grades and under-call low grades. We are implicitly sampling with teaspoons and mining with buckets.

Implications are we need to adjust estimates to reflect the volume we will be mining to when we report or apply selectivity criteria to a resource model. In addition, we should state the degree of selectivity we are using for reporting.

Birth of a new technique

Krige's observation of the volume-variance effect using empirical data from the Witwatersrand led him to work with Georges Matheron on creating an estimation technique that would accurately reflect the selectivity at mining.

They set out to create a linear estimation technique that, of all possible linear estimation methods, would have the least overall difference between predicted and actual mined grades. This technique also had to be unbiased.

A linear estimation means the weights are applied directly to the sample grades to produce an estimate. No fancy transformations required!

So two conditions:

least overall difference between predicted and actual
and
unbiased

In mathematical terms, these can be rewritten as:

$(\text{predicted} - \text{actual grade})^2$ is minimised
and
sum of weights is equal to one

Let us rewrite this in a bit more detail:

$((\text{sum of weights} \times \text{grade}) - \text{actual grade})^2$ is minimised
and
sum of weights is equal to one

We can combine these two conditions to:

$$[((\text{sum of weights} \times \text{grade}) - \text{actual grade})^2 - \mu(\text{sum of weights minus one})]$$

which we want to minimise.

Notice the "μ" that has crept in. This is called a LaGrange Multiplier and it essentially measures the degree of bias that exists without this condition to get the weights to sum to one. When the surrounding sample points are clustered, sparse or there is significant extrapolation required, "μ" will be larger.

You may be wondering where we get the "actual grade". This is where the variogram comes in handy. Recall a variogram is calculated by comparing pairs of actual sample values, and so the variogram inherently carries information about the "actual grades".

Now, using a bit of calculus, we can "differentiate" or optimise this system to find the weights that result in these two conditions being met.

Without going into the detail of the differentiation, the result is a series of equations where the weights are the values we are trying to calculate. These equations can be summarised in matrix form:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \ldots \\ n \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & \ldots & n & \end{array} \\ \begin{bmatrix} C_0 & \gamma_{12} & \gamma_{13} & . & \gamma_{1n} & 1 \\ \gamma_{12} & C_0 & \gamma_{23} & . & \gamma_{2n} & 1 \\ \gamma_{13} & \gamma_{23} & C_0 & . & \gamma_{3n} & 1 \\ . & . & . & . & . & . \\ \gamma_{1n} & \gamma_{2n} & \gamma_{3n} & . & C_0 & 1 \\ 1 & 1 & 1 & . & 1 & 0 \end{bmatrix} \end{array} \times \begin{bmatrix} weights \\ weight_2 \\ weight_3 \\ weight_4 \\ . \\ weight_n \\ \mu \end{bmatrix} = \begin{bmatrix} sample\ to\ block \\ \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \bar{\gamma}_3 \\ . \\ \bar{\gamma}_n \\ 1 \end{bmatrix}$$

A bit of explanation …

The first matrix summarises the variogram values between each sample near the block to be estimated. The symbol γ is shorthand for "the variogram" and the numbers attached to the γ symbol conveys the number of the sample being compared - γ_{12} is the variogram at a separation distance between sample #1 and sample #2.

The C_0 is shorthand for the nugget effect.

$\bar{\gamma}_3$ represents the average variogram between sample #3 and the discretisation[12] points in a block. By taking the average of the variability between a sample and all discretisation points within a block, the kriging system is applying a volume-variance correction. The discretisation points within small blocks will be closer together than discretisation points within larger blocks. This means a more similar span of separation distances between sample and discretisation point and, therefore, more similar variogram values. In contrast, discretisation points within larger blocks are further apart and thus use a wider range of variogram values to calculate the average variability between the sample point and the block.

Ultimately, the system is reduced to a calculation of the weights (and μ). These weights are applied to the respective sample grades to produce an ordinary kriged estimate.

So, the kriging system generates weights that ensure an unbiased linear estimate that, of all possible linear estimates (yes even inverse distance), has the least overall variability between predicted and actual.

Notice the weights are essentially a function of the variogram model. This means each domain's unique character can be used to control the weights. Consequently, the variogram model should accurately reflect the variability observed in the data. For this reason, I prefer to review variogram models against the calculated variogram values as a plot rather than a table of variogram parameters. The variogram plot highlights the variogram fit and how reasonably the model reflects the calculated variability.

[12] Discretisation points are regular points within a block – typically at least two in each direction (2 x 2 x 2) that provide a sense of change in volume between the samples and a block.

Kriging variance

The kriging variance can also be calculated based on the kriging system. Historically, the kriging variance was used as a selling point for kriging since it provides a measure of confidence in the estimate. This measure is based on the sample configuration surrounding a block and the variogram.

The kriging variance is calculated as the weighted sum of the variograms between the sample and the block, plus the LaGrange multiplier (μ) less variability contained within the block.

$$\begin{aligned} Kriging\ variance = sum(&variogram(point\ to\ block\ distance) \times kriging\ weight\ ) \\ &- average(variogram(between\ each\ and\ every\ discretisation\ point\ in\ block\ size)) \\ &+ LaGrange\ multiplier \end{aligned}$$

The kriging variance can be summarised as =

- sum of the point to block variability weighted by the kriging weights
- minus variability within a block
- plus LaGrange Multiplier

Let us take a closer look at each of these components:

point to block variability: This is the average of the variogram values for each of the sample to discretisation points (Figure 136).

sum of the point to block variability weighted by the kriging weights: The contribution of each sample to variability (or confidence) in the block is summarised by adding up all the sample to block variabilities, but weighting them according to the influence each sample has had on the estimated (i.e. the kriging weights).

variability within a block: This is the average of the variogram values based on the distances between each and every discretised point inside the block (Figure 137). For large blocks, this average will be large and, since it is subtracted in the formula, will mean a reduced kriging variance.

LaGrange multiplier: The LaGrange multiplier increases when the data surrounding a block has a sub-optimal geometry (clustered, sparse or resulting in an extrapolated estimate). Inadequate data configuration results in an increased LaGrange multiplier and, therefore, a higher kriging variance.

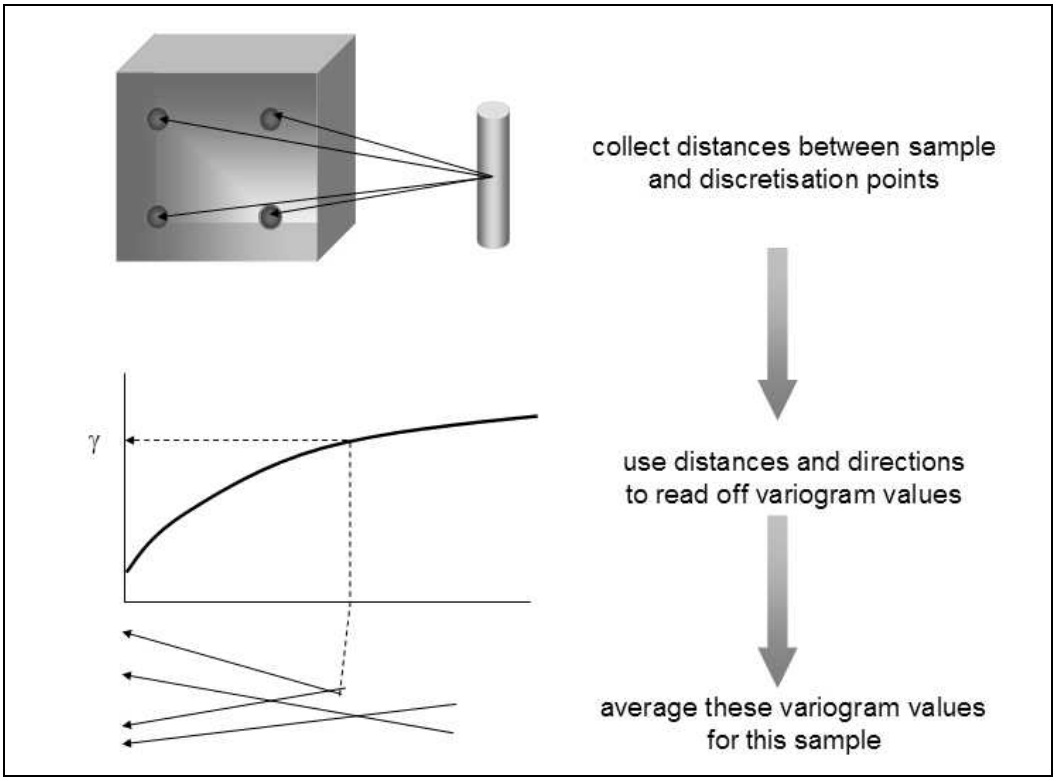


Figure 136 Sample to block variability

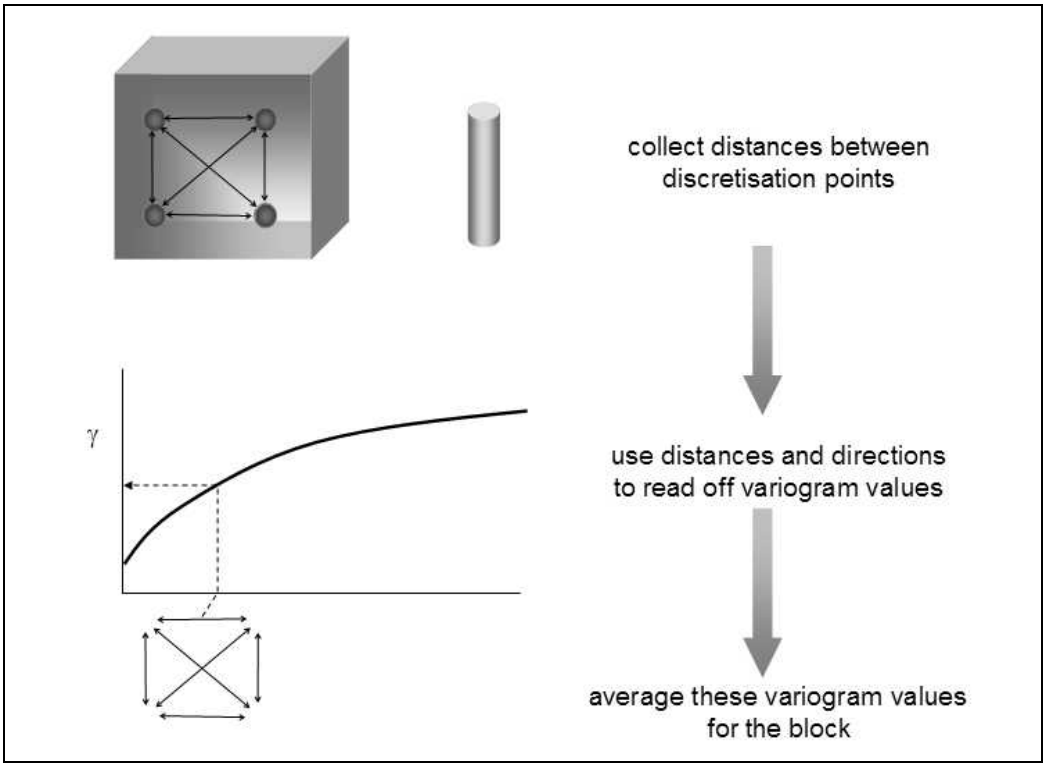


Figure 137 Variability within a block

At no stage are sample values incorporated into the kriging variance. The true variance or uncertainty surrounding the estimate will depend on the sample values used to calculate the estimate. This means the two blocks in Figure 138, which are in the same domain and therefore using the same variogram, will result in identical kriging variances.

Conditional simulation is an alternative to generating some measure of grade variability on a local scale.

The kriging variance, however, provides a relative measure of data coverage surrounding blocks, which provides a useful guide for resource classification.

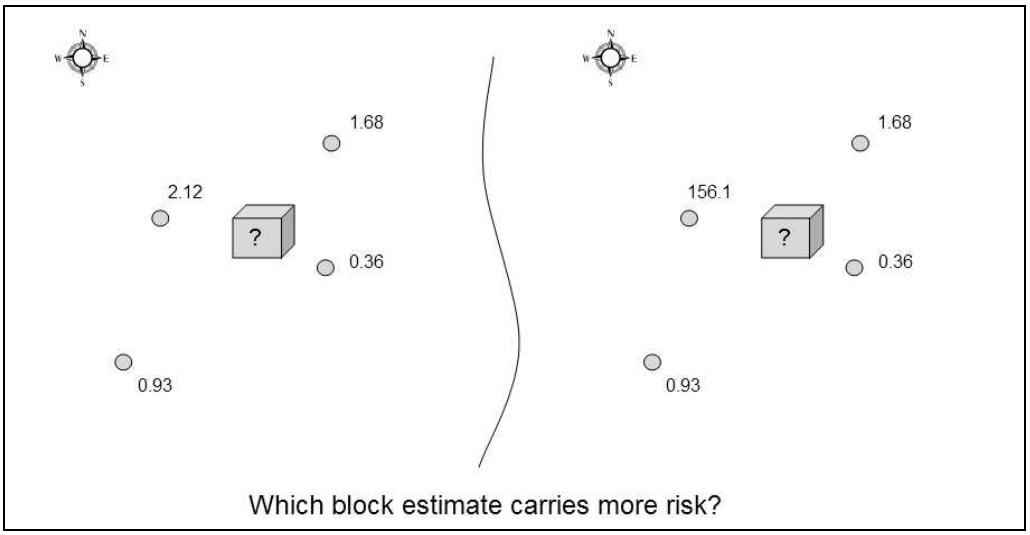
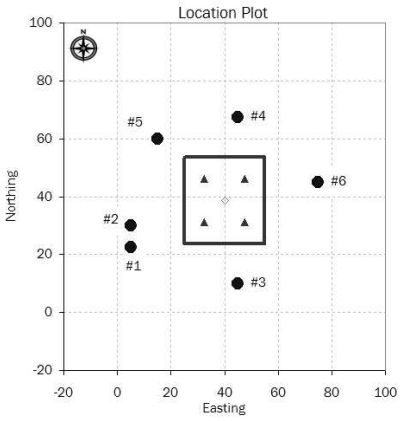


Figure 138 Kriging variance for two blocks (same sample geometry, but different grades)

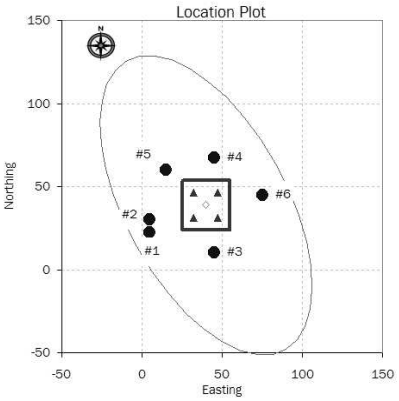
A kriging example in action

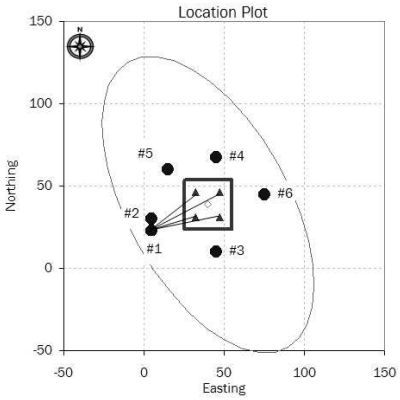
Let us step through an example …



Six samples surround a block, which has been discretised into 2 x 2 in the X and Y directions respectively.

The first step in any estimation process is to search the database to find the closest sample points. These points should come from the same domain and may include samples from adjacent domains if the boundary conditions are gradational. Note the search ellipsoid is centred on the block for the search process.



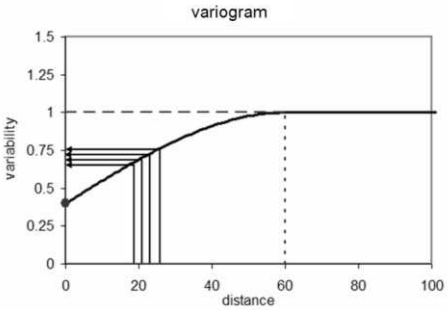


The distances between each sample and all the discretisation points are then measured.

These distances are used to read of the corresponding variogram values (based on the variogram model parameters)

These variogram values are averaged to get a sample-to-block variogram value for each sample. This is repeated for every sample selected within the search ellipsoid.

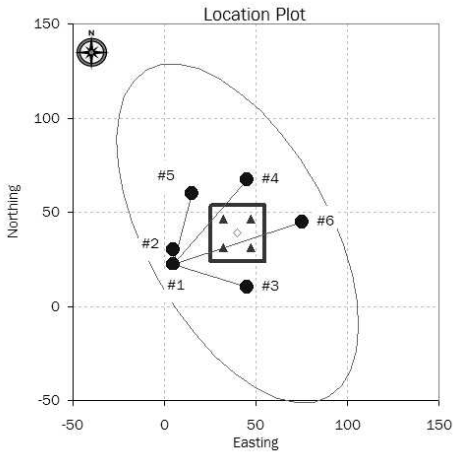
The distance between each sample and every other sample is also measured and used to look up the expected variogram value for every pair of samples.



This information is placed in the kriging system, which calculates the weights.

The weights are applied to the respective sample grades to generate an ordinary kriged estimate for the block.

This process is repeated for every block within the domain. Each domain may use a unique set of parameters.



Selecting estimation parameters

When selecting estimation parameters, common sense should prevail.

Consider the following when selecting parameters

- **Domaining -** The way the data is domained will have a significant impact on the quality of the model. The boundary conditions (sharp or gradation change in grade at the boundaries) are a guide to identifying how the data will be used for estimation.

- **Block size -** The block size is critical to the selectivity and the degree of volume-variance correction. When blocks are too small, there is a false sense of selectivity built into the model. Adjacent blocks tend to have very similar grades, which means the model is too smooth and the selectivity in the model is actually for a larger bulkier scenario. Kriging Neighbourhood Analysis (see page 178) is useful for establishing confidence in the block size. Typically, the industry standard is for blocks to be no smaller than half the drill spacing.

- **Discretisation -** Typically more than four discretisation points in any direction is more than sufficient.

- **Search ellipsoid -** The anisotropy and orientation of the search ellipsoid can be based on the variography. Pay careful attention to the continuity you expect to see based on your understanding and interpretation of the geology. The variogram range is a useful guide to selecting the search range.

- **Minimum and maximum number** of closest samples can influence the degree of smoothing in the model. Consider the selectivity you would like the model to reflect – for high selective zones, you may want to limit the maximum number of samples. Remember you are building a MODEL of the grade, so take time to establish your expectations of what the model should reflect of the data.

- **Samples per drillhole** – some packages allow you to control the number of samples per drillhole. This constraint is useful for controlling smearing in alternating bands of grades.

In selecting parameters, be guided by the data geometry relative to the grade continuity. Draw a picture of a block with the surrounding samples. How far are the closest say 40 samples? What does this mean for the search criteria?

Remember, the objective is to build a three-dimensional model of what you believe the mineralisation to look like, given all the available facts, data, information and inferences you have collected during the data analysis phase.

A note about negative weights

Negative weights result from the declustering process within kriging. When samples are screened by samples closer to a specific block, the samples effectively add no value to the estimate. This screening and the forced unbiasedness (reflected in the weights summing to one) can necessitate negative weights.

The negative weights are typically very small negative values and usually do not create any problems. Sometimes, however, the small negative weights are applied to relatively high grades. This means a high negative contribution to the overall kriged value, which can result in a negative estimate.

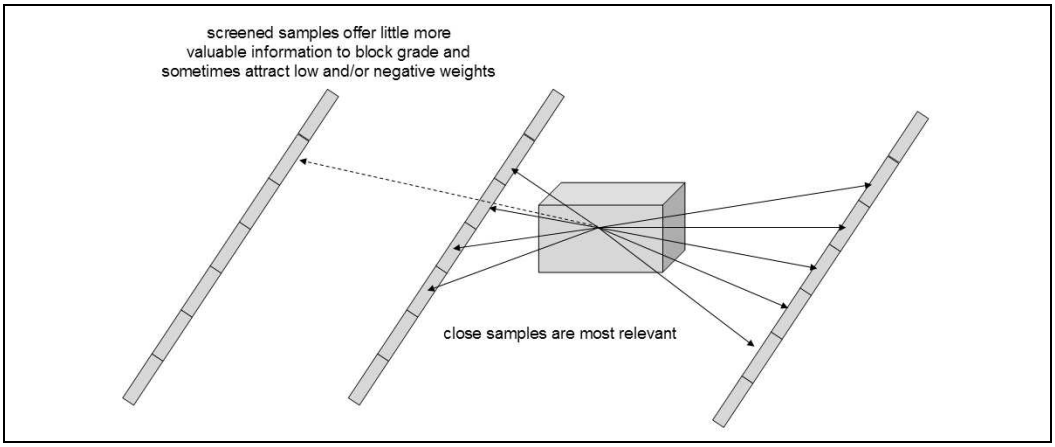


Figure 139 Negative weights result from screened samples

There are some practical, though not necessarily theoretically correct, ways to deal with negative weights:

1. The most common is to set the negative weights to zero – they are typically very small. The impact of this is that the weights do not end up summing to one. Theoretically, this produces a biased result. I like to run a model without this setting as well as with the setting and then to compare the two approaches in a scatterplot between the two resulting block models. Most block estimates should lie on the one-to-one line, while those affected by the "set negative weights to zero" setting will lie off the one-to-one line. The degree of difference is the measure of the impact of this setting. Ideally, only a few blocks should be affected and the grade of those blocks should be carefully validated against the surrounding sample data.

2. An alternative is to re-run the estimation for the affected blocks (i.e. blocks with negative estimates) with a reduced number of samples (either by limiting the number of samples per drillhole, or reducing the maximum number of samples).

3. A setting I would like to see in commercial packages is the option to ignore samples with negative grades. When this is done, the remaining samples are treated as the samples within the search and their weights then add up to one, thereby creating an unbiased estimate. This would require a re-run of the kriging process for all blocks where negative grades are encountered. We have the technology and the computer power to be open to this option.

4. When all else fails the block grades could be manually adjusted to a grade the resource analyst believes represents the volume. Reporting codes require principles of materiality, transparency and competence. Within this spirit, block grades can be manually identified and classified accordingly.

Indicator kriging

Indicator kriging is useful when we have spatially integrated populations (say cross cutting structures with multiple phases of mineralisation). Indicator kriging should NEVER be used in place of good geology and domaining.

Data analysis for indicator kriging

Since the underlying data distributions are mixed, the standard statistics used to describe the data tend not to be representative. Consider a data set made up from two different distributions. The mean of the data set represents neither population and, whilst central, does not say much about the two individual populations.

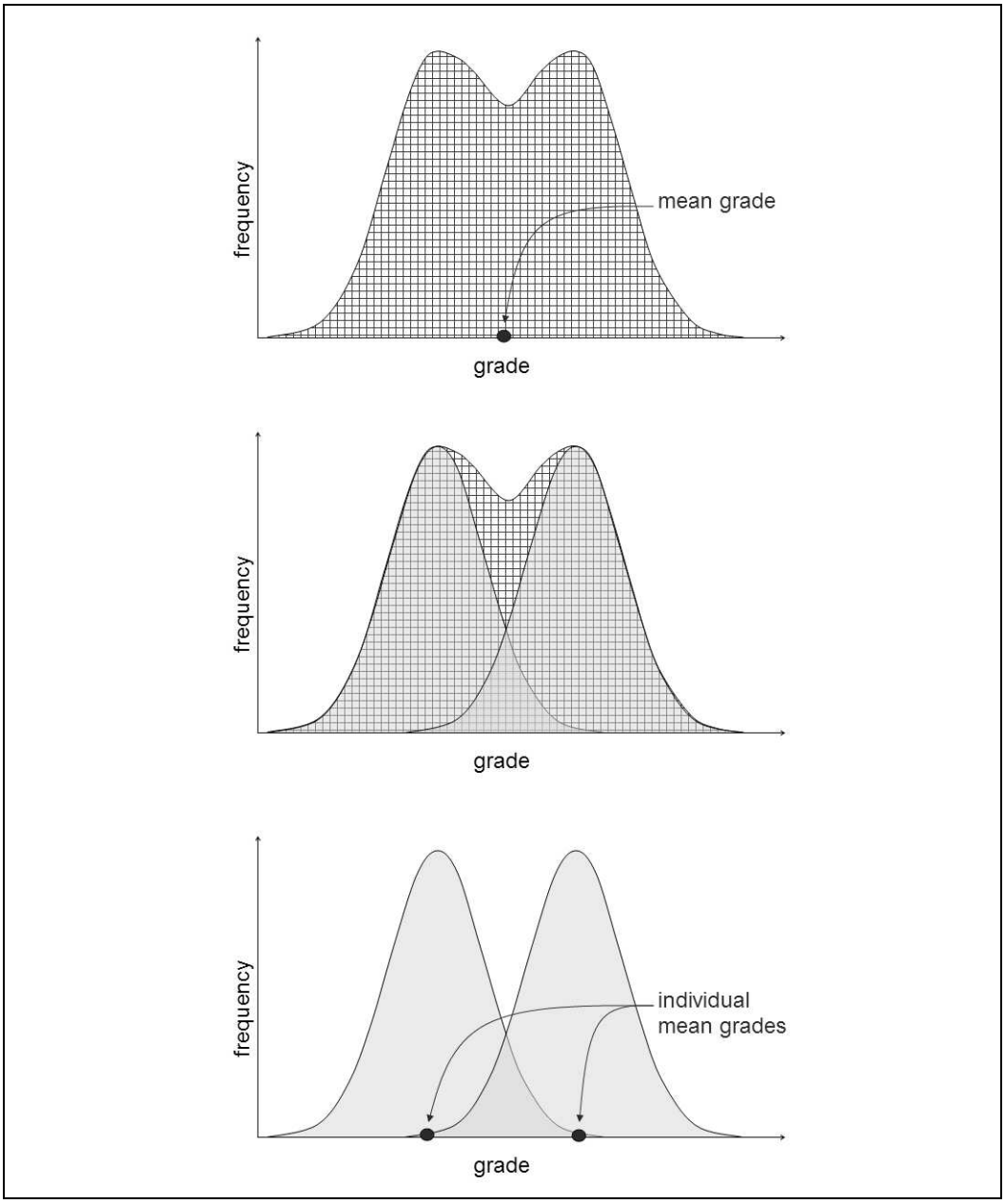


Figure 140 Mixed populations

When populations cannot be separated spatially (and sufficient domaining has been attempted!), then an indicator is typically adopted to allow variable weighting of the samples according to their most likely population characteristics.

The data analysis required for indicator kriging includes:

- adequate demonstration of spatial integration of populations
- description of modes – what are the typical grades for each of the mixed populations?
- description of inflections – at what grade does one population become more dominant?
- demonstration of spread of higher grades relative to lower grades in 3D
- the deciles (10 percentiles) based on ranking the sample grades, or the grade corresponding to 10 percentile increments in metal
- the inflection percentiles
- additional percentiles at the high grade tail
- average and median grade between the reported percentiles

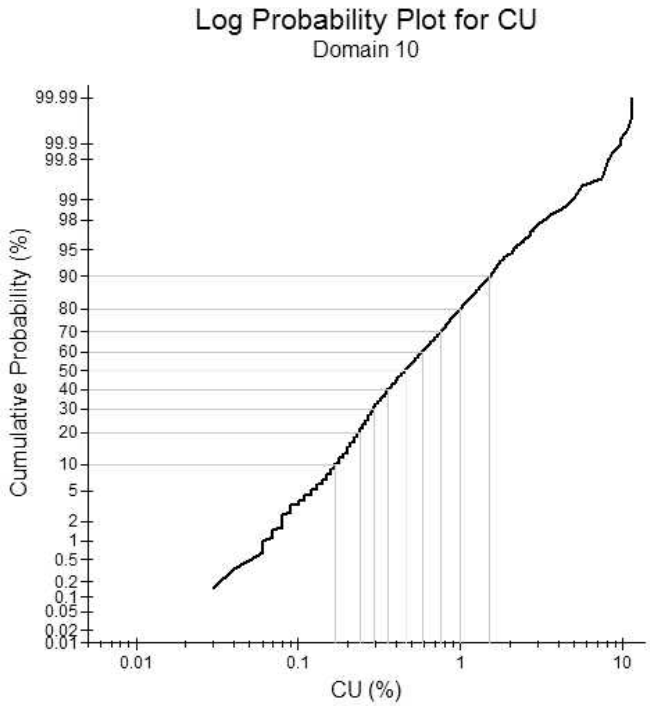
Indicator coding

The indicator approach requires generating a set of binary codes for each percentile reported in the data analysis (for example at the 10 percentile increments). These percentiles are called Indicators or Thresholds.

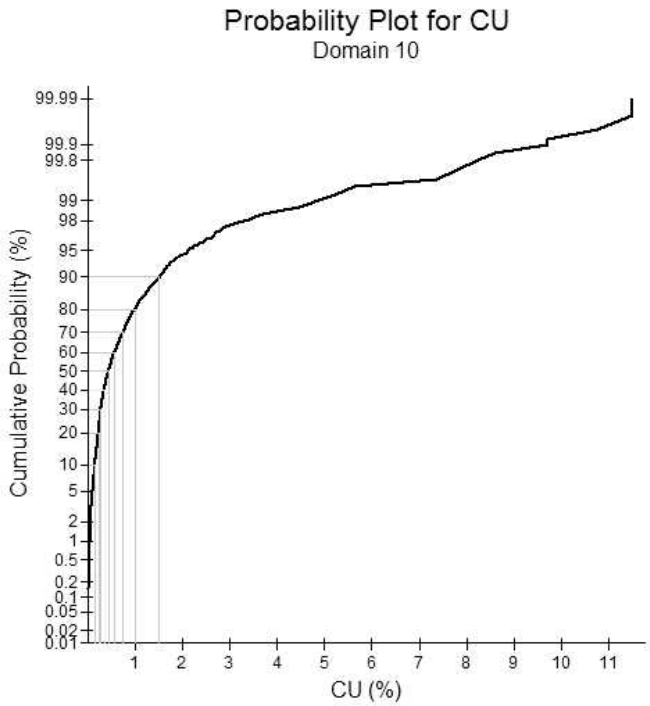
The coding function is:

- If a sample grade is less than the indicator grade set the code to "1".
- If a sample grade is greater than or equal to the indicator grade set the code to "0".

All data within a domain is coded according to a series of thresholds (sometimes called indicators or cut-offs). Ideally, the range of thresholds should adequately describe the spread of the data. Figure 142 highlights location of indicator thresholds using only the nine grade deciles. Additional indicators at the 95^{th} and 97.5^{th} percentiles provide better high grade tail definition (Figure 142).

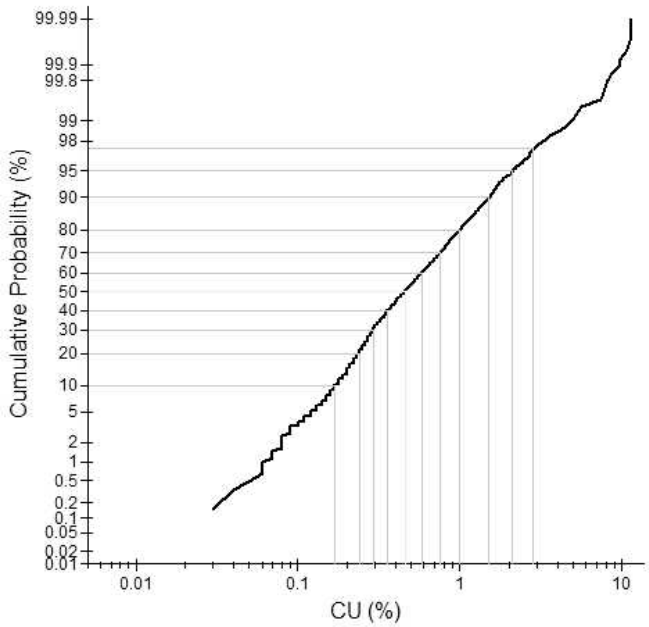


(a) nine deciles only (log scale)

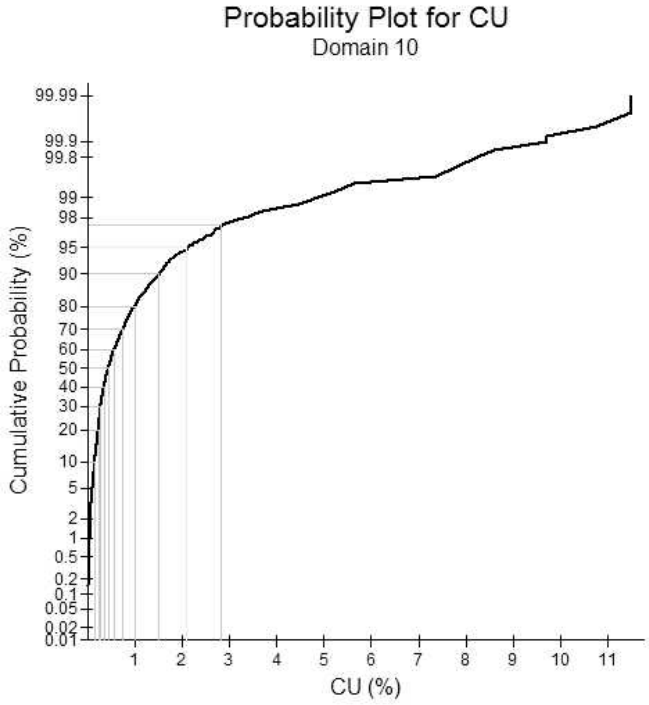


(b) nine deciles only (normal scale)

Figure 141 Location of indicator thresholds on probability plots – nine indicators only



(a) additional indicators (log scale)



(b) additional indicators (normal scale)

Figure 142 Location of indicator thresholds on probability plots – additional indicators

To help us understand indicator kriging, let us follow through the steps using a simple data set with nine samples (Figure 143), and we are interested in the grade for the block located in the centre of the data.

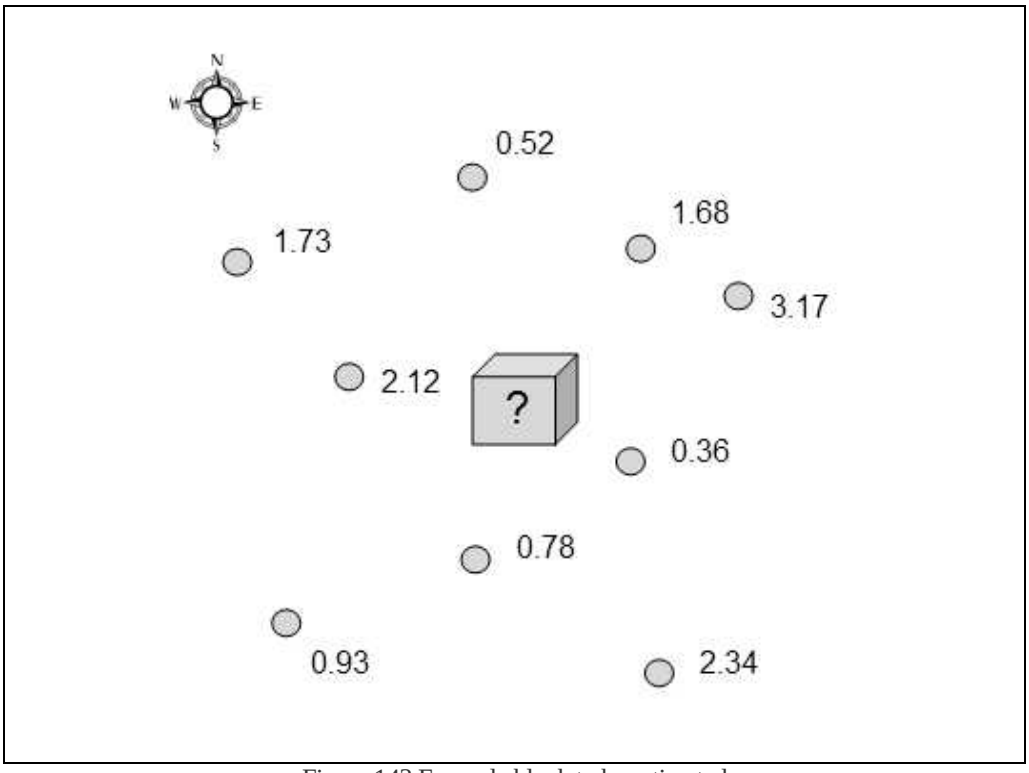


Figure 143 Example block to be estimated

When we code the data at indicators[13] (for illustration purposes we will only look at three threshold grades 0.95 g/t, 2.10 g/t and 2.93 g/t (Figure 144)).

[13] In practice typically one would use between nine and 12 indicators

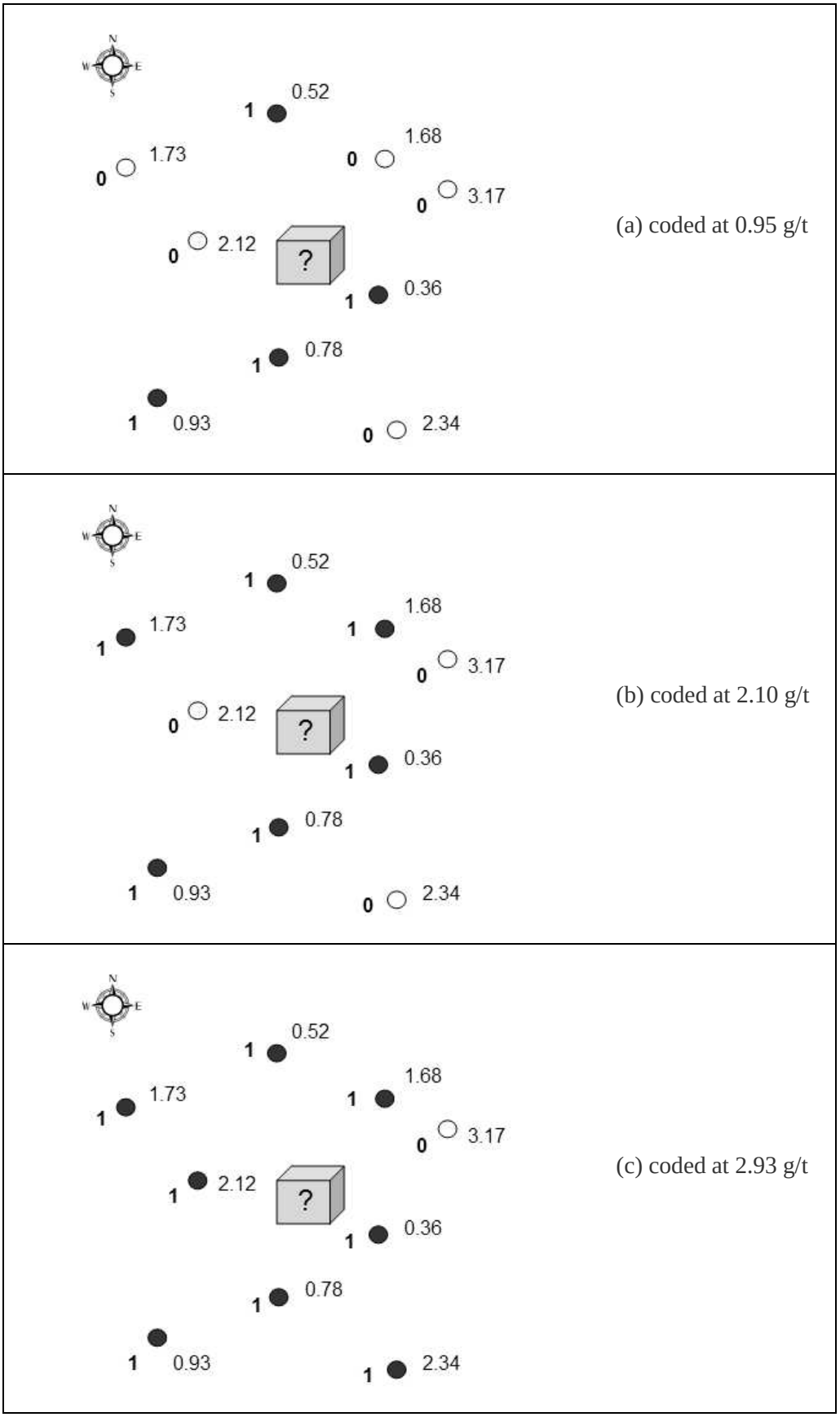


Figure 144 Indicator coded samples for three indicators

Indicator variography

Indicator variography is identical to the process used for any other variogram study. We use the same modelling techniques and philosophies we use for traditional variogram modelling – the only difference is that instead of calculating the typical difference between sample grades for different separations, we use the codes in place of the grades. This means that if there are 11 thresholds, there will be 11 sets of indicators and therefore 11 sets of variography analyses (one for each set of codes per threshold).

Importantly, the parameters should vary gradually between indicators. This will prevent order relation problems during estimation. If parameters are erratic, then the differences between estimated indicators can cause negative variances. Typically, one would expect the nugget effect to increase with increasing indicators and the ranges to decrease within increasing indicators.

Following our example through, let us suppose the indicator variogram models for the 0.95 g/t, 2.10 g/t and 2.93 g/t coded data are the ones presented in Figure 145.

Indicator estimation

Indicator estimation involves repeatedly running ordinary kriging on the coded data: one run per indicator. Let us work through an example:

The variograms are used to weight the codes at each indicator, using the respective indicator variograms (Figure 146).

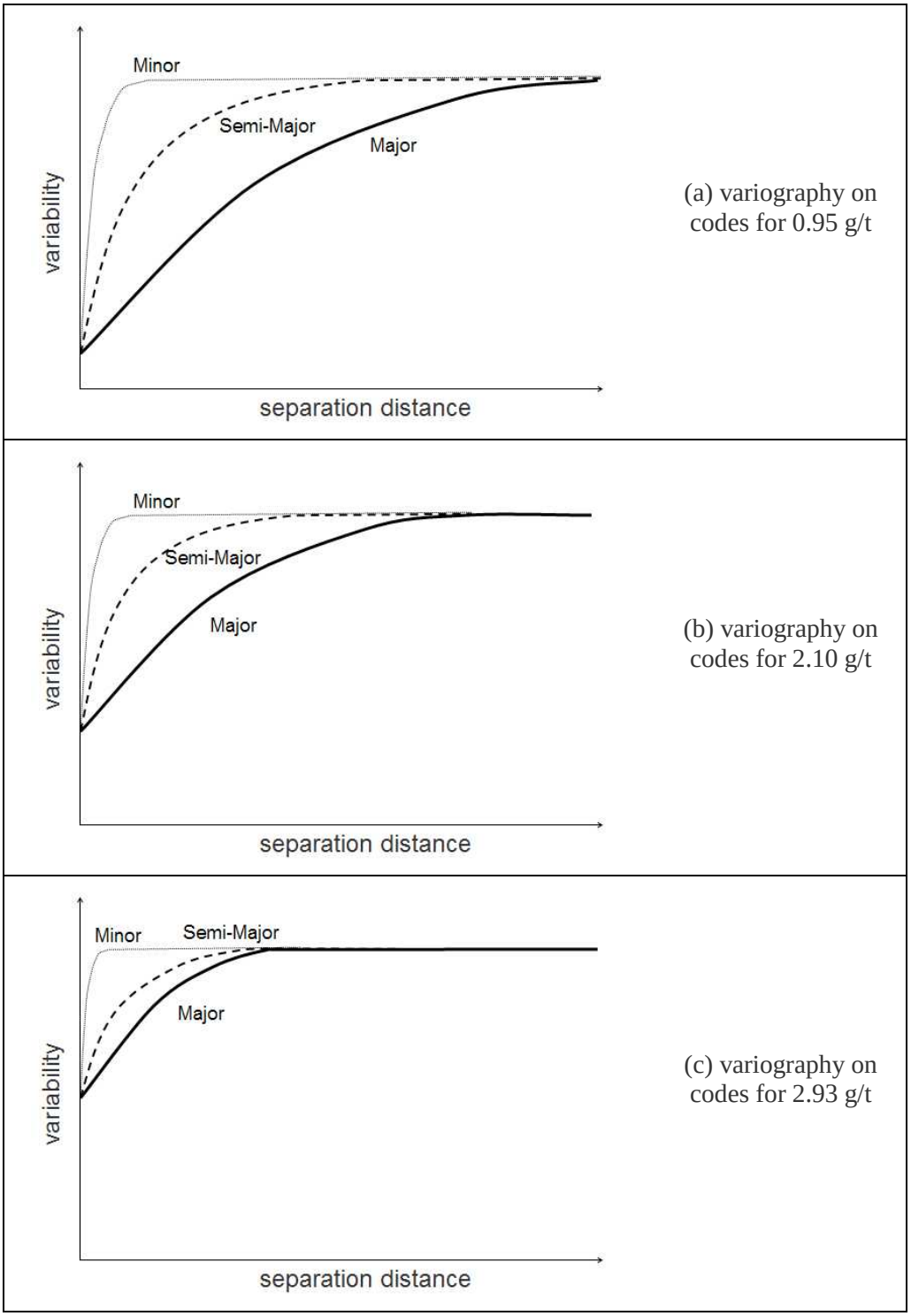
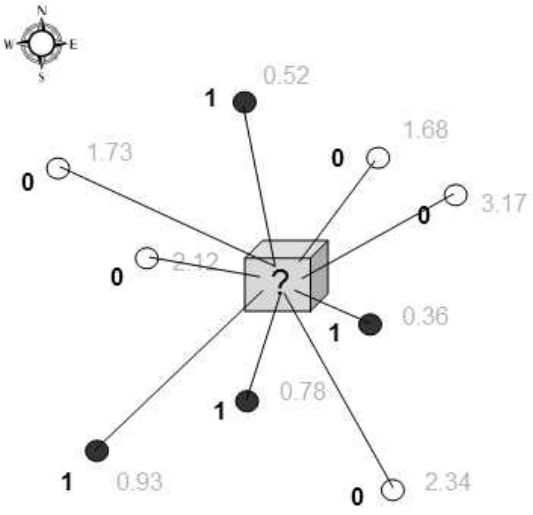
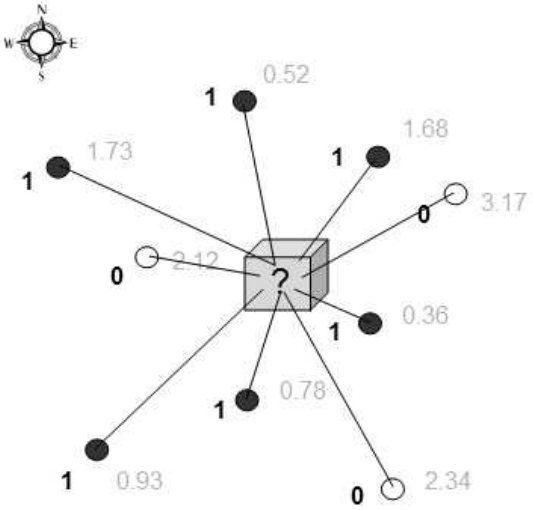


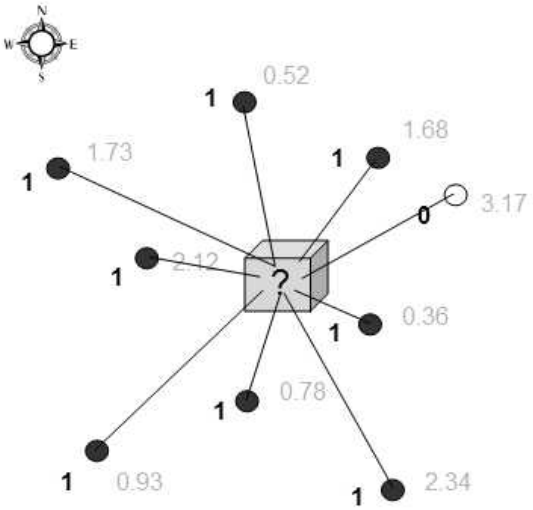
Figure 145 Variograms for each of the three indicators



(a) kriging codes at 0.95 g/t using respective variogram



(b) kriging codes at 2.10 g/t using respective variogram



(c) kriging codes at 2.93 g/t using respective variogram

Figure 146 Ordinary kriging for each of the three indicators

The estimated codes will be between zero and one and represent a probability that the unknown grade is less than the indicator grade. Notice that the proportion of "1" codes increases with increasing indicator and so the raw average of the codes increases with increasing indicator. The kriged codes are essentially a similar average, only each sample code is weighted according to the respective indicator variogram.

Let us assume the estimates for each indicator are as follows:

- At the 0.95 indicator the estimated probability is **0.275.**
- At the 2.10 indicator the estimated probability is **0.695.**
- At the 2.93 indicator the estimated probability is **0.925.**

This means there is

- a 27.5% probability that the grade is less than 0.95,
- a 69.5% probability that the grade is less than 2.10, and
- a 92.5% probability that the grade is less than 2.93.

Converting these to the probability of the unknown grade being between the indicators means

- a 27.5% probability that the grade is less than 0.95,
- a 42.0% probability that the grade is greater than or equal to 0.95 and less than 2.10,
- a 23.0% probability that the grade is greater than or equal to 2.10 and less than 2.93, and
- a 7.5% probability that the grade is greater than or equal to 2.93.

Now, suppose we know from our data analysis that the average grades for each interval between indicators are: 0.52, 1.76, 2.45 and 4.5 respectively. We can apply the probabilities of each interval to the average grades to estimate the overall expected grade.

Working this through:

27.5% x 0.52

+ 42.0% x 1.76

+ 23.0% x 2.45

+ 7.5% x 4.5

= 1.783

This is the indicator kriged grade.

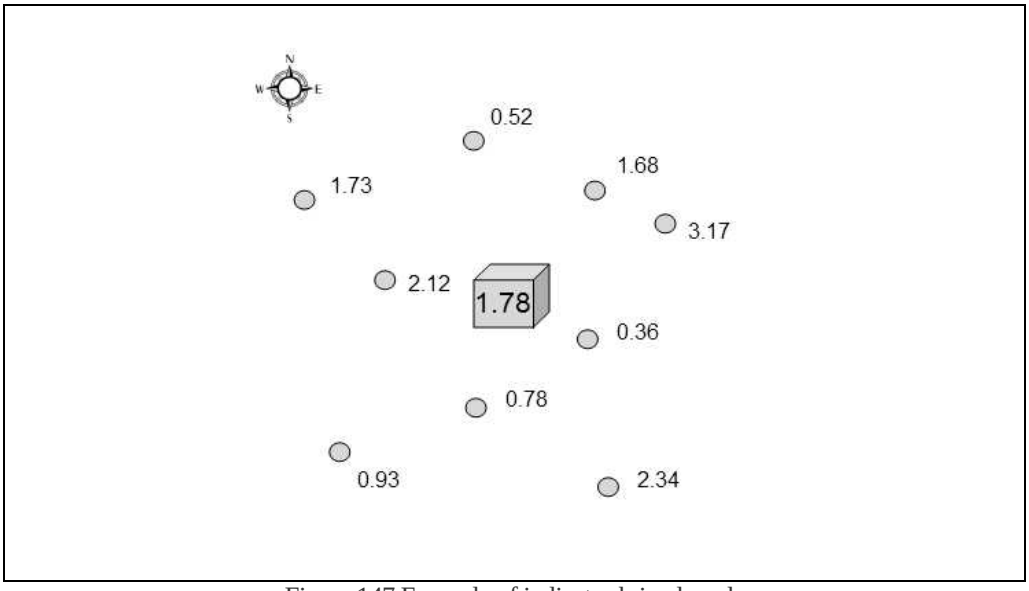


Figure 147 Example of indicator kriged grade

Often the average of the top interval is biased due to a few extreme grades. One way to manage this is to use the median instead of the mean of the sample data within the last interval.

Order relations

Order relation problems occur when the estimation parameters change dramatically with increasing indicators. An order relation occurs when the probability of the estimate being below a higher indicator is lower than the probability of being below the next smaller indicator. This will result in a negative probability (interval probability = probability less than a higher indicator less the probability of being below the lower indicator). This is nonsensical. Most programs build in a smoothing mechanism to correct for order relation problems and report the size and number of corrections.

It is best to minimise order relation problems by ensuring gradual changes in indicator variogram parameters with increasing indicators (Figure 148 and Figure 149).

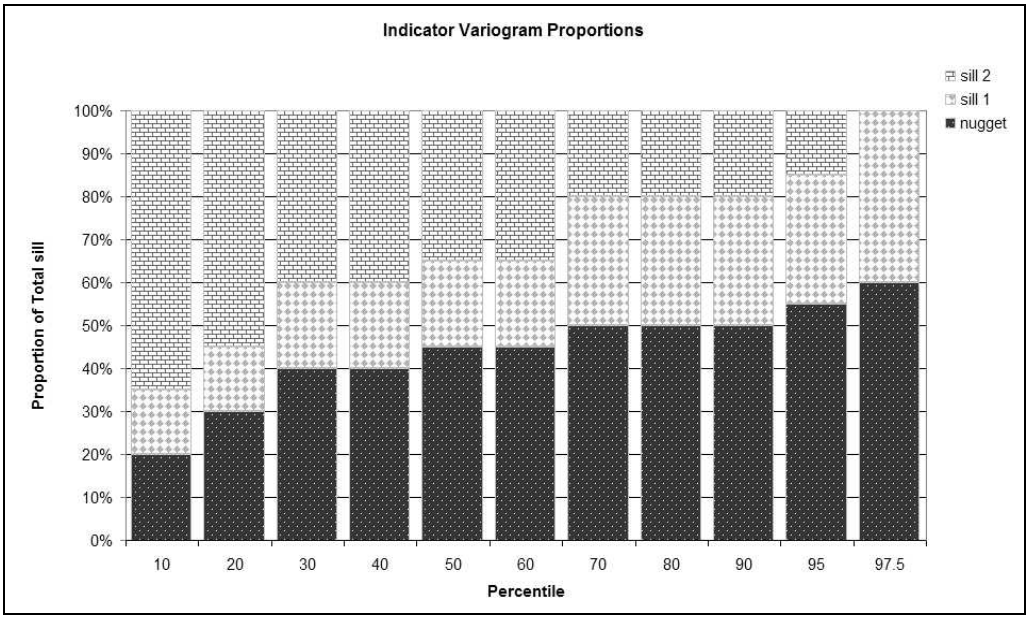


Figure 148 Nugget and sill proportions should vary gradually between indicators

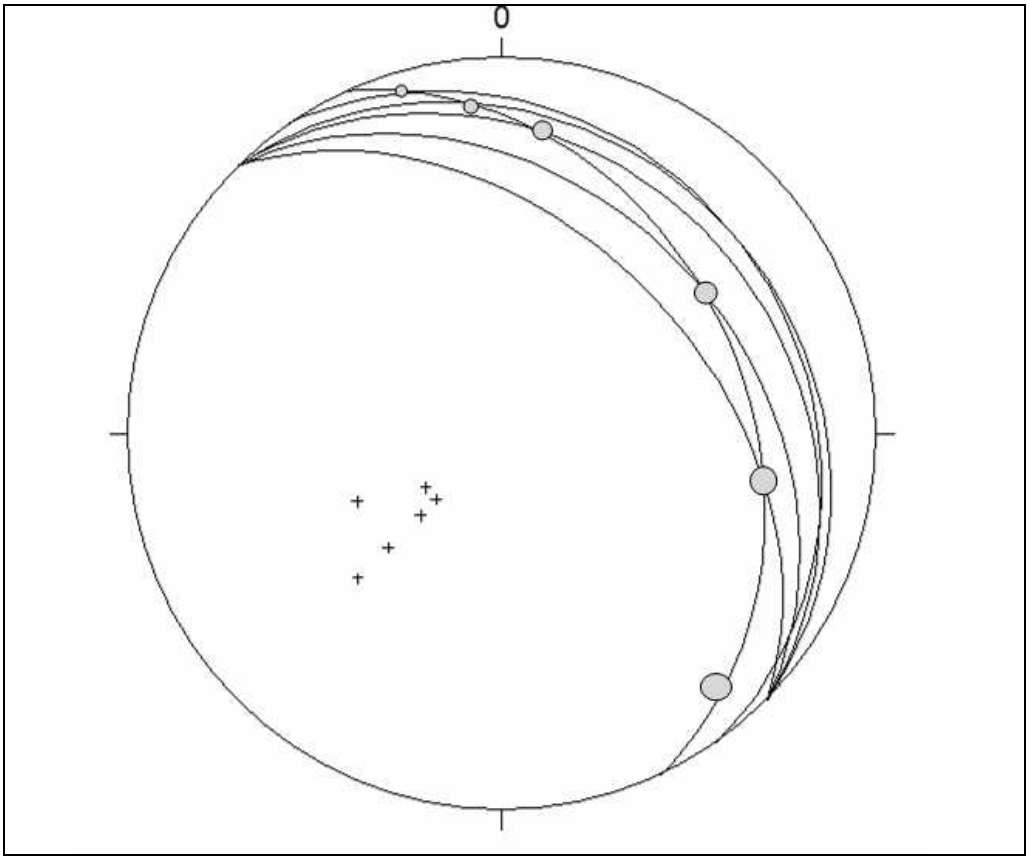


Figure 149 Stereonet illustrating gradual and systematic change in continuity directions

Selecting an estimation method

Simplicity. I recommend you use the simplest method necessary for your domained data set. With each new complication in a method comes a new layer of parameters as well as a new set of assumptions.

Ordinary kriging works well in most situations where there is sufficient domaining. The advantages over inverse distance include the built in declustering process and the weighting of samples according to observed and modelled variability rather than an arbitrary power.

Indicator kriging is useful when populations are spatially integrated. However, indicator kriging should not replace sound domaining.

Other non-linear methods are useful for maintaining good volume-variance control. These methods, however, tend to be confined to highly specialised software.

Types of Grade Arrangements

There are two typical arrangements of grade continuity:

Mosaic and Diffuse

The mosaic model describes disjointed grade continuity – much like mosaic tiles. In a Mosaic model high grades can juxtapose low grades in a seemingly disorderly pattern (Figure 150).

Where grades are more gradational, high and low grades are separated by medium grades the pattern is called a diffusion-type model (Figure 150).

A mix of the two models is created when the higher grades follow a mosaic model while the lower grades are more diffuse. This pattern is called an indicator residual-type model.

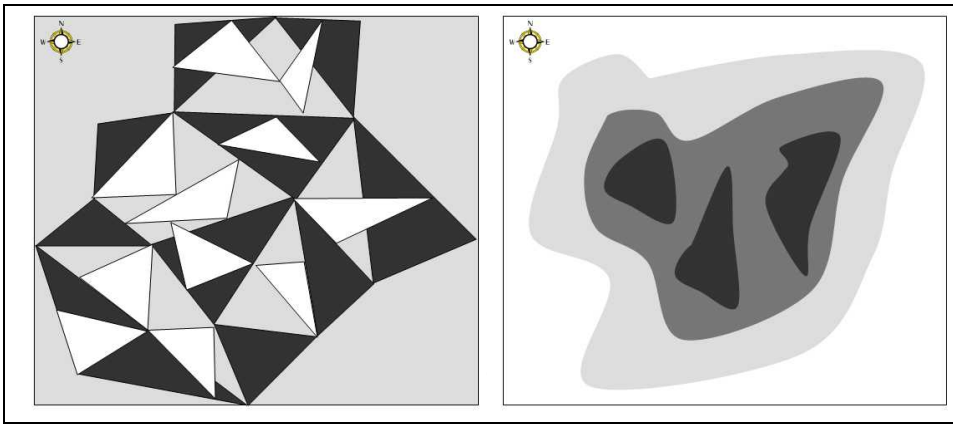


Figure 150 Example of (a) Mosaic Model (b) Diffusion-Type Model

Indicator estimation inherently assumes a mosaic model. This assumption allows each indicator to be kriged independently.

How can we test this assumption? One way is to measure the correlation between the indicators – if there is a correlation then the assumption of a mosaic model is flawed. If there is no correlation, then the assumption holds.

The way to measure the correlation is by generating a cross-indicator variogram between indicators. Standardising this cross-indicator variogram against the variogram of the lower indicator provides the probability that sample pairs are either side of the indicator, for each lag. Under the mosaic model this probability will be close to constant across all lags.

A diffusion model exists when the probability of a difference between the pairs gradually increases with increasing lag. The lower the probability for short lags, the more likely is a diffusion model applies. In this case, the mosaic model assumption is discredited and a Gaussian approach is more appropriate.

The Gaussian method of disjunctive kriging is often recommended. This is the kriging approach through a hermite polynomial transformation. Conceptually, the disjunctive kriging involves kriging of the factors of the hermite polynomials and back-transforming to produce a block estimate of grade. Strict stationarity is required for this approach.

Density modelling

A three-dimensional density model allows us to estimate the tonnage of mineralisation in the resource. With sufficient information, a tonnage can be estimated into each block from density samples in the same way as grade is estimated.

Often there is insufficient density data and density values are assigned to blocks based on the position relative to geological contacts (for example weathering profiles).

In cases where the density is a function of a sampled variable (say sulphide or lead), a regression equation is often used to estimate density in blocks based on the relationship between density and the more widely sampled variable.

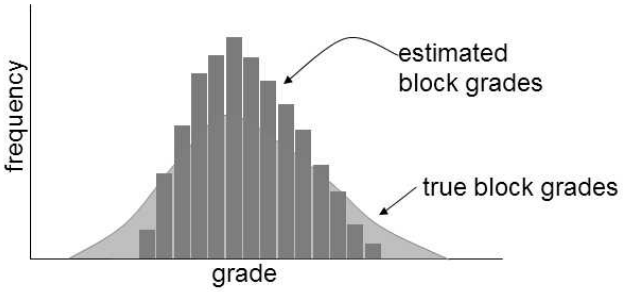
Whatever your approach to density modelling you should be explicit in your description of how you have generated a density model. This is particularly important when reporting to principles based code such as JORC, SAMREC or NI43-101, where Transparency and Materiality are two of the three guiding principles.

Optimising processes

Measures of goodness of fit

Danie Krige (1996) proposed goodness of fit statistics to evaluate the appropriateness of the set of parameters used for estimation.

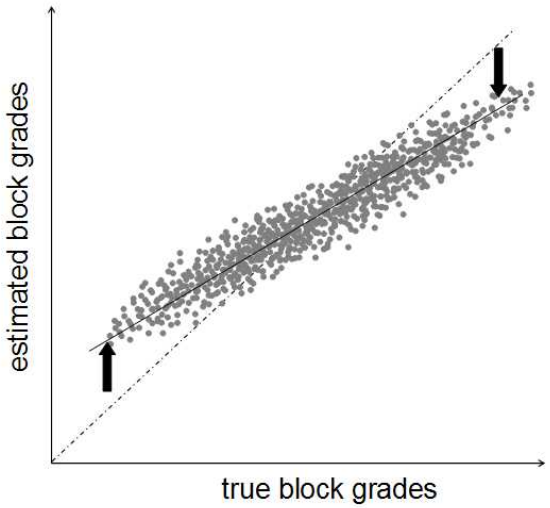
Central to the argument was the concern of conditional bias. Estimated blocks are conditionally biased when higher grades are under-estimated and lower grade blocks are over-estimated. One extreme situation of this occurs when the estimated blocks are much too small to represent accurately the grades at the locations of the small blocks.



Here we have a distribution of block grades, but when we compare them with the true block grades the tails of the estimated histogram show that the estimates are too smooth.

If we compare the estimates to the true grades on a block-by-block basis, we see a good relationship between the estimated and actual high-grade blocks and the estimates and low-grade blocks.

However, there appears to be a consistent bias – the estimated high grades are lower than the actual high grades and, conversely, the estimated low grades are higher than the actual low grade blocks. So instead of a 1:1 relationship, which would produce a slope of "1" in a regression between the estimated and actual data, we have a conditional bias where the extreme grades tend to be smoother than they should be. The regression between the estimated and actual produces a slope that is less than one. The lower the slope, the more smooth the tails of the distribution of the estimated block grades.



The goodness of fit statistics used to approximate the phenomena described above are:

- the kriging efficiency, and
- the slope of regression.

The kriging efficiency estimates the percent overlap expected between the estimated block histogram and the histogram of the true block grades. When a block has a kriging efficiency of 100%, we expect a perfect match between the estimated and true grade distributions. As the data becomes more sparse, is clustered or blocks are extrapolated more than interpolated, the kriging efficiency drops. Sometimes the kriging efficiency can even be negative, signalling extremely poor estimates.

The slope of regression estimates the slope of the regression equation between the estimated and true block grades. When the slope is "1.0", the estimated high grades and estimated low grades correspond accurately to the respective true high and low grades.

The kriging efficiency (KE) and slope of regression (slope) are estimated as:

$$KE\% = \frac{\text{block variance} - \text{kriging variance}}{\text{block variance}}$$

where the block variance is the total sill less the variance contained within a block.

$$\text{Slope} = \frac{\text{block variance} - \text{kriging variance} + |\mu|}{\text{block variance} - \text{kriging variance} + 2|\mu|}$$

Note when an estimated block has good sample coverage, the kriging variance and μ are low. An extreme is a kriging variance of zero. In this case, the kriging efficiency becomes 100% and, when the μ is negligible, the slope becomes one, a perfect estimate.

Optimising block size

The kriging efficiency and slope statistics provide a useful means of estimating the block accuracy and conditional bias ahead of estimation. Estimates at a block size that maximise the kriging efficiency and the slope statistic are expected to be more accurate. In the process described below, the objective is to test a range of parameters and then to select the parameter that maximises the kriging efficiency and slope statistic.

Consider this process for optimising the block size:

Decide which block sizes you believe are reasonable for the model.

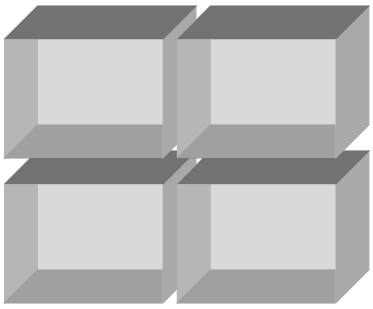
At this stage, you should have a variogram model and the sample data (this can be either the actual data or pseudo data at planned drilling intervals).

Fix the following estimation parameters to extremely high values:

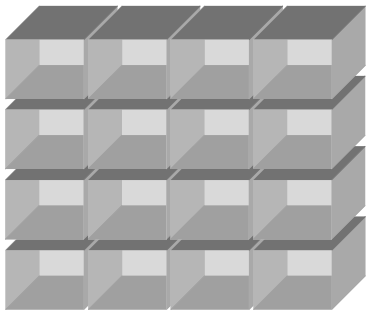
- Discretisation[14] (say to 8x8x8),
- Search ranges (for example to three time variogram range), and
- Maximum number of samples (say 150).

Define a test volume (say 100m x 100m x 100m) and set up one block model for each block size you would like to test. Calculate kriging efficiency (KE) and slope for all blocks in each test block model.

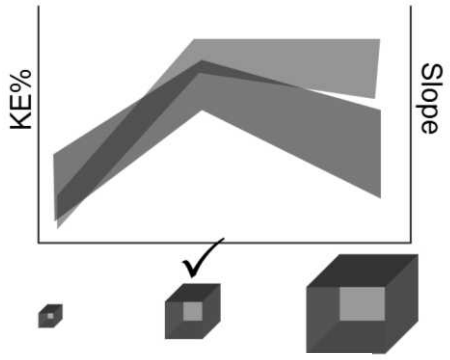
Test block size 1



Test block size 2



Test block size 3



block size options

Plot the range of KE and slope values against block size and select the block size that maximises KE and slope.

[14] My preference is to optimise the block discretisation after the search parameters are optimised.

Optimising search parameters

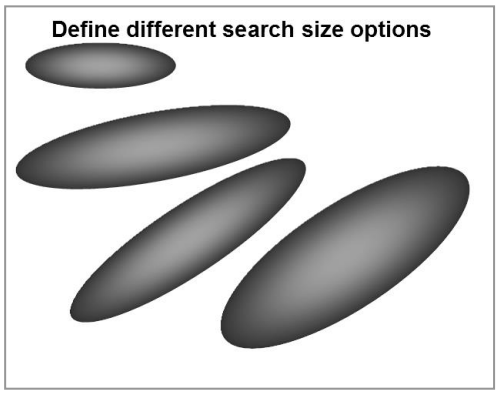
The two search parameters used to collect samples prior to estimation are the search ellipsoid (as defined by the search range and the rotation) and the maximum number of samples within the search to retain for estimation.

Consider this process for optimising the search parameters:

Search ellipsoid

Decide which search ellipsoids are reasonable for estimation.

At this stage, you should have a variogram model and the sample data (this can be either the actual data or pseudo data at planned drilling intervals) as well as the block size.



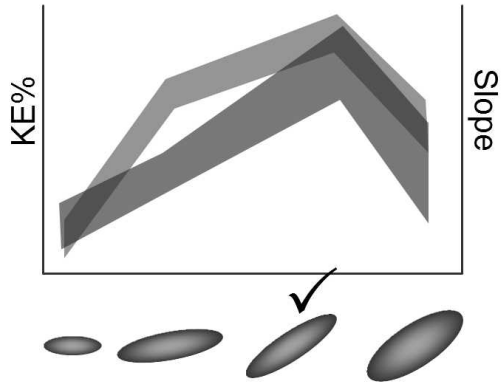
Fix the following estimation parameters to extremely high values:

- Discretisation (say to 8x8x8), and
- Maximum number of samples (say 150).

Define a test volume (say 100m x 100m x 100m) and set up one block model for each search ellipsoid you would like to test.

Calculate KE and slope for all blocks in each test model.

Plot the range of KE and slope values against search ellipsoid and select the search ellipsoid that maximises KE and slope.



Search ellipsoid options

Maximum number of samples per block

Next, decide what the least maximum number of samples are needed to produce a reasonably accurate estimate.

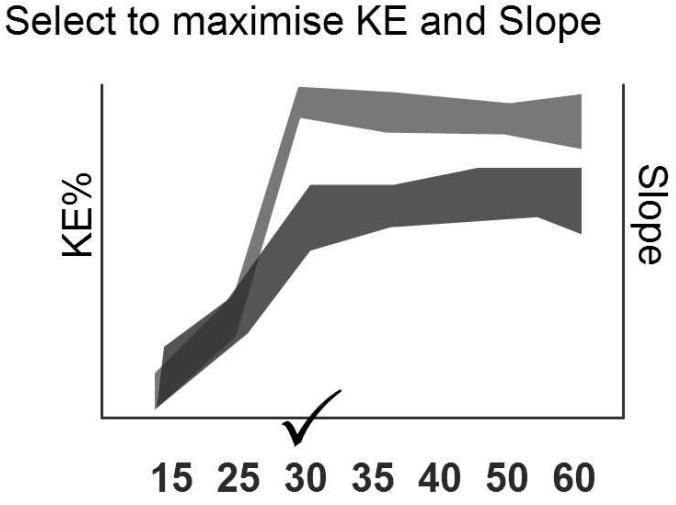
Example options for
maximum number of samples …
15, 25, 30, 35, 40, 50, 60

Using the variogram, sample data, block size used above; and the search as selected above, fix the discretisation to extremely high values (say to 8x8x8).

Based on the same test volume (say 100m x 100m x 100m), set up one block model for each value of maximum samples you would like to test.

Calculate KE and slope for all blocks in each test model.

Plot the range of KE and slope values against number of samples and select the number of samples that maximises KE and slope. Here my preference would be to choose the lowest maximum number of samples that still provides reasonable KE and slope. This allows the estimation to respond to local variations – too many samples can overly smooth out local patterns.

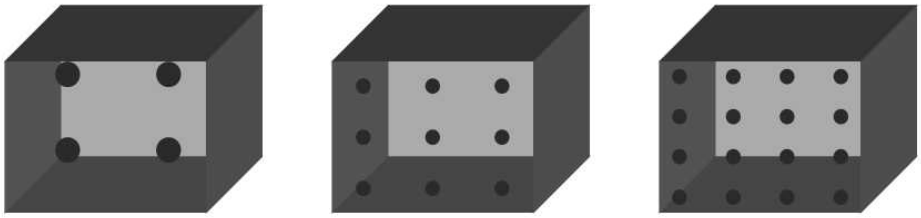


Options for maximum number of samples

Discretisation

Finally, optimise the discretisation, using the same philosophy as above.

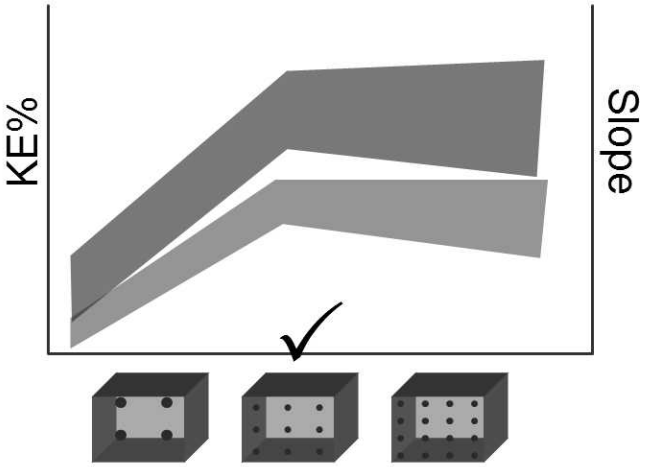
Decide on a series of discretisation options to test.



Set all other parameters according to the selection criteria as selected above. Define a test volume (say 100m x 100m x 100m) and set up one block model for each discretisation option you would like to test.

Calculate KE and slope for all blocks in each test model.

Plot the range of KE and slope values against the discretisation scenario and select the discretisation scenario that maximises KE and slope.



Discretisation options

Getting practical

We need to temper the optimisation process described above with reality. The first practical aspect to consider is the volumetric definition. The "optimisation" process described above is performed within a test volume.

If, for example, the orebody is tightly constrained, then a theoretically optimal block size may be too large for the wireframe and whilst sub-celling does a fine job of honouring a wireframe volume, the estimates are generally run on the "parent" blocks. This can result in estimation based on extrapolations to discretisation points inside the parent block, but outside the wireframe. One way to check that this is not the case is to plot up the kriging efficiency and slope model as part of the model validation process. If these statistics are reasonable, the issue may be minor.

Consider incorporating a check on the kriging efficiency and slope block models are part of your model validation process.

Another practical issue to consider are the mining limits. In an ideal world, the block volumes should mimic the minimum mining volume. This is not always possible when data is still too widely spaced. However, being reasonable in selection of dimensions can help the mine planning process (for example setting a block height equivalent to the bench height).

In my opinion and experience, the current approaches to optimising search parameters using kriging efficiency and slope offer limited value in truly optimising parameters. Until we are able to use computer grunt to calculate the kriging efficiency and slope on a continuum of parameters, and then to do this for each continuum against each and every other parameter's continuum, we are testing the best of a limited choice. When we truly optimise we test for an overall optimum set. In the meanwhile, I believe the various forms of kriging efficiency and slope testing processes currently used in the mining provide guidance until the impacts of each parameter are understood.

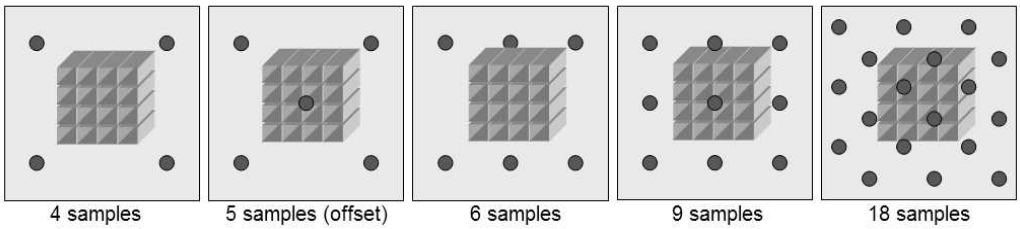
Kriging efficiency and slope are more valuable when testing and selecting between drill patterns and spacing (see page 183).

Optimising drill spacing

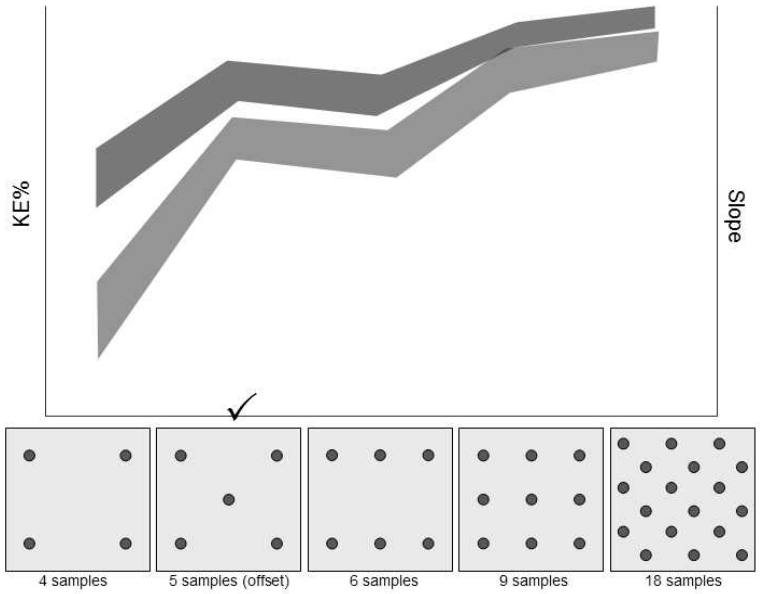
Kriging efficiency and slope can be used to optimise drill spacing and/or patterns. Again, we can adopt the same philosophy as described for optimising estimation parameters.

Let us say you have an idea of the selectivity (i.e. the block size). Other parameters you will need include the variogram model and a set of search parameters.

Set up several test patterns. These could include different orientations, drilling density and sampling frequency (note: there is no need to allocate a grade). Below we have five test patterns.



Now estimate kriging efficiency and slope in the blocks for each scenario. Since the scenarios are based on different number of samples and each sample has approximately the same unit cost, the kriging efficiency and slope statistic can be plotted against the number of samples to provide cost/benefit information. Ideally we would like the lowest number of samples that still provides good KE and slope statistics.



Sample configuration options

Conditional simulation

Conditional simulation is a process for describing grade uncertainty within a geological context.

Conditional simulation models are used to evaluate the risk associated with mining decisions such as:

- Delineation of ore/waste in grade control
- Impact on grade, tonnes and metal (and subsequent consequences) of changing a bench height
- Risk associated with pit optimisations and designs
- Variability/uncertainty in estimated grade and metal of stope designs
- Quantified risk classification

The process of generating conditional simulation models is closely related to the resource estimation or grade control processes. Conditional simulation models require just as much QAQC and geological input as resource estimation.

The data preparation and interpretation steps that precede the conditional simulation process are identical to the steps that precede an estimation run. These include:

- QAQC
- Geological analysis and interpretation
- Domaining and wireframing
- Data analysis
- Variography
- Parameter selection (including choice of method)

The same assumptions regarding stationarity still hold and are probably more important than for estimation.

The outcomes from conditional simulation are a series of equally likely models (say 50), which can be used to test consequences of decisions. Rather than represent smoothed estimates of grade like the resource and grade control models do, the conditional simulation models reflect the variability or grade texture as described by the variogram model. Since this is a better representation of the variability in adjacent grades, the conditional simulation models provide an opportunity to trial different decisions on the same model. There is also the opportunity to measure the range of impact of the grade variability on the decision over several equally likely models.

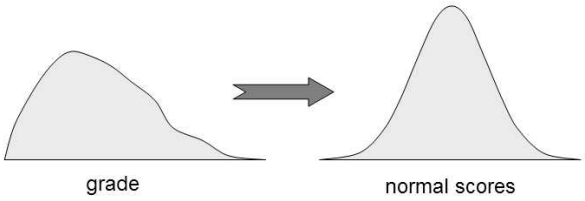
Here we will describe the two more popular methods of conditional simulation for a single variable:

- Sequential gaussian simulation (SGS), and
- Sequential indicator simulation (SIS).

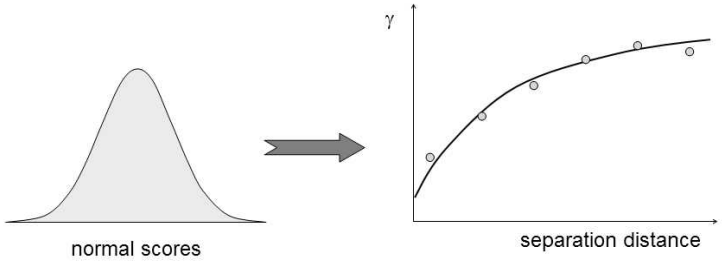
Sequential gaussian simulation

Let us follow through an example to understand the mechanics of sequential gaussian simulation. Note that this process is applied within a single domain, although samples may be accessed across a domain boundary.

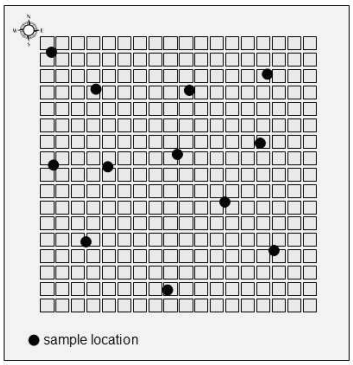
Step 1: Transform input data to normal distribution (NSCORE transform – see 127)



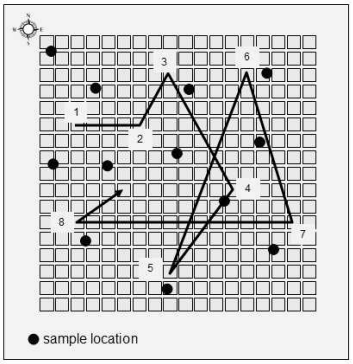
Step 2: Model NSCORE variogram



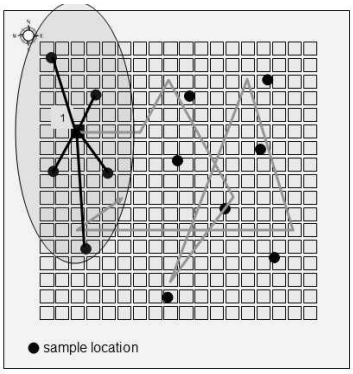
Step 3: Create a grid of very small nodes



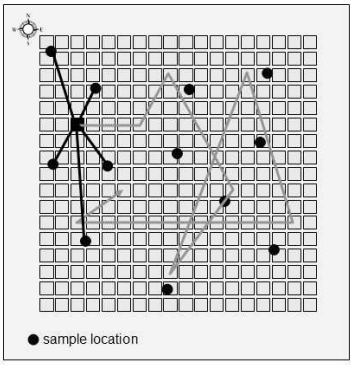
Step 4: Set up a random path to visit each node once only

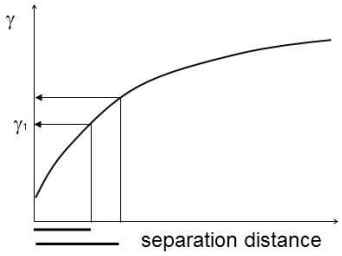


Step 5: Go to first node and search for nearest samples

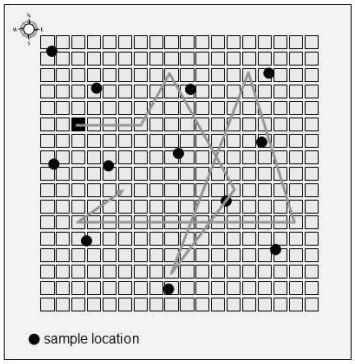


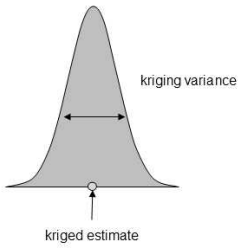
Step 6: Krige using the nearest samples



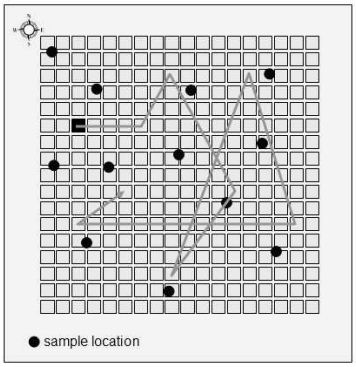


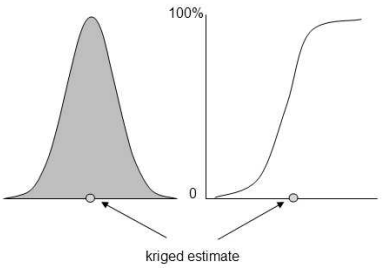
Step 7: Use kriged estimate and kriging variance to define a normal distribution



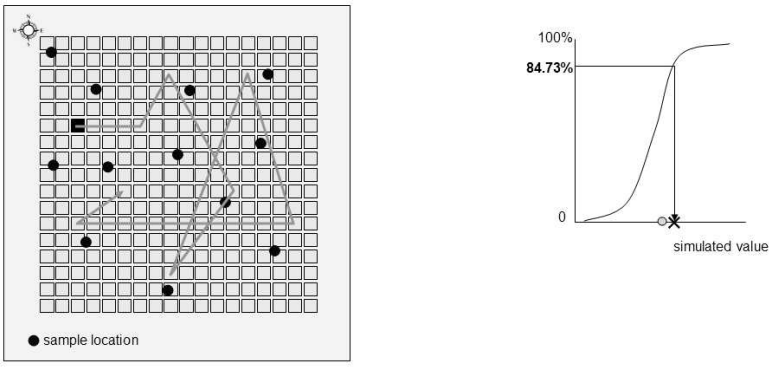


Step 8: Convert distribution to cumulative distribution



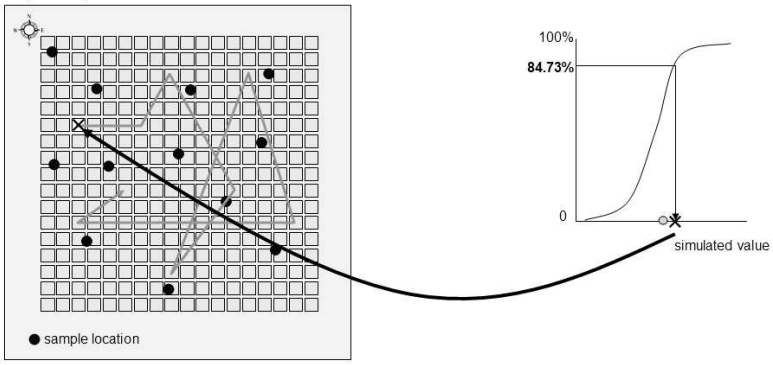


Step 9: Use Monte Carlo to randomly sample the cumulative distribution

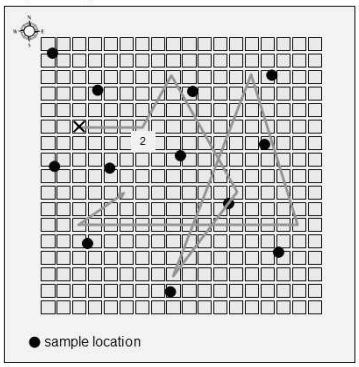


Note: if there are many close samples, the cumulative distribution will be steep and the range of possible simulated values small. However, in areas of less certainty, the distribution will be less certain, flatter and result in a wider range of possible values.

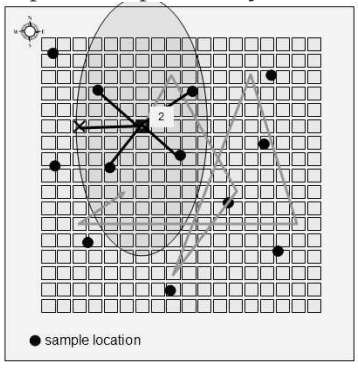
Step 10: Place simulated value at node 1



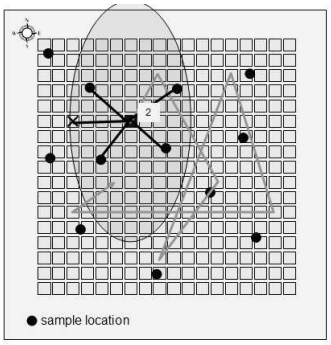
Step 11: Go to node 2



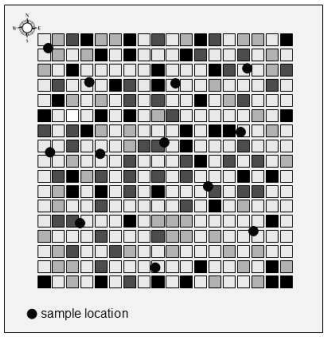
Step 12: Repeat search on samples **and** previously simulated nodes



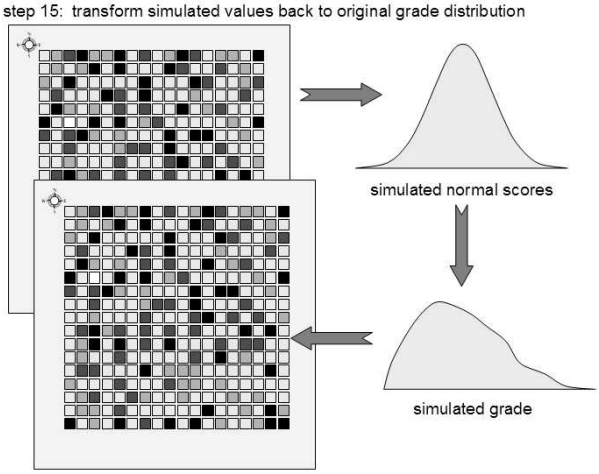
Step 13: Repeat estimation and Monte Carlo sampling to generate simulated value for node 2



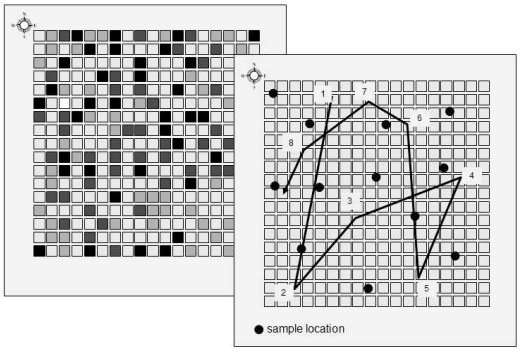
Step 14: Repeat for all nodes in path



Step 15: Transform simulated values back to original grade distribution



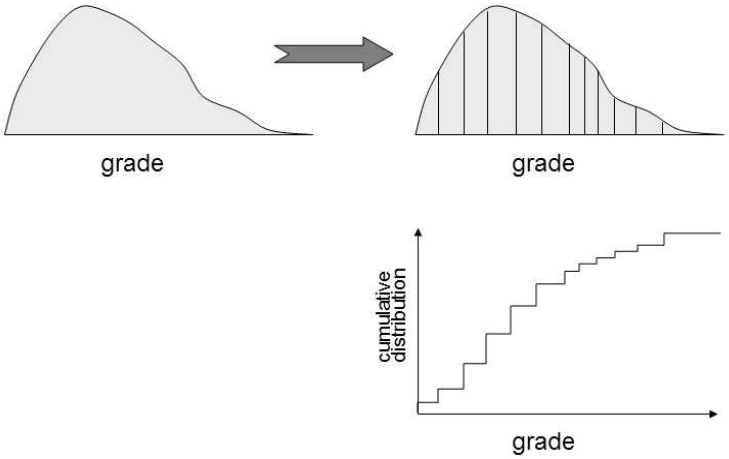
Step 16: Repeat conditional simulation process using a new random path



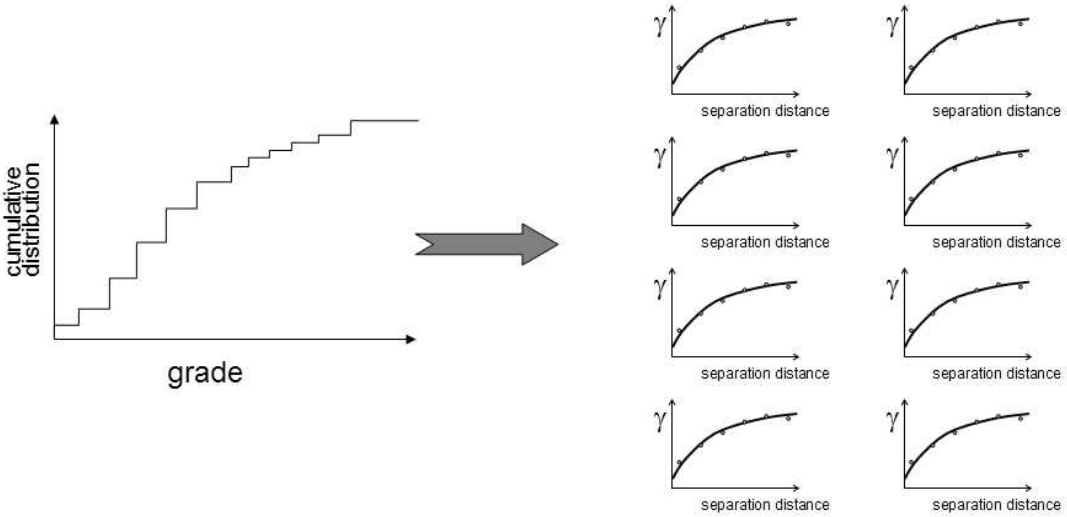
Sequential indicator simulation

Sequential indicator simulation follows the same process as Sequential Gaussian Simulation, except for the following steps …

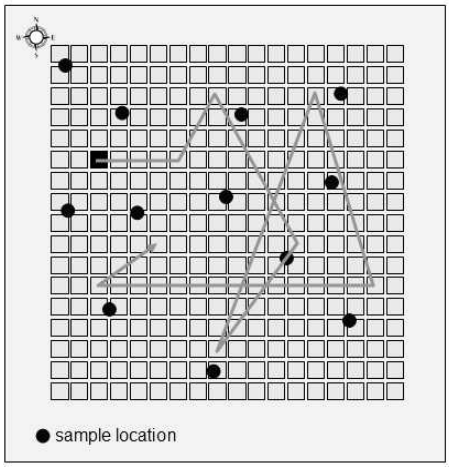
Step 1: Transform input data using indicators

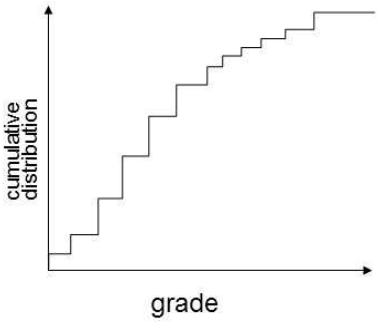


Step 2: Model indicator variograms



Steps 7 and 8: Use indicator kriging to define cumulative distribution





Step 15: There is no need to back transform since the simulation is directly on the distribution.

Multi-variate simulation

Multi-variate stepwise simulation is a neat process for building correlated simulation models. The technique works by transforming correlated data into uncorrelated data (and storing the transformation); then simulating the transformed variables independently before re-introducing the correlations.

Stepwise transformation

The method for generating correlated conditional simulation models incorporates a stepwise transformation (Leuangthong and Deutsch, 2003).

The steps for creating correlated conditional simulation models are:

1. Select sequence of elements for transformation
2. For each domain:
    a. validate correlation
    b. stepwise transformation of multivariate data
    c. validate transformation
3. Generate several (say 50) conditional simulations for each of transformed variables
4. Validate conditional simulations of transformed data
5. Reverse stepwise transformation
6. Validate correlated conditional simulations against input data.
7. Re-block simulations to required selectivity

This process depends on the stepwise transformation.

A practical implementation of the stepwise co-simulation on the Vermelho nickel laterite project, Brazil, is presented in “Exposing Uncertainty in Schedules for Proactive Stockpile Planning” (Coombes et al, 2005).

Stepwise transformation in action

Let us look at an example of the stepwise transformation. There are two elements in this example: nickel and iron. Since the project is a nickel project, nickel is selected as the primary variable for the transformation process.

There is a high correlation (0.81) between nickel and iron (Figure 151).

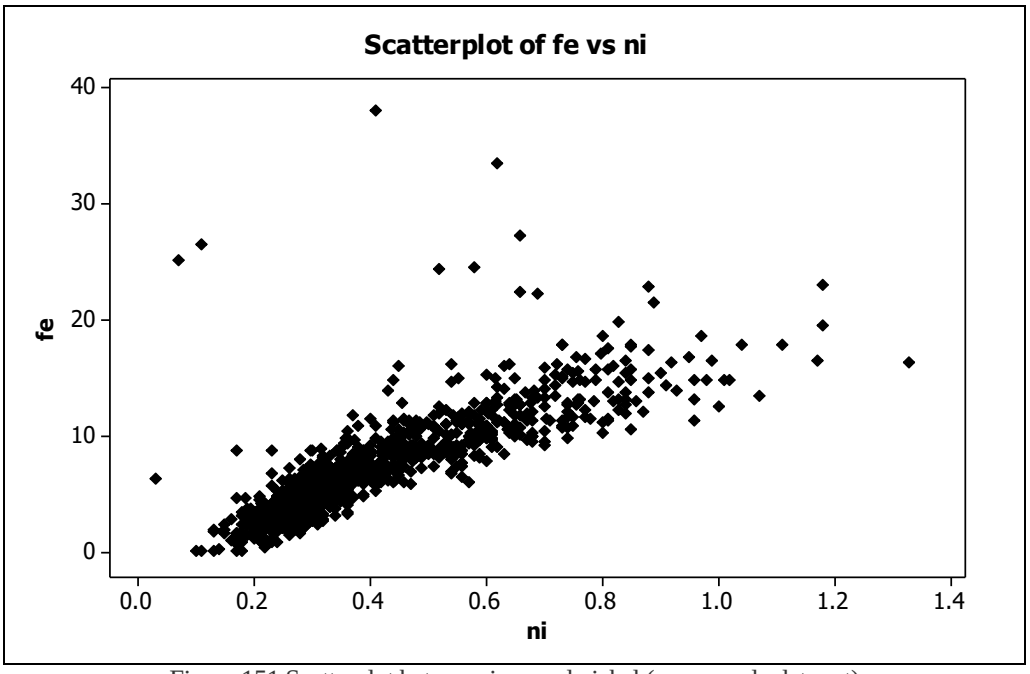


Figure 151 Scatterplot between iron and nickel (an example data set)

Next, we transform nickel to a normal distribution using a normal score transform (Figure 152)[15]. This is achieved by sorting the data into ascending order; calculating each sample's percentile and then looking up the corresponding normal score value for each percentile.

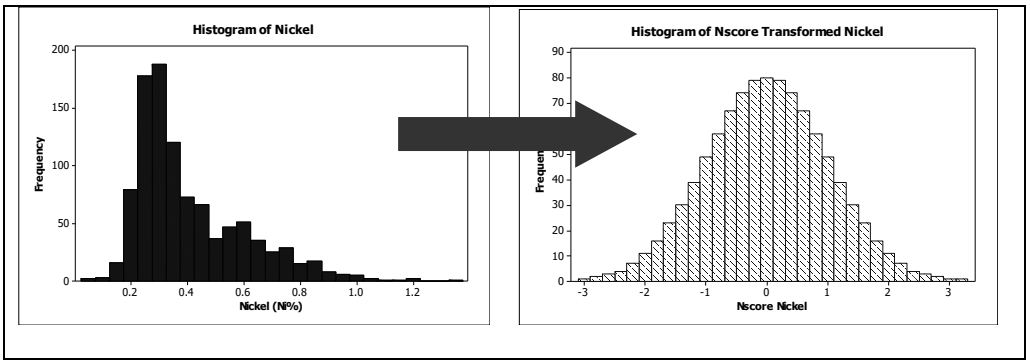


Figure 152 Normal score transform of nickel

[15] See Normal score explanation on page 127

Next, we transform the iron data to a normal distribution. The difference is we do this in slices of nickel grade. In other words, we break the nickel distribution up into intervals and independently normal score transform the iron values within each interval (Figure 153).

Therefore, a value of iron (for example 7.22%) will have a different percentile in each of the nickel intervals (Figure 154 and Figure 155).

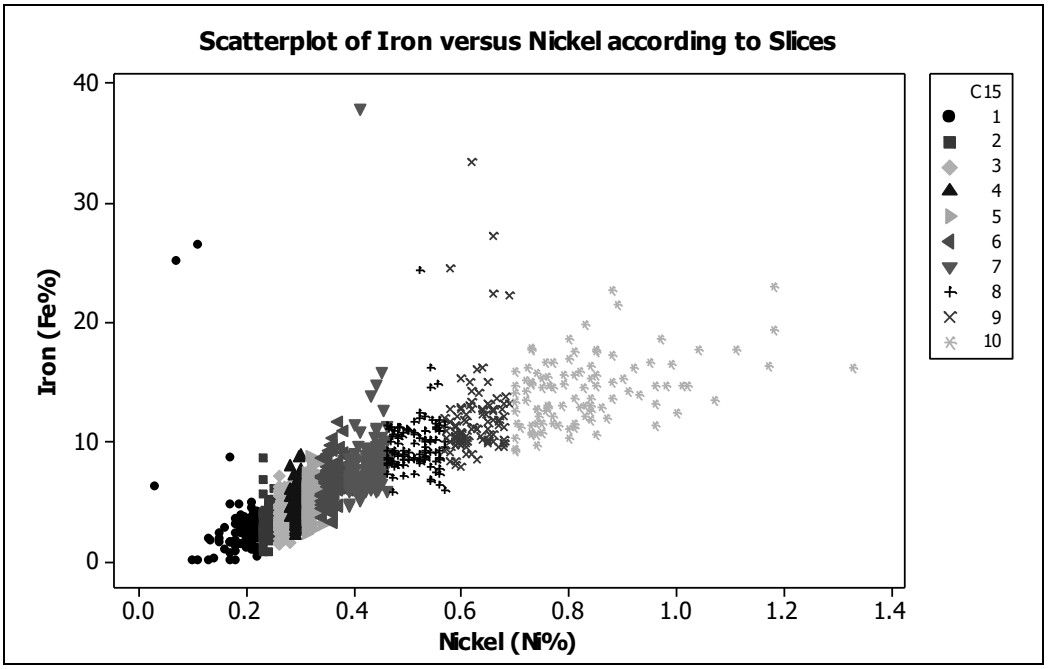


Figure 153 Iron values within intervals of nickel

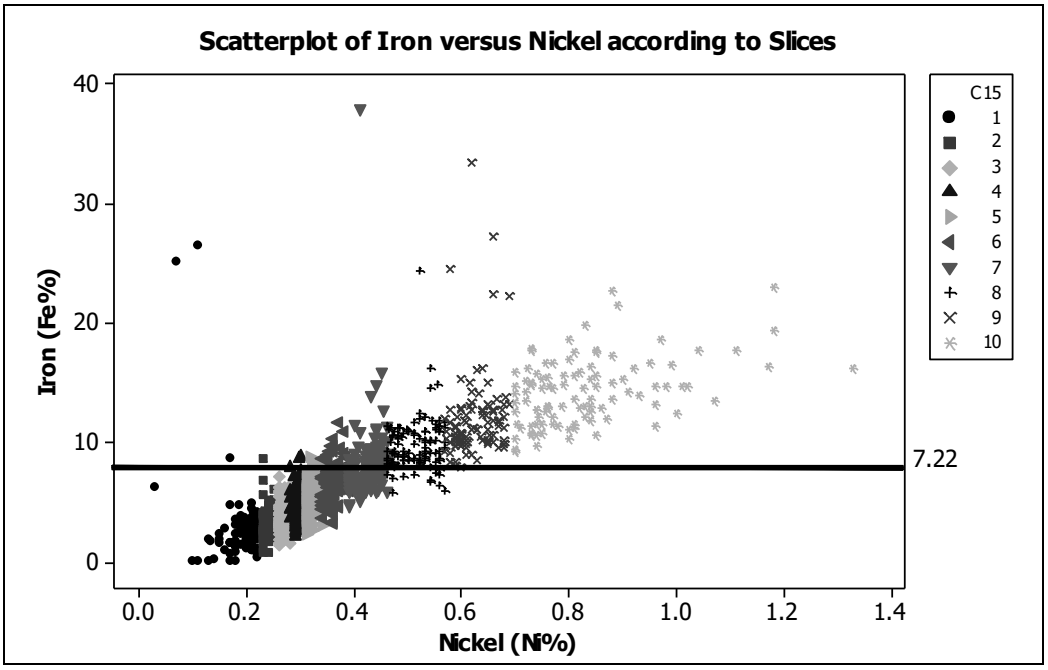


Figure 154 Iron values lie differently depending on nickel interval

Since these sub-sets are each transformed to a normal distribution with a mean of zero and a variance of one, the final distribution of transformed iron is normal. In addition, the relationship between the transformed nickel and transformed iron shows no correlation (Figure 158).

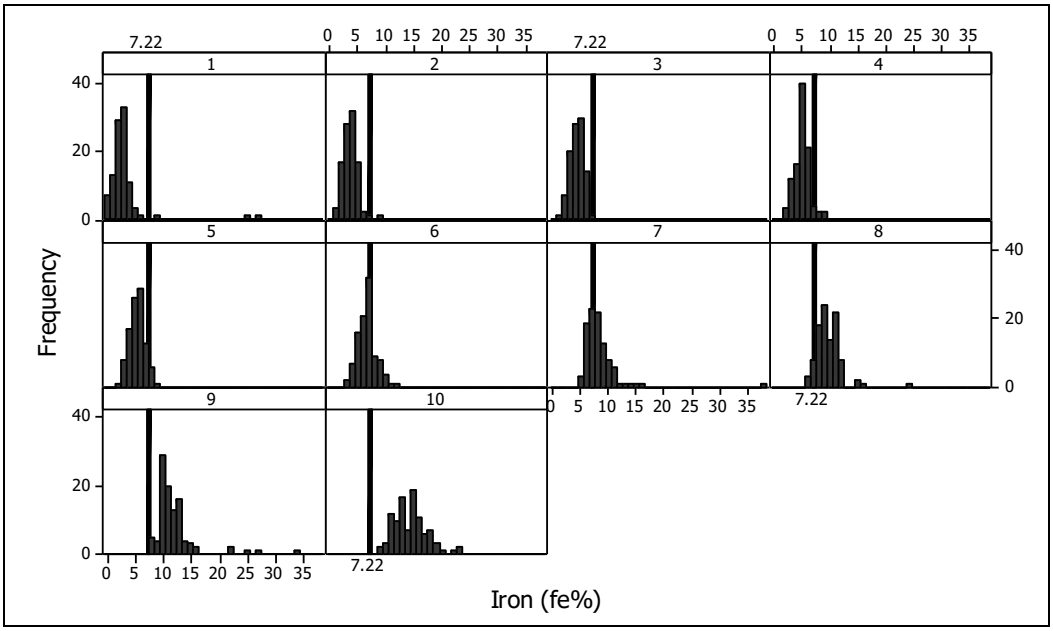


Figure 155 Iron distributions within nickel intervals

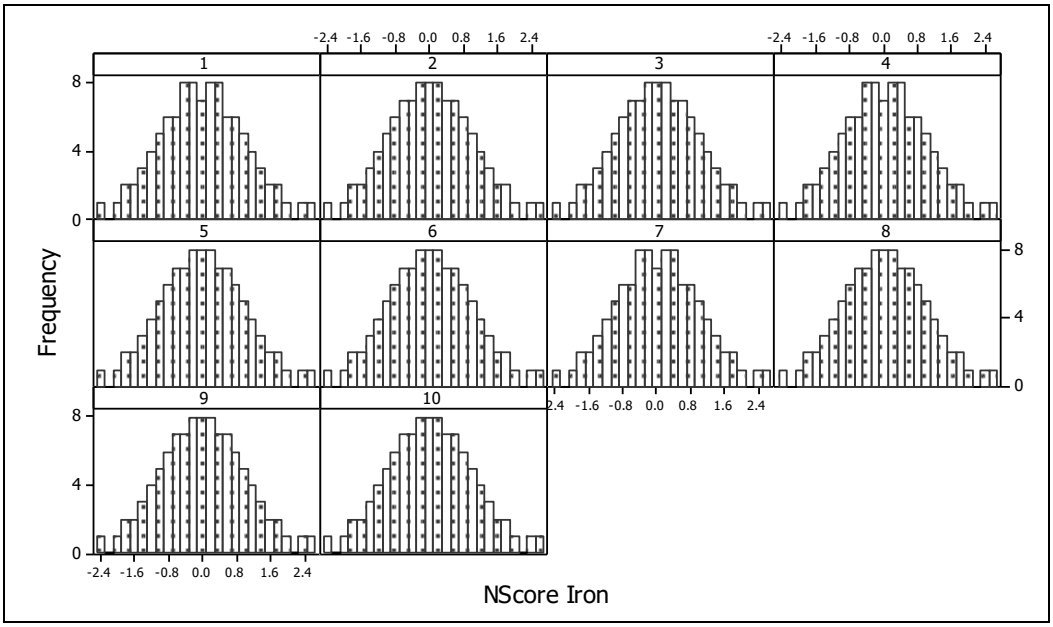


Figure 156 Each iron distribution is transformed to normal independently

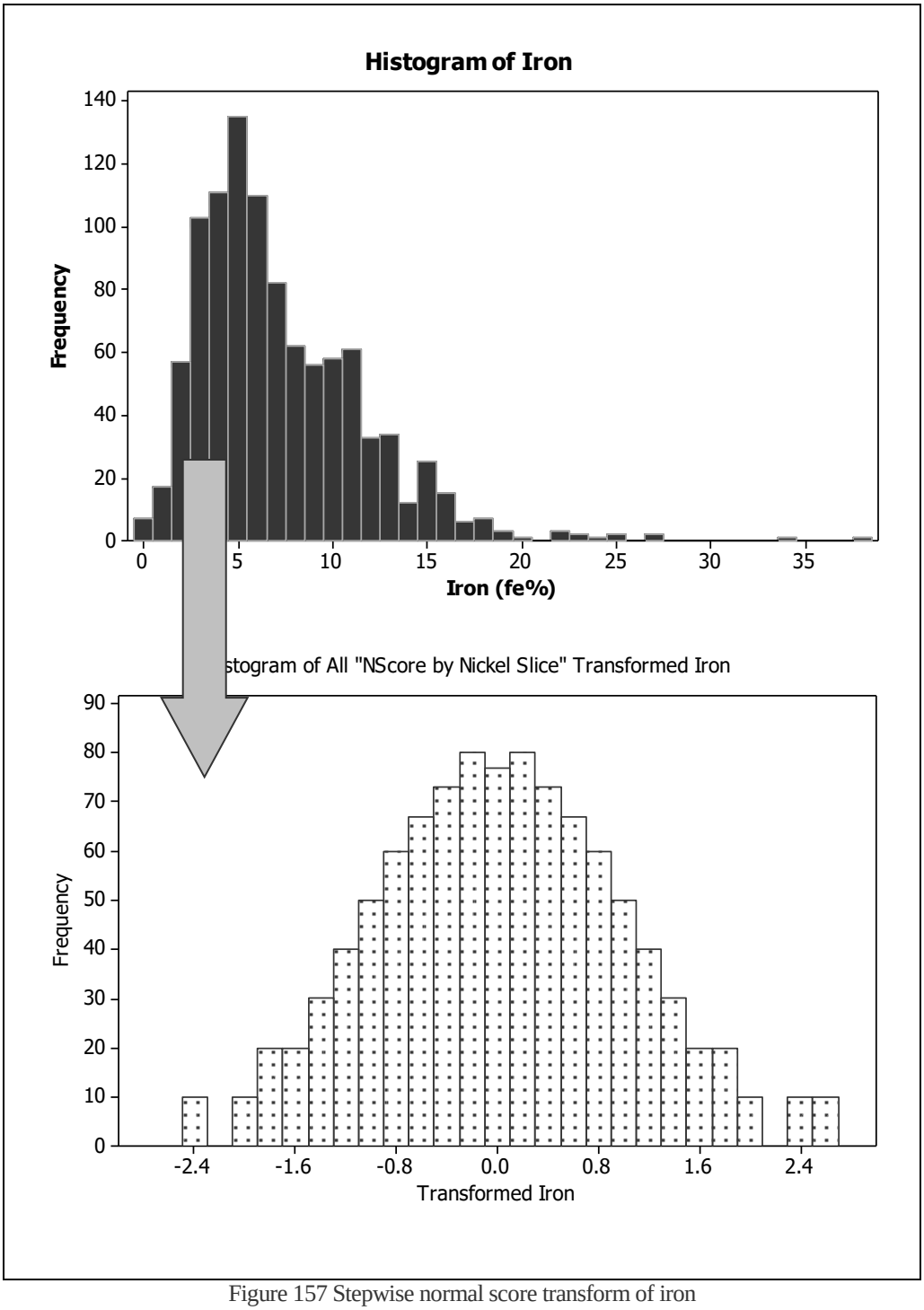


Figure 157 Stepwise normal score transform of iron

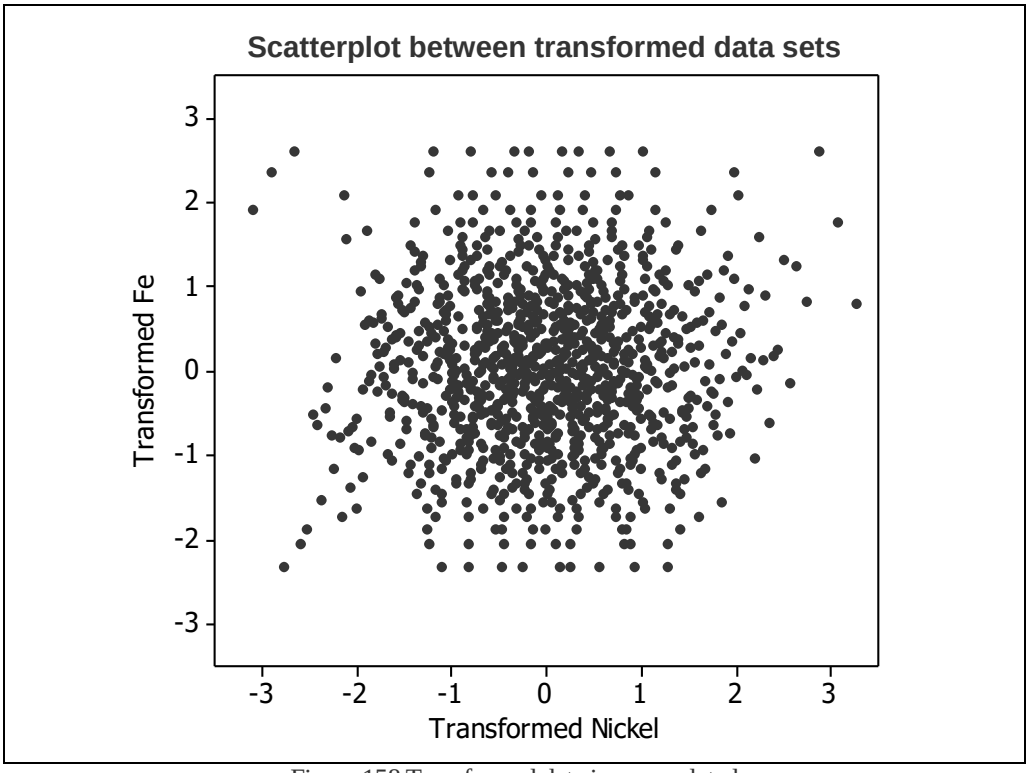


Figure 158 Transformed data is uncorrelated

Since the transformed data is uncorrelated, simulations for the two transformed variables can be generated independently. This is typically done using Sequential Gaussian simulation (see page 185).

Once the simulations of the transformed data are validated and complete, the simulation models are back-transformed. This is done by reversing the normal score process for the transformed nickel data first.

Next simulated transformed iron samples are cross-referenced to the nickel data to identify a nickel interval. The normal score process used for the specific slice is used to back-transform the simulated transformed iron value to an iron value.

Once this is complete, the correlations between the models are validated by plotting a scatterplot between the simulated nickel and simulated iron grades.

Conditional simulation applications

Conditional simulation is a tool for a completely different set of problems to those requiring estimation. Estimation is equivalent to the mean, median and mode. Estimation models are therefore useful when we want to know typical grades at various locations. If, however, we are interested in how different we believe the true grade is likely to be to the estimated grade, we would generate conditional simulation models.

Situations that would require conditional simulation models include:

- Evaluating the impact of short term grade variation on scheduled grades.
- Evaluating the range of possible economic impacts of changing mining equipment.
- Evaluating the economic risk associated with thickness variations.
- Evaluating the variation in multi-element stockpiles and subsequent risks for product quality management.
- Evaluating the degree of uncertainty or risk associated with Measured, Indicated and Inferred resources.

As these examples imply, the ultimate value of conditional simulation models is in the communication of uncertainty by the resource analyst to the engineer. Instead of assuming a typical ±10% error on an estimated resource, the engineer is able to transfer the *in situ* risk through a mine plan and ultimately measure the expected risk associated with any portion of the mine plan.

Recoverable resource estimation

The dilemma

The cost constraint of data collection means exploration data or data available at the time of resource estimation tends to be as widely spaced as possible for the assumed grade and geological model of the deposit. This means the block optimisation tends to suggest larger blocks for estimation than the selectivity that will be used during mining. Additional grade control drilling is necessary to estimate local grades accurately to facilitate the mine/waste decision.

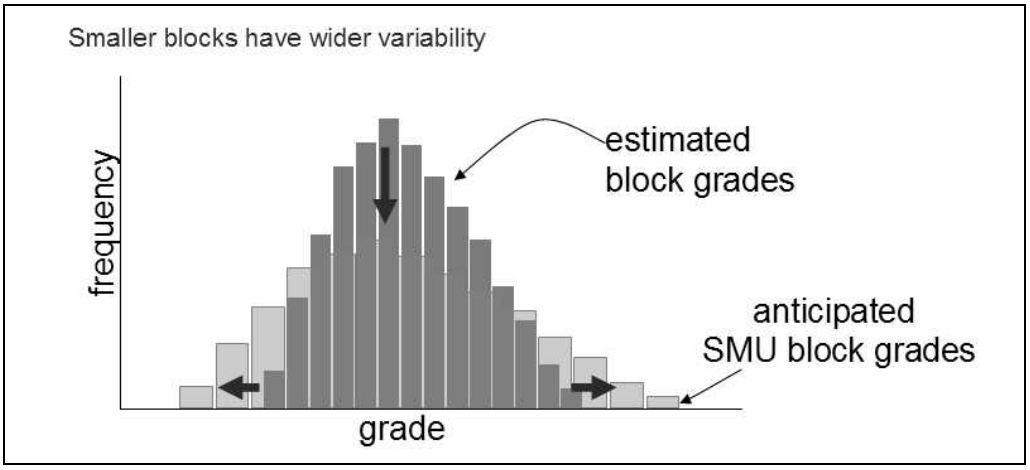


Figure 159 Mining selectivity and variability

Recall the volume-variance effect. Large blocks are less variable than the smaller Selective Mining Units (SMU). Therefore, the variability in grades observed in the resource model is less than the variability that is likely to occur at the time of grade control. Consider how the histogram of grade changes as the selectivity changes. At the sample scale, the grades have a wide low histogram. As the volume increases, the histogram narrows and peaks more.

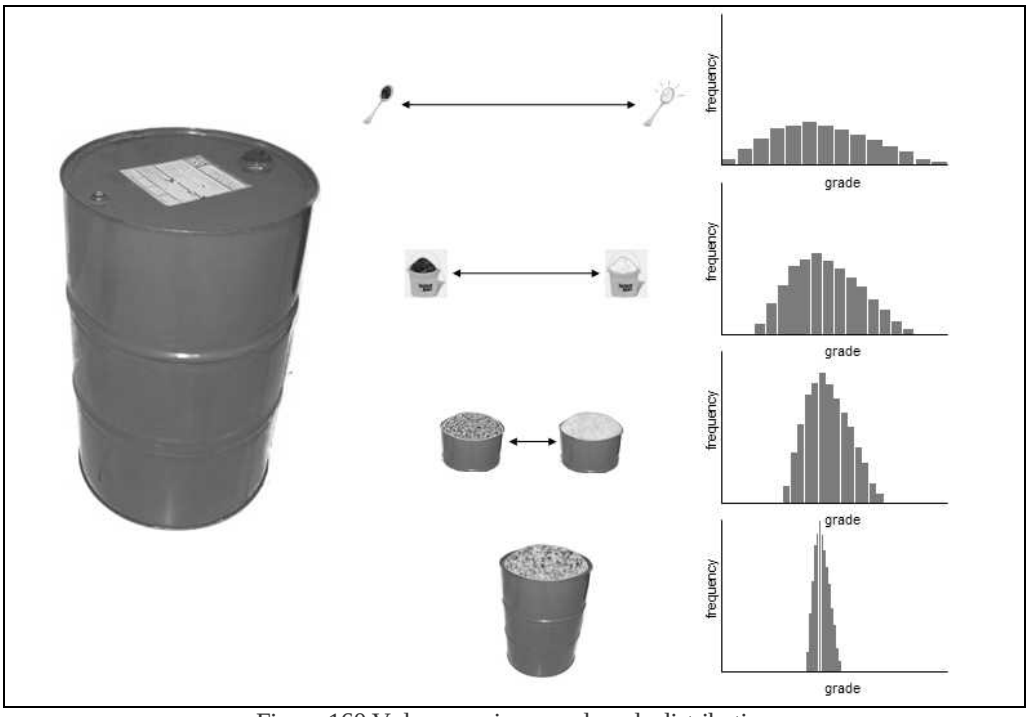


Figure 160 Volume-variance and grade distribution

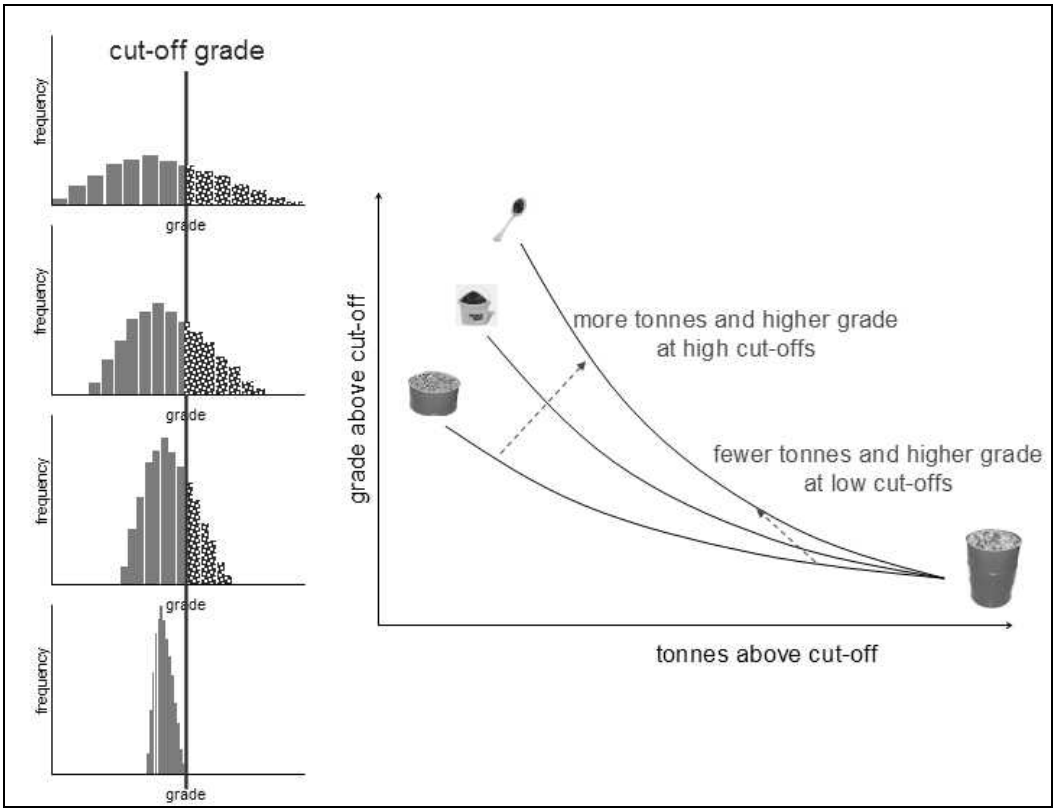


Figure 161 Impact of volume-variance effect on grade-tonnage relationship

Consider the differences in reported tonnes and grade above the same cut-off grade (Figure 161). Large volumes have no estimates above the higher cut-off grades. Conversely, the entire volume is selected at low cut-offs. This pattern is evident when the tonnes and grades above cut-off are presented on a grade-tonnage curve.

Recoverable resource estimation is the process of adjusting the variability in the resource model to provide an estimate of likely tonnes and grade at the time of grade control (i.e. for a smaller block size).

Approaches to adjusting the volume-variance

There are several approaches to adjusting a resource model at one scale to reflect the likely recoverable resource at the mining scale. These include the global approaches:

- Global affine correction
- Global indirect log-normal approach

and the more local approaches

- Local affine correction
- Local indirect log-normal approach
- Uniform conditioning

Another approach is to use conditional simulation to simulate the likely changes in variability for different scales of selectivity.

Each of these is discussed below.

Global approach

The global approach is to take the large block estimates and stretch the distribution by adjusting to the SMU variance before reporting the likely proportion and tonnes above cut-off. Note there can be no spatial context to this style of report.

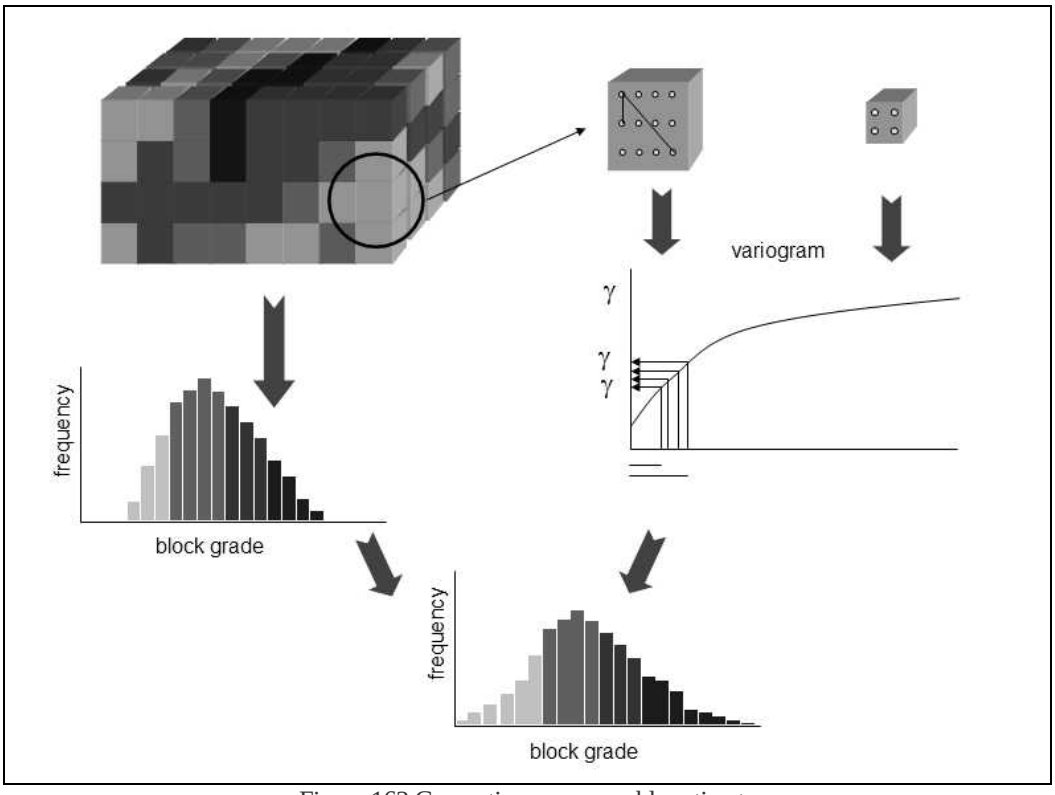


Figure 162 Generating a recoverable estimate

Global Affine Adjustment

The steps for the global affine transformation are:

1. Calculate the variance correction factor.
2. Calculate the theoretical block variance at the scale of the large blocks

   Total sill - (the average variogram value between discretisation points within a block at the resource model scale)

3. Calculate the theoretical block variance at the mining scale.
4. The change in variance is the ratio between the SMU variance and the variance at the large block scale.
5. For each estimated block calculate:

   SMU grade = mean grade + (model grade – mean grade) x variance ratio

6. Report the tonnes and grade above cut-off grade.

NOTE: These SMU grades have no spatial context and SHOULD NOT be used as a spatial grade model.

Global Indirect Log-Normal Adjustment

The indirect log-normal approach follows the same philosophy as the Affine approach. The shape of the distribution, however, is assumed to become more normal (Gaussian) as the block size increases. This is sensible in light of the Central Limit Theorem, which describes how the distribution of averages becomes more normal (Gaussian) as the number of samples within the sample sets increases, regardless of the sample distribution shape.

Each block grade is adjusted to an equivalent SMU grades as:

$$\textbf{SMU-temp grade} = a \times (\textbf{Block grade})^b$$

where:

$$a = [\text{mean} / \sqrt{(f \times COV^2 + 1)}] \times [\sqrt{(COV^2 + 1)}/\text{mean}]^b$$

and

$$b = \sqrt{[\ln(f \times COV^2 + 1)/ \ln(COV^2 + 1)]}$$

Then to get the new distribution centred on the mean …

$$\textbf{SMU grade} = [\text{mean}/\text{average of SMU-temp grades}] \times \textbf{SMU-temp grade}$$

This is repeated for each block grade. The tonnes and grades are then reported above cut-off grade to present the expected recoverable resource at the SMU scale.

NOTE: These SMU grades have no spatial context and SHOULD NOT be used as a spatial grade model.

Local approach

The local approach is to take each block estimate, impose an uncertainty distribution around the estimate and adjust the variance of the local distribution according to a volume-variance correction. These estimates present a proportion and expected grade above cut-off for each block.

A key assumption (or leap of faith!) behind this approach is that probability and proportion are equivalent. Is this possible?

Variances of lower confidence estimates are wider than estimates that have greater confidence (for example due to closer data). When variances are wide, there is more chance of presenting a probability of a high grade.

This is not the same as being certain of the presence of a small proportion of high grade.

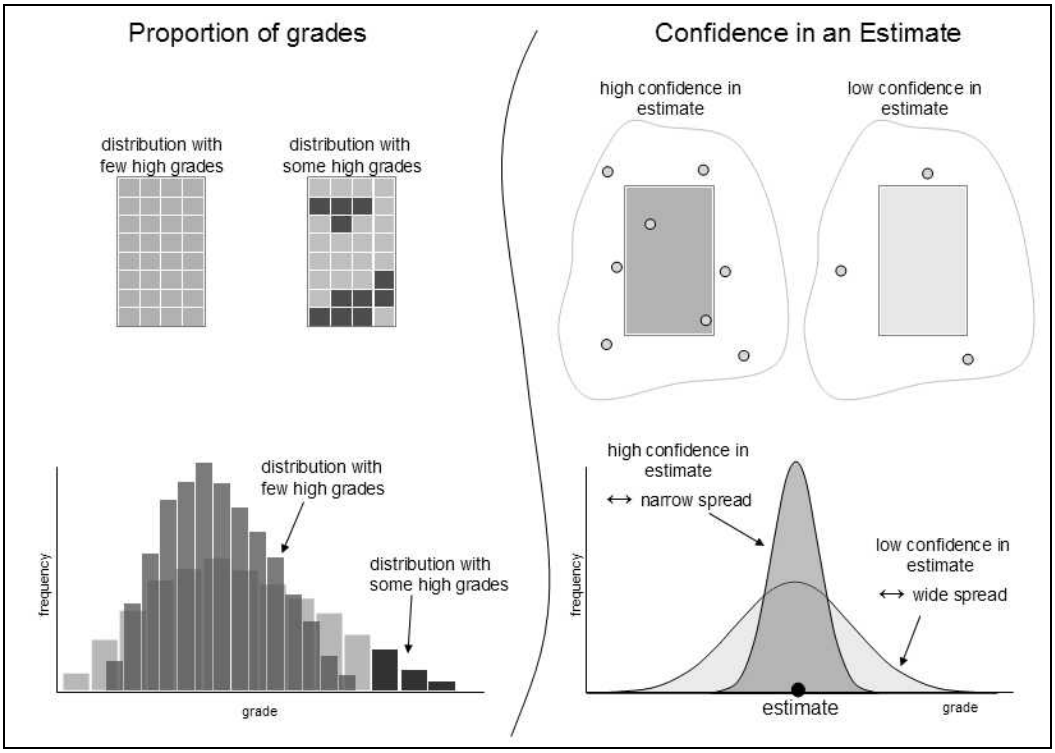


Figure 163 Proportion or probability?

The steps for the Local recoverable adjustments are identical to the Global approach, expect the process is repeated for each block:

1. Calculate the variance correction factor

2. Calculate the theoretical block variance at the scale of the large blocks

   Total sill - (the average variogram value between discretisation points (within a block at the resource model scale)).

3. Calculate the theoretical block variance at the mining scale

4. The change in variance is the ratio between the SMU variance and the variance at the large block scale

5. For each block:

   a. Assume a distribution of grades within the block. For indicator kriging, the indicator distribution is assumed while for an ordinary kriging approach a normal distribution with a fixed variance is assumed.

   b. Adjust the variance according to the variance correction factor.

   c. Report the proportion of block and associated grade above cut-off.

   d. If the proportion is less than a minimum proportion set the proportion to zero.

   e. Adjust proportions to multiples of SMUs by "diluting" the SMU grades.

6. Report the tonnes and grade above cut-off grade.

Note: While these models can be used in a spatial context, there is an inherent assumption that probability reflects proportion.

The Affine approach adjusts each block by a simple ratio change to the variance of the distribution, while the indirect log normal approach adjusts the distribution shape according to the affect the central limit theorem would have on a log-normal distribution shape.

Uniform conditioning

Uniform conditioning tends to be applied as a local correction. The steps in uniform conditioning are:

1. Use Hermite polynomials to model the grade distribution and transform it to a normal distribution.
2. Calculate and model the variogram based on the transformed data.
3. Check the data is bi-variate normal by observing concentric circles on the h-scatterplot (see page 108).
4. Calculate the anamorphosis function (the variance factor) for the SMU and for the large block size.
5. Transform the estimated resource model blocks to a normal distribution through the anamorphosis function.
6. Adjust each transformed block distribution according to the SMU anamorphosis variance.
7. Report the proportions and values above cut-off.
8. Back-transform the estimates to grades.
9. Accumulate the tonnes and grades above cut-off for each block to present a global recoverable resource.

Again, note the key assumption here is that probability reflects proportion.

Conditional simulation approach

Conditional simulation allows close points to be simulated. Whilst conditional to the variogram model, they do provide an estimate of the likely change in tonnes and grades for a change in selectivity.

The approach is simply to simulate on a fine grid and to reblock the simulations to both the resource model and the mining scale. For each simulation report the percent change in tonnes and grade at the cut-off of interest. This can be used to guide the expected change in tonnes and grade of the resource model to the SMU scale selectivity.

Journel and Kyriakidis (2004) provide a detailed application of the conditional simulation approach.

Multi-variate recoverable estimation

When multi-elements are correlated, the relationship between the samples can be carried through to the recoverable estimation stage.

Consider an example of correlated elements - say iron and nickel in a nickel laterite data set (Figure 164). The correlation is about 68% and tends to have a curvilinear relationship (Figure 165).

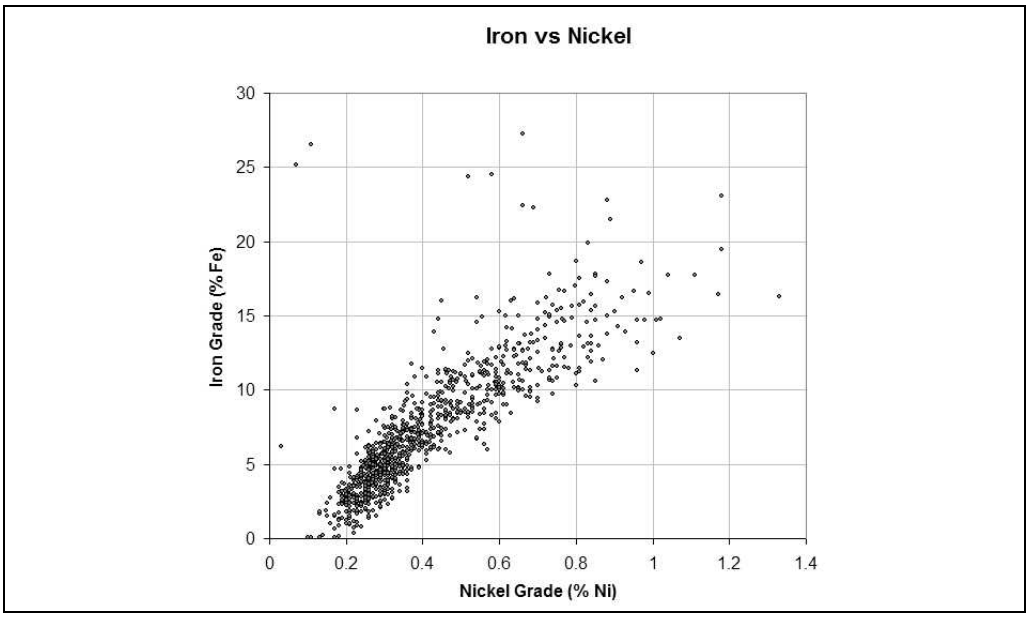


Figure 164 Scatterplot between iron and nickel (example data set)

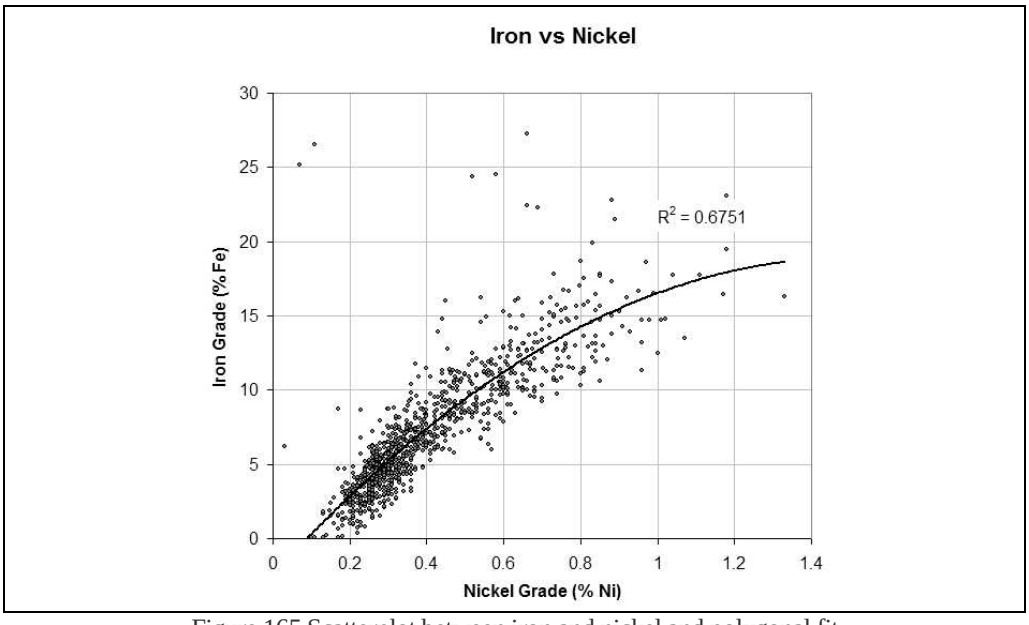


Figure 165 Scatterplot between iron and nickel and polygonal fit

Consider applying a volume-variance correction to each of the two data sets from point distribution to the selective mining unit according to their respective variograms (Figure 166 and Figure 167).

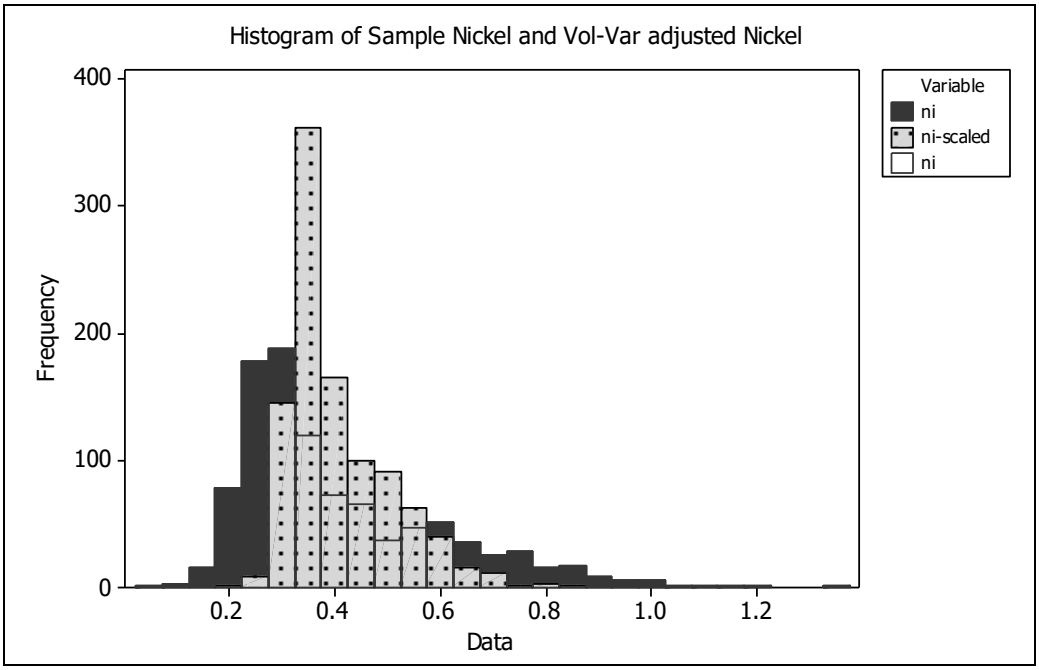


Figure 166 Histogram of nickel (sample and SMU volumes)

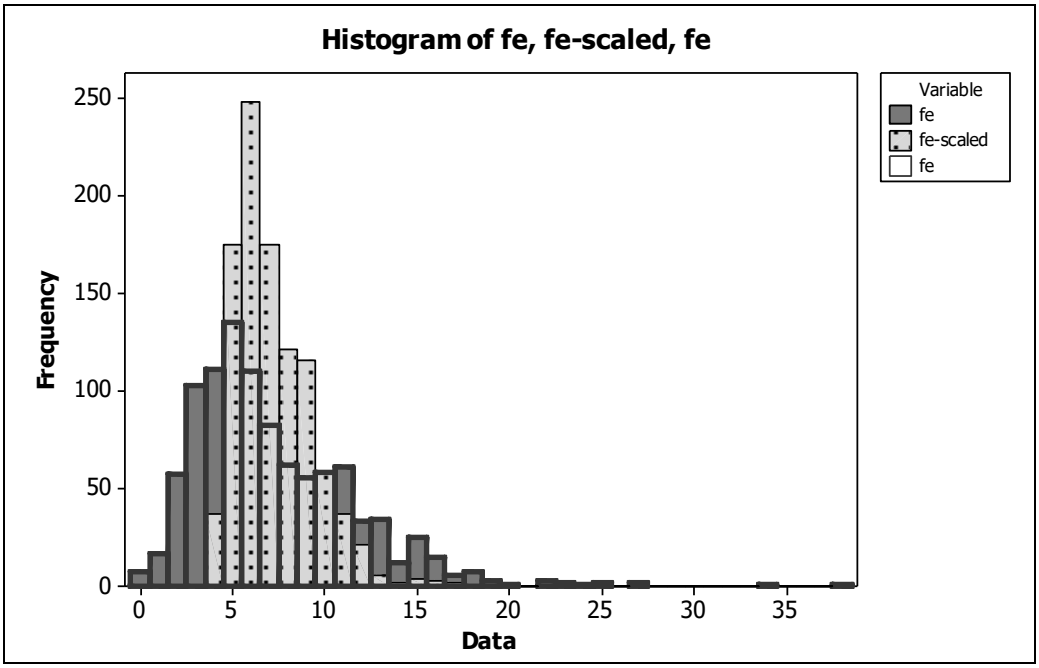


Figure 167 Histogram of iron (sample and SMU volumes)

If the volume-variance adjustments made to the sample grades are done in such a way that the pair relationship is maintained, then the SMU-adjusted distributions would plot on a scatterplot as in Figure 168.

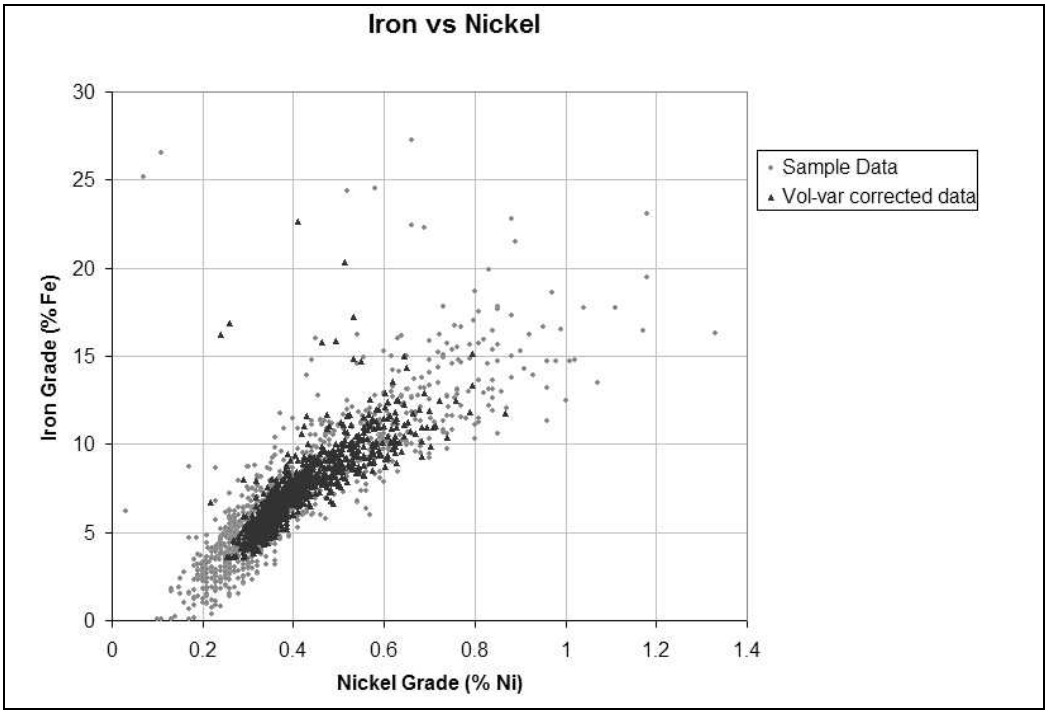


Figure 168 Scatterplot of volume-variance adjusted data

Now we have a measure of the expected relationship at the scale of the Selective Mining Units. We call this relationship the Reference SMU relationship between nickel and iron. This means that for a given SMU nickel grade we have an idea of the likely iron grade at that selectivity.

The next step is to run a standard recoverable estimate on our primary variable (nickel in our example above). This means generating a block model for nickel and producing a recoverable estimate (see pages 200 and 202).

Let us suppose we have a resource model block whose nickel grade is 0.4 % Ni. When we adjust the block to report a recoverable estimate above a cut-off of 0.5% Ni, the proportion is 37.6% of the block with an SMU grade of 0.72% Ni.

To look up the corresponding iron grade we feed the recoverable nickel grade into the reference distribution and report the nearest corresponding iron grade (Figure 169).

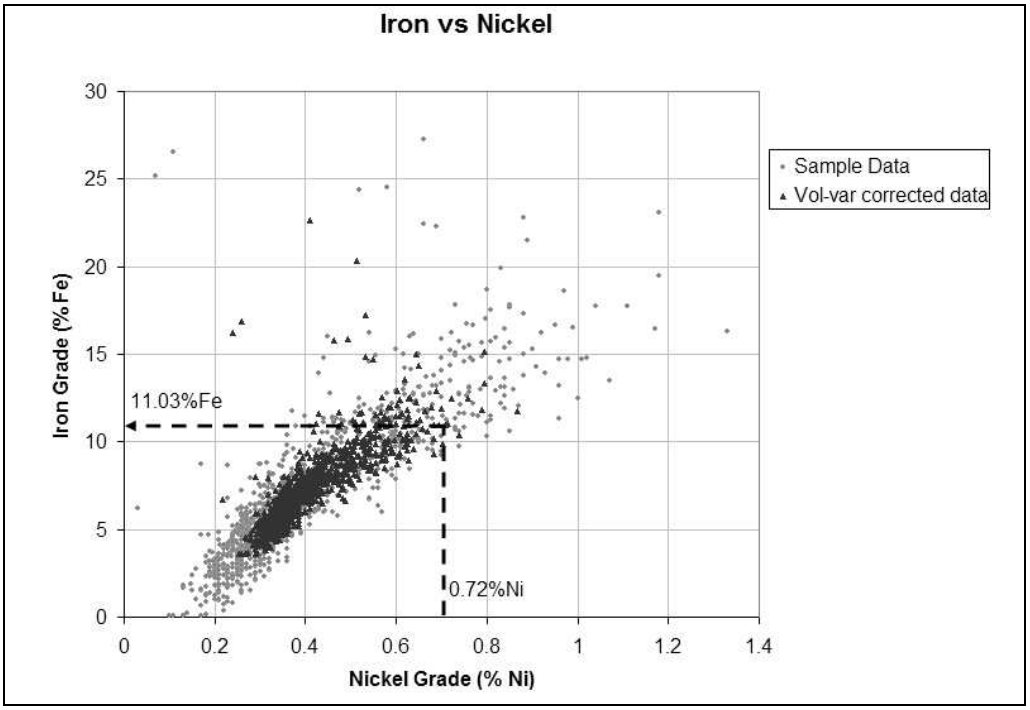


Figure 169 Look up SMU iron value for given SMU estimated nickel value

This approach can be expanded to more than two elements. The steps are:

1. Apply the volume-variance correction to each element's sample distribution (maintain the order of the data).

2. Generate a recoverable estimate for the primary element. Use each primary recoverable estimate to look up the corresponding recoverable secondary elements from the volume-variance adjusted distributions.

Phase 4: Validation

Purpose

Validation should be a natural part of every stage of your work, rather than a retrospective force fit:

- When you wireframe, calculate the volume and apply a nominal density and check whether the tonnes you get make sense for the dimensions of the orebody.
- When the blocks are coded, check the volume of the blocks against the wireframe to see whether the volumes are comparable.
- When data is composited, check whether the total length of the composites is comparable with the total lengths of the samples.

Here we look at specifically how to validate an estimated block model and conditional simulation models.

Estimation validation

The four-point check

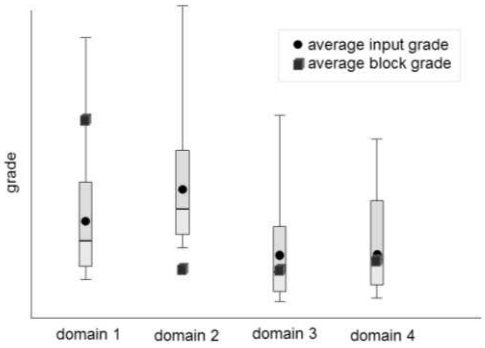
Model validation is the process of confirming the model you produce is an accurate reflection of the data you supply to the system. An ideal situation occurs when we can validate our model (at least partly) against a degree of reality through reconciliation with a grade control model and production.

A "4-point check" for validating a resource model against the input data is presented in Figure 170.

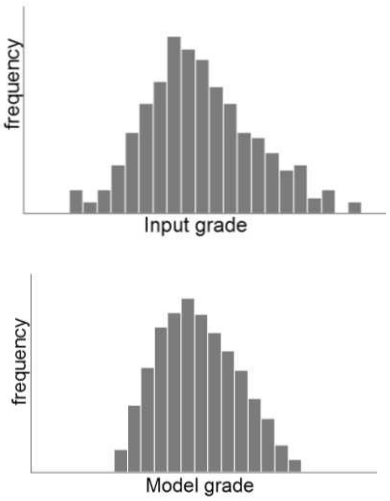
Remember to include only the blocks that make sense to validate. Extrapolated blocks will bias the first three checkpoints. Similarly, sub-blocks should be volume weighted if the comparisons between the input and output are to be fair.

Evaluation of a model's kriging efficiency values provides an indication of the degree of accuracy in the model.

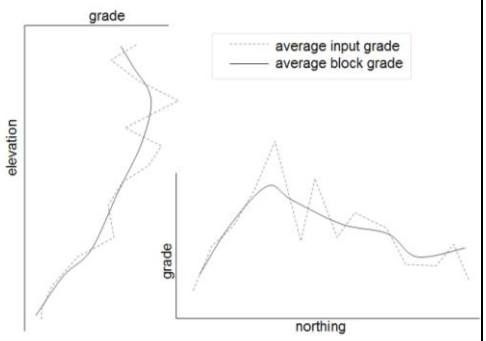
1. Output average should resemble input average for each domain



2. Output histogram shape should resemble input histogram shape for each domain, with some degree of smoothing and normalising



3. Output trend in depth, northing and easting should reflect the corresponding trends in the input data



4. Block grades should reflect the tenor of local drillhole grades and boundary conditions evident in drilling

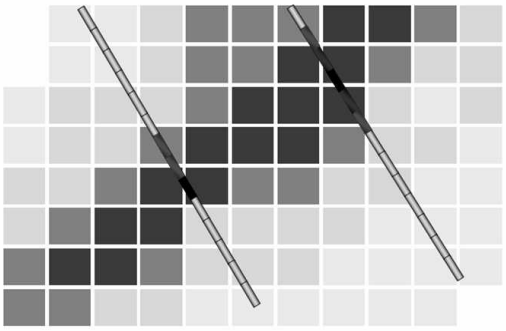


Figure 170 Four-point model validation

Conditional simulation validation

A basic assumption in conditional simulation is that the simulated models accurately reflect (in addition to the typical input grades and trends) the variability as described by the spread of the input data and the spatial variability as described by the variogram.

The assumptions need to be verified by plotting the conditional simulation models against the input data in Q-Q plots to verify the spread in the simulations reflects the spread in the sample data (Figure 171 and Figure 172).

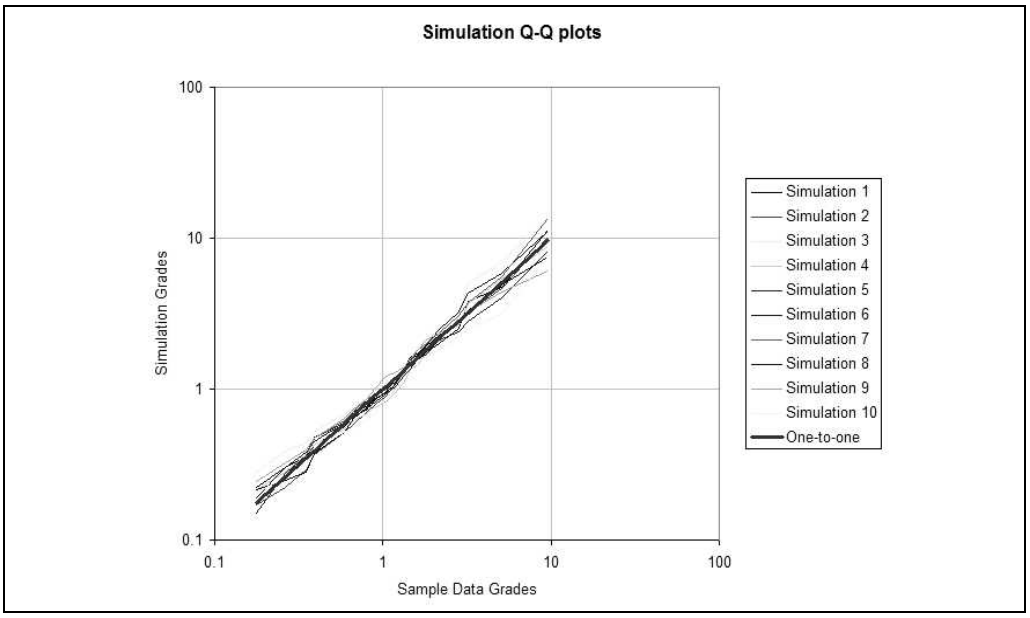


Figure 171 Q-Q plot validation of simulations against input sample data

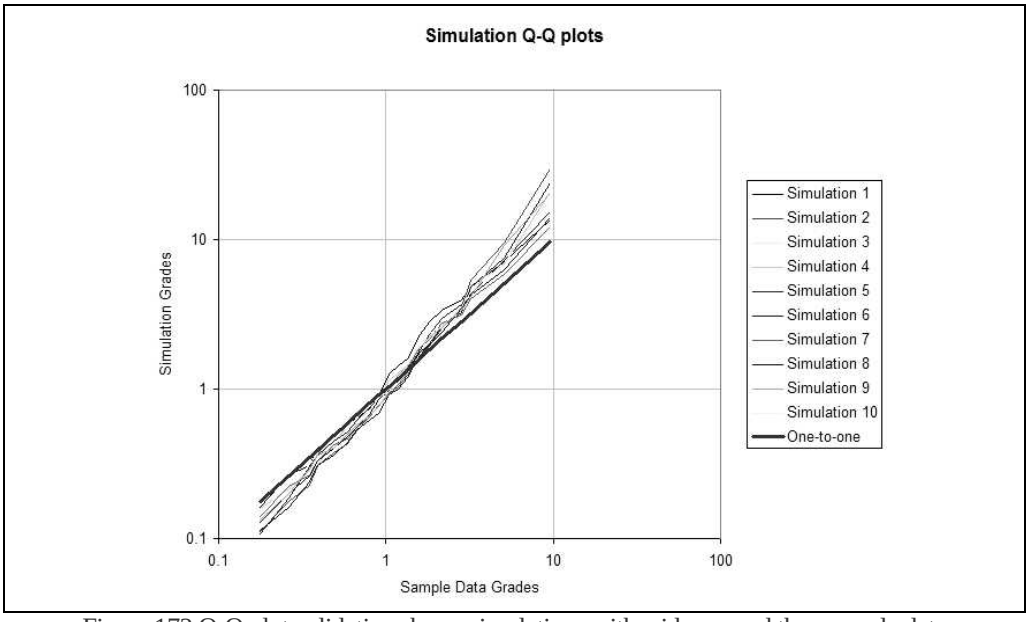


Figure 172 Q-Q plot validation shows simulations with wider spread than sample data

The variograms calculated on the simulations are plotted against the input variogram model to test whether the spatial patterns have been reproduced (Figure 173).

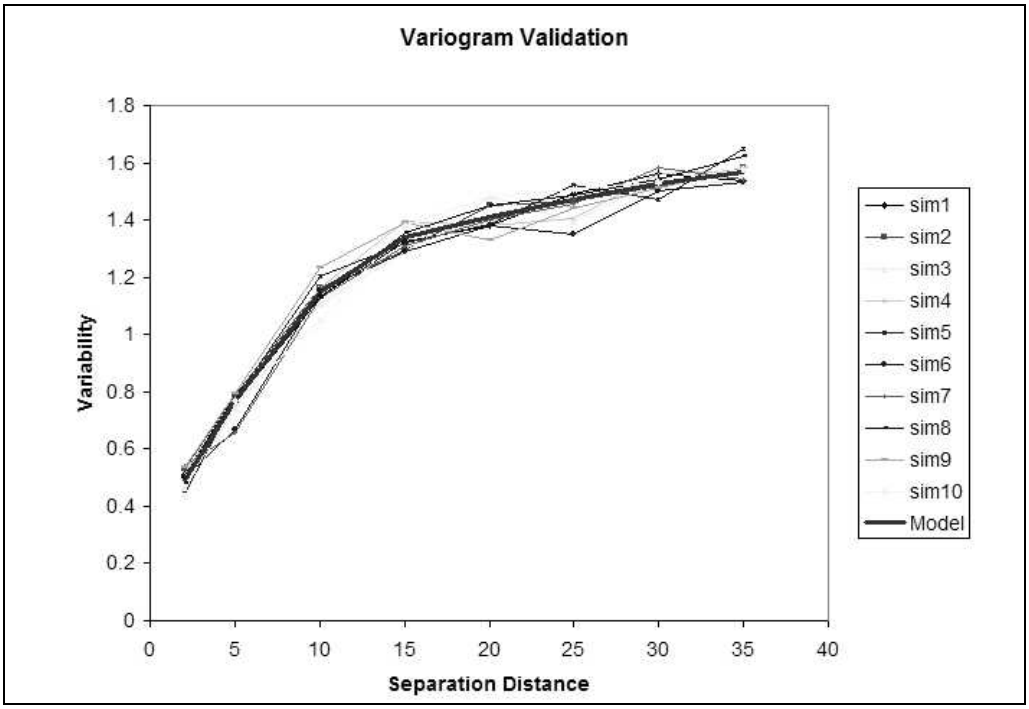


Figure 173 Variogram validation shows simulations reflect model variogram used for simulations

A graph of the average simulated grade plotted against the simulation number verifies random variability about the overall sample average and ensures there are sufficient simulation runs (Figure 174).

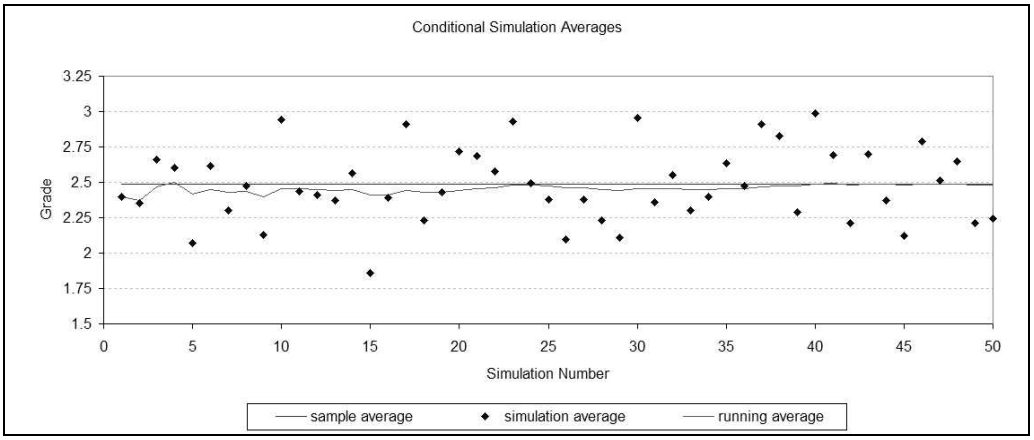


Figure 174 Average simulation value and cumulative simulation average against sample average

A note about classification

Recall our original objective to generate a three-dimensional model of mineralisation that is useful to mining engineers. Whilst we aim to estimate accurately, some portions of the model will be more accurate than others. Often there are volumes where we believe we need more information – either more drilling or improved information. The vehicle for communicating our confidence is the reporting code (for example the JORC guidelines, SAMREC guidelines and National Instrument 43-101).

These codes provide the scaffolding for communicating the confidence we have in our estimates. For example, Table 1 in the JORC code provides a series of items to consider whilst building our case for the classifications we apply to a resource model.

The confidence categories are subjective and rely on our competence, transparency in our intentions and the scale or materiality of the report. The three resource classification levels are: Measured, Indicated and Inferred. Very conceptually we can think of these criteria as describing the amount of work necessary to be confident in the accuracy of the resource estimate.

The highest confidence category is Measured and essentially describes the portion of the resource that is exhaustively drilled, has a high degree of quality in the sample collection, assaying, logging and surveying, and there is sufficient evidence to support both the interpreted geological controls on mineralisation as well as the spatial estimates of mineralisation. In addition, the density values applied to establish tonnages are accurate.

When any one or more of these criteria is lacking, we need to downgrade the respective volumes of the resource to either Indicated or Inferred - according to the degree of confidence in the criteria. Indicated is a higher confidence category than Inferred and typically refers to that portion of the resource model that is on the brink of conversion to Measured, but lacks data, information and/or quality for a few criteria. Inferred refers to that portion of the resource model that has the lowest confidence and requires significant amounts of data, information and/or quality to convert to Indicated resources.

Concluding comments

My philosophy to resource estimation is to build models that accurately reflect our understanding of the orebody. This means we cannot be prescriptive about the technique, nor about the parameters that we choose to use. Yes, people have their favourites, but this has no bearing on whether the set of parameters or approach works for every situation.

I believe when our choice of approach, method and parameters is evidence-based and we can support our choices through a thorough analysis of the data, we are better able to predict resources accurately. We need to focus on creating a three dimensional representation of what we believe the orebody to look like – this means there is no one-size fits all recipe, but rather a process of exploring the data and gathering evidence to support parameters and decisions.

Modelling a resource is a process for communicating mineralisation expectations to engineers who ultimately decide on the method, the timing and extraction of the ore. Resource analysts are responsible for reflecting an orebody's characteristics and subtleties in a three-dimensional model in a way that limits the element of surprise when it comes to actually mining the orebody. This responsibility is best taken up when we spend more time reflecting on the information and data at hand than force-feeding data through pre-scripted macros.

Give the geology a chance. Give the data a chance. Moreover, respect the individuality of these orebodies. There is so much to learn about them – no two orebodies are the same. Yet each reflects an opportunity to enjoy the thrill of mining. You have a chance to present a fair representation of each orebody and, in so doing, play your role in preventing disappointments or lost opportunities.

George Box, noted statistician, explains

"All models are wrong but some models are useful".

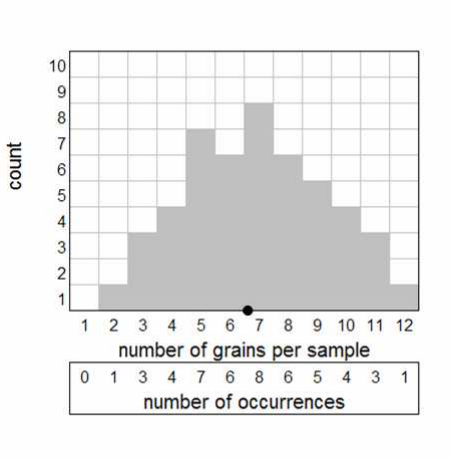
Let us do our part in creating really useful models.

See www.coombescapability.com.au for more hints and tips for resource estimation.

Selected model answers

Activity page 11

2	6	7	10	8	6
10	6	4	3	11	7
8	7	8	10	7	9
5	9	3	5	8	11
9	5	11	8	10	7
6	7	12	7	9	8
5	4	4	3	4	5
6	9	6	7	5	5



Overall average of the samples is 6.92 grains per kilogram. This average represents the grade of the lot.

The standard deviation is calculated by subtracting the mean from each sample and squaring the difference (see below). These are then summed and divided by the number less one to get a standard deviation value of 2.43.

(2 - 6.92)^2	(6 - 6.92)^2	(7 - 6.92)^2	(10 - 6.92)^2	(8 - 6.92)^2	(6 - 6.92)^2
(10 - 6.92)^2	(6 - 6.92)^2	(4 - 6.92)^2	(3 - 6.92)^2	(11 - 6.92)^2	(7 - 6.92)^2
(8 - 6.92)^2	(7 - 6.92)^2	(8 - 6.92)^2	(10 - 6.92)^2	(7 - 6.92)^2	(9 - 6.92)^2
(5 - 6.92)^2	(9 - 6.92)^2	(3 - 6.92)^2	(5 - 6.92)^2	(8 - 6.92)^2	(11 - 6.92)^2
(9 - 6.92)^2	(5 - 6.92)^2	(11 - 6.92)^2	(8 - 6.92)^2	(10 - 6.92)^2	(7 - 6.92)^2
(6 - 6.92)^2	(7 - 6.92)^2	(12 - 6.92)^2	(7 - 6.92)^2	(9 - 6.92)^2	(8 - 6.92)^2
(5 - 6.92)^2	(4 - 6.92)^2	(4 - 6.92)^2	(3 - 6.92)^2	(4 - 6.92)^2	(5 - 6.92)^2
(6 - 6.92)^2	(9 - 6.92)^2	(6 - 6.92)^2	(7 - 6.92)^2	(5 - 6.92)^2	(5 - 6.92)^2

6 kilogram sample:

						Total within a 6 kg sample	6 kg sample (grains per kilo)
2	6	7	10	8	6	39	6.50
10	6	4	3	11	7	41	6.83
8	7	8	10	7	9	49	8.17
5	9	3	5	8	11	41	6.83
9	5	11	8	10	7	50	8.33
6	7	12	7	9	8	49	8.17
5	4	4	3	4	5	25	4.17
6	9	6	7	5	5	38	6.33

The overall average is still 6.92 grains per kilogram, but the standard deviation is lower at 1.37.

Losing a grain per sample:

						Total within a 6 kg sample	6 kg sample (grains per kilo)
1	5	6	9	7	5	33	5.50
9	5	3	2	10	6	35	5.83
7	6	7	9	6	8	43	7.17
4	8	2	4	7	10	35	5.83
8	4	10	7	9	6	44	7.33
5	6	11	6	8	7	43	7.17
4	3	3	2	3	4	19	3.17
5	8	5	6	4	4	32	5.33

The overall average drops by a grain per kilogram to 5.92 grains per kilogram, whilst the standard deviation remains at 1.37.

Control chart activity on page 36

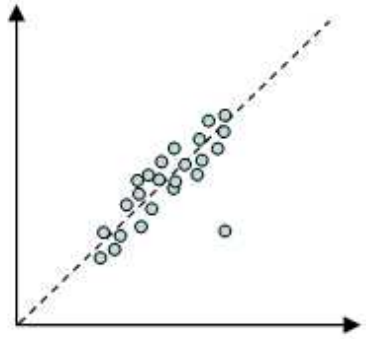
The control charts show a myriad of problems including:
Chart 1: Changes in variability, bias below the expected value, values out of the control limits
Chart 2: Value outside control limit, changes in variability, 6 values trending down
Chart 3: Sudden trend upwards to a run of values outside control limit, run of identical values

QAQC activity on page 46

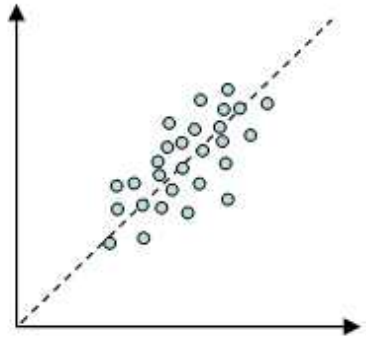
The iron data is behaves poorly with poor precision. There is, however, no bias (see the Q-Q plot).

The SiO_2, in contrast, shows high precision as well as no bias.

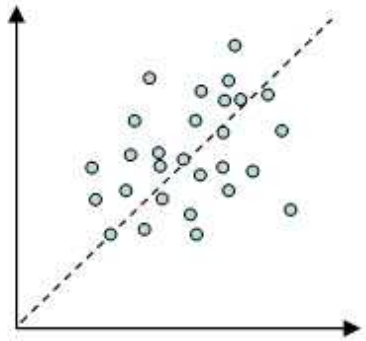
H-scatterplots for page 108



h-scatterplot of samples 20m apart.

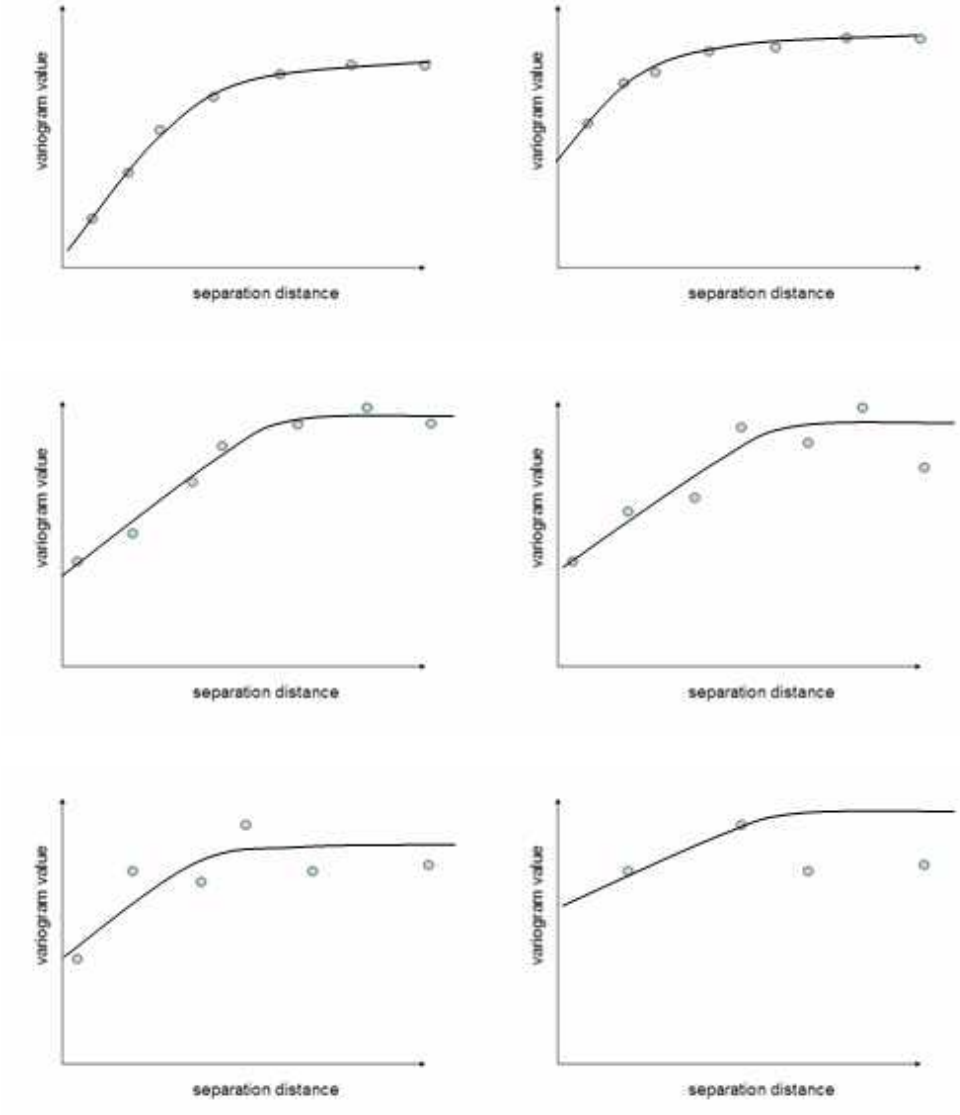


h-scatterplot of samples 2x20m apart.



h-scatterplot of samples 3x20m apart.

Suggested results for page 134



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Pictures on pages 53, 59 and 75 sourced from Microsoft Clipart.

Reporting codes can be found at:

CRIRSCO	www.crirsco.com
JORC	www.jorc.org
NI43-101	http://www.osc.gov.on.ca/Regulation/Rulemaking/Current/Part4/rule_20051007_43-101_sd-mineral-projects.jsp
PERC	www.percreserves.com
SAMREC	www.samcode.co.za

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About the author

Jacqui graduated with a Masters degree in Statistics in 1991. Her master's thesis (an extension of her honours thesis that had won her South Africa's coveted National Best Honours Thesis in 1989) examined spatial estimation techniques and the associated analysis required to build meaningful estimates. This provided a strong basis for Jacqui's interest in spatial data patterns and her yearning to understand the intricate geological reasons for these patterns.

Whilst completing her Masters degree, Jacqui joined an oil and gas exploration company, Soekor, to introduce and share spatial statistical techniques. Soekor provided Jacqui with opportunities to work with multiple sources and densities of spatial information. Jacqui introduced multi-variate estimation and conditional simulation to Soekor. The highlight for Jacqui was working with a team of geophysicists and geologists to develop a depth-conversion program that allowed users to combine well and seismic data and estimate the depth to horizon of surfaces. Other projects included estimating and simulating parameters such as porosity and permeability.

After almost four years in Cape Town, Jacqui migrated to Australia and joined Snowden Mining Industry Consultants.

The mining industry provided Jacqui with the chance to examine more directly where statistical techniques worked. Reconciliation is an amazing teacher! And, with the relatively short turnaround time between modelling and mining, Jacqui was able to work with others to learn which parameters and techniques for grade estimation were most sensitive and successful in the various geological settings. Added to this was the opportunity to work on a variety of commodities in different geological settings globally.

After more than 12 years with Snowden, Jacqui set up Coombes Capability at the end of 2006 to foster talent in the mining industry. Jacqui's love of teaching, sharing and practical approach to problem solving has helped bring her into contact with talented geologists and engineers as they build resource models across commodities (including iron ore, nickel, gold, uranium, zinc, lead, mineral sands).

Jacqui's work includes evaluating and reviewing resource estimates, resource and grade control auditing, drilling optimisation, resource risk assessment, reconciliation, QAQC analysis, multi-variate data analysis, resource classification, training and technical mentoring.

Jacqui lives in Perth, Australia. Jacqui has been lucky enough to travel to some amazing mine sites around the world.